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Ori Belkind

Physical Systems

Conceptual Pathways between Flat Space-time and Matter

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Conceptual Pathways between Flat Space-time and Matter

by

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This book is dedicated to my parents,
Ilana *and* Naaman.

Preface

This book originates with the first course I took in graduate school. The course was in the Philosophy of Science, and it was taught by Arthur Fine at Northwestern University. It was not the first time I have read Kuhn's *The Structure of Scientific Revolutions*, but this time reading it I became intrigued with Kuhn's claim that the concept of mass is a primary example of a concept that has undergone a scientific revolution. According to Kuhn, the parameter *m* in Newtonian physics and in the Special Theory of Relativity might be referred to with the same word, mass. However, in fact, the mass concepts in the two theories operate within radically different paradigms, to the extent that they share no meaning and are incommensurable. Kuhn's view struck me as absurd, and I set out to refute him in a short seminar paper. To refute the incommensurability thesis, I tried to articulate what I took to be the overlap in meaning between Newtonian and relativistic mass. The task turned out to be more challenging than I originally thought, but I did come up with a vague account of the shared geometric-dynamic role that mass has in the two theories. It slowly dawned on me that my interpretation of mass deviates from the way in which mass was presented in physics textbook, and so I conceived the idea to write a dissertation on the concept of mass, which will make precise the account vaguely conceived for the seminar paper. I soon found myself reading and thinking about the nature of spacetime, since the semi-geometric interpretation of mass required an analysis of the foundations of spacetime, or at least a decent understanding of the philosophical debates about the topic. While preparing for my project, I began reading seventeenth and eighteenth century texts surrounding the publication of Newton's *Principia*, a reading that took off in its own direction, since I realized that knowing the rich history of Newtonian concepts is key to understanding the various subtle roles that mass originally had for Newton's physics and philosophy of science. Reading into the history and pre-history of Newton's physics also reshaped my project, since I now believed that central to understanding the concept of mass (and physical concepts in general) is the notion of physical system. Inspired by seventeenth century physics, I articulated a philosophical account of physical systems, which takes motion to be a fundamental entity, and systems to be structures in which the motion of composite systems are constructed out of motions of their parts.

The dissertation I ended up writing comprised of a strange mix of historical and philosophical analyses of the foundations of spacetime and mass in both the contexts of Newton's theory and the Special Theory of Relativity. The project, given its peculiar history, ended up touching on various topics in the history and philosophy of physics, from the philosophy of space and time, to the concept of mass, to the scientific method. The concept of mass was like a tied knot at the center of Newtonian physics and the Special Theory of Relativity. Untying this knot required a slow and careful process of disentangling various threads, until the foundation of Newtonian and relativistic physics came into view. (Or, at least what I hope is a novel way of viewing the foundations of classical theories.) Unwinding each thread required the tackling of thorny conceptual problems.

After completing the dissertation, I realized the many infelicities of the work I came up with. I was unhappy with the vague articulation of the connection between spacetime geometry and mass, and took upon myself to articulate my views of spacetime more clearly, beginning with the simplest of axioms that take uniform unidirectional motions to be the fundamental entities of spacetime, and proceeding to derive the known Galilean Spacetime and flat relativistic spacetime. I ended up rewriting most chapters, adding some that were not in the original project, and revising the rest to sharpen the views and make the arguments more cogent.

The project now lies in its final form. It no doubt reflects my peculiar way of thinking and idiosyncratic combination of historical and philosophical inclinations. But I hope the project would be of use to philosophers of physics, who might be interested in the notion of physical system (Chapter 1), the foundations of spacetime (Chapters 2 and 3), my semi-geometric interpretation of the concept of mass (Chapter 6) and the foundations of the Special Theory of Relativity (Chapter 8). The work might also be of interest to philosophers and metaphysicians who are interested in the metaphysics of time (Chapter 4), to historians of physics working on Newton's physics (Chapters 5 and 7), and to philosophers of science and epistemologists interested in scientific methodology (Chapters 2, 7 and 8).

I would like to express my ocean-deep gratitude to Arthur Fine, who made this project possible on many levels, professional and personal. As my dissertation advisor, Arthur provided the bulk of intellectual freedom, institutional support and personal encouragement I needed to conceive, write, rewrite, and complete the project. I would also like to thank the Faculty Research Committee at the University of Richmond for providing financial support during the summers of 2005, 2006, 2008 and 2009 which aided in completing various parts of the project. Gratitude is owed to my colleagues at the philosophy department at the University of Richmond, Geoff Goddu, David Lefkowitz, Miriam McCormick, Del McWhorter, Nancy Schauber and Gary Shapiro, who provided a congenial and supportive intellectual environment in the last 5 years. I benefited tremendously from conversations with and comments from Marc Lange, Andrea Woody, Geoff Goddu, Brannon McDaniel, Miriam McCormick, Wayne Myrvold, Bill Harper, Nick Huggett, Amit Hagar, Eric Schliesser, Andrew Janiak, Sona Gosh, Kevin Scharp, Natan Berber, Ken Chung, Rob DiSalle, John Norton, Amy Au, Gideon Freudental, Craig Callender, Ed Slowic, and Paul Teller. Last but not least I would like to express my appreciation and gratitude for Michele Bedsaul's help in copy editing the manuscript and preparing it for publication.

Contents

List of Figures

Chapter 1
Physical Systems and Physical Thought

1.1 Introduction

The notion of *physical system* is so ubiquitous it is mentioned in almost every work in physics. Scientists use the term, without much reflection, to refer to an aggregate of physical objects. Attention is sometimes directed to a system when one is interested in the system's components and their distinct states and properties. But more often, physicists are concerned with the arrangement of the parts and interactions between the parts. They use various theoretical constructs to single out states and properties of the composite system; states and properties that either supervene on the particular configuration of the parts or constitute non-supervening, emergent features.

Even though the notion of physical system does not necessarily presuppose any metaphysical stance, physicists often think that properties of composite systems are reducible to the properties of their parts. Take for example a neutron comprised of one "up" quark and two "down" quarks. One way of describing the neutron is to think of it as a physical system comprised of distinct components. The states and properties attributed to the neutron as a whole are determined by the states and properties of the quarks and the strong interaction between them. The zero net charge of the neutron, for example, arises from the $+\frac{2}{3}e$ charge of the "up" quark and the $-\frac{1}{3}e$ charges of each of the "down" quarks. The neutron's description, according to this view, is merely an economic representation of the quarks. When details concerning the quarks are not relevant to the problem at hand the neutron's internal structure is ignored, and the neutron is treated as a single entity with its distinct states and properties.

Even though states and properties of composite systems are indispensable to physics, the tendency is to think of composite bodies as non-existing. A perfect theory would then include no composite objects and would admit only complex descriptions of simple parts. One way to make this approach tenable is to assume that material objects have ultimate parts and that these ultimate constituents of matter have material properties and are located in particular points in spacetime. An aggregate of such localized parts may then be described with the help of composite

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states and properties.[1] A common practice is therefore to visualize physical systems as being assembled out of infinitesimally small objects endowed with material properties such as mass, charge and spin, and with states such as velocity, momentum and energy. Classical fields are also constructed from localized properties that vary continuously from one location in space to a neighboring point. In this way, field theories also satisfy an "eliminativist" approach towards composite objects. According to this attitude all references to composite entities should be eliminated and replaced with complex descriptions of the entities' parts.

The notion that composite objects are nothing more than a collection of localized physical parts is referred to in the literature as *local physicalism* or *particularism* (see Teller, 1986, 1989; Lewis, 1986). Moreover, it is also believed by many that particularism is not only a metaphysical belief but an explanatory ideal. According to this approach, a genuine scientific account will strive to construct the description of a composite entity from localized objects endowed with various properties. Many commentators think of Newtonian physics and classical electromagnetism as exemplifying this metaphysical stance and methodological ideal, but the claim that classical theories entail particularism is far from obvious. Consider, for example, the electric potential of a system comprised of two charges. Since the potential depends on the distance separating the charges, it is a relational property irreducible to the components' non-relational states.[2] If one is committed a priori to particularism, one would be inclined to replace electric potentials with localized entities – such as the electric field – and argue that the electric potential is not a genuine physical property. However, classical theory in itself does not dictate the removal of irreducible relational properties or composite objects. Whether or not electric potential is reducible to some underlying field is a theoretical question, or perhaps even a matter of taste. Particularism still remains merely a blueprint or an explanatory ideal and is not dictated by classical physics.

There is a deeper question about the relation between classical theories and particularism, which goes beyond the problem of finding a reductive particularist account for every physical concept. It is standard practice in classical theories to compile composite properties and interactions from various properties instantiated in physical parts. Almost every physical problem relies on various summation rules for calculating a composite property, whether one relies on discrete sums or on integration over a continuous medium. But what justifies the summation rules themselves? Presumably they are merely mathematical devices used to replace a set of properties instantiated in the parts of a physical system with an overall sum attributed to the composite system. For example, the simplest summation rule conceivable is the one in which the volumes of infinitesimal parts of a continuous body

[1] A complication arises when we think of the position of an object as one of its properties. The object's position could be taken as either a relational property between the object and space or a relational property between material objects. We shall postpone discussion of spatial and temporal attributes until the next section.

[2] See Teller (1986, p. 74).

are summed up to yield the volume of the composite object. Ordinarily, integration over volumes appears in the formalism as a mere mathematical device, a standard rule in which numbers are added up. Nevertheless, a brief reflection will demonstrate that integration over volume reflects a physical assumption about the nature of space: viz., that the volume of a composite body is the sum of the volumes of the parts. It is such an intuitive assumption underlying our concept of space that one becomes unaware of the physical origin of this rule and thinks of it as a formal device. But the rule for integrating over volumes of space is not merely a formal rule, reflecting the mathematical function of addition; rather, it is a mathematical rule representing a structural assumption about the spatial properties of physical bodies.

Paradoxically, reductive explanations are made possible via assumptions about the structure of physical systems. For compound descriptions to be reducible to their simpler parts, one must have at one's disposal various measures and calculational devices that sanction inferences from the simple to the compound and vice versa. Consider, for example, additive properties such as mass and charge, which underwrite the summation over the properties of the parts. These measures are used so seamlessly that one forgets that they are measures of physical facts describing relations between parts and wholes. For 200 years mass was thought to be an additive property, but the Special Theory of Relativity demonstrates that rest mass is not an additive property. When a system decomposes into parts, the rest mass of the composite system is not necessarily the sum of the rest masses of the products of decay. The standard interpretation of such decays is that rest mass was "converted" into energy, but the truth of the matter is that rest mass is simply not an additive property.[3] While Newtonian mass is additive, the rest mass of a composite system in STR is not the sum of the rest masses of the parts. Similarly, there is a possible world in which the volume of a composite system is not the sum of the volumes of its components, although such a world is very difficult to conceive. These reflections make it clear that the ability to sum over properties of the parts in producing properties of composite systems reflects a physical assumption.

Another very important reductive tool is Newton's parallelogram law for the composition of forces.[4] According to this law, one can derive the composite force impinging on a body from the component forces. There is, however, a possible world in which the composite force is not the vectorial sum of the component forces.

When a composite description is constructed from the description of simpler elements, substantive structural assumptions about physical systems are implicitly employed. The universal validity of Newton's parallelogram law seems natural because one often thinks of it as merely expressing a mathematical rule governing vectors. However, the physical content of the law is striking when it is recognized that the law expresses a Rule of Composition. Assume bodies A_2 and A_3 exert forces $\vec{F}_{21}$ and $\vec{F}_{31}$ over body A_1. The parallelogram law allows us to calculate the

[3] See Lange (2001)

[4] Corollary 1 to the Laws of Motion (Newton, 1999, p. 417).

total force operating on A_1: $\vec{F}_t = \vec{F}_{21} + \vec{F}_{31}$. In Newton's theory, the total force $\vec{F}_t$ may be thought of as the causal influence of the composite object comprised of A_2 and A_3 over A_1. This composite object is located at the center of mass of A_2 and A_3 and possesses a total mass of $m_t = m_2 + m_3$. The relation between the component forces imposed by A_2 and A_3 separately, and the composite force imposed by A_2 and A_3 together, is a *physical* fact, one which is shown through the appropriateness of representing forces with vectors. The decision to treat forces as belonging to a vector field is the result of a commitment to a general Rule of Composition governing forces.[5]

Using a vector field to represent forces has as much physical content as the Galilean rule for adding velocities. When it is assumed that a vector is appropriate for describing a force, one is also implicitly assuming that mathematical rules governing the summation of vectors is appropriate for describing the composite forces. Vector summation rules have physical content in much the same way that Galilean rule for the summation of velocities has physical content. A new theory might convince us that composite forces are not constructed from component forces using the linear rules governing vectors. Because a summation rule is disguised in the form of a mathematical rule, one gets the impression that the decomposition of interactions into their components is merely a decomposition of complex descriptions into the descriptions of simple parts. But what is hidden from view is that the summation rules governing forces involve concrete assumptions about relations between parts and wholes.

The notion of physical system requires philosophical reexamination. Debates regarding the nature of physical systems do not have a clear account of how parts of systems retain their identities and what relates the properties of the parts to the properties of the whole. Part of the difficulty in understanding stems from a long habit of disguising assumptions about physical structure in various "calculational" devices or "laws of nature," which forms a significant part of our physical knowledge. A law of nature is most commonly interpreted as a rule that governs the behavior of objects. Laws of nature are interpreted as governing how a particle or a field evolves over time. But I argue that momentum and energy conservation laws implicitly encode assumptions about the *structure* of physical systems, i.e., they implicitly encode rules for constructing the description of composite systems from descriptions of their parts. To further current discussions about the nature of physical systems, I will take on the task of bringing to the foreground the structural assumptions implicit in momentum and energy conservation laws. Moreover, I will argue that the assumptions regarding the structure of physical systems are more fundamental than spacetime structure, the existence of material properties such as mass, and laws of nature. Much of the work will consist of reconstructing Newton's physics and the Special Theory of Relativity using the notion of physical system as a philosophical guide.

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5 For a more detailed discussion of these part-whole inferences see Lange (2002, pp. 234–35).

A key move will be to reinterpret the law of momentum conservation (and in a wider context, energy conservation) as a "structural assumption." The core of the argument will be that momentum-energy conservation laws are not merely external laws that dictate the behavior of bodies, but constitute core assumptions about the structure of physical systems. At their heart, conservation laws articulate basic assumptions about the relation between parts of physical systems and their composite. One can better see the role of conservation laws as structural assumptions when one reflects on their role in isolating a physical system from its environment. It is often said that energy and momentum are conserved for closed systems. One ordinarily thinks of this claim as merely empirical: if a system is closed, then it is an empirical fact that the total momentum and energy of that system is conserved. Thus, it is imagined that, had the empirical facts been different, a closed system *could* turn out to have total momentum and energy that is not conserved. But do we have a criterion for determining whether a system is closed, independently of the conservation laws? In other words, do we have a criterion for causally determining that a system is isolated from the environment? Is this criterion independent of momentum-energy conservation laws? A short reflection would demonstrate that there is none. And so a system is *defined* as closed whenever the total momentum and energy of that system is conserved. In other words, the conservation laws themselves provide us with a criterion of isolation, or the criterion by which a system is shown to be causally isolated from the rest of the world. Momentum-energy conservation laws differentiate for us between the system, and what is not part of the system or is part of "the environment." Given their role in providing a Criterion of Isolation, it is clear that these conservation laws are crucial for individuating physical systems, and for differentiating between one system and another. Moreover, the simple rules for summing the momentums and energies of the parts to calculate the momentum and energy of the composite, isolated whole, are in effect rules that describe the structure of physical systems – they are inferences from the motions of the parts to the motion of the whole. Momentum-energy conservation laws are therefore not simply empirical laws governing the behavior of bodies. Rather, they constitute fundamental assumptions about the structure of physical systems. As such, they hold an important epistemic role in relation to our physical knowledge. Momentum-energy conservation laws underwrite the inferences from observed phenomena to causal laws that generate the phenomena, as they provide the background assumptions relative to which causal relations between parts of a system reveal themselves and are made apparent.

Another key move in demonstrating the role of structural assumptions will be to revisit commonplace metaphysical assumptions about physical reality. A widespread metaphysical stance separates physical reality into three layers: spacetime, material objects and their properties, and laws of nature. According to this view, spacetime provides the backdrop relative to which physical bodies receive their locations and their trajectories are defined. Material properties such as mass are then thought of as properties somehow instantiated in bodies. In much the same way in which Aristotle believed his forms to be instantiated in material bodies, so does the modern philosophical view think of material properties as correlated with predicates that describe the objects. Thus attributing mass to a particle is something like attaching a property to the body located in a particular position in spacetime. One can then speak of a particle "having" or "possessing" a certain property. For example, an electron is thought to be a body located at a particular point in spacetime, and "possessing" a mass of 9.109×10^{-31} kg. Laws of nature are then taken to be external decrees that govern the behavior of these objects, given certain states they have. The metaphysical stance that separates physical reality into three distinct realms is one of the obstacles in recognizing the role of structural assumptions. The analysis that will be carried out here suggests that there is no clear distinction between spacetime, bodies and their material properties, and laws of nature. For example, in the following it will be shown that the spacetime geometry is not so clearly demarcated from material properties attributed to bodies, as the standard account has it. It will be shown that mass has a geometric origin, which falls out directly from the geometric descriptions of motions that will be developed here. On the other hand, what is known as the "law of conservation of mass" – in the Newtonian context – cannot be clearly demarcated from the conservation of momentum. The upshot is that material properties such as mass are not clearly demarcated from the laws that govern the bodies' motions. Thus we shall see conceptual pathways that connect spacetime with material properties on the one hand, and between material properties and laws of nature on the other hand. In the metaphysical view endorsed here, presuppositions regarding the structure of physical systems are more fundamental than spacetime, material properties, and laws of nature. All three physical parts of reality – spacetime, properties and laws – fall out when structural assumptions regarding the nature of physical systems are made fully explicit.

To see the connection between spacetime structure and the other dimensions of physical reality, a reconstruction of physical geometry will be taken up. The purpose of this reconstruction is to provide a philosophical framework for bridging spacetime structure and momentum-energy conservation laws. The reconstruction of spacetime is made possible by taking uniform rectilinear motions as fundamental entities of spacetime. This reconstruction does not describe a set of spacetime points or a manifold with independent spatial and temporal metrics, but instead introduces potential uniform rectilinear motions (PUMs) as fundamental entities; intersections between motions are taken to be events. This approach has affinities with relational accounts of spacetime. However, the description of spacetime does *not* begin with spatial and temporal relations between bodies. Rather, the description takes PUMs to be the fundamental entities. The meaning of this assertion will become clearer in later chapters. At this stage, it is enough to say that spatial and temporal relations are not defined *prior* to motions on a set of spacetime events. Instead, spatial and temporal relations are shown to be the decompositions of motion intervals. The decomposition of a motion interval into spatial distance and temporal duration is given when a PUMs is projected onto a set of parallel PUMs.

The geometry of PUMs will demonstrate a surprising connection between spacetime symmetries and the concept of inertial mass. It will be shown that inertial mass can be given a geometric interpretation, such that instead of thinking of inertial mass as *causing* different kinds of accelerations in bodies, the variation in accelerations will stem from various symmetries implicitly present in a geometry of PUMs. This alternative approach to spacetime and the geometric interpretation of inertial mass will be combined with a structural interpretation of momentum-energy conservation laws, yielding the complete physical theory, including the various aspects of the mass parameter and Newton's laws of motion.

The resulting metaphysical picture takes physical systems to include only a geometry of PUMs governing the motion of parts and structural assumptions for constructing the motion of composite systems from the motions of the parts. Thinking of physical systems as moving parts and moving wholes, I hope, will open up a fresh perspective into the foundations of Newtonian mechanics. In particular, it will establish new conceptual pathways with which to investigate the relations between spacetime, material properties such as mass, and laws of motion.

To show that the analysis provided here is not limited to Newtonian physics, an analogous reconstruction of the Special Theory of Relativity will be offered. It will be shown that a flat relativistic spacetime stems from a relativistic account of a geometry of PUMs. Similarly, a geometric interpretation of rest mass will be presented. A structural definition of relativistic systems of the special theory will then be used to derive the concept of relativistic mass and relativistic laws of motion governing the energy-momentum four-vector.

The central theme in what follows is that momentum-energy conservation laws hold a much deeper and central role in physical knowledge than what was previously imagined. Heretofore, it was held that momentum-conservation laws are very general and very basic empirical laws that govern all physical systems. While these conservation laws are given central place in standard interpretations, their metaphysical and epistemic role is not considered any more fundamental than other fundamental laws, such as the Universal Law of Gravitation or the fundamental laws of electromagnetism. However, I will argue that the assimilation of momentum-energy conservation laws to other empirical laws conceals the physicists' commitment to certain very general assumptions about the structure of physical systems. Metaphysically, these conservation laws are like the central nucleus of physical knowledge. These structural assumptions are more fundamental than spacetime, material properties, and laws of nature. Epistemically, they underwrite inferences from the phenomena to the force-laws that explain the phenomena.

The purpose of this chapter is to give a general overview of the whole project rather than to provide convincing argument for each of the philosophical positions endorsed here. By the end of the chapter, most of the battles will still be unsettled, but I hope the reader will be better able to see her way through the battlefield by keeping in mind the master plan.

1.2 Quantum Mechanics and Particularism

The project of demonstrating the role of structural assumptions in Newtonian physics and the theory of relativity, I believe, is interesting in its own right, despite the somewhat antiquated feel that such a project might have. I do not take on, in this book, the task of reconstructing classical field theories, the General Theory of Relativity, quantum theories, quantum field theories, or Quantum Gravity. Moreover, most of the technical arguments I will introduce are downright simplistic, if not simple-minded. However, I would like to emphasize the potential significance of this project for current philosophical debates. In recent decades, debates on particularism were revived in the context of Quantum Mechanics (QM). According to some philosophers, the case of quantum entanglement suggests that there might be cases in which the state of a composite physical system is not reducible to the states of the system's components. Thus, for those who believe that particularism fails in QM, the theory requires revising our previous philosophical commitment to particularism. A big part of the philosophical debate on QM focuses on whether the mathematical formalism of QM, together with the empirical confirmation that QM gets from experiments, demonstrates the failure of particularism. Parties to the debate, however, often take for granted that Newtonian physics is committed to particularism. But without an analysis of the foundations of Newtonian physics it seems difficult to assess whether or not QM introduces new reasons to reject particularism. If Newtonian physics relies on structural assumptions regarding the nature of physical systems, are there alternative structural assumptions in QM? Assuming that particularism fails in QM, how do these alternative assumptions impose the non-reducibility of composite entangled states? Simply looking at the formalism of QM without developing a general account of structural assumptions would make it difficult to come to any ultimate conclusions in the philosophical debate on holism in the context of QM.

The failure of particularism is thought to stem from the following analysis of quantum entanglement. The Bohm-EPR pair is often given as a paradigm example. A pair of particles l and r are selected each with a spin of $\frac{1}{2}$. Depending on the direction of the magnetic field used to measure the particles' spin, the spin may turn out to be "up" or "down" when the particle moves "upwards" or "downwards" while moving through the magnetic field. Thus each particle may be in one of two spin states, represented as $|\uparrow_l\rangle$, $|\downarrow_l\rangle$, $|\uparrow_r\rangle$ and $|\downarrow_r\rangle$. Since the particles move in a determinate direction upwards or downwards while the spin is being measured, the states $|\uparrow_l\rangle$ and $|\downarrow_l\rangle$ are mutually exclusive, as are $|\uparrow_r\rangle$ and $|\downarrow_r\rangle$. It is possible to prepare two particles to have either the state $|\phi_1\rangle = |\uparrow_l\rangle|\downarrow_r\rangle$ or the state $|\phi_2\rangle = |\downarrow_l\rangle|\uparrow_r\rangle$. In this case, the spins are anticorrelated. The result of measuring spin "up" implies that state of l is $|\uparrow_l\rangle$, which in turn implies that the spin of r is "down" (and vice versa). Because spin is a conserved property, prior to measurement the particles' spins remain correlated even if they get separated over time.

The unique feature of QM is that it is possible to get the pair to be in a superposition of $|\phi_1\rangle$ and $|\phi_2\rangle$. For example, the two particles may end up in the following "singlet" state:

$$|\phi\rangle = \frac{1}{\sqrt{2}}|\uparrow_l\rangle|\downarrow_r\rangle - \frac{1}{\sqrt{2}}|\downarrow_l\rangle|\uparrow_r\rangle \tag{1.1}$$

The singlet composite state assigns probability $\frac{1}{2}$ to both $|\phi_1\rangle$ or $|\phi_2\rangle$. There is then an equal chance that the particles will be in one of these two states. Whether or not the two particles are in fact in the state $|\phi_1\rangle$ or $|\phi_2\rangle$ is not determined until someone measures the spin of one of the particles. The states of the two particles are somehow "entangled," since measuring the spin of one particle seems to make the spin of the other particle determinate by forcing the state of the composite system to reduce to either $|\phi_1\rangle$ or $|\phi_2\rangle$.

According to Howard (1985, 1989), Einstein believed that Quantum Mechanics is incomplete since it violates the "separability principle." This principle asserts that two spatially separated systems have distinct and independent states. The separability principle fits naturally within a particularist framework. If it is assumed that the properties of composite systems are reducible to the properties of their parts and in addition it is assumed that the ultimate parts can only include intrinsic properties that are localized in space, the separability principle follows as a consequence.[6] The separability thesis should be distinguished from the locality principle, which asserts that there is no "action at a distance." According to the locality principle, there exists no causal influences that propagate instantaneously from one physical system to a remote one.

If both the separability and locality principles hold, the spin of l should be determined by the state of l and the causes acting in l's immediate vicinity alone. However, measuring the spin of r in the direction parallel to the particles' spin reduces $|\phi\rangle$ instantaneously into either $|\phi_1\rangle$ or $|\phi_2\rangle$. The measurement therefore collapses the state of the entangled system, after which the spin of l becomes determinate. The conjunction of the separability and locality principles suggests that the direction of measurement apparatus and r's spin could not have determined the state of l, given the spatial separation between the particles. But since the collapse of the quantum state seems to make the spin of l determinate, we have to give up something: either the state of l was not made determinate by the measurement of r's spin, and it was made determinate by some unknown local factor; or we have to give up one of the two assumptions, the separability or locality principle.[7] The first option would suggest that QM is incomplete. The second option would suggest that some well-entrenched philosophical expectations have to be abandoned.

As is well-known, Bell's famous inequalities seem to confound any hope for a theory of local hidden variables that would explain the quantum correlations. If there were such hidden variables, one could assume that the possible spin states of l are determined by some local factor λ and the direction of the magnetic field, $\mathbf{a}$, used to measure the spin of l. Similarly, the possible states of r would be determined by a local factor and the direction of the magnetic field, $\mathbf{b}$, used to measure the spin of r.

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[6] See Healey (1991) for a careful analysis of the distinction between particularism and separability.

[7] As Arthur Fine pointed out to me in a private communication, if we give up separability, then essentially we are also giving up on the locality principle having any determinate meaning. If the states of two particles are inseparable, it does not make sense to ask whether there is a local or non-local interaction between the particles.

If the states of the two particles are separable and there is no non-local interaction between the particles, it follows that the two states are statistically independent. The statistical correlation between the spin measurements of the two particles (i.e., the probability $\mathbf{P}(\mathbf{a}, \mathbf{b})$ for measuring opposite spin values for the two particles) would depend only on the probability distribution of the local hidden factors and on the direction of the magnetic fields **a** and **b** measuring the spins of the particles. But such a probability leads to inequalities that are not consistent with the confirmed predictions of QM.[8] Thus, it seems as if we are in a position to reject that a hidden variables theory which presupposes the separability and locality principles is consistent with the confirmed predictions of QM.

Many conclude from Bell's inequalities that one has to give up either the separability or the locality principles. But an apparent conflict with the theory of relativity convinces some commentators that QM should not be taken to violate the locality principle. According to relativity, causal influences cannot occur through instantaneous action at a distance. The "natural" conclusion is that QM must violate the separability principle.

For some, the philosophical lesson to learn from Bell's Theorem is that quantum systems are not always separable. According to this interpretation, the state $|\phi\rangle$ ascribed to the composite entangled system is non-reducible to the inherent states describing l and r independently, even if there is a distance separating the two particles.[9] But there are also dissenting voices questioning whether violations of the assumptions in Bell's Theorem necessarily imply non-separability.[10]

There are various ways to interpret quantum non-separability. One may think, in the case of entangled systems, that each part of the system loses its individual existence. According to such a view, the components of the entangled pair are "blended" into the composite whole and the physical system is smeared throughout space with no distinguishable parts.[11] Another interpretation is that the individual parts of the system retain their identities but there is a relation between them that

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[8] Bell (1987, pp. 152–53).

[9] Such a conclusion can be found in Bohm (1981, chapter 1), Teller (1986, 1989), French (1989), Shimony (1989), Jarrett (1989), and Healey (1991). Although Bohm's theory seems to violate locality rather than non-separability, he endorses holism in his philosophy of QM (but also in Relativity). This holism exhibits a certain "formative" cause (in distinction from an efficient cause):

> Evidently, the notion of formative cause is relevant to the view of undivided wholeness in flowing movement, which has seen to be implied in modern developments of physics, notably relativity theory and quantum theory. Thus . . . each relatively autonomous and stable structure (e.g., an atomic particle) is to be understood not as something independently and permanently existent but rather as a product that has been formed in the whole flowing movement and that will ultimately dissolve back into this movement. How it forms and maintains itself, then, depends on its place and function in the whole (Bohm, 1981, p. 14).

[10] See for example Maudlin (2002, p. 98), Winsberg and Fine (2003), and Belousek (2003).

[11] See Belousek (2003) for such a possible interpretation.

is irreducible to the intrinsic properties of the parts.[12] Teller (1986) calls this latter ontology “Relational Holism.”

I shall not pursue these issues in the philosophy of QM any further in this book. The purpose of this cursory survey of some philosophical problems in QM is to suggest that there is a need to analyze how structural assumptions are embedded into our physical thinking. If it were easy to “read off” from its mathematical formalism whether a theory violates particularism or separability, there would not be any philosophical discussion. So to advance our understanding of current theories like QM, we have to go back to the origins of our physical concepts and analyze their connection to structural assumptions. It will be argued in the rest of the book that it is not possible to separate our understanding of spacetime structure, physical properties, and laws of nature from our understanding of structural assumptions implicit in our physical thinking. The preoccupation with particularism requires widening the discussion into the very nature of our physical concepts. This book is a first attempt at this widening of the debate.

1.3 Structural Assumptions and Conservation Laws

The most significant assumptions about the structure of physical systems are hidden in energy and momentum conservation laws. In the Newtonian context, momentum and energy are defined as $\vec{P} = m\vec{v}$ and $\mathcal{E} = m\frac{v^2}{2}$, where m is the mass of the body and $\vec{v}$ is its velocity. One ordinarily takes Newtonian particles to be localized, pointlike entities. It is therefore natural to assume that the general conservation laws, $\dot{\vec{P}}_{tot} = 0$ and $\dot{\mathcal{E}}_{tot} = 0$, are rules that dictate the behavior of distinct, localized objects. These rules predict the evolution of physical states.

But a more thoughtful examination reveals that the total momentum and total energy of a system play a significant role in individuating the various components as a system. One can see this by examining the epistemic role of momentum and energy conservation. To empirically verify the truth of these conservation laws one has to be reminded of the qualification that these laws apply only to closed systems. Of course, if a system interacts with other systems, it exchanges momentum and energy with them and so conservation laws do not apply to it. Thus, an experiment confirming momentum-energy conservation laws will begin by isolating the system from external disturbances. For example, the experimenter may block electromagnetic interference, compensate for the gravitational pull, and reduce forces of friction as much as possible. After insulating the system from all external influences, the experimenter will measure the momentum and energy of each component at various times, calculate the total momentum and energy in the system, and verify that these totals are indeed conserved.

[12] See Teller (1986), Teller (1989), and French (1989) for such an interpretation.

But the initial process (of isolating a system from external disturbances) itself takes for granted the conservation laws. After all, a system is determined to be closed by verifying that it doesn't lose or gain momentum or energy. No independent criterion exists for identifying a system as closed. One may object to this claim by asserting that, with the help of a particular force-law, one may simply verify that the conditions for impressed forces do not apply in a particular case (at least to a good approximation). However, the experimenter only knows about the various external forces operating on the system through previous studies, which presupposed that forces are present only when momentum and energy are increased or diminished. Or, to be more precise, one can learn about the presence of a force only when momentum and energy are exchanged. Thus, it is far from surprising that conservation laws are "confirmed" by experience. The only experiments that could be considered as relevant for testing the law presuppose its validity.

The aforementioned should not be taken to imply that conservation laws do not have any empirical validation and are a priori valid. The contention is not that conservation laws are valid independently of experience and are necessarily true in a metaphysical sense. Instead, one should think of conservation laws as providing the necessary presuppositions for converting observed phenomenal laws into physical knowledge. If the experimenter discovers that momentum or energy is not conserved in her experiment, she is more likely to believe that the system examined is not truly closed, despite her initial confidence that it is. It could be that some hidden external force has escaped the experimenter's attention or that there are components of the system that have left the boundaries of the system without being noticed, taking momentum and energy with them.[13] If the problem persists across many experiments, the refutation of momentum and energy conservation would require the articulation of some other theoretical and practical procedure for isolating a system. The experimenter would then need other criteria through which she can isolate her system and measure the interactions between subsystems. She would need to presuppose another means of identifying an aggregate of discrete objects as causally isolated from the rest of the world. This Criterion of Isolation must be some property or state attributed to a physical system which is comprised of many parts, and the property or state must somehow be constructed from the properties and states of the parts. One might imagine various ways in which such a Criterion of Isolation would be given, but some alternative to conservation laws must be in place for a system to be considered as causally isolated.

[13] The famous case of the Neutrino particle demonstrates how relevant this hypothetical scenario is. To explain the missing continuous energy spectrum in beta decay, Bohr proposed to limit the validity of energy conservation. But Pauli's intuition was that the principles of conservation are too deep-seated to be questioned. In a famous letter from 1930 to participants in a conference at Tübingen, Pauli proposed the existence of a hitherto unobserved particle that carries the missing energy. The theory of the neutrino was developed by Fermi in 1933 and the particle discovered in 1955, 25 years after Pauli initially proposed the idea. Pauli's letter can be viewed at the online Pauli archive at CERN.

The reconstruction of Newtonian physics therefore begins with rethinking momentum conservation and recasting it as a structural definition of physical systems. One may analyze momentum conservation with the help of two presuppositions regarding the structure of physical systems. The first presupposition is the Criterion of Isolation, which provides the physicist with a criterion for isolating a physical system from the rest of the world. The assertion that the total momentum of an isolated system is conserved, is in effect a *Criterion* of Isolation, not an empirical law governing a system that was isolated via some other Criterion of Isolation prior to testing the conservation law. One *identifies* the state of being isolated via the state wherein a system's momentum does not change. The second presupposition is the Rule of Composition that dictates the relation between the properties of the system's components and the property of the composite system. In the case of momentum, the Rule of Composition is the summation rule according to which the momentum of the composite system is the vectorial sum of the momentums of the parts. This Rule of Composition amounts to an inference from properties of the parts to properties of their composite. When the Criterion of Isolation applies to a simple system with no parts, such as structureless particles, the criterion yields trajectories of particles that are free from external influences, i.e., inertial motion. When the Criterion of Isolation applies to a composite system, it identifies for us the state wherein the parts of the system do not interact with any external objects. Conservation of momentum in the case of composite systems, is a combination of the Rule of Composition, according to which the momentum of the composite system is the vectorial sum of the momentums of the parts, and the Criterion of Isolation, which identifies conserved momentum with the state of being isolated.

One ordinarily thinks of the total momentum of a system as a state of the composite system reducible to the properties of the system's components. It is assumed that the trajectory of the center of mass of a system is some average of the various component trajectories, which is only helpful as a mathematical device and does not represent any real object. But it may be useful to think of the center of mass as describing the *actual* trajectory of the composite system comprising of the various interacting parts. One benefit of taking this metaphysical view is that one now has a clear analogy between free particles and isolated composite systems. According to the Law of Inertia, a free particle moves with uniform rectilinear motion. According to the law of momentum conservation, an isolated system moves with uniform rectilinear motion. The two cases can be summarized with a single criterion:

1.3.1 The Criterion of Isolation

A system is isolated if and only if it remains in the same state of uniform rectilinear motion.

The criterion of isolation establishes a conceptual connection between the state of "being isolated" and the state of "uniform rectilinear motion."

Treating the conservation of momentum as providing a Criterion of Isolation also clarifies the conservation laws' epistemic status in inferences between observed physical phenomena and derived laws. The Criterion of Isolation is a necessary presupposition for interpreting the phenomena, since the state of uniform rectilinear motion distinguishes between open and closed systems. Such a distinction is necessary for correlating observed motions with the external causes that bring them about. It is standard practice to use the state of uniform rectilinear motion as a counterfactual trajectory relative to which the deviations of particles from their natural force-free motions are correlated with some external influence. When applied to a system's center of mass, the state of uniform rectilinear motion is used to demarcate a composite system of interacting parts and separate it from the rest of the world. Once a system is isolated, one can then conclude that forces operating on one of the system's components must originate from the other components of the system. The Criterion of Isolation is therefore a necessary presupposition for correlating motions with external causes and for finding the relevant sources of these causes.

There are affinities between our treatment of the Criterion of Isolation and Friedman's relativized a priori (see Friedman, 1983, p. 286; Friedman, 1999, pp. 59–70). Friedman popularized Reichenbach's *Axioms of Coordination* introduced in Reichenbach (1920). In his early work, Reichenbach's thinking was still influenced by the neo-Kantian school. According to Reichenbach, the *Axioms of Coordination* are a priori axioms of physical theory. These axioms are a priori in the sense that they are valid "independently," or "prior" to experience. They are akin to framework principles that form the concept of object – which is a rational form imposed by cognition on intuitions. But they are not a priori in Kant's sense of being necessarily true. Given new experiences one may revise the overall framework, resulting in new *Axioms of Coordination*. In contradistinction to the *Axioms of Coordination*, the *Axioms of Connection* function as laws established directly by experience.

In a vein similar to that of Friedman's relativized a priori, the argument here is that the Criterion of Isolation is presupposed as valid prior to observations and experiments. One relies on the Criterion of Isolation both to design experiments and interpret their results. The Criterion of Isolation is not metaphysically necessary or discoverable by the mere operation of the mind. This criterion is inspired by experience and thought experiments, and can be revised in the light of irreconcilable experiences when replaced by a new Criterion of Isolation or alternative presuppositions regarding the structure of physical systems. The a priori validity of this criterion is reflected in the fact that there is no direct way to refute it, since it is presupposed in the inference from the phenomena to the interactions that explain them.

Despite the similarities between the interpretation of the Criterion of Isolation as required for interpreting phenomena and Friedman's relativized a priori, there are important differences as well. According to Reichenbach, *Axioms of Coordination* constitute the "subjective" form of knowledge. The *Axioms of Coordination* are "organizing principles" that constitute the contribution of Reason to the object of knowledge. One important characteristic of these *Axioms of Coordination* is they introduce an element of arbitrariness, as the truth of these axioms is not determined by the objects being studied. But contrary to Reichenbach's early philosophical views, the contention here is that one cannot neatly separate the "contribution of reason" from the contributions of experience. There is an important sense in which the Criterion of Isolation is not arbitrary; it is not merely stipulated that isolated systems are identified with uniform rectilinear motion or with PUMs.

Reichenbach's neo-Kantian views and Friedman's relativised a priori have a difficult time explaining the status of discarded *Axioms of Coordination*. If, for example, the absolute nature of time and Euclidean geometry were taken to be valid a priori in Newtonian physics, they are no longer a priori valid in the Theory of Relativity. They are just empirically inadequate assumptions about the nature of time and space. The Theory of Relativity is empirically superior to Newtonian physics, which suggests that there are empirical reasons to reject the absolute nature of time and Euclidean geometry. If there are empirical reasons to reject those principles, they are no longer valid prior to experience. The neo-Kantian might respond by asserting the discarded a priori principles must somehow be approximations of newer *Axioms of Coordination*. The absolute nature of time is approximately true when considering velocities much slower than the speed of light; and the Euclidean nature of space is approximately true when the gravitational field is more or less uniform. But if these discarded principles are only approximately true, and are literally false, then they are no longer valid a priori, and they never were.

Thus a new-Kantian framework for understanding the Criterion of Isolation is inadequate, given that the transition from one theory to another suggests that revisable a priori principles must somehow be sensitive to experience, and be falsified by it, even if it is valid prior to single, isolated experiences. The a priori nature of the Criterion of Isolation can be understood when one moves away from the rationalist gloss that the Neo-Kantian school attributes to a priori principles. In the neo-Kantian school, experience is somehow produced in the mind when intuitions are placed under concepts. The a priori elements of experience arise from the conceptual structure that cognition imposes on experience; perception is "infected," so to speak, by the rational form the mind imposes on experience. However, the Criterion of Isolation is not merely a conceptual structure imposed by human cognition.

The Criterion of Isolation is valid prior to experience, because it is presupposed both when experience is interpreted *and* when experience is produced. One cannot carry out any physical experiment without presupposing the criterion through which the experimenter can identify a closed system. Any experiment that measures the effect of causes, must form a valid inference from the cause to the effect. But this inference is destroyed when the cause and the effect cannot be isolated from other external causes. A Criterion of Isolation must therefore be correctly embedded in the experimental practice if one is to acquire empirical confirmation for theoretical predictions. The problem with the neo-Kantian account is that it posits a rational structure, determined by a stipulation on our part as to the form of intuition. But the Criterion of Isolation guides the experimental practice as much as it is an abstract conceptual connection. If the experimentalist is able to consistently isolate systems with the help of the Criterion of Isolation, she learns to rely on it more and more until she incorporates it seamlessly into her art of experimentation. The world must therefore cooperate with the Criterion of Isolation and with what the experimentalist does for the experimental practice to be effective, even though the experimental practice presupposes it. Systems must be, at least roughly, isolated, when they conform to the Criterion of Isolation, if the experiment is to have any value.

One therefore needs to differentiate between a principle that is valid prior to any individual experience, and a principle that is valid prior to *all* experiences. Given any particular observation or experiment, the Criterion of Isolation is valid prior to it, because it is presupposed both at the level of conceptual structure and the level of experimental practice. But the Criterion of Isolation is not valid prior to all experiences, taken together as a whole. One can form a judgment, based on the experimenter's ability to isolated systems and investigate the causal connections between their parts, that the Criterion of Isolation is empirically true, or at least roughly true. But this judgment is a holistic one, and requires the assessment of the theory and the entire range of experiments and observations as a whole. Such an assessment may bring in holistic epistemic virtues such as simplicity, explanatory power, and the unification of disparate domains of inquiry. But once a Criterion of Isolation is evaluated in relation to all experiences, it is deemed as empirically true or false.

Thus the neo-Kantian school needs to revise even further Kant's notion of a priori validity. The a priori principles are valid prior to any *individual* experiences, but are revisable when all experiences are considered as a whole. The Criterion of Isolation is neither strictly speaking a priori, nor strictly speaking a posteriori. It is a priori valid in the context of performing a single experiment, but a posteriori valid when the merits of the entire theory and its predictions are considered. One should be careful in applying that distinction as if it applies in all contexts. Moreover, the general rationalistic gloss of a priori principles should also be given up. The Criterion of Isolation is not merely a rational construction imposed on experiences, but is embedded in the experimental practice and is therefore not dissociated from the successes experimenters have in isolating causal relations. The experimental and theoretical aspects of the Criterion of Isolation are inseparable.

The revisions I propose here to neo-Kantian epistemology may remind readers of conventionalist-positivist accounts. Reichenbach abandoned his early neo-Kantian views when he became convinced that there could not be a distinct category of *synthetic* a priori judgments. The conventionalist-positivist[14] accounts emphasize both the a priori nature of linguistic representations, and the role of experimental practice in connecting theoretical terms with physical objects. According to positivist approaches, the truth of a priori propositions is determined by semantic conventions, since the meaning of the terms in these propositions is determined via an arbitrary semantic stipulation. These semantic conventions are supplemented

[14] I am referring to conventionalist accounts of science such as Einstein in his 1905 relativity paper and in his 1921 paper, Carnap (1922), Reichenbach (1927, 1969), Grünbaum (1963), and to Wittgenteinian accounts such as Hanson (1965).

with extra-linguistic rules for relating concepts with physical objects (what the later Reichenbach called *principles of coordination*). Thus according to the positivist approach, conceptual interrelations and rules for grounding propositions in experience become completely dissociated.

For example, positivist accounts take Newton's laws of motion to be valid a priori because of their role as axioms. Other propositions are derived from the laws, while the laws are not derived from anything else. Thus, according to the positivist approach, the Law of Inertia is a semantic convention; the truth of the law expresses a linguistic rule. As a result, the conceptual connections established between the terms in the Law of Inertia constitute implicit definitions. Poincaré, for example, asserts that the status of this law is that of a convention:

> ... this law [of inertia], verified experimentally in some particular cases, may be extended fearlessly to the most general cases; for we know that in these general cases it can neither be confirmed nor contradicted by experiment. (Poincaré, 1905, p. 97)

Hanson (1965) emphasized in a similar fashion the connection in meaning established between the terms appearing in the Law of Inertia. According to him, the concept of being "force-free" is "built into" the concept "uniform rectilinear motion." Since the truth of the law is stipulated, one cannot exclude the possibility of other adequate conceptual systems in which these semantic connections do not exist. These alternative systems would still be able to provide adequate interpretations of our experiences even if they do not take Newton's Law of Inertia to be true.[15] According to the positivist account, the a priori validity of the Law of Inertia derives from a semantic convention, not from any physical fact concerning material bodies.

The positivists argued that the experimental practice establishes a connection between formal, mathematical terms of the theory and physical objects. For example, when an experiment is performed, a coordinate system is selected, and length and duration are measured relative to this coordinate system. But the selection of one coordinate system produces one model of the theory; another equally appropriate model is based on the selection of another coordinate system. Thus, while the physical theory is merely a formal, empty linguistic structure, the selection of a coordinate system provides the formal theory a particular interpretation. The coordinate system also provides the formal theory with physical and empirical content. I argue in Chapter 2 that the positivist account falls short of providing a satisfactory

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[15] In Poincaré's words:

> The Law of Inertia, as I have said above, is not imposed on us a priori; other laws would be just as compatible with the principle of sufficient reason. If a body is not acted upon by a force, instead of supposing that its velocity is unchanged we may suppose that its position or its acceleration is unchanged. Poincaré (1905, p. 92)

When Poincaré uses the notion of a priori he means it in the rationalist sense (i.e., as an irrefutable and necessary truth that is arrived at independently of experience). But the notion of a priori can be given an empiricist slant, so that a proposition is taken to be necessary only in relation to a particular inquiry, relative to a particular domain or an area of inquiry.

account of the Principle of Relativity. Here I would like to emphasize that one of the main problems with the positivist account, is that according to their account, the Law of Inertia establishes an "empty," arbitrary linguistic connection between the state of being force-free (or the state of being isolated) and the state of moving with uniform rectilinear motion. However, their account makes it seem as if the implicit definition (of the state of being force-free) is a mere syntactic relation between linguistic terms, that stands over and above our experimental practice. However, the Law of Inertia does not express a mere linguistic rule, as it provides the bedrock of our experimental practice as much as it is also at the heart of theory. The success of the experimental practice in isolating systems suggests that the Criterion of Isolation is not an arbitrary linguistic convention.

Before moving on to the Rule of Composition, I would like to amend the above formulation of the Criterion of Isolation. At first glance, the formulation of the Criterion of Isolation implicitly presupposes the existence of a background space, relative to which the uniform rectilinear motion of a body is defined. But as Chapter 3 will make clear, it is possible to construct spacetime structure from a geometry of PUMs, which is a geometry that takes uniform unidirectional motions as the fundamental entities, rather than spatiotemporal points. Thus, in the language preferred here, the Criterion of Isolation should be articulated as follows:

Criterion of Isolation*. A system is isolated if and only if it instantiates a PUM (potential uniform motion).

The next chapter will provide a somewhat more detailed critique of widespread approaches to the philosophy of spacetime. At this point, I will only put forward the claim that taking uniform, unidirectional motions (PUMs) to be fundamental entities of spacetime, and constructing spacetime from PUMs, provides a new approach to understanding the structure of spacetime. Thus, when the Criterion of Isolation asserts that a system is isolated if and only if it instantiates a PUM, it asserts that the state of being isolated is identified with the most fundamental entity in physical theory – a PUM. The Criterion of Isolation* therefore connects directly with the foundation of spacetime structure, and does not presuppose a spacetime structure as a background. The claim that a system is isolated if and only if it instantiates a PUM should be understood as associating the state of being causally isolated with the most fundamental entity in spacetime theory. This comment will only be understood once a reconstruction of spacetime is presented.

As was stressed before, there is empirical significance to the *practical* success in relating the state of being causally isolated with a PUM. A "relativistic" PUM is more adequate than a "Galilean" PUM because it more accurately describes events along the trajectory of an isolated system. The empirical adequacy of a Criterion of Isolation increases when a new geometry of PUMs is used to construct a more powerful and fruitful theory. The Newtonian law of inertia and its counterpart in the Special Theory of Relativity do not stand on equal footing from an empirical point of view. At the same time, the Law of Inertia is not merely true a posteriori, when individual experiments are considered. The typical strategy for those who argue that the Law of Inertia is an empirical generalization[16] is to reify the structures of spacetime. When spatiotemporal points and geometric structures are taken to exist independently of material bodies and processes one may then view the actual motion of a free particle along a geodesic as corroborating a law about free particles. But the reification of spacetime carries unfortunate metaphysical consequences, and generates an obscure understanding of spacetime and its relation to dynamic laws. Thus, since the Criterion of Isolation* involves the most fundamental entity in spacetime theory, and since this Criterion of Isolation treats both free particles and systems comprised of many interacting parts as analogous entities, it is clear that it provides the most fundamental criterion for interpreting phenomena.

In the following, I will assume that the Criterion of Isolation (and the accompanying Rule of Composition) hold a special role in physical reasoning. Together, the Criterion of Isolation and the Rule of Composition constitute structural assumptions about the nature of physical systems. These structural assumptions are a priori principles, whenever local, isolated experiments are considered, but are a posteriori principles when the theory and its predictions are considered as a whole. This approach is similar to the neo-Kantian school, in that it identifies certain principles as a priori valid, but it is also distinct from the Neo-Kantian school since it takes the revisable nature of structural assumptions to be evidence of their connection to experience. But the approach is also distinct from positivist accounts, since these structural assumptions are not mere linguistic conventions, nor are they mere coordinative definitions. They are at the heart of both theory and experiment. The conservation of momentum and energy implicitly asserts that the Criterion of Isolation applies to physical systems. A particular theoretical articulation of the Criterion of Isolation is not metaphysically necessary. In the case of Newtonian mechanics (and the Special Theory of Relativity), this criterion establishes a conceptual connection between being isolated and instantiating a PUM. This connection is established prior to the interpretation of phenomena, but this connection is not a mere linguistic definition, since it embedded in the experimental practice.

1.3.2 The Rule of Composition

In the previous section, I argued that the Criterion of Isolation holds a primitive, constitutive role in the definition of physical systems. However, to fully describe the structure of a physical system one must also assume certain inferential relationships between the composite physical system and its parts. It is often difficult to be aware of these inferences, since they are deeply embedded in mathematical structures and in various calculational devices for forming descriptions of composite systems. For example, in the context of the kinetic theory of gases, certain mathematical inferences establish the connection between the microstructure of the physical system, such as the kinetic properties of molecules, and its macroscopic

[16] See Earman and Friedman (1973) and Friedman (1983).

properties, such as temperature and pressure. The conservation of momentum and energy is used as a bridge between microscopic and macroscopic states. The conservation of momentum and energy is taken to be laws of motion governing the distinct parts of which the system is made. However, the total momentum and energy of the composite system is not merely a concise way to describe the parts of the system, but a means of referring to a state belonging to the composite system. The conservation of momentum and energy provides the conceptual tool for reducing states of the composite system to states of its parts, and establishes the inferences between parts and wholes.

Ordinarily, the law of momentum conservation is taken to be a *law* of motion. According to this conception, the law dictates how various distinct bodies interact and how their states evolve over time. This account of the conservation law *defines* the momentum of each particle as the product of mass and velocity:

$$\vec{P}_i(t) \equiv m_i \vec{v} \tag{1.2}$$

where m_i is the mass of a body and $\vec{v}$ is its velocity. It is then asserted that the total momentum

$$P_\Pi(t) = \sum_i P_i(t) = \sum_i m_i \vec{v}_i \tag{1.3}$$

is conserved, or

$$\frac{d\vec{P}_\Pi}{dt} = 0 \tag{1.4}$$

This presentation of the law of momentum conservation makes it seem as if the law takes in as the initial state the velocities and positions of the various interacting particles, and then dictates how these states evolve over time. The description of this evolution also depends on the details of the interaction, such as the particular force that operates between the parts. The law is likened to an external decree that "governs" the behavior of individual localized bodies. Material properties such as mass represent inherent causal powers or the dispositions to behave according to the conservation law.

If one takes the standard approach to conservation laws, i.e., as laws that dictate the evolution of physical states, one has difficulty in establishing the logical status of the mass parameter. On the one hand, mass is supposed to be an intrinsic property to be found "within" each material body (i.e., this parameter is instantiated in the body independently of any other body). On the other hand, the nature of this property is revealed only through the disposition of a body to obey laws that are imposed from without. Newton imagined mass to be associated with some inherent causal power or "inertial forces" that propel a body to move in a straight line. However, these intrinsic forces appear to spring into existence in reaction to impressed forces (i.e., the intrinsic forces come into being in response to external influences). This incoherent metaphysical story signifies a certain lack of clarity as to the role of the mass parameter. This unclarity is increased when one takes mass to be describing the quantity of matter, which underlines the assumption that the mass of a composite system is the sum of the masses of the components, and that isolated systems do not gain or lose mass. Textbooks often refer to the "law" of conservation of mass, as if mass itself is governed by a law that predicts how mass-states evolve over time. But mass is itself a parameter that is required to *define* the momentum-state of a body, so it is not clear how the state of body (i.e., the mass-parameter of a body) is governed by one law (the conservation of mass), while it is presupposed in the definition of another state (the momentum state), which is governed by an independent law (the conservation of momentum). The two laws seem logically independent, yet momentum conservation somehow presupposes the mass parameter.

One way to avoid the conundrums concerning the mass parameter is to take the view that momentum is a fundamental state of bodies, and the mass parameter is determined solely by its role in the articulation of momentum conservation. Conservation of momentum is the presupposed law, and the mass parameter attributed to each body *is a logical consequence* of their momentums being conserved. Something like this was proposed by Weyl:

> . . . the inertial mass is no perceivable characteristic of a body, but can only be determined by allowing the body to react with others and then applying the impulse law to these reactions. This law asserts: to every isolated body a momentum may be assigned, this momentum being a vector with the same direction as the velocity; the positive factor m, by which the velocity must be multiplied in order to give the momentum, is called the mass. If several bodies react on each other, the sum of their momenta after the reaction is the same as before. It is only through this law that the concept of momentum, and with it that of mass, attains a definite content. (Weyl, 1989, p. 39)

Weyl's approach is superior to Newton's account, since the concept of mass is no longer seen as a causal agent that gives rise to inherent forces. The notion of mass as some inherent agent in bodies seems opaque, and is not genuinely explanatory. One may begin with the assumption that laws of motion dictate the behavior of bodies, and mass is merely the disposition of the body to behave as the laws dictate. There is no reason to suppose that mass is anything more than the disposition to obey the laws.

But Weyl's approach generates the puzzle that momentum seems to presuppose mass in its definition, and it is not clear how one can conceive of momentum as being the fundamental state while at the same time taking it to be the product of mass and velocity. The priority of momentum conservation over mass is difficult to conceptualize, especially since there is a law, i.e., the conservation of mass, which governs the mass parameter independently of its role in momentum conservation. Since one defines the momentum-states of distinct bodies as the product of mass and velocity, it is not clear how mass could be the logical consequence of momentum conservation, rather than the causal agent that gives rise to it.

One may avoid the difficulties faced by Weyl's approach if the conservation of momentum is reconceptualized as a structural assumption, rather than a law dictating the behavior of distinct parts. Such an account assigns a determinate velocity

$\vec{v}_\Pi$ to the isolated composite system comprised of n interacting particles, and treats this velocity as a composite state that arises from the states of individual parts. If such an approach is feasible, then the mass parameter attributed to each body would be the result of the relation between the motions of the parts and the motion of the composite.

The proposal is therefore to think of the conservation of momentum as consisting of assumptions about the structure of physical systems, rather than external laws that dictate the behavior of bodies. Assume that an isolated system containing n particles has a velocity $\vec{v}_\Pi$ attributed to it. It would follow from the Criterion of Isolation that the velocity of this composite system remains uniform and unidirectional, given that it is isolated from the rest of the world. Alternatively, this velocity of the composite could be taken to be a function of the velocities of the parts:

$$\vec{v}_\Pi \equiv f(\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n) \tag{1.5}$$

If one takes the above function to be a linear vectorial sum, one can find a way to conceptually relate the state of the isolated composite system to the velocities and states of the individual parts:

$$\vec{v}_\Pi \equiv \sum_i \alpha_i \vec{v}_i \tag{1.6}$$

The rule for relating the motions of parts to the motion of a composite system will be referred to as a Rule of Composition. This rule, in addition to the Criterion of Isolation*, completes the description of physical systems:

Rule of Composition. The motion of the composite system is a linear vectorial sum of the motions of the components.

On the assumption that the Criterion of Isolation* holds for closed systems, one may identify with each coefficient the relative mass of the body, so that:

$$\alpha_i = \frac{m_i}{\sum_i m_i} \tag{1.7}$$

This would allow us to write down the equality:

$$\left(\sum_i m_i\right) \vec{v}_\Pi = \sum_i m_i \vec{v}_i \tag{1.8}$$

Each body is therefore attributed a mass parameter m_i and the composite system a parameter $M = \sum_i m_i$. One may treat the function in (1.6) as part of the *definition* of the structure of physical systems. The existence of determinate mass-ratios would then result from the claim that the posited structure applies to physical systems. When one presupposes a linear function correlating the velocities of the components with the velocity of the composite system, one is able to derive the conservation of momentum from the Criterion of Isolation*. Once the relation in (1.6) is defined, the definition of momentum as $\vec{P} = m\vec{v}$ follows as a consequence.

The conservation of momentum can be considered as the consequence of a relation between the velocities of part of a physical system and the velocity of the whole. This account takes velocity to be the fundamental state, and the conservation law as a consequence of the Criterion of Isolation and a compositional relation between velocities of parts and the velocity of the whole. Thus, unlike Weyl's account, there is no need to account for a fundamental state of momentum, which presupposes the mass parameter. Velocity is fundamental, and the conservation law arises from a relation between parts and wholes.

The account which takes conservation of momentum to consist of a structural assumption therefore has a potential for making velocity fundamental, while rendering mass a product of the velocity-state of the isolated composite system. However, the account of mass as the logical consequence of the Criterion of Isolation* and the Rule of Composition needs to account for the following facts:

1. **Fundamental particles have a constant mass parameter.**
   If the mass parameter is not inherent in bodies, but instead stems from a part-whole relation governing the motions of bodies, why do we attribute a single mass parameter to a fundamental particle throughout its entire life?
2. **Newtonian mass is a quantity of matter.**
   The Newtonian mass of a composite system is always the sum of the masses of the components, and does not change for an isolated, composite system. But what *justifies* the assumption that mass is an additive quantity? What justifies the conservation of mass in closed Newtonian systems?

It is tempting to view these two facts as based in experience. Textbooks often cite the "law" of mass conservation as an additional empirical law implicit to Newtonian physics. According to this approach, the *meaning* of mass is identical with the role of the concept in Newton's second law of motion. Mass represents the inertial "slugishness" of bodies, or the resistance of bodies to external forces. The conservation of mass is another empirical fact that is true of the property.

The reconstruction of mass introduced here will show how inferences about Newtonian mass are derived from the structure of Newtonian systems. The reconstruction will demonstrate that the single mass parameter we attribute to non-composite bodies is implicit in the structure of Galilean spacetime, when this spacetime is derived from a geometry of PUMs. The surprising fact our reconstruction reveals about mass is that it is inherently related to the symmetries of Galilean spacetime, or to the equivalence between inertial reference frames. Once it is shown how the Galilean spacetime is derived from a geometry of PUMs, it will become apparent that the *mass parameter is a logical extension of the equivalence between inertial reference frames!* The choice of a mass parameter for the description of a body's acceleration is similar in nature to the choice of an inertial reference frame. Thus, a significant consequence of the proposed account of physical systems is the conceptual pathway it erects between the spacetime structures and material properties. The theory of spacetime describes the various possible motions of physical systems.

The concept of mass is similarly another component in the structures that govern the motions of systems.

This approach has the benefit of treating the Criterion of Isolation as a single basic principle that governs Newtonian or relativistic systems, whether one has free particles in mind or systems of interacting parts. According to this procedure, assigning mass parameters to each individual body is the consequence of presupposing a geometry of PUMs, the Criterion of Isolation, and the Rule of Composition. The "structure" of physical systems posited by Newton's mechanics and Einstein's Special Theory of Relativity is more primitive than the property of mass ascribed to bodies.

This approach is very economical from the ontological point of view. According to this approach there is, in nature, nothing but moving parts and moving wholes. One first assumes a geometry describing relations between PUMs is assumed, then rules relating the motions of parts and wholes. The material property of mass is not instantiated in individual bodies like a universal that is instantiated in some particular substance. Rather, mass is a logical consequence of the geometry of PUMs and of rules governing the structure of physical systems. Replacing the notion of physical objects with the notion of physical systems simplifies the ontology of the world and does away with "inherent" physical properties. However, there is a cost to pay. One needs to think of composite systems as objects fully existing on par with the existence of the individual parts. If conservation laws express the structure of physical systems, one has to treat the composite system as being present. Since the trajectory of a composite system is best represented by the trajectory traced by the center of mass, one may think of the composite system as present at the center of mass.

The analysis will not be limited to Newtonian physics, as the role of structural definitions will also be demonstrated in the context of the Special Theory of Relativity. In the context of the Special Theory of Relativity, one may take the rule of composition implicit to the notion of physical system to apply to four-dimensional velocities. Thus, in the context of relativity, both the conservation of momentum and the conservation of energy are incorporated into a structural definition governing four-momentums. The analysis of the Special Theory of Relativity will enable us to analyze the conceptual relations between rest mass, relativistic mass, flat relativistic spacetime, and the conservation of four-momentums.

1.4 Structural Definitions

To clarify the role of assumptions about the structure of physical systems, one needs to differentiate between a standard definition, which may be explicit or implicit, and the definition of physical structure. An explicit definition concerns the syntactical relation between terms. For example, one may define a triangle as a three-sided polygon. This explicit definition presupposes that parts of the definition acquire their meaning independently of the defined term. An implicit definition is one where the terms of the theory are implicitly defined by the axioms of the system. For example, part of the meaning of the term “point” could be given by the first axiom of Euclidean geometry, which asserts that between any pair of points there exists a straight line. In contradistinction with explicit and implicit definitions, an assumption about the structure of physical systems concerns the relations between the parts of the system and their composite. A composite structure is a description of a particular way in which various parts are put together. This inferential relation does not necessarily describe relations between different properties, as is the case for implicit and explicit definitions. Rather, it is a relation between different instantiations of the property; the instantiation in parts and wholes.

Standard accounts of definitions takes them to be the syntactical relation between terms. For example, consider the definition of a triangle as a three-sided polygon. The definition asserts that whenever a geometric figure has the property of being a polygon, and in addition is three-sided, by definition the figure is a triangle. The definition is a logical relation erected between predicates describing properties of the same figure. Other definitions might consider relations as reducible to monadic properties or to other relations. In short, explicit definitions can be represented as logical relations between predicates, disregarding the objects to which the predicates apply. Implicit definitions work similarly by stipulating a set of propositions as true, thereby erecting logical relations between the predicates of the system. A structural assumption works analogously to implicit definitions, in that it provides part of the meaning of a term. For example, part of the meaning of the term “extension” is the fact that the extension of the composite object is the sum of the extensions of the parts. Similarly, part of the meaning of “momentum” is determined by the compositional nature of momentums, where the momentum of the composite system is the sum of the momentums of the parts. But it is difficult to assimilate the compositional aspect of extension or momentum to the notion of explicit or implicit definition. In articulating the compositional nature of a property, one has to think of the parts of the system, on the one hand, and the composite, whole system on the other hand, as distinct objects in which the properties inhere. Thus one is often led to think of structural assumptions as laws bridging between the properties of the parts and the properties of the composite rather than definitions.

At first glance, structural assumptions do not seem to function like definitions, since they relate properties that belong to different objects (parts and wholes), and so they are assimilated to laws that determine causal relations between properties belonging to different objects. However, there is a sense in which the bridging laws account of structural assumptions is misleading. The account suggests that the identity of the composite system is determined independently of the identity of the parts. The account implies that one may go through the following process. First, one can consider each independent part and identify its properties. Then, the properties of the composite system are determined. Finally, patterns relating the properties of the parts to the properties of the composite system are empirically detected, and the bridging laws are articulated. However, when one refers to the composite system one is implicitly relying on the structure of the system to *identify* the composite system. For example, imagine a cube with six sides. I can refer to each of the cube’s sides and consider its geometric properties. However, to refer to a cube and any of its properties, I am already taking into consideration the structure of the cube and the geometric relations between the parts. Thus identifying the composite object already makes reference to structure; it is circular to assume that one can identify the composite system and then articulate the bridging laws. Thus one cannot assimilate relations between parts and whole to bridging laws; the structural assumptions are relied upon in identifying the composite system. Identifying the composite whole and attributing properties to it already implies the existence of structure.

On the one hand, there are reasons to think that structural assumptions are distinct from implicit and explicit definitions. Given that structural assumptions relate properties of parts to properties of composite systems, they are not mere syntactical relations between different predicates, since they often relate the same property that is instantiated in parts and wholes. On the other hand, one cannot assimilate structural assumptions to bridging laws, since the composite system cannot be identified independently of the presumed structure of the system.

One way to articulate the problem more clearly, is to consider formal mereology, which reduces the parthood relation itself to an implicit definition. Consider the axioms governing the most basic structures of mereology.[17] To articulate these axioms, one has to consider the various cases in which the part-whole relation P is satisfied. The first axiom of basic mereology expresses the intuition that each object is part of itself. Therefore, the part-whole relation must be reflexive:

P 1. Pxx Reflexivity

The second axiom of mereology corresponds to the intuition that if an object x is part of object y, and object y is part of object x, we must accept that the two objects are identical. The contraposition of this proposition is that two non-identical objects x and y cannot both be part of each other. Thus, the part-whole relation must be antisymmetric for non-identical objects:

P 2. $(Pxy \& Pyx) \rightarrow x = y$ Antisymmetry

Finally, the third intuition is that the part-whole relation is transitive. If y contains x as a part, and z contains y as a part, then our intuition is that z contains x as a part.[18]

P 3. $(Pxy \& Pyz) \rightarrow Pxz$ Transitivity

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[17] Here I follow Varzi (2004) in formalizing part-whole relations.

[18] The transitivity axiom has been disputed in certain cases in which we think organizational levels prevent the part-whole relation from being transitive. If a human cell is part of a human body, and the human individuals are counted as part of the national census, then it does not follow that the human cell is counted as part of the national census. These failures of transitivity will not be considered here.

Using the above axioms one can *define* the overlapping relation, i.e., the relation between two objects that share a part. An overlap between two objects amounts to the claim that there exists a common part that satisfies that parthood relation with both objects:

$$Oxy =_{df} (\exists z)(Pzx\&Pzy) \tag{1.9}$$

Also, one may define the notion of *proper part* using the above axioms. This notion differentiates between an object x that is a part of y and an object x that is part of y, but is not identical with it. Thus, the notion of proper part can be defined as follows

$$PPxy =_{df} (Pxy\& \sim Pyx) \tag{1.10}$$

These axioms and definitions therefore characterize the part-whole *relation*.

The above three axioms provide the minimal set of inferences governing part-whole, relations which is commonly referred to as basic mereology **M**. The mereological literature considers various additional axioms, but I will postpone consideration of some of them until the end of the section. The problem with the formal character of **M** is that it is too abstract to describe the part-whole relation governing physical systems. These axioms hold necessarily for any partial ordering relation, such as the $\leq$ relation among the real numbers. Intuitively, one can sense that a number x is "contained in" or is part of any number y that is greater than or equal to x. The axiom system **M** makes this intuition more precise. But, notice that with regard to the real numbers, their very nature determines whether x is contained within y or y is contained within x.

If a randomly selected pair of physical objects x and y is examined, whether x is a physical part of y cannot be determined solely by the nature of the distinct physical objects x and y. If x is an electron and y is an atom one still does not know whether x is part of y, even if the electron and the atom overlap in space. One must examine the particular *states* of x and y to determine whether x is part of y. For that, one needs a non-formal criterion, or a physical property, with which one can analyze the parthood relation in physical systems. This is where the Criterion of Isolation and the Rule of Composition come in. Given the motions of the components, the Rule of Composition allows us to find potential composites by summing over the motions of the parts and calculating the motion of the composite system. The Criterion of Isolation, or the assertion that a system is isolated if and only if it instantiates a PUM, provides a line of demarcation between systems.

One therefore needs to replace the above formal axioms with a *structural assumption*. Assume that x_1 is one of the proper parts of the physical system y. To define the relation between x_1 and y one can specify the property T_μ, where $\mu = 0 - 3$. This property has four values, corresponding to the four components of the motion of a body. Thus, in the context of relativity theory, the property T_μ represents the four-momentum. The energy property T_0 ranges over the positive reals, $T_0 \in [0, \infty)$. The momentum property T_i ranges over three dimensional reals, $T_i \in \mathbb{R}^3$. The relation between the proper part x_1 and y is represented by certain algebraic relations among the predicates T_μ that apply to x_1 and y. The first part of the structural definition is the Rule of Composition, which asserts that the T_μ of y is the sum of the T_μ's of the parts $x_1, x_2, \ldots, x_n$ so that $T_\mu^y = \sum_i T_\mu^{x_i}$. This composition rule identifies the properties T_μ ascribed to the composite system. The Criterion of Isolation demarcates between systems. So to say that x_1 is a proper part of y, one has to say that the state T_μ attributed to y is a sum of various states $T_\mu^{x_i}$ attributed to various bodies, including x_i, and that either y is isolated or y is a proper part of a system that is isolated. In other words, according to the Criterion of Isolation, either the T_μ^y is conserved or the T_μ^z of a composite z which contains y is conserved. The following summarizes the two components of the structural definition:

Structural Definition of Physical Systems

$(\forall x_1)(\forall y) PPx_1 y \equiv$

I. Rule of Composition

$$(\exists x_2) \quad (T_\mu^y = T_\mu^{x_1} + T_\mu^{x_2})$$

II. Criterion of Isolation

$$\dot{T}_\mu^y = 0 \qquad \text{or}$$

$$(\exists z)(\exists x_3) \quad (T_\mu^z = T_\mu^{x_1} + T_\mu^{x_2} + T_\mu^{x_3}) \quad \text{and}$$

$$\dot{T}_\mu^z = 0$$

The above structural definition makes explicit the sense in which conservation of momentum and energy defines the relation between parts of a physical system and their composite.

Once the relation of proper part is defined using the states T_μ, one can define the part-whole relation in virtue of the proper part relation:

$$Pxy =_{df} (PPxy \vee x = y) \tag{1.11}$$

It can now be shown that the parthood relation, defined by the above structural definition, satisfies the formal axioms of the mereological system **M**. The part-whole relation in (1.11) is reflexive because of the identity relation in (1.11). It is antisymmetric because x_1 is a proper part of y only if there are additional objects whose T_μ of the parts determine the T_μ of the whole. The relation in (1.11) is transitive because the operation of additivity is transitive.

It is important to briefly compare our notion of a structural definition with other accounts of part-whole relations. Notice, first, that the structural definition is not a law bridging between properties of parts and properties of wholes. To identify the composite system, one relies on the composition rule that combines the properties of the parts to form the property of the composite. Thus, one cannot discuss the part-whole relation without taking the Rule of Composition and the Criterion of Isolation as valid. The very *existence* of the relata in the relation between part and whole depends on the validity of the definition. On the other hand, the structural definition is not a standard implicit definition, where the meaning of the predicate is determined via its conceptual relations to other predicates or conceptual relations that are stipulated to be true via the axioms of the system. The structural definition clarifies the meaning of momentum and energy by erecting a compositional relation between the same property applied to both parts and wholes.

The structural definition is therefore a unique kind of implicit definition. The meaning of the notions of momentum and energy are partly determined by the relation they erect between parts and wholes. On the other hand, the parthood relation is itself partly determined by the concepts of momentum and energy. Thus, the structural definition is an implicit definition of a very distinct kind, one that incorporates the notion of parthood into the meaning of a physical term. In the philosophy of mathematics, an implicit definition that introduces several new terms is also called a structural definition, since the implicit definition introduces a mathematical structure. However, in our case, the structural definition (in the mathematical sense) establishes the structure of physical systems. Thus, it is a very unique kind of definition, one in which the mathematical structure of physical structure is laid out. Henceforth, I shall refer to "structural definition" in the restricted physical sense introduced here.

The above structural definition, I believe, may also settle disputes in the formal theory of mereology. For example, some argue whether the following additional axiom should complete the basic mereological system **M**:

P 4. $\sim Pxy = (\exists z)(Pzy \& \sim Ozy)$

This axiom asserts that if x is not a part of y, then y has a remainder which does not overlap with x. This axiom is called the *strong supplementation* of basic mereology and the resultant mereological system is labeled extensional mereology or **EM** for short. The controversy with axiom **P4** is that one of the theorems of **EM** asserts that two objects which have identical proper parts are identical. That is, the identities of the proper parts completely determine the identity of the composite object and there is no "remainder" when one takes into account all the proper parts of a system. Not all philosophers accept the *supplementation principle* since one often has the intuition that a composite object is distinct from the aggregate of its material components, and is not completely determined by them. This intuition comes about when one considers the notion of a biological organism, which retains its identity even if the material components are replaced over time. It is then tempting to say that there must be a remainder to the material constitution of an object, which perhaps can be associated with the "form" of the organism or its principle of organization. In the context of physical systems, one can accept the strong supplementation principle, and the claim that there is no "remainder" to the proper parts of which the composite system is comprised. One can still accommodate this intuition *and* the notion that the physical system may remain the same while its material constituents are replaced.

It is important to distinguish between two aspects of the part-whole relation, as is made clear in structural definitions. On the one hand, a consequence of the above structural definition is that the property of a composite system is determined by the properties of the proper parts. This notion is underwritten by the Rule of Composition, which assembles the momentum and energy of the composite system from the momentums and energies of the components. But *whether* an object is a proper part of a physical system depends on a property belonging to the composite system as a whole – in particular, it depends on whether the composite system is isolated. If bodies x and y form an isolated system at time t_1, and bodies y and z form an isolated system at time t_2, then body y is part of two different composite systems at t_1 and t_2. These composite systems are distinct from the aggregate of x, y, and z, therefore they are conceptually distinct from their material components. But this does not undermine the view which takes the properties of the composite systems at each time t_1 and t_2 to be completely determined by the properties of the components: and that two composite systems that have the same parts are identical to each other.

1.5 Conclusion

In this chapter I introduced the notion of a structural definition that is valid a priori, whenever individual experiments are carried out. This notion was used to make explicit the structural assumptions implicit in the laws of momentum and energy conservation. I argued that one can reduce the structural definition into two assumptions, the Criterion of Isolation and the Rule of Composition, and claimed that a similar structural definition exists for the conservation of four-momentum in the Special Theory of Relativity.

The image the reader should keep in mind in what follows is that of a physical system whose description is given at the level of both parts and whole. The only states one should include are the states of motion of the parts and the state of motion of the composite. The next chapters will demonstrate that the various structures of Newtonian physics and the Special Theory of Relativity stem from various ways in which the motion of parts are related to the motion of the whole. This is the basic structure, and we do not assume that there is a more fundamental structure in which these motions are embedded.

Chapter 2
Interpretations of Spacetime and the Principle of Relativity

Chapter 3 will introduce an interpretation of flat spacetime theories I call *Primitive Motion Relationalism* (PMR). According to this interpretation, motion should be thought of as a primitive entity, more fundamental than spatial points and temporal instants. Events are taken to be coincidences between motions; the identity of events depends on the identity of the underlying motions. The other main feature of this approach is that spacetime consists of a set of *potential* trajectories. The spacetime manifold and the metric defined on it should not be thought of as a field analogous to other material fields. Rather, spacetime determines the form of actual trajectories and relations between motions, in analogy with Aristotelian formal causes that determine the essence of a substance. One of the main advantages of PMR is that it explains the restricted Principle of Relativity (i.e., the equivalence between inertial reference frames), without presupposing the Principle of Relativity as a postulate.

To differentiate PMR from standard accounts of spacetime, this chapter delineates some of the dominant interpretations of spacetime. The chapter outlines three common approaches: the conventionalist, the geometric, and the dynamic interpretations of spacetime. While the conventionalist account is mostly out of favor today, the geometric interpretation is the accepted doctrine. Dynamic accounts constitute a minority view that wishes to undermine official doctrine. Each account of spacetime has important advantages, however each also carries some weaknesses and liabilities.

Primitive Motion Relationalism has affinities with the dynamic approach, as it does not suppose the independent existence of spacetime. Like dynamical relationalism, PMR does not separate between dynamic and kinematic aspects of physical knowledge. However, PMR also has affinities with the geometric approach, since it attempts to provide a unifying account for spacetime symmetries, while current versions of dynamical relationalism do not seek to do so.

I will restrict my attention to flat spacetimes, and will leave discussion of curved spacetime for future work. PMR argues that spacetime constitutes a range of possible trajectories, and is not itself an actualized entity. This possibilist conception of spacetime faces difficulties in the context of the General Theory of Relativity. If spacetime consists of a range of possible trajectories, it is difficult to conceive of spacetime as a contingent structure that is determined by how matter is actually

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distributed. While I do not think this problem is beyond resolution, I shall not consider it here and instead focus on the positive reasons for believing in PMR. The possibilist conception of spacetime and the fundamental nature of motion helps explain the restricted Principle of Relativity. Thus PMR has an important advantage over traditional approaches that either assume the Principle of Relativity as a postulate or provide an inadequate explanation for it. To demonstrate the benefits of adopting PMR, I devote this chapter to a brief and sketchy assessment of existing interpretations of spacetime. Section 2.1 will introduce the restricted Principle of Relativity and some recent discussions regarding its appropriate interpretation. I shall then consider how each of the three common approaches, i.e., the conventionalist (Section 2.2), geometric (Section 2.3) and dynamic (Section 2.4) interpretations of spacetime, accommodates the Principle of Relativity. I shall note what I think is the central weakness in each account, in this way preparing the way for evaluating the merits of PMR.

2.1 The Restricted Principle of Relativity

Einstein introduced the Principle of Relativity as a postulate of the theory of relativity. According to Einstein, the restricted Principle of Relativity, which is the equivalence between inertial reference frames moving with uniform rectilinear motions, is modeled after the "classical" Principle of Relativity, which is articulated in Corollary V to Proposition I in Newton's *Principia*. The restricted Principle of Relativity is to be distinguished from the General Principle of Relativity, which according to Einstein amounts to the covariance of equations of motions under general coordinate transformations.[1]

Einstein argued that the restricted Principle of Relativity and the Light Postulate have an empirical basis. The laws describing Newtonian mechanics and electrodynamics do not include any property that makes reference to absolute rest. But in Einstein's theory, the Principle of Relativity assumes the status of a postulate, which he explicates as follows:

> **Einstein's Principle of Relativity**. The laws by which the states of physical systems undergo change are not affected, whether these changes of state be referred to the one or the other of two systems of coordinates in uniform translatory motion. (Einstein, 1952, p. 41)

According to Einstein's Principle of Relativity, the laws are the same whether they are defined relative to one coordinate system, or to a coordinate system moving with uniform rectilinear motion relative to the first. A coordinate system assigns a 4-tuple x^μ to any event that takes place, where $\mu = 0, 1, 2, 3$, and comes equipped with a set

[1] This interpretation of the General Principle of Relativity was later contested by many physicists and philosophers, since the General Principle of Relativity is violated by some theories of spacetime that are nevertheless invariant under general coordinate transformations. I shall not pursue this interpretive problem here.

of measuring rods and clocks that are relatively at rest.[2] His theory of relativity, he claims, is about the relations between these measuring devices and electromagnetic processes.

Einstein's account of coordinate systems begs for further explanation. Clocks and rigid rods are macroscopic systems, and a clock that measures time at a particular infinitesimal point can only be a highly abstracted idealization. Nevertheless, the benefit of this idealization is that it provides Einstein with a conceptual tool for grounding his Principle of Relativity. One first imagines a coordinate system K consisting of a set of clocks and rigid rods that are relatively at rest. One then imagines another coordinate system K', whose clocks and rigid rods are relatively at rest, but move with uniform rectilinear motion relative to the clocks and rods at K. The origin in the coordinate system K', described with the 4-tuple x'^{μ} moves uniformly in a straight line in the coordinate system $K = x^{\mu}$. Thus $x^{\mu} =< \alpha, 0, 0, 0 >$ coincides with $x'^{\mu} =< \beta, -v_1\beta, -v_2\beta, -v_3\beta >$ (assuming that their origins coincide at $x^0 = x'^0 = 0$). Once coordinate systems are given, the Principle of Relativity can be articulated. Any laws describing changes in states of a physical system in K will be the same in K'.

The Principle of Relativity describes an isomorphism between laws articulated relative to different coordinate systems. However, Einstein also used the Principle of Relativity in deriving the Lorentz transformations, which are laws that transform *between* measurements performed relative to different coordinate systems. In the first step of his argument, Einstein derives generalized Lorentz transformations from the light Postulate, assuming that the translatory motion of K' is in the x_1 direction[3]:

$$
\begin{aligned}
x'^0 &= \phi(v)\gamma\left(x^0 - \frac{v^2}{c}x^1\right) \\
x'^1 &= \phi(v)\gamma\left(x^1 - vx^0\right) \\
x'^2 &= \phi(v)x^2 \\
x'^3 &= \phi(v)x^3
\end{aligned}
\tag{2.1}
$$

where $\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$ and $\phi(v)$ is an unknown function of the relative velocities between the frames. To reduce (2.1) to the standard Lorentz transformations, Einstein looks at the transformation between K' and K'' that moves with a velocity $-v$ relative to K' (so that K'' and K are at rest relative to each other). Let a rod of length 1 lie on the x^1 axis of system K'. In the frame K, the length of this rod will appear

[2] A tacit assumption is that if rods in one coordinate system are boosted until they moved with uniform rectilinear motion relative to the first system, they would represent the same length in the new coordinate system. Similarly, the implicit assumption is that boosted clocks would also represent the same unit of time in the moving frame. See Brown (2005, p. 81) and footnote 41 for supporting quotations.

[3] In the following I replace Einstein's notation with the more readable modern notation.

contracted by a factor of $\frac{\phi(v)}{\gamma}$. In the frame K'' the same rod will appear contracted by a factor of $\frac{\phi(-v)}{\gamma}$. Thus, the rod in K'' will appear contracted in K by a factor of $\frac{\phi(v)}{\phi(-v)}$. Given the symmetry of the situation, Einstein argues, the transformations $\Lambda : K \mapsto K'$ and $\Lambda' : K' \mapsto K''$ should look exactly the same, so one should conclude that

$$\phi(v)\phi(-v) = 1 \tag{2.2}$$

Einstein argues that since K and K'' are in fact at rest relative to each other, the transformations $\Lambda : K \mapsto K'$ and $\Lambda' : K' \mapsto K''$ should also be considered as inverse transformations, so that $\Lambda'(-v) = \Lambda^{-1}(v)$. It therefore follows that $\phi(v) = \phi(-v) = 1$, since otherwise the contraction parameter for the rods will depend on factors other than the relative velocities between frames. From this Einstein concludes that the generalized Lorentz transformations reduce to the restricted Lorentz transformations[4]:

$$\begin{aligned} x'^0 &= \gamma\left(x^0 - \frac{v^2}{c}x^1\right) \\ x'^1 &= \gamma\left(x^1 - vx^0\right) \\ x'^2 &= x^2 \\ x'^3 &= x^3 \end{aligned} \tag{2.3}$$

The notion is that the Lorentz transformations should form a group, and so the equality $\Lambda(v) = \Lambda^{-1}(-v)$ is referred to as the "Reciprocity Principle" by some commentators. But in asserting that the Lorentz transformations should conform to a group structure, Einstein in effect applies the Principle of Relativity to laws transforming *between* coordinate systems. As it is articulated, the Principle of Relativity describes an isomorphism between laws articulated relative to different frames; Einstein's application of the principle, in deriving the Lorentz transformations, is to the coordinate systems themselves. It is at least logically possible to separate the laws articulated in each frame from the laws transforming between them. One could, for example, assume that the frame K'' is not the same as K.

[4] Einstein justifies this conclusion as follows:

> From reasons of symmetry it is now evident that the length of a given rod moving perpendicularly to its axis, measured in the stationary system, must depend only on the velocity and not on the direction and the sense of the motion. The length of the moving rod measured in the stationary system does not change, therefore, if v and $-v$ are interchanged. Hence follows that $\frac{l}{\phi(v)} = \frac{l}{\phi(-v)}$ and
>
> $$\phi(v) = \phi(-v) = 1$$

Einstein uses the Principle of Relativity to justify the Reciprocity Principle, or the notion that rods and clocks are warped only as a result of the relative velocity boosts they experience. But it is not clear why the violation of the Reciprocity Principle is a violation of the Principle of Relativity. According to the Reciprocity Principle, it is supposed that if one has two sets of rods and clocks that are at rest relative to each other, they both provide the same "natural" units of length and time relative to clocks and rods at rest in another inertial reference frame. However, one could imagine, for example, rods and clocks made of different substances; one kind of substance would be appropriate for measuring the length in K, and one would be appropriate for measuring the length in K''. Or, one could imagine different procedures by which clocks and rods in each frame were prepared. These two coordinate systems would have different units of length that are natural to them, even though they are at rest relative to each other. Because both K and K'' are coordinate systems, the fact that they are mutually at rest does not in itself violate the Principle of Relativity, since all one needs is for the laws of nature to be the same in K and K'' for the Principle of Relativity to hold. These considerations have led some to include the Reciprocity Principle as a separate axiom of relativity theory and to avoid appealing to the Principle of Relativity in justifying the restricted Lorentz transformations, given that the Principle of Relativity is only articulated for dynamic laws defined *relative* to frames (see, for example, Madarász et al., 2007).[5]

The application of the Principle of Relativity to the derivation of the specialized Lorentz transformations is logically independent of the isomorphism between laws articulated relative to different frames; one is a symmetry governing transformations between frames, the other is an isomorphism between laws articulated in different frames.

Einstein's derivation makes it seem as if the kinematical results of relativity theory are directly derived from Einstein's two postulates. When writing about the status of these postulates Einstein often compared them to the postulates of thermodynamics, suggesting that STR is a "principle theory" rather than a "constructive theory."[6] Constructive theories begin from a relatively simple formal scheme and construct from these elementary components the complex phenomena. For example, in the kinetic theory of gases, the macroscopic states of gases are constructed from the microscopic states of molecules and the laws governing these microscopic states. On the other hand, principle theories begin from empirically discovered principles, which describe general characteristics of natural phenomena. These principles give rise to criteria which the separate processes have to satisfy. Einstein provides

[5] Brown and Sypel (1995) argue that Einstein's application of the Relativity Principle to derive the Lorentz transformations should not be surprising, since "the rods and clocks are themselves to be viewed not as primitive, structureless objects, but as solutions of the basic equations, treating clocks and rods as composite entities whose parts are governed by dynamic forces." Thus given a dynamic approach to spacetime, Einstein's application of the Principle of Relativity in this context is unproblematic. I shall consider dynamic approaches and their problems in Section 2.4.

[6] See Brown (2005, section 5.2).

thermodynamics as an example of a principle theory and places STR in the same category.

The distinction between principle and constructive theories is important in this context since it seems to provide an account of the kinematical results of STR. While one does not have a complete constructive account of composite structures such as clocks and rods in STR, Einstein argues, one can derive their behavior from broad phenomenological principles like the Light Principle and the Principle of Relativity. However, it is not clear, given that the Principle of Relativity itself seems to presuppose the existence of coordinate systems, how it applies to clocks and rods themselves. Either the presupposition is that an underlying dynamic account of clocks and rods is yet to be supplied, in which case the Principle of Relativity would be articulated as a symmetry of underlying dynamic laws that makes no reference to coordinate systems. Or, there is a constructive account of a different kind, perhaps involving a geometric account of manifolds, that should replace the phenomenological principle. In either case, the Principle Theory account of kinematic effects of relativity seems incomplete. The analogy to thermodynamic theory is illustrative. While it is legitimate to think of thermodynamic theory as a successful theory, the kinetic theory of gases does complete the thermodynamic theory and "grounds" it in fundamental facts.

One may feel as if a genuine constructive account is required for kinematics. For example, a possible explanation for length contraction may be that moving rods experience various forces due to their motion. These forces are velocity-dependent and they affect all rigid rods in the same way. Similarly, a moving clock will experience forces that make its parts move more slowly. The initial attempts to explain the Michelson-Morley experiment involved an interaction between the ether and charged bodies. Ether theorists such as Lorentz (1881, 1904) hypothesized that rigid bodies undergo some contraction after they are accelerated to some motion. While these proposals for a dynamic account of clocks and rods appear to violate the Principle of Relativity, it is not necessary that they do so. As Brown (2005, chapter 4) points out, one can hypothesize that these forces arise in proportion to the velocity of an object relative to an inertial reference frame, in this way keeping in tact the Principle of Relativity.

Einstein's account of length contraction seems to supersede Lorentz's theory and other ether theorists, since it doesn't require an additional explanation over and above the axioms of relativity.[7] The notion that dynamic forces explain the

[7] It took a while until the philosophical community came to grips with the status of the Lorentz-Fitzgerald contraction hypothesis (LCH). Popper (2003, p. 62) argued that the LCH is an ad-hoc hypothesis, since the prediction of Maxwell's theory together with Newtonian mechanics regarding the motion through the ether was falsified by the Michelson-Morely experiment. The LCH was just introduced in order to avoid facing the falsification of accepted theories, and produced no new predictions. Grünbaum (1959) argued that the LCH in isolation was *falsified* by the Kennedy-Thorndike experiment. In this experimental setup, the interferometer used was similar to that of Michelson and Morley's, only it had arms of different lengths. The difference in time between the two arms did not depend on the orientation of the interferometer. This shows that the LCH by itself

deformation of rigid rods and clocks is odd, given that they apply universally to all rigid rods and clocks, no matter what they are made of. But still, how does one justify the Reciprocity Principle if a constructive account of clocks and rods is not forthcoming?

So far I argued that it seems odd to subsume laws governing rods and clocks in different inertial reference frames under the same Principle of Relativity which governs dynamic laws. Another way to see the disparity between the two applications of the Principle of Relativity (the first to kinematic laws and the second to dynamic laws), is to consider the distinction between global and local versions of the Principle of Relativity. Einstein's formulation of the Principle is articulated for "changes in the state of physical systems." Einstein is implicitly referring to isolated systems, as the Principle of Relativity does not hold for a subsystem that interacts with another subsystem. However, it is logically possible for a universe to have indistinguishable dynamic models, but that the same symmetries do not apply locally to isolated subsystems of the universe.[8]

The global version of the Principle of Relativity is exemplified by Anderson (1967). Anderson articulates a precise formulation of the Principle of Relativity based on modern mathematical theories of differential geometry. According to Anderson's approach, a physical theory consists of classes of kinematically and dynamically allowed models. Each model M consists of a manifold with spacetime structures and matter fields:

$$\langle M, O_1, \ldots, O_n \rangle$$

Symmetry principles can now be introduced via the notion that various models are acceptable kinematic and dynamic models of the theory. Thus, the covariance of the theory is given via the equivalence between one model and another model where a diffeomorphism $d : M \mapsto M$ acts on the manifold, so that if $\langle M, O_1, \ldots, O_n \rangle$ is a model of the theory and $d*$ is the drag on the objects of the theory, then so is $\langle M, d * O_1, \ldots, d * O_n \rangle$. One can distinguish between spacetime symmetries and dynamic symmetries by considering whether any diffeomorphism of the manifold leaves the geometric structures invariant, so that the models $\langle M, A_1, \ldots, A_n \rangle$ and $\langle M, d * A_1, \ldots, d * A_n \rangle$ remain the same, i.e. $d * A_i = A_i$. The objects A_i would then be the absolute (geometric) objects of the theory. A dynamic symmetry is a diffeomorphism which renders the shifted dynamic model acceptable, so that $\langle M, A_1, \ldots, A_n, D_1, \ldots, D_n \rangle$ and $\langle M, A_1, \ldots, A_n, d * D_1, \ldots, d * D_n \rangle$ are both dynamically acceptable.

Anderson's account generates a natural connection between spacetime symmetries and the dynamic symmetries. On the one hand, one considers whether a certain transformation leaves a geometric object invariant, and then considers whether the

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is not sufficient to cohere with the data, and one needs to assume time dilation in addition to length contraction. See Grünbaum (1959), Evans (1969), and Erlichson (1971).

[8] See Treder (1970, p. 86), Brown and Sypel (1995), and Budden (1997).

same transformation renders acceptable the transformed dynamic objects. Assume that one defines frames on the manifold using a set of parallel time-like straight worldlines. Assume that there is a group of transformations, for example, the Lorentz transformations, for which each element of the group transforms one frame to another. One may think of the global Principle of Relativity as the notion that the group of transformations leaves the absolute objects of the theory invariant (in which case the group is a spacetime symmetry), or it leaves the dynamic objects of the theory invariant (in which case the group is a dynamic symmetry).

In this formulation of the Principle of Relativity, a direct connection appears to have been made between the spacetime symmetries and the dynamic symmetries. However, when the Principle of Relativity is formulated globally in this way, it is an inter-world principle describing the equivalence between various possible worlds. The global Principle of Relativity is not an inter-world symmetry between frames. Assume we have frames K_1 and K_2 in world w_1, and frames K_1' and K_2' in world w_2. If the worlds w_1 and w_2 are related through a kinematic symmetry, the frames K_1 and K_2 are mapped onto K_1' and K_2' via the spacetime symmetry. If the symmetry is also that of the dynamic objects of the theory, the results of experiments performed in K_1 are indistinguishable from experiments performed in K_1'. Moreover, experiments in K_2 are indistinguishable from experiments in K_2'. But this does not imply an intra-world equivalence between K_1 and K_2 or K_1' and K_2'. The global Principle of Relativity does not imply the local Relativity Principle, which is an intra-world relation between frames (or the equivalence between laws applied to isolated systems).

Budden (1997) provides an example of a theory that many take as satisfying the global but not the local Principle of Relativity. Budden introduces a couple of theories with anisotropic spacetime that are analogous to the isotropic Minkowski spacetime. The theory includes the structures $\langle \mathbb{R}^4, \lambda, N \rangle$, where $\mathbb{R}^4$ is the manifold, λ is the relation of lightlike connectibility, and N is a set of parallel lines in the spacetime, which amounts to a preferred frame. In one of Budden's theories, inertial clocks do not obey the Lorentz dilation factor. Instead, the theory defines a temporal congruence relation $\sim_1$, such that $ab \sim_1 ac$, if and only if b and c lie in one of the null hyperplanes picked out by N. Thus, Budden's theory picks a preferred direction in spacetime (i.e., a frame), and defines anisotropic congruence relations on spacetime intervals relative to this frame. The congruence relation does not define a unit of time, which implies that the clocks do not obey the Reciprocity Principle between frames. Budden's theory enables one to define a dilation effect between the preferred frame and other frames:

$$D = D(1 + v)^{-1} \tag{2.4}$$

where v is the relative velocities between the frames. In all other respects, the anisotropic theory satisfies the global symmetries of the theory of relativity, since a transformation from one possible world with a preferred frame to another will yield indistinguishable dynamic models. But clocks behave differently than in relativity, temporal measures sometimes dilating and sometimes contracting depending on the relative velocities between the preferred frame and the frame boosted relative to it.

Thus, Budden's theory shows that one can have both a global kinematic and dynamic symmetry, e.g., Lorentz covariance, without having local kinematic symmetry. In Budden's theory, dynamic laws relative to one inertial reference frame are isomorphic to dynamic laws of the frame which has actively been transformed under the Lorentz transformations. But time measurements in different frames do not obey the Lorentz transformations.

Recently Skow (2008) argued that Budden's anisotropic theory fails to demonstrate that the global Principle of Relativity does not imply the local Principle of Relativity. According to Skow, the anisotropy of Budden's spacetime implies that temporal processes in a non-preferred frame are correlated with remote temporal processes on the preferred frame. This somehow suggests that systems in the non-preferred frame that appear to be isolated are not genuinely isolated. Given that their evolution in time depends on its relation to the preferred frame, one cannot take them to be isolated. There must be some non-local interaction to account for Budden's non-isotropic spacetime, perhaps mediated by spacetime itself, between systems residing in the non-preferred frame on the one hand and systems residing in the preferred frame on the other hand. Thus, what looks like a violation of the local Principle of Relativity, is not genuinely a violation of the principle, because in an anisotropic spacetime one cannot truly isolate a system from another system if a preferred direction of spacetime is given.

Perhaps what is at stake is a distinction that commentators fail to make, and that is the distinction between the failure of the Reciprocity Principle (that kinematic contraction and dilation effects only depend on the relation between frames, and not on the existence of a preferred frame), and the failure of the local Principle of Relativity (according to which the same dynamic laws apply to all isolated systems). Einstein argued that the Reciprocity Principle follows from the local Principle of Relativity. Given Einstein's claim that the local Principle of Relativity implies the Reciprocity Principle, one is tempted to take Budden's theory as a violation of the local Principle of Relativity. Skow is correct to argue that the failure of Reciprocity Principle does not imply the failure of the local Principle of Relativity. Skow's argument amounts to the claim that when the Reciprocity Principle fails, the local Principle of Relativity is not shown to be false, because somehow an anisotropic spacetime suggests that no system can be genuinely isolated from another. And if no system can be isolated, one cannot argue that isolated systems in different frames disobey the local Principle of Relativity. But the correct response to Budden's theory is that the Reciprocity Principle and the local Principle of Relativity are logically distinct, and the local Principle of Relativity could still hold despite the failure of the Reciprocity Principle.

To summarize the above discussion, despite Einstein's claim, there is no direct logical connection between the restricted Principle of Relativity, and the Reciprocity Principle. While Einstein attempts to derive the relation $\Lambda'(v) = \Lambda^{-1}(-v)$ from the Principle of Relativity, he is not justified in doing so. In recent decades work has been done to demonstrate the gap between global, or purely geometric accounts of the Principle of Relativity, and the local Principle of Relativity, which asserts the dynamic equivalence between isolated systems. While this work is correct to point out the gap between global and local symmetries, much of it confuses the failure of the Reciprocity Principle with the failure of the local Principle of Relativity. What these arguments in fact point to is the distinction between the Reciprocity Principle and the Principle of Relativity.

2.2 Conventionalism

The conventionalist approach to spacetime was espoused by Einstein in his early work and by philosophers such as Poincaré (1905), Reichenbach (1927, 1969), Carnap (1937), Schlick (1920), and Grünbaum (1963).

One may detect a variety of conventionalists accounts, ranging from fairly modest claims about theories being underdetermined by the phenomena to radical claims about any axiomatic system amounting to an arbitrary linguistic construct. Poincaré, for example, famously believed that the nature of space gives rise to an underdetermined geometric structure. According to Poincaré, the choice between Euclidean, hyperbolic and spherical geometries is underdetermined by our measurements of spatial relations. The implication is that the axioms of geometry are neither empirical claims – since they are underdetermined by observations – nor are they necessary claims – since alternative axiomatizations of space can be given. According to Poincaré, there is a separate category for propositions that are neither empirical nor a priori or necessarily valid; these would be conventions. However, according to Poincaré the conventional nature of geometry does not extend to other branches of mathematics; the nature of arithmetics does not lend itself to a conventional choice about the axioms. Thus Poincaré's conventionalism is fairly conservative in its scope.

Another version of conventionalism is neo-Kantian Conventionalism. According to this version, the interpretation of experience requires that our minds impose a rational form on intuitions. Kant believed that the rational form of intuition is necessary, but the neo-Kantian school allowed for those forms to change over time. In his earlier work, published in 1920 and entitled *The Theory of Relativity and A priori Knowledge* (1920), Reichenbach attempted to reconcile the insights of Kantian epistemology with the lessons of relativity theory. Reichenbach argued that some Kantian principles could be preserved in the light of the new theory, and could illuminate the epistemological nature of the theory; but other principles need to be revised. He argued, for example, that one could give a Kantian account of the relativistic principle that the speed of light provides an upper limit to all physical velocities. He claimed that there being an upper limit to the velocity of causal signals is a consequence of the principle of no action at a distance. The locality of causal action is itself a consequence of Kant's a priori principle of permanence of substance (Reichenbach, 1920, p. 12). On the other hand, some of Kant's claims about certain propositions being a priori must be given up in the light of relativity theory, including the absolute nature of time and the Euclidean character of space.

Reichenbach's earlier work therefore attempts to salvage some of Kant's claims. However, beyond accepting certain Kantian principles as being valid a priori, and rejecting others as invalid, Reichenbach also revised Kant's notion of a priori itself. Kant equated a priori judgments with necessary truths. Reichenbach felt that the advent of relativity theory clearly demonstrated that a priori propositions are not necessary. Some principles, like the absolute nature of time, were held to be valid a priori in Newtonian physics, but were then taken to be false in the theory of relativity. However, Reichenbach argued, a certain interpretation of the a priori can be salvaged, as long as a priori judgments are not equated with necessary truths. But knowledge presupposes the cognitive coordination of individual terms of the theory with individual objects of experience. This coordination depends on the definition of the concept of object given by cognition. Without the concept of object, cognition would not be able to interpret intuitions, so that intuitions must have rational form. The philosophical problem is to identify those cognitive coordinations that are *unique*, so that no contradiction arises between theory and experience (1920, p. 47).

Reichenbach calls the principles of coordination that constitute the concept of the object *Axioms of Coordination*. Examples for *Axioms of Coordination* are the axioms of arithmetics with which the concept of a mathematical vector is defined. Without the mathematical theory of vectors, physical forces could not be conceptualized or identified in experience. Another *Axiom of Coordination* is the principle of genidentity, according to which the trajectory of a particle determines its identity. Euclidean geometry functions as an *Axiom of Coordination* in Newtonian physics, since it is not possible to identify in experience Newtonian physical objects without assuming Euclidean geometry as valid. But it is no longer an *Axiom of Coordination* in the General Theory of Relativity. *Axioms of Connection*, on the other hand, describe empirical connections between terms in the theory.

Axioms of Coordination have an a priori status in the theory, since without these axioms the theory is not able to receive empirical content. However, unlike Kant's synthetic a priori judgments, these axioms are not necessarily valid for all theories. When new theories are formulated, different *Axioms of Coordination* are used to bridge between the formal system and experience. Friedman (1991, 1999) has coined the term "relativized a priori" to convey the epistemic status of *Axioms of Coordination*. The a priori nature of certain propositions arises from the "constitutive" role of principles that bridge between theory and experience. Nevertheless, over time, these principles can be revised. Thus, these principles are only a priori relative to a particular theoretical context. Euclidean geometry, according to Reichenbach, is constitutive in the context of classical mechanics, but only topology is constitutive in the context of the general theory of relativity. When a new principle of coordination is introduced, discarded principles of coordination can be shown to be approximations of newer principles, so that new theories can be shown to be an improvement over the older, less adequate theories.

What distinguishes a principle of coordination from any other empirical proposition, is that one can detect an element of arbitrariness in the principle. "*The* *contribution of reason is not expressed by the fact that the system of coordination contains unchanging elements, but in the fact that arbitrary elements occur in the system*" (1920, p. 89). Thus, the contribution of reason is made felt, so to speak, through the existence of conventional systems of representation; all adequate for the representation of experience, but none preferable. According to Reichenbach, the Principle of Relativity, since it allows for a conventional choice of inertial reference frames, demonstrates the contribution of reason to the object of understanding:

> The theory of relativity teaches that the four space-time coordinates can be chosen arbitrarily, but that the ten metric function $g_{\mu\nu}$ may not be assumed arbitrarily; they have definite values for every choice of coordinates. Through this procedure, the subjective elements of knowledge are eliminated and its objective significance formulated independently of the special principles of coordinates. Just as the invariance with respect to transformations characterizes the objective nature of reality, the structure of reason expresses itself in the arbitrariness of admissible systems. (Reichenbach, 1920, p. 90)

Thus for Reichenbach, relativized a priori propositions introduce conventional, subjective elements to our theories. A convention is required because experience has to be understood via its rational form. Since this rational form is not necessary, one can bring to bear alternative rational forms to the same experience, introducing an element of arbitrariness to our representations.

But Reichenbach did not preserve his neo-Kantian view for long, and was influenced by Schlick to revise his account, thus forming a third kind of conventionalism of a positivist kind. Reichenbach slowly came to believe that Kant's notion of synthetic a priori no longer holds if experience is to determine which principles of coordination are acceptable. If the theory of relativity is empirically superior to Newtonian physics, one can no longer treat the discarded principles of Newtonian physics, and the accepted principles of the Theory of Relativity, as equally valid. Thus, they could no longer be taken as a priori in good faith. The consequence is a retreat to a Humean-like fork: propositions must strictly be divided into analytic a priori, and synthetic a posteriori propositions. Reichenbach was also influenced by Schlick, who objected to the notion of the object of understanding being constituted by reason and to the neo-Kantian form of idealism. According to Schlick (1920), one may think of the totality of physical facts as objective claims that are distinguished from subjective experiences. Physics in fact describes mind-independent reality. Nevertheless, it is also possible to indicate the same set of facts by means of various systems of judgment (Schlick, 1920, p. 86). Thus, one can articulate various theories expressing the same set of facts, such that the choice between these theories is a matter of convention. While there is an indefinite number of conventional representations, one may use simplicity as a criterion for selecting between the various systems of judgments, but there is nothing in reality to tie us down to a specific representation as the correct one.

Influenced by Schlick's positivist-conventionalism, Reichenbach (1927, 1969) revised his account of the a priori elements of theory. The cleavage between analytic a priori propositions and synthetic a posteriori claims demand that Reichenbach's *Axioms of Coordination* be reevaluated. In a work published in 1924, entitled *Axiomatization of the Theory of Relativity*, Reichenbach introduced his new account of principles of coordination. He distinguished between two aspects of coordination. First, there are the linguistic conventions and definitions of the theory. In this, Reichenbach and Schlick were following Poincaré's and Hilbert's notions of implicit definition. According to Poincaré (1905, p. 92), for example, the Law of Inertia is not necessarily true. However, it is not an empirical claim, either. The notion that a force-free body will move with uniform rectilinear motion is partially determined by the meaning of "force" (and vice versa). One may associate a force with a change in position, or a change in acceleration; both would allow one to articulate alternative laws of inertia. Nothing in the observations will imposes Newton's Law of Inertia. But once the Law of Inertia defines the force-free state of the particle as the uniform motion of an object, the Law of Inertia acquires the status of necessary truth within Newton's axiomatic system. Similar views about the Law of Inertia were held by Reichenbach (1927, p. 116) and Hanson (1965).

However, the more significant transition in Reichenbach's thinking is the revision of his account of coordination between theory and experience. Reichenbach is still arguing that for a mathematical theory to be applied to experience, individual terms of the theory must be coordinated with individuals of experience. However, now the coordination is not done from within the theory, and the concept of object is no longer determined by reason. Instead, the coordination between individual terms and individuals of experience is carried out via some experimental practice. The coordination of theory to experience is performed via what physicists do, not by the rational form of their intuitions. Thus Reichenbach is no longer speaking of *Axioms of Coordination*, but instead referring to them as coordinative definitions. In Reichenbach (1927), he articulates the notion of coordinative definition:

> The mathematical definition is a *conceptual definition*, that is, it clarifies the meaning of a concept by means of other concepts. The physical definition takes the meaning of the concept for granted and coordinates to it a physical thing; it is a *coordinative definition*. Physical definitions, therefore, consist in the coordination of a mathematical definition to "a piece of reality"; one might call them *real definitions*. The concept of a unit of length is a mathematical one; it asserts that a certain particular interval is to serve as a [standard of] comparison for all other intervals. From this nothing can be inferred, however, as to which physical interval is to serve as the unit of length. The latter is first accomplished by the coordinative definition which designates the Paris standard meter as the unit of length. In this physical definition, the mathematical definition of the concept is presupposed. (Reichenbach, 1969, p. 8)

According to Reichenbach, complementing the linguistic conventions are definitions that coordinate between linguistic terms and individual objects of experience. In the case of physical geometry, concepts such as "unit of length" and "unit of time" have to be coordinated with physical objects such as measuring rods and clocks used to generate the relevant measures of length and duration. It is not that reason constructs for us the object of experience, rather, the objects are simply pointed out ostensibly. A coordinate system therefore consists of a set of physical measuring rods and clocks that are relatively at rest, and functions in our scientific practice as a coordinative definition. The implication of Reichenbach's approach is that geometric axioms do not represent a single theory but a family of models. The symbols of the theory can be interpreted only after the coordinative definition is made (i.e., a coordinate system is selected) and the propositions of physical geometry gain physical (i.e., empirical) content.

Reichenbach's transition from neo-Kantian to Positivist Conventionalism carries important epistemological consequences. In the neo-Kantian account, the Principle of Relativity marks the constitutive role of reason in constructing the concept of object. In the positivist account, the Principle of Relativity describes an isomorphism between the various models of the theory. In the positivist account, each model of the theory relies on a different coordinative definition. There is a crucial difference between an interpreted formal theory, and its interpretation after the individual terms are taken to represent specific objects. The formal theory is only a symbolic, syntactic structure, lacking any empirical or physical content. The interpreted theory has empirical or physical content in virtue of the objects to which the formal theory is referring. That is why logical positivists distinguish between formal and physical geometry. Formal geometry is a mathematical structure; the propositions of physical geometry are about real, physical objects.

The upshot of logical positivism is that two different interpretations of a formal theory include propositions that are not directly comparable. For example, one could conceive of Hilbert's axiomatization of Euclidean geometry as a formal theory that is then given different interpretations. In one interpretation of the theory, the notion of a "point" could be taken to represent a point in physical space, and the notion of a "line" could be taken to represent a line in physical space. But the formal terms could be interpreted differently. For example, the notion of a "point" could be taken to represent a line in physical space, and the notion of a "line" could be taken to represent a point in physical space, leading to what is known as projective geometry. The consequence is that claims made within the context of one interpretation are not comparable or are not commensurable with claims made in the context of another interpretation, despite them being interpretations of a single theory. The physical content of these claims crucially depends on the coordinative definition, or on the interpretation of the formal system.

The logical positivists argue that Einstein's restricted Principle of Relativity is a prime and central example of their epistemology. Einstein's 1905 paper seems to follow conventionalist epistemology. The two postulates of the theory of relativity, the Principle of Relativity and the Light Postulate, provide the axioms of the theory. From these axioms one can deduce the theorems of relativity. In addition to the pure axiomatic system, the applied theory uses coordinate systems to bridge between the abstract conceptual system and physical experiences. Each inertial reference frame provides a particular interpretation of the theory of relativity. The frame coordinates between abstract concepts of "unit of length" and "unit of duration" and the physical objects used as standards of length and duration in experiments (i.e., rigid measuring rods and clocks).

According to the Principle of Relativity, whether a law is articulated relative to a coordinate system that is at rest, or whether it is articulated relative to a system that moves with uniform rectilinear motion relative to the first coordinate system, it assumes the exact same form. If one is reading into the Principle of Relativity a positivist epistemology, one is asserting the equivalence between different models of the theory. Each coordinate system is in effect a different interpretation of the formal system. If a certain coordinate system K is given, the term "unit of length" is coordinated with a unit of length marked on the measuring rods of K. One does not have to have the actual rods placed in the coordinate system, but at least in principle, for each unit of length in the relativistic geometry there should be a rod corresponding to it. Similarly, the term "unit of time" in the formal theory is coordinated with a period of a clock. The set of rods and clocks that are at relative rest comprise the coordinate system.

However, the logical positivist interpretation for the Principle of Relativity is untenable. Assume that one has at his disposal two coordinate systems K and K'. One can choose to interpret the formal geometry by taking its terms to refer to rods and clocks in K, or one can take them to refer to rods and clocks in K'. In each case, a different interpretation of the formal theory is provided. If the positivists take seriously the notion that physical geometry receives content only once the coordinative definition is made, then different coordinate systems in effect introduce alternative interpretations of the formal theory.

Given such a positivist gloss, the Principle of Relativity ceases to be a theoretical proposition. The principle asserts that different interpretations, stemming from different coordinate systems, produce the same dynamic laws. This type of equivalence is not a *theoretical* equivalence, because the different interpretations are not directly comparable. At most one can say that the different models are *empirically equivalent*, in that the same measurements will be predicted by each model. The upshot is that the Principle of Relativity is not itself an empirical claim within a particular model, but a meta-theoretical principle for constructing models. This implication of the positivist epistemology is not always emphasized or even recognized by positivist philosophers, since it is always stressed that the origin of the Principle of Relativity is empirical. Einstein argues that no physical theory ever makes reference to anything but relative velocities, which seems to provide the principle with great empirical support. But given the positivist reading, it may be that the Principle of Relativity received much empirical support, but the Principle itself is not a theoretical or a directly empirical claim. The Principle of Relativity asserts an isomorphism between different models of the theory, but is not itself part of the theory. Carnap (1937) seems to recognize the implication and treats the Principle of Relativity itself as a convention *about* the syntactical rules of the language of physics (see p. 328).

The problem with the positivist-conventionalist account of the Principle of Relativity is that the principle seems to be justified by empirical and theoretical considerations, but the principle itself cannot have any physical or empirical content. There seems to be an incongruity between the *justification* the Principle of Relativity receives from experiments and from theoretical considerations (these include, for example, the unification of the electric and magnetic fields), and the epistemic role the principle receives within the positivist account. If it is possible for the Principle of Relativity to receive empirical and theoretical justification, it seems odd to assert that the principle itself has no empirical or physical content. If the principle has no empirical and physical content, then all the justification it receives is irrelevant to its stipulated truth. If the Principle of Relativity is a mere convention, a meta-theoretical principle for the construction of models, then no justification for the principle is necessary.

The positivist-conventionalist seems untenable when nature of equivalence between models of the theory is considered. While the different models produce isomorphic structures, the equivalence between models is strictly *empirical*, i.e., there is no underlying theory that unifies the models into one overall interpreted theory. But if all models of the theory are isomorphic to one another, there is reason to suspect that they consist of different representations of the same world state. It would just be an enormous coincidence to have such a deep symmetry governing models which are based on conventional coordinative definitions. Friedman (1983, pp. 277–94) argues that the replacement of empirically equivalent models with equivalent representations is the process by which the vocabulary of a theory is revised in order to decrease the conventional and arbitrary elements of a theory. One may think of the Minkowski's approach to spacetime as an attempt to do just that, i.e., to provide a four-dimensional geometric structure that unifies the various models of relativistic spacetime into one overall theory.

A similar development concerned the conventionalist interpretation of the Principle of Equivalence in the context of the General Theory of Relativity. At first, conventionalists viewed Einstein's Principle of Equivalence as proof of the conventionalist thesis. According to Einstein, the equality of inertial and gravitational mass implies the equivalence between two descriptions; one is a frame of reference at rest with a uniform gravitational force, the other is a frame of reference accelerating uniformly but experiencing no gravitational force. Conventionalists argue that the choice between the two descriptions is conventional. However, as later interpreters have realized, the equivalence is only valid locally, and Einstein's incorporation of the gravitational force into the spacetime metric does not allow the global elimination of gravitational effects. Thus, in Einstein's theory there is no longer a conventional choice about the nature of gravitation, and the gravitational field could not be dispensed with throughout spacetime or be treated as an eliminable universal force.

While the conventionalist approach to spacetime is mostly out of favor today, the program for axiomatizing the theory of relativity is still in full force.[9] According to this approach, one can gain insight to the foundations of a physical theory by reconstructing it within first order logic. Those who endorse this approach do not always insist on the conventionalist nature of the axioms of spacetime theory (by putting emphasis, for example, on the empirical origins of the Light Postulate). Neither do these modern revivals discuss the epistemological nature of different models of the theory. But they still view spacetime theory as a collection of empirically equivalent models. In Madarász et al. (2007), the project of constructing an first

[9] The program of axiomatizing spacetime theory has a long and interesting history. For a modern revitalization of the program see Andréka et al. (2006) and Madarász et al. (2007).

order logic axiomatization of relativity consists of constructing observable motions. A model of this theory includes a universe of bodies B and sets of four quantities Q describing the spatiotemporal location of bodies. But for the spatiotemporal locations to make sense, they must "belong" to a certain structure – the inertial system of an "observer," which is a particular kind of body. Thus, each model of the theory includes a "worldview," which is a six-place relation $W(o, b, x, y, z, t)$ stating that a body b has a spatial location x, t, z at time t in o's inertial reference frame. There are also special axioms fixing the motions of these observers. This approach to the axiomatization of relativity leads to various empirically equivalent models, depending on the "observer" or the inertial frame chosen as reference. The axiomatization of spacetime theory may be useful for gaining insight into the foundations of the theory.[10]

But such an axiomatization of spacetime remains an unsatisfying view of the nature of spacetime. As long as our axiomatization leads to empirically equivalent models, where each model is defined relative to a coordinate system or an "observer," the Principle of Relativity itself becomes a non-theoretical principle *about* models – asserting the equivalence between them. It may be that spacetime theory is inherently limited, and that it is simply not possible to articulate a spacetime theory that eliminates observers or coordinates systems from the foundations of spacetime theory. However, if there exists an interpretation of spacetime that is able to unify the empirically equivalent models, the positivist-conventionalist interpretation of the restricted Principle of Relativity should be abandoned.[11]

Positivist conventionalism was abandoned in the second half of the twentieth century, as a result of many factors. Some include post-positivist critiques by Quine and Kuhn, other factors perhaps include sociological ones. In any case it is apparent that Einstein's Principle of Relativity is not best understood by the positivist-conventionalist view. Some commentators, most notably Michael Friedman, recommend a return to the neo-Kantian views of early Reichenbach. But then it is not clear how one can retain the synthetic a priori nature of principles that are demonstrated to be empirically inadequate. Others attempt to revive the axiomatized approach to the theory of relativity, taking each observer to be constituting its own "worldview." These commentators seem to be unaware of the epistemological weakness faced by the early conventionalists of the positivist school who took each inertial reference frame to be a different model of the formal theory.

[10] For example, the analysis of the logical foundations of relativity theory shows that it is possible to replace the Principle of Relativity with a weaker axiom (i.e., that observers agree on which events take place). The price for weakening the relativity postulate is that it is necessary to add a reciprocity relation between frames, i.e., time dilation and length contraction must be the same for any boosted frame relative to the frame at rest.

[11] See Friedman (1983, chapter VII) for a similar complaint against conventionalism.

2.3 The Geometric Approach to Spacetime

The conventionalist account of spacetime theory has by and large been superseded by the geometric approach to spacetime (see, e.g., Minkowski, 1952; Earman and Friedman, 1973; Nerlich, 1979; Mellor, 1980; Healey, 1995; Balashov and Janssen, 2003; Baker, 2005). One can trace the origins of this approach to Minkowski's geometric formulation of the Special Theory of Relativity. The world, according to Minkowski, is a collection of worldpoints designated with a system of values x, y, z, t, where x, y, z are spatial coordinates and t is a temporal coordinate (Minkowski, 1952, p. 76). A body's motion through this world is described as a worldline; it is a set of worldpoints in which infinitesimal variations dx, dy, dz correspond to infinitesimal variations dt. Minkowski erects an analogy between the transformation group G_c that governs the spacetime structure of STR and the Euclidean group governing Euclidean space. The transformation group G_c contains the Euclidean group of rotations and translations in the spatial dimensions (i.e., the transformations leaving $x^2 + y^2 + z^2$ invariant). But the group G_c also includes velocity boosts that leave invariant the spacetime interval $ds^2 = c^2t^2 - x^2 - y^2 - z^2$. Minkowski argues that invariance in relation to G_c is a general principle governing natural phenomena, and raises this property to the status of world-postulate.

Minkowksi concludes that the main lesson of relativity is that space and time can no longer be taken to exist independently:

> We should then have in the world no longer *space*, but an infinite number of spaces, analogously as there are in three-dimensional space an infinite number of planes. Three dimensional geometry becomes a chapter in four-dimensional physics. Now you know why I said at the outset that space and time are to fade away into shadows, and only a world in itself will subsist. (Minkowski, 1952, p. 79)

Thus Minkowski believes that underlying the kinematics of relativity is an objective four-dimentional world with spacetime points and objective spacetime intervals defined between them. The Principle of Relativity is therefore not a phenomenological principle asserting an isomorphism between different models of the theory, but a geometric symmetry governing the underlying four-dimensional spacetime. Different inertial reference frames produce alternative representations of the same world-state.

Minkowski argues that his account of relativistic kinematics is more intelligible than that of Lorentz's dynamic account of length contraction. Imagine a rod at rest in x, y, z, t. Such a rod looks like a band (see Fig. 2.1). The length of the rod PP is l. If one looks at a rod moving with uniform rectilinear motion relative to the original, its band will look slanted in the Minkowski diagram. The length of this rod is also l, but this length is measured relative to a rotated axes x', y, z, t' so that $Q'Q' = l$. Since the length of this moving rod in x, y, z, t is the cross-section along the axes of x, y, z, t, its length is l/γ in this system.

According to Minkowski, there is no dynamic account of length contraction and time dilation; these phenomena result from rods and clocks conforming to a four-dimensional underlying geometry. Each inertial reference frame, because of

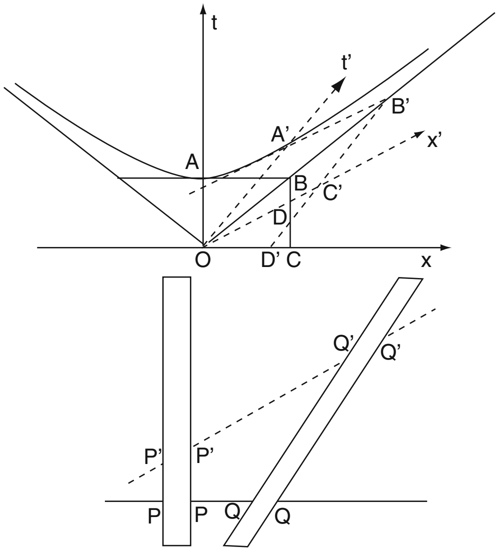

Fig. 2.1 Length contraction in Minkowski's geometric approach

its orientation in spacetime, introduces different standards of length and duration, thereby producing different representations of the same spatiotemporal intervals. Moving rods appear contracted in the rest frame, since one is not taking the set of events that are considered simultaneous along the moving reference frame as defining a unit of length in the rest frame. A cross-section of non-simultaneous events produced by the moving rod *is* shorter when measured in the rest frame.

In Minkowski's geometric approach, the lessons of relativity appear entirely different than Einstein's conventionalist-leaning remarks. According to this new approach, space and time have to be fused together to form a four-dimensional structure. Understanding that space is only a substructure of spacetime

> ... is indispensable for the true understanding of the group G_c, and when [this further step] has been taken, the word *relativity-postulate* for the requirement of an invariance with the group G_c seems to me very feeble. Since the postulate comes to mean that only the four-dimensional world in space and time is given by the phenomena, but that the projection in space and in time may still be taken with a certain degree of freedom, I prefer to call it the *postulate of the absolute world* (or briefly, the world-postulate). (Minkowski, 1952, p. 83)

Thus in Minkowski's account, the Principle of Relativity is not a phenomenological postulate of the theory, but a symmetry inherent in the geometric structure of a four-dimensional world. Rather than thinking of the Principle of Relativity as requiring an arbitrary coordinative definition, Minkowski thinks of it as a postulate of the absolute world, i.e., as a degree of freedom implicit in selecting arbitrary systems of reference used to construct representations of an objective world.

The view, which takes spacetime to describe an objective spacetime structure, has a philosophical advantage over conventionalist accounts. In the conventionalist account, different models of the theory are considered *empirically equivalent*. But in the geometric account, different representations of the spacetime are *theoretically equivalent*.

Minkowski's view of relativity came to dominate relativity textbooks and philosophical explications of relativity theory. It also came to dominate Einstein's own thinking while working on the General Theory of Relativity.

The trouble with the geometric approach is that it tempts us to think of spacetime as a material field that causally influences the behavior of bodies. Momentum and energy conservation laws assert that closed systems move along geodesics of the spacetime. However, what causes free particles and light rays to follow the geodesics of the spacetime? Since the path of a free particles is not caused by any other material body, the image of a four-dimensional manifold of events leads commentators to think of spacetime as the cause or the "origin" of kinematics. The view of spacetime as guiding inertial motions has its historical origins in Mach's critique of Newton's absolute space in *The Science of Mechanics*. Mach was concerned to show that Newton's bucket experiment does not necessarily rule out every possible definition of relative motion as responsible for inertial effects (Mach, 1893, p. 300). But his other argument against Newton's absolute space was that Newton *could have* posited the existence of a material medium, like the ether, which lies throughout space and directs bodies in their inertial motions (Mach, 1893, p. 282). Newton's concept of absolute space is a metaphysical notion, precisely because it is a physical entity that has no causal influence on material bodies.

Einstein came to think of spacetime in much the same way as Mach proposes. According to Einstein, spacetime should be thought of as an entity that *acts* on bodies as this structure determines the inertial behavior of bodies. The problem with Newton's mechanics and STR, according to Einstein, is that spacetime seems to act on matter without being acted upon in return. This "violation" of the principle of action equals reaction is corrected in The General Theory of Relativity where matter is said to be acting on spacetime by curving it. Thus, spacetime can be compared to an ether that causally influences the behavior of bodies:

> The inertia-producing property of this ether [Newtonian spacetime], in accordance with classical mechanics, is precisely *not* to be influenced, either by the configuration of matter, or by anything else. For this reason we may call it "absolute". That something real has to be conceived as the cause for the preference of an inertial system over a noninertial system is a fact that physicists have only come to understand in recent years . . . Also, following the special theory of relativity, the ether was absolute, because its influence on inertia and light propagation was thought to be independent of physical influences of any kind . . . The ether of the general theory of relativity differs from that of classical mechanics or the special theory of relativity respectively, insofar as it is not "absolute", but is determined in its locally variable properties by ponderable matter. (Einstein, 1921, pp. 55–56)

Thus the difference between absolute and relative conceptions of spacetime, according to Einstein, is whether the spacetime structure is "mutable" or not.

Following Mach and Einstein, philosophers understood arguments for absolute space and time as an inference from inertial phenomena to the best explanation thereof. One first begins with inertial effects as observations in need of explanation.

One next argues that no relative motion could provide a reasonable explanation for these effects. Then it is supposed that a spacetime background is needed relative to which inertial motions are defined. It is then argued, that without a spacetime structure, it is not possible to explain the motion of force-free particles.[12] Finally, the argument concludes that because spacetime is indispensable for explaining physical phenomena, it is also real, and should actually be taken to have causal influence over physical bodies, in directing force-free particles and lights rays along the geodesics of the spacetime.

The reification of spacetime suggests that free particles could have failed to move along a geodesic. So the truth of the Law of Inertia is contingent and is analogous to any observed law of nature. Thus, the geometric interpretation of spacetime helps commentators replace the conventionalist account of the Law of Inertia with what appears like an empiricist's account. Earman and Friedman (1973) argue that inertial reference frames are reducible to the independent structures of spacetime.[13] If one has good reasons to think that the manifold and the spacetime structures are real, then one should treat the Law of Inertia as a directly verifiable prediction.

However, there are many drawbacks to reifying the spacetime structure and no benefits. Taking the spacetime itself as "explaining" the inertial behavior of bodies leads interpreters to say that the spacetime causally influences the behavior of isolated systems. Spacetime acts on bodies in that it "guides" them to move through geodesic lines. A useful way to think of these geodesic lines is to think of them as analogous to ruts that make the passage through spacetime easier. But this causal account of spacetime seems to stretch too far our common intuitions about explanation. One ought to feel uncomfortable when spatiotemporal points are attributed causal powers. For one, an entity which has causal powers seems to require some substance-like existence. It seems natural to think that causal powers must be inherent in some substratum. But spatiotemporal points by definition are not the kinds of things that persist. So in what sense do they have substance? Do we not need to think of an entity with substance as persisting? For these reasons, even Newton's account of absolute space specifically precludes spatial points from being substance-like as Newton takes for granted that they are without causal efficacy.[14]

A typical example of an argument that reifies space is given by Nerlich:

> Without the affine structure there is nothing to determine how the [free] particle trajectory should lie. It has no antennae to tell it where other objects are, even if there were other objects . . . *It is because space-time has a certain shape that world lines lie as they do*. (Nerlich, 1976, p. 264)

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[12] For the history of this reading of the argument, see Reichenbach (1927, pp. 210–18), Burtt (1954, pp. 244–55), Jammer (1994, p. 106), Lacey (1970), and Westfall (1971, p. 443).

[13] A reference frame F is defined by a time-like vector field X, i.e., $dtX \neq 0$. The trajectories of X could be interpreted as the worldlines of points in the spacetime. If there is a coordinate system $\{x^\mu\}$ in which the components of the affine connection vanish, or $\Gamma^\gamma_{\mu\nu} = 0$, and the coordinate system is adaptable to F, i.e., the spatial coordinates x^α, $\alpha = 1, 2, 3$ of the trajectories of X are constant, then F is an inertial reference frame (Earman and Friedman, 1973, section 3).

[14] See Newton's account of absolute space in the *De Gravitatione*, Newton (2004).

Nerlich's imagery suggests that because spacetime has a certain shape, the trajectories receive a certain structure as a result. The implication that the shape of spacetime is explanatory in some way, i.e., it gives rise to the way in which free-particles, clocks and rods behave.[15]

However, the causal language describing the relation between spacetime and matter violates certain ingrained intuitions about causal explanations. Brown puts the objection as follows:

> If free particles have no antennae, then they have no space-time feelers either. How are we to understand the coupling between the particles and the postulated geometrical space-time structure . . .? In what sense then is the postulation of absolute space-time doing more explanatory work than Molière's famous dormative virtue in opium? (Brown, 2005, p. 24)

Brown's worry is therefore with the cogency of taking spacetime to have causal powers analogous to the causal powers a physical substance or field may have. If the assumption – that spacetime has a causal efficacy – is cogent, one needs to consider whether the assumption provides a genuine explanation for inertial motion. To do so, one may analyze the causal relation between spacetime and matter with the help of counnterfactuals. Assume that the state ϕ of A causes the state θ of B. The causal relation between A and B implies the counterfactual statement "if A had not existed with property ϕ, then B would not have existed with property θ." It is tempting to describe the relation between spacetime and matter as causal since it supports a similar counterfactual. If spacetime did not have certain properties (e.g., a pseudo-Riemannian structure, or a curvature), rigid rods and clocks would not behave as they do, and trajectories of free particles would not have been as they are.

One can parse Brown's objection as follows. It is not always the case that such a counterfactual underwrites a causal relation. In the case where A having property ϕ states the condition for the *possibility* of B having property θ, the counterfactual does not describe causation between two independently existing things. For example, suppose we say that the Nobel Prize committee had awarded Einstein the Nobel Prize. One may say that a counterfactual claim is supported – had the Nobel Prize committee not given Einstein the title, he would not have been a Nobel Prize laureate. However, the act of giving the prize is not the cause of Einstein's getting it. The act of giving the prize is part of the conditions for the possibility of earning the prize. The act of giving the prize and the event of earning it are one

[15] Even though Einstein described the relation between spacetime and matter as causal, he also thought that having two independent fields existing side by side is problematic. This is one of his motivations for searching for a unified field theory. In a lecture delivered at the Nobel Prize ceremony, he asserted the following:

> The mind striving after unification of the theory cannot be satisfied that two fields should exist which, by their nature, are quite independent. A mathematically unified field theory is sought in which the gravitational field and the electromagnetic field are interpreted as only different components or manifestations of the same uniform field . . . The gravitational theory, considered in terms of mathematical formalism, i.e., Riemannian geometry, should be generalized so that it includes the laws of the electromagnetic field. (Einstein, 1923, p. 489)

and the same; they are two different descriptions of the same thing. Similarly, that a counterfactual claim connects spacetime properties with material properties does not imply that an efficient causal relation is involved. Since spacetime defines the very distinction between trajectories of free particles and trajectories influenced by some dynamic force, the spacetime structure provides the condition for the possibility of rods, clocks and free particles having the properties that they have. Thus, the counterfactual does not underwrite a genuine efficient causal relation. There is no reason to suppose that spacetime causes free particles to move as they do in analogy to a billiard ball which causes another billiard ball to move. And since the counterfactual does not underwrite a genuine causal relation, the reification of spacetime is not a genuine explanation of inertial motions.

Now the response to such an argument may be that a geometric mode of explanation is non-causal, but may still be an independent means of explaining the behavior of bodies (Nerlich, 1979). But it then becomes a mystery why one ought to posit that the spacetime manifold and its metric should be thought of as a field existing independently and alongside matter fields. A geometric mode of explanation, at the end of the day, merely attempts to describe how bodies behave, and it is unnecessary to think that points exist independently of bodies and relations between them. Another reaction to the argument questioning the causal efficacy of spacetime might be that the mere existence of a counterfactual connecting properties of spacetime with properties of matter may underwrite the causal relation between spacetime and matter (Mellor, 1980). But if this line of argument is taken, then spacetime is explanatory in a way that an Aristotelian formal cause is explanatory of particular bodies. Assume that the explanation amounted to the counterfactual, "if spacetime M had not existed with property ϕ, then a free particle would not have existed with property θ." Since the spacetime structure provides the means for describing θ, then spacetime is merely the articulation of the shape that a free particle has, in much the same way that a formal cause is merely a description of the form that a certain Aristotelian substance has. A square figure in Euclidean space has the form of a square, but in describing the square one is not giving an independently existing structure which is causally responsible for the existence of the square, one is describing the spatial form of the object.

It is worth noting another difficulty with the geometric approach. In the context of the General Theory of Relativity, the reification of the manifold gives rise to the Hole problem (Earman and Norton, 1987). The problem consists of there existing indistinguishable dynamic models of the theory, $\langle M, g, T\rangle$ and $\langle M, h * g, h * T\rangle$, where M is the spacetime manifold, g is the metric, T is the stress-energy tensor, and h is a diffeomorphism on the manifold. While indistinguishable from a dynamic perspective, these models differ in their mapping of empty regions of the manifold to the metrics g and $h*g$. This situation leads to indeterminism, since a manifold realist cannot predict which trajectory in the manifold will be realized by an object moving into the hole. The upshot seems to be that points in the manifold cannot retain their identity independently of the dynamic objects of the theory (Hoefer, 1996).

The geometric approach has a clear advantage over the conventionalist approach, in that the restricted Principle of Relativity can be viewed as stemming from a four-dimensional spacetime symmetry. The geometric approach was also instrumental in the construction of the General Theory of Relativity, and facilitated the merging of the gravitational field with the spacetime metric. However, there is a clear lacuna in the attempts to conceive of spacetime as another field lying alongside matter fields. The coupling between spacetime and matter becomes obscure if it is compared to the coupling between two material fields, and attempts to attribute the spacetime field efficient causal powers come close to being unintelligible. Thus, there is a steady stream of voices that attempt to argue against the official doctrine, which the geometric interpretation has become. Although it is difficult to overcome the appeal of the geometric approach, since Minkowski's analogy between Euclidean and four-dimensional spacetime symmetries has strong intuitive appeal.

2.4 The Dynamic Approach to Spacetime

The difficulty in articulating the nature of the causal relation between spacetime and matter compels some to doubt the causal roles attributed to spacetime (see Stein, 1967; DiSalle, 1995; Brown, 2005; Brown and Pooley, 2006). Thus some commentators take the geometric interpretation to be untenable, and offer a program for reducing spacetime theory to dynamic laws. According to one articulation of dynamical relationalism (developed by Teller, 1987; Dieks, 2001a,b), one ought to think of spacetime as a collection of physical quantities actualized by material bodies, in a manner analogous to that of inherent properties such as mass or charge. The range of possible spacetime positions and velocities of each body is determined through the dynamical laws which characterize the theory. When a physical system actualizes a certain dynamic theory, it necessarily actualizes a set of possible trajectories and their relations. Thus, if the Hamiltonian of a system comprising of two particles is $H = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} - V(q_1, q_2)$, the allowable coordinates and their variation would be those that satisfy the dynamics described by the Hamiltonian. The trajectories of the particles are physical possibilities that must conform to the dynamic laws. The spacetime symmetry, i.e., the equivalence between inertial reference frames, is determined by the symmetries of the dynamic laws.

However, there is a metaphysical difficulty with Teller's and Dieks' version of dynamical relationalism, which is made clear by the analogy they erect between spacetime quantities and other physical quantities. The plausibility of Teller's dynamical relationalism turns on whether it is reasonable to treat spatiotemporal quantities as physical quantities. Is being at location x and then being at location x' assimilable to the instantiation in the body of the physical quantity x, and then the instantiation of physical quantity x'? Presumably, whether a body possesses location x depends on the relation between this location and all the other spatiotemporal points. When one attributes a mass parameter to a body, one seems to be able to do so without any reference to other bodies. Thus, it is possible to conceive of a body that exists in empty space with a mass parameter, without assuming that any other body exists.[16] That a body instantiates a certain mass parameter can be conceived independently of mass parameters actualized by other bodies. A spacetime position, in contradistinction, is necessarily relational. Treating the notion of spatiotemporal position as a monadic property therefore seems unintelligible.

Teller (1991) later clarified his account of spacetime positions as physical quantities, and reduced them to potential relations to actual bodies, endorsing a kind of liberal relationalism (to be distinguished from narrow relationalism, which reduces spacetime to actual relations between bodies). In this account, indistinguishable Leibnizian models of spatiotemporal relations are conceptualized as monadic physical quantities, *whose values* depend on the values of quantities instantiated by other bodies. Teller attempts to make this notion plausible by comparing spatiotemporal locations to the values of masses. The particular mass value a body instantiates is not entirely independent of mass values instantiated by other bodies. The particular mass value instantiated by the body would arbitrarily change if all masses and forces are scaled in the same proportions. However, it is not clear that this elaboration of dynamical relationalism renders it more plausible. It is not only that the *value* of the coordinate of a spatiotemporal point is determined in relation to other points. The *identity* of the point is also determined in relation to other points. There is some connection between the identity of objects and the identity of the points they occupy. Some physical models do not allow for two bodies to be in the same place. In these models, the location of the body identifies the body for us. The location of the body helps differentiate between *this* particle-mass over here from *that* particle-mass over there. One might explain the fact that two bodies cannot reside in the same place by positing a spatial exclusion principle, in analogy with Pauli's Exclusion Principle, according to which two particles of spin 1/2 cannot be in the same state. According to the spatial exclusion principle, two particles cannot instantiate the same spatial quantity. However, it seems clear that the analogy between coordinates and physical quantities is stretched here. It is perfectly reasonable to take two bodies as having the same mass value, but mass values do not determine the identity of the body. However, the identity of spatiotemporal point is crucial for determining the identity of objects that occupy it, so that spatiotemporal positions do not seem to function like physical quantities. While the disanalogy between spacetime points and other physical quantities does not prove dynamical relationalism false, it does weaken the force of the analogy between a spatiotemporal point and other physical quantities.

Another version of dynamical relationalism is articulated in a stimulating book by Brown (2005). Brown argues that relativity theory was at first conceived by Einstein as a Principle Theory. Einstein compared the principles of STR to classical thermodynamics theory in that these principles are based on broad phenomenological principles. Thus, while kinematic effects of length contraction and time dilation are provided as consequences of the theory, these effects are merely described by the

[16] In the following chapter I will argue for a different account of the mass parameter, which undermines the view that mass is an inherent property. However, I am using the popular (though false) understanding of mass to make a philosophical point.

theory without any proper explanation. A fuller account of these kinematic effects would require a constructive theory that captures the behavior of macroscopic bodies such as clocks and rods. A constructive theory of spacetime would appeal to fundamental dynamic theories that explain the structure of composite material systems. According to this neo-Lorentzian strategy, spacetime effects are implicit in the dynamic theories explaining the fundamental structure of matter.

One of the difficulties with Brown's strategy is that kinematical effects seem not to describe composite physical systems such as clocks and rods, but physical processes themselves. Thus, when one measures the half life of a decaying atom, its half life is dilated when it moves relative to the clocks and rods in the lab frame. But this dilation is a property of the decaying process, and a constructive account of clocks and rods would be irrelevant to the delay ascribed to the decaying process. Moreover, the dilation effect does not appear related to the specific dynamic laws governing the process of decay and the structure of the decaying atom, but a universal property of all processes taking place within time. All dynamic laws appear to have the general symmetries of relativistic spacetime, and so it seems as if one needs to find a unifying account for these effects that does not rely on the specific dynamic details. Relativistic effects are not a product of the structure of matter or the composite devices one uses for measuring time and length, but of time itself. This is the main reason the Lorentzian strategy for explaining relativistic effects seems beside the point, and that a direct account of spatiotemporal relations is needed independently of any dynamic theory explaining the structure of matter.

A more serious difficulty with both versions of dynamical relationalism is that they treat the Principle of Relativity and the symmetries of spacetime as brute, "accidental" features of the underlying dynamics. Brown readily admits as much:

> In the dynamical approach to length contraction and time dilation that was outlined in the previous chapter, the Lorentz covariance of all the fundamental laws of physics is an unexplained, brute fact. This, in and of itself, does not count against the approach: all explanation must stop somewhere. What is required if the so-called space-time interpretation is to win out over this dynamical approach is that it offer a genuine explanation of universal Lorentz covariance. This is what is disputed. Talk of Lorentz covariance "reflecting the structure of space-time posted by the theory" and of "tracing the invariance to a common origin" needs to be fleshed out if we are to be given a genuine explanation here, something akin to the explanation of inertia in general relativity. Otherwise we simply have yet another analogue of Molière's dormative virtue. (Brown, 2005, p. 143)

Brown therefore thinks that explanation ends exactly at the point where one finds an astounding symmetry governing all known dynamic laws. But the Principle of Relativity seems to beg an independent explanation since it would be a miraculous accident if it just happened that all dynamic laws are Lorentz-covariant. Why is it that future laws which are yet to be discovered are expected to have the property of being Lorentz-covariant? Either this expectation is unfounded or one has to find an explanation – a property that all dynamic laws share independently of their specific form.[17] On the other hand, Brown is correct to doubt the substantivalist account

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[17] See Balashov and Janssen (2003) for a similar argument against Craig's neo-Lorentzian interpretation of relativity.

of spacetime as giving a proper explanation. It is not clear why taking spacetime points to be real explains why the laws reflect the same symmetries as the underlying spacetime.[18]

2.5 Conclusion

The conventionalist account of spacetime dominated the philosophical literature initially. However, it soon gave way to the geometric interpretation of spacetime, which is philosophically superior. The geometric interpretation was inspired by Minkowski's geometrization of Einstein's theory and his description of relativistic effects as resulting from the structure of a four-dimensional spacetime manifold. If spacetime is taken to be a four-dimensional spacetime manifold, and various inertial reference frames are taken to be mere cross-sections of the same spacetime, then one has an intuitive grasp of what it means to have *different representations* of the same spatiotemporal structure, rather than *empirically equivalent* models. However, the geometric interpretation of spacetime, while providing an improvement over the conventionalist account, carries its own interpretive difficulties, since it compels commentators to "breathe life" into the shadowy spacetime and render it substance-like. Reifying spacetime leads commentators to the notion that the spacetime manifold is an independent physical entity, which results in dubious metaphors that credit the manifold with efficient causal powers. The efficient causal metaphors seem inappropriate, since they lend spacetime the appearance of a physical entity analogous to other material bodies. Spacetime provides the framework for interpreting causal relations, so it seems incongruous to take spacetime itself as an entity that has independent causal powers. Thus, while the geometric approach is dominant, there are critics of the geometric approach who argue that the kinematic effects of spacetime are implicit in the dynamic laws. However, it is not clear why the various dynamic laws conform to the same spacetime symmetries. It is difficult to see how various dynamic laws do not receive these symmetries from an underlying spacetime structure that is somehow "responsible" for these symmetries.

Einstein's interpretation of spacetime shifted throughout his career, reflecting perhaps the appeal of each approach. His early work is couched in the conventionalist approach. His treatment of relations of simultaneity as a convention arising from the arbitrary choice of reference frame is explicated with the help of conventionalist epistemology. Initially, Einstein's reaction to Minkowski's geometrization of spacetime was not enthusiastic, but he later embraced Minkowki's approach and made innovative use of a geometrized spacetime in the development of the General Theory of Relativity, incorporating the gravitational field into the curvature of spacetime. Finally, while he realized that spacetime was in fact treated by him as some kind of

[18] I should point out that Brown is not endorsing the adoption of an ether or an absolute frame of reference. Thus he is not attempting to resuscitate Lorentz's specific strategy for explaining kinematic effects. Rather, he claims that dynamic laws should explain the kinematic effects of a set of rods and clocks moving relative to another system of clocks and rods. Brown argues that dynamics should explain kinematics, not that the Principle of Relativity is false.

field or as some kind of ether, he also recognized the threat posed by the claim that clocks and rods are somehow determined by the geometry of spacetime, and flirted with a dynamic interpretation of spacetime.

An assessment of the three main interpretations of spactime suggests that none of these interpretations is fully satisfying. The difficulty with the conventionalist interpretation is that it permits a range of empirically equivalent models without seeking a theoretical framework for viewing these models as equivalent representations of the same world-state. The difficulty with the geometric account is that it lends commentators the impression that the spacetime manifold exists independently of material processes and that it causally interacts with material bodies. Finally, the difficulty with the dynamic interpretation is that it fails to recognize the unifying role of spacetime. In the next chapter, I introduce an alternative to these three approaches, that bears some similarity to both the geometric and the dynamic approaches, but is also distinct from both. The approach will attempt to provide a theoretical way for unifying the various inertial reference frames into one geometric theory, without assuming the independent existence of spacetime or an efficient causal relation between spacetime and matter.

Chapter 3
Primitive Motion Relationalism

3.1 Introduction

Chapter 1 offered a reading of the law of momentum conservation, which takes it to consist of structural assumptions about physical systems. Structural assumptions include a Criterion of Isolation and a Rule of Composition. One benefit of thinking of conservation laws as structural assumptions is that it makes clear the epistemic role of conservation laws. The Criterion of Isolation appears to be central to the scientific practice, since in order to attribute certain properties to parts of a system, physicists need a criterion for isolating the composite system from the environment. If a system is not approximately isolated, one cannot investigate the system, either theoretically or experimentally. Without a criterion for isolating the system, it is not possible to discern the causal processes that flow from one part of the system to another, and dissociate them from causal processes that arises from external factors.[1] A Criterion of Isolation therefore holds a certain a priori place when it comes to individual experiments. However, structural assumptions are not metaphysically necessary. If the experimental physicist is able to succeed in isolating systems in the laboratory, thereby controlling interactions between parts of the system, the a priori criterion of isolation becomes embedded in the experimental practice and recognized as making such a practice possible. Thus, structural assumptions do not function only in the construction of interpretations of experience. They are validated by the success of the experimental practice.

The Criterion of Isolation governing Newtonian systems asserts that a system is isolated if and only if it moves with uniform unidirectional motion. The Criterion of Isolation applies to fundamental particles, in which case the particle is free and moves uniformly along the geodesic of the spacetime. But the same Criterion of Isolation applies to a composite system, which is represented by the trajectory of the center of mass of interacting particles. Rather than thinking of the free particle

[1] I should qualify these remarks and say that the *initial* investigation into causal processes requires a Criterion of Isolation. One can also apply the Criterion of Isolation in one dimension (for example, horizontally to the force of gravitation on the surface of the earth), or isolate a system only to a good approximation. Once, for example, a certain force is discovered, one can assume it exists without applying the Criterion of Isolation.

O. Belkind, *Physical Systems,* Boston Studies in the Philosophy of Science 264,
DOI 10.1007/978-94-007-2373-3_3,

and the center-of-mass frame as two different applications of a conservation law, the suggestion is to treat these two cases as conforming to the same Criterion of Isolation, and to think of this criterion as applying either to simple or composite systems.

A common way to interpret the Law of Inertia is to think of it as the consequence of laws of conservation, which are articulated relative to a background spacetime manifold. In this approach the background spacetime, or the bedrock level of physical reality, is differentiated from material bodies and the dynamic laws governing their behavior. Spacetime locations tell us where the bodies are; their properties, states of motion, and dynamic laws determine where they will be next. Given this metaphysical approach, spacetime and matter are understood to be distinct elements of reality. Conservation laws, since they are not part of spacetime, are assimilated to dynamic laws. Thus when the Law of Inertia is viewed as the logical consequence of conservation laws, it is subsumed under dynamic laws. The result is a causal interpretation of the Law of Inertia. One imagines the Law of Inertia to be forcing the free particle along the geodesic. Newton himself was led to think of inertial forces as inherent forces compelling a body to move from one location in space to another. Other commentators attribute causal powers to spacetime itself.

I am inclined to say that the trajectory of a free particle is not "governed" by any dynamic law, and that a free particle instantiates a certain motion. This motion is not caused by anything else, or by an independently existing spacetime. Rather, spacetime provides a *description* of fundamental motions which also happen to be inertial motions. If this intuition is to be insisted on, one must find a way to reinterpret conservation laws as partly involved in shaping geometric structures. That is, one must find a conceptual means of undermining the distinction between dynamic and geometric laws.

To some extent, the General Theory of Relativity blurred the distinction between dynamic and geometric laws. The inertial structure and gravitational force are both incorporated to the spacetime metric in this theory and subsumed under a single metric. But even in the General Theory of Relativity there is still a conceptual separation between geometry and dynamics, or between the inertial-gravitational structure and the other forces. Fields and laws governing the evolution of fields are thought to be "embedded" in a spacetime structure. Conservation laws and the symmetries governing dynamic laws are thought to inherit the symmetries of the underlying spacetime, but the conceptual relation between the two is not clear. Conservation laws are still expressed as additional dynamic laws that govern bodies in spacetime, even though conservation laws dictate that free particles should move along the geodesics of spacetime.

If the Criterion of Isolation holds an a priori place in carrying out any single experiment, and if uniform unidirectional motion functions as a Criterion of Isolation, it seems logical to take uniform unidirectional motion as a basic, fundamental state of physical systems. This intuition suggests a way to rethink the ontology of spacetime. If Einstein claimed that theories of spacetime should begin with a set of coincidence points, I take the opposite view; spacetime should begin with geodesic motions as the fundamental entities. Points of coincidence are then conceived as the intersections between geodesic motions. Rather than thinking about motions as changes in the spacetime location of a body, locations in spacetime are thought of as the intersections of bodies that have uniform unidirectional motions. Since spacetime theories describe *possible* events, spacetime points should be taken as intersections between possible geodesic motions. The connection between the geometry of uniform motions and physical geometry would then be to assert that uniform unidirectional motions are the same as inertial motions – free particles and isolated systems in Galilean spacetime and free particles, isolated systems or light signals in flat relativistic spacetime. I call these motions Paradigms of Uniform Motion (PUMs).

The account here suggests that *possible* events and the spatiotemporal points in which they are located are not real. In this respect the account resembles traditional relational accounts. However, unlike traditional relationalism, this account does not admit the priority of spatial and temporal relations over motions, since it reduces spatiotemporal relations to the structure of PUMs. Moreover, this account takes the description of spatiotemporal relations between possible events to be meaningful, even if the spatiotemporal points in which they are located do not exist. One can still discuss potential intersections between possible PUMs, even if these events are not actual and the points in which they are located are not real.

The description of empty spacetime is analogous to the basic principles governing architectural plans of buildings that have not been constructed yet. There is a range of architectural plans that describe buildings that are physically possible, given that they adhere to certain laws of nature. But it is yet possible to design plans of buildings that are not physically possible. Both kinds of plan do not describe anything existing in reality. For those architectural plans that describe feasible buildings, one can distill some basic principles common to all possible buildings. However, these principles do not describe real entities that influence buildings through efficient causal processes. They are common formal elements one would find in all possible buildings and can be seen as their formal cause, if causal language is to be insisted on. Analogously, theories of spacetime describe various physical constraints on the motions of physical systems. Some theories of spacetime, like Galilean spacetime, allow for processes that are not possible and do not allow for some processes that are. Relativistic theories of spacetime are closer to describing the correct limitations on physical processes. Thus, theories of spacetime carry genuine empirical content even if the spacetime points they describe are not real. The manifold of events which forms the basis of these theories does not represent anything in reality. Moreover, the identity of spacetime points depends on the identity of intersecting motions.[2]

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[2] Thus *Primitive Motion Relationalism* has conceptual ties to possibilist relationalism – see Manders (1982), Mundy (1986) and Teller (1991). Someone may worry that the analogy between spacetime theories and architectural plans is vitiated by the similarity between architectural plans and the buildings they describe. Architectural plans are often drawn on paper and are simply scaled spatial representations of the buildings themselves. This would seemingly undermine the purpose of the analogy, which is to argue that spacetime theories do not describe anything real. Drawings of buildings may not describe an existing building, but are themselves actualized spatial structures.

In flat spacetimes, constraints on possible motions are given independently of the actual distribution of matter. In the General Theory of Relativity, the description of possible spacetime trajectories depends on the actual distribution of matter. But the dependency of the structure of spacetime on the distribution of matter does not require that we view spacetime as some kind of ether that acts on matter and is acted on by matter. While spacetime depends on the actual distribution of matter, this dependency is not necessarily a causal dependency, but is the dependency of possibilities on actualized entities. Just as the creation of new entities generates new possibilities, so does the actual distribution of matter shape possible trajectories. The dependency of matter on spacetime is, again, not a relation of efficient causation, but the dependency of actual trajectories on possible forms that trajectories have.

To summarize the above view of spacetime, i.e., the notion that spacetime points are not real, that spacetime structure can be constructed from ideal counterfactual uniform motions, and that spacetime points are intersections between motions, I will give it the label *Primitive Motion Relationalism*.

To describe a spacetime of more than one spatial dimension, one needs to define rigid bodies and the possibility of rotating these bodies from one spatial dimension to another. The behavior of infinitesimal rigid bodies determine the Euclidean structure of a hypersurface in spacetime and the curvature of spacetime. I will not consider curved spacetimes, and will leave that task to future work. Instead I will present the program of reducing spacetime points to counterfactual PUMs by reconstructing from PUMs a flat $\{1 + 1\}$ Galilean spacetime and a flat $\{1 + 1\}$ relativistic spacetime, without going into the Euclidean geometry defined on a hypersurface in the spacetime or into curved spacetimes.

Before presenting this reconstruction of spacetime, I should clarify a conceptual point regarding the relationship between space, time and motion. Ordinarily, motion is taken to be the derivative of the function $x_1(x_0)$, where x_1 is the spatial dimension and x_0 is the temporal dimension. According to this definition, instantaneous motion is defined as the limit in a series of ratios, so that:

$$v(x_0) =_{df} \frac{x_1(x_0 + \Delta x_0) - x_1(x_0)}{\Delta x_0} \lim_{\Delta x_0 \to 0} \tag{3.1}$$

This definition of motion takes for granted the existence of a “trajectory,” which is a continuous function from temporal instants into the spatial dimension. This definition assumes that space and time form independent parameters, and that the metrics of these dimensions are also defined independently. However, I argue here that the very structure of the spatial and temporal dimensions is determined by primitive PUMs, and that the spatial and temporal metrics are not independently defined. I argue that the notion of trajectory is defined relative to a set of uniform, unidirectional motions. Thus (3.1) is not the definition of instantaneous motion but

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However, this objection is easily rebutted if we think of computer programs that encode architectural plans. The printouts or screen simulations are not essential components of the plans, but are mere aids for humans in understanding the plans.

a calculation that *reveals* one of the instantaneous parts of the trajectory, which is identical with an instantaneous part of a uniform, unidirectional motion.[3]

To show that it is possible to construe motion and events in this manner, an axiomatic system describing a geometry of uniform motions, or PUMs as I call them here, will be introduced in Section 3.2. The axiomatic system will confer a metric of motion intervals on these PUMs, which is determined independently of any underlying spatial and temporal metric. In Section 3.3, I demonstrate how to decompose motion intervals into spatial and temporal components by "projecting" a PUM onto a class of parallel PUMs. First a "Galilean" decomposition of PUMs is articulated and a $\{1+1\}$ Galilean spacetime is then derived. Then a "relativistic" decomposition of PUMs is articulated and a flat $\{1+1\}$ relativistic spacetime is derived. In Section 3.5 I consider the similarities and differences between the approach introduced here and some of the existing interpretations of spacetime. I argue in this section that *Primitive Motion Relationalism* offers an intuitive and compelling explanation of the restricted Principle of Relativity, which overcomes some of the weaknesses of existing approaches. Section 3.6 concludes with a few remarks.

3.2 A Geometry of PUMs

The axiomatic system for *Primitive Motion Relationalism* takes uniform, unidirectional motions (PUMs) to be the basic entities of spacetime. Such motions are represented in the system with Greek letters $\alpha, \beta, \gamma \ldots$. The geometry therefore assumes that PUMs are undefined primitives. An event is an intersection between motions, so in effect an event is a relation between two motions. The approach here therefore constructs spacetime from geometric relations between motions, first defining events as intersections between motions, and then defining geometric relations between those events. Since an event is a relation between PUMs, it is not a primitive entity. In simple geometries where spacetime is flat, one can deduce that

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[3] The account here sheds new light on recent debates about the nature of motion. In recent discussions (see Tooley, 1988; Jackson and Pargetter, 1988; Arntzenius, 2000; Carroll, 2002; Lange, 2005), some have argued that instantaneous velocities cannot be defined as neighborhood properties or the derivative of a trajectory. Our normal scientific practice is based on the intuition that instantaneous motions are *states* of physical systems. The notion of a *state* of motion is required to explain the dynamic evolution of physical systems. For example, the Law of Inertia asserts that the state of motion of a body will remain unchanged if no external forces are impressed on the body. Thus, it is not enough to define motion as the limit on a series of ratios, since that limit is not a genuine property of the body which exists at a particular instant. We need instantaneous motions as properties that *explain* the dynamics of a physical interaction. But if we take instantaneous motions to be primitive states of a physical system, it is not clear what the relation is between primitive instantaneous motion and the derivative of the trajectory, which seems to be providing the *definition* of instantaneous motion. Our account alleviates this tension since it is assumed here that the derivative of the trajectory can only be defined relative to the Paradigm Uniform Motion, which is a paradigm for instantaneous primitive motions.

two PUMs are parallel from coincidence relations they have with other PUMs. Thus, the notion of being parallel can be defined in such a geometry using the notion of intersection. Before articulating the axioms for the geometry of PUMs, the definitions of event, of being parallel and of PUM event classes, are given.

The axioms will lay out the necessary structures for conferring a metric of motion intervals onto the PUMs. Relations of determinateness and betweenness are implicitly defined through the determinateness and betweenness axioms, so that a causal structure is implicitly defined through these axioms. Finally, a metric of motion intervals is defined through congruence relations between pairs of events on PUMs. The motion intervals are not dependent on underlying spatial and temporal intervals, but are given directly as a relation between events (coincidences) on PUMs.

Basic Definitions[4]

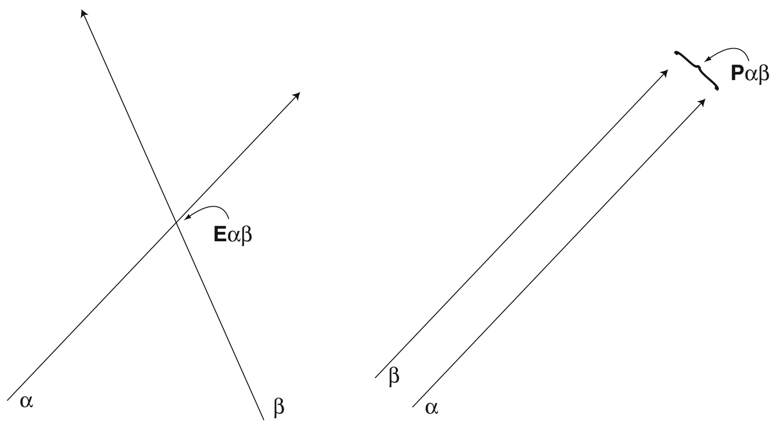


Fig. 3.1 Event and parallel relations between PUMs

Definition 1 (Event) An event is defined as the intersection between PUMs (see Fig. 3.1). Since it consists of a meeting between two underlying motions, an event is a two place relation between PUMs. The relation is designated as $\mathbf{E}\alpha\beta$, and is true if the motions intersect, or false if they do not. For flat spacetimes, an event p is *identical* with a true relation $\mathbf{E}\alpha\beta$, since it is assumed that there is no more than one event in which two PUMs coincide.[5] It is supposed that coincidence relations

[4] Throughout the account the reader should keep in mind what is meant by the existential operators. Underlying the geometry are PUMs, which describe possible uniform motions. When the existential quantifier describes PUMs (in conjunction with Greek letters α, β, ...) it is describing the existence of a possible uniform motion. The assumption is of a fixed domain interpretation of modality; all statements of predicate logic are referring to possible states of affairs.

[5] The extension of the approach here to curved spacetime will complicate matters, since it will require more than one coincidence point between PUMs in the case of spherical curvature. But

are irreflexive, since a PUM does not coincide with itself. Coincidence relations are also symmetric:

1. $\sim \mathbf{E}\alpha\alpha$
2. $\mathbf{E}\alpha\beta \leftrightarrow \mathbf{E}\beta\alpha$

Whenever convenient, a true coincident relation will be replaced with the event name. Thus, if $< \alpha, \beta >\in \mathbf{E}$, the pair can be identified as an event:

$$(< \alpha, \beta >\in \mathbf{E}) \rightarrow \left(p =_{df} < \alpha, \beta >\right)$$

Definition 2 (Parallelism) In the context of spacetime, two uniform, unidirectional motions are parallel when they have the same velocity and move in the same direction (see Fig. 3.1). In a flat time-space plane with only one spatial dimension, one may deduce the relation of being parallel from the fact that the two motions do not intersect. Infinite uniform PUMs in a single spatial dimension would eventually intersect if they do not have the same velocity. Thus, the notion seems promising that one can *deduce* the relation of being parallel from coincidence relations. The first condition for two motions being parallel is that they do not intersect. Thus $\mathbf{P}\alpha\beta$ (α is parallel to β) only if $\sim \mathbf{E}\alpha\beta$. In addition to non-intersection, it is assumed that parallelism forms an equivalence class. Since it was argued that no PUM can intersect itself, it is natural to infer that every PUM is parallel to itself. Moreover, since coincidence is a symmetric relation, non-coincidence is also a symmetric relation. But coincidence is not a transitive relation, and neither is non-coincidence. Thus to define parallelism as an equivalence class one needs to add the condition that parallelism is a transitive relation. To establish parallelism one needs to impose the condition that if both α and γ are parallel to β, then α and γ do not intersect. The definition of parallelism is therefore the combination of the following two criteria:

1. $\mathbf{P}\alpha\beta \rightarrow \sim \mathbf{E}\alpha\beta$
2. $[\mathbf{P}\alpha\beta \wedge \mathbf{P}\beta\gamma] \rightarrow \mathbf{P}\alpha\gamma$

Definition 3 (PUM events) It will be expedient to think of all points of intersection with α as events belonging to the motion. Thus let the set $\mathcal{S}(\alpha)$ include all the intersection points of α with other motions. In formal notation the definition looks as follows:

$$\mathcal{S}(\alpha) : (\alpha \times \beta) \in \mathbf{E}\alpha\beta$$

the uniqueness of the coincidence relation is still true locally in curved spacetimes. The problem of how to define a neighborhood of a coincidence relation without relying on distance relations between points will be left for future work.

Axioms

Following these definitions are the axioms. A list of incidence axioms describe coincidence relations between PUMs. These relations guarantee the existence of coincidences $\mathbf{E}\alpha\beta$ for motions and articulate the existence of classes of parallel motions. Determinateness, Betweenness, and Congruence axioms are then given for events. In these latter axioms the discussion shifts from coincidence relations between PUMs, to relations between events. It should be kept in mind that relations between pairs of events are in effect four-place relations between PUMs. For example, the relation $\mathbf{D}pq$ is the relation of determinateness, where event p determines event q. The determinateness relation describes an asymmetrical causal relation between events along motions. Nevertheless, each event is a coincidence relation between two motions, so in effect the determinateness relation is a relation between four motions, two motions α_1 and α_2 must intersect to produce event p, and two motions β_1 and β_2 must intersect to produce event q before it could be said that p determines q.

The determinateness relation is then used to define a three-place betweenness relation $\mathbf{B}pqr$ between events on a particular PUM. The three-place relation of betweenness is in effect a four-place relation between PUMs, since it is a relation between three events on a particular PUM. It is therefore a relation between a PUM α and three other PUMs, β_1, β_2 and β_3, that intersect it. The betweenness relation is then used to articulate the betweenness axioms. Finally, a congruence relation $\mathbf{C}pqrs$ between motion segments is defined, which sets a metric of motion intervals. A segment is the interval between two events on a particular motion, so a segment is defined by a motion α and two motions γ_1 and γ_2 that intersect it. Thus the congruence relation involves six PUMs.

Axioms of Incidence

The axioms of incidence state the existence of motions, coincidence relations, and parallel relations. The axioms are stated for every PUM α:

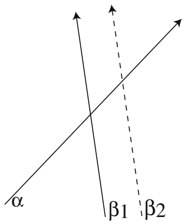
I 1. This axiom asserts that every PUM α is intersected by at least one other PUM.

$$(\exists\beta)\mathbf{E}\alpha\beta$$

I 2. This axiom asserts that for every motion α, there is an intersection point between two motions not on α.

$$(\exists\beta_1)(\exists\beta_2)[(\alpha \neq \beta_1 \neq \beta_2) \wedge \mathbf{E}\beta_1\beta_2]$$

I 3. This axiom is the current system's version of the parallel axiom. For every α which is intersected by β_1, there is another motion β_2, which is parallel to β_1 and intersects α. Thus, the axiom looks as follows:



$$\mathbf{E}\alpha\beta_1 \rightarrow (\exists\beta_2)[(\beta_1 \neq \beta_2) \wedge \mathbf{P}\beta_1\beta_2 \wedge \mathbf{E}\alpha\beta_2]$$

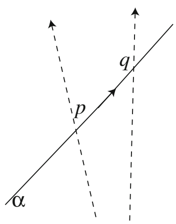
I 4. This axiom guarantees that if a PUM α intersects another PUM β_1, it intersects all other PUMs parallel to β_1. This axiom determines the dimension of the spacetime as a {1+1} spacetime.

$$\mathbf{E}\alpha\beta_1 \rightarrow (\forall\beta_2)[\mathbf{P}\beta_1\beta_2 \rightarrow \mathbf{E}\alpha\beta_2]$$

Axioms of Determinateness

A two-place relation of determinateness can now be imposed between events on a particular motion. The determinateness relation amounts to a causal relation between two events. The full articulation of this relation involves four PUMs, so that $\mathbf{D}pq$ asserts that the intersection $p =< \alpha_1, \alpha_2 >$ determines the intersection $q =< \beta_1, \beta_2 >$.

D 1. This axiom asserts that any pair of events p and q belonging to a motion stand within an antisymmetric causal relation, so that if $p, q \in \mathcal{S}(\alpha)$, then either p determines q or q determines p but not both.



$$[p, q \in \mathcal{S}(\alpha) \wedge (p \neq q)] \rightarrow (\mathbf{D}pq \vee \mathbf{D}qp) \wedge \sim (\mathbf{D}pq \wedge \mathbf{D}qp)$$

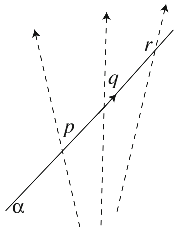
D 2. This axiom asserts the transitivity of the determinateness relations, so that if p determines q, and q determines r, then p determines r. Notice that this axiom does not require that the three events belong to the same PUM.

$$(\mathbf{D}pq \wedge \mathbf{D}qr) \rightarrow \mathbf{D}pr$$

Axiom of Betweenness

To articulate the "betweenness" axiom we first define the "betweenness" relation using the determinateness relation. The betweenness axiom could therefore be seen as further articulating the determinateness relation.

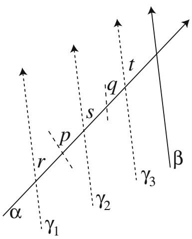
Definition 4 (Betweenness) The "betweenness" relation is a three place relation $\mathbf{B}xyz$ between events. An event q is between events q and r if and only if p, q and r belong to a single motion α, and either p determines q and q determines r or r determines q and q determines p.



$$\mathbf{B}pqr \equiv_{df} \left[p, q, r \in \mathcal{S}(\alpha) \wedge (p \neq q, q \neq r, p \neq r)\right.$$
$$\left.\wedge \left[(\mathbf{D}pq \wedge \mathbf{D}qr) \vee (\mathbf{D}rq \wedge \mathbf{D}qp)\right]\right]$$

There is one additional axiom involving the betweenness relations.

B 1. Assume that there are two events p and q belonging to the motion α. If there is a motion β that intersects α, there are three motions parallel to β that intersect α. One event intersects α prior to p and q, another intersects α between p and q, and the third intersects α after p and q. This axiom guarantees that each motion β that intersects a PUM α belongs to a class of parallel motions forming a dense set of intersection points with α.



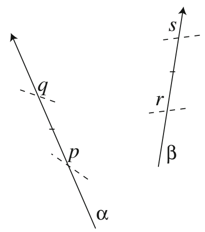
$$[p, q \in \mathcal{S}(\alpha) \wedge (\exists\beta)\mathbf{E}\alpha\beta] \rightarrow$$
$$(\exists\gamma_1)(\exists\gamma_2)(\exists\gamma_3)\left[\mathbf{P}\gamma_1\beta \wedge \mathbf{P}\gamma_2\beta \wedge \mathbf{P}\gamma_3\beta \wedge (\mathbf{E}\alpha\gamma_1 \wedge \mathbf{E}\alpha\gamma_2 \wedge \mathbf{E}\alpha\gamma_3)\right] \wedge$$
$$\left[\mathbf{B}rpq \wedge \mathbf{B}psq \wedge \mathbf{B}pqt\right]$$
$$r = <\alpha, \gamma_1>, s = <\alpha, \gamma_2>, t = <\alpha, \gamma_3>$$

Axioms of Congruence

Consider a particular PUM α and events generated by this PUM as a result of its intersections with other PUMs. The progress from one event on α to the next is determined by some internal process. When considering a PUM independently of any other PUMs, it is impossible to define a metric on it since there are no distinct events on it. But when one takes a particular PUM α and a class of PUMs with which it intersects, then the separation between events generated by intersections can be assessed as congruent or incongruent motion intervals.

The congruence relation can be defined as a four-place relation between events $\mathbf{C}pqrs$, where the first pair of events resides on motion α – i.e., $p, q \in \mathcal{S}(\alpha)$ – and the second pair of events resides on β, so that $r, s \in \mathcal{S}(\beta)$.

C 1. This axiom asserts that for any segment formed by two events on α, there is a congruent segment on β beginning with any event on β.



$$\left[p, q \in \mathcal{S}(\alpha), r \in \mathcal{S}(\beta) \rightarrow (\exists\gamma)[\mathbf{E}\beta\gamma \wedge \mathbf{C}pqrs\right], \textit{where } s = <\beta, \gamma>$$

C 2. This axiom asserts the transitivity of the congruence relation:

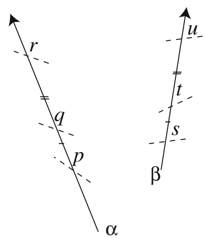
$$(\mathbf{C}pqrs \wedge \mathbf{C}rstu) \rightarrow \mathbf{C}pqtu$$

C 3. This axiom asserts the additivity of congruent segments (i.e., the sums of congruent segments are congruent):

For three events p, q, r assume that $\mathbf{D}pq$ *and* $\mathbf{D}qr$*. Also, assume that there are three events s, t, u such that* $\mathbf{D}st$ *and* $\mathbf{D}tu$*.*

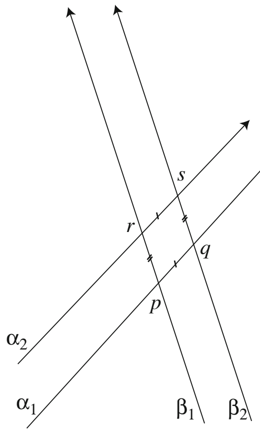
$$(p, q, r \in \mathcal{S}(\alpha) \wedge \mathbf{D}pq \wedge \mathbf{D}qr) \wedge (s, t, u \in \mathcal{S}(\beta) \wedge \mathbf{D}st \wedge \mathbf{D}tu) \rightarrow$$

$$(\mathbf{C}pqst \wedge \mathbf{C}qrtu) \rightarrow \mathbf{C}prsu$$



C 4. To simplify the account, an axiom is added to guarantee that the resulting spacetime is flat. The flatness condition asserts that any two parallel motions take the same motion interval to cross the distance between two other parallel motions.

$(\mathbf{P}\alpha_1\alpha_2 \wedge \mathbf{P}\beta_1\beta_2)\wedge$
$(\mathbf{E}\alpha_1\beta_1 \wedge \mathbf{E}\alpha_1\beta_2 \wedge \mathbf{E}\alpha_2\beta_1 \wedge \mathbf{E}\alpha_2\beta_2)\wedge$
Define $p = <\alpha_1, \beta_1>, q = <\alpha_1, \beta_2>, r = <\alpha_2, \beta_1>, s = <\alpha_2, \beta_2>$
$(\mathbf{D}pq \wedge \mathbf{D}rs \wedge \mathbf{D}pr \wedge \mathbf{D}qs) \rightarrow \mathbf{C}pqrs \wedge \mathbf{C}prqs$



The congruence relation allows us to pick a unit of motion interval ΔI and to define a congruence relation $\cong$ between motion intervals $\overline{pq}$ connecting events belonging to a PUM.

Axioms of Continuity

The axioms of continuity are now formulated:

R 1. This axiom guarantees the finitude of every segment $\overline{pq}$ on a PUM.

If $p, q \in \mathcal{S}(\alpha)$ *and* $\mathbf{D}pq$*,* $r, s \in \mathcal{S}(\beta)$*, then there exist n events* $t_i \in \mathcal{S}(\alpha)$ *producing n segments congruent to* $\overline{rs}$*, so that a segment of the length* $n\Delta I(r, s)$ *covers the segment* pq*. More formally, this axiom looks as follows:*
$(p, q \in \mathcal{S}(\alpha) \wedge \mathbf{D}pq \wedge r, s \in \mathcal{S}(\beta)) \rightarrow$
$(\exists\gamma_1)(\exists\gamma_2)\ldots(\exists\gamma_n)(\mathbf{E}\alpha\gamma_1 \wedge \mathbf{E}\alpha\gamma_2 \wedge \ldots \mathbf{E}\alpha\gamma_n)$
Assuming $t_1 = <\alpha, \gamma_1>, t_2 = <\alpha, \gamma_2>, \ldots t_n < \alpha, \gamma_n >,$
$(\mathbf{D}pt_1 \wedge \mathbf{D}t_1t_2 \wedge \ldots \mathbf{D}t_{n-1}t_n)\wedge (\mathbf{C}pt_1rs \wedge \mathbf{C}t_1t_2rs \wedge \ldots \mathbf{C}t_{n-1}t_nrs) \wedge \mathbf{D}qt_n.$

R 2. This axiom guarantees that there is a one to one correspondence between events on a PUM and points on a real line.

For $\mathcal{S}(\alpha)$ assume that there are two non-empty sets Σ_1, Σ_2 such that:

1. $\mathcal{S}(\alpha) = \Sigma_1 \cup \Sigma_2$
2. $p \in \Sigma_1 \rightarrow \sim (\exists q)[q \in \Sigma_2 \wedge \mathbf{D}qp]$
3. $p \in \Sigma_2 \rightarrow \sim (\exists q)[q \in \Sigma_1 \wedge \mathbf{D}pq]$

$(\exists\beta)[\mathbf{E}\alpha\beta \wedge [Assuming\ \ o = <\alpha, \beta>, \mathbf{B}poq \leftrightarrow (p \in \Sigma_1) \wedge (q \in \Sigma_2) \wedge (p \neq o \neq q)]]$

So far a geometry of PUMs has been articulated. But this geometry is not complete, since it does not explain the congruence relation between two different paths for connecting a pair of events. The geometry is missing the analogue for Hilbert's congruence relations for angles between segments, which complete the system for Hilbert's Euclidean geometry. In the context of spacetime geometry, the congruence relations for "relative velocities" or the "angle" separating two motions is missing. This requires that one would introduce the decomposition of motion intervals into their spatial and temporal components, which is carried out in the next section. It is the definition of these "angles" between motions that will ultimately lead to either Galilean or relativistic spacetimes.

3.3 Galilean Spacetime

To complete the PUM geometry, a PUM is projected onto a class of parallel motions. Once events are generated, the decomposition of the motion interval into its spatial and temporal components can be carried out. When a motion α intersects a class of parallel motions, the motion intervals on the parallel motions function as the time intervals dt for the intersecting motion α. The separation between parallel motions functions as the spatial interval. The distance between parallel motions can be assessed by the "number" of PUMs that are crossed by α. But notice that the decomposition of a motion interval into spatial and temporal components does not presuppose the independent existence of the spatial and temporal metrics. What is presupposed is that one can define the notion of relative velocity, or the "angle" between the PUMs, which is the "ratio" between the spatial and temporal progression of a PUM when it is projected onto a class of parallel motions. Thus, motion intervals and relative velocities are the basic metrics of the spacetime, not the spatial and the temporal metrics.

3.3.1 Reconstructing Galilean Spacetime

The decomposition of motion intervals into their spatial and temporal components requires the definition of a spatial and temporal metric for a particular set of parallel

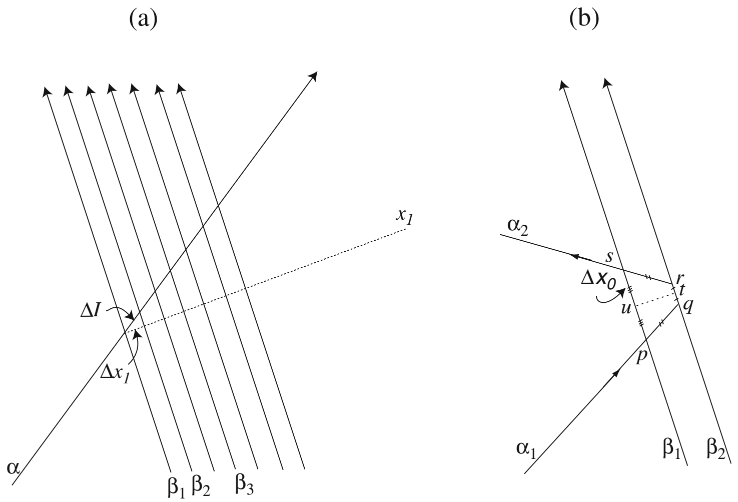


Fig. 3.2 Projection onto the spatial dimension

motions $\mathcal{V}$. A spatial metric on the separation between parallel motions can be defined using a randomly picked α that intersects the parallel motions. By definition, the reference α is treated as a PUM moving with unit velocity relative to $\mathcal{V}$. The motion intervals on α can then be used to define the spatial distances between members of $\mathcal{V}$ (see Fig. 3.2a).

Definition 5 (Spatial Metric) Assume that $\mathcal{V}$ is a set of parallel PUMs β_i, so that $\mathbf{P}\beta_i\beta_j$ for every $\beta_i, \beta_j \in \mathcal{V}$. Assume that a motion α is intersecting all members of $\mathcal{V}$. Let $p_i = < \alpha, \beta_i >$ (i.e., p_i is the intersection of α with motion β_i). Then let $x_1(\beta_i, \beta_j)$ be the motion interval on α connecting p_i and p_j.

The flatness condition **C4** guarantees that the spatial metric does not depend on the motion chosen as reference, up to an arbitrary unit of length.

Relations of simultaneity on parallel motions will now be defined. This is done in a manner similar to Einstein's definition of simultaneity which used light signals. Take two parallel motions $\mathbf{P}\beta_1\beta_2$ and let a PUM run from β_1 to β_2, and another PUM from β_2 to β_1. If motion intervals between intersection points are the same, one can define a relation of simultaneity between half-points on the parallel motions (see Fig. 3.2b).

Definition 6 (Simultaneity) Assume that a motion α_1 intersects parallel motions β_1, β_2 (i.e., $\mathbf{P}\beta_1\beta_2$) at events $p = < \alpha_1, \beta_1 >$ and $q = < \alpha_1, \beta_2 >$ and that α_2 intersects these motions at $r = < \alpha_2, \beta_2 >$ and $s = < \alpha_2, \beta_1 >$, and that $\mathbf{D}pq$, $\mathbf{D}qr$ and $\mathbf{D}rs$. If $\overline{pq} \cong \overline{rs}$, events $u \in \mathcal{S}(\beta_1)$ and $t \in \mathcal{S}(\beta_2)$ are defined simultaneous if and only if $\overline{pu} \cong \overline{us}$ and $\overline{qt} \cong \overline{tr}$.

The definition of relations of simultaneity now enables the definition of a temporal metric. First, it can be easily shown relying on the flatness condition **C4** that the relations of simultaneity forms an equivalence class on all events belonging to motions in the class of parallel motions $\mathcal{V}$. The flatness condition also guarantees a universal metric of motion intervals on motions belonging to $\mathcal{V}$. (I omit the proof for brevity.) One can then define a temporal metric on motions in $\mathcal{V}$.

Definition 7 (Temporal Metric) Let β_i, β_j be parallel motions, so that $\mathbf{P}\beta_i\beta_j$. Also let $u_0, u_1 \in \mathcal{S}(\beta_1)$ and $t_0, t_1 \in \mathcal{S}(\beta_2)$. Let u_0 and t_0 be simultaneous, and u_1 and t_1 be simultaneous. It follows that the motion intervals $\overline{u_0u_1} \cong \overline{t_0t_1}$. Define $\overline{u_0u_1}$ as the time that elapsed between events u_0 and u_1.

The previous definitions demonstrate that given a geometry of PUMs, one can use motion intervals to define spatial and temporal metrics on each class of parallel motions. But the spacetime structure is incomplete since one has yet to define the relation between the spatial metric and the temporal metric when a motion progresses relative to a class of parallel motions. The various PUMs that cross the same pair of parallel PUMs may "take more time" to traverse the distance. Thus a geometry of PUMs ought to determine the function correlating the temporal and spatial metrics. The basis of Galilean spacetime is the "Galilean" decomposition of a PUM. According to the Galilean PUM, the temporal duration Δx_0 correlates linearly with the spatial interval Δx_1.

C 5. Galilean Paradigm of Uniform Motion (GPUM)
$\mathbf{E}\alpha\beta_1 \wedge \mathbf{E}\alpha\beta_2 \wedge \mathbf{P}\beta_1\beta_2 \rightarrow$
$\Delta x_0(p, q) = a\Delta x_1(p, q)$, *where* $p = <\alpha, \beta_1>$ *and* $q = <\alpha, \beta_2>$

The meaning of the GPUM is that the spatial progression of a motion is linearly related to the temporal progression of the parallel motions it intersects. Notice the temporal and spatial distances along a PUM are not independent parameters – they arise "together" from the more basic phenomenon of motion and its projection on a class of parallel motions. According to this approach, motion is not the *transition* from one potential event to another, it is the generation of coincidences via the unfolding of motions. The linear relation in **C5** describes the various possible PUMs connecting the parallel motions β_1 and β_2 (see Fig. 3.3a).

Using the relation in the GPUM, potential events may now be labeled in the spatial and temporal "dimensions". Once potential events are labeled in the spacetime, one can trace the trajectories of *actual* objects that are not moving uniformly as in Fig. 3.3b. Each infinitesimal part of the actual trajectory is identical to an infinitesimal part of a PUM. This would enable one to describe the trajectory of the object with a function $x_1(x_0)$. Each infinitesimal section of a body's trajectory will actualize an infinitesimal part of a PUM.

The geometry of PUMs describes potential trajectories and intersections between them. This geometry underwrites the temporal and the spatial metrics. The actual measurements of space and time require the use of clocks and measuring rods. Thus one requires a "scheme" for translating the measurements of length and duration to the underlying spacetime. The actual measurements of length and duration destroy

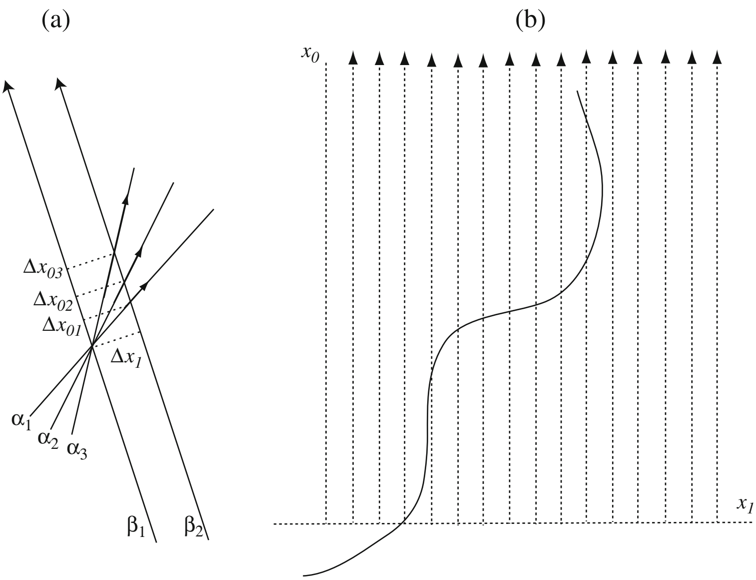


Fig. 3.3 Galilean paradigm of uniform motion

the implicit conceptual connection between space and time that is established via the PUMs.

To re-establish the conceptual connection between measurements of length and duration and the PUMs geometry, imagine a hypothetical process via which clocks and measuring rods are calibrated. Einstein's initial presentation of STR *presupposes* that one has at a particular location periodic clocks in which the same unit of time repeats itself. It is this presupposition, that one has a calibrated clock at a particular location, and that each reference frame has a set of calibrated measuring rods, that is now being examined.

For the purpose of calibrating the clock, one needs to use a body that approximates a PUM α as closely as possible.[6] Assume that the motion α intersects two parallel motions PUMs β_1 and β_2, so that $\mathbf{P}\beta_1\beta_2$. The body intersects β_1 at p_1, then travels and intersects β_2 at p_2. The body is reflected back to approximate a PUM γ_1 that intersects β_2 at p_3. The body travels again from β_1 to β_2, approximating a motion α_2, parallel to α_1, until it intersects β_2 at p_4. It then reverses its course and approximates a PUM γ_2, which is parallel to γ_1, until it intersects β_1 at p_5, at which point it reverses its course. The object then approximates a PUM α_3 which is parallel to α_1, until it hits β_1 at p_6, and so forth. Since PUMs are traveling between

[6] In practice when calibrating clocks physicists often rely on objects that actualize, with good approximation, inertial motion. For example, the earth's rotation around its axis is governed by the conservation of angular momentum to a good approximation. The sun and the earth orbiting the sun provide another good approximation. While these motions do not actualize a linear progression of a PUM, they do resemble PUMs between one instant and the next.

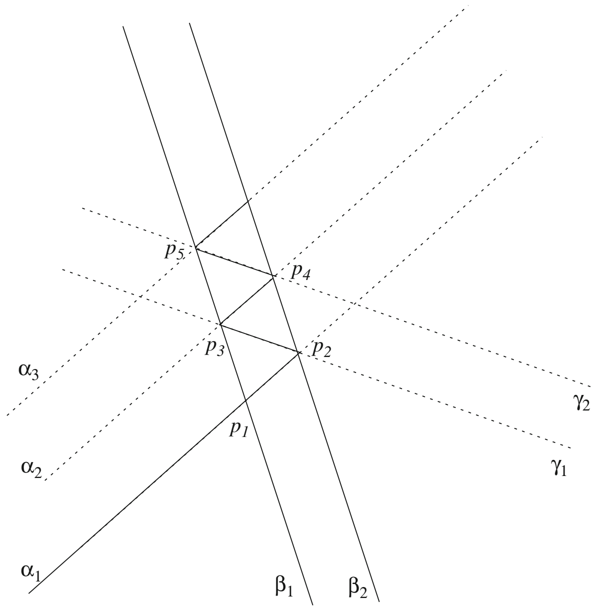


Fig. 3.4 Decomposition of motion intervals

two parallel PUMs β_1 and β_2, the object is traveling the same distance. Thus, if each turn is defined as a unit of time, the events can be marked with coordinates $p_1 = < 0, x_1 >$, $p_2 = < 1, x_2 >$, $p_3 = < 3, x_1 >$, $p_4 = < 4, x_2 >$, $p_5 = < 5, x_1 >, \ldots$, etc. If the uniformly moving object approximates the PUMs well, a clock that is at rest relative to β_1 and produces events coincident with $p_1, p_3, p_5 \ldots$ will be a good clock (see Fig. 3.4).

The calibration of the clock's period crucially depends on two assumptions. First, that the objects used for calibrating the clock actualize PUMs, and that the two PUMs β_1, β_2 are parallel. Both assumptions are trivial from the point of view of our abstract geometry, but complex to verify from a practical point of view. The physical procedure for verifying these assumptions require full knowledge of the dynamic laws. Thus one has an epistemological problem in understanding the relation between abstract geometry and the use of sensible measures that rely on clocks and rods. The validity of abstract geometry is ascertained through the behavior of clocks and rods. On the other hand, the calibration of clocks and rods requires full knowledge of dynamic laws, and empirical knowledge of dynamic laws presupposes the validity of geometric laws. I shall not go into the epistemological discussion here in any detail, except to say that while full knowledge of the dynamic laws is required for the calibration of clocks and rods, it is still valid procedure to differentiate between fundamental and complex motions. The geometry of PUMs should be considered as the laws which lay out the structure of the fundamental motions; dynamic laws as those which describe how to combine complex motions from fundamental ones. The separation between geometry and dynamic laws then becomes the heuristic distinction between the fundamental building blocks and the details of particular structures. Both geometric and dynamic laws are verified in each experiment, but geometric laws governing fundamental motions are given epistemological priority, since they provide a common fundamental structure for all motions.

To calibrate a measuring rod consider a similar process relying on PUMs. To calibrate a rod one has to verify that the rod's "0" and "1" marks are coincident with parallel PUMs β_1 and β_2. Formally this implies that the marks actualize paths of parallel PUMs. What is significant is that a PUM would travel a certain amount of time before it traverses the distance between the "0" and "1" marks. Taking into account the GPUM, the events produced by the marks would be $p_1 =< 0, 0 >$, $p_2 =< 1, 1 >$, $p_3 =< 2, 0 >$ and so forth. If this calibration is to work, one has to assume that the rod is at rest relative to the parallel motions β_1 and β_2. The calibration of the distance between the marks can now be done by letting the uniformly moving object travel back and forth between "0" and "1". If the object traverses the distance while the same amount of time elapses, then one may conclude that the measuring rod is calibrated (see Fig. 3.4).

The first conclusion from the above analysis, is that the notion of time and distance are abstracted from the particular events produced by PUMs. One should distinguish between the duration that elapsed from one event to another, a duration that corresponds to the cycles of the clock, and the time one takes the clock to represent. The former duration is conceptually linked to the process by which the clock is calibrated, and it is inseparable from the PUM. The concept of time one takes the clock to be *representing* is abstracted away from the particular way the clock is calibrated. The consequence is the metaphysical view which thinks of time as "flowing" independently of any particular motion. Similarly, one should distinguish between the distance between the events produced by the marks on the rod and the abstracted distance one takes to exist between them. The former is conceptually linked to the process by which the rod is calibrated, and it is inseparable from PUMs. The distance one takes to be *represented* by the rod is abstracted from any particular motion or time.

The abstraction of duration and length from the process by which clocks and rods are implicitly calibrated generates the impression that duration and length exist independently of one another and independently of the motions of bodies. However, if the geometry of PUMs describes spacetime reality, the concept of duration and length should always be assessed as the duration and length that elapse between *events generated by uniformly moving objects*.

3.3.2 Galilean Transformations

There is a conceptual relation between the GPUM and the equivalence between inertial reference frames. The structure of the GPUM ensures that selecting an alternative set of parallel motions, while keeping the same GPUM, yields equivalent representations of the evolution of PUMs.

In the previous section it was assumed that a coordinate system is based on clocks and rods at rest relative to the class of parallel motions that forms the reference for decomposing the spatial and temporal dimensions. This coordinate system was erected with the help of the GPUM, which assumed a linear relation between the spatial and temporal metrics.

Consider the multiple ways in which a motion could be decomposed. First, a motion α is decomposed relative to a set of parallel motions $\mathcal{V}$. Relative to $\mathcal{V}$, α traversed a distance Δx_1 for every duration Δx_0. But there is also the possibility of there existing a set of parallel motions $\mathcal{V}'$ relative to which the traversing PUM crosses a larger distance, $\Delta x_1'$ by a factor of μ during the same amount of time:

$$\begin{aligned} \Delta x_1 &\mapsto \Delta x_1' = \mu \Delta x_1 \\ \Delta x_0 &\mapsto \Delta x_0' = \Delta x_0 \end{aligned} \tag{3.2}$$

This transformation is the mapping from a pair of reference parallel motions $\mathbf{P}\beta_1\beta_2$ to another pair of motions β_1', β_2' that are also parallel but belong to a different set $\mathcal{V}'$ (see Fig. 3.5). Since the axiom **C4** assumes a linear relation between the spatial and the temporal infinitesimals, it also guarantees that the same relation holds relative to another class of parallel motions. The linear relation between spatial and temporal units determines important aspects of the spacetime. It is clear that the relative velocity between PUMs is not bounded because of the structure of GPUM.

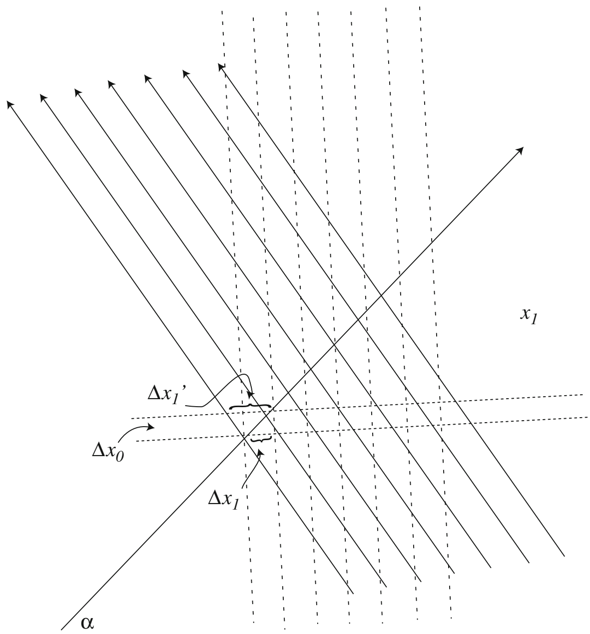


Fig. 3.5 Stretching the spatial dimension relative to the temporal dimension

There is nothing preventing us from letting the "stretching parameter" μ approach infinity, which implies that relative velocities between PUMs can approach infinity as well. Moreover, the unbounded relative velocity also implies absolute simultaneity. Assume that p_1 is the intersection between a PUM β_1 and a signal α, p_2 is the intersection between α and another PUM β_2, and that the signal α returns to β_1 and intersects it at p_3. The time defined as $\frac{t(p_3)-t(p_1)}{2}$ is identified as the instant on β_1 that is simultaneous with p_2. If the relative velocity between α and β_1, β_2 approaches infinity, the duration between p_1 and p_3 will approach zero, which implies that one can approach as close as one wants a single, absolute definition of simultaneity. Thus, the linear nature of the GPUM, because it enables unbounded relative velocities, is also conceptually linked to absolute relations of simultaneity.

The transformation between the old and the new coordinate systems $K_G = \langle x_0, x_1 \rangle$ and $K'_G = \langle x'_0, x'_1 \rangle$ can now be defined. This relation between these coordinate systems is described with the following transformation:

$$\Delta = \begin{pmatrix} 1 & 0 \\ 0 & \mu \end{pmatrix} \tag{3.3}$$

If the transformation Δ were to describe a stretch (or a contraction) in a spatial dimension that was completely independent of the temporal dimension, then Δ would merely be a stretch of spatial units. But ordinary coordinate systems represent the spatial and temporal intervals as measured by clocks and rods, not the rescaled distance between the *events* produced by the PUMs. Thus the transformation in (3.2) does not necessarily reflect the length measured with our rods.

To represent the transformation Δ in the coordinates measured by rods and clocks, one has to rely on the GPUM. Assume that the distance separating two parallel β_1 and β_2 which was covered by a motion during the time Δx_0 now covers the distance $\Delta x'_1 = \mu \Delta x_1$ separating two parallel motions β'_1, β'_2 of a different class of parallel motions. To calibrate a measuring rod in K'_G, one takes the signal to be first coincident with the "0" mark and then with the "1" mark of the rod. This could only be possible if the rod is moving at a velocity v in order to traverse the distance $(\mu - 1)\Delta x_1$ during the time Δx_0, which is the distance in K_G between the "1" mark on the rod at rest in K_G, and the place where the mark "1" is coincident with β_2. If one takes the motion of the "0" mark on the rod to be indicating the origin of K'_G, it follows that the origin of K'_G moves with a velocity:

$$v = (\mu - 1)\frac{\Delta x_1}{\Delta x_0} \tag{3.4}$$

relative to K. Thus, a "stretching" of the spatial dimension Δx_1 relative to the temporal dimension Δx_0 leads to the requirement that the measuring rods travel at a uniform motion relative to the original rods. But it is the particular *structure* of the GPUM, i.e., the linear relation between the motion interval and the spatial and temporal displacement, that allows one to move from the initial description of the transformation Δ to an account of the transformation in terms of measurement rods.

The clock in the new reference frame will be calibrated in the same way as the old reference frame, only now it will be at rest relative to the motions in $\mathcal{V}'$.

In other words, the transformation $\Delta^\dagger$ between the coordinates systems, that represents the transformation between measurement results, is:

$$\Delta^\dagger = \begin{pmatrix} 1 & 0 \\ -v & 1 \end{pmatrix} \tag{3.5}$$

One knows that the length and the duration in the new coordinate system must be measured using clocks and rods moving with a velocity v relative to the previous clocks and rods.

The above is essentially a reconstruction of a $\{1+1\}$ Galilean (or neo-Newtonian) spacetime from the GPUM. This reconstruction explains several facts about spacetime theory. First, it provides an explanation of the "relativity of motion," which differs from traditional accounts that appeal to metaphysical or empiricist principles. The traditional argument for the relativity of motion begins with the assumption that only change in the relative positions between bodies can be observed (or conceptualized), and thus motion is inherently a relative concept. But this argument presupposes that spatial relations between objects can be determined prior to the measurement and even conceptualization of motion. The approach here turns the argument for relativity of motion on its head; it is meaningless to posit a distance separating two objects without considering the counterfactual uniform motion that would connect the two bodies. Without the notion of uniform motion we do not have a conceptual comparison between distances measured in various locations.

Finally, the Galilean rule for adding velocities stems from the GPUM, since the process of adding velocities v_1 and v_2 is equivalent to the process of boosting velocity v_1 by a velocity $-v_2$. Since the GPUM describes a linear relation between Δx_0 and Δx_1, the velocity boosts also result in a linear sum between the original velocity v_1 and the velocity boost v_2.

In subsequent chapters it is assumed that a $\{1+3\}$ spacetime can be constructed from a geometry of PUMs. It will not be shown here how this can be constructed, but it will be assumed that it is possible to do so. First one assumes that there is a class of parallel motions $\mathcal{V}$. One takes three PUMs moving in three different orthogonal directions to define spatial distances between the parallel PUMs (the above axiom **I4** will have to be given up). It is assumed that on a particular hyperplane of this set of parallel motions, distances obey the Euclidean relation $\Delta r = \sqrt{(\Delta x_1)^2 + (\Delta x_2)^2 + (\Delta x_3)^2}$. Thus, when another set of parallel motions $\mathcal{V}'$ is chosen, the most general transformation between the two coordinate systems is:

$$\Delta = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \mu_1 & 0 & 0 \\ 0 & 0 & \mu_2 & 0 \\ 0 & 0 & 0 & \mu_3 \end{pmatrix} \tag{3.6}$$

Relative to a set of rods and clocks the transformation will look as follows:

$$\Delta^\dagger = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -v_1 & 1 & 0 & 0 \\ -v_2 & 0 & 1 & 0 \\ -v_3 & 0 & 0 & 1 \end{pmatrix} \tag{3.7}$$

The transformation (3.7) is the standard Galilean transformation between equivalent reference frames.

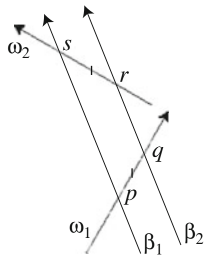
3.4 Flat Relativistic Spacetime

The above reconstruction of Galilean spacetime from the Galilean PUM gives support to the claim that spacetime is derived from the structure of uniform motions and their decomposition to spatial and temporal metrics relative to sets of parallel motions. To make this a viable thesis that extends beyond Galilean spacetime, it should be possible to give an analogous reconstruction of the spacetime theory implicit in the Special Theory of Relativity. The initial clue is Einstein's elevation of light waves into universal clocks and rods. One may think of Einstein's Light Postulate as the introduction of a new, relativistic Paradigm of Uniform Motion.

3.4.1 Reconstructing Flat Relativistic Spacetime

Galilean spacetime was reconstructed in the previous section from the geometry of PUMs. In addition to motion intervals defined in Section 3.2, the spacetime was articulated further using the GPUM, described in **C5**. According to the GPUM, the progression of a motion α relative to a class of parallel motions involves a linear relation between the spatial progression, across the parallel motions, and the temporal progression along the parallel motions. When a motion is decomposed relative to another set of parallel motions, the GPUM dictates that the spatial separation $\Delta x'_1$ traversed by α relative to the "boosted" frames is the separation Δx_1 multiplied by a factor μ in the non-boosted frame. This alternative decomposition of a PUM is in essence a change from one inertial reference frame to another. In relativistic spacetime, a similar geometry of PUMs provides the basis for the spacetime, so that the axiomatic system in Section 3.2 is taken as a structure common to both spacetimes. However, the Galilean linearity assumption in **C5** is given up. Instead, two privileged sets of parallel motions $\mathcal{V}_1^0$ and $\mathcal{V}_2^0$ are presupposed to exist. The motions $\omega \in \mathcal{V}_1^0$ represent light waves going in one direction, while motions $\omega \in \mathcal{V}_2^0$ represent light waves going in the opposite direction. These privileged motions set the paradigm for the spatial and metric decompositions of motion intervals. The first assumption in relativistic spacetime guarantees that motions in $\mathcal{V}_1^0$ and $\mathcal{V}_2^0$ have equivalent motion intervals no matter which set of parallel motions is used as reference. This assumption amounts to asserting that motions in these privileged classes of motion provide an absolute standard of motion intervals. This assumption is our reconstruction of the Light Postulate. To ensure this presupposition, another congruence axiom is added to those given in Section 3.2:

C 6. There exist two classes of parallel motions $\mathcal{V}_1^0$ and $\mathcal{V}_2^0$ such that members of these classes represent absolute motion intervals. These privileged motions in $\mathcal{V}_1^0$ and $\mathcal{V}_2^0$ move in opposite directions, and motion intervals on intersections between two parallel motions intersecting $\mathcal{V}_1^0$ and $\mathcal{V}_2^0$ are always the same.



$$\mathbf{P}\beta_1\beta_2 \rightarrow (\exists\omega_1)(\exists\omega_2)[\omega_1 \in \mathcal{V}_1^0, \omega_2 \in \mathcal{V}_2^0]\wedge$$
$$\wedge[\mathbf{E}\beta_1\omega_1 \wedge \mathbf{E}\beta_2\omega_1 \wedge \mathbf{E}\beta_1\omega_2 \wedge \mathbf{E}\beta_2\omega_2] \wedge [\mathbf{D}pq \wedge \mathbf{D}rs] \rightarrow \mathbf{C}pqrs$$
where $p = <\beta_1, \omega_1>, q = <\beta_2, \omega_1>, r = <\beta_2, \omega_2>$ *and* $s = <\beta_1, \omega_2>$

The privileged motions in $\mathcal{V}_1^0$ and $\mathcal{V}_2^0$ can be used to define relations of simultaneity on members of a class of parallel motions. This is carried out by sending a light wave from PUM β_1 to a parallel motion β_2, and then another light wave from β_2 to β_1:

Definition 8 (Relativistic Simultaneity) Assume that a motion $\omega_1 \in \mathcal{V}_1^0$ intersects parallel motions β_1, β_2, i.e., $\mathbf{P}\beta_1\beta_2$ at events $p = <\omega_1, \beta_1>$ and $q = <\omega_1, \beta_2>$ and that $\omega_2 \in \mathcal{V}_2^0$ intersects these motions at $r = <\omega_2, \beta_2>$ and $s = <\omega_2, \beta_1>$, and that $\mathbf{D}pq$, $\mathbf{D}qr$ and $\mathbf{D}rs$. Since $\overline{pq} \cong \overline{rs}$, events $u \in \mathcal{S}(\beta_1)$ and $t \in \mathcal{S}(\beta_2)$ are defined simultaneous if and only if $\overline{pu} \cong \overline{us}$ and $\overline{qt} \cong \overline{tr}$.

Once relations of simultaneity are defined on a class of parallel motions $\mathcal{V}$, the motion intervals on members of this class can be defined as a temporal metric (see Fig. 3.6a).

Definition 9 (Temporal Metric) Let β_i, β_j be parallel motions, so that $\mathbf{P}\beta_i\beta_j$. Also let $u_0, u_1 \in \mathcal{S}(\beta_1)$ and $t_0, t_1 \in \mathcal{S}(\beta_2)$. Let u_0 and t_0 be simultaneous, and u_1 and t_1 be simultaneous. It follows that the motion intervals $\overline{u_0u_1} \cong \overline{t_0t_1}$. Define $x_0(u_0, u_1)$ as the time that elapsed between events u_0 and u_1.

To define a spatial metric between members of a set of parallel motions $\mathcal{V}$ we let the motion α intersect the members $\beta_i \in \mathcal{V}$. Define the spatial metric using motion intervals on α.

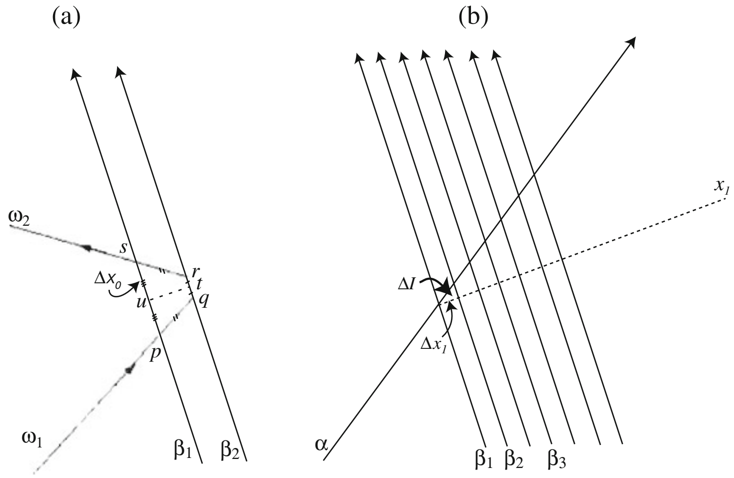


Fig. 3.6 Simultaneity in relativistic spacetime

Definition 10 (Spatial Metric) Assume that $\mathcal{V}$ is a set of parallel PUMs β_i, so that $\mathbf{P}\beta_i\beta_j$ for every $\beta_i, \beta_j \in \mathcal{V}$. Assume that a PUM α is intersecting all members of $\mathcal{V}$. Let $p_i = <\alpha, \beta_i>$, i.e., p_i is the intersection of α with motion β_i. Then define $x_1(\beta_i, \beta_j)$ as the motion interval on α connecting p_i and p_j.

Once the spatial and temporal metrics are given by decomposing motions onto a class of parallel motions, it is possible to define the Relativistic Paradigm of Uniform Motion. According to this additional congruence axiom, the spatial and temporal displacement of motions in $\mathcal{V}_1^0$ and $\mathcal{V}_2^0$ are related as follows:

C 7 (Relativistic Paradigm of Uniform Motion (RPUM)).
$\mathbf{E}\omega\beta_1 \wedge \mathbf{E}\omega\beta_2$ *and* $p = <\omega, \beta_1>, q = <\omega, \beta_2>$
$c^2(\Delta x_0(p,q))^2 - (\Delta x_1(p,q))^2 = 0$

To satisfy the RPUM, we take the motions in $\mathcal{V}_1^0$ to be waves moving in the positive direction of x_1, and motions in $\mathcal{V}_2^0$ to be waves moving in the opposite direction. Thus the following are wave solutions for each class of motions:

$$\psi_+(x_{0+}, x_{1+}) = e^{i\omega x_{0+} + ikx_{1+}}$$
$$\psi_-(x_{0-}, x_{1-}) = e^{i\omega x_{0-} - ikx_{1-}} \tag{3.8}$$

where $\omega = \frac{2\pi}{T}$, $k = \frac{2\pi}{\lambda}$, c is the wave's velocity, T is the wave's period, λ is the wavelength, and $cT = \lambda$. The spatial and temporal metric are not yet universal temporal and spatial metrics, since each wave equation describes the spatial progression of the wave relative to its temporal progression. It is still required to construct spatial and temporal metrics from the wave progressions. The spatial distance measured by

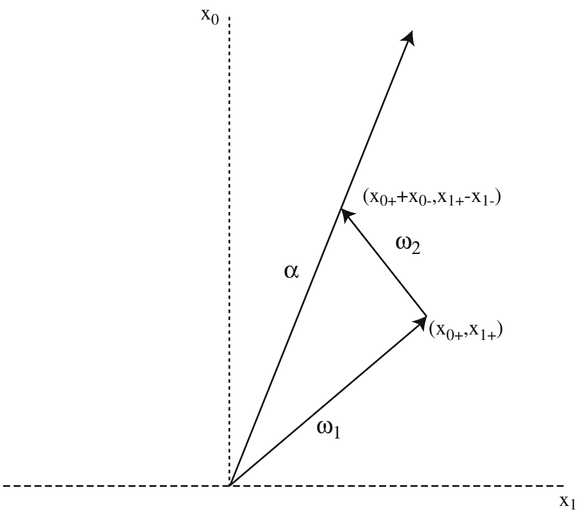


Fig. 3.7 Constructing spatial and temporal measures from light rays

rods and temporal duration measured by clocks should be constructed from the two wave solutions moving in opposite directions.

One can use the above solutions to the RPUM to define the units of length and duration. To do this, assume that a PUM travels from the origin of spacetime $< 0, 0 >$ to a point in spacetime $< x_0, x_1 >$. The same spatiotemporal distance can be described with the help of two privileged motions w_1 and w_2 (see Fig. 3.7). It is clear that the time elapsed is the sum of the times traversed by the two privileged motions, and the distance is the difference between the distances covered by each motion. The result is the following relation:

$$
\begin{aligned}
x_0 &= x_{0+} + x_{0-} \\
x_1 &= x_{1+} - x_{1-} = cx_{0+} - cx_{0-}
\end{aligned}
\tag{3.9}
$$

The equations in (3.9) describe the relation between the waves in (3.8) and the time and length measured by clocks and rods.

3.4.2 The Lorentz Transformations

We define events in our spacetime according to the RPUM. This set of possible events forms a coordinate reference frame $K_R = \langle cx_0, x_1 \rangle$. It is possible to decompose the same light signals relative to a different set of parallel motions $\mathcal{V}'$. Since the congruence axiom **C6** is still valid, the motion interval on a segment of $\omega_1 \in \mathcal{V}_1^0$ intersecting two motions in $\mathcal{V}'$ is congruent to the segment on $\omega_2 \in \mathcal{V}_2^0$ (see Fig. 3.8).

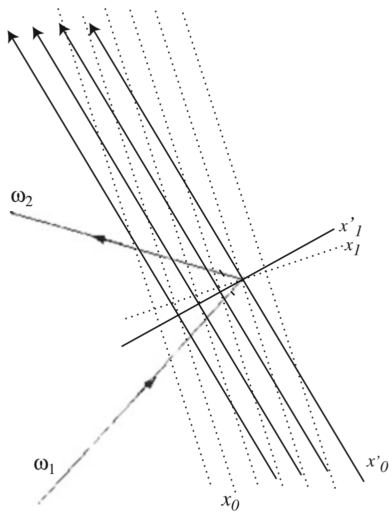


Fig. 3.8 Stretching and contracting wavelengths

Decomposing the same light signals relative to a different set of parallel motions $\mathcal{V}'$ is equivalent to expanding ω_1's wavelength by a factor $\mu_\alpha > 0$ and ω_2's wavelength by a factor $\mu_\beta > 0$. Assume therefore that:

$$\lambda_+ \mapsto \lambda'_+ = \mu_\alpha \lambda_+ \qquad (3.10)$$
$$\lambda_- \mapsto \lambda'_- = \mu_\beta \lambda_-$$

Assuming that wavelength velocity c remains constant, from $\lambda = cT$ one may conclude that the periods of the wave transform in the same way:

$$T_+ \mapsto T'_+ = \mu_\alpha T_+ \qquad (3.11)$$
$$T_- \mapsto T'_- = \mu_\beta T_-$$

Such a stretching of the wavelength should result in an alternative coordinate reference frame, which is defined as:

$$x'_0 = x'_{0+} + x'_{0-} = \mu_\alpha x_{0+} + \mu_\beta x_{0-} \qquad (3.12)$$
$$x'_1 = x'_{1+} - x'_{1-} = \mu_\alpha x_{1+} - \mu_\beta x_{1-}$$

Equation (3.12) could be rewritten in terms of a transformation Λ between $K_R = \langle cx_0, x_1 \rangle$ and $K'_R = \langle cx'_0, x'_1 \rangle$, so that:

$$\Lambda = \frac{1}{2} \begin{pmatrix} \mu_\alpha + \mu_\beta & \mu_\alpha - \mu_\beta \\ \mu_\alpha - \mu_\beta & \mu_\alpha + \mu_\beta \end{pmatrix} \qquad (3.13)$$

The inverse transformation Λ^{-1} has the following form:

$$\Lambda^{-1} = \frac{1}{2\mu_\alpha \mu_\beta} \begin{pmatrix} \mu_\alpha + \mu_\beta & -(\mu_\alpha - \mu_\beta) \\ -(\mu_\alpha - \mu_\beta) & \mu_\alpha + \mu_\beta \end{pmatrix} \tag{3.14}$$

For the transformations to be symmetric, impose the condition that $|\Lambda| = |\Lambda^{-1}|$. This imposes the condition that a unit of length and a unit of time transform in the same way by Λ and Λ^{-1}. To get this equality one has to set

$$\mu_\alpha \mu_\beta = 1 \tag{3.15}$$

Solving (3.14) for $x'_1 = 0$ and $x_1 = vx_0$ one gets the following solution for the relative velocities between the coordinate reference frames:

$$v = \left(\frac{\mu_\alpha + \mu_\beta}{\mu_\alpha - \mu_\beta} \right) c \tag{3.16}$$

One can separate between two cases, $\mu_\alpha < 1, \mu_\beta > 1$ on the one hand, and $\mu_\alpha > 1, \mu_\beta < 1$ on the other hand. In the first case, it follows from (3.16) that $v \leq c$. In the second case it follows that $v \geq c$. The discussion here shall be restricted to $v \leq c$, and will leave discussion for relative velocities greater than the speed of light to another time. Whether or not these transformations are physical depends on the nature of the entities one takes to travel with a velocity higher than the speed of light. This implies that there is a restriction on relative velocities between PUMs, as they could only be less than or equal to c. Finally, substituting (3.15) and (3.16) into (3.14), and defining $\beta =_{df} \frac{v}{c}$, one gets

$$\beta = \left(\frac{\mu_\alpha + \mu_\beta}{\mu_\alpha - \mu_\beta} \right) = \frac{v}{c} \tag{3.17}$$

Also define $\gamma =_{df} \sqrt{(1 - \beta^2)}$. The usual Lorentz Transformations can be derived:

$$\Lambda = \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \tag{3.18}$$

The situation with the flat relativistic {1+1} spacetime is analogous to that of the Galilean {1+1} spacetime. The priority of the RPUM over the structure of the spacetime explains the equivalence between inertial reference frames, as these various representations of the spacetime stem from taking the RPUM as basic. One also thereby gets an account of Einstein's restricted Principle of Relativity, without taking this principle to be a postulate *from* which the structure of spacetime is derived. Rather, the Principle of Relativity is shown to be a consequence of the thesis that uniform motion is prior to spatial and temporal relations and the structures which decompose motions into their spatial and temporal components. Moreover, the Einsteinian rule for adding velocity also stems from the transformation given in (3.18). One of the most revealing parts of the above reconstruction of relativistic spacetime is that it explains the result that no body can travel faster than the speed of light. Since a wave-like phenomena describes the PUM intersections with a class of parallel motions, the symmetries in (3.14) essentially give the possible definitions of relative velocities between objects moving uniformly relative to light. The restriction is therefore a direct consequence of the RPUM. Given that this restriction seems to be an important fundamental thesis in relativity, again one gets confirmation for the thesis that uniform relative motion is decomposed according to the RPUM.

A further and significant lesson to take from these reconstructions of Galilean and flat relativistic theories of spacetime is that they provide an alternative explanation for the Principle of Relativity and its dual role in both kinematic and dynamic laws. If one assumes that laws of nature describe changes in states of physical systems, and if fundamental states are infinitesimal PUMs, then changes in states of physical systems must obey the PUM symmetries. Galilean and Lorentzian covariance seem to be derived from the PUM structure of each spacetime and its decomposition to spatial and temporal metrics. If one assumes that the evolution of a state of a physical system actualize the temporal evolution of PUMs in infinitesimal segments, one must accept that dynamic laws governing this evolution conform to the same symmetries underlying Lorentzian PUMs. That is, dynamic laws must be Lorentz covariant. Thus, unlike the classical and relativistic Principles of Relativity, which have to be postulated both at the level of the theory of spacetime and at the level of dynamic laws, *Primitive Motion Relationalism* accounts for both kinematic and dynamic applications of the Principle of Relativity. Again, the Principle of Relativity seems to be derived from the assumption that uniform, unidirectional motion is an undefined primitive. The geometry of PUMs takes the metric of relative motions as basic, while spatial and temporal distances are derivative notions.

For the account to apply to real spacetime, one needs to generalize the above results for a $\{1+3\}$ spacetime. The solution to **C 7** is a wave function moving with the velocity of light. The solution can be generalized as follows:

$$\begin{pmatrix} \psi_{1+}(x_{0+}, x_{1+}) = e^{i\omega x_{0+} + ikx_{1+}} & \psi_{1-}(x_{0-}, x_{1-}) = e^{i\omega x_{0-} - ikx_{1-}} \\ \psi_{2+}(x_{0+}, x_{2+}) = e^{i\omega x_{0+} + ikx_{2+}} & \psi_{2-}(x_{0-}, x_{2-}) = e^{i\omega x_{0-} - ikx_{2-}} \\ \psi_{3+}(x_{0+}, x_{3+}) = e^{i\omega x_{0+} + ikx_{3+}} & \psi_{3-}(x_{0+}, x_{3+}) = e^{i\omega x_{0+} - ikx_{3-}} \end{pmatrix} \tag{3.19}$$

Define the spatial and temporal measures in three dimensions:

$$\begin{aligned} x_0 &= x_{0+} + x_{0-} \\ x_i &= x_{i+} - x_{i-} = cx_{0+} - cx_{0-} \end{aligned} \tag{3.20}$$

The stretching of a wavelength in all directions may look as follows:

$$
\begin{aligned}
T_{i+} &\mapsto T'_{i+} = \mu_i T_{i+} \\
T_{i-} &\mapsto T'_{i-} = \mu'_i T_{i-}
\end{aligned}
\tag{3.21}
$$

Define:

$$
\mu_0 \equiv \frac{1}{\sqrt{\mu_1 \mu'_1}} \tag{3.22}
$$

To simplify discussion assume that:

$$
\mu_2 = \mu'_2 = \mu_3 = \mu'_3 = 1 \tag{3.23}
$$

The transformation in (3.21) leads to the transformation Λ between $K_R = \langle cx_0, x_1, x_2, x_3 \rangle$ and $K'_R = \langle cx'_0, x'_1, x'_2, x'_3 \rangle$, so that

$$
\Lambda = \frac{1}{2} \begin{pmatrix} \mu_1 + \mu'_1 & \mu_1 - \mu'_1 & 0 & 0 \\ \mu_1 - \mu'_1 & \mu_1 + \mu'_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3.24}
$$

Given the transformation between the coordinate systems, the relative velocity between these coordinate systems can be defined. The coordinate $x = \langle cx_0, 0, 0, 0 \rangle$ will transform to:

$$
x' = \left\langle (\mu_1 - \mu'_1) cx_0, (\mu_1 + \mu'_1) cx_0, 0, 0 \right\rangle. \tag{3.25}
$$

Thus, the relative velocity of an object at rest in Δ will be moving uniformly in Δ' with the velocity:

$$
\frac{v}{c} = \frac{(\mu_1 + \mu'_1)}{(\mu_1 - \mu'_1)} \tag{3.26}
$$

A constraint will now be imposed on Λ to derive the ordinary Lorentz transformations. Assume that $\Lambda : K_R^{1111} \mapsto K_R^{\mu_0 \mu_1 11}$ equal the inverse transformation, $\Lambda^{-1} : K_R^{\mu_0 \mu_1 11} \mapsto K_R^{1111}$, except for the relative velocity changing signs. (The superscripts include the relevant "stretching" parameters, μ_0, μ_1, and $\mu_2 = \mu_3 = 1$.) The transformation Λ^{-1} looks as follows:

$$\Lambda^{-1} = \frac{1}{2\mu_1\mu_1'} \begin{pmatrix} (\mu_1 + \mu_1') & (\mu_1 - \mu_1') & 0 & 0 \\ (\mu_1 - \mu_1') & (\mu_1 + \mu_1') & 0 & 0 \\ 0 & 0 & 2\mu_0 & 0 \\ 0 & 0 & 0 & 2\mu_0 \end{pmatrix} \tag{3.27}$$

Since it is assumed that $\Lambda(v) = \Lambda(-v)^{-1}$, it follows that $\mu_0 = \sqrt{(\mu_1\mu_1')} = 1$. The relation between the velocity and the scaling factor μ_1 would then be:

$$\beta = \frac{v}{c} = \frac{\mu_1^2 - 1}{\mu_1^2 + 1} \tag{3.28}$$

The scaling factor could be defined as a function of β, so that:

$$\mu_1 = \sqrt{\frac{1+\beta}{1-\beta}} = \frac{(1+\beta)}{\sqrt{1-\beta^2}} \tag{3.29}$$

Let $\gamma \equiv \frac{1}{\sqrt{1-\beta^2}}$. The transformation $\Lambda : K_R^{1111} \mapsto K_R^{1\mu_1 11}$ then becomes:

$$\Lambda = \begin{pmatrix} \gamma & \beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3.30}$$

which is the ordinary Lorentz transformations. In this case the subscript μ_1 describes the stretch in the wavelength of the set of PUMs that move in the x direction. Thus an arbitrary wave expansion μ_1, μ_2 or μ_3 in one of the spatial directions leads to the Lorentz transformations in one of the three spatial directions.

The assumption behind the above derivation of the Lorentz transformations is that $\Lambda(v) = \Lambda^{-1}(-v)$. To justify this part of the derivation, Einstein appealed to the Principle of Relativity, arguing that the transformations between inertial reference frames should not depend on anything but the relative velocities. Einstein's appeal to the Principle of Relativity is suspect, since his articulation of the Principle of Relativity is, strictly speaking, inapplicable to the transformations between inertial reference frames. Einstein's formulation of the Principle of Relativity asserts that the laws of physics are the same *in* all inertial reference frame. But Einstein applied the Principle of Relativity to the generalized Lorentz transformations, which describe laws of transformation *between* inertial reference frames. To say that a rod will measure the same length when at rest in all reference frames does not follow logically from the Principle of Relativity. It is possible for a rod not to measure the same length in different inertial reference frames, and for the laws of physics to be the same relative to the different coordinate systems. Instead the assumption of $\Lambda(v) = \Lambda^{-1}(-v)$ amounts to a convention that separates expansion parameters such as μ_0, which "inflate" the whole inertial reference frame and transformations that do not involve a change in the units of length and time. In Chapters 6 and 8, I will explore the more generalized transformations and their relation to the concept of mass.

3.5 Primitive Motion Relationalism vs. Standard Interpretations of Spacetime

In the previous section, I introduced a reconstruction of spacetime based on the assumption that PUMs are more fundamental than their decomposition into space and time. If I am correct in claiming that the Principle of Relativity is a direct consequence of taking motions as primitive, then we have philosophical motivation for giving up the priority of time and space over motion and existence. The main motivation for accepting *Primitive Motion Relationalism* is that it provides a better philosophical framework for understanding the foundations of spacetime.

We can now estimate the precise sense in which PMR overcomes some difficulties inherent in alternative approaches to spacetime. The main weakness of positivist-conventionalism was that the positivist assumed that all models of spacetime are empirically isomorphic to one another, yet she does not explain this impressive symmetry. But PMR is able to view all the different models as various representations, or alternative decompositions, of the same PUMs. We assumed that the decomposition of a motion into its spatial and temporal components, or the projection of a motion onto a class of parallel motions, is described with either the Galilean or the relativistic PUMs. Each PUM structure leads to an infinite set of equivalent decompositions, depending on the set of parallel motions we take as reference. The structure of the PUM decomposition determines the transformation between inertial reference frames and explains the equivalence of models produced in the various frames.

The approach here also provides an alternative to the geometric interpretation of spacetime. PMR is based on a geometry of possible motions, not a geometry of real spacetime points. In this account the identity of spacetime points depends on the identity of motions. Thus there do not exist indistinguishable dynamic models that produce different mappings between spacetime points and the dynamic objects of the theory. More importantly, the account here obviates the need to utilize causal language to discuss the relation between spacetime and material bodies, such as clocks, rods, free particles, and light signals. The geometry of PUMs gives us a range of possible trajectories (and their relations), and physical objects simply realize these possible motions. Thus the relation between spacetime and matter should not be described with the language of efficient causation, but is better understood as the formal cause of actual trajectories in analogy with Aristotelian forms. However, unlike Aristotle's analysis of motion, one should not think of motion as a substance shedding one form and acquiring another. Rather, the instantiation of a certain trajectory through spacetime *is* the realization of the form of a motion.

PMR has conceptual ties to the dynamic interpretation of spacetime (or dynamic relationalism). PMR is similar in spirit to dynamical relationalism, since it attempts to replace a substantival spacetime with a set of possible trajectories implicit in the laws governing the evolution of physical systems. But in our account the notion of primitive motion replaces the notion of coordinates as physical quantities realized by material bodies. This seems like a philosophical improvement, since it is difficult to make sense of the notion of coordinates as physical quantities. In the geometry of PUMs, bodies actualize motions, and geometric properties arise out of a certain configuration of motions, i.e., from relations between motions. Thus beginning spacetime from a description of possible PUMs retains the general strategy of dynamical relationalism without committing the approach to an unintelligible metaphysical conception.

Another version of dynamical relationalism is Brown (2005) and his neo-Lorentzian attempt to derive spacetime symmetries from dyanmic laws. Brown thinks that explanation ends exactly at the point where we find an astounding symmetry governing all known dynamic laws. But the Principle of Relativity seems to beg an independent explanation, since it would be a miraculous accident if it just happened that all dynamic laws are Lorentz-covariant. Brown is correct to doubt the substantivalist account as giving a proper explanation. It is not clear why taking spacetime points to be real explains why the laws reflect the same symmetries as the underlying spacetime. But it seems as if PMR provides a natural explanation for the unifying symmetry of dynamic laws. The fundamental motions from which all dynamic evolutions are constructed are given by the Lorentzian PUMs. When all motions and changes of states are reduced to infinitesimal Lorentzian PUMs, the Principle of Relativity falls out as a consequence. Thus, regardless of which dynamical laws we discover, they are going to be Lorentz-covariant, as the dynamic laws take Lorentzian PUMs as the basis for the dynamic description. One does not require an independent spacetime, which is fully real and causally efficacious to explain this fundamental feature of physical theories. Instead, one may simply think of any complex motion as reducible to fundamental PUMs, in this way providing an explanation for the common spatiotemporal structure of dynamic laws. In providing a unifying account of dynamic symmetries, the approach here resembles the geometric approach more than it does the dynamic approach to spacetime.

3.6 Conclusion

I offer *Primitive Motion Relationalism* as an interpretation of spacetime that is not subject to difficulties plaguing other interpretations of spacetime. The novelty of this proposal is the assertion that a spatiotemporal event cannot be described independently of the process of generating events. Thus we begin with PUMs to describe the most basic spatiotemporal relations, and then proceed to investigate the complex trajectories formed from these PUMs for the various dynamic laws that govern the evolution of physical systems.

When the identity of spatiotemporal points is reduced to relations between motions, we can think of the Principle of Relativity as a statement about the nature of fundamental motions. The PUM consists of an objective process of generating events, one that does not depend on our mode of representation. However, the *decomposition* of motion into its spatial and temporal components requires that a PUM would be projected onto a class of parallel PUMs.

Chapter 4
The Metaphysics of Time

4.1 Introduction

For the most part, contemporary philosophy of time is governed by the distinction between Presentism and Eternalism. In understanding the nature of time, the choice seems to be between a moving present and a frozen history, laid out along the time-line. According to Presentism, the only concrete time that exists is the present. Everything that was past no longer exists and that which is in the future is yet to happen. The present moment keeps "flowing," so that every instant is followed by a new instant in which part of the future becomes the present and the present becomes past. One frequently distinguishes between the past, present and future based on the types of action that are available. It is not possible to influence the past; actions take place only in the present. The future is pregnant with possibilities that might or might not be realized, depending on what we do in the present. Thus, Presentism is presupposed whenever one forms plans for the future or allows for the possibility that things could have been otherwise than they are.

Commonsense intuition, linguistic habit, and practical reason render Presentism the natural position to take. But many philosophers believe that current scientific theories are in conflict with Presentism. According to the Eternalist view, or as it is sometimes called the block-universe view, everything in the past, present and future exists. Eternalism is based on an analogy between the spatial and temporal lines. It is uncontroversial to believe that everything that exists at present exists in three-dimensional space, even if one can only view existing things from a particular point in space. Similarly, the analogy goes, if one occupies a certain instant in time, one is only immediately aware of events that happen at that instant. But this limited awareness does not entail that past and future events do not exist. Some interpretations of the Special Theory of Relativity (STR), most notably that of Minkowski, suggest that space and time are abstracted from a more basic four-dimensional structure of spacetime. According to this approach time should be treated as if it were just another dimension for describing the manifold of events. If the temporal dimension is completely analogous to the spatial dimension, there is reason to believe that existence at a particular instant in time is analogous to existence at a particular point in space. Instants spread along the temporal dimension all exist, in analogy with points along a spatial line.

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Many philosophers of time believe that it is not possible to reconcile relativistic spacetime with Presentism.[1] In STR, relations of simultaneity are defined relative to an inertial reference frame, making it impossible to define an objective "present" in which the whole world comes into and then goes out of existence. Without a single, absolute definition of simultaneity, it is not possible to define a universal "now." If the distinction between Presentism and Eternalism is accepted as exclusive and exhaustive, one is faced with three choices. Either the notion of "the present" is to be revised so as to accommodate the lessons of STR, or one should conclude that Eternalism is true, or one should reject STR as relevant to the metaphysics of time. The most promising attempt to revise presentism in the light of STR seems to be that of Stein (1968, 1991) who defines the "here-now" as the only event that exists. The "present" according to Stein is therefore reduced to a single point in spacetime relative to which the future light-cone is the future and the past light-cone is the past. This enables Stein to preserve a presentist intuition, namely, the intuition that the present is a point that demarcates between two kinds of events; those that can causally influence the present (and are therefore in the past), and those events that can be influenced by the present (and are therefore in the future).

Einstein's discovery – that there doesn't exist a single, absolute definition of simultaneity – is not itself in conflict with the intuition that some events may causally influence the present, and some events are influenced by it. If one accepts Stein's method for defining the present instant, one might be able to salvage the causal asymmetry of the present. But as I argue in Section 4.3, following many other commentators, Stein's proposal confounds another intuition I take to be central to the presentist view. According to this presentist assumption, the movement of the now must track the processes by which events materialize and *become* present. Since Stein's revision of Presentism fails to accommodate the evolution of the now, it seems reasonable to reject Stein's Presentism and accept Eternalism, assuming there are no other workable varieties of Presentism and assuming that Eternalism is the only viable alternative.

The problem with the eternalist position is that it is in conflict with the fundamental intuition that the future is open, and that we, human beings, have various incompatible yet distinctly possible futures. This intuition is so ingrained in our habits of mind that it seems impossible to deny its validity. It is an assumption that is presupposed every time humans take action. The presupposition is that humans can somehow make a difference as to how things turn out, and so it matters to what goal we exert our efforts. Another, more general worry, concerns the nature of causal relations. In the context of a classical theory such as STR, a causal relation between events seems to imply a certain relation of production. According to this conception of causation, an event that takes place in the present brings about events that will happen in the future. There is a causal relation between events, which relates the

[1] See Putnam (1967); Rietdijk (1966, 1976); Maxwell (1985); Rea (1998); Savitt (2000); Sider (2001); Saunders (2002); Hales and Johnson (2003); Gibson and Pooley (2006); Petkov (2006).

existence of one event to another. If event e_1 causes event e_2, then one assumes that e_2 would not have existed had e_1 not existed. But it is not clear how this causal relation coheres with the eternalist view. In the eternalist view, there is a complete symmetry in the manner of existence of the cause, e_1, and the effect, e_2. Both simply exist, and nothing can ground the causal asymmetry.

What makes true, according to the eternalist, the causal relation between events? The eternalist might respond by comparing the actual, four-dimensional world, w_1, with a similar possible world w_2. Assume that in the actual world w_1, the event e_1 is a billiard ball that hits another billiard ball, and the event e_2 is the second billiard ball that begins to move. One can compare this chain of events of w_1 with another very similar world w_2, which has exactly the same events prior to e_1, except that in w_2 the event e_1 did not take place. According to the eternalist view, there is no inherent difference in w_1 between events e_1 and e_2. All events in w_1 exist in the same way. However, one may still consider the causal relation between e_1 and e_2. The evidence for the causal relation would be to show that in world w_2 in which the billiard ball is somehow prevented from hitting the second billiard ball, one can *infer* that the event e_2 does not take place.

However, the eternalist account of how to reconcile causation with Eternalism is uncompelling. The causal relation seems to suggest that the existence of e_1 helped produce the event e_2, so that the coming into existence of e_2 was somehow the consequence of the existence of e_1. But if the causal relation is justified via inference relations – i.e., inferring the non-existence of e_2 from the non-existence of e_1 – one is not able to quite capture the notion that e_1 brought about the existence of e_2. When two protons collide, it seems reasonable to assume that the collision brought about the annihilation of the protons, and the creation of new particles such as photons and muons. It is not just that the existence of the photons and muons can somehow be inferred from the collision. Rather, the collision initiates the process by which the new particles come into existence, the photons and muons are produced as a result of the collision. In short, there is the intuition that there is a genuine difference between a causal relation and a correlation. And if the eternalist is pointing out the mere inference from the non-existence of e_1 to the non-existence of e_2, the eternalist merely accounts for the correlation between cause and effect. However, the eternalist has no resources to account for the relation of production between cause and effect.

One may interject here and argue that the production account of causation is questionable. Some alternative accounts exist in the literature. However, it seems as if the account of causal relation as a relation of production fits naturally with classical theories such as STR. Thus, if an eternalist points to a conflict between STR and Presentism, one needs to remind the eternalist that a conflict also exists between STR and eternalism, if STR suggests an asymmetrical relation between cause and effect. I do not wish to settle the dispute with the eternalist here. I only wish to raise a few worries to suggest that there is something unintuitive about the eternalist view. My main contention is that before concluding that Eternalism is the only alternative to Presentism, one must examine the common assumptions underlying Presentism and Eternalism. Contenders to the metaphysics of time share two presuppositions that are rarely, if ever, questioned. These presuppositions are held by all current proposals regarding the nature of time, including some unpopular alternatives to Presentism and Eternalism. The first presupposition is that time is more fundamental than motion, and that any definition of motion is ontologically secondary to the fundamental notion of time. The second presupposition is an object or an event exists if and only if it is present at a temporal instant t.

Perhaps there is a strategy that could provide for a third alternative that is neither entirely presentist, nor quite eternalist. First, an unreflective assumption held by contemporary accounts is that time is ontologically prior to motion. The standard account of motion takes it to be the transition of an object from being present at position x_1, at temporal instant t_1, to being present at x_2 at temporal instant t_2 (the at-at theory of motion). This widespread account of motion takes temporal instants (and locations in space) to exist prior to changes from one spatiotemporal location to another. Since both Presentism and Eternalism presuppose the priority of time over motion, one will disrupt the sharp dichotomy between the two if one assumes that motion is more fundamental than time. According to *Primitive Motion Relationalism*, time consists of adopting a PUM as the paradigmatic standard for change. When events unfold relative to this PUM, one has the impression that motions occur within time. However, one can select a different PUM (or a set of parallel motions) as a standard of temporal change. In that case, the new set of PUMs function as "time." When a new standard of change is selected, the original PUM selected as a temporal standard is taken to be a motion occurring within time, rather than time itself. Such an approach undermines the distinction between Presentism and Eternalism, since if motions are more fundamental than time, one cannot infer from the existence of a particular motion the existence or the presence of its temporal parts. The existence of a motion is not predicated on the existence of any of its temporal parts or its presence at a particular location in space and time.

The second presupposition that both Presentism and Eternalism (and all existing alternatives) share is a strong conceptual link between being *present* at a temporal instant t and *existing*. According to both Presentism and Eternalism, events are the fundamental entities of reality. The Presentist believes that any object that exists, exists only at the present. This implies that the existing entities which inhabit the world are of infinitesimal temporal duration, i.e., the world is constructed out of present events, or of infinitesimal time-slices of three-dimensional objects. Similarly, Eternalism is also committed the view that objects have various temporal parts, or time-slices, whose duration are instantaneous. Thus, since objects are constructed out of their temporal parts, the most fundamental entity is an event, whose duration is instantaneous.[2] Furthermore, one concludes that the event exists based on the

[2] I am here ignoring the distinction between perdurantists and endurantists, which may cause consternation for some metaphysicians of time. Perdurantists believe that objects are four-dimensional, and that the time-slices of an object form genuine parts of the object. Endurantists believe that objects are three-dimensional, and that these objects endure throughout time, with different properties instantiated in the object at different times. While eternalists are ordinarily perdurantists, it is certainly consistent for an eternalist to be an endurantist about objects, although this is not an easy position to hold. I will not go into this debate about the nature of objects, except to say that

temporal instant at which the event is present. The presentist assumes that there is a privileged temporal instant t which is "the" present. An event is taken to exist if and only if it is present at t. Those events that were present in the past or will be present in the future, at temporal instants other than "the" present instant, are not present, and so one can infer that they do not exist. In contradistinction, Eternalism does not privilege any instants. As long as an event is present at some temporal instant, the event should be taken to be present, and one therefore concludes that it exists. Thus according to Eternalism, if a temporal part of a body is present at t, that does not preclude other temporal parts from being present (and existing) at temporal instants other than t. Presentism and Eternalism disagree on whether the scope of the existential quantifier ranges over events present at "the" present instant, or whether it should range over events present at any instant. But both Presentism and Eternalism agree that being present at a temporal instant is a marker of existence.

It is difficult to pry apart the notions of "being present" and "existing," even though existence is an all or nothing affair, while being present is a relation. The modern intuition is that an event exists if and only if it is present at t. At first glance this inference from being present to existing seems unproblematic, as if existing and being present are two different expressions of the same notion. However, a closer examination reveals that the notion of existing and being present carry different connotations, and that equating being present with existing is not free from implicit assumptions. Part of the reason for the conflation between being present and existing is that one often infers that a certain object exists from the fact that it is present. When one says of a certain object that it is present, one ordinarily implies a certain relation – the object is present to something else, whether it is a mind, or another object that "feels" the presence of the object via some causal chains. The presence of an object is "felt" through various causal chains that emanate from it. For example, an object is present to the mind when there is a causal chain that stems from the object and ends up producing an impression in the mind. An object is present to another object when the other object can "feel" its presence via some causal relation. It is as if the presence to our mind, or presence to another object is proof that it exists, and if it is not present to something else, one no longer has reason to think that the object exists.

Severing the connection between existing and being present at instant t is justified when one considers the slight difference in meaning between the notions of existing and being present. While being present is a relation, existence is simply the existence of an object or an event whether or not it bears any relation to another thing. While it is meaningful to say that "the object exists but is not present to anything," it is meaningless to say that "the object is present but does not exist."

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both perdurantists and endurantists take time-slices as the basic entities from which objects are comprised. Either one believes that an object is wholly present when a single time-slice is present (which makes the view endurantist), or one believes that the object is wholly present in virtue of all time-slices being present (which makes the view perdurantist). I am trying to undercut the distinction between endurantism and perdurantism as well, since I am suggesting that being present at t is not the same as existing.

The former statement seems to border on being senseless, but it is clear that it is not self-contradictory, while the second statement is self-contradictory. Thus one may think of existence as describing the actualization of a certain entity independently of any other entity, while being present is a relation: between an object and a mind, between two objects, or between an object and a temporal instant. Now, being present at a temporal instant t captures a different meaning from being present to a mind or being present to another object. The relation of being present at a temporal instant is more abstract, since it does not involve a causal chain that emanates from the object to the temporal instant that "feels" the presence of the object. There is some similarity between the notion of an object being present to God's mind and the notion of an object being present at a temporal instant t. Both relations of being present do not rely on any causal chain via which God's mind or a temporal instant can "feel" the presence of the object. But it is clear that being present at t is a relation and existence is not. Generally speaking, being present presupposes existence, but existence does not presuppose (or does not presuppose necessarily) the object being present.

The identification of being present with existence runs deep in western philosophy, and it is difficult to imagine how one can conceive of existence without grounding it in the presence of objects. The identification of the two notions leads to surprising conclusions. For example, idealist approaches have taken the epistemic connection between being present to the mind and existence to the extreme. The Idealist presupposes that to be present, is to be present to a mind, i.e., it is to be perceived. The first premise of the Idealist argument is therefore that being present is a relation between an object and a mind; it is the relation of being perceived. The second premise of the Idealist argument is that being present is equated with existence. An object exists if and only if it is present (to a mind). The conclusion of the two premises is the Idealist assertion that to be is to be perceived. In case an object is not perceived by a finite human mind, it must be perceived by God's mind.

Most of the arguments leveled against the Idealist do not question the second premise of the Idealist argument, i.e., the premise which equates existence with being present. Rather, the arguments against Idealism attempt to reinterpret the notion of being present in such a way that it becomes implausible to claim that being present is a relation between an object and a mind. One standard way to do this is to assert that being present is a relation between an object and a temporal instant; an event is present if it is present at a particular temporal instant t. This conception of being present confines one to the notion that temporal instants are given prior to any articulation of the presence relation. The biconditional between existing and being present at t and the priority of time over motion therefore reinforce each other. If one defines the notion of being present as a relation between an event and one particular instant of time, an instant one takes to be "the present," one ends up with the presentist view. Because of the relation of being present to t, the unique temporal instant which is "the present," the event becomes actual, i.e., it is real. Based on the notion that presence is equated with existence, one may also conclude that all present events exist and non-present events, such as past and future events, do not exist.

Presentism is a view that the Idealist considers as a direct threat, and it is no wonder that McTaggart's Idealism compelled him to articulate the argument against a robust version of Presentism. As McTaggart has shown, if the presentist attempts to include an account of the evolution of the "now," her account will run into difficulties. There is no coherent account of the process by which a present temporal instant becomes non-present (or past), and a future non-present instant becomes present (see Section 4.2).

Another way to avoid the Idealist conclusion is Eternalism. According to Eternalism, all temporal instants render present those events that are located at them. The conclusion is that all events that are located at temporal instants are present, and therefore also exist, and there is no conceptual difference between past, present, and future events. But as I shall argue, this eternalist position runs counter to our most basic intuitions about dynamic processes. Eternalism does not accommodate the intuition that the future is open and that there is an asymmetrical causal relation, according to which the cause leads to the effect, but not vice versa. This principle of causality seems to form a fundamental part of our scientific theories, at least in the context of classical theories such as STR. Eternalism is therefore another flawed alternative to the Idealist conclusion.

It is possible to conceptualize being present at t, and only at t, but still *existing* at various instants other than t. If motion is taken to be a primitive entity, one may attribute existence to the fundamental process that generates events. According to this conceptual possibility, the existence of processes is not predicated on the existence of process-stages, which are themselves predicated on being present at a particular instant t. Rather, it is the other way around – the existence of stages is predicated on the existence of the underlying process. Thus, one can take different stages of the process to be present at different temporal instants, but this does not deny the ontological dependency of stages on the underlying process. According to this view, one does not take motion to be a four-dimensional object that is composed of ontologically distinct "time-slices." A four-dimensional object in that case would not be defined as the mereological sum of its "time-slices." While the existence of process-stages may depend on the existence of the underlying process, the process-stage may be considered as present due to relations of the underlying process with other processes. Thus, the four-dimensional underlying process exists, but its stages are present at different temporal instants.

The presentist or eternalist may object to the notion that being present is a relation between two things, as I have suggested. One may think about being present to the mind is a relation between an object and another substance, mental in character. But, one may argue, being present at a particular instant t is not a relation between an object, and another thing, i.e., an instant of time. One may simply consider the propositions describing the world as *being true* at t. Whatever exists is an existential proposition implied by the propositions that are true at t. However, if this is what the notion of being present at t amounts to, one seems to be divorcing true propositions from the entities that make them true. It seems as if propositions that are true at t, are true in virtue of the entities that exist. And those entities exist in virtue of the fact that they are present at the temporal instant t. Thus if one is a presentist or an eternalist, one is committed to a basic relation of being present at t as the one that grounds the existence of events.

Taking into consideration our reconstruction of spacetime in Chapter 3, it is possible to think of motions as the fundamental entities of spacetime. According to this approach, there is no prior grid of spatial points and temporal instants relative to which the progress of motions is measured. Spatial and temporal distances "arise together" from the basic process. In such a spacetime, events are not the basic physical entity, but instead arise from relations between motions, i.e., relations of intersection. Thus, one may distinguish between the existence of a motion α, which is ontologically prior to any events which belong to this motion, and the presence of α to another motion β. If α intersects β (i.e., $\mathbf{E}\alpha\beta$), then α is present to β at the event of intersection. Thus, the claim that the intersection event is present depends on first, whether the motion α it belongs to exists, and second, whether there is another motion to which α is present. Thus, instead of thinking of α as the transition from being present (and existing) at (x_1, t_1), and then being present (and existing) at (x_2, t_2), instead think of these events as the consequence of one motion intersecting, or being present to two other motions. Events that take place at particular spatiotemporal locations "owe their existence" to the mutual presence of two motions (or the potential mutual presence of a motion to another motion).

Thus *Primitive Motion Relationalism* is an attempt to resuscitate the idea that Becoming is more fundamental than Being. But this Becoming is not identical to a present instant that moves along the temporal line. Such a notion would reduce Becoming to Presentism. Rather, the idea is that the process of generating events has its own mode of existence, and this existence is ontologically prior to being present at a particular instant. This view is the "third alternative" to Presentism and Eternalism. According to this account, there are two modes of existence. At the fundamental level, a motion exists independently of any other motion. This motion exists and evolves prior to any temporal determination, and its existence is independent of being present at a particular instant t. The view suggests that Presentism is false (and Eternalism is partially true), since the existence of the underlying motion precedes being present at any instant. However, another mode of existence concerns *the relations* between motions, i.e., intersections between them that constitute events. Whether two motions intersect, or whether an event has actualized, is still open prior to the event of intersection. Along a particular motion, one can single out a "present instant" for each particular motion. It is always clear at what stage the process is in; there is only one stage that is present, because it must be present to some other motion, i.e., it bears a relation to the other motion. The process is not the sum of its stages. On the one hand, the present instant owes its existence to a more fundamental level, i.e., to the underlying motion or process. On the other hand, the present instant also owes its existence to the relation between motions, i.e., to the stage in which the motion becomes present to another motion. Given that only one event along a particular motion is present, one can also take Eternalism to be false (and Presentism as being partially true).

The thesis that motion exists prior to space and time provides a new philosophical approach to the study of time and its relation to relativity theory. First, I will examine the conceptual difficulties concerning the flow of time (Section 4.2). I argue that these difficulties stem from the assumed priority of time over motion, and that these difficulties dissolve if this priority is given up. Second, I agree with the widespread assessment that the structure of relativity theory is in conflict with Presentism and will defend this claim in Section 4.3. However, even if Presentism is false it does not follow that Eternalism is true (Section 4.4). Thus I disagree with the conclusion most philosophers of time take from the conflict between Presentism and relativity. If motion is taken to exist prior to space and time, there is an alternative to Presentism other than Eternalism. In Section 4.5 I sketch the alternative account of the metaphysics of time, which gives up the two main assumptions of Presentism and Eternalism. First, I argue that *Primitive Motion Relationalism* provides a framework where motion can be taken as ontologically prior to time. Second, I argue that one can make sense of this view when one differentiates between the existence of motion and the presence of motions to other motions.

4.2 The Flow of Time and Motion

Problems with presentist theories, at least those that presuppose the flow of time, exist independently of any conflict there might be between Presentism and STR. It would be pointless to consider whether Presentism is consistent with STR if it is not even a coherent view. Moreover, if it is difficult to formulate a coherent account of Presentism independently of STR, perhaps the tension with STR stems from the same conceptual obscurities plaguing the presentist position itself rather than a clash between metaphysics and science.

In the following, it will be useful to differentiate between the existential presentist presuppositions and the attempt to ground temporal becoming in the movement of the now. Arguments against the notion that the "now" is moving towards the future often get the reply that the minimalist presentist view is concerned merely with the notion that the only events or objects that exist are the ones that exist at the present time. Presentism is not concerned with the flow of time or with how events come into existence. Thus one ought to separate the minimalist, austere presentist view, with the full-blown notion that the "now" evolves from the present to the future. According to the austere presentist view, one is solely concerned with the presentist's existential claims. The "robust" presentist view is also interested in explaining the dynamics of change, and in accounting for the evolution of the present moment.

Presentism carries with it an existential assumption. According to Presentism, only a single instant t is present, and therefore exists; instants other than t are either past or future, and are therefore non-present instants that do not exist. The central presentist assumption is therefore that the present instant t is also a marker of existence. For any object or event, it exists if and only if it is present (and it can only be present now). There are some varieties in articulating the nature of this existential claim. For example, some presentists attempt to be precise about the nature of the objects that exist at the present instant t, and the modal character of this existence. At minimum, the presentist argues that concrete objects only exist if they are present (now). But if at a present instant t it is coherent to discuss and think about past and future objects, then one has to recognize the existence of propositions, at t, that describes times other than t. Thus it may be that at present time t, one should also recognize the existence of abstract entities or propositions that represent objects that exist in the past and in the future. Thus some presentists erect an analogy between descriptions of non-present times and discussions of possible worlds. The difference between non-present and present times would then be analogous to the distinction between possible and actual worlds. Furthermore, if it is not possible for propositions describing the present instant t to be false, one may say that these propositions are necessarily true. That is, if the propositions describing the present instant are true, it is not possible for them to be false. Thus, Sider (1999) for example, equates Presentism with the claim that "necessarily, it is always true that everything is (then) present" (p. 326). The point of this definition is to rely on a certain indexical (then) to refer to a single instant in time, and to argue that everything that exists, is necessarily present at that temporal instant.

It is a common worry in the philosophy of time whether our language correctly represents our existential assumptions. If claims about the past or the future are true, and events in the past and in the future no longer exist, one needs semantic tools to articulate true propositions about entities that no longer or do not yet exist. Prior (2003) introduced semantic operators for paraphrasing statements about past or future events to tensely modified statements. The statement "I was having my breakfast" is paraphrased as "It was the case that I am having my breakfast." And the statement "I will have my breakfast" is paraphrased as "It will be the case that I am having my breakfast." The semantic operators "It was the case that p" and "It will be the case that p" are indexed to a particular present instant. Thus tensed talk is a speaker's assertion that a certain state of affairs was the case or a certain state of affairs will be the case. There may be traces in the present about past events, or description of events that have not yet happened, but only present events exist at each present instant. Events are *spoken* of as past or future, but there are no past or future events, strictly speaking.

There are some conceptual difficulties involving the presentist existential assumption. There is the problem of cross-temporal relations such as the causal relation. It is not clear how the presentist would treat relations between two events, one in the past and the other in the present, if only the present event exists.[3] Presumably a true cross-temporal internal relation can only hold if both terms of the relation exist. However, when a relation holds between a present and a non-present entity, it is not clear which conditions make the proposition expressing the relation true. It is also not clear how, according to the presentist's approach, statements such as "Abraham Lincoln was tall" and "David Lewis admires Ramsey" are able to contain names referring to objects not currently existing. Such statements should undergo

[3] See Bigelow (1996), Sider (1999) and Markosian (2004) for an attempt to defend Presentism against the problem of cross-temporal relations, and Davidson (2003) for a critique of such attempts.

some appropriate paraphrasing if one were to analyze their truth conditions. Since a proper name must genuinely refer to an existing object, it is not exactly clear how a singular proposition could be true and yet refer to an object which is not presently existing, or to two different objects existing at different times.

There are various ways of defending the presentist existential assumptions, but I will not consider these difficulties here. Focusing on these difficulties obscures a deeper problem with Presentism, which arises when one attempts to *explain* the movement of the now. I.e., Presentism faces problems when it is offered as part of a robust form of Presentism that tries to account for dynamic change.

The presentist conception becomes even more contentious when it is assumed that the movement of the present instant ("robust" Presentism) accounts for the various processes of becoming, which are the processes by which various events and objects come into existence. A common way to describe the movement of the now is to think of certain instants of time as first being future, then becoming present, and finally becoming past. Another way to describe this dynamic of the present instant is to think of descriptions of future times as becoming true, and propositions describing the present as becoming false. A visual metaphor for this changing present is a razor-thin, three-dimensional surface which progresses along the temporal line, where the progression marks the movement or the flow of time.[4]

The paradigm example for an account which connects the movement of the present with the flow of time is Newton's account of absolute time:

> Absolute, true, and mathematical time, in and of its own nature, without reference to anything external, flows uniformly and by another name is called duration. (Newton, 1999, p. 408)

The "flow" metaphor used to describe the passage of time provides a certain visual representation. One imagines a river flowing and gets a feel for what it means for time to move from the present into the future. But despite the intuitive appeal of the river metaphor, it is conceptually inadequate. Since any motion seems to presuppose the passage of time, it is not clear how the passage of time can itself be represented with some metaphorical motion. The passage of time grounds any change, so it is not clear what conceptually grounds changes in time itself.

In his well-known argument against the reality of time, McTaggart (1908) distinguished two ways of describing the series of temporal instants. The A-series consists of temporal instants that have the properties of being past, present and future. One ordinarily conceives of instants of time as first being in the future, then becoming present, and finally residing in the past. But there is another way to describe a series of temporal instants, i.e., through the relations of *earlier than* or *later than*. This

[4] This image of the present moving from one instant to the "next" may be seriously misleading, since the assumption is that the temporal dimension can be modeled by a real line. Since the instants along this line are dense, between any two temporal instants there is another. Thus, strictly speaking there is no single instant that can be designated as the "next" one, just as there is no single spatial point that is the next point to the right of the point marked 0. But one may speak loosely in this way to describe the movement of the now.

series of temporal instants is characterized by an asymmetrical, transitive ordering relation. McTaggart designated this latter structure governing temporal instants as the B-series. While the B-series captures the order between temporal instants, it is not able to account for genuine notions of change. Thus according to McTaggart, an account of time – given the requirement that it should explain change – must include the A-series.

McTaggart points out that the A-series is untenable, thus leading him to conclude that time is unreal. A temporal instant according to the A-series is first in the future, then becomes present, and finally ends up in the past. But an instant cannot have the property of being in the future, the present and past, since these are incompatible properties; an object with incompatible properties cannot exist. If one argues in response that it was only in the past that the event had the property of being in the past, and that only in the present did the event have the property of being in the present, etc., then it seems as if one runs into an infinite regress. Another A-series is needed to differentiate between the various times in which events have the properties of being in the past, present and future. To avoid an infinite regress one has to accept that the A-series is impossible. One is left with the B-series, or with the claim that time is reducible to relations of earlier and later than, with no particular instant that is distinguished as the present and no passage of time.

McTaggart's argument is an attempt to make precise the vicious circle that arises in trying to conceptualize the passage of time. The infinite regress begins as soon as one attempts to describe the process by which a future instant *becomes* present, and a present instant *becomes* past. Changes in time itself seem to require a higher temporal dimension in which to describe the movement of the now, but such stratification of various levels of change gives rise to an infinite regress. One way to avoid McTaggart's regress is to retreat to an austere presentist conception in which there is no genuine change in time, but the annihilation of everything that exists in one temporal instant and the creation of everything that exists in the new present instant. The presentist then argues that events do not become, they simply exist at certain present instants. One could distinguish between things, such as an electron, a person, or a city, which could undergo changes, and events that either exist or do not exist.[5] Craig (1998, 2001a), for example, argued that the notion that a future instant "becomes" present does not amount to a pure A-series, since the transition from a future to a present instant requires the alignment of A-series instants relative to the B-series. McTaggart's argument is in fact directed at a hybrid of A and B-series of temporal instants. A "pure" A-series of temporal instants is one where there is never a flow from events existing in the future to events existing in the present. Events do not carry monadic properties such as "being past" and "being future" – their being past or future is merely asserted from the point of view of some present speaker.

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[5] Mellor (1981) argues that it is possible to think of things evolving in time and their events as existing within a B-series. But Mellor is a B-theorist who wants to deny the existence of the A-series. See Dagys (2008) for further analysis.

In addition to McTaggart's argument suggesting that the "flow" of time leads to an infinite regress, Newton's definition of time suggests metaphysical alternatives to the actual *rate* in which time flows. If time in the actual world flows uniformly, one might conceptualize other possible worlds in which time does not flow uniformly. Thus there is a conceptual difference between the uniform and non-uniform flow of time. This again suggests a second time dimension relative to which we can compare the rate in which the first time dimension flows. Clearly this is not a coherent account, since it leads to an infinite regress. If the second time dimension can be described as "time," then it flows too, requiring another dimension relative to which one can describe the rate of its time-flow, etc.[6]

The response to McTaggart's argument ordinarily amounts to a retreat to an austere form of Presentism, where the presentist view is reduced to an existential claim about the present, and there is no positive claim about the process by which a future instant becomes present, and a present instant becomes past. But the retreat to a pure A-series, like the reduction of time to the B-series, fails to capture the experience one has of time passing and of there being genuine change written into the fabric of the universe. If only the present exists at any particular instant, and there are no processes that extend throughout time, not even the passage of time itself, then the presentist view does a poor job of explaining the intuition that change is reducible to the passage of time. Moreover, the austere presentist is committed to an unexplained harmony between the evolution of the universe, on the one hand, and the order governing temporal instants, on the other hand. Assume that at different present instants, different times (and perhaps representations of non-present times) exist. On the one hand, one may describe the laws of nature that govern the evolution of entities that exist at a particular time and the tendency of future states of those entities to become present. On the other hand, one can describe the ordering relation between temporal instants. It seems a matter of pure coincidence to suggest that the temporal instants are ordered in a way that conforms to the laws of nature that are made true by the world existing at a particular instant t. One would not be able to differentiate between two ways of ordering temporal instants, whether by its standard ordering, or by some random ordering. For example, if the present instant is 5:00 am, January 1, 2010, it is perfectly consistent with the presentist view to think that the next present instant existing is 3:34 pm, January 1, 1840, and the next one to exist is 11:32 am, January 1, 5230. If there is only the present instant that exists, there is nothing to recommend a correlation between the actual order of temporal instants and the various laws that seem to govern the evolution of physical states of entities existing solely in the present. The laws are described as governing the actual world at the present instant and other non-present worlds. But it only makes sense to say that t_1 is later than t_0 if there is a causal process that is responsible for producing a change from states of objects existing in t_0 into states of objects existing in t_1.

[6] See Smart (1949) for an examination of the problem of the rate of time's flow. Smart argues that since it is not clear what rate time flows, it is not coherent to talk of the flow of time. See Markosian (1993) for an attempt to respond to Smart.

Without a process that begins at one time and ends in another, it does not make any sense to insist that temporal instants conform to a certain order, or that the actual order of temporal instants conforms to laws governing entities that exist only at the present instant. It would just be a miraculous accident to suppose that the times line up to a certain temporal order in just the same way that traces of past times appear in the present time.

An austere presentist view is unable to ground both the ordering of temporal instants and the evolution of physical states of the entities that exist. The austere presentist view, since it takes the movement of the now to be the annihilation of the present and the creation of another present, is in tension with the asymmetry of the temporal order. If the presentist takes the past to be fixed and the future open, then it is not clear to what this asymmetry could be reduced. Given the ontological symmetry between the past and the future it is not clear what justifies the notion that the future is different in kind than the past.[7] A philosophy of time would undermine itself if it were not able to account for the relation between the passage of time and motion.

The austere presentist view might be internally consistent. However, whenever a presentist attempts to go beyond the confines of austere presentism, and explain the flow of the present instant, she is bound to fail. Presentism is not a coherent view whenever it attempts to provide an explanation for temporal change. An attempt to provide a robust form of Presentism must accept a dual view of motion. On one level the motion of bodies is given within time, on the other hand the present instant itself is moving. As soon as one tries to make sense of the movement of the "now," then one is pressed to suppose the existence of a higher temporal dimension relative to which the movement of the now is described. However, retreating to a coherent, austere, presentist account results in a metaphysical view that fails to explain temporal change, which robs the presentist view of its intuitive appeal. Of course, presentists often insist that Presentism is merely the existential commitment to objects only existing in the present, and so by definition Presentism is the austere view and the McTaggart argument is irrelevant. But if one confines Presentism to the view that only the present time exists, and does not explain how the future time comes into existence, then Presentism becomes a stale view to hold, and an impoverished metaphysics of time.

A presentist is shackled to a dilemma between an untenable robust form of Presentism, and an austere form of Presentism that is explanatory deficient. She is bound to this dilemma by the presupposition that time is given, ontologically speaking, prior to motion. As long as the presentist thinks of the motion of bodies as occurring within time, she will be forced to think of the present instant as moving if she would like to give an account of change, leading to an incoherent picture. She might avoid committing herself to a movement of the "now," but then her position

[7] See Diekemper (2005) for an argument that a pure presentist position is inconsistent with the asymmetric fixity of the temporal order.

becomes too austere to provide a convincing account of change. A "frozen" present is not a very satisfying account of change.

Once the logical priority of time over motion is given up, it is no longer tempting to think of time as *grounding* change on the one hand, and time itself as changing. Assuming that motions exist, not in time but as grounds for time and space, then there is no need to conceive of a dual layer of change; change with respect to time and the change of time itself. If the world consists of various motions, time is simply the privileging of one motion over all others and the comparison of all motions to this paradigmatic motion. Change relative to time would be change relative to this paradigmatic motion. Change of time itself would be the change of the paradigmatic motion, relative to an alternative motion selected as the paradigm of change. Change therefore is the unfolding of one motion relative to another, and is a relative concept. Motions display change in relation to one another, but there is no such thing as a pure change of time.

4.3 The Conflict Between Presentism and Relativity

Many philosophers of time believe that current scientific theories, in particular STR, are in conflict with Presentism. Since relations of simultaneity have to be defined relative to an inertial reference frame, it is not possible to define a single instant as "the present," and think of this instant as extending over all of space. Presentism seems to rely on dated assumptions carried over from Newtonian physics and Galilean spacetime, where relations of simultaneity are objective and independent of the inertial reference frame.

Putnam (1967) and Rietdijk (1966, 1976) utilize the conflict between Presentism and STR to argue for Eternalism. According to Putnam, whose argument is considered first, one may safely assume that whatever happens here and now to an observer S_1 exists. For simplicity sake, it is assumed that this observer S_1 is not accelerating, i.e., one can define the inertial reference frame relative to which S_1 is at rest. Putnam assumes that the event p produced by the observer S_1 here and now is real.

According to Putnam it is reasonable to assume that the coexistence relation Rxy – which is interpreted as "the event y is real if x is" – can be defined on a particular spacetime using spatial and temporal relations between events. In Galilean spacetime, the relation R of coexistence supervenes on relations of simultaneity. Thus, relative to the here-now p experienced by an observer, all events x simultaneous with p are considered real, so that $\{\forall x, t(x) = t(p)|Rpx\}$. In Galilean spacetime one can therefore define a singular present instant for which all objects exist. However, in relativistic spacetime, the relation R cannot supervene on relations of simultaneity without running into problems. Attempting to reduce R to spatiotemporal relations seems to result either in taking R as holding on all event-pairs in the spacetime, or accepting counterintuitive assumptions about R. Assume that there is an observer S_2 moving uniformly in relation to S_1, and that S_2 produced the event q which is simultaneous with p in S_1's rest frame (see Fig. 4.1). If R supervenes on relations of simultaneity defined in S_1's rest frame, Rpq holds. Since p is real, q is also real. But

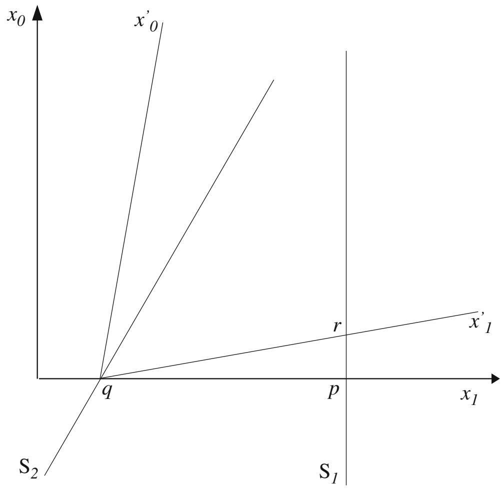

Fig. 4.1 Relations of coexistence in various reference frames

then if S_2 is moving relative to S_1, one can define R on events simultaneous with q in S_2's rest frame. Thus, take an event r that took place on S_1's worldline which is simultaneous with q in S_2's rest frame. Two options are now available. Taking Rpr to be true means that r is real if p is. If that is the case, Presentism is false since the future event r is real as soon as p is real. Or, in other words, a future event is as real as the "now." (Analogous arguments can be constructed for any event in the future or the past light cone to p). The alternative is to think that Rpr is false despite Rpq and Rqr being true, i.e., R is not transitive. This alternative is implausible since it implies that relations of coexistence depend on the observer.[8]

Rietdijk follows an argument analogous to that of Putnam's. According to Rietdijk, one can think of the relation Bxy, which means that y is determined if x is. Thus, in Galilean spacetime, Presentism implies that events in the past are determined in relation to the present moment. Assume that an event p is in the present. All events belonging to the past must have already been determined. Thus $\{\forall x, t(x) \leq t(p)|Bpx\}$. All events in the future are yet to be determined given that p is determined so that $\{\forall x, t(x) > t(p)| \sim Bpx\}$. If an event p is determined, one can try to specify all the events that are determined in relation to p. In the context of relativistic spacetime, it is clear that all events in the past light cone are determined relative to the here-now. Thus, one can say that $\{\forall x, s^2(x, p) > 0, \Delta t(x, p) < 0|Bpx\}$, where s is the relativistic spacetime interval defined as $s^2(x, y) = \eta_{\mu\nu}x^{\mu}y^{\nu}$ and η is the Lorentzian metric. Now, like Putnam's relation of co-existence, the question is whether a space-like separated event is determined in relation to the here-now p. Riedjijk's proof follows the same reasoning as that of Putnam's. *If* events simultaneous with p are determined in relation to p, then either the relation Bxy is true on

[8] See Hinchliff (1996) for an attempt to resist the transitivity of the coexistence relation.

all the spacetime or we have to give up the transitivity of this relation. But giving up the transitivity of this relation is counterintuitive, and so one has to accept that for any event p, all other events in the spacetime are determined in relation to it. If this is the case, Eternalism is the only account of time consistent with relativity.

Putnam and Rietdijk both assume that an event exists if and only if it is present at a particular spatiotemporal point p. This assumption leads to the assertion that if events e_1 and e_2 co-present, then the events e_1 and e_2 coexist. The question is whether two events are co-present when they are present relative to the same inertial reference frame. However, if one separates the notion of being present at a spatiotemporal point from existing, then there may be a distinction between two events being co-present, and two events coexisting. If one assumes that the notion of being present involves a relation between two motions (that are present to each other when they intersect), then one may conclude that two space-like separated events are neither co-present nor are they not co-present. If one takes the relation of being present to involve a relation between two motions, the notion of co-presence is not well defined if it is intended to be a relation between two remote events. On the other hand, all events, since they belong to a fundamental motion, exist in virtue of the fundamental motions to which they belong and the relations between motions. The fundamental motions all coexist, since all exist at a fundamental level.

An important presentist intuition may be salvaged if Presentism is revised in an appropriate way. Given that relations of simultaneity are defined relative to an inertial reference frame, one cannot abstract from the particular worldlines and define a present instant common to all worldlines. As Stein (1968, 1991) argued, there is no need to posit an objective instant that demarcates between determined and undetermined events running throughout all of space in a particular hyperplane. Similarly, one need not suppose that there is such a line underwriting relations of co-existence. It's coherent to think that, *relative* to the here-now, events in the future light cone are undetermined and events in the past light-cone already are.

Stein's suggestion captures an important intuition that drives Presentism. If a particular point in spacetime is present, one may clearly distinguish between those events that could causally influence this point and those events that may be influenced by it. Stein argues that:

> In the context of relativity . . . we cannot think of evolution as the development of the world *in time*, but have to consider instead (as above) the more complicated structure constituted by, so to speak, the "chronological perspective" of each space-time point. The leading principle that connects this mathematical structure with notions of "process" and "evolution" (and justifies the use of our notion of "becoming" in relativistic space-time) is this:
>
> At a space-time point a there can be cognizance – or information about influence propagated from – only such events as occur at points in the past of a. (Stein, 1968, p. 16)

Thus according to Stein one may revise the notion of present in STR to include a single spatiotemporal point. Each point in the spacetime carries its own perspective on processes of becoming. Instead of attributing becoming to the world as a whole, one focuses instead on differentiating between points that can initiate influence that propagates until it effects change in the present and chains of influence that initiate from the present.

Notice that Stein's language reflects the distinction between existing and being present. For him, only the here-now exists and there is only a single spacetime point that actually exists. But what Stein is really interested in is the notion of being present, which contains certain relations to the past and to the future. A present instant "is cognizant" or "contains information" about the influence propagated from the past. Events lying in the future cone would contain information about the present instant. Thus, Stein implicitly uses the notion of being present as marking a certain causal relation to past events, a relation that results from certain events being present and the causal chains that emanate from those present events. A certain event is present, when it is "produced by" or generated by events that were present in other instants. But Stein goes further to conclude that, since one can only define the relation of causal connectedness from the perspective of a single point in spacetime, the here-now is the only present point and therefore is also the only one existing. Thus, Stein is adhering to the strong metaphysical connection between being present and existing.

Clifton and Hogarth (1995) have shown a way to reduce Stein's notion of causal influence to processes of becoming that govern individual worldlines. One can use becoming relations along particular worldlines to derive a general determinateness relations between the "here-now" and the past light cone (assuming plausible assumptions about worldline transitivity of becoming relations). According to this approach, there is one point on the worldline which exists, and that part is "present." The difference between classical and relativistic spacetime is that the present, which is well defined for a particular worldline in relativistic spacetime, cannot be extended unambiguously to other spatiotemporal points. Instead of the world as a whole undergoing a process of becoming, one may instead think of individual worldlines as undergoing their own processes of becoming.[9]

But does it make sense to restrict the present to the here-now? What is then the status of events that are space-like separated from the here-now? Do they exist or do they not exist? Stein's suggestion is to take only the here-now as obeying the relation of coexistence to itself.[10] But it seems odd to say that none of the space-like separated events are coexistent with the present here-now.[11] As Stein himself

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[9] See Clifton and Hogarth (1995, pp. 382–83).

[10] See Callender (2000) for an argument claiming that this definition of Presentism steers too far from our standard presentist intuition.

[11] Sider, for example, argues that limiting existing events to a single spacetime point is extremely counterintuitive:

> . . . the presentist might banish all of space-time other than a single point. (A related proposal would be to banish all of spacetime other than a single point plus its past light cone.) Note that the right way to assert here-now-ism is to say that only a single point of the spacetime is real, that there exist no spatiotemporally distant events. The wrong way is to say that *at any point in the spacetime, only a single point of spacetime is real.* This suggests a misleading picture, that there *are* multiple points in spacetime, but somehow, from the perspective of one of them, the others are not real. Unless the presentist is involved in a Meinongian distinction between being and existence, this can only be a confusion. (Sider, 2001, p. 44)

recognized, for his proposal to count as a revision of Presentism it has to preserve some intuition that the present “becomes cognizant” only of processes occurring in its past. But what is the connection between the here-now “becoming cognizant” of the processes in its past and the here-now coming into existence? Can we use the occurrences taking place at various here-nows to infer how the present instant evolves over time?

One of the main benefits of the presentist position is that it provides an intuitive framework for understanding the relation between time and becoming. In traditional presentist accounts, the evolution of the now is conceptually linked to processes of Becoming. There are two interpretations that could ground the relation between the evolution of the now and coming into existence. The first interpretation thinks of the temporal instant as becoming present, which then grounds the going out of existence of present events and the coming into existence of future events. Another interpretation is that the becoming present of the temporal instant merely marks the fact that a certain set of events have come into existence. Thus, according to this latter interpretation, the present instant is no more than a marker of existence. One way to understand the distinction between the two interpretations is to consider whether it is possible for the whole world to freeze, and for time to march on. Can one conceive of time passing without the world as a whole changing? If the answer to this question is “yes,” then one takes the evolution of the present instant to be making possible processes of becoming, and the presence of temporal instants ground existence. According to this interpretation, existence follows an underlying process of temporal instants becoming present. If one cannot imagine time to progress without events coming into existence, then the second interpretation applies, and one believes that there is no more to the present instant than the coming into existence of the world.

Whether or not time grounds existence, or existence grounds time, Stein’s proposal falters as a genuine presentist account. Stein’s method is based on the notion of spatiotemporal perspective. His proposal does not consider the ways in which our spatiotemporal perspective evolves. Presumably, one simply inhabits various spatiotemporal perspectives at will. The consequence is that it is not possible to imagine the evolution of the present instant as making possible processes of becoming, nor can one consider the evolution of the present instant as marking processes of becoming.

Sider’s argument is based on the assumption that the relation of coexistence must be either true or false. If spatiotemporally distant events do not *coexist* with the here-now, then they do not exist when the here-now exists. On the other hand, if one looks at spacetime from the perspective of the spatiotemporally distant event, then the distant event exists and the here-now does not. Is it reasonable to assume that existence depends on the perspective of the spatiotemporal point? Isn’t existence simply a brute fact independent of the particular perspective? Sider’s worry may be resisted by insisting that relations of coexistence are *not well defined* on space-like separated events. One way to avoid defining relations of coexistence is to restrict the predicate “being present” to a particular point on a particular worldline. According to this proposal, it is meaningless to ask whether space-like separated events on different worldlines coexist. While this solution *feels* a little strained, there is no way to reject it out of hand.

Consider the possibility that the evolution of the present instant makes possible the existence of events. Assume that the here-now p is present. From p one may consider causal chains that go into the future light-cone, so that various "new" here-nows may be $p' = p + dx_{\mu}$. But which instant is the "next" present depends on the inertial reference frame. Thus it is not possible for the change from p being present to p' being present to allow for the possibility of an event coming into existence at p', because one may have considered many different future p's, which are potentially the new present instant. One cannot utilize the notion of spatiotemporal perspective to describe the evolution of the present instant. The evolution of the present instant cannot underwrite the process of events coming into existence.

Stein may argue in response that it was never a core intuition of Presentism to suggest that the movement of the now underwrites the process of becoming. It may be that in traditional presentist accounts it seemed as if existence is indexed to a particular present instant, but this is not an ontological grounding of existence, only an epistemic way to access it. The movement of the now was only supposed to reflect a process of becoming of the world as a whole. Similarly, a revised version of Presentism such as Stein's asserts that the becoming of individual worldlines is given, and the here-now is present as a result of this process of becoming. But if this is the proposal, then it is not clear why Stein is allowed to treat events that are space-like separated from the here-now as non-existing. When a remote worldline that is space-like separated from the here-now becomes, its process of becoming is independent of the here-now; and whether a point on the remote worldline is present depends on that worldline becoming and on nothing else. If a point on the remote becoming worldline exists, it violates Stein's proposal. If it doesn't, there is no evolution of the present instant that can track processes of becoming. The consequence is that Stein's proposal cannot accommodate a core presentist assumption, namely that the movement of the "now," or the present instant, is conceptually linked to processes of becoming.

The above dialectic regarding the consistency of Presentism and STR is a recurring one. In light of the conceptual difficulties in reconciling Presentism with STR, various proposals are given as to how to revise Presentism to accommodate the "lessons" of relativity. In the process a form of Presentism is proposed that is consistent with the structure of STR. When critics point out the weaknesses in such revisions, presentists respond by saying that this or that aspect of traditional Presentism is not essential to the view. For example, one may argue that the "core" of Presentism is merely the claim that the now exists and that non-present events do not exist. The presentist does not concern herself with "how" the present comes into existence. But such anemic versions of Presentism, while not entirely indefensible, lack the substance that links Presentism with genuine concepts of change. And without this conceptual connection Presentism becomes an unstable position to take. A similar dialectic governs other proposals for revising Presentism in the light of STR. For example, Godfrey-Smith (1979) and Hinchliff (2000) argue that one is free to define the present as the set of events on the surface of the light cone leading up to a particular event. The justification for this definition is that light signals do not only leave their traces on the spatiotemporally distant events, they actually connect the events and make them co-present. The rearward light cone definition of the present is perhaps a consistent way to use the word "present," but it suffers the same weaknesses that trouble Stein's proposal, since the view relies on relations of causal connectedness to define the present, while the process of becoming is left unanalyzed and disconnected from the concept of being present.[12]

There are more desperate attempts to salvage Presentism, which involve a retreat into the claim that metaphysical views are immune from scientific advances. According to this approach, there could be no direct conflict between a scientific theory and a metaphysical view. Any scientific theory is merely an economical summary of phenomena, or at least it may be that metaphysical systems are underdetermined by scientific theories, and there is no reason to think that the metaphysical nature of entities such as time can simply be read from the theory without further interpretation.[13] Given that much of our experience of time is presentist in nature, and given that denying Presentism would undermine our self-understanding as freely acting agents, some philosophers feel compelled to resist the Eternalism that seems to be implied by STR. It is often the case that resistance stems from religious and ethical motivations or from certain views regarding the philosophy of mind. Without action influencing the future, there is not much left to the notion that a human agent is responsible for his or her actions. One way to avoid the eternalist position is to argue that despite the equivalence between inertial reference frames in STR, there is a privileged frame as a matter of metaphysical fact, or as a matter of undetectable physical fact.[14] Whether or not one has means of assessing metaphysical claims that go beyond the scientific theory I leave aside, since I believe there is more philosophical work to do in making sense of the implicit metaphysical assumptions "involved" in STR. While I am convinced that there is little chance of reconciling the spirit of Presentism with the structure of STR, I am unconvinced that Eternalism is the proper and only alternative to Presentism.

4.4 But Eternalism Is False Too

The trouble with Eternalism is not so much the conflict it generates with our pre-philosophical experience of time, but that it undermines the notion that time is asymmetric, and that the flow of time also reflects the propagation from cause to effect. Assume that event e_1 caused event e_2. For argument's sake, one assumes that e_1 happened prior to e_2. Ordinarily, to accept a causal relation between two events implies the acceptance of the counterfactual "e_2 would not have existed if e_1 did not exist." It is not entirely clear what could make such a counterfactual true, if one adopts the eternalist view. An eternalist has more of a challenge in providing an

[12] See Savitt (2000) for a critique of cone-Presentism.

[13] See Hawley (2006) for a discussion on the various strategies of defending metaphysical claims from conflicts with scientific theories.

[14] See, for example, Craig (2001b) and Hinchliff (2000) for such a line of argument.

account for such a counterfactual than the presentist. The presentist believes that before e_2 is present, it does not exist; thus one can conceive of the existence of e_2 as dependent on the existence of e_1. Since the presentist believes that e_2 came into existence at some instant, she can also say that the coming into existence of e_2 would have been prevented had e_1 not existed. The presentist has the problem of articulating what grounds the causal relation, given that when the effect comes into existence, the cause no longer exists. But the eternalist has a more acute problem, since according to the eternalist all events exist, and so they never come into existence nor do they go out of existence. It seems to follow that if no event comes into existence, the existence of any single event does not depend on the existence of any other particular event.

One way to make sense of a causal claim in an eternalist context is to argue that there is a possible world w_2 similar in all relevant respects to the actual world w_1. All events leading up to e_1 in w_1 have their identical counterpart events in w_1. But in w_2 the event e_1 does not exist. Thus w_2 is identical to w_1 in all respects except that e_1 does not exist. If one can *infer* from the previous remarks that the event e_2 does not exist in w_2, one has a means of stating the truth conditions for the counterfactual causal claim, i.e., that e_2 in w_1 would not have existed had e_1 not existed in w_1. However, it is not clear what precludes us from saying that there is no similar world w_3, that is identical in all respects to w_2, except that in w_3 the event e_2 exists and e_1 does not. Such a world is certainly conceivable to us. The world w_3 might violate some law of nature that dictates the causal relation between e_1 and e_2. However, it seems that the causal connection between e_1 and e_2 is not merely the suggestion that having e_2 actualized without e_1 actualized violates a certain law of nature. Rather, the causal connection suggests that the existence of e_1 brings about the existence of e_2. It is a connection between the existence of one event and the existence of another. Thus it is not clear how one could use counterfactual claims based on alternative worlds similar to ours to ground inferences between the existence of one event and the existence of another. The causal relation involves just the two events, not the general question which events belong together in a possible world similar to ours.

There seems to exist a genuine tension between the eternalist account and the notion that causal relations establish a counterfactual between a cause and its effect. Without a future that contains unrealized possibilities, it is difficult to make sense of a cause whose existence is responsible for the existence of the effect. But the notion of a causal relation is fundamental in STR. First, fundamental to relativistic spacetime is the analysis of spacetime according to various regions that are either causally connected or causally unconnected with the here-now. This analysis is intended to clarify how spacetime structure can be used to trace causal relations, and so demonstrates the importance of causal relations in analyzing relativistic phenomena. Second, the theory of relativity embodies the scientific ideal that there is no action at a distance. The relativistic framework incorporates interactions that are locally delineated. What would be the point of utilizing localized field interactions if causal relations do not integrate well with the proper metaphysical interpretation of STR? While some are skeptical about the role of causation in science (Norton, 2003, 2007), the skepticism stems from a lack of uniform restrictions imposed by the principle of causality. But it is clear that in the context of STR, the restriction to local distribution of field interactions is a manifestation of a certain ideal of local causation. Moreover, given that in STR the local evolution of the energy-momentum coheres very well with the production account of causation, the eternalist should be concerned to explain how Eternalism is consistent with the production view of causation.[15]

By focusing on localized causal relations, I am not offering locality as an overarching universal principle. It is quite plausible to argue that some non-local influence exists in the context of quantum theories. But the argument is that *if* Presentism is undermined by STR, then so is Eternalism, given that the structure of relativity is meant to incorporate a certain ideal of causal relations. It is very plausible that future theories will replace STR, in much the same way that the General Theory of Relativity replaced STR, and that our arguments regarding the nature of time should be revisited. But it is likely that future theories would have structures that are similar to STR, at least in their local approximations, and so the hope is that a conceptual analysis of STR would reveal something important about the theories that are yet to be articulated.

Thus, while STR is certainly in conflict with the spirit of Presentism, it is also in conflict with the spirit of Eternalism. And given that so far the metaphysics of time is viewed as having only these two options, perhaps one should try an alternative approach that is neither presentist nor eternalist to accommodate "the lessons" of STR.

4.5 Primitive Motion Relationalism and the Metaphysics of Time

It may be beneficial to consider again the conceptual space of possibilities in the metaphysics of time. The standard distinction governing debates in the metaphysics of time is that between Presentism and Eternalism. While there are alternatives to these two views, they ordinarily fail to gain support beyond a few adherents. The presentist is faced with conceptual difficulties whenever it attempts to explain the process of future temporal instants coming into existence. On the other hand, Presentism becomes explanatory deficient when it limits its scope to the existential claim, i.e., when it merely asserts that the only objects that exist are the ones that are present now. Such a view cannot explain the evolution of the now, and it cannot explain the harmony between the evolution of time and the traces of past worlds that exist in the present. Another serious objection to Presentism stems from the tension

[15] Frisch (2009) argues that causality principles govern the derivation of dispersion relations for the electromagnetic field. This application of a causal principle seems to provide evidence that in the context of relativistic field theories, principles of causation are scientifically relevant. As Frisch points out, the relevance of causal principles in this limited context does not imply an *a priori* commitment to causal principles for all scientific theories, which is Norton's main source of objection to fundamentalism about causation.

it faces when one tries to reconcile it with STR. When philosophers take these conceptual and empirical difficulties to heart, they often embrace Eternalism. However, Eternalism is not an attractive view if a productive account of causal relations is taken to be implied by STR.

Presentism and Eternalism consist of two unattractive alternatives in the metaphysics of time. But one can examine two central presuppositions both positions hold; namely, that time is ontologically more fundamental than motion, that an object exists if and only if it is present at a temporal instant t. If PMR is a plausible account of spacetime, one can take this approach to be a direct challenge to both presuppositions. According to *Primitive Motion Relation*, motions are more fundamental than time, and they exist independently of whether the motion is present to another object. Motions provide the grounds for temporal change, and do not occur *within* time. But when two motions intersect, one may say that one motion is present to another motion, or in other words that an event produced by the motion becomes present. Thus there is a distinction between existing and being present, and one does not reduce existence to being present at a particular temporal instant. Moreover, it is impossible to discuss the notion of being present at a particular instant of time, since there are no temporal instants that exist prior to the intersection of motions.

The proposal to sever the conceptual connection between being present and existing is difficult to make sense of. It is difficult to resist the notion that an event becoming present amounts to an event coming into existence. Before two motions intersect, it seems safe to assert that the event of intersection did not happen, and so before the intersection between the two motions the event did not exist. Once the event becomes present, it exists. One is able to resist this inference (from presence/non-presence of an event to its existence/nonexistence) when one thinks of intersection events as relations between motions. If the intersection between two motions happens, then a certain relation holds, the proposition describing the intersection (the event) becomes true, and the event becomes present. However, one should not think of the intersection as an event that *comes* into existence, since whether two motions intersect is what defines the spatiotemporal location. There is no spatiotemporal location prior to the actual intersection at which place the event *could* materialize. There is no sense in which one could say that the event *came* into existence.

According to the geometry of PUMs articulated in Chapter 3, for any two events on a motion α, there is a determinateness relation. Either $\mathbf{D}pq$ or $\mathbf{D}qp$, but not both. Assume that p determines q. This implies that one can associate an asymmetric causal relation between two events belonging to a single motion. If p is present, then q is not yet present. When one takes the perspective of motion α, the event q is yet to happen when p is present. If q is present, then from the perspective of α the event p has already happened. Thus for each motion there is a clear demarcation between events that were present, that are present at this moment, and are yet to be present. For each motion there is a clear demarcation between past, present and future. However, this clear demarcation is not translatable to the claim that events in the past no longer exist and events in the future are yet to exist. Existence is not a attached to singular events, but to motions as a whole. Whether or not a motion exists is independent of the question whether it intersects another motion or whether another motion is present to it. Thus existence and being present are distinct notions.

It is conceivable to think of motions whose intersections with other motions after a certain present event *p* are yet to be determined. Thus, one may think, for any particular motion, that the future is open. How to conceive of an existing motion whose future intersections with other motions are yet to be determined will require much conceptual work to make sense of. One distinction that one needs to make is that between an existing motion and an actual event. While the same motion exists, it may be that a certain event either becomes actualized or is made present independently of whether the motion exists. Thus, one should consider separating two realms of possibility. First, there is the range of possible motions; some motions may remain strictly a conceptual possibility, and others may exist. Another realm of possibility concerns those events that are yet to actualize, given that a certain set of motions exists. There is a distinction, according to this approach, between being actual and existing. Or in other words there is a distinction between two modes of existence; the presence of events and the existence of motions.

My position is therefore that neither Presentism, nor Eternalism captures the nature of time. Each position is to some extent true, and to some extent false. Once the biconditional between existing and being present at *t* is severed, it is possible to relate to the underlying motions as existing, and to particular events as being present. The existence of the underlying motion does not entail the presence of all events belonging to the motion, since the presence of an event presupposes the intersection between motions.

Chapter 5
The History of Newtonian Mass

The standard interpretation of Newtonian physics takes the concept of mass to be a primitive property of bodies. The most elementary Newtonian object is a particle that occupies a point in space at a particular time. The particle has a trajectory, or a continuous mapping from instants of time into three-dimensional space. According to the standard interpretation, each point-particle carries with it an inherent property of mass which compels a body to resist the action of impressed forces. Thus, one often finds reference in expositions of Newtonian physics to a "point mass," or a "particle of mass *m*," as the most primitive material entity.[1]

I believe there is still work to be done in clarifying the concept of mass, particularly if one is to find a clear presentation of the interconnections between Newtonian concepts. The concept of mass played an important role in the two scientific revolutions that led to the Special and the General Theory of Relativity. The conceptual interrelationship between mass and energy proved helpful in understanding the interaction between radiation and matter. These insights led Einstein to the Special Theory of Relativity. The equivalence between inertial and gravitational mass provided Einstein with his initial insights into the General Theory of Relativity. The accepted interpretation of mass has undergone major revisions with each revolution in physical theory. Thus a clarification of the philosophical significance of this concept might provide important insights into the metaphysical and conceptual underpinnings of our physical theories. My aim is therefore to analyze the historical and conceptual origins of the concept of mass in classical physics. Another aim is to bring forth a new interpretation and framework for solving some of the inconsistencies and obscurities plaguing this concept. To prepare the way, this chapter first traces the history of the concept, paying close attention to the way in which

[1] Bertrand Russell describes the Newtonian system in this way:

> There is an absolute space, composed of points, and an absolute time, composed of instants; there are particles of matter, each of which persists through all time and occupies a point at each instant. Each particle exerts forces on other particles, the effect of which is to produce accelerations. Each particle is associated with a certain quantity, its "mass," which is inversely proportional to the acceleration produced in the particle by a given force. (Russell, 1957, p. 14)

O. Belkind, *Physical Systems,* Boston Studies in the Philosophy of Science 264,
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Newton introduced it. Chapter 6 introduces a new interpretation of mass, one that will undermine the accepted view that takes mass to be a primitive, inherent property of material bodies. Another widespread assumption that will be questioned is the view which restricts the meaning of mass to its inertial role.

Newton originally conceived of mass as a geometric concept, one that is intimately related to the nature of space. Newton arrived at the concept of mass when he realized, via his critical examination of Descartes' physics, that there is a conceptual distinction between absolute space and matter. To make this distinction viable, matter was conceived as a region of space that is impenetrable to other regions of space. Matter was therefore simply the property of impenetrability located within a moveable region of space. The size of an impenetrable place, or the "quantity of matter," is the quantity that later came to be associated with the concept of mass. But Newton also argued that the quantity of matter contributes to the conservation of quantity of motion (momentum). Thus the notion of quantity of matter also carries a dynamic role, which ultimately leads Newton to introduce his notorious inherent or inertial forces.

Contemporary interpretations of mass emphasize its inertial role. This emphasis is largely the result of Mach's positivist critique of Newtonian physics. Mach's *Science of Mechanics* is mostly known for its critique of Newton's absolute space and time and for its influence on Einstein. But Mach also changed the way in which commentators interpret the concept of mass. Newton defined "mass" as the quantity of matter, i.e., he initially thought of it as a quantity that reflects how much matter there is in a body, or the body's "bulk," in analogy with size, which reflects how big a body is.

Mach's positivist epistemology compelled him to be suspicious of metaphysical entities such as absolute space and causal agencies that cannot be directly observed. This philosophical approach compelled Mach to criticize Newton's concept of absolute space. He was also very suspicious of inherent forces of inertia, and Newton's concept of quantity of matter. Mach proposed to define the concept of mass by reducing it directly to mechanical experiences, avoiding Newton's inherent forces of inertia. While Mach's definition of mass was never accepted by the scientific community, his critique of absolute space, inertial forces, and quantity of matter had lasting influence. In particular, his critique left its mark on Einstein's physical thinking. A consequence of Mach's critique is that in future generations the geometric origin of the mass was forgotten, and the dynamic role of mass assumed the essential meaning of the concept. Since one of the aims of this work is to revive the geometric origin of the concept of mass, I take Mach's critique of mass to be a significant event in the history of the concept.

This chapter will first focus on the early history of the concept of mass. The primary concern in this part is to delineate the reasons and the philosophical motivations that led to the Newtonian conception of mass. Section 5.1 sketches the geometric origins of Newtonian mass. Section 5.2 reconstructs the dynamic conception of mass and its role in Newton's three laws of motion. I claim in this section that Newton's three laws of motion attempt to articulate various agencies in bodies that give rise to the conservation of quantity of motion, and that this attempt ultimately fails. Section 5.3 focuses on Ernst Mach's critique of Newton's quantity of matter.

5.1 The Geometric Conception of Mass

Twentieth century interpreters and historians ordinarily emphasize the inertial role of mass over its role as quantity of matter. For example, Jammer (1997) places Newton's concept of mass within a tradition that differentiates between the active and passive principles of matter. According to Jammer, a key development in the evolution of the concept is Kepler's introduction of inertial mass. When Kepler discovered that the motion of the planets could be described by ellipses rather than by perfect circles, he had to find a dynamic explanation to replace the "naturalness" of circular motion. The planets were no longer moving according to the path they were supposed to trace; their motion depended on some external influence. Force consisted of the powers that drove planets to motion, and the "inertness" of matter – its tendency to remain at rest – was the resistance they offered to this external influence. Jammer places Kepler's inertial mass within the tradition of Neoplatonic conceptions of matter. According to this tradition, the inertness of matter is associated with the non-active component of natural processes and things. The inertness of matter differs in kind from active principles, or spiritual essences that are able to produce change. In Kepler's conceptual scheme the cause of change (i.e., of motion) is the force acting on matter, and the factor resisting change is the material essence of bodies and their natural "tardiness." As Jammer explains, Kepler's concept of inertial mass does not yet carry the meaning of "inertia" we associate with the Newtonian concept, since Kepler only speaks of the bodies' tendencies to remain at rest, not their tendency to continue moving in uniform, rectilinear motion. "Kepler, by associating inertia with *copia materiae*, made the metaphysical notion of *inactivity* ("plumpness") into what at his time might be considered as a scientific concept" (Jammer, 1997, p. 58).

Jammer's narrative is typical to twentieth century historiography. This narrative ignores completely Newton's definition of mass as a quantity of matter. The focus on the inertial role of mass seems to continue Mach's argument that the definition of mass as a quantity of matter is incoherent. Jammer lists the influences that led to Newton's systematization of the concept of mass. These include Huygens' study of centrifugal forces and his and others' studies of impact phenomena (Jammer, 1997, pp. 60–62). Descartes' physics, according to Jammer, was simply a hindrance to the development of the concept (Jammer, 1997, p. 59). Descartes' aversion to material properties that are not derived from spatial extension is simply an impediment. After all, Descartes claimed that "all the properties we perceive in [matter] are reducible to its divisibility and consequent mobility of the parts" (Descartes, 1985, p. 232). The concept of inertial mass was either completely rejected by Descartes or had a minor role.[2]

[2] Descartes considers reasons to doubt the claim that extension is the only attribute of matter, and then refutes the objection from the consideration of rarefaction and condensation with the following infelicitous remarks:

Jammer's reading overlooks Descartes' key influence on Newton's thinking.[3] One can reconstruct Newton's thinking in the *De Gravitatione* text, which in all likelihood predates the *Principia*. Preliminary conceptual investigations led Newton to critique and revise Descartes' definition of quantity of motion. This critical examination of Descartes' physics prepared the way for Newton's synthesis of Descartes' conservation of momentum with Kepler's concept of inertial mass. As Koyré notes, one of Descartes' influences on Newton's *Principia* is the treatment of uniform rectilinear motion as a *state*, equivalent in all respects to the state of being at rest. The extension of inertia to states of motion crucially depends on Descartes' formulation of a conservation law and his treatment of motion as a "natural state" in which an undisturbed body perseveres.[4]

> The first is the widespread belief that many bodies can be rarefied and condensed in such a way that when rarefied they possess more extension than when condensed. Indeed, the subtlety of some people goes so far that they distinguish the substance of a body from its quantity, and even its quantity from its extension . . . But to invent something unintelligible so as to provide a purely verbal explanation of rarefaction is surely less rational than inferring the existence of pores or gaps which are made larger, and supposing that some new body comes and fills them . . . Moreover, it is very easy for us to see how rarefaction can occur in this way, but we cannot see how it could occur in any other way. Finally, it is a complete contradiction to suppose that something should be augmented by new quantity or new extension without new extended substance, i.e., a new body, being added to it at the same time. For any addition of extension or quantity is unintelligible without the addition of substance which has quantity and extension. (Descartes, 1985, p. 225)

Thus Descartes is unable to conceive of a difference between extension and matter, and argues that no quantity can be added to a substance other than its size. Descartes is unable to conceive of a difference between space and matter since he has convinced himself that matter is inconceivable without extension and that extension is inconceivable without matter.

[3] Alexander Koyré emphasizes the role of Descartes' physics in Newton's thinking:

> [Newton] did not mention the Cartesian origin of the concept *quantity of motion* (mv), which he stubbornly maintained as a measure of force against the Huygenian and Leibnizian *vis viva* (mv^2), even while he rejected the Cartesian assertion of the conservation of motion in our world. Nor did he mention that it was Descartes' formulation of the principle of inertia, which placed motion and rest on the same ontological level, that inspired his own.
>
> We shall not judge the Newtonians, nor even Newton, for being unfair to Descartes. Human thought is polemic; it thrives on negation. New truths are foes of the ancient ones which they must turn into falsehoods. It is difficult to acknowledge one's debts to one's enemies. Now Newton's thought, nearly *ab ovo*, had been formed and developed in opposition to that of Descartes. Accordingly, we cannot expect to find praise, or even historical justice, for Descartes whose title, *Mathematical Principles of Natural Philosophy*, contains an obvious reference to, and rejection of, his *Principles of Philosophy*. (Koyré, 1965, p. 65)

[4] Koyré explains the importance of conceiving of motion as a *status*:

> *Status* of motion: by using this expression Newton implies or asserts that motion is not, as had been believed for about 2000 years – since Aristotle – a process of change, in contradistinction to *rest*, which is truly a *status*, but is also a *state*, that is, something that no more implies change than does rest. Motion and rest are, as I have just said, placed by this

Treating motion as a state enabled Descartes and Newton to accept uniform rectilinear motion as a natural state, namely as a state that does not require an external physical cause. Descartes described the inclination of bodies to continue moving in a straight line as their *conatus*, while the ultimate cause of a body's *conatus* was God's continual intervention in the created world. Newton instead posited the existence of a *vis insita* and described it as the power a body possesses inherently and independently of God's intervention. The replacement of *conatus* with *vis insita* is the first move that allows Newton to combine the Keplerian property that resists change with the Cartesian conservation of motion. Thus, the *vis insita* is also a *vis inertia*, and indicates the power of material bodies to resist impressed forces, whether they are in a state of rest or of uniform rectilinear motion. However, the synthesis of Kepler's concept of inertial mass with Descartes' *conatus* is not the complete story of Newton's innovation. The synthesis goes hand in hand with Newton's conceptual analysis and critique of Descartes' physics.

Newton's magnus opus, entitled *Mathematical Principles of Natural Philosophy*, can be seen as a work intended to replace Descartes' main scientific text, *Principles of Philosophy*. Newton believed there were important flaws in Descartes' work, which he intended to correct in his own. The most important flaw in Descartes' work is a methodological one. Descartes does not begin his system of the world with observed phenomena, and does not base his conclusions on careful observations and chains of reasoning that take those observations as the starting point. Descartes' faulty methodology has led Descartes and his followers to a false theory of gravitation, which relied more on the intuitive nature of their explanations than empirical evidence. But there is another very serious conceptual flaw in Descartes' physics, since it entails an incoherent account of motion. There is a conceptual inconsistency between Descartes' kinematics – his definitions of motion, and Descartes' dynamics, primarily his definition of quantity of motion and its conservation. While Newton is concerned with exposing the incoherencies in Descartes' physics, he is also concerned with salvaging Descartes' quantity of motion. The conservation of quantity of motion will prove essential to Newton's project.

The conservation of quantity of motion is the grain of truth that Newton would like to salvage from Descartes' physics. It is the conservation of quantity of motion that is the main conceptual tool for analyzing causal chains and understanding the forces of nature. But Descartes' insistence that there is no distinction between matter and space, and his definition of true motion, render the notion of quantity of motion unusable. The conceptual reworking of Cartesian metaphysics, with an eye towards

word on the same level of being, and no longer on different ones, as they were still for Kepler, who compared them to darkness and light, *tenebrae et lux*. Now precisely and only because it is a *state* – just like rest – that motion is able to conserve itself and that bodies can persevere in motion without needing any force or cause that would move them, exactly as they persist at rest. (Koyré, 1965, p. 67) retaining Descartes' notion of quantity of motion, compels Newton to introduce both absolute space and the concept of mass.

Descartes' physics is inconsistent, since it articulates a definition of true motion and a definition of quantity of motion that are incompatible.[5] On the one hand, Descartes defines the true motion of a body as "the transfer of one piece of matter, or one body, from the vicinity of the other bodies which are in immediate contact with it, and which are regarded as being at rest, to the vicinity of others" (Descartes, 1985, p. 233). Thus according to Descartes, the true motion of a body is defined as the motion of a body relative to the containing bodies, which are taken to be at rest. This definition of motion is relative, since the motion of each body is defined in relation to bodies that enclose it; there is no common reference frame for all motions. But it should be emphasized that despite true motion being defined relative to other bodies, Descartes' definition does *not* arbitrarily depend on the reference body. For each body, since according to Descartes there is no empty space, there is a unique set of bodies that immediately enclose it. Thus for each motion, there is a unique frame of reference that is used to define the motion of the body. And if one takes those enclosing reference bodies to be at rest for the purpose of defining the motion of the enclosed body, there is a unique motion that is attributed to it.

In addition to the kinematic definition of motion, Descartes defines quantity of motion. According to Descartes, motion "has a certain determinate quantity; and this, we easily understand, may be constant in the universe as a whole while varying in any given part" (Descartes, 1985, p. 240). Thus, Descartes argues that while true motion is defined relative to the surrounding bodies, the quantity of motion in the whole universe is conserved. The quantity of motion may vary in the parts, while it is preserved in the universe as a whole. This implies that there is a quantity of motion that is conserved for a closed system of bodies. If the quantity of motion is in each part of a solid body, the quantity of motion of a composite body is proportional to the body's size. The consequence is that the product sv, or the product of size and velocity, is conserved. According to Descartes, "... if one part of matter moves twice as fast as another which is twice as large, we must consider that there is the same quantity of motion in each part; and if one part slows down, we must suppose that some other part of equal size speeds up by the same amount" (Descartes, 1985, p. 240).

The problem with Descartes' physics is a conceptual inconsistency between his definition of true motion and his definition of quantity of motion. Newton alludes to this inconsistency in the *De Gravitatione*, wherein Newton argues that Descartes both takes the earth to be moving and not moving. Given that, according to Descartes, the earth drags some of the ether surrounding it, the definition of the earth's true motion implies that the earth is at rest. On the other hand, attributing the earth a *conatus*, implies that the earth has a quantity of motion, i.e., it requires that the earth have speed. Descartes' kinematic and dynamic claims about the motion of the earth cannot be reconciled. This inconsistency is a conceptual one, and goes to

[5] See Belkind (2007) for a fuller account of Newton's critique of absolute space.

the core of Descartes' physics, as a close examination of the Scholium to Newton's definitions in the *Principia* shows. In the Scholium, Newton critiques the same inconsistency, i.e., the conflict between Descartes' definition of true motion and his quantity of motion. Newton assumes in his argument that there is a distinction between true and apparent motion, and that true motion is either relative or absolute. On the one hand, the parts of a moving solid body are at rest relative to other parts of the body. Thus, according to Descartes' definition of true motion the parts of a solid body are at rest, since they do not change their place relative to their immediate surroundings. On the other hand, a solid body that moves has a *conatus* when it moves as a whole. But if each part of the moving solid body is at rest, it is not clear how the composite body can be given a quantity of motion. Presumably, the quantity of motion of a composite body arises from the quantities of motion of each of the parts. If each of the parts is at rest according to Descartes' definition of true motion, it is inconsistent to attribute any composite solid body a non-zero quantity of motion.

The upshot of Newton's critique of Descartes' physics is that, to preserve a notion of quantity of motion, one needs to provide a definition of true motion that is consistent with a quantity of motion. To correct Descartes' physics a central Cartesian tenet must be given up. First, Newton argues that true motion of a particular body cannot depend on a reference frame constructed from movable bodies. Newton therefore introduces absolute space, which will provide a common frame of reference for the true motion of bodies. The essential characteristic of this reference frame is that it consists of a set of non-movable places (space), which are distinguished from movable places (material bodies). This will resolve the tension between Descartes' kinematics and dynamics. But in order to provide a set of immovable places, i.e., to acknowledge the existence of absolute space, one needs to distinguish between absolute space and matter. An essential material property must be introduced.

The original impetus for introducing the concept of mass stems from the Newtonian project of revising and correcting Cartesian physics. Newton is convinced that in order to articulate a coherent notion of quantity of motion, one needs to introduce a distinction between movable and non-movable places, which is at the same time the distinction between space and matter. This distinction is made possible with the concept of impenetrability. In the *De Gravitatione*, Newton tells a story about the way in which God could have created material bodies:

> Thus we may suppose that there are empty spaces scattered through the world, one of which, defined by certain limits, happens by divine power to be impervious to bodies, and by hypothesis it is manifest that this would resist the motions of bodies and perhaps reflect them, and assume all the properties of a corporeal particle, except that it will be regarded as motionless. If we should suppose that that impenetrability is not always maintained in the same part of space but can be transferred here and there according to certain laws, yet so that the quantity and shape of that impenetrable space are not changed, there will be no property of body which it does not possess. (Newton, 2004, p. 28)

Thus it is enough for God to designate parts of space to be impervious to others and to allow these impenetrable regions to move hither and thither, to create bodies that are indistinguishable from the ones we experience. Bodies can therefore be defined as "*determined quantities of extension which omnipresent God endows with certain conditions*" (Newton, 2004, p. 28, emphasis in original). These conditions are (1) that bodies are mobile; (2) that two bodies do not coincide (i.e., that they are impenetrable); and (3) that they excite various perceptions of the senses and the imagination in created minds.[6]

In the *De Gravitatione* Newton thinks of bodies as impenetrable regions of space that can move from one place to another. Thus Newton thinks of bodies in geometric terms, as certain regions of space that have an essential property that distinguishes them from space. In this geometric conception, a body is *nothing but* an impenetrable region of space. The material properties one associates with bodies is derived from the single property of impenetrability. The connection between mass and impenetrability can now be examined. If bodies are impenetrable regions of space, then the *size* of an impenetrable region should give us its quantity of matter. But there are a couple of complications involved. First, it is the size of the impenetrable region, rather than the size of the containing space (which includes pores and empty regions) that should replace size in Descartes' original definition of quantity of motion as the product of size and speed. Second, since some bodies are porous, it is clear that the overall size of a body is not the same as the size of its impenetrable region.

The notion of density is already present in the *De Gravitatione*. In definitions 5–13 Newton defines the notion of force, its extension and intension. Definition 5 asserts that "Force is the causal principle of motion and rest" (Newton, 2004, p. 36). Newton proceeds to define the intension of the force, which is "the degree of its quality" and the extension of the force, which is "the quantity of space and time in which it operates." These definitions are not very clear, but they become clearer when Newton discusses the absolute quantities of the force, which are the product of its extension and intension. Newton explains:

> And thus motion is either more intense or more remiss, as the space traversed in the same time is greater or less, for which reason a body is usually said to move more swiftly or more slowly. Again, motion is more or less extended as the body moved is greater or less, or as it is diffused through a larger or smaller body. And the absolute quantity of motion is composed of both the velocity and the magnitude of the moving body. So force, conatus, impetus, or inertia are more intense as they are greater in the same or an equivalent body: they are more extensive when the body is larger, and their absolute quantity arises from both. (Newton, 2004, p. 37)

[6] Newton's account suggests that the distinction between space and bodies is that between penetrable and impenetrable places. However, there may also be another distinction at work in Newton's thinking. Newton seems to follow More in distinguishing between mathematical and physical divisibility. While space is infinitely divisible, there seems to be a limit to divisibility in physical bodies and material bodies must comprise of indivisible atoms. See Janiak (2000) for an account of Newton's views on mathematical and physical divisibility.

In this passage Newton is thinking of the absolute force of *conatus*, which is also known as impetus or inertia. Newton defines this absolute force as the product of "the magnitude of the moving body" and its velocity. The force is the product mv, where m is the extension of the impetus force; and v is the intension of the force. Newton here is not referring explicitly to the notion of mass; rather, it seems he has Descartes' definition of the quantity of motion in mind, sv, where s is the size of the body; and v its speed. Here in the *De Gravitatione* it is not yet clear whether by v Newton, like Descartes, means speed, or he has the notion of velocity in mind, which is speed together with the inclination to go in a particular direction. However, it seems clear from the context that an important difference between Descartes' and Newton's definition is that Newton takes bodies to consist of impenetrable places, rather than just bounded regions in space. Thus, "the magnitude of the moving body" is the amount of impenetrable place the body occupies and is the extension of the force of inertia mv. In the *Principia* the "magnitude of the moving body" will be replaced with mass, thus there is a direct line of thinking that connects impenetrability with mass.

Definition 15 of the *De Gravitatione* describes the notion of density (Newton, 2004, p. 37). To clarify the notion Newton discusses a body that is shaped like sponge with pores. The body has regions that are impenetrable and pores that do not contain matter. The inertia of a body increases or decreases in proportion to the density as the pores diminish or increase in overall size. Thus, density is defined as the amount of impenetrable volume a body occupies relative to its overall volume, pores included. This is where Newton's definition of the quantity of motion departs from that of Descartes, since for Descartes the notion of density does not make sense. There can be no difference for Descartes between impenetrable and non-impenetrable places – no body has regions that are empty of matter.

A theoretical consequence of viewing matter as impenetrable regions of space, is that the ultimate parts of matter are all alike. If one breaks a material body into its ultimate parts, one would arrive at small impenetrable regions of space, and impenetrability itself does not come in varying degrees. Thus, the ultimate parts of matter appear on this conception to be of a uniform, non-varying density. Moreover, it also seems to follow that there is an upper limit to the value of density. There is historical evidence that Newton actually believed that all atomic parts of matter are indistinguishable. But he took care not to endorse this view explicitly in the *Principia,* since he could provide no empirical evidence that this is in fact the case.[7]

According to the Newton's geometric conception of matter, there are no uniformly distributed bodies of different densities. However, Newton seems to think that from a mathematical perspective, this consequence is not important. In the *De Gravitatione*, Newton says the following:

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[7] See Biener and Smeenk (2011) for a discussion of Newton's geometric concept of mass.

> But in order that you may conceive of this composite body as a uniform one, suppose its parts to be infinitely divided and dispersed everywhere throughout the pores, so that the whole composite body there is not the least particle of extension without an absolutely perfect mixture of infinitely divided parts and pores. Certainly such reasoning is suitable for contemplation by mathematicians. (Newton, 2004, p. 38)

From a mathematical perspective, one could take a non-uniformly distributed body, and think of it as if it consisted of a uniform distribution of impenetrable regions and empty regions of space. Thus from a physicists' perspective – given that one cannot perform experiments at the level of ultimate parts of matter – there is no practical difference between uniform bodies of varying densities and non-uniform bodies of a single density for its ultimate impenetrable parts. To imagine uniformly distributed bodies of varying densities, think of an impenetrable region of space as uniformly contracted or expanded to occupy different volumes. In that case one could conceive of different uniform densities where every least particle "is a perfect mixture of infinitely divided parts and pores."

The concept of impenetrability may also explain how Newton conceived of the causal roles of mass. An impenetrable place resists another body's attempt to "enter" the impenetrable region, so that impenetrability gives rise to the power of a body to resist impressed contact forces. Thus there is an analogy between the resistance of impenetrable space to external contact forces and the inertial force that resists external impressed forces. But the notion of impenetrable places does not explain why a body resists the action of forces that act at a distance (since no external body is really attempting to "enter" the impenetrable region of the body), nor does it explain why the resisting force is proportional to the acceleration of a body. However, some evidence that the notion of impenetrability still guides Newton's thinking in the *Principia* is the fact that he lists impenetrability as one of the qualities that should be assigned to all bodies universally, discussed in Rule 3 for the Study of Natural Philosophy, at the beginning of Book III of the *Principia*.

To summarize the argument so far: the origin of Newton's concept of mass is Newton's critical examination of Descartes' physics. Newton identifies an internal inconsistency between Descartes' definition of true motion and his conservation of quantity of motion. It is not possible to define the true motion of a body relative to the containing bodies, while insisting that quantity of motion is conserved and can be used to analyze forces. Thus, Newton introduces the distinction between movable and non-movable places. The set of non-movable is absolute space, and the movable places will be the impenetrable regions of space that can move around. The notion of mass and density comes about when one recognizes that in porous bodies there is a distinction between the overall space a body occupies and its impenetrable region. Density is the ratio between the size of the impenetrable region and the overall size of the body, pores included. While Newton seems to believe that ultimate parts of matter have uniform density, he does allow for a mathematical conception of density, where the ultimate parts of matter come in varying degrees of density.

The upshot of this historical reconstruction is that mass has an important origin in geometric conceptions of matter. While Newton distinguishes his physics from Descartes', and while he introduces absolute space and mass, his original conception of mass identified it with an impenetrable region of space. This geometric origin of the concept was put aside by historians of physics and later physicists. In the next chapter I shall try to revive the geometric underpinnings of mass, and argue for their relevance to understanding the nature of the concept.

The initial formulations in the *De Gravitatione* provide the background for Newton's remarks about mass in the *Principia*. The size of the impenetrable region of which a body is comprised can be thought of as a quantity of matter:

> **Definition 1** *Quantity of matter is a measure of matter that arises from its density and volume jointly.*
>
> If the density of air is doubled in a space that is also doubled, there is four times as much air, and there is six times as much if the space is tripled. The case is the same for snow and powders condensed by compression or liquefaction. I am not taking into account any medium, if there should be any, freely pervading the interstices between the parts of bodies. Furthermore, I mean this quantity whenever I use "body" or "mass" in the following pages. It can always be known from a body's weight, for – by making very accurate experiments with pendulums – I have found it to be proportional to the weight as will be shown below. (Newton, 1999, p. 403)

What Newton called "the magnitude of the body" in the *De Gravitatione* is now termed quantity of matter. The definition in the *Principia* which takes this quantity to be the product of density and volume resembles Newton's remarks in the *De Gravitatione* about density arising from the spread of the same impenetrable place over different volumes. Newton associates the quantity of matter with the size or volume of the impenetrable region. Whether or not the ultimate parts of matter are indistinguishable remains an unverified consequence of Newton's conception of matter.

5.2 The Dynamic Conception of Mass

Newton introduced the concept of mass in order to distinguish between movable and non-movable places. But his main objective is to provide the conservation of quantity of motion with a coherent conceptual framework. Thus mass has another significant conceptual role, which connects it to the dynamic concepts of the *Principia*, i.e., quantity of motion, acceleration and force. In the *Principia*, Newton defines the quantity of motion as follows:

> **Definition 2** *Quantity of motion is a measure of motion that arises from the velocity and the quantity of matter jointly.*
>
> The motion of the whole is the sum of the motions of the individual parts, and thus if a body is twice as large as another and has equal velocity there is twice as much motion, and if it has twice the velocity there is four times as much motion. (Newton, 1999, p. 404)

It is often assumed that Definition 2 provides the meaning of quantity of motion. However, a brief examination of the explanation of this quantity shows that Definition 2 does not give us an explicit definition, but is instead a mathematical quantity that is a consequence of one of motion's properties. Newton asserts in the explanation that the motion of the whole is the sum of the motions of the parts. Newton obviously shared with his readers the assumption that "motion" is additive, so that the velocities found in parts of a body add up to comprise the motion of the composite.

Newton's use of the notion of motion is obscure to the contemporary reader, since motion is neither simply the velocity of a body, nor is it exactly the same as the modern concept of momentum. The modern conception of momentum takes it to be a parameter that is *defined* as the product of mass and velocity, and is conserved. But in Newton's thinking, motion is a compositional property of bodies. Each ultimate part of matter has a certain velocity, which is also the motion of that part. The motion of a composite system is the sum of the motions of the parts. Thus, while motion is identical to velocity when the object is an indivisible particle, it is not the same as velocity for a composite body, since motion is compositional and velocity is merely the change in place relative to the time elapsed. (This notion corresponds to Newton's belief that all atomic parts of matter are indistinguishable, and of the same density.) Thus one can estimate the *quantity* of motion in solid bodies by the product of quantity of matter and velocity, since the quantity of motion reflects the amount of motion one finds in a composite body.

Newton's goal is to salvage Descartes' quantity of motion. In doing so he also introduces material agencies that give rise to its conservation. Descartes claimed that the conservation of quantity of motion is derived from God's act of creation (Descartes, 1985, p. 240). According to Descartes, motion has a quantity that is conserved in the universe as a whole, and it is conserved directly by God. This is not to deny the existence of secondary causes, i.e., the existence of forces which bodies exert on each other. However, the existence of secondary causes is dependent on a primary cause, which is God's conservation of quantity of motion. For Descartes, God is therefore the external agency responsible for conserving the quantity of motion, and one need not attribute to bodies agencies that bring about the conservation, other than examine the particular forces that cancel out due to the conservation law. However, Newton wants to limit God's involvement with created things to the original act of creation (and perhaps some infusion of motions to account for the diminishing of forces through inelastic collisions). This approach requires that he find powers within bodies that bring about this conservation.

To find the material agencies that are responsible for the conservation of quantity of motion, Newton introduces forces that "conspire" to conserve this quantity. For this purpose, Newton defines the notions of inherent and impressed forces, and articulates the three laws of motion. Consider first his definition of inherent force:

> **Definition 3** *Inherent force of matter is the power of resisting by which every body, so far as it is able, perseveres in its state either of resting or of moving uniformly straight forward.*
>
> The force is always proportional to the body and does not differ in any way from the inertia of the mass except in the manner in which it is conceived. Because of the inertia of matter, every body is only with difficulty put out of its state either of resting or of moving. Consequently, inherent force may also be called by the very significant name of inertia. Moreover, a body exerts this force only during a change of its state, caused by another

force impressed upon it, and this exercise of force is, depending on the viewpoint, both resistance and impetus: resistance insofar as the body, in order to maintain its state, strives against the impressed force, and impetus insofar as the same body, yielding only with difficulty to the force of a resisting obstacle, endeavors to change the state of that obstacle. Resistance is commonly attributed to resting bodies and impetus to moving bodies; but motion and rest, in the popular sense of the terms, are distinguished from each other only by point of view, and bodies commonly regarded as being at rest are not always truly at rest. (Newton, 1999, p. 404)

Newton's definition of inherent force makes clear that each material object contains a force "proportional to the body," which in Definition 1 Newton asserts is the same as mass. Thus, the definition lays out the inertial role of mass. Each body, in virtue of possessing a certain quantity of matter, has the power of resisting impressed forces and of preserving its state of uniform rectilinear motion. The inherent force is the material agency that compels a body to conserve the quantity of motion, since every body is deflected from its uniform rectilinear motion only with difficulty and preserves its state of uniform rectilinear motion if no external force is impressed on it.

The inherent force of matter is meaningless without supposing the existence of impressed forces, since without those there would be nothing to resist. So even though the definition of inherent force seems to stand on its own, it only makes sense in conjunction with the definition of impressed forces:

Definition 4 *Impressed force is the action exerted on a body to change its state either of resting or of moving uniformly straight forward.*

This force consists solely in the action and does not remain in a body after the action has ceased. For a body perseveres in any new state solely by the force of inertia. Moreover, there are various sources of impressed force, such as percussion, pressure, or centripetal force. (Newton, 1999, p. 405)

The impressed force acts as some action exerted on the body to change its state of being at rest or moving uniformly straight forward. But the explication makes clear that the impressed force is only present during changes in rectilinear motion, not in between such changes. This creates an interpretive problem for Newton, since Definition 3 asserts that the inherent force is essentially a force of reaction to external forces: "a body exerts this force only during a change of its state."

Newton proceeds to articulate the Three Laws of Motion that, taken together, imply the conservation of quantity of motion. The First Law of Motion is the Law of Inertia:

Law 1. *Every body perseveres in its state of being at rest or of moving uniformly straight forward, except insofar as it is compelled to change its state by forces impressed.* (Newton, 1999, 416)

The First Law of Motion takes the viewpoint of an isolated body undisturbed by some external influence. When no impressed forces exist, the force of inertia preserves the state of uniform rectilinear motion. Thus, the phenomenal consequence of possessing the force of inertia is that a body is propelled to continue in its state of being at rest or of moving uniformly straight forward. In the Second Law of Motion, Newton addresses the consequence of applying force to a body:

> **Law 2.** *A change in motion is proportional to the motive force impressed and takes place along the straight line in which that force is impressed*. (Newton, 1999, p. 416)

The modern vectorial formulation takes the second law to be $\vec{F} = m\vec{a}$. But notice that the law itself does not mention the quantity of matter. What does Newton mean when he says that the change in motion is proportional to the force? Given the modern perspective, one is inclined to take change in motion to be the same as acceleration, and the law as asserting that the acceleration is proportional to the force, i.e., that the force is the product of mass and acceleration. Nevertheless, it is odd that Newton would omit reference to quantity of matter. Given that Newton defined the quantity of motion in Definition 2, it is plausible to assume that Newton made the claim that the change in quantity of motion is proportional to the force, so that $\Delta(m\vec{v}) \propto \vec{F}$.

The modern notion of force takes the change in quantity of motion to be the product of force and time, so that $\Delta(m\vec{v}) = \vec{F}t$. In case Newton limited his discussion to a unit of time, and in case the body is solid and its mass does not change, the formulation reduces then to the familiar $\vec{F} = m\vec{a}$. But why does Newton fail to mention the unit of time necessary for correlating change in quantity of motion with force? Many commentators argue that Newton modeled the continuous applications of force with successive applications of impulsive forces operating during infinitesimal periods of time.[8] Evidence for this reading is present throughout the proofs of the *Principia*, where Newton utilizes geometric methods instead of infinitesimal analysis. For example, Proposition 1 in Book 1 proves that the area law is equivalent to the operation of a centripetal force by appealing to impulsive forces operating at regular periods of time.[9]

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[8] See Ellis (1962), Dolby (1966), Cohen's introduction to the *Principia* (Newton, 1999, section 5.3), and Erlichson (1991).

[9] There are some problems with interpreting the notion of impressed motive force as an impulsive force. First, in Definition 8 Newton defines the motive quantity of centripetal force as the measure that is proportional to the motion which it generates in a given time. Thus, the notion of motive force in Definition 8 seems much closer to our modern understanding which defines force as the rate of change of momentum, so that $\vec{F} = \frac{d(m\vec{v})}{dt}$. However, it may be that the motive quantity of a centripetal force is not the general notion of motive force described in Newton's Second Law. (See Ellis, 1962; Dolby, 1966.) Second, an instantaneous impulse force operating over an infinitesimal period of time is a discontinuous function, and its derivative and integrals are not necessarily well behaved. One may think of some limiting process by which a discrete application of impulsive forces is used to recover a continuous force. However, it is difficult to see how such a limiting process can easily capture a smooth constant rate of change in momentum (Cohen, 1971, p. 181). In a draft of the Second Law, written after the second edition of the *Principia*, Newton reworded the Second Law to indicate that it also applies in the case of continuous forces (Newton, 1981, Vol. VI, pp. 539–42). From Newton's explication it is clear that the effect of applying a continuous force oblique to the motion of a body is "measured" directly by the deviation of the body from its inertial motion. It is obvious that such a deviation can only correlate with the force if it is measured "in a

Whatever the correct interpretation of Newton's Second Law, one may assume that force is understood by him as identical with either change in quantity of motion, i.e., that $\Delta m\vec{v} = \vec{F}$, or according to the modern understanding as the change in quantity of motion in a given period of time $\frac{d(m\vec{v})}{dt} = \vec{F}$.[10] In any case, in the Second Law the concept of mass functions as the constant that relates the impressed motive force with the change in velocity a solid body experiences. And it is to ground this function of the concept of mass that Newton introduces in Definition 3 the force of inertia which resists the action of an impressed force. The intuitive meaning of mass in this context is that the more massive a body is, the harder it is to deflect it from its uniform rectilinear motion. The quantity of the endeavor to continue moving in a straight line is proportional to the quantity of matter.

The three Laws of Motion together imply the conservation of quantity of motion, and it is difficult not to assume that they were articulated just so that quantity of motion would be conserved. The Third Law states as follows:

> **Law 3.** *To any action there is always an opposite and equal reaction; in other words, the actions of two bodies upon each other are always equal and always opposite in direction.* (Newton, 1999, p. 417)

Thus, if force is defined as the change in quantity of motion, and action equals reaction, then any change in quantity of motion in one body is counteracted with a change in quantity of motion in another body. The point of the laws of motion, it seems, is to break down the law of conservation of quantity of motion, via the actions of forces, to individual exchanges of quantity of motion. When a body does not exchange quantity of motion with another body, its state of motion is conserved (Law 1). When a body A does transfer quantity of motion to body B, the change in quantity of motion in A must be the opposite of the change in quantity of motion in B (Law 3), and an individual exchange (force) results in change in quantity of motion (Law 2). Thus Corollary 1 and 2 to the Laws of Motion explain how to decompose and combine forces, which are individual transfers of quantity of motion, and Corollary 3 expressed the guiding concept behind the laws:

> **Corollary 3** *The quantity of motion, which is determined by adding the motions made in one direction and subtracting the motions made in the opposite direction, is not changed by the action of bodies on one another.*

Thus it is natural to assume that Newton composed the Laws of Motion and intended them to be Axioms *so that* the conservation of quantity of motion would be

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given amount of time." See Pourciau (2006) for an elaboration of this interpretation of Newton's concept of force.

[10] I do not agree with Pourciau (2006) when he argues that force should not be understood as change in quantity of motion, but as change in the motion of a body in a given amount of time, where motion is simply the deflection a body experience from its inertial motion in a given amount of time. Thus Pourciau relates force to the quantity $m\frac{\overrightarrow{LQ}}{h}$, where m is the mass of the body, $\overrightarrow{LQ}$ is the deflection a body experiences from its inertial motion, and h is the time during which the force operates on the body. It is not clear to us why this quantity cannot be considered as the calculated change in quantity of motion in the *particular* context in which a continuous force is applied.

recaptured as one of the first few Corollaries. One of the reasons for proceeding in this indirect way was that Newton was searching for material agencies *responsible* for the conservation of quantity of motion, agencies that would replace God's role in Descartes' physics. Another very important reason for proceeding this way was Newton's success in classifying various exchanges of quantities of motion under a single law of nature. Newton's derivation of the Universal Law of Gravitation allowed him to place projectile motions, earthly gravitational acceleration, and the planetary orbits under a single law describing the force of gravitation. This unifying description suggested to Newton a scientific method by which forces in nature can be unveiled and classified. This scientific method greatly expands the power of Newton's science beyond that of Descartes', who only recognized one kind of force – a mechanical force of push and shove. Newton's concept of force therefore allowed Newton to dissociate his physics from the metaphysical background of Descartes' philosophy, and to establish a scientific method for studying causal interactions.

However, the allocation of causal agencies to material bodies that are responsible for the conservation of quantity of motion is not fully coherent. Ernan McMullin argues that the attempt to reduce the various aspects of inherent force into one agency results in conceptual inconsistency. The problem is to find a single causal agency that is responsible both for the impetus of a body and the force of inertia, which is a force of resistance. When the body continues to move with uniform rectilinear motion, one says that some inherent agency compels the body to keep moving as if this agency pushes the body forward. During such periods of time, the inherent force functions as a *vis conservans*, which is the endeavor of the body to preserve its motion. When some impressed force operates on the body, one says that the inherent force resists the attempt to change the state of a body. When a force is impressed on the body, the inherent force functions as a *vis resistens*, which is equal and opposite to the impressed force. Newton equivocates about the nature of the agency that bodies carry, since the *vis conservans* is simply present in the body independently of any interactions with other bodies, while the *vis resistens* springs into action depending on contextual factors (McMullin, 1978, p. 37).

Newton attempts to alleviate this equivocal nature of inherent forces by distinguishing between the two roles of inherent force. According to Newton, there are two different viewpoints from which one can estimate the effects of inherent forces. When the body is at rest and some impressed force changes its state, the inherent force acts as a power to resist external influences. When the body is moving with uniform rectilinear motion, the inherent force acts as the power to overcome obstacles, i.e., it acts as the impetus of bodies. But these two viewpoints are ill attempts to reconcile the nature of the inherent force as a force that resists, with its nature as a force that conserves. The two roles of inherent forces do not cohere.

The inconsistency in Newton's account stems from his attempt to articulate a set of agencies in bodies that would be responsible for the conservation of quantity of motion in a system of bodies. It is difficult to see how various inherent forces conspire to conserve a quantity in a system of bodies. Given a system of bodies, an increase in quantity of motion in one part requires the decrease of quantity of motion in another part. If the system is viewed from without, it is clear that an overall balance must be kept. The equilibrium in quantity of motion is attributed to an isolated system as a whole. But it is not clear how to allocate the various agencies that give rise to this equilibrium. According to Descartes' account, a divine agency is responsible for the conservation of quantity of motion, but Newton is seeking material agencies that bring about this conservation.

Newton's solution is to think of mass as causally responsible for the preservation of quantity of motion in each part. When no forces operate on the body, the inherent force acts to conserve the quantity of motion. In this context, the inherent force acts to conserve the state of uniform motion; it compels the body to continue moving and can be viewed as an impetus force. When the inherent force acts as an impetus force it operates independently of any external factors. When a force is impressed on the body, some quantity of motion is "poured" into the body, and the inherent force becomes a resistance force, only allowing for a change in quantity of motion that corresponds to the pouring in of quantity of motion. In this role the force of inertia is "resisting" the deflection of a body from its state of uniform rectilinear motion. When the inherent force acts as a force of inertia (i.e., as resistance), it responds to contextual factors (i.e., it resists in proportion to the force impressed).

It is difficult to reconcile the two aspects of inherent forces. On the one hand, Newton takes the viewpoint of the system as a whole, wherein the combined inherent forces of bodies conspire to conserve the quantity of motion. In this role the inherent force becomes a conserving force, present throughout time. This force is present whether or not individual forces operate on some of the parts and conserves the overall state of motion in the whole. On the other hand, Newton takes the viewpoint of a single body, wherein the inherent force is reacting to some external influence (i.e., an impressed motive force) to mitigate the effects of changing the quantity of motion of the part. In this role the inherent force becomes a force resisting the work of some external influence. But the dual role of inherent forces does not provide a cogent account of how conservation of quantity of motion comes about.

Newton's definitions of inherent and impressed forces, and his three laws of motion, articulate the dynamic role of the concept of mass. Newton imagines mass to be the locus of inherent forces that contribute to the conservation of quantity of motion, and he lays out the three laws of motion so that this quantity would be conserved. Thus in Newton's account, the *definition* of quantity of matter relies on Newton's geometric conception of mass. In the three laws of motion Newton connected the geometric origin of mass with its dynamic role in the conservation of quantity of motion. Newton imagined the force of inertia, which is the inherent force existing in each body, as establishing the connection between the geometric concept of mass and its causal, dynamic role. But as we've seen, the inherent force is not a coherent concept, and confusedly mixes the tendency to *conserve* the state of motion with the tendency to *resist* changing it.

5.3 Mach's Critique of Newtonian Mass

Newton's definition of mass as a quantity of matter came under attack by many commentators in the twentieth century, and as a result this notion receded into mere historical curiosity. Instead of quantity of matter, modern presentations articulate the "law of conservation of mass," supplanting Newton's definition with a law of nature. However, Newton never articulated such a law, and took the quantity of matter to be the central meaning of "mass." The notion that mass describes a certain quantity of matter seemed intuitive to him, and did not require a "law" for its conservation. For him the mass of a material body is analogous to the volume of a certain region of absolute space. When one discusses the size of a region of space, one thinks of this size as a quantity that is both additive and indestructible. When one combines two regions of absolute space, the size of the combined volumes is the sum of the volumes of the parts. One also does not think that a volume of absolute space spontaneously decreases in size. Nevertheless, there is no empirical law governing the conservation of spatial volumes. Similarly, since the mass of a body is a chunk of an impenetrable region of space, it is additive and indestructible. The mass of a composite body is the sum of the masses of the parts, and the mass of an isolated system does not increase or diminish. The "conservation" of mass follows directly from its definition as an impenetrable region of space.

Mach's motivation for dismissing the role of mass as a quantity of matter stems from his positivist epistemology, and the desire to reconstruct the foundations of physics directly from experience. In *The Science of Mechanics* Mach famously objects to Newton's definition of mass:

> . . . it is to be observed that the formulation of Newton, which defines mass to be the quantity of matter of a body as measured by the product of its volume and density, is unfortunate. As we can only define density as the mass of unit of volume, the circle is manifest. Newton felt distinctly that in every body there was inherent a property whereby the amount of its motion was determined and perceived and this must be different than weight. He called it, as we still do, mass; but he did not succeed in correctly stating this perception. (Mach, 1893, p. 237)

Thus Mach interprets Definition 1 of the *Principia* as an explicit definition of quantity of matter. If mass is defined as the product of volume and density, one is left in the dark as to the meaning of density. If one defines density as the ratio of mass and volume, then one still would like to know the nature of mass. The definition therefore seems circular. However, we have seen in Section 5.1 that the definition of mass as the product of volume and density is the result of Newton's geometric concept of mass, which takes it to be the size of the impenetrable region of space, which is the body. Thus Newton's definition is not an explicit definition, but a quantity that results from his underlying conception of bodies. The quantity of matter is analogous to volume, except that bodies are porous and therefore come in various densities.

At first glance, Mach's criticism of Newton's definition of mass is simply that Newton's definition is circular. However, his motivation in criticizing the notion of quantity of matter is Mach's empiricist leanings. According to Newton, quantity of matter is a property that gives rise to inherent forces which are also forces of inertia. But for Mach the very notion that there are properties inherent in matter is an ill-conceived metaphysical abstraction. According to Mach, scientific theories are merely economic descriptions of sensations; a complete and concise summary of all possible experiences. Any permanency one attributes to material properties is only a provisional aspect of the specific inquiry one is engaged in – "a body is one and unchangeable only so long as it is unnecessary to consider its details" (Mach, 1911, p. 6). We are deceived by the sense of touch that gives the illusion that something is tangible and solid, and one is led to think that a "durable nucleus" is hidden behind all evanescent properties. The force of habit then leads us to think of this nucleus as "the vehicle of the more fugitive properties annexed to it" (Mach, 1911, p. 7).

According to Mach, despite the habit of positing the existence of a "durable nucleus" or a "substance" which underwrites physical experiences, one ought to remember that any property, including quantity of matter, should be reduced to elements of sensation. According to this approach, any property is merely a connection between elements of experience, and could never be thought of as holding the status of "higher reality" over and above the rest of the properties one finds in his or her experiences.

To remain honest in our empiricist commitments one has to relinquish the habit of thinking of objects as "substances" that are causally responsible for our experiences. Part and parcel of this view is a distrust of causal explanations. To say that property A is the cause of property B is to say something more than can be known. Thus, the notion of inherent properties such as mass that are causally responsible for our experiences of bodies extends what can be known from experience.

Mach supplants his critique of Newton's definition of mass with a reconstruction of mass from elements of sensation. This project of reconstruction is intended to purge the concept of mass from its metaphysical overtones and to clarify its scientific meaning. To make clear the role of mass Mach begins with the bedrock of "mechanical experiences" (Mach, 1911, p. 265). Bodily trajectories are what we experience, and so trajectories in space and time should be used to define the property that is taken to be unchanged. But "mass" is nothing other than an indirect way of describing trajectories, and so one should never "project" the concept into the essence of matter or to think of it as an inherent property of substances.

Mach's first step is to argue against Newton's notion of inherent force, i.e., the force of inertia. The supposed inherent force of inertia does not leave any detectable traces in our experiences. To see this, divide all trajectories into those that consist of uniform rectilinear motion, and those in which bodies accelerate. All bodies that move with uniform rectilinear motion behave in the same way, so mass does not figure in explaining the tendency to continue moving uniformly straight forward. The notion of mass is helpful in classifying various *accelerated* motions, but other than the mass parameter doing the work of classifying these trajectories, it is not possible to observe the action of an inherent force. Thus, Newton was mistaken in concluding that the same inherent force operates when bodies accelerate and when they do not (Mach, 1911, p. 173). Mach's criticism prefigures McMullin's analysis of the various agencies associated with Newton's inherent forces. Except that for

Mach the notion of agency is suspect in the first place, so he attempts to move away from the notion of agency to the notion of mass as a parameter for classifying trajectories.

Mach therefore proceeds with his project of reconstructing the concept of mass. Mach needs to define mass without appealing to the notion of force. Newton's Second Law of Motion cannot be used to define mass since it would again lead to a circular definition. In the *Science of Mechanics*, Mach begins his reconstruction of mass by imagining pairs of bodies isolated from the perceptual experiences of other bodies. Assuming that no observable property is determinative of accelerations, one can use the mutual acceleration of interacting pairs of bodies to *define* the determinative property. In case bodies experience the same accelerations, the two bodies are taken to possess the same mass. In case they experience different accelerations, one can take them to have different masses, in proportion to the ratio of their accelerations. The mass parameters are used to *classify* these accelerations, so one need not assume that some inherent force *caused* these accelerations. One only needs to assume that there is "interdependence of phenomena." Thus Mach believes mass should be treated as a functional connection between observed phenomena (Mach, 1893, p. 267).[11]

An isolated pair of interacting bodies A and B can be used to define directly the ratio of accelerations they experience. The acceleration ratios are then used to attribute mass parameters to individual bodies:

$$\frac{m_B}{m_A} = -\frac{\phi_A}{\phi_B} \tag{5.1}$$

where ϕ_A is the acceleration of A and ϕ_B is the acceleration of B. If it is stipulated that body A has a unit of mass, then the mass of B is defined according to:

$$m_{BA} = -\frac{\phi_A}{\phi_B} \tag{5.2}$$

One can drop the A subscript from B's mass if all bodies in the universe are determined according to Eq. (5.2). Hypothetically one may subject all bodies to a direct interaction with body A, and classify them according to the observable ratio in Eq. (5.2).

Mach argues that "the true definition of mass can be deduced only from the dynamical relations of bodies" (1893, p. 301). However, the relation between mass,

[11] This interdependence can be simply a functional relationship for Mach:

> In a lecture delivered in 1871, I outlined my epistemological point of view in natural science generally, and with special exactness for physics. The concept of cause is replaced there by the concept of function; the determining of the dependence of phenomena on one another, the economic exposition of actual facts, is proclaimed as the object, and physical concepts as a means to an end solely. (Mach, 1893, p. 325)

acceleration and force is somewhat complicated by the assumptions that figure in Mach's definition of mass. To argue that one can attribute a unit mass to body A, and a different mass parameter to body B according to (5.2), one has to presuppose a certain procedure for "separating out" the two mass parameters. The "definition" of the mass ratio presupposes the equality $m_B\phi_B = -m_A\phi_A$, since only this equality would allow the clear attribution of distinct mass parameters to the two bodies. "In the concept of mass and the principle of reaction . . . the same fact is twice formulated; which is redundant" (1893, p. 269). Thus Mach's definition presupposes the equality of action and reaction or Newton's Third Law of Motion.

Newton took the notion of mass to be a quantity of matter – a parameter that represents the "bulk" or the magnitude of the material body. This aspect of Newtonian mass allows us to take any pair of bodies, say A and B, and lump them together to form the quantity of matter of the composite body. Thus, quantity of matter presupposes that mass is an additive quantity:

$$m_\Pi = m_A + m_B \tag{5.3}$$

Mach has to give an account of the additive and conserved nature of mass, since those are not implicit in his definition. It is possible that mass-ratios for two parts of a system would not add up to the mass-ratio of the composite system. Mach claims that the additive nature of mass is implied by yet another fact of experience:

> We place by the side of each other the bodies A, B, C in the proportion of weight a, b, c in which they enter the combination AB and AC. There exists, now, no *logical* necessity at all for assuming that the same propositions of weight b, c of the bodies B, C will also enter into the chemical combination BC. Experience, however, informs us that they do. If we place by the side of each other any set of bodies in the proportions of weight in which they combine with the body A, they will also unite with each other in the same propositions of weight. But no one can know this who has not tried. And this is precisely the case with the mass-values of bodies. (Mach, 1893, p. 268)

Thus for Mach, once mass is defined using ratios of acceleration, one can make the further discovery that the same mass-ratios are additive. If one compares Mach's procedure with Newton's, a certain reversal of priorities becomes apparent. Newton considered the quantity of matter as the essential meaning of mass, and its inertial role as a secondary causal function of the basic concept. Mach reverses the priority and takes the inertial role of mass as the one to derive directly from experience; the additive and conserved nature of mass is a secondary experiential fact not essential to the meaning of the concept.[12] It seems somewhat arbitrary to take the inertial role of mass as essential while the additive and conserved nature of the concept as secondary, when both are derived from our "mechanical experiences." There is

[12] Narlikar (1939) attempted to add the conservation of mass as another experimental principle that should be made explicit.

no set of experiences that is more fundamental than another, and all experiences that are relevant to articulating the meaning of the concept are equally significant. Perhaps Mach imagined that the conservation of mass is further remote from directly observed experiences than inertial mass, since the inertial role of the concept relates to directly observed accelerations.

According to Mach, his definition of mass surpasses Newton's in simplicity and economy. He summarizes his reformulation of Newtonian physics in three "experimental propositions" and two definitions. The first experiential proposition is that, "Bodies set opposite each other induce in each other, under certain circumstances to be specified by experimental physics, contrary accelerations in the direction of their line of injunction. (The principle of inertia is included in this.)" (1893, p. 304). He then introduces his definition of mass, which is "The mass-ratio of any two bodies is the negative inverse ratio of the mutually induced accelerations of those bodies" (1893, ibid.). Mach then proceeds to define force as the product of mass and accelerations.

Mach's *Science of Mechanics* had tremendous influence over scientists and philosophers alike. Mach's forceful critique of Newtonian physics encouraged Einstein in his push towards the special and the general theories of relativity. Mach's account of science also influenced the positivist school and was an inspiration to members of the Vienna Circle, so that Mach indirectly influenced twentieth century philosophy. But Mach's critique also had an indirect influence on the way in which Newtonian physics was interpreted and taught in the generations that followed. The notion of quantity of matter was mostly erased from textbook presentations of Newton's physics. Mach's critique of Newton's force of inertia as an inherent force was instrumental in thinking of inertial motions as "natural," i.e., as motions without need for a causal explanation or as caused by some interaction with space. This transformation in thinking undoubtedly aided physicists in reconceptualizing inertial motion as following the geodesics of spacetime.

The first objection one might level against Mach's reconstruction of mass is that he assumes that his experimental propositions are direct statements of fact. Mach had in his mind the view that bodily trajectories are directly observed, and that one can directly measure their accelerations. However, there are a lot of theoretical assumptions and experimental procedures one needs to take into account before one can ascertain the mutually induced accelerations in bodies. Mach seems to relegate these considerations to the experimental practice when he qualifies his claim about bodies inducing accelerations in each other in "certain circumstances to be specified by experimental physics." When these circumstances are unraveled a bit, it becomes clear that Mach was wrong to assume that this experimental proposition is isolable from a wider set of theoretical and practical presuppositions.

Contrary to Mach's assumptions, pairs of interacting bodies do not come isolated from the environment. One never simply observes pairs of bodies inducing accelerations in each other; experimental physicists carry out long and complicated tasks to isolate systems from the environment.[13] There is no experimental setup that can guarantee that a system of bodies is causally isolated from the environment. The experimental physicist has to presuppose that an isolated system exhibits certain properties. In practice, experimentalists presuppose that the center of mass of an isolated system would continue to be at rest or to move uniformly straight forward. A system is isolated from its surrounding if it does not exchange momentum or energy with it. Thus, Mach's first experimental proposition has content only if the experimental practices implicitly take into account the conservation of momentum and energy.[14] In his definition of mass, Mach assumes the acceleration of any body is simply "given." However, his own understanding of motion is that it is relative, and that acceleration should be defined relative to a reference body. Assume, however, that this reference body C is uniformly accelerating relative to another reference body D. Relative to D Mach's definition of mass would be:

$$m'_{BA} = -\frac{\phi'_A}{\phi'_B} = -\frac{\phi_A - a}{\phi_B - a} \tag{5.4}$$

Where a is the uniform acceleration of body C relative to body D.[15] Mach's definition is only valid if velocities are measured relative to inertial reference frames, since only in those reference frames is the definition unique. To prepare measurements so that they are performed relative to inertial reference frames, one already has to suppose that momentum and energy are conserved, since inertial reference frames are identified through the uniform rectilinear motion of the center of mass of the lab system.[16]

---

[13] Several writers criticized Mach for implicitly assuming that the interacting particles are isolated, including Pendse (1937, 1939, 1940), and Simon (1938).

[14] Pendse (1937, 1939, 1940) has shown that Mach's assumption, i.e., that pairs of interacting bodies appear isolated, falsely creates the impression that mass values can be determined empirically. Pendse showed that there exist cases wherein an isolated system with more than seven bodies does not allow the determination of mass-values if Mach's procedure is followed, even if the accelerations are measured during more than one instant.

[15] I am here following Simon (1938), Pendse (1939), and Jammer (1997, pp. 93–102) who raise a similar objection to Mach's definition.

[16] Poincaré was aware of this problem when he examined Mach's definition a few years after the *Science of Mechanics* was published:

> Now, for someone who only knows the relative motion of the two particles considered, it is impossible to speak of the acceleration of these two particles; these words are devoid of meaning. The two particles have accelerations only if we assume that their combined motion is referred to a certain set of coordinate axes. But then these accelerations, their direction, and their relation will essentially depend on the coordinate axes that have been chosen. If the preceding proposition is correct after choosing a certain set of axes, it becomes false, in general, when we choose another, moving in an arbitrary motion with respect to the first. (Poincaré, 1903)

Another problem with Mach's definition is the inference from measuring the mass-ratio relative to a reference object, to the assertion that this mass-ratio is found to be the same whether mass-ratios are mediately or immediately arrived at. Mach himself was aware of this problem:

> One difficulty should not remain unmentioned in this connection, inasmuch as its removal is absolutely necessary to the formation of a perfectly clear concept of mass. We consider a set of bodies, A; B; C; D ... and compare them all with A as a unit.
>
> $$\begin{matrix} A & B & C & D & E & F \\ 1 & m & m' & m'' & m''' & m'''' \end{matrix}$$
>
> We find the respective mass-values, 1, m, m', m'', m''', m'''' ... and so forth. The question now arises: If we select B as our standard of comparison (as our unit), shall we obtain for C the mass-value $\frac{m'}{m}$, and for D the mass-value $\frac{m''}{m}$, or will perhaps wholly different values result? More simply, the question may be put thus: Will two bodies B, C, which in mutual with A have acted as equal masses, also act as equal masses in mutual action with each other? No *logical* necessity exists whatsoever, that two masses are equal to a third mass should also be equal to each other. (Mach, 1893, p. 268)

The problem is how to justify taking the mass-ratio, determined according to interactions with a particular body A, as applicable to bodies B, C, etc. in all circumstances and all interactions. It is conceptually possible that if one takes body B as our reference in determining mass-ratios, that mass-ratios would not line up as they did in interactions with A. Assume that bodies C and D had the same mass-ratio m when interacting with A; what guarantees that they would have the same mass-ratios $\frac{m}{m_{BA}}$ and experience the same accelerations while interacting with B?[17]

The most general way of stating this objection is the following: what phenomenological fact guarantees that the following equation holds:

$$m_{CA} = m_{CB} \times m_{BA} \tag{5.5}$$

where m_{CA} and m_{BA} are the mass-ratios of bodies C and B respectively, measured by subjecting these bodies to an interaction with body A, where A is stipulated to have a unitary mass-ratio, and m_{CB} is the mass-ratio of C measured when interacting with B? The relation in Eq. (5.5) is necessary if mass-ratios in Eq. (5.2) are to form an equivalence class for all bodies. Moreover, Eq. (5.5) also guarantees that the number field governing mass-ratios is scalable.

However, what experimental fact guarantees that Eq. (5.5) holds? Mach provides the following physical argument:

> If we were to assume that the order of combination of the bodies, by which their mass-values are determined, exerted any influence on the mass-values, the consequences of such an assumption would, we should find, lead to conflict with experience. Let us suppose, for

[17] Robert Musil (1982, p. 42), the engineer-philosopher who later became a renowned novelist, argued similarly that Mach failed to account for the ordinary meaning of mass which takes it to be a property of bodies independent of the particular interactions it has with other bodies.

> instance, that we have three elastic bodies, A, B, C, movable on absolutely smooth and rigid ring . . . We presuppose that A and B in their mutual relations comport themselves like equal masses and that B and C do the same. We are then also obliged to assume, if we wish to avoid conflicts with experience, that C and A in their mutual relations act like equal masses. If we impart to A a velocity, A will transmit the velocity by impact to B, and B to C. But if C were to act towards A, say, as a greater mass, A on impact would acquire a greater velocity than it originally had while C would still retain a residue of what it had. With every revolution in the direction of the hands of the watch, the *vis viva* of the system would be increased. If C were the smaller mass as compared with A, reversing the motion would produce the same result. But a constant increase of *vis viva* of this kind is of a decided variance with our experience. (Mach, 1893, p. 269)

Mach therefore secures the transitive nature of equality of mass by arguing that conservation of energy, which is derived from experience, does not allow an interaction in which the mass-ratio is not conserved. It is not clear why Mach picks out conservation of energy rather than conservation of momentum as the fact from experience that supports the transitivity of mass-ratios across interactions. A constant increase in the momentum of a system is at variance with experience, and a circular interaction – like the one described by Mach – will end up violating momentum conservation if mass-ratio are not transitive.

Mach's definition of mass seems to presuppose the conservation of momentum or energy at various levels – at the level of isolating pairs of interacting bodies, at the level of the definition of mass itself in which it is presupposed that action equals reaction, and at the level of assuming that mass-ratios are transitive irrespective of the reference bodies relative to which mass-ratios are measured. Perhaps a better strategy is simply to assert that conservation of momentum and energy is supported by experience, and that mass parameters are necessary for articulating this law.

Despite the various problems with Mach's project of reformulating Newtonian physics according to empiricist principles, there are important lessons to take from Mach. First, Mach's insistence that one should make explicit the various conceptual connections between fundamental concepts of physics is well-justified. Newton's notion of force of inertia is problematic. The notion of inherent force does not add anything to our understanding, and it obscures the essential connection between mass and the conservation of quantity of motion. Mach is right, therefore, in arguing that the inertial role of the concept of mass should be viewed as part of the meaning of the concept, rather than the result of some force inherent in bodies. However, while Mach's systematic analysis of the foundation of physical thinking is illuminating, his analysis is tainted by his staunch empiricism. Mach's positivist leanings lead him to search for the bedrock mechanical experiences from which physical concepts can be constructed, and his insistence that isolated pairs of interacting bodies are somehow simply "given" is misguided.

Our approach in the next chapter and in the rest of the book shall be that the central concept in Newtonian dynamics is the conservation of momentum. The law of momentum conservation is presupposed when pairs of interacting bodies are isolated from the rest of the world. Without assuming that momentum is conserved in isolated systems there cannot be a correlation between observed accelerations and mutual interactions between bodies.

In the wake of Mach's work, Newton's geometric conception of mass became obsolete. Later generations took the inertial role of mass to be grounded in experience, and the conserved nature of mass to be a further law of nature governing the mass-parameter. In the next chapter, I argue that the conservation of mass is inseparable from conservation of quantity of motion, and that the conservation of mass is the logical consequence of the conservation of momentum. The nature of mass is derived from the additive and conserved nature of quantity of motion. If this is the case, there is no "law of conservation of mass" that is logically distinct from the "law of momentum conservation." The twentieth-century attempt to reduce the additive and conserved nature of mass to an empirical law distinct from the conservation of momentum is misguided. Furthermore, the next chapter will revive the geometric interpretation of the concept of mass, and will show that the concept of mass can be in part derived from the foundations for spacetime theory that were articulated in Chapter 3. Thus a surprising result is that the quantity of matter can receive a genuinely geometric interpretation, which can explain various puzzling facts about the concept of mass, without resorting to obscure inherent agencies, such as inertial forces.

Chapter 6
Physical Systems and Mass

Chapter 5 examined the history of Newtonian mass and Ernst Mach's critique of quantity of matter. Leaving aside the gravitational role of mass, the concept of mass carries two significant connotations: the quantity of matter and the inertial mass. Newton first conceived of matter as impenetrable places. He then dubbed the size of the impenetrable place as the quantity of matter, and took it to represent the amount of "stuff" there is in a body, in analogy to the volume that bodies occupy. Today such a conception seems outdated. First, particles such as electrons are structureless particles, and carry distinct mass parameters while only occupying a single point in spacetime. The notion of mass as an impenetrable region of space does not make sense if the mass parameter is assigned to a dimensionless object. Second, Mach's critique of Newtonian mass had a lasting influence on interpretations of Newtonian physics. Mach's overall strategy of emphasizing the inertial role of mass over its role as quantity of matter is widely accepted. The essential character of the concept of mass is said to be determined by Newton's Second Law of Motion. According to this approach, mass relates the force impressed on the body to the acceleration it experiences. The additive and conserved nature of the mass parameter is an additional law of nature, a law that does not express the meaning of the concept but some experimental truth about it. Taking the inertial role of mass as the essential meaning of mass seems to be in line with empiricist principles, according to which measured accelerations of bodies are directly observed empirical facts (or if they are not directly observed are at least very close to the bedrock of our experiences). Third, the preference toward the inertial role of mass seems in line with the advent of the Special Theory of Relativity, where the notion of rest mass is not necessarily conserved given the supposed equivalence between mass and energy. Mass can no longer be considered as the quantity of matter if some of it can transform into energy, or vice versa, energy transformed into mass. (The role of mass in the Special Theory of Relativity will be considered in Chapter 8.)

It is by now established tradition to reduce mass into its inertial role, and take its geometric origins to be a mere historical curiosity. However, the geometry of PUMs introduced in Chapter 3 offers a way to revive the conceptual relation between mass and geometric laws. In this chapter I will offer a geometric interpretation of mass, without necessarily introducing the notion of impenetrable places, which is indeed dated. I argue that there is a close connection between geometric laws governing

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spacetime and the mass parameter, and that the established distinction between kinematic and dynamic laws should be undermined. The geometry of PUMs reveals an essential connection between spacetime and the concept of mass. Once PUMs are taken to be the fundamental entities of spacetime, the basic symmetries of spacetime become apparent. The decomposition of a PUM into its spatial and temporal components allows for alternative decompositions based on the set of parallel PUMs selected as references. These alternative decompositions of PUM intervals lead to the equivalence between inertial reference frames. However, it can be shown that a natural extension of those symmetries is intimately connected with the concept of mass. One may recognize the possibility of expanding or contracting local regions of spacetime; operations that expand the spatial and temporal dimensions while the relative velocities between bodies remain the same. These operations can be likened to "spacetime bubbles" incorporated into the very fabric of spacetime. This method of expanding or contracting a local region of spacetime, which is analogous to the global method of selecting an inertial reference frame, is shown to capture essential aspects of the mass parameter. Thus, while Newton's notion of impenetrable regions of space is dated, the geometric conception of mass can be given a modern formulation. This interpretation opens the door to a new understanding of mass and of the connection between spacetime and matter.

In this reconstruction of mass, the property of mass is not taken to be an inherent property of bodies which causes their inertial behavior. Instead, the geometric notion of expanded regions of spacetime is supplemented with rules governing the relations between a body and the composite system of which it is part. Since every body participates in the motion of the isolated system that contains it, every body "contributes" to the uniform rectilinear motion of the isolated system. It will be shown that the Rule of Composition and the Criteria of Isolation governing momentum completely determine the conserved nature of mass. There is no law of mass conservation independent of the law of momentum conservation: the latter logically implies the former. The role of Newtonian mass as quantity of matter is therefore derived from the geometry of PUMs and the structural assumptions regarding the nature of physical systems.

6.1 Primitive Motion Relationalism and the Expanded Reference Frames

In Chapter 3 I proposed to derive the structure of spacetime from ideal motions conforming to a Paradigm of Uniform Motion (PUM). The geometry of PUMs was used to derive a $\{1+1\}$ Galilean spacetime. The procedure consisted of using PUMs and intersection relations between them to define a metric of motion intervals ΔI along the PUMs. To decompose motion intervals into their spatial and temporal components, a motion α is "projected" onto a set of parallel motions $\mathcal{V}$. The motion intervals on the parallel motions are then defined as the time intervals for that reference frame. The motion intervals on some reference α are then used to define the spatial interval "separating" the parallel motions. The Galilean spacetime is determined when the mathematical relation between the spatial and temporal intervals is given.

C5. Galilean Paradigm of Uniform Motion (GPUM)

$\mathbf{E}\alpha\beta_1 \wedge \mathbf{E}\alpha\beta_2 \wedge \mathbf{P}\beta_1\beta_2 \rightarrow$
$\Delta x_0(p, q) = a\Delta x_1(p, q)$, where $p = < \alpha, \beta_1 >$ and $q = < \alpha, \beta_2 >$

A Galilean spacetime is characterized by a linear relation between the progression of a motion "across" a set of parallel motions and the motion intervals "along" the set of parallel motions, which represent time in that reference frame.

Assume that one can define a relation of perpendicularity between two time-space planes using motion intervals, and that the dimensionality of space is determined by the largest number of time-space planes that can be perpendicular to each other. In addition, assume there are three perpendicular time-space planes, and that there are three sets of parallel motions $\mathcal{V}_1, \mathcal{V}_2, \mathcal{V}_3$ that form perpendicular time-space planes. Assume as well that spatial separations between parallel motions obey the Euclidean rule for the spatial separation, so that if $\Delta x_1, \Delta x_2, \Delta x_3$ are the spatial separations between motions belonging to three perpendicular time-space planes, a time-space plane running "across" the three planes will have a spatial separation of $\Delta x = \sqrt{(\Delta x_1)^2 + (\Delta x_2)^2 + (\Delta x_3)^2}$. Finally, assume that one selects an alternative system of parallel motions in which each set of parallel motions $\mathcal{V}_1, \mathcal{V}_2, \mathcal{V}_3$ receives a "boost" forming a system of motions $\mathcal{V}'_1, \mathcal{V}'_2, \mathcal{V}'_3$ that are either faster or slower than the original set. That is, we have:

$$
\begin{aligned}
\Delta x_0 &\mapsto \Delta x'_0 = \Delta x_0 \\
\Delta x_1 &\mapsto \Delta x'_1 = \mu_1 \Delta x_1 \\
\Delta x_2 &\mapsto \Delta x'_2 = \mu_2 \Delta x_2 \\
\Delta x_3 &\mapsto \Delta x'_3 = \mu_3 \Delta x_3
\end{aligned}
\tag{6.1}
$$

The original frame in which spatial and temporal relations are measured receives the label K_G^{1111}. The meaning of the superscripts will become apparent shortly. The transformation $\Delta : K_G^{1111} = \langle x_0, x_1, x_2, x_3 \rangle \mapsto K_G^{1\mu_1\mu_2\mu_3} = \langle x'_0, x'_1, x'_2, x'_3 \rangle$ can be defined as:

$$
\Delta = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \mu_1 & 0 & 0 \\ 0 & 0 & \mu_2 & 0 \\ 0 & 0 & 0 & \mu_3 \end{pmatrix} \tag{6.2}
$$

The transformation in (6.2) does not necessarily reflect the length measured with our rods. A rigid rod whose "0" and "1" marks coincide with the stretched unit in each time-space plane must move in $K_G^{1\mu_1\mu_2\mu_3}$ with uniform velocities of:

$$
v_i = (\mu_i - 1)\frac{dx_i}{dx_0} \tag{6.3}
$$

relative to K_G^{1111}. Thus, a "stretching" of the spatial dimension dx_i relative to the temporal dimension dx_0 leads to the requirement that the measuring rods that are stationary in $K_G^{1\mu_1\mu_2\mu_3}$ travel at a uniform motion relative to the original rods that are stationary in K_G^{1111}.

The Δ^* transformation, may now be expressed as the transformation between coordinate systems in which measurements are made relative to rods and clocks, not relative to the PUMS:

$$\Delta^* = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -v_1 & 1 & 0 & 0 \\ -v_2 & 0 & 1 & 0 \\ -v_3 & 0 & 0 & 1 \end{pmatrix} \tag{6.4}$$

Thus we have seen how a {1+3} Galilean spacetime can be derived from our geometry of PUMs (i.e., the Galilean spacetime is derived from the structure governing ideal uniform motions).

The transformation in (6.2) led to the transformation between inertial reference frames. The priority of motion over space and time (or, more precisely, the interdependence of space and time through the concept of motion) provides a natural interpretation of inertial reference frames. But the transformation in (6.2) is not the most general one, and one may imagine the stretching of both the spatial and the temporal unit by the same factor. Thus we may describe a transformation analogous to the one in (6.2), to be labeled $\Delta : K_G^{1111} \mapsto K_G^{\mu_0 111}$:

$$\Delta = \begin{pmatrix} \mu_0 & 0 & 0 & 0 \\ 0 & \mu_0 & 0 & 0 \\ 0 & 0 & \mu_0 & 0 \\ 0 & 0 & 0 & \mu_0 \end{pmatrix} \tag{6.5}$$

where $\mu_0 > 0$. At first glance it is not clear how one should interpret a transformation of all temporal and spatial coordinates with a factor of μ_0 and how this transformation is analogous to the transformation between inertial reference frames. First, one should recognize that this is an active transformation of the spacetime, not a passive transformation of coordinate systems. The transformation describes a physical stretching or contracting of spacetime by a factor μ_0. A point in K_G^{1111} will transform into $K_G^{\mu_0 111}$ as follows:

$$\Delta \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \mu_0 x_0 \\ \mu_0 x_1 \\ \mu_0 x_2 \\ \mu_0 x_3 \end{bmatrix} \tag{6.6}$$

To elaborate the meaning of the reference frame $K_G^{\mu_0 111}$ one needs to examine how various points of the original frame K_G^{1111} behave in the expanded/contracted frame.

First, from (6.6) it follows that a point at rest in K_G^{1111} will remain at rest in $K_G^{\mu_0 111}$. Thus it appears as if all the rods and clocks that are at rest relative to K_G^{1111} are also at rest relative to $K_G^{\mu_0 111}$. One can use the same clocks and rods for both frames, with a scaling factor to transform from one reference frame to another. This symmetry is essentially the symmetry in which one multiplies the size of all objects and the duration measured by all clocks by a factor μ_0. The change would not be revealed by measurements directly performed by clocks and rods, only through second or larger time derivatives of spatial displacements. Thus when a set of rigid rods and clocks is used, since they are calibrated in relation to PUMs, one knows that there is a degree of freedom that correlates the *actual* measurements of length and duration with their proper representation. One may either represent them in K_G^{1111} or in $K_G^{\mu_0 111}$, and the representation is not completely determined by the way in which actual measurements are performed. If measurements are represented in K_G^{1111}, there is a 1-1 correspondence between the measurement results and the values representing distance and duration. If measurements are represented in $K_G^{\mu_0 111}$, one ought to multiply all distances and durations by a factor μ_0. The possibility to rescale spatial and temporal units demonstrates that one cannot discern, using clocks and rods that are relatively at rest, between members of an infinite set of reference frames $K_G^{\mu_0 111}$. But the existence of this set of reference frames has important theoretical consequences and will be the basis for a surprising conceptual connection between spacetime and the concept of mass.

The velocity $\vec{v}$ measured in K_G^{1111} remains invariant when transformed into $K_G^{\mu_0 111}$ or vice versa:

$$v_i' = \frac{dx_i'}{dx_0'} = \frac{\mu_0 dx_i}{\mu_0 dx_0} = \frac{dx_i}{dx_0} = v_i \tag{6.7}$$

Thus the transformation $\Delta : K_G^{1111} \mapsto K_G^{\mu_0 111}$ preserves the velocity components. However, the acceleration $\vec{a}$ transforms as follows:

$$a_i' = \frac{d^2 x_i'}{dx_0'^2} = \frac{\mu_0 d^2 x_i}{\mu_0^2 dx_0^2} = \frac{a_i}{\mu_0} \tag{6.8}$$

In transforming from $K_G^{\mu_0 111}$ to K_G^{1111}, the acceleration is multiplied by a factor of μ_0.

The most general transformation from K_G^{1111} to $K_G^{\mu_0 \mu_1 \mu_2 \mu_3}$ is one that combines Eqs. (6.2) and (6.5):

$$\Delta = \begin{pmatrix} \mu_0 & 0 & 0 & 0 \\ 0 & \mu_0 \mu_1 & 0 & 0 \\ 0 & 0 & \mu_0 \mu_2 & 0 \\ 0 & 0 & 0 & \mu_0 \mu_3 \end{pmatrix} \tag{6.9}$$

Since measuring rods and clocks undergo the same expansion, the same rods and clocks can be used in both systems $K_G^{\mu_0\mu_1\mu_2\mu_3}$ and $K_G^{1\mu_1\mu_2\mu_3}$. Thus, the measuring rods in $K_G^{\mu_0\mu_1\mu_2\mu_3}$ are moving with uniform rectilinear motion, with a velocity $\vec{v}$, with components $v_i = (\mu_i - 1)\frac{dx_i}{dx_0}$, relative to the measuring rods and clocks in K_G^{1111}. There is a set with an infinite number of reference frames $K_G^{\mu_0\mu_1\mu_2\mu_3}$ which all have clocks and rods at rest relative to each other, and all move with the same uniform rectilinear motion in relation to K_G^{1111}. Since fundamental PUMs are represented with $dx_i = dx_0$ in K_G^{1111}, one may represent the transformation in (6.9) as follows:

$$\Delta = \begin{pmatrix} \mu_0 & 0 & 0 & 0 \\ \mu_0(\mu_1 - 1) & \mu_0 & 0 & 0 \\ \mu_0(\mu_2 - 1) & 0 & \mu_0 & 0 \\ \mu_0(\mu_3 - 1) & 0 & 0 & \mu_0 \end{pmatrix} \tag{6.10}$$

In other words one may define a "quantity of motion" $\vec{P} = \mu_0\vec{v}$, where $\vec{v}$ is the velocity of rigid rods in the coordinate reference frame $K_G^{\mu_0\mu_1\mu_2\mu_3}$ relative to rods that are stationary in K_G^{1111}, and $P_i = \mu_0(\mu_i - 1)\frac{dx_i}{dx_0}$. The quantity of motion is the relative velocity between points in these two frames even if the *measured* velocity of the rigid rods and clocks stationary in $K_G^{\mu_0\mu_1\mu_2\mu_3}$ is the same as the *measured* velocity relative to those clocks and rods stationary in $K_G^{1\mu_1\mu_2\mu_3}$.

At our disposal there is a new dimension of spacetime μ_0 that seems to be merely a function of the particular choice of representation. But as will be shown, this spacetime dimension can be used to describe the Newtonian mass parameter. One can also define the "standard acceleration" of a body as $\vec{a}_s = \mu_0\vec{a}$, which is the acceleration of a body measured in the "standard," unexpanded frames K_G^{1111} or $K_G^{1\mu_1\mu_2\mu_3}$. The "standard accelerations" will be interpreted as the force operating on a body, so that the product of μ_0 and the acceleration can be the transformation between two different representations of a body's accelerations. The force operating on a body is simply another representation of the body's acceleration.

6.2 The Stretching Parameter μ_0 and Newtonian Mass

A rescaling of the spatial dimension relative to the temporal dimension leads to the selection of an alternative inertial reference frame. An analogous transformation, which expands the reference frame both in the spatial and the temporal dimension by a factor μ_0, is implicitly present in the structure of spacetime. Thus a new set of reference frames depending on the expansion parameter μ_0 is added to each inertial reference frame. I will now argue that this expansion parameter can be interpreted as Newtonian mass. The claim seems entirely absurd, since I am arguing that a characteristic parameter of a reference frame can be associated with each Newtonian particle. It is not clear how a property that is a feature of the measurement apparatus or of spacetime itself can be taken to be a property ordinarily understood as inherent in the body and related to the material nature of bodies moving through spacetime.

Consider two particles, p_A and p_B, whose trajectories are observed while clocks and rods are calibrated in K_G^{1111}. That is, the trajectories of these particles are described from the point of view of a "standardized" frame. Contrary to ordinary interpretations of classical physics, one should not presuppose that these two particles have "mass." But if the reconstruction of spacetime in Chapter 3 is correct, a coordinate system that includes a set of rigid rods and clocks cannot differentiate between members of an infinite set of reference frames $K_G^{\mu_0 111}$. Thus there is a degree of freedom that translates the actual measurement of distances and durations to their true representation. If one measures the accelerations $\vec{a}_A$ and $\vec{a}_B$ of p_A and p_B, the actual accelerations are in fact measured in $K_G^{\mu_{0A}111}$ and $K_G^{\mu_{0B}111}$ respectively and do not represent the "true" accelerations in the standard frame K_G^{1111}. Because each particle is moving through spacetime, using a single scale $\mu_0 = 1$ unconsciously distorts the representation of the particles' actual accelerations. The spacetime scale μ_0 may vary from one particle to another. To find a coherent representation of these accelerations one must rescale the measurements, once for p_A and another time for p_B, so that the true accelerations in K_G^{1111} are now $\vec{a}_{sA} = \mu_{0A}\vec{a}_A$ and $\vec{a}_{sB} = \mu_{0B}\vec{a}_B$. Thus properties of systems of representation become properties of bodies. If there is a degree of freedom that translates the actual measurement of a property to its true representation (or, as is the case here, the degree of freedom translates incoherent representations into a coherent system of representations), each body may be assigned a different parameter depending on which system is appropriate for its representation.

So far no relation was assumed to exist between particles p_A and p_B, and the actual measurements of accelerations may be attributed to two reference frames $K_G^{\mu_{0A}111}$ and $K_G^{\mu_{0B}111}$. A mechanism is now supposed for describing a system comprised out of these two particles. It should be possible to describe each part independently, with its own trajectory and motion; but it should also be possible to treat the two particles (or any set of bodies) as a composite system with a single trajectory and motion. Assume that $\vec{v}_A$ and $\vec{v}_B$ are measured in $K_G^{\mu_{0A}111}$ and $K_G^{\mu_{0B}111}$. If $\vec{a}_A$ and $\vec{a}_B$ are the accelerations in $K_G^{\mu_{0A}111}$ and $K_G^{\mu_{0B}111}$ respectively, one may define the "standardized" accelerations of the particles, $\vec{a}_{sA}$ and $\vec{a}_{sB}$, relative to the non-expanded frame K_G^{1111}. Since $\vec{v}_{sA} = \vec{v}_A$ and $\vec{v}_{sB} = \vec{v}_B$, it follows that $\vec{a}_{sA} = \mu_{0A}\vec{a}_A$ and $\vec{a}_{sB} = \mu_{0B}\vec{a}_B$. The Rule of Composition governing these motions can now be articulated:

Rule of Composition. *For a set of simultaneous events, the quantity of motion $\vec{P} = \mu_0\vec{v}$ of a composite system is the sum of the quantities of motion of the individual parts.*

Consider the two particles p_A and p_B. The state of each of these particles consists of their quantities of motion $\vec{P}_A$ and $\vec{P}_B$. The state of the composite system Π of which these particles are comprised is described with the state:

$$\vec{P}_\Pi = \vec{P}_A + \vec{P}_B \tag{6.11}$$

Thus the additive rule describing the summation of quantities of motion describes the relation between parts of physical systems and their composite. In addition to the Rule of Composition, we can also articulate the following principle:

Criterion of Isolation. *The motion of whole, isolated systems, instantiates a PUM.*

The Criterion of Isolation applies the Galilean PUM to the motion of a composite system, determined by Eq. (6.11). The Criterion of Isolation implies that the time derivative of the quantity of motion of a composite isolated system vanishes. It follows that in case the two particles are isolated from the world, from (6.11) we get:

$$\frac{d\vec{P}_\Pi}{dx_0} = \frac{d\vec{P}_A}{dx_0} + \frac{d\vec{P}_B}{dx_0} = \vec{a}_{sA} + \vec{a}_{sB} \tag{6.12}$$

In other words the following equation holds:

$$\mu_{0A}\vec{a}_A = -\mu_{0B}\vec{a}_B \tag{6.13}$$

where $\vec{a}_A$ and $\vec{a}_B$ are the accelerations of p_A and p_B measured in $K_G^{\mu_{0A}111}$ and $K_G^{\mu_{0B}111}$ respectively, i.e., these are the accelerations actually measured by clocks and rods. Implicit in Eq. (6.13) are Newton's Second and Third Laws of Motion. Newtonian Laws of Motion are therefore shown to be derived them from the geometry of PUMs in Chapter 3 and the rules governing physical systems, the Criterion of Isolation and the Rule of Composition. To consider the relation between our reconstruction and the fundamental concepts of Newtonian physics each fundamental concept of Newtonian physics and its relation to our reconstruction is now explained.

The fundamental concepts of Newtonian mechanics include space, time, mass and force. The viewpoint which considers physical systems as structures of moving parts and moving wholes can be used to reconstruct the fundamental Newtonian concepts. In Chapter 3 the structure of Galilean spacetime was derived from a geometry of PUMs. The same PUM structure governs both simple, unstructured systems, such as the free particles, and events generated by isolated composite systems of interacting particles. To say that these isolated systems instantiate a PUM, is merely to say that they take the form of a particular, fundamental motion in the geometry of PUMs. Thus, there is no need for forces of inertia or for the idea that spacetime somehow influences the behavior of bodies. In the case of systems of interacting particles, the process governs the motion displayed by the center of mass of the system. The center of mass of a system is not treated as a fictional point that aids in the calculation of trajectories, but as a point describing the trajectory of the composite system. A system comprising of interacting parts is as substantive as the description of the parts, and can be seen as a unifying description of the parts.

If a geometry of PUMs is assumed, there exist two kinds of symmetries leading to two types of transformations: $\Delta : K_G^{1111} \mapsto K_G^{1\mu_1\mu_2\mu_3}$, and $\Delta : K_G^{1111} \mapsto K_G^{\mu_0 111}$. The first transformation reduces to the Galilean transformation between inertial reference frames. The second transformation suggests the existence of another degree freedom μ_0 correlating actual measurements of accelerations with their "true" representations in K_G^{1111}. This degree of freedom stems from the symmetry in which both spatial and temporal dimensions are multiplied by the same stretching parameter μ_0: transformations that leave the PUMs invariant. Now, with the Rule of Composition in place, and keeping in mind that the Criterion of Isolation is taken to apply to isolated systems of interacting particles, it will be shown that the stretching parameter μ_0 captures the essential meanings of the concept of mass in Newtonian physics, without taking on board the redundant metaphysical connotations of the old Newtonian concept.

I shall divide my comments in relation to these two conceptual roles, relating the expansion parameter μ_0 to each of these roles.

6.2.1 The Quantity of Matter

The concept of Newtonian mass exhibits the following characteristics:

1. Mass is the product of density and volume, i.e., $m = \rho V$.
2. Mass is additive – the mass of a composite body is the sum of the masses of the parts, i.e., $m_\Pi = \sum_i m_i$, where m_Π is the mass of the composite system and m_is are the masses of the parts.
3. Mass is conserved.

The reconstruction offered here establishes a surprising conceptual connection between modern accounts of spacetime and Newton's original thinking about mass as a geometric concept, while still preserving the modern intuition that dimensionless particles carry mass. Assume there is a lump of matter of volume V measured in frame $K_G^{\mu_0 111}$. Assume now that one divides this lump of matter into individual parts, where each component occupies an infinitesimal region of space. One assumes that a Rule of Composition governs the quantities of motion of these components, so that:

$$\vec{P}_\Pi = \sum_i \vec{P}_i \qquad (6.14)$$

If the body is continuous, one has to replace the discrete sum with an integral, but the idea is the same. A stretching factor μ_0 can be associated with each infinitesimal part forming a continuous scalar function ρ to be called density. The product $\rho(x_1, x_2, x_3)dV$ gives us the expansion parameter μ_0 associated with the infinitesimal part (or the reference frame $K_G^{\mu_0 111}$ from which the trajectory of the part is measured). That is, the density function describes a continuous field of transformations that "correct" the spatial and temporal measurements for each infinitesimal part of the composite object piecemeal.

Once the continuous density function is articulated, one may derive the following equation:

$$\mu_{0\Pi}\vec{v}_\Pi = \int_V \rho\vec{v}dV \tag{6.15}$$

where $\vec{P} = \rho\vec{v}$ is the quantity of motion of each infinitesimal part and $\mu_\Pi\vec{v}_\Pi$ is the quantity of motion of the composite body. Equation (6.15) is simply the Rule of Composition applied to a continuous body. If the body is rigid, one can then derive the relation between the expansion factor μ_0 of the composite body and the expansion parameters describing each infinitesimal part $\rho(x_1, x_2, x_3)$. In case the composite body is rigid, $\vec{v}_\Pi = \vec{v}$, i.e., the velocity of the composite body is the same as the velocity of each infinitesimal part. From (6.15) and the rigidity assumption it follows that:

$$\mu_{0\Pi} = \int_V \rho dV \tag{6.16}$$

For a body with uniform density $\rho = \rho(x_1, x_2, x_3)$, one may conclude that:

$$\mu_{0\Pi} = \rho V \tag{6.17}$$

Thus the additive nature of the stretching parameter μ_0 in (6.16) is a logical consequence of the geometry of PUMs and the Rule of Composition applied to quantities of motion. The frame of reference appropriate for the description of the composite system $K_G^{\mu_{0\Pi}111}$ is determined by adding up the stretching parameters of the frames of reference $K_G^{\mu_{0i}111}$ appropriate for describing each part.

However, the assumption of rigidity is not necessary for deriving the additive nature of the stretching parameter μ_0. Consider a discrete number of n bodies, each described from a frame of reference $K_G^{\mu_{0i}111}$. The reference frame $K_G^{\mu_{0\Pi}111}$ appropriate for describing the system "comprising" the n bodies is determined by $\mu_{0\Pi} = \sum_i \mu_{0i}$. To see this, consider the rule governing the quantities of motion:

$$\vec{P}_\Pi = \sum_i \vec{P}_i \tag{6.18}$$

According to the definition of quantity of motion, one derives the following relation from:

$$\mu_{0\Pi}\vec{v}_{\Pi} = \sum_i \mu_{0i}\vec{v}_i \tag{6.19}$$

So far it was assumed that the "standardized" frame of reference is K_G^{1111}. Each body is associated with a frame of reference $K_G^{\mu_{0i}111}$ for which corrections are made so that its accelerations are measured in relation to K_G^{1111}. The standardized reference frame is now transformed into $K_G^{1\mu_1\mu_2\mu_3}$. Thus a normal Galilean transformation is being carried out. The only difference is that each measured velocity is transformed according to (6.4) and receives a velocity boost, so that $\vec{v}' = \vec{v} - \vec{V}$, where $\vec{V}$ is the relative velocity between the frames. If quantities of motion are defined according to $\vec{v}'_{\Pi}$, $\vec{v}'_i$, it follows that:

$$\mu_{0\Pi}\vec{v}'_{\Pi} = \sum_i \mu_{0i}\vec{v}'_i \tag{6.20}$$

Replacing the original velocities for the transformed ones it follows that:

$$\mu_{0\Pi}(\vec{v}_{\Pi} - \vec{V}) = \sum_i \mu_{0i}(\vec{v}_i - \vec{V}) \tag{6.21}$$

From (6.19) and (6.21) one concludes that

$$\mu_{0\Pi}\vec{V} = \sum_i \mu_{0i}\vec{V} \tag{6.22}$$

which leads to the additive nature of the expansion parameter:

$$\mu_{0\Pi} = \sum_i \mu_{0i} \tag{6.23}$$

The rigidity assumption is not necessary for deducing the additive nature of the expansion parameter μ_0. The additive nature is derived from the assumption that the Rule of Composition applies to quantities of motion in all reference frames $K_G^{1\mu_1\mu_2\mu_3}$. Or in other words, the additive nature of μ_0 follows from the additive nature of the quantity of motion.

It now remains to be shown that the μ_0 parameter is "conserved." First, consider a fundamental particle, which is a point that is moving through spacetime. Once the frame of reference $K_G^{\mu_0 111}$ appropriate for describing the trajectory of this point-particle is determined, the same frame of reference should be referred to throughout the life of the particle. Thus, a fundamental particle should have a constant parameter μ_0 associated with it, given that μ_0 belongs to the system of representation appropriate to the particle. In this account, the constancy of mass for fundamental particles stems not from them "possessing" some inherent property that resists external forces. Rather, it stems from a degree of freedom implicit in the structure of spacetime. Once this degree of freedom is determined, it does not require further verification and is "constant" throughout the life of the particle. The proper analogy for understanding the role of the expansion parameters μ_0 is the process of finding the appropriate inertial reference frame for describing the motion of bodies. If bodies were to be described from different inertial reference frames, or if one did not know how to relate measured trajectories to some single inertial reference frame, one would not be able to form a consistent representation or to analyze interactions between bodies. Similarly, the expansion parameter μ_0 enables the translation of a particle's actual measurements to those representations that cohere with representations of other bodies. The μ_0's are not inherent properties of bodies but functions of divergent systems for representing motion.

Fundamental particles retain the same expansion parameters μ_0 throughout their life, given that this parameter is implicit in the transformation $\Delta : K_G^{\mu_0 111} \mapsto K_G^{1111}$. But it is possible to show that this parameter is also conserved for composite bodies. The conservation of μ_0 follows from the assumption that it is constant for fundamental particles and from its additive nature expressed in (6.23). Assume a body is comprised of n discrete parts, each part is assigned the same parameter μ_{0i} throughout its life. The expansion parameter $\mu_{0\Pi}$ appropriate for describing the trajectory of the composite system is now determined through the additive rule. Thus, unless the system loses one of the fundamental parts or gains new parts, the composite system's expansion parameter will remain the same.

The expansion parameter μ_0 fully captures the conceptual role of quantity of matter, when the Rule of Composition and the Criterion of Isolation are presupposed. We gained a new insight to the connection between spacetime structure and the mass parameter, and avoided some of the difficulties of this conceptual role. We have shown that the additive nature of mass follows from the Rule of Composition governing the quantity of motion, that the nature of mass is such that fundamental particles must carry a constant mass parameter, and that mass is conserved because of the conservation of quantity of motion in all reference frames. We have shown all this without presupposing that mass is an inherent property, or that it describes some essential nature of material bodies. This reconstruction avoids Mach's criticism of mass as representing some substantial reality, or a material nature behind the phenomena that gives rise to the phenomena. It also avoids Mach's argument that Newton's definition of mass is circular. According to the interpretation introduced here, mass is a characteristic of systems of representation and is a consequence of momentum conservation, not a property inherent to material bodies.

6.2.2 Inertial Mass

The expansion parameter μ_0, together with the Criterion of Isolation and Rule of Composition, also captures the role of mass as the property of inertia. Chapter 5 presented the standard interpretation of mass, which takes it to be the power of a body to resist external forces. This power is expressed via Newton's Second Law of motion, which in modern formulations is represented as $\vec{F} = m\vec{a}$. The more mass a body has the more difficult it is to deflect it from its rectilinear motion. Thus, it seems as if there are two causal agencies operating on a body – one causal agency is external to it and attempts to deflect the body from its uniform rectilinear motion. The other agency is located within the body, and it counteracts the external impressed force. The product of this battle between external and internal causal agencies produces the final acceleration the body experiences.

Mach attempted to cleanse Newtonian physics from causal agencies, since he believed that scientific concepts should be reduced to immediately perceived facts. Instead of considering mass as giving rise to some inherent force of inertia, he attempted to reduce the concept to what is directly perceived. Mass is understood by Mach as the ratio between mutually induced accelerations in pairs of interacting bodies.

The reconstruction of Newtonian physics introduced here offers another interpretation for Newtonian mass that bears some resemblance to Mach's interpretation without relying on his positivist epistemology. Like Mach's interpretation, this interpretation does away with inherent causal agencies and thinks of mass as derived from the structure of spacetime together with "structural assumptions," i.e., assumptions about the structure of physical systems.

There is an infinite number of reference frames $K_G^{\mu_0 111}$ implicit in the structure of spacetime. For each body, there is a degree of freedom that correlates between actual measurements of the accelerations $\vec{a}$ of the body and the "standardized representation" $\vec{a}_s$ of this acceleration in K_G^{1111}, so that the following relation holds:

$$\vec{a}_s = \mu_0 \vec{a} \tag{6.24}$$

In this reconstruction there is a relation that bears resemblance to the relation $\vec{F} = m\vec{a}$ known as Newton's Second Law of Motion. If the analogy is correct, the "standard acceleration" $\vec{a}_s$ replaces the force impressed on the body; the expansion parameter μ_0 replaces the body's inertial mass; and the "measured acceleration" $\vec{a}$ replaces the acceleration of a body, which is directly measured according to the traditional interpretation of Newtonian physics. The reconstruction offered here takes $\vec{a}_s$ and $\vec{a}$ to be two representations of the same thing, i.e., they are both representations of acceleration in different reference frames and there is no ontological difference between them.

According to this account, what one ordinarily calls the "force" operating on a body is the acceleration of a body measured in the standardized frame K_G^{1111}. What one ordinarily calls the "acceleration" of a body, is the acceleration as it is measured in the frame $K_G^{\mu_0 111}$. This interpretation is in variance with traditional interpretations of Newtonian physics that take force and accelerations to be entirely different entities, ontologically speaking.

The traditional interpretation that distinguishes between force and acceleration has two main arguments for it. First, a force may describe a general tendency to produce change that is independent of particular bodies and the actual accelerations that are produced in them. Thus, it seems as if there is reason to distinguish between the causal agent, i.e., force, that gives rise to the acceleration, and the effect of the action of the force, which is the acceleration of a body. Another reason to distinguish between forces and accelerations is that while various forces may combine according to the parallelogram law, there is only one acceleration a body experiences. Thus it is very tempting to take force to be the causal agent responsible for change, and acceleration as the product of this external force after it has met the resistance of the body's inertial force.

Consider the distinction between the force generated and the acceleration produced in a body. For example, assume a force exists at a certain point $x = (x_1, x_2, x_3)$. This force is defined according to the acceleration a test particle of unit mass *would have* experienced were it placed at x. What is the reason for taking the force to be the *cause* of accelerations, and the actual accelerations as their effect? In case the body has a unit mass, the force and acceleration are represented with the same vector. In that case it is not possible to differentiate between the two vectors, except that they are given in different dimensions. But the different dimensions are merely conventional names one gives to mathematical magnitudes, and there is no reason other than our customary way of thinking that prevents us from thinking of forces and accelerations as possessing the same magnitude. The counterfactual acceleration of the test particle is taken to be the agency that causes these accelerations whenever the force is conceived as a causal agent, but the magnitude of this force is nothing but the acceleration of the test particle.

The inference from $\vec{F}$ to $\vec{a}$ is the inference from a counterfactual acceleration of a test particle to the actual acceleration of the particle that experiences the force. Assume the particle's acceleration is measured as $\vec{a}$. If there were only one reference frame K_G^{1111} in which to measure the acceleration of bodies, the measured acceleration *would have been* the same as the true representation of the acceleration, i.e., $\vec{a}_s = \vec{a}$. In such a case one could have predicted the trajectory of the particle without calibrating the clocks and rods to reflect the expansion parameter μ_0. But since there is a degree of freedom μ_0 that correlates the measured acceleration with its true representation in K_G^{1111}, one still has to convert the measured acceleration into its standardized representation to calculate the exchange of quantity of motion. Thus, instead of thinking of the mass parameter as resisting the external force in determining the final acceleration of a body, one ought to think of it as calibrating the measured accelerations so that a consistent representation of accelerations can be found. The standardized accelerations are also useful in calculating the *counterfactual* accelerations a reference body would have experienced in case its expansion parameter were $\mu_0 = 1$. This representation is useful since it allows a quick inference from the standardized acceleration to the acceleration one actually measures. Thus, instead of differentiating between force and acceleration, one should think of them as two different modes for representing acceleration. The force "operating" on a body is the counterfactual acceleration a body would have experienced were its acceleration measured in the standardized frame K_G^{1111}. The "actual" acceleration of a body is the acceleration of the body as it is measured in the frame $K_G^{\mu_0 111}$ from which one is describing the body's motion.

Another argument for distinguishing between forces and accelerations is the fact that component forces operating on a body can be combined, while the actual acceleration it experiences is the result of the combined forces. For example, consider a drop of charged oil that floats stationary in a tube and is under the influence of both electric and gravitational forces. It is legitimate to say that this body experiences two kinds of forces at the same time. However, the zero acceleration of the body is only one. One can deduce the acceleration from the forces operating on the body by calculating the total force via the Rule of Composition for forces. But one cannot deduce the forces that operate on a body from the actual accelerations it experiences. One therefore cannot assume that forces and accelerations are equivalent descriptions of the same thing.

The distinction between component forces and the actual acceleration generated by a combination of forces could be understood via the distinction between counterfactual and actual accelerations. The ability to calculate composite forces crucially depends on the Rule of Composition, according to which the quantity of motion of a composite system is the sum of the quantities of motion of the component parts. For example, assume that the earth, the capacitor generating an electric field and a drop of charged oil are bodies A, B and C, respectively. If one puts together bodies A and C, with B not present, the bodies will exchange quantities of motion according to the Universal Law of Gravitation. This means that body C will accelerate in that case downwards. In the standard frame K_G^{1111} this acceleration will be represented as $\vec{a}_{sg} = \mu_0 \vec{a}_g$, where $\vec{a}_g$ is the measured acceleration of C as a result of its gravitational interaction with A. If one puts bodies B and C together, with A not present (for example, if the drop of oil and capacitor are examined very far from the surface of the earth), the electric interaction between B and C will produce a measured acceleration $\vec{a}_e$ "upwards" in C. This acceleration will be represented as $\vec{a}_{se} = \mu_0 \vec{a}_e$ in the reference frame K_G^{1111}. To calculate the standard acceleration of C when A, B and C are present, one relies on the Rule of Composition to add up the two interactions. In order for the total quantity of motion to remain the same in the composite system, the quantity of motion in C cannot change. As a result C will not experience any acceleration and the drop of oil will hover. One can then compare this situation with the acceleration measured in the two other counterfactual cases. The comparison leads us to assert that the accelerations "canceled" out. But in fact there is no cancelation of forces or accelerations, only a comparison between actual and counterfactual cases. When one recognizes that the Parallelogram Law for the composition of forces is the same law that governs the addition of quantities of motion, one realizes that the same Rule of Composition that informs us about the structure of physical systems is the one governing interactions between bodies. Moreover, this Rule of Composition aids us in comparing actual cases to counterfactual ones and in making quick inferences from counterfactual accelerations to actual ones.

The identification of force with “standardized accelerations” immediately does away with Newton’s attempt to reduce the inertial role of the mass parameter to some inherent causal agency located within bodies. Chapter 5 argued that such an interpretation leads to inconsistencies, since the inherent force is supposed to act both independently of the context in which it is operating (as a *vis conservans*) and in relation to external impulses (as a *vis resistens*). The account introduced here replaces Newton’s inherent causal agencies with the notion that a body’s motion is governed by a certain geometry. The ideal motion is a PUM, which is uniform, unidirectional motion. Various bodies participate in this motion when they are part of a single physical system. The material nature of bodies is expressed through the various roles they take as parts of physical systems. In particular, the property of mass is derived from the symmetries of the underlying geometry of PUMs, from the Criterion of Isolation which asserts that an isolated system instantiates a PUM, and from the Rule of Composition. Thus the inertial role of mass is the logical consequence of the geometry of PUMs and the relations between parts of a physical system and their composite.

The approach here undermines prevalent distinctions that often govern physical thinking. First, this approach undermines the distinction between spacetime structure and dynamic laws. If one takes the Criterion of Isolation – or the assertion that an isolated system instantiates a PUM – to be a geometric fact, then a geometric fact forms an integral part of momentum conservation, which is ordinarily taken to be dynamic law. Since the trajectory of an isolated system instantiates a PUM, there is no causal agency, such as the inherent forces within matter, or the causal influence of spacetime over material objects, that propels bodies to move in uniform rectilinear motion. Rather, there is the “shape” that spacetime structure imposes on the trajectories of isolated systems. Second, the approach undermines the distinction between geometric and material properties. The stretching parameter μ_0 captures the essential meanings of the concept of mass, but this parameter is intimately connected to the symmetries of the underlying spacetime. On the other hand, the inertial role of mass, and its role as quantity of matter are also intimately connected with the presumed structure of physical systems. Thus, according to the interpretation provided here, both the inertial role of Newtonian mass and its role as quantity of matter are derived from the geometry of PUMs and the structure of physical systems.

Our approach has resonance in a 1959 paper written by Schlesinger about the relation between material properties and laws governing the behavior of physical systems:

> It may appear then, that if the behavior of a part of a system is determined by laws governing the system as a whole, then the different parts must so to speak be aware of each other’s state and in a planned and concerted effort act in a manner to satisfy the laws imposed by nature on the aggregate. Consequently the attitude that while we may show that a particular property of a portion of a system *follows* from general laws appertaining to the whole system, it does not mean that we have discovered that it is *determined* or *caused* by those. This means that, in spite of all warnings and exhortations, we do at times unconsciously associate with the concept of cause the notion of “compulsion”, and are not prepared to view it as a mere functional relationship between events. Therefore in order to “achieve insight” into

> the mechanism of a situation and be able to see "how things really work" we feel that we must break down the system under consideration into its smaller elements, the properties of which give rise to the collective properties of the whole. (Schlesinger, 1959, p. 247)

Schlesinger argues that the inference from structural properties of physical systems to inherent causal agencies "explaining" the structural properties is often a tempting strategy in explaining how physical systems "work." But the reduction of dynamic laws to inherent or external causal agencies may not add to our understanding and is based on a scientific prejudice towards reducing dynamic structures to efficient causal processes. The suggestion here is to interpret physical laws as governed by geometries of motion and as describing the structure of physical systems. The laws give us relations between parts of a physical system and the composite whole, and one does not need a list of bedrock causal agencies to ground this structure.

6.3 Conclusion

The reconstruction of Newtonian mass introduced here, as partly implicit in the structure of spacetime, and partly in the presumed structure of physical systems, provides a new interpretation of the concept. On the one hand, a strong analogy is erected between inertial reference frames, i.e., the Galilean transformations, and the parameter represented by the expansion parameter μ_0. The reconstruction of spacetime in Chapter 3 showed that the Principle of Relativity and the equivalence between inertial reference frames can be derived from a geometry of PUMs, where uniform motions are taken to be the fundamental entities of spacetime. If certain structures are presupposed as governing ideal uniform motions, then different inertial reference frames provide equivalent means of representing these motions. The geometry of PUMs allows for various equivalent inertial reference frames $K_G^{1\mu_1\mu_2\mu_3}$, but it also leads to another spacetime symmetry, and a set of reference frames $K_G^{\mu_0 111}$. The expansion parameter μ_0, which is implicit to those frames of reference, is a parameter that transforms measurements of length and duration performed in different frames. The different frames $K_G^{\mu_0 111}$ agree on the measured velocities, but disagree on the accelerations defined in each frame. Since this expansion parameter μ_0 was shown to be closely tied to our ordinary assumptions about mass, we now have a new interpretation of mass as a function of our systems of representation rather than an internal property with causal powers.

While a crucial analogy exists between inertial reference frames and the property of mass, another important component in this reconstruction are the Criterion of Isolation and the Rule of Composition, which are essential for describing how motions of components of physical systems combine to describe the motion of the composite whole. Once those two presuppositions are in place, the various conceptual roles of mass, including its additive role as the quantity of matter and its inertial role, are derived. The inertial role of mass arises from the role of the expansion parameter μ_0 in relation to the structure of spacetime (correlating the measured acceleration of bodies with their "proper" representation) and from the structural assumptions governing physical systems. The additive and conserved nature of mass is a logical consequence of the same structures. These two important conceptual connections demonstrate that mass has important conceptual links to the structure of spacetime, on the one hand, and to laws of motion, on the other hand. It is therefore not surprising that commentators have found it difficult to find the "true" essential meaning of mass, and why it was difficult to find a metaphysical or reductive account that would bring all these conceptual roles into one interpretive system.

The reconstruction of Galilean spacetime and Newton's Laws of Motion undermines a certain metaphysical picture that ordinarily guides our thinking about physical reality. According to the standard interpretation, physical reality can be divided neatly into three components: spacetime, material bodies with their essential and accidental properties, and laws of nature governing the behavior of those bodies. According to this metaphysical picture, the three "layers" of reality reside in relatively independent metaphysical realms. In particular, space and time seem to hold a more fundamental reality than that of material properties, such as mass. According to this picture, it is metaphysically possible to conceive a universe with the same spacetime structure but with bodies possessing different kinds of material properties. In particular it is possible to conceive a universe with Galilean spacetime but with bodies that do not possess mass. The reconstruction here suggests that this metaphysical picture is misleading, and that providing a proper description of spacetime structure is intimately connected with providing a proper conceptual understanding of mass. Mass is not an essential property of bodies, but is an essential property of trajectories through spacetime.

Chapter 7
Structural Assumptions, Newton's Scientific Method, and the Universal Law of Gravitation

In previous chapters Galilean spacetime and Newton's Laws of Motion were reconstructed from a geometry of motions and the structure of physical systems. Once a geometry of PUMs and a structure governing physical systems were assumed, the basic physical concepts and laws of motion of Newtonian mechanics were derived. The main benefit of this reconstruction so far is in providing an economic presentation of the foundation of Newtonian mechanics, and in revealing new conceptual connections between material properties such as mass and the structure of spacetime.

This chapter examines the epistemic role of structural assumptions. I argue that structural assumptions regarding the nature of physical systems hold a unique epistemic role in scientific inferences. An inference is considered scientific when it is able to deduce general scientific laws from a few empirical observations. But there is a difficult methodological problem regarding such inferences. On the one hand, scientific research aims at limiting itself to those claims that are suggested by the empirical evidence. The more a scientific theory outsteps the boundaries of empirical evidence, the more likely it is to lose its scientific value. However, science must go beyond that which is merely given in our experience. One must be able to anticipate patterns, provide idealized models of complex systems, and evaluate the extent to which error has crept into the experimental practice. Scientific practice must rely on laws of nature for it to be able to function. But it is not clear how scientific method is both able, on the one hand, to strictly adhere to the evidence provided by experience, and on the other hand to deduce laws of nature that far outstep the mere deliverances of experience.

I argue in this chapter that Newton provides an intriguing solution to the methodological problem, relying in his account on structural assumptions regarding the nature of physical systems. Structural assumptions, I claim, hold for Newton a central place in his scientific methodology. On the one hand, when empirical evidence suggests that a certain structural assumption holds, Newton takes this as evidence that the law implied by the structural assumption holds exactly and universally for all physical systems. For example, when empirical evidence suggests that the

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conservation of momentum holds, Newton takes momentum conservation to hold exactly and universally, even for systems for which no empirical evidence can be given. The reason for accepting conservation of momentum as a law of nature is that the conservation of momentum can be interpreted as a structural assumption, or as a property of composite systems (force of inertia) that is reducible to the same property applied to the system's ultimate parts. The complete reducibility of a property renders the law that governs it a universal law of nature.

The notion of structural assumption has a crucial role in inferring laws of nature from empirical evidence that is always more limited and incomplete than what the law of nature covers. However, the laws based on structural assumptions also have a crucial role in inferring *new* laws of nature from the phenomena. For example, the conservation of momentum functions as a crucial background assumption in Newton's scientific method and in his argument for the Universal Law of Gravitation. In the case of gravitation, structural assumptions function as Archimedian points from which Newton was able to erect his inference from Kepler's phenomenal laws to his gravitation law. But the law of gravitation was taken to be universally valid only when the force of gravitation itself was shown to obey a distinct strcutrual assumption governing gravitation.

I shall argue that structural assumptions, both the ones underwriting the conservation of quantity of motion and the one governing the force of gravitation, hold a key role in Newton's scientific method and in turning observed regularities into causal laws. Given Newton's central role in establishing the practice of modern physics, one should take this discussion of Newton's method as a historical analysis aimed at clarifying the epistemic role of structural assumptions in Newton's inferences. Whether or not this epistemic role can be shown to be relevant in contemporary science remains to be seen.

7.1 Hypotheses and Scientific Propositions

The publication of Newton's *Principia* was received with admiration. Newton's contemporaries were astonished by his extraordinary mathematical skills and keen physical intuitions. However, Newton's argument in Book III for the Universal Law of Gravitation was met with criticism. Huygens, one of the leading scientists of the day, thought that Newton's argument bordered on scientific irrationality. Since gravitational attraction contradicts the principles of mechanical philosophy, Newton's theory seemed counterintuitive and even absurd.[1] Leibniz was critical of Newton's argument as well.[2] After all, how is one to accept action at a distance without relying on contact forces or whirling fluids? Newton on his part argued that his theory was based on impeccable reasoning. Even if his gravitational force violates the scientific sensibilities of the day one still has to accept it as fact.

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[1] See Maglo (2003) for an account of the reception of Newton's gravitational theory.

[2] See Leibniz's letters to Newton from March 7, 1692/3 in (Newton, 2004, p. 106).

Newton explains his attitude towards hypotheses in the General Scholium to Book III of the *Principia*:

> For whatever is not deduced from the phenomena must be called a hypothesis; and hypotheses, whether metaphysical or physical, or based on occult qualities, or mechanical, have no place in experimental philosophy. In this experimental philosophy, propositions are deduced from the phenomena, and are made general by induction. The impenetrability, mobility, and impetus of bodies, and the laws of motion and the law of gravity have been found by this method. (Newton, 1999, p. 943)[3]

According to Newton the nature of a hypothesis does not matter; hypotheses have no place in experimental philosophy. Newton is primarily concerned to undermine the Cartesian explanation for gravitation as a legitimate alternative to his attraction force. According to Newton, mechanical explanations are no more scientific than occult qualities if one cannot deduce them from the phenomena. The empiricist rhetoric gives Newton an important advantage over his Cartesian opponents.

In Rule 4 for the Study of Natural Philosophy, Newton again codifies his approach:

> In experimental philosophy propositions gathered from phenomena by induction should be considered either exactly or very nearly true notwithstanding any contrary hypotheses, until yet other phenomena make such propositions either more exact or liable to exceptions. (Newton, 1999, p. 796)[4]

Thus even if scientific intuitions contradict the propositions derived from the phenomena, one does not have good reasons to reject them. The phenomena should dictate what is taken as true. If the scientific proposition is derived from the phenomena, it is not completely safe from refutation. Scientists may extend the investigation into new domains or discover new phenomena that demonstrates the proposition to be false. According to Newton one must *take* the scientific proposition deduced from the phenomena to be true, without denying the possibility of it being refuted in the future.[5]

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[3] It is important to note that the Scholium to Book III was added to the second edition of the *Principia* published in 1713, some 26 years after the first edition in 1687. In these long years between the first and second edition, Newton was criticized for failing to include a proper explanation of gravity, and we can see the Scholium as an attempt to answer critics. See I.B. Cohen's introduction to the *Principia* (Newton, 1999, pp. 274–80).

[4] This Rule was only added in the third edition of the *Principia* from 1726, almost 40 years after the initial publication of the *Principia*! It is possible that this Rule reflects only Newton's latest thought on methodology. See I.B. Cohen's introduction to the *Principia* (Newton, 1999, p. 200).

[5] In the preface to the second edition of the *Principia* Cotes defends Newton from the charge that he treats gravity as an occult force:

> . . . occult causes are not those causes whose existence is very clearly demonstrated by observations, but only those whose existence is occult, imagined, and not yet proved. Therefore gravity is not an occult cause of celestial motions, since it has been shown from phenomena that this force really exists. (Newton, 1999, p. 393)

While Cotes may not have the same philosophical views as Newton, the defense Cotes provides here is very much in line with the wording of Rule 4.

However, it is not exactly clear what Newton means by "propositions gathered from phenomena by induction." Is there a rule of induction that informs the scientist how to produce general statements from observations? What does this rule look like? According to a long tradition in the philosophy of science, formal rules of induction are either extremely hard or impossible to formulate. Perhaps by "experimental demonstration" Newton means simply that causal laws are derived from the phenomena through deductive reasoning. Duhem (1991, pp. 190–95) famously argued that it is impossible for Newton to have used logical deduction in deriving law of gravitation from the phenomena. Newton started with the elliptical orbits of the planets and deduced from them the inverse square nature of the law of gravitation.[6] He then used the law of gravitation to calculate corrections in the planets' orbits, using the law of gravitation to determine the planets' deviations from pure elliptical orbits. Following a strict deductive method cannot reach conclusions that demonstrate a premise to be false or show it to be only approximately true. Thus, Newton must have followed some other strategy in deriving his law of gravitation.

Proponents of the Hypothetico-Deductive (HD) method worry that general scientific propositions always extend what can be shown with a few observation or experiments. Moreover, they argue that for any favored hypothesis, there may be others that are consistent with the phenomena. Thus there is no foolproof procedure which can be given for generating a scientific proposition that extends the empirical basis. If the process of arriving at general hypothesis is fallible, it seems unreasonable to suppose that a rule of induction exists. Thus proponents of the HD method argue that one must begin with a conjecture, or some educated guess, and then examine whether logical consequences of the hypothesis are consistent with the phenomena. Newton warned in Rule 4 that one ought to be careful when deriving a proposition from the phenomena. In case a new phenomenon deviates from the scientific proposition, it should be rejected. Thus, Newton was well aware that induction is fallible, and yet he still rejected the HD method. Newton seemed to think that there is a rule of induction that charts the course from a given set of phenomena to the scientific proposition derived from it.

But how can Newton resist having his method collapse into the HD method? If there were a mechanical rule of induction that allows one to directly infer the hypothesis from the phenomena, this rule of induction would resemble deduction in its strength. Valid inductive inferences that start from true premises would be infallible. However, it is difficult to find a rule of induction that operates in the same way, in all contexts. If the procedure for generating hypotheses from the phenomena is not foolproof, can it even be formulated? It seems plausible to conclude that the *process* by which a hypothesis is generated could never be universalized and made into a rule, and so any inductive procedure followed must be merely a contingent

[6] A superficial reading of the third book demonstrates that the claim that the law of gravitation is derived from the elliptical orbits of the planets mischaracterizes Newton's argument in Book III of the *Principia*. However, Duhem's point can be made using the actual argument in Book III. See Smith (2002b) and Section 3 for a full discussion.

one with no universal validity. One ought to confine the process of generating a hypothesis to the "context of discovery." But in case there is no rule of induction, all hypotheses are essentially conjectures as there is an ineliminable gap between the phenomena and the hypotheses.[7] To find a gray area between these two choices, i.e., between there being a rule of induction and there being no rule, one needs to show how an inductive form of reasoning, starting from a *particular* set of phenomena, follows a specifiable procedure while still being fallible.

If proponents of the HD method are right, they must explain Newton's assertions that he was following an inductive method. Either Newton's methodological claims were misinterpreted, and he in fact was endorsing the HD method; or, the method Newton endorsed explicitly is not the one he followed as a matter of fact.[8] According to Hanson, for example, when Newton expresses his commitment to inductivism, he is actually endorsing the HD method. Newton's use of the word "hypothesis" is simply meant as "an expression of some philosophical or metaphysical prejudice" (Hanson, 1970, p. 32). This pejorative use of "hypothesis" merely describes impeding metaphysical stances or prejudices that have no testable conse-

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[7] This is the reasoning that led Karl Popper to his falsificationism:

> ... it is obvious that this rule or craft of "valid induction" is not even metaphysical: it simply doesn't exist. No rule can ever guarantee that a generalization inferred from true observations, however often repeated, is true.... And the success of science is not based upon rules of induction, but depends upon luck, ingenuity, and the purely deductive rules of critical arguments. (Popper, 2003, p. 70)

This leads Popper to articulate his methodological rule. First a scientist, in virtue of some leap of the imagination, formulates a hypothesis. Then, he or she derives a testable implication from the hypothesis. Finally, if the testable implication is shown to be false when compared with observations, the scientist concludes by *modus tollens* that the hypothesis is refuted. Otherwise, we have reason to take the hypothesis seriously. Popper articulates a methodological rule that encodes this approach:

> Once a hypothesis has been proposed and tested, and has proved its mettle, it may not be allowed to drop out without "good reason". A "good reason" may be, for instance: replacement of the hypothesis by another which is better testable; or the falsification of one of the consequences of the hypothesis. (Popper, 2002, p. 53)

Popper's HD method therefore suggests that no valid distinction can be made between a mere hypothesis and the propositions that are deduced from the phenomena. We have to start with hypotheses. We then use deductive rules to see if they cohere with our observations.

[8] According to Hanson (1970), one should be careful to interpret correctly Newton's use of the word "hypothesis," since Newton did not use this word very consistently. Some occurrences of the word "hypothesis" in the first edition of the *Principia* were replaced by the word "phenomenon" in the second edition. The "hypothesis" that the solar system is at rest is explored by Newton in both the first and the second editions of the *Principia* to settle the controversy between the geocentric and heliocentric systems. Hanson differentiates between four different kinds of scientific propositions: a supposed observational claim, which functions like the initial conditions we specify when solving a physical problem, a confirmed observational claim, a supposed theoretical claim, and a confirmed theoretical claim. All of these may be referred to as hypotheses, with varying meanings, depending on the context.

quences and therefore cannot be confirmed or refuted. Newton was being pestered by the Cartesians about Newtonian gravity being a force that attracts at a distance. The metaphysical prejudice of his day was that all physical forces are reducible to mechanical forces of push-and-shove. But the Cartesian whirling fluids hypothesis, used to explain gravity, had no testable implications. So Newton's methodological remarks do not suggest that hypotheses have no place in experimental philosophy (Hanson, 1970, p. 31), only those hypotheses that have no testable implications ought to be excluded. However, as Worrall (2000, p. 47) argued, Hanson's attempt to mitigate Newton's inductivism renders his methodological remarks inconsistent with his scientific practice. There are examples of hypotheses Newton excludes solely on the basis that they were not derived from the phenomena, even though they had empirical implications consistent with the phenomena.[9]

According to another reading of Newton's methodological remarks, one should distinguish between Newton the scientist and Newton the rhetorician. Newton the scientist made conjectures and hypothesized that the motions of the planets are governed by a force of gravitation obeying the inverse square law. Newton the rhetorician claimed to have followed a strict inductive method.[10] However, no charitable interpretation of Newton's work can accept this interpretation. While Newton's actions as a public figure are not moral exemplars – he was not charitable to his contemporaries – it seems unlikely that he would use a Baconian spin on his scientific theories just to render them more acceptable. Newton resisted publication of his work for many years precisely because he feared being embroiled in public disputes over his theory, so it seems implausible that he would characterize his scientific method in a misleading way just to get the upper hand in a scientific dispute.

During the last third of the twentieth century, some philosophers of science started rehabilitating Newton's inductive method. Jon Dorling (1973, 1990) proposed a method he coined "Demonstrative Induction" (DI). Dorling argued that general propositions may be inferred deductively from the phenomena if additional background assumptions are used. According to Dorling, the history of science demonstrates that the DI method was used in many cases to derive new theoretical claims from the phenomena. These deductions conferred on their conclusions

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[9] Worrall claims that

> ...Newton's famous attitude toward material emission ("corpuscular") theory of light would be irreducibly mysterious if this Hanson-style view were correct. As is well known, Newton many times and very heatedly insisted that this emission theory was a *mere* hypothesis because it could not be deduced from the phenomena; and yet the theory is clearly testable. (Worrall, 2000, p. 47)

[10] This is how Imre Lakatos put it:

> The schizophrenic combination of the mad Newtonian methodology, resting on the credo quid absurdum of "experimental proof" and the wonderful Newtonian method strikes one now as a joke. But from the rout of Cartesians to 1905 nobody laughed. (Lakatos, 1978, p. 212)

scientific credibility that led to the exclusion of other hypotheses consistent with the phenomena. Norton (1993, 1994, 1995) and Harper (1990, 2002) also demonstrated the importance of the DI method and of the closely related method of Eliminative Induction.[11] According to Norton, the background assumptions used in these demonstrations belong to the very core of the theories held by the scientific community. They constitute the most basic and general assumptions about systems. Thus one may take these background assumptions to be nearly certain, given that it would take a scientific revolution to seriously deny this core set of beliefs. Indeed, as Norton argues, some of these assumptions are so general and weak that they survive scientific revolutions. It is not that these assumptions do not carry some inductive risk, but one should not forget their (almost) universal validity in the eyes of the scientific community.

However, the DI method faces several challenges. The first challenge is Duhem's logical analysis of DI inferences. While Kepler's laws are inexact, incomplete observations, Newton's Universal Law of Gravitation is exact. Moreover, Newton was able to find corrections to Kepler's laws based on models in which the law of gravitation was held to be true. How can any deductive inference lead to conclusions that could then be used to find corrections in the premises? Duhem's analysis is a logical one – deductive inferences can only yield propositions that contain the same content as was given in the premises of these arguments, only in different form. Thus if premises are then taken to be only approximately true, and corrections to the premises assume the conclusion as true, the argument cannot be merely deductive.

Another challenge concerns the difference in kind between phenomenal laws and laws of nature. Conclusions of scientific arguments often count as "laws of nature." These are taken to be exact and physically necessary, i.e., it is not *physically* possible for these laws to be false.[12] If laws of nature were discovered using DI arguments, then the necessity of laws of nature must be derived from the necessity to be found in the premises. It is natural to assume that the origin of the conclusions' necessity can be traced to the background assumptions used in DI arguments. However, by doing so one has merely pushes the problem one step back. One can immediately ask, what renders the background assumptions necessary? To avoid a possible regress, one needs to find a scientific proposition which naturally presents itself as universally valid and exact.

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[11] According to Eliminative Induction, the additional background assumption is a disjunctive statement (with the "exclusive or" connective). One requires only a few observations to confirm one of the disjunct and eliminate all the other disjuncts. The methods of Demonstrative and Eliminative Induction are equivalent from a logical point of view.

[12] By treating laws of nature as "necessarily true," I do not mean propositions whose negation is inconceivable. It is perfectly conceivable for Newton that bodies could have been created without impenetrability, inertia or gravity; we can imagine God creating material bodies without those properties. However, there is a sense of "necessity" that is relevant in this context, which is the applicability of laws of nature to both actual and counterfactual scenarios. It is obvious that Newton thinks his laws of nature apply "universally," in the sense that we can compare the actual trajectories of bodies to their counterfactual ones. Laws of nature dictate what the counterfactual trajectories would have been.

Some of the DI background assumptions may be justified on a priori grounds, since, for example, much could be learned from the phenomena through mathematical reasoning. Some of the assumptions may themselves be phenomenal laws. However, both these cases are not interesting from a methodological perspective, since the propositions derived are themselves merely phenomenal laws. Moreover, the conclusions of scientific arguments frequently involve additional theoretical entities that were not present in the phenomena. From the observed motions of the planets, for example, Newton derived the existence of a gravitational force. This force goes "beyond" or "behind" the phenomena, and can be taken as the cause that generates the phenomena.[13] Thus another challenge that the DI method faces is how exactly a deductive argument is able to introduce new theoretical entities when these are not present in the observations themselves.

If John Norton's account is to be believed, it seems the necessity of background assumptions of DI arguments stems from their central role in a scientific paradigm. However, if the near certainty of the background assumptions stems from their role as core assumptions in a scientific paradigm, then it is not clear to what extent the DI method is distinct from the HD method. As Worrall (2000, pp. 69–76) argued, the background assumptions of DI arguments can be overturned during scientific revolutions. Thus, even though they hold a significant place in a paradigm, these propositions function as hypotheses. Scientists presume these assumptions are true and rely on their truth whenever they investigate the phenomena. Thus the final challenge that the DI method faces is how it is able to introduce exact laws of nature with new theoretical entities that can be used to find corrections to approximated phenomenal laws, without collapsing into the HD method.

On the surface of things, the HD and DI methods conceive of empirical confirmation differently. According to the DI method, a new scientific proposition is deductively inferred from the phenomenal laws together with background assumptions. In the HD method, the observable implication is logically derived from the hypothesis together with the background assumptions. Thus, it seems as if the methods differ in how they conceive of scientific inferences; either the scientific proposition is the conclusion of a deductive inference (the DI method) or one of its premises (the HD method). But where the inference begins is not really essential, since at the end what we are concerned with is the logical consistency of a set of propositions, some of them theoretical, some of them empirical. In both methods background assumptions pose constraints on the new propositions one incorporates into the accepted system of beliefs.[14]

[13] Granted, Newton did not think that his account of the gravitational force was the whole story. His search for a mechanical explanation of this force suggests that further elaboration of the mechanism which leads to the inverse square law may still be discovered. But this does not undermine the fact that the notion of force by itself is not present in the phenomena.

[14] The distinction between the DI and the HD methods is blurred even further when we recognize that a conclusion of a DI argument may get additional empirical support from other domains of the phenomena. When evidence gathered from various domains converges in support of a theoretical claim, we take this convergence as increasing the plausibility of the claim. Someone endorsing the

The DI method therefore faces several challenges. First, the DI method should meet the Duhemian challenge. If the DI argument is a deductive argument beginning with phenomenal laws, then an account needs to be given of how it was possible to derive exact laws of nature from approximated phenomenal laws. Second, the DI method needs to account for the nomic necessity one attributes to laws of nature. How is one able to derive a law that carries necessity from phenomenal laws that seem anything but? Third, it is not clear how a DI argument is able to introduce new theoretical terms. Conclusions of scientific arguments ordinarily carry theoretical terms not present in premises of DI arguments, and pure deductive reasoning seems incapable of achieving this feat. Finally, the DI method needs to meet all of these challenges, i.e., of deriving exact, physically necessary laws from inexact phenomenal laws, and of introducing new theoretical terms, without collapsing into the HD method. All these challenges to the DI method appear to require a "gap," which transcends the phenomenal laws when causal laws are introduced. But if this gap from the phenomena to causal laws is present, then it is not clear how the DI method is able to bridge this gap without collapsing into the HD method.

A cursory examination of Book III of the *Principia* suggests that Newton's argument for the law of gravitation was following the DI method. But his method does not collapse into the HD method. In deriving the Universal Law of Gravitation Newton is following what is mostly a deductive argument which begins with Kepler's phenomenal laws and ends with the Universal Law of Gravitation. The additional background assumptions Newton brings to bear are not hypotheses, but *structural assumptions* such as the one underlining the conservation of quantity of motion. Structural assumptions are properties of physical systems that erect inferences between parts of the system and the composite system as a whole. The Criterion of Isolation and the Rule of Composition from previous chapters, which underwrite the conservation of quantity of motion, consist of structural assumptions, and are therefore uniquely qualified to function as background assumptions in DI arguments.

Structural assumptions are grounded in experience, but they take on universal validity, i.e., they are taken to be true in both actual and counterfactual cases, once they are elevated into general assumptions about the structure of physical systems.[15]

DI method would justify the added credibility of converging evidence with the help of a common cause principle. That is, all else being equal, we would prefer a theory which does not posit multiple causes for the various phenomena, over a theory which posits separate causes. In fact, the purpose of Newton's Rules 1 and 2 for the Study of Natural Philosophy is to guarantee that evidence gathered from different phenomena in favor of a scientific proposition will bolster its plausibility. But the common cause principle would be redundant, if we simply assume a hypothesis to be true, and then try to get confirmation for it from as many domains of the phenomena as possible. The process of confirmation seems to boil down to the same procedure (Worrall, 2000, pp. 66–67).

[15] Some may object to my use of the notion of "universal validity." Ordinarily, the notion of validity is taken to be a property of arguments. A valid argument is one where the conclusion must be true given that the premises are. According to this view, only the notion of truth is a property of propositions. But I need a notion that would describe the difference between the generalization "All A's are B's," which is only true for actual cases, and the necessary statement "All A's are B's,"

The near certainty attributed to these background assumptions stems not from their widespread acceptance, but from their unique role in reducing descriptions of composite systems into descriptions of their ultimate parts.

This chapter describes the process of taking a claim that is initially a mere empirical generalization and turning it into a universally valid proposition. First Newton replaces the empirical claim with an assumption which relates the properties of composite systems to properties of their parts. Once this assumption has the original empirical claim as its logical consequence, Newton takes it to be universally valid, i.e., he takes it to be valid in both actual and counterfactual circumstances.

After a structural assumption is taken to be universally valid, it provides a reliable Archimedian point for turning other phenomenal laws into causal laws of nature. The upshot is that Newton followed carefully constructed inferences throughout the derivation; i.e., he deduced his universal law of gravitation from the phenomena. However, this inference is not as strong as a deductive inference. The process of elevating an empirical claim into a structural assumption is not foolproof. Structural assumptions can still be revised if other more encompassing or accurate assumptions are found. Thus, that structural assumptions have natural necessity does not imply that they are metaphysically necessary. Newton would not claim that it is impossible for him to have stumbled upon the wrong structural assumptions. Nevertheless, even though the process of elevation is not foolproof, it is still governed by a particular type of reasoning, and so we may think of it and derivations based on it as some form of inductive reasoning.

7.2 Structural Assumptions and Their Role in Inductive Reasoning

Throughout his argument in Book III of the *Principia* Newton relies heavily on his three laws of motion as background assumptions. The laws of motion are partly justified through experiments and observations. Nevertheless, Newton applies these laws in domains far exceeding their empirical support. Also, applying these laws in the context of an attraction force outsteps the conceptual paradigm in which these laws were introduced. Stein (1990) argued that Newton's application of the Third Law of Motion (equality of action and reaction) to the system of a central body and a rotating satellite exceeds the empirical basis of this law. The Third Law of Motion was confirmed for collisions performed on the surface of the earth where there is contact between the bodies. In collisions it is reasonable to assume that bodies act on one another equally and in opposite directions. Moreover, when Newton explicates the Third Law of Motion in the *Principia*, he gives various examples where

which is true in both actual and counterfactual cases. Kant describes the latter kind of propositions as a priori true, but his use of it carries the prejudice that necessary propositions could be arrived at independently of experience. I will therefore use "universally valid" to describe a necessarily valid proposition, without prejudging what sort of necessity is involved.

this law holds; when pressing a stone with a finger; when a horse draws a stone tied to a rope; and when a body impinges on another body. All these examples are ones where contact occurs. In the Scholium to the Laws of Motion, Newton also mentions pendulum experiments he performed to test the third law (Newton, 1999, p. 426). Newton does argue as well that the Third Law of Motion applies to forces of attraction. However, this is a conceptual argument which presupposes the conservation of momentum and not an empirical argument. Thus, according to Stein (1990, p. 217) Newton's Third Law of Motion functions as a hypothesis. Newton assumed the law to be true in all circumstances and in all contexts, beyond the domains it was empirically tested and beyond the scientific paradigm in which the law was introduced.

Many of Newton's contemporaries were astounded with Newton's bold application of laws of motion to the solar system. Leibniz, for example, thought it was absurd to apply these laws *without* presupposing they were caused by interplanetary fluids or some other mechanical cause. In the context of contact forces, it is reasonable to assume that action equals reaction. But without supposing forces are grounded in mechanical explanations, how is one to *explain* the validity of the Third Law of Motion? Huygens, while accepting Newton's inverse-square result for celestial gravitation, did not believe that Newton adequately showed the universal nature of the force of gravitation. Applying the Third Law of Motion to a central body with a distant satellite seemed non-sensical to him.[16]

One may think Newton's three laws of motion are universally valid due to their foundational role in Book I of the *Principia*. At least according to Newton's presentation of these laws it appears that they are not "deduced" from the phenomena. Newton asserts these laws before any phenomena is mentioned. A common strategy during the first half of the twentieth century was to treat the laws of motion as implicitly defining the meaning of the terms used in them.[17] According to this approach, one may think that the truth of these laws is stipulated. The Law of Inertia implicitly defines the state of being force-free as uniform rectilinear motion. Similarly, part of a definition of "force" implies the equality of action and reaction. Given that the use of the notion of "force" presupposes the stipulated truth of the axioms, Newton may legitimately apply the laws of motion to every place it is appropriate to use the notion of "force."

A proper critique of the above conventionalist approach is beyond the scope of this work. I shall only say that the main problem with this view is that the laws seem to have been produced by some arbitrary process. In effect the stipulated laws have the same epistemic status as a conjecture or a guess, only one cannot get any confirmation or refutation of these laws because the terms are used to interpret the evidence. According to this view, the laws of motion are the "free creations" of the scientists who came up with them, and their treatment as axioms is not grounded in

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[16] He also thought he had good empirical reasons to reject Newton's reasoning. See Schliesser and Smith (1996).

[17] See for example Poincaré (1905, p. 97).

any reasoning process that could be reconstructed or analyzed according to scientific principles. A scientific inference that relies on such stipulations is essentially a version of the HD method.

Stein does not argue that the laws of motion are conventional. But he does argue that Newton's application of the Third Law of Motion carries with it some irreducible element of stipulation. By affirming the universal validity of the law of equality of action and reaction, particularly in the context of action at a distance, Newton simply presupposed the law to be valid in all circumstances. This stipulation renders the third law a hypothesis – other propositions are compatible with the empirical evidence and the law is not dictated by the phenomena.

It seems as if Stein is presupposing that any stipulation of universality is *arbitrary*, at least in the sense that it is not dictated by the phenomena, and hence any proposition which extends its empirical and explanatory basis is a hypothesis. To be sure, the stipulation becomes less and less arbitrary the more the hypothesis is tested. One can even see the argument in Book III as an overall justification of the initially stipulated hypothesis. However, a close reading of Newton's methodological remarks suggests that he articulated a criterion for elevating empirical statements into universally valid propositions that *precedes* subjecting these propositions to further empirical tests. If such a criterion can be reconstructed, then much of the arbitrariness of stipulated hypotheses can be shown to have been eliminated prior to any empirical tests the hypothesis is subjected to. This criterion for elevating empirical claims carries inductive risks, nor can it be completely formal. However, it does place severe limitations on the type of propositions that are accepted as universally valid.

Newton's inductive method proceeds as follows. To receive their status as universal laws of nature, empirical claims have to be re-conceptualized as assumptions about the structure of physical systems. A "structural assumption" is a rule governing the relation between parts of a physical system and their composite. For example, the property of "extension" is a structural assumption. Part of the meaning of "extension" is the relation it describes between parts of a physical system and the system as a whole. By definition, the extension of a composite body is the sum of the extensions of the parts, so one may say that the extension of the whole arises from the extensions of the parts.[18]

When one moves from parts of a physical system to the composite whole, one is guided by assumptions about relations between parts and wholes. The three laws of motion are logically entailed by conservation of momentum, since force is identified with the change in quantity of motion.[19] One should keep in mind how Newton explicates the quantity of motion:

[18] The notion of "part" and "whole" is used in many contexts with varying meanings. (See Nagel, 1961, pp. 381–83.) Here the notion of part means something *like* the spatial part of a physical system. I do not identify part with a spatial part since it seems as if Newton takes "mass" to be the quantity of matter. Thus, for Newton an ultimate part of a physical system is given by an infinitesimally small volume of unit mass. This is obviously an idealization.

[19] See Chapter 5 for the debate on the relation between force and change in quantity of motion.

> The motion of the whole is the sum of the motions of the individual parts, and thus if a body is twice as large as another and has equal velocity there is twice as much motion, and if it has twice the velocity there is four times as much motion. (Newton, 1999, p. 404)

The structural assumption in this case is very simple since it is represented with the rule of addition governing quantities of motion. The structural assumption therefore includes a Rule of Composition. In the case of a solid body, the quantity of motion of a composite system is proportional to the mass of the body, i.e., the "number of parts" in the system. In the case of a physical system with parts moving in different directions, the quantity of motion of the whole is the vectorial sum of the quantities of motion of the parts.

While it is not difficult to see why quantity of motion is a structural assumption for solid bodies, it is quite another claim to suggest that the conservation of momentum amounts to a structural assumption in dynamic cases. While writing the *Principia*, Newton had good reasons to think that the conservation of quantity of motion for collisions is empirically well-confirmed. Huygens, Wren and Wallis were able to show that collisions are well-described by the assertion that the quantity of motion of a closed system is conserved. Moreover, Newton himself conducted pendulum experiments to show that the equality of action and reaction does not depend on the material nature of the object. In these experiments, Newton let the bobs of two pendulums collide, and then measured the change in quantity of motion in each bob. It did not seem to matter whether the bobs were made out of gold, silver, string or iron, the change in quantity of motion in one bob corresponded to the exact opposite change in quantity of motion in the other bob. It is therefore reasonable using enumerative induction to conclude that conservation of quantity of motion applies in collisions. However, Newton applied the Third Law of Motion audaciously to regions far removed from the domain of experiments, and in the context of an attraction force. Thus either he stipulated the third law to be universally valid (i.e., as applying in all actual and counterfactual interactions), or he had some inductive argument for it.

An important inductive step takes place when Newton universalizes structural properties. His argument is that since the property of a composite body is reducible via a clear and unambiguous rule to the same property attributed to the ultimate parts, then it must be a universal property. To understand why it is possible to universalize structural properties, compare this process with enumerative induction. Assume one observes that all examined crows are black. One would be tempted to think that the proposition "All crows are black" is universally valid, i.e., unexamined crows would be black in the same way that examined crows are. However, this assumption carries great inductive risk, since both the property "crow" and the property "black" are composite properties. If these properties are not reducible to properties describing the ultimate parts of the object, there is no guarantee that the property "crow" cannot be instantiated without the property "black." Some configuration of parts may lead to the instantiation of the property "crow" without the instantiation of the property "black." In contradistinction, consider the claim that the quantity of motion of a closed system is conserved. In all examined cases this quantity was conserved; does one have reason to believe that it would be conserved in unexamined cases? One does if it is possible to construct the quantity of motion of any physical system from quantities of motion belonging to the ultimate parts. Assuming that all ultimate parts of matter are alike, reducing properties of composite systems to properties of ultimate parts suggests a reason to universalize the property. Because the quantity of motion of any observed system arises only from the quantities of motion of the parts, it is reasonable to assume that the quantities of motion of all closed systems arise from the quantities of motion of their parts. Moreover, if each part does not change its motion over time unless it transfers quantity of motion to another part, one may conclude that quantity of motion is universally conserved over time.

That structural assumptions are significant for taking propositions to be universally valid is evident in Newton's Rule 3 for the Study of Natural Philosophy. The rule states as follows:

> Those qualities of bodies that cannot be intended and remitted [i.e., qualities that cannot be increased and diminished] and that belong to all bodies on which experiments can be made should be taken as qualities of all bodies universally. (Newton, 1999, p. 795)

In his explication of this rule Newton begins by insisting that the qualities that can be universalized must have a basis in experiments. But he also asserts that "qualities that cannot be diminished cannot be taken away from bodies." It is difficult to make sense of this criterion, since all properties, including extension, hardness and mobility, seem to be capable of being increased or diminished in magnitude. Newton explicates what he means in the following:

> The extension, hardness, impenetrability, mobility and force of inertia of the whole arise from the extension, hardness, impenetrability, mobility and force of inertia of each of the parts; and thus we conclude that every one of the least parts of all bodies is extended, hard, impenetrable, movable, and endowed with a force of inertia. And this is the foundation of all natural philosophy. (Newton, 1999, p. 795)

Thus for Newton, a quality cannot be intended or remitted when it is recognized as being governed by a structural assumption. If properties divide neatly into the parts of a system whenever a process of division occurs, one has to assume that the ultimate parts of matter have these properties. Furthermore, new composite bodies will be composed of the same kind of ultimate material parts. Thus any material body would carry the properties one was able to reduce to the ultimate parts of matter. The reason for taking extension, hardness, etc. as universal properties is that for each of them the property of the composite object is compounded from the same property (i.e., extension, hardness, etc.) attributed to the parts. It requires only a few observations to confirm that a property survives the division of an object into parts. Once this structural assumption is confirmed, one may take it to apply universally.

In Rule 3, Newton argues that the properties of impenetrability, mobility and force of inertia apply universally because of their role as structural properties. The impenetrability of a composite body arises from the impenetrability of the parts, and so this property has to apply universally. The mobility and force of inertia of the composite body also arises from the mobility and force of inertia of the parts, as is clear from the concept of quantity of motion.

One can now see the analogy between quantity of motion and the extension of bodies. The force of inertia of a composite body is reducible to the force of inertia of its ultimate parts. And because the tendency to continue moving in a straight line is reducible in such a way, one finds that the quantity of motion is proportional to the quantity of matter, i.e., one finds the motivation for the equation $\vec{P} = m\vec{V}$. Since the force of inertia of a composite body is governed by a compositional rule and can be reconstructed from the motion of microscopical parts, one may take it to be a universal property. This implies that even two remote bodies that interact have a conjoined force of inertia that is reducible to the force of inertia of each body (in just the same way that two remote bodies have a conjoined extension). The two remote bodies, unless disturbed, would continue to move in a straight line (taking the center of mass as their common trajectory). Because the conjoined force of inertia of the composite system is comprised of the forces of inertia of each part, in case the two bodies are isolated, an increase in quantity of motion in one body must imply a decrease in quantity of motion in the other body, so as to conserve the tendency of the composite system to move in a straight line.

The consequence of universalizing the force of inertia and taking it to be governed by a composition rule implies that the conservation of quantity of motion must be valid in both actual and counterfactual scenarios. Since the three laws of motion are a logical consequence of momentum conservation, they are presumed to be valid in every possible circumstance. For Newton, therefore, the laws of motion have natural necessity justified through Rule 3. Nevertheless, that the conservation law applies to all physical systems universally does not imply that his reasoning is infallible, and that it is not possible that he has derived the wrong structural properties. It is quite possible that newly found phenomenal laws or exceptions to known phenomenal laws would "dictate" alternative structural assumptions. Structural assumptions are not metaphysically necessary, even if they are taken to have natural necessity.[20]

Newton also utilizes Rule 3 to argue for the universal nature of the gravitational force:

> Finally, if it is universally established by experiments and astronomical observations that all bodies on or near the earth gravitate toward the earth, and do so in proportion to the quantity of matter in each body, and that the moon gravitates toward the earth in proportion to the

[20] In a recent paper Ducheyne (2005) argued that Newton used autonomous models to investigate the various properties of forces and interactions. These models are based on the laws of motions, the definitions in the beginning of the *Principia* and various initial conditions and force laws. Only after these models were developed to various degrees of complexity were they compared with the phenomena. On the one hand, Ducheyne argues that the models are developed independently of the phenomena. On the other hand he argues that the laws of motion on which the models were based were deduced from the phenomena. Ducheyne's account is problematic since it is not clear how the laws of motion can be derived from the phenomena and then used to construct counterfactual models.

> quantity of its matter, and that our sea in turn gravitates toward the moon, and that all planets gravitate toward one another, and that there is a similar gravity of comets toward the sun, it will be concluded by this rule that all bodies gravitate toward one another. (Newton, 1999, p. 796)

This explication of gravity shows the significance of the structural assumption governing the force of gravitation. Newton derives from the phenomena the empirical claim that gravitational acceleration does not depend on the mass of a body. The distance between two bodies, no matter what their shape, mass or chemical constitution, is enough to determine their rate of gravitational acceleration. The empirical fact regarding gravitational acceleration is then redescribed by Newton as a Rule of Composition governing the force of gravitation, which asserts that the total gravitational force is the sum of the gravitational forces operating on each part. The Rule of Composition is expressed in the formula $f_m = mf_a$ where f_m is the overall gravitational force operating on the body, m is the body's mass, and f_a is the gravitational force operating on each part. Since the gravitational force operating on a composite body survives the division of the body into parts, one may conclude that the gravitational force exhibits a structural property. The gravitational force operating on a composite body arises from the gravitational force operating on the ultimate parts.

Newton uses Rule 3 to argue that the susceptibility to the gravitational force cannot be separated from any physical body, and thus the gravitational force is shown to have universal validity. The inductive step involved therefore depends on Rule 3 and on the universalizable nature of structural assumptions.

7.3 Newton's Argument for the Universal Law of Gravitation

Once empirical claims are reconceptualized as structural assumptions, they function in Newton's argument as background assumptions in DI arguments. To show this, I will follow Bill Harper's (2002) division of Newton's argument for the Universal Law of Gravitation into three significant parts. The first step of the argument relies on Kepler's *Area Law*, which asserts that the radius from the sun to the planets or from a planet to one of the moons sweeps equal areas in equal times. The second step utilizes Kepler's *Harmonic Rule* which asserts that for all gravitating satellites, the period of rotation T is related to the radius of rotation as $T^2 \propto R^3$. The *Area Law* and the *Harmonic Rule* are provided as the Phenomena at the beginning of Book III. Phenomenon 1 describes the motion of Jupiter's moons relative to Jupiter. Phenomenon 2 describes the motion of Saturn's satellites relative to Saturn. Phenomena 3–5 describe the motion of the planets relative to the sun. Finally, Phenomena 6 describes the *Area Law* applied to the earth's moon. Newton's first step of the argument deduces from Kepler's *Area Law* the centripetal nature of the gravitational force. In the second step of the argument, Newton deduces from the centripetal nature of the force of gravitation and the *Harmonic Rule* the inverse square nature of the force of gravitation ($f \propto \frac{1}{R^2}$). In the third step of his argument, Newton deduces the universal nature of the force of gravitation, and the formula:

$$f = G\frac{m_1 m_2}{R^2} \tag{7.1}$$

I shall demonstrate the role of structural assumptions in each part of the derivation.

7.3.1 From the Area Law to the Centripetal Nature of the Force of Gravity

The first step of Newton's argument infers the centripetal nature of the force of gravitation from the *Area Law*. The equivalence between the *Area Law* and the centripetal nature of the law is proven in Book I, Propositions 2 and 3.

Figure 7.1 describes Newton's idealized model for a body traversing equal areas in equal amounts of time. Newton's idealization consists of taking the motion of such a body as governed by an instantaneous force operating at points B, C, D and E at equal intervals of time. The distances AB, BC, etc. represent the velocity of the object if one takes the force to act at integral multiples of the unit of time. The Law of Inertia implies that, had the force not acted on the body at point B, it would have traveled uniformly and would have reached the point c at the same time the body

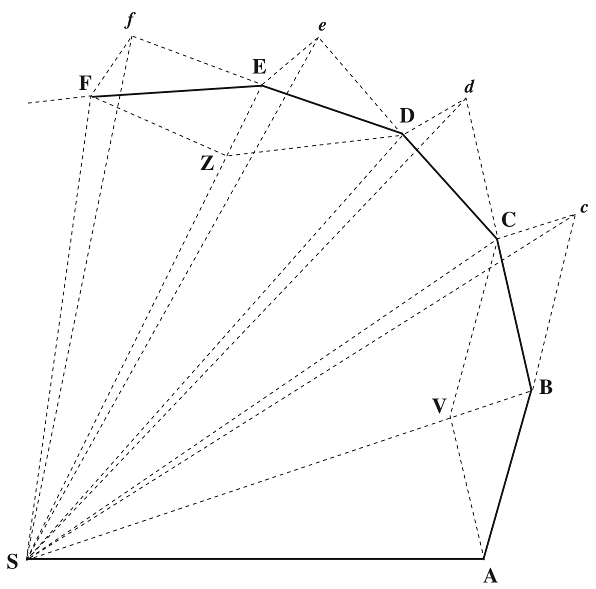

Fig. 7.1 From the *Area Law* to the centripetal nature of the gravitational force

has reached C in its actual motion. The conceptual link between the *Area Law* and the Law of Inertia is established when one compares the areas of triangles *SAB* and *SBc*. Because these triangles are of equal heights and bases, they are of equal areas. And since the model was constructed to retrieve the *Area Law*, by definition the area of triangle *SBC* equals the area of *SAB*. It follows that the area of *SBC* equals the area of *SBc*. The following equivalence holds: the area of the triangle *SBC* is equal to the area of *SBc* if and only if the change in motion Cc is parallel to (i.e., in the direction of) BS (Book I, Propositions 2 and 3). Thus, Kepler's *Area Law* is equivalent to the claim that change of motion is always directed at the immobile center S.

One may take the *Area Law* to be "measuring" the direction of the force. If the area traversed by the radius from S were to increase or decrease at B, the direction of the force would have been off the line SB. Thus it is clear that Newton was able, in this part of the argument, to translate a phenomenal law into a statement about the force generating the phenomena.

However, notice that Newton presupposes here the universal validity of momentum conservation in both actual and counterfactual cases. First, the Law of Inertia is taken to apply counterfactually. The trajectory Bc is taken as the trajectory the body would experience had a force not operated on it. Second, Newton presupposes that the direction of the force is identified with the change in momentum. The universal validity of momentum conservation far extends the empirical support of this claim, especially in the case of celestial bodies. But Newton argues in Rule 3 that mobility and the force of inertia are structural properties. The Law of Inertia must apply in both actual and counterfactual cases. Thus crucial to the argument is the natural necessity Newton attributes to the Law of Inertia.

The first step of the argument can be summarized as follows:

Argument I

Premise 1.	Kepler's *Area Law*.	*(Phenomenal Law)*
Premise 2.	Momentum conservation.	*(Structural Assumption)*
2.1.	The Law of Inertia applies counterfactually.	
2.2.	The force equals the change in momentum.	
Premise 3.	Euclid's geometry.	*(Background Assumption)*
Conclusion I	The gravitational force operates in the direction of an immobile center.	

The first inductive step, therefore, follows the method of Demonstrative Induction. At this stage the background assumption used as a premise has already gone through the process of being elevated into a structural assumption. The natural necessity Newton attributed to the Law of Inertia enables him to "measure" the direction of the gravitational force. By comparing the actual trajectory of the body relative to its counterfactual one, Newton is able to conclude that the gravitational force operates towards the center of rotation.

7.3.2 *The Harmonic Rule and the Inverse Squared Distance Nature of the Gravitational Force*

The second step in Newton's argument derives the inverse-squared nature of the gravitational force from the *Harmonic Rule*, which is:

$$T^2 \propto R^3 \tag{7.2}$$

where T is the period of the rotation around the center, and R is the radius. To carry out this part of the derivation, Newton makes an approximating assumption by taking the planets to be moving in perfect circular motion instead of ellipses. Newton proves in Book 1 Proposition 4 that the centripetal acceleration of the body that rotates in a perfect circle is the following:

$$a = \frac{v^2}{R} \tag{7.3}$$

where the velocity v is the instantaneous velocity of the body and R is the radius of rotation. The proof relies on taking the polygon described in Fig. 7.1 and reducing the length of the segments until they are indistinguishable from motion in a circle.[21] One can follow Newton's reasoning by examining Fig. 7.1 again taking the radiuses SA, SB, etc., to be all the same since the body is now taken to be moving in a circle. The triangle *SBC* is again compared with the triangle *SBc*. Since AB and BC represent the velocity of the body, BV represents the change in velocity, and SB the radius of rotation, one can deduce the centripetal acceleration with the help of Euclidean theorems.[22] Since the instantaneous velocity of the body is related to the period of rotation through the equation $vT = 2\pi R$, one concludes from (7.2) to (7.3) and the conclusion of Argument 1 the centripetal acceleration of the body:

$$a = \frac{v^2}{R} = \frac{(2\pi)^2 R}{T^2} \propto \frac{1}{R^2} \tag{7.4}$$

Thus Newton utilizes the *Harmonic Rule* to "measure" the gravitational acceleration. The assumptions that were employed in the first step of the derivation were employed in this step as well. In deriving the centripetal acceleration Newton relied on the Law of Inertia and on the identification of force with the change of momentum. However, here an important approximating assumption was used in the derivation, namely that the bodies are moving with perfect circular motion. This part of the deduction could be summarized as follows:

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[21] See the Scholium to Proposition 4, (Newton, 1999, p. 452).

[22] See Brackenridge and Nauenberg (2002) for a history of these calculations.

Argument II

Premise 1.	Kepler's *Harmonic Rule*.	*(Phenomenal Law)*
Premise 2.	The gravitational force is centripetal.	*(Conclusion 1)*
Premise 3.	Momentum Conservation.	*(Structural Assumption)*
Premise 4.	Satellites move with circular motion.	*(Approximating Assumption)*
4.1.	$a \propto \frac{v^2}{R}$	
Conclusion II.	$a \propto \frac{1}{R^2}$	

A complication in the derivation is the approximating assumption. According to Kepler's laws, the planets are moving with ellipses around the sun, and so the above argument applies to the motion of these planets only crudely. This complication is compounded by the fact that the phenomenal laws too hold only approximately, and one may find different curves to describe the data. The premises of Newton's derivation are known to hold only approximately, or in Newton's words, they hold *quam proxime* (very nearly). This reminds us of Duhem's objection to Newton's claim that he derived his Universal Law of Gravitation from the phenomena using rules of deduction. George E. Smith (2002a, b) explicated the seemingly perplexing Newtonian procedure of beginning with phenomenal laws that hold only approximately, and then using the result of DI arguments to assess the origin of possible discrepancies between the observed phenomena and the ones predicted by the theory together with various idealized conditions.

Part of the story has to do with various calculations carried out by Newton to show that the approximations carried out in the premises of DI arguments cannot lead us too much astray in deriving the conclusions. Newton showed the biconditional between small deviations in the *Area Law* leading to small deviations of the force from the central gravitating body. Even if the *Area Law* does not hold precisely, Newton showed that small deviations would not have produced significant deviations in the conclusion. The DI argument is "stable" under small perturbations. Another example is the calculation Newton carried out to show that small deviations from the inverse-square law lead to a precession of orbits. I.e., he calculated that the apsidal angle θ is related to the index n of the exponent of centripetal acceleration (where $a = r^{(n-3)}$, and r is the radius of acceleration) as $n = (180/\theta)^2$. With this calculation Newton demonstrated that no precession implies the inverse-square law precisely. Because no precession in the planets' orbits are observed, one has indirect confirmation of the inverse square law. One should therefore be careful in evaluating DI arguments. Newton's confidence in the conclusions of DI argument is not only based on the relation between premises and the conclusion of the argument. It also relies on estimating the approximating assumptions made in the premises; one needs to gauge the extent to which errors might have crept in. Newton could have deduced from the elliptical orbits the inverse-square nature of the gravitational force. However, if small deviations from the elliptical orbits yield significant errors in the calculation of the force's power, such a derivation cannot be trusted. There must be some mechanism for mitigating the potential for errors.

Newton's detailed calculations show that his DI derivations are not sensitive to small deviations and inaccuracies. However, in order to assess such deviations, he relied on background assumptions that were taken to be valid in both the actual case and the idealized conditions presupposed in the argument. For example, the centripetal nature of the law of gravitation is deduced from the *Area Law* on the assumption that the First Law of Motion is valid universally. The Law of Inertia helps Newton bolster the validity of his conclusion by considering counterfactual scenarios and comparing these scenarios to the actual ones. But only by assuming that the Law of Inertia holds *universally* and *exactly*, can Newton show that small deviations from the *Area Law* correlate with small deviations from the centripetal direction of the force. One cannot estimate the inductive risk unless one takes certain laws to be valid necessarily.

Smith is well aware of the paradoxical nature of approximating procedures, which seem to presuppose certain scientific claims as exact in order to calculate the errors that may arise from approximating assumptions (Smith, 2002b, p. 45). Smith calls this common scientific approach the *exact-approximate duality*. He argues that the procedure is that of providing successive approximations, where each DI argument leads to results that yield further detail that help in evaluating deviations. What criterion do we use to designate certain scientific propositions as unquestionable background assumptions? Smith comes close to our account of structural assumptions in the following passage:

> Not just any old first approximation will permit such a sequence of successive refinements, as it might if this were merely tantamount to curve-fitting. The theoretical claim for which Newton requires the first-approximation phenomena to provide crucial evidence is a generic force law – the law of gravity in the case of orbital motions and his law for the resistance force arising from the inertia of the fluid in book 2. Moreover, when the force in question is a net force acting on a macroscopic body, he requires a compositional account of it in terms of forces acting on the individual parts of the body – in terms of microgravitational forces or, in his resistance case, in terms of the forces of impact of fluid particles on parts of the body. Finally, once inductively generalized, the force law ought to have as its consequences a host of idealized phenomena reaching beyond those providing the original evidence for the law. . . . These idealized consequences are expected to agree with observation to increasingly high approximation, and to the extent they do, they of course provide further evidence of the law of force. (Smith, 2002b, p. 48)

Smith identifies in passing Newton's attempt to provide a "compositional" account of the forces operating on a composite body. Thus when Newton provides an account of how composite forces are reduced to microphysical forces arising from interactions between individual parts of the body, he is safer in assuming that force-laws hold universally. The compositional account thus bolsters the robustness of scientific propositions one takes to hold universally and exactly. In this Smith comes very close to our account of Newton's reliance on structural assumptions. The problem with Smith's account is that he doesn't recognize the theoretical connections of the compositional accounts to Rule 3, and Newton's philosophical attempt to provide a methodological justification for taking such compositional accounts as evidence that laws hold universally and exactly. Smith also does not recognize how Rule

3 was intended to secure exactly those background assumptions Newton took as universally valid in his derivations and assessments of approximating assumptions.

The scientific process of taking the conclusions of a DI argument and using it to create models of increasing accuracy is crucial and must supplement the initial derivation. Thus one should concur with Smith's claim that there is a risk that the conclusion will lead to a "garden path," since not all the inductive risk is located within the initial derivation. But this process of finding more accurate models depends on finding laws that are valid in all actual and counterfactual cases. One cannot estimate the errors that creep into our observations without presupposing some law that is valid in all circumstances.

The answer to Duhem's challenge is therefore that an approximating assumption can be gauged for the possible error it introduces when certain laws are taken to be exact and to hold necessarily. Thus, for the scientist to gauge the possible error introduced by the DI argument, the DI method is supplanted with a universalizing criterion. Once an empirical proposition is replaced by a law that arises from a structural property, one can take this law to be exact and necessary. This law can then be used to assess the errors introduced by the idealizing assumptions in the original derivation, and an iterative process of finding more exact phenomenal laws is initiated.

7.3.3 Deriving the Universal Nature of Law of Gravitation

In his third step of the derivation, Newton concludes that the gravitational force operates between any pair of masses, and is proportional to the product of the masses according to the following equation:

$$f = G\frac{m_1 m_2}{R^2} \tag{7.5}$$

Newton's argument for the universal nature of gravitation occurs in Book III, Propositions 6 and 7. The argument in Proposition 6 begins with the observation that all earthly material objects move with the same gravitational acceleration. This fact was first discovered by Galileo. Newton describes pendulum experiments he conducted to show that all earthly matter gravitates towards the center of the earth with an acceleration of $g = 9.8\frac{\text{cm}}{\text{s}^2}$. Moreover, since all of Jupiter's moons obey the *Area Law* and the *Harmonic Rule* relative to Jupiter, it follows that their acceleration toward Jupiter depends only on their distance from the planet. By the same argument, the acceleration of the planets toward the sun depends only on their distance from the sun. Moreover, the motions of Jupiter's moons relative to Jupiter are very regular, which implies that their acceleration toward the sun is the same as that of Jupiter and independent of their relative mass. Thus, an important empirical claim to be deduced from all observations is that gravitational acceleration is independent of the mass of the body or its chemical constitution.

An important question remains. How does Newton derive the universal nature of the force of gravitation from this empirical claim? Some form of reasoning has enabled Newton to move from a claim that is valid for all observed bodies, to a universal law of nature asserting that a force of gravitation would operate between any pair of masses. According to a Howard Stein (1970, 1990) Newton utilized the concept of field to make the inductive leap. Even though Newton did not explicitly use the notion of field, there are indications that he invented a very similar concept. In the beginning of the *Principia* Newton made an important distinction between the "absolute," "accelerative" and "motive" forces. The various forces are given in Definitions 6–8 of the *Principia*:

> The quantities of forces, for the sake of brevity, may be called motive, accelerative, and absolute forces, and, for the sake of differentiation, may be referred to as bodies seeking a center, to the places of the bodies, and to the center of the forces: that is, motive force may be referred to a body as an endeavor of the whole directed toward a center and compounded of the endeavors of all the parts; accelerative force, to the place of the body as a certain efficacy diffused from the center through each of the surrounding places in order to move the bodies that are in those places; and absolute force, to the center as having some cause without which the motive forces are not propagated through the surrounding regions, whether this cause is some central body . . . or whether it is some other cause which is not apparent. (Newton, 1999, p. 407)

The absolute measure of the force refers to its causal origin located at the center towards which the force of gravitation is directed. The motive force is defined as the force a composite body experiences. The accelerative force is the force experienced by each of the body's parts. Moreover, Newton asserts that the motive force is related to the accelerative force as momentum is related to velocity, i.e., $f_m = mf_a$. The motive force f_m is the product of the mass of the body and the accelerative force f_a operating on each part.

Stein argues that Newton's notion of accelerative force functions as an acceleration field. This acceleration field describes the disposition of any body to accelerate according to the inverse-square law, and it is clear that Newton ascribes this disposition to the place a body occupies rather than to the body itself. Newton also describes how these dispositions are distributed from the center of the attracting body to the surrounding places, and so that the "accelerative quantity of force" describes the efficacy of the gravitational force at these places.

An important inductive step, according to Stein, is Newton's hypothesis that the acceleration field exists:

> Newton's inductive conclusion is that the accelerations toward the sun are *everywhere* – i.e., even where there are no planets – determined by the position relative to the sun . . . that argument cannot be made without the notion of a field. (Stein, 1970, p. 268)

Stein's account suggests that the disposition of the gravitational force to generate accelerations, where that disposition is attributed to particular *places* rather than particular existing bodies, enables Newton to generalize from the particular cases observed to a universal rule. Thus according to Stein it is the notion of a field, describing a set of dispositions spread out throughout space, which provides the gravitational force its universal validity, including its validity in counterfactual cases. Only if we assume that the attracting body generates an acceleration field, can we say that a body *would* experience a gravitational force had it been placed at a certain distance from the attracting body.

However, contrary to Stein's assertion, it seems as if the notion of acceleration field is not a necessary conceptual tool for making the generalization. If one takes Newton's laws of motion to hold in counterfactual cases, then every time the *Area Law* and the *Harmonic Rule* apply, the body's acceleration would be proportional to $\frac{1}{R^2}$, independently of its mass or material constitution. If Newton is able to justify the claim that all gravitating bodies are likely to obey the *Area Law* and the *Harmonic Rule*, then a body would accelerate in proportion to the "intensity" of the gravitational force at that particular position. Thus, the notion of field is not *necessary* for taking the inductive step. If extending the premises of the Argument I and II to counterfactual cases can be justified, the acceleration field would be the *result* of Newton's inductive argument, not an aid in making it.

In fact there is an alternative explanation for Newton's inductive step. It seems as if the inductive leap occurs when Newton takes the empirical claim, i.e., that gravitational acceleration does not depend on the mass of the body, and re-conceptualizes it as a structural assumption. In the concluding remarks in Proposition 6 Newton states as follows:

> But further, the weights [or gravities] of the individual parts of each planet toward any other planet are to one another as the matter in the individual parts. For if some parts gravitated more, and others less, than in proportion to their quantity of matter, the whole planet, according to the kind of parts in which it most abounded, would gravitate more or gravitate less than in proposition to the quantity of matter of the whole. (Newton, 1999, p. 808)

The theoretical fact which best accounts for the empirical fact is the assertion that the gravitational (motive) force operating on a composite body is the sum of the gravitational (accelerative) force operating on each part. Moreover, one knows that the accelerative force f_a operating on each part is independent of the nature of the part. It does not matter whether a body is made of gold or of coal, each of its parts will experience the same gravitational acceleration. Newton therefore articulates a Rule of Composition governing the gravitational force analogous to the Rule of Composition governing quantities of motion:

> **Rule of Composition for the Gravitational Force.**
>
> The gravitational force operating on a composite body is the sum of the gravitational forces operating on each part. Or, in other words, $f_m = mf_a$, where f_m is the motive force operating on the composite body and f_a is the accelerative force operating on each part.

Because Newton was able to articulate a structural assumption governing the gravitational force and because the force does not take into account any property restricted to a particular kind of body, Newton reaches the conclusion that a structural assumption applies to all bodies in all circumstances. The argument can be summarized as follows:

Argument III-1.

Premise.	Gravitational acceleration is independent of mass.	*(Conclusion II)*
Conclusion III-1.	The motive force f_m is the sum of the accelerative forces f_a operating on the parts; $f_m = mf_a$	

Argument III-1 is not a deductively valid argument. Rather, it is an argument where an empirical claim is re-conceptualized and elevated to the status of a structural assumption. Thus, the inductive step that Newton takes does not rely on the notion of a field; rather, it relies on the criterion for universalizing properties implicit in Rule 3. This criterion is not unique for gravitation and is employed for all universal properties such as extension, impenetrability and inertia. Since the gravitational force operating on the composite body can be reduced to the forces operating on each ultimate part, one cannot separate the gravitational force from the ultimate parts of matter. Thus, the gravitational force operates on all ultimate parts of matter in the same way.

The second part of the third step of the derivation concludes with the universal nature of the force of gravitation. As step 2 of the argument showed, the gravitational acceleration is proportional to $\frac{1}{R^2}$ independently of the mass of the body, so that:

$$f_a \propto \frac{1}{R^2} \tag{7.6}$$

But from argument III-1, it follows that motive force is proportional to the mass:

$$f_m \propto \frac{m}{R^2} \tag{7.7}$$

Newton then uses the structural assumption governing the gravitational force with the Third Law of Motion to conclude the universal nature of the force of gravitation:

> Since all the parts of any planet A are heavy [or gravitate] toward any planet B, and since the gravity of each part is to the gravity of the whole as the matter of that part to the matter of the whole, and since to every action (by the third law of motion) there is an equal reaction, it follows that the planet B will gravitate in turn toward the whole of the planet as the matter of that part to the matter of the whole. (Newton, 1999, p. 810)

The reasoning here may be described as follows. If body A gravitates toward body B, then the motive force operating on A is proportional to the mass of A over the distance squared, so that:

$$f_A = k_A \frac{m_A}{R_{AB}^2} \tag{7.8}$$

where K_A is some constant. But according to Newton's Third Law of Motion, the gravitational force operating on A is equal in magnitude and is opposite in direction to the force operating on B. This force is gravitational in nature, so it too is the composite of the forces operating on B's parts. Thus, the force operating on B is:

$$f_B = k_B \frac{m_B}{R_{AB}^2} \tag{7.9}$$

From the Third Law of Motion it therefore follows that $f_A = -f_B$, which implies that the gravitational force is proportional to the product of the bodies' masses:

$$f_G = G \frac{m_1 m_2}{R_{AB}^2} \tag{7.10}$$

The second part of the third step of Newton's argument can be summarized as follows:

Argument III-2

Premise 1.	$f_m = m f_a$.	*(Structural Assumption – Conclusion III-1)*
Premise 2.	$fa \propto \frac{1}{R^2}$	*(Conclusion II)*
Premise 3.	Momentum Conservation	*(Structural Assumption)*
3.1.	Newton's Third Law of Motion	
Conclusion III-2	$f = G \frac{m_1 m_2}{R^2}$.	

Arguments I, II and III-2 all follow the DI method. These arguments use phenomenal laws and background assumptions as premises and deductive reasoning to conclude the nature of the force generating the phenomena. However, it is important to note that the background assumptions used in these arguments are of a very particular nature. Other than mathematical and phenomenal propositions they are all structural assumptions.

It seems tempting to think of these two structural assumptions, i.e., the conservation of momentum and the Rule of Composition governing the force of gravitation, as de facto hypotheses. In a heuristic sense, they are. These are theoretical propositions that are not deductively entailed by empirical claims. It is conceivable that these structural assumptions will be replaced in the future with new, more adequate assumptions. The universal validity attributed to these assumptions is not metaphysical in nature, since it may be that a more adequate structural assumption will be introduced in a future theory.

However, these structural assumptions are *not* hypotheses in the sense used by philosophers of science. First, they are not arbitrary conjectures governed solely by the imagination and luck of individual scientists. It may have required the imagination and courage of Newton to conceive of these structural assumptions and to take them as applying universally, but these assumptions are certainly not arbitrary, as we have spelled out what singles out these assumptions over others. Second, it is clear why these assumptions acquire the universal validity that is attributed to them. Unlike enumerative induction, where universal propositions link composite properties, these structural assumptions enable us to argue that we have stumbled on the properties of the ultimate parts over which it seems safer to generalize. If structural assumptions are valid, they are valid universally in both actual and counterfactual cases if it can be assumed that ultimate parts of matter are all alike. Finally, structural assumptions are not hypotheses because they are closely related to the results of experiments and observations. Only after carefully assembling all the evidence, can structural assumptions be introduced into the theory.

7.4 Newton's Scientific Method

For Newton, the conservation of momentum and the compositional nature of the gravitational force are more than just hypotheses; they are structural assumptions. Newton was well-justified in perceiving himself as deducing his universal law of gravitation from the phenomena. He was not employing in his inductive method hypotheses that function as inspired guesses. He introduced structural assumptions based on a careful procedure. First, an empirical fact universally confirmed by all observations is singled out. Then, this empirical fact is re-conceptualized as a structural assumption, in which no property restricted to a particular kind of material is utilized. Finally, the structural assumption is recognized as universally valid due to its role in making composite physical processes intelligible.

It is clear that this procedure does not depend on the sociological role of background assumptions as standardizing rules for solving scientific problems. Newton was creating a new paradigm through the introduction of structural assumptions. Once Newton started using the conservation of momentum as a universally valid rule, it gave the impetus to the generations that followed to emulate him. But it is not their currency in the eyes of his peers which gave Newton the confidence to apply these scientific propositions universally, it is their nature as structural assumptions. The universal validity of structural assumptions are derived from Newton's belief that the properties of composite systems must be constructed from properties belonging to the ultimate parts of matter.

Our analysis of Newton's argument also indicates that Popper is right in that there is no *universal* "rule of induction" that applies to all inductive arguments. The reason why Newton takes his structural assumptions to hold universally is that it seems unlikely that composite physical processes follow different rules of composition depending on the context. However, it may be that the particular theoretical and experimental context determines which structural assumption is "suggested" by the evidence. It is very possible that new structural assumptions will end up replacing older ones. Thus, these structural assumptions should be treated as local rules of inductive inference. The generation of a structural assumption follows a regulated procedure. However, one should not think that this procedure is mechanical nor is it incorrigible. Nature does not dictate to the scientist which structural assumption is implied by the phenomena, nor does it show how to read them from the phenomena. There is an element of stipulation in formulating these structural assumptions, however, this stipulation is not entirely arbitrary and is not a linguistic convention.

It is now clear that Newton had a valid method for distinguishing between hypotheses and propositions that are derived from the phenomena. However, one has to be clear that the derivation does not rely exclusively on deductive rules. The DI method gets part of the story right, but not the whole story. There is an important inductive step Newton utilizes to secure the background assumptions which serve as premises in his DI arguments. The strength of these DI arguments crucially depends on the strength of these background assumptions. Only by introducing this additional criterion can the DI method meet the challenges it faces. Without providing an account of which properties can be universalized, it is not possible for the DI method to meet the Duhemian challenge, explain the necessity and exactness of laws of nature, and introduce new theoretical terms. It cannot meet those challenges without collapsing into the HD method completely.

The Criterion of Isolation and Rule of Composition governing the quantity of motion consist of a structural assumption. In this chapter we have seen the importance of structural assumptions in Newton's scientific method. These structural assumptions provided Newton with local rules of induction with which he was able to take phenomenal laws and derive from them laws of nature. Perhaps there is hope to separate out analogous structural assumptions in more contemporary arguments that illuminate the nature of inductive inference and the success of empirical arguments in securing laws of nature that extend the strict empirical basis on which they stand. I leave such work to future research.

Chapter 8
The Special Theory of Relativity

The reconstruction of classical physics in previous chapters unveiled a conceptual relation between Galilean spacetime and Newtonian mass. Once the Galilean geometry of PUMs was assumed, the basic structure of Galilean spacetime was derived. The parameter μ_0, which was later used to reconstruct mass, was derived from an implicit spacetime symmetry. The full meaning of mass was captured when the reconstruction introduced the "classical" Criterion of Isolation and the Rule of Composition governing motions.

The analogy between Galilean and relativistic spacetimes suggests that a similar relation between relativistic spacetime symmetries and relativistic mass might be established. If the analogy holds, one should be able to derive relativistic mass from extending the spacetime symmetries implicit in the equivalence between inertial reference frames. In addition to spacetime structure, a reconstruction of relativistic dynamics should presuppose a Criterion of Isolation and a Rule of Composition for relativistic systems. This chapter will proceed with reconstructing relativistic mass, keeping in mind the theoretical complication stemming from an ambiguity in the reference of mass. In the Special Theory of Relativity, mass could be interpreted as referring either to rest mass or relativistic mass, and so one no longer has the same parameter capturing both roles of mass; as the property of inertia and as the quantity of matter. However, the analogy between classical and relativistic physics suggests that mass in relativity also has its geometric origins, and that the dynamic structures of the theory are not separable from its kinematic structures. To demonstrate these claims the next sections proceed with a reconstruction of relativistic mass from the relativistic geometry of PUMs and presuppositions governing relativistic systems.

8.1 The Expansion Factor μ_0 and Mass in STR

In Chapter 3, relativistic spacetime was derived from a relativistic geometry of PUMs. The assumption was that a PUM is projected onto a class of parallel PUMs according to the following rule:

O. Belkind, *Physical Systems,* Boston Studies in the Philosophy of Science 264,
DOI 10.1007/978-94-007-2373-3_8,

C 7. Relativistic Paradigm of Uniform Motion (**RPUM**)
$\mathbf{E}\omega\beta_1 \wedge \mathbf{E}\omega\beta_2$ and $p =< \omega, \beta_1 >, q =< \omega, \beta_2 >$
$c^2(\Delta x_0(p,q))^2 - (\Delta x_1(p,q))^2 = 0$

The solution to **C 7** is a wave function moving with the velocity of light. The solution can be generalized to a {3+1} spacetime:

$$\begin{pmatrix} \psi_{1+}(x_{0+}, x_{1+}) = e^{i\omega x_{0+} + ikx_{1+}} & \psi_{1-}(x_{0-}, x_{1-}) = e^{i\omega x_{0-} - ikx_{1-}} \\ \psi_{2+}(x_{0+}, x_{2+}) = e^{i\omega x_{0+} + ikx_{2+}} & \psi_{2-}(x_{0-}, x_{2-}) = e^{i\omega x_{0-} - ikx_{2-}} \\ \psi_{3+}(x_{0+}, x_{3+}) = e^{i\omega x_{0+} + ikx_{3+}} & \psi_{3-}(x_{0+}, x_{3+}) = e^{i\omega x_{0+} - ikx_{3-}} \end{pmatrix} \tag{8.1}$$

Define the spatial and temporal measures in three dimensions as follows:

$$\begin{aligned} x_0 &= x_{0+} + x_{0-} \\ x_i &= x_{i+} - x_{i-} = cx_{0+} - cx_{0-} \end{aligned} \tag{8.2}$$

Define the relation between the wave equation and the time and length measurements performed with clocks and rods. The stretching of a wavelength in all directions may look as follows:

$$\begin{aligned} T_{i+} &\mapsto T'_{i+} = \mu_i T_{i+} \\ T_{i-} &\mapsto T'_{i-} = \mu'_i T_{i-} \end{aligned} \tag{8.3}$$

Define:

$$\mu_0 \equiv \frac{1}{\sqrt{\mu_1 \mu'_1}} \tag{8.4}$$

To simplify discussion assume that:

$$\mu_2 = \mu'_2 = \mu_3 = \mu'_3 = \frac{1}{\mu_0} \tag{8.5}$$

The transformation in (8.3) leads to the transformation Λ between $K_R^{1111} = \langle cx_0, x_1, x_2, x_3 \rangle$ and $K_R^{\mu_0\mu_1 11} = \langle cx'_0, x'_1, x'_2, x'_3 \rangle$, so that:

$$\Lambda = \frac{1}{2} \begin{pmatrix} \mu_1 + \mu'_1 & \mu_1 - \mu'_1 & 0 & 0 \\ \mu_1 - \mu'_1 & \mu_1 + \mu'_1 & 0 & 0 \\ 0 & 0 & \frac{2}{\mu_0} & 0 \\ 0 & 0 & 0 & \frac{2}{\mu_0} \end{pmatrix} \tag{8.6}$$

Given the transformation between the coordinate system, the relative velocity between these coordinate systems can be defined. A point stationary at the origin of K_R^{1111} will transform as follows:

$$\Lambda \begin{pmatrix} cx_0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \mapsto \begin{pmatrix} (\mu_1 + \mu_1')cx_0 \\ (\mu_1 - \mu_1')cx_0 \\ 0 \\ 0 \end{pmatrix} \tag{8.7}$$

Thus the relative velocity of an object at rest in K_R^{1111} will be moving uniformly in $K_R^{\mu_0\mu_1 11}$ with the velocity:

$$\frac{v}{c} = \frac{(\mu_1 - \mu_1')}{(\mu_1 + \mu_1')} \tag{8.8}$$

A constraint will now be imposed on Λ to derive the ordinary Lorentz transformation. Assume that $\Lambda : K_R^{1111} \mapsto K_R^{\mu_0\mu_1 11}$ equals the inverse transformation, $\Lambda^{-1} : K_R^{\mu_0\mu_1 11} \mapsto K_R^{1111}$, except for the relative velocity changing signs. The transformation Λ^{-1} looks as follows:

$$\Lambda^{-1} = \frac{1}{2\mu_1\mu_1'} \begin{pmatrix} (\mu_1 + \mu_1') & (\mu_1 - \mu_1') & 0 & 0 \\ (\mu_1 - \mu_1') & (\mu_1 + \mu_1') & 0 & 0 \\ 0 & 0 & 2\mu_0 & 0 \\ 0 & 0 & 0 & 2\mu_0 \end{pmatrix} \tag{8.9}$$

Since it is assumed that $\Lambda(v) = \Lambda(-v)^{-1}$, it follows $\mu_1\mu_1' = 1$ and that $\mu_0 = 1$. The relation between the velocity and the scaling factor μ_1 would then be:

$$\beta \equiv \frac{v}{c} = \frac{\mu_1^2 - 1}{\mu_1^2 + 1} \tag{8.10}$$

The scaling factor could be defined as a function of β, so that:

$$\mu_1 = \sqrt{\frac{1+\beta}{1-\beta}} = \frac{(1+\beta)}{\sqrt{1-\beta^2}} \tag{8.11}$$

Let $\gamma \equiv \frac{1}{\sqrt{1-\beta^2}}$. The transformation $\Lambda : K_R^{1111} \mapsto K_R^{1\mu_1 11}$ then becomes:

$$\Lambda = \begin{pmatrix} \gamma & \beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{8.12}$$

which is the ordinary Lorentz transformation. In this case the subscript μ_1 describes the stretch in the wavelength of the set of PUMs that move in the x direction. Thus an arbitrary wave expansion μ_1, μ_2 or μ_3 in one of the spatial directions leads to the Lorentz transformations in one of the three spatial directions.

The assumption behind the above derivation of the Lorentz transformation is that $\Lambda(v) = \Lambda^{-1}(-v)$. To justify this part of the derivation, Einstein appealed to the Principle of Relativity, arguing that the transformations between inertial reference frames should not depend on anything but the relative velocity between inertial reference frames. However, in the reconstruction of relativistic spacetime offered in Chapter 3, the Principle of Relativity was shown to be the logical consequence of basing the geometry of spacetime on geometry of PUMs and relations between them. The Principle of Relativity does not function here as a postulate. Moreover, Einstein's appeal to the Principle of Relativity is suspect, since his articulation of the Principle of Relativity is, strictly speaking, inapplicable to the transformations between inertial reference frames. Einstein's formulation of the Principle of Relativity asserts that the laws of physics are the same *in* all inertial reference frames. But Einstein applied the Principle of Relativity to the generalized Lorentz transformations, which describe laws of transformation *between* inertial reference frames. To say that a rod will measure the same length whenever it is at rest relative to a reference frame does not follow logically from the Principle of Relativity. It is possible for a rod not to measure the same length in different frames, and for the laws of physics to be the same relative to the different frames. Instead of Einstein relying on the Principle of Relativity to justify the Reciprocity Principle, one may adopt a convention that separates expansion parameters such as μ_0 which "inflate" the reference frame from transformations in which the Reciprocity Principle holds. The following examines the generalized transformations that accept the RPUM (**C7**) as valid.

Consider the transformation $\Lambda : K_R^{1111} \mapsto K_R^{\mu_0\mu_1 11}$. The relative velocity between these reference frames is:

$$\beta = \frac{\mu_1 + \mu_1'}{\mu_1 - \mu_1'} = \frac{\mu_1^2\mu_0^2 + 1}{\mu_1^2\mu_0^2 - 1} \tag{8.13}$$

Define μ_1 as a function of β and μ_0:

$$\mu_1 = \frac{(\beta + 1)}{\mu_0\sqrt{1-\beta^2}} = \gamma\frac{(1+\beta)}{\mu_0} \tag{8.14}$$

With this equality, the transformation in (8.9) reduces to:

$$\Lambda = \begin{pmatrix} \frac{\gamma}{\mu_0} & \frac{\gamma\beta}{\mu_0} & 0 & 0 \\ -\frac{\gamma\beta}{\mu_0} & \frac{\gamma}{\mu_0} & 0 & 0 \\ 0 & 0 & \frac{1}{\mu_0} & 0 \\ 0 & 0 & 0 & \frac{1}{\mu_0} \end{pmatrix} \tag{8.15}$$

To make the significance of (8.15) clear, consider first the transformation Λ : $K_R^{1111} \mapsto K_R^{\mu_0 111}$. In case the relative velocity between the two reference frames is $v = 1$, the transformation reduces to:

$$\Lambda = \begin{pmatrix} \frac{1}{\mu_0} & 0 & 0 & 0 \\ 0 & \frac{1}{\mu_0} & 0 & 0 \\ 0 & 0 & \frac{1}{\mu_0} & 0 \\ 0 & 0 & 0 & \frac{1}{\mu_0} \end{pmatrix} \tag{8.16}$$

Assume that a body is at rest in K_R^{1111}, so that it has the following trajectory:

$$\mathbf{x}(cx_0) = \begin{pmatrix} cx_0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{8.17}$$

The four-velocity of this body is then:

$$\mathbf{u}(cx_0) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{8.18}$$

The trajectory of the body in $K_R^{1\mu_1 11}$ is then:

$$\mathbf{x}'(cx_0) = \begin{pmatrix} cx_0\gamma \\ -cx_0\gamma\beta \\ 0 \\ 0 \end{pmatrix} \tag{8.19}$$

Thus the four-velocity of the body in $K_R^{1\mu_1 11}$ is:

$$\mathbf{u}'(cx_0) = \begin{pmatrix} \gamma \\ -\gamma\beta \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{8.20}$$

Thus the body moves with velocity $-\beta$ relative to $K_R^{1\mu_1 11}$ when it is at rest in K_R^{1111}.

Now consider the trajectory of the body in $K_R^{\mu_0\mu_1 11}$:

$$\mathbf{x}''(cx_0) = \begin{pmatrix} \frac{\gamma c x_0}{\mu_0} \\ -\frac{\gamma\beta c x_0}{\mu_0} \\ 0 \\ 0 \end{pmatrix} \tag{8.21}$$

This implies that the body has the following four-velocity in $K_R^{\mu_0\mu_1 11}$:

$$\mathbf{u}''(cx_0) = \begin{pmatrix} \frac{\gamma}{\mu_0} \\ -\frac{\gamma\beta}{\mu_0} \\ 0 \\ 0 \end{pmatrix} \tag{8.22}$$

The velocity of the body relative to $K_R^{\mu_0\mu_1 11}$ is still $-\beta$. Thus $K_R^{1\mu_1 11}$ and $K_R^{\mu_0\mu_1 11}$ are indistinguishable with regards to velocity measurements. The frames only disagree with regards to a factor μ_0 that multiplies both the spatial and the temporal metrics. The two frames use exactly the same measuring rods and clocks, but they differ with regards to the significance of their measurements. If each trajectory is assigned an arbitrary μ_0 parameter, associated with the degree of freedom each trajectory has in terms of the appropriate frame $K_R^{\mu_0\mu_1 11}$, it is clear that the four-velocity needs to transform from the measured four-velocity $\mathbf{u}''(cx_0) = \langle\gamma, -\gamma\beta, 0, 0\rangle$ to the "standardized" four-velocity $\mathbf{u^0}(cx_0) = \langle\mu_0\gamma, -\mu_0\gamma\beta, 0, 0\rangle$. Thus, it follows that μ_0 can be identified with the rest mass of the body, since the energy and momentum of the body can now be defined as follows:

$$E = \mu_0\gamma \;\; p = \mu_0\gamma\beta \tag{8.23}$$

One can also see that $\mu_0 = \sqrt{E^2 - p^2}$, which is what we expect from multiplying the length of the velocity four-vector by a factor μ_0.

The energy-momentum vector is simply an alternative way to represent the four-velocity of a body, and the two representations reflect a non-determined choice of reference frames; either $K_R^{1\mu_1 11}$ or $K_R^{\mu_0\mu_1 11}$ are coherent with the actual devices used to measure the trajectories of bodies. Since velocity measurements and coordinate systems are physically indistinguishable for these frames, one assigns the value μ_0 to each body based on the discrepancy between standard and non-standard representations of velocity, in $K_R^{1\mu_1 11}$ and $K_R^{\mu_0\mu_1 11}$ respectively. Thus the energy-momentum is a rescaled, "standardized" representation of motion, and does not differ physically from the ordinary four-velocity of a body.

In previous chapters the Criterion of Isolation and Rule of Composition were articulated. According to this rule, the momentum of the composite body is the sum of the quantities of motion of the parts. Once this rule was articulated the various expanded/contracted Galilean reference frames $K_R^{\mu_0 111}$ were differentiated. When various trajectories of different bodies are treated as interacting components of a physical system, various expansion parameters are attributed to bodies. Consider now the Rule of Composition in the relativistic context:

Rule of Composition. *The standardized four-velocity of a composite system is the sum of the standardized four-velocities of the parts.*

Assume that the velocities of n subsystems, $p_1, p_2, \ldots, p_n$ are measured as $\mathbf{u}_1$, $\mathbf{u}_2, \ldots, \mathbf{u}_n$. One does not know which reference frame each of these velocities were measured in, so that there is an infinite range of possible reference frames $K_R^{\mu_{01}\mu_{11}11}, K_R^{\mu_{02}\mu_{12}11}, \ldots, K_R^{\mu_{0n}\mu_{1n}11}$. The "standardized" velocities of each subsystem in K_R^{1111} is:

$$\mathbf{u}_i^0 = \begin{pmatrix} \mu_{0i}\gamma_i \\ \mu_{0i}\gamma_i\beta_i \\ 0 \\ 0 \end{pmatrix} \tag{8.24}$$

The four-velocity $\mathbf{u}_\Pi$ of the composite body that comprises the two subsystems may now be considered:

$$\mathbf{u}_\Pi^0 = \sum \mathbf{u}_i \tag{8.25}$$

Assume, moreover, the relativistic version of the Criterion of Isolation:

Criterion of Isolation. *A physical system is isolated if and only if it instantiates a PUM.*

If the overall system is isolated from the rest of the world, the implication is that the four-velocity of the composite system is constant, so that

$$\frac{d\mathbf{u}_\Pi^0}{dx_0} = 0 \tag{8.26}$$

The consequence is the conservation of energy and momentum for $p_1, p_2, \ldots, p_n$, so that:

$$\begin{aligned} E_{tot} &= \textstyle\sum_i \mu_{0i}\gamma_i \\ p_{tot} &= \textstyle\sum_i \mu_{0i}\gamma_i\beta_i \end{aligned} \tag{8.27}$$

One can also derive from the Rule of Composition the additive nature of relativistic mass $m_R = \mu_0\gamma$, so that an expanded reference frame $\mu_{0\Pi}$ can be associated with the composite system comprised of $p_1, p_2, \ldots, p_n$. Take for example a composite system comprising of two subsystems p_1 and p_2. One may begin the analysis in the reference frame K_R^{1111}. The four-velocities of the two systems are then

$\mathbf{u}'_1 = \langle \gamma_1, \gamma_1\beta_1, 0, 0\rangle$ and $\mathbf{u}'_2 = \langle \gamma_2, \gamma_2\beta_2, 0, 0\rangle$. These velocities are measured in $K_R^{\mu_{01}111}$ and $K_R^{\mu_{02}111}$. To find the standardized velocities of the two subsystems, the velocities are multiplied by μ_{01} and μ_{02}, so that $\mathbf{u}_1^0 = \langle \mu_{01}\gamma_1, \mu_{01}\gamma_1\beta_1, 0, 0\rangle$ and $\mathbf{u}_2^0 = \langle \mu_{02}\gamma_2, \mu_{02}\gamma_2\beta_2, 0, 0\rangle$. The standardized four-velocity $\mathbf{u}_\Pi^0$ of the composite system can now be calculated, since it is the sum of the standardized velocities of the parts:

$$\mathbf{u}_\Pi^0 = \begin{pmatrix} \mu_{01}\gamma_1 + \mu_{02}\gamma_2 \\ \mu_{01}\gamma_1\beta_1 + \mu_{02}\gamma_2\beta_2 \\ 0 \\ 0 \end{pmatrix} \tag{8.28}$$

According to the Criterion of Isolation, the velocity of an isolated composite system is constant. So there exists a frame $K_R^{1\mu_1 11}$ in which the four-velocity of the composite system is $\mathbf{u}'_\Pi = \langle 1, 0, 0, 0\rangle$. Thus, if the relative velocity between $K_R^{1\mu_1 11}$ and K_R^{1111} is β_Π, the four-velocity of the composite system in K_R^{1111} is:

$$\mathbf{u}_\Pi^0 = \begin{pmatrix} \mu_{0\Pi}\gamma_\Pi \\ \mu_{0\Pi}\gamma_\Pi\beta_\Pi \\ 0 \\ 0 \end{pmatrix} \tag{8.29}$$

Thus we see that the Rule of Composition for four-velocities implies the additive nature of relativistic mass, since:

$$\gamma_\Pi\mu_{0\Pi} = \gamma_1\mu_{01} + \gamma_2\mu_{02} \tag{8.30}$$

In case $\gamma_\Pi = 1$ – or in the rest frame of the composite system – the above relation reduces to:

$$\mu_{0\Pi} = \gamma'_1\mu_{01} + \gamma'_2\mu_{02} \tag{8.31}$$

Thus in general, the expansion parameter μ_0 of the composite system is more than the sum of the expansion parameters μ_0 of the parts.

This section completes our reconstruction of relativistic equations of motion. A geometry of PUMs could be used to derive the equivalence between inertial reference frames. An implicit symmetry μ_0 was uncovered in the spacetime structure which allowed us, together with the Criterion of Isolation and the Rule of Composition for relativistic systems, to derive the conservation of the relativistic four-momentum and the concept of rest mass. Thus the analogy between the reconstruction of Newtonian mass and the reconstruction of rest mass in relativity is fully established. In both cases, the mass of a body could be traced to reference frames analogous to inertial reference frames, in which the spacetime is expanded or contracted by a factor of μ_0. The additive nature of mass is derived from a Rule of Composition governing the motions of parts of physical systems.

8.2 A New Interpretation of Mass in STR

The derivation in the previous section allows for a new interpretation of mass in the context of STR. According to this interpretation, mass has both a geometric origin and a connection to structural assumptions regarding the nature of physical systems.

The transition from Newtonian to relativistic physics is ordinarily understood as resulting from the consolidation of space and time into a single spacetime structure. According to this view, the objective quantity in relativistic physics is the interval between two events, and spatial and temporal measures are non-invariant properties; i.e., they are quantities defined only relative to observers. This view treats rest mass as an objective (i.e., invariant) property which does not depend on the observer. Relativistic mass, on the other hand, is taken to be dependent on the observer, i.e., it is not an objective property. Traditional interpretations of STR therefore tend to discredit the additive role of mass in relativity. This reading insists that in STR, the essential meaning of mass is captured by its inertial role. Relativistic physics seems to continue Mach's critical stance towards quantity of matter, since rest mass is no longer conserved in relativity.

The interpretation of mass introduced here is able to capture both roles of mass; in its roles as the property of inertia and the quantity of matter. These aspects of mass are derived from the geometry of PUMs and structural assumptions about motions. The two aspects of mass are consequences of the deeper structure of physical theory, which only admits moving parts and moving wholes.

The interpretation of mass offered here also solves interpretive problems in the context of STR. Some of these are dealt with in the next section. Our brief remarks will be consolidated according to various claims made in the literature about Newtonian and relativistic mass. First, I consider the claim that Newtonian and relativistic mass are incommensurable concepts. Second, I consider the claim that the reference of Newtonian mass is indeterminate, and that relativistic concepts capture some of the meaning of Newtonian mass but not the full meaning of the concept. Third, I consider the claim that frame-invariant properties such as rest mass and spacetime intervals are "objective" properties, while frame-dependent properties such as velocity, momentum and energy are "subjective," or depend somehow on the observer. Fourth, I consider the claim that Einstein's equation $E = mc^2$ is indicative of a metaphysical thesis about the nature of mass and energy. Since there are, supposedly, interactions in which energy is converted to mass (and vice versa), mass and energy are often taken to be different manifestations of the same material substance.

8.2.1 Kuhn's Thesis of Incommensurability

The new interpretation of relativistic mass offered here highlights the similarities and differences between the concepts of mass in Newtonian physics and STR. The analogous role of the expansion parameter μ_0 in both theories suggests a conceptual continuity between them. This semi-geometric interpretation of mass conflicts with claims made by Kuhn (1962) in his influential work, *The Structure of Scientific Revolutions*. According to Kuhn, the shift from Newtonian physics to relativity is a prime example of a paradigm shift. According to him, the change from Newtonian to relativistic mass is not the evolution of a concept, but a replacement of one concept with another bearing a similar name but no commensurable meaning. I shall not delve here into the general problem of incommensurability, but will provide a few remarks comparing the notion of physical systems analyzed here and Kuhn's notion of a paradigm. The claim here is that presuppositions regarding the nature of physical systems cannot receive direct confirmation from experience, since these presuppositions are relied upon in interpreting experience. Presuppositions regarding the structure of physical systems provide localized rules of induction for interpreting and evaluating phenomena. The notion of physical systems and its fundamental structure therefore seems to function as the core of a physical paradigm.

One way to recognize the central role of structural assumptions (i.e., assumptions regarding the structure of physical systems), is to consider the role of conservation of momentum and energy in interpreting the phenomena. In any experimental situation where one attempts to study the causal interaction between subsystems A and B, one must find ways to isolate the two subsystems from the rest of the world. However, in order to isolate the interacting subsystems a Criterion of Isolation is required, which will articulate a property of the two subsystems that identifies them as isolated (or at least separates known external causes from those causes that flow from A to B and vice versa). To infer that a system is isolated, both in theory and in practice, there is only one criterion available, and that is that the momentum and energy of the composite system comprising of A and B is conserved. Thus, when the nature of causal interactions is first investigated, one must assume the universal validity of energy and momentum conservation laws.

The upshot of these considerations is that some assumptions about the nature of physical systems have to be in place before causal laws can be derived from the phenomena. Chapter 7 described the role of conservation of momentum and the compositionality of gravitation in deriving the Universal Law of Gravitation from the phenomena. However, while the Criterion of Isolation is valid a priori, in the sense that this criterion is necessary for interpreting the phenomena, one needs to be careful in drawing philosophical conclusions from its universal validity. First, there is the misconception that a priori propositions are unrevisable, i.e., that they carry metaphysical necessity and are true in all possible worlds. According to this reading, a priori assumptions about physical systems are analogous to logical truths or to linguistic conventional definitions. However, the replacement of Newtonian structural assumptions with relativistic assumptions demonstrates that structural assumptions are revisable without being merely conventional; relativistic assumptions are empirically more adequate than Newtonian assumptions. The difference between an ordinary empirical statement and a structural assumption, however, is that the empirical adequacy of structural assumptions is not evaluated directly vis-à-vis specific observations, but is evaluated holistically, in terms of the complete set of empirical claims one takes to be true. Various virtues are involved in making such a holistic evaluation, including simplicity and adequacy in multiple domains of verification.

According to Newton, structural assumptions acquire natural necessity, given that they provide the rules for reducing composite properties into the properties of the ultimate parts of matter. A structural assumption provides the *means* of reducing composite properties. Thus, if all observations can be explained with a single rule for reducing composite descriptions, one has legitimate reasons for universalizing the structural properties and to take the laws of nature that they support to be universally valid.

The a priori nature of structural assumptions does not imply that Newtonian and relativistic physics involve conceptual schemes with altogether incommensurable *meanings*. If our reconstructions of Newtonian and relativistic physics are to be trusted, one can detect the analogies between the varying structural assumptions while recognizing that they amount to different standards for evaluating phenomena. One can recognize the analogy between structural assumptions governing Newtonian systems and those governing relativistic systems, even though one cannot apply both sets of assumptions at the same time to the same phenomena and derive comparable results. Thus, even though there might be different conceptions of physical systems that lead to incommensurable *practices* of measurement and induction, there need not be incommensurability at the level of *meaning*.

Thus structural assumptions can be taken to be core assumptions of physical paradigms. To revise a physical paradigm, one needs to revise core assumptions that involve the whole field of physical knowledge. However, there is no support for Kuhn's Thesis of Incommensurability. Kuhn went too far when he argued that scientists operating in different paradigms live in altogether different worlds.

8.2.2 Field's Indeterminacy of Reference

The framework introduced here for reconstructing mass in Newtonian physics and relativity provides a useful conceptual setting for exploring the role "mass" plays in both theories. A common reading of the transition from Newtonian to relativistic mass argues that there is ambiguity or indeterminacy regarding to true reference of mass. One cannot single out a single aspect of the concept of "mass" in relativity and view it as the analog of Newtonian mass. According to this interpretation, there are two mass concepts in relativity, rest mass and relativistic mass. Each of these concepts captures part of the meaning of mass in Newtonian physics. Thus, the question whether mass has the same meaning in the two theories is unanswerable; the reference of mass in Newtonian physics is indeterminate, and there is no clear, unambiguous analog to Newtonian mass in STR.

Hartry Field (1973) utilized the notion of mass in order to support a semantic theory about the reference of concepts. Some semantic theories argue that the reference of singular terms is determined through the descriptions in which they appear or implicitly replace. Since descriptions of objects are made possible via the vocabulary of the language used to describe the objects, the reference of theoretical terms also shifts whenever radical changes in scientific theories take place. According to Field, during scientific revolutions concepts undergo a process of refinement. Whenever a radically new scientific theory is introduced, determinate references arise out of indeterminate ones. Before Einstein's STR, claims Field, the concept of Newtonian mass partially denoted two different properties. In relativity the two partial referents became explicit and were split into two different significations. The ambiguous referent of mass in Newton's theory, became unambiguous in STR when two concepts were introduced; relativistic mass, defined as $M_R = \frac{E}{c^2}$, and proper mass (or rest mass) defined as $m_0 = \frac{E-E_k}{c^2}$, where E is the total energy of the body and E_k is the kinetic energy. Relativistic mass is the product $m_0\gamma$.

In some sense, the reconstructions of Newtonian and relativistic mass introduced here echo Field's interpretation. Two conceptual roles were identified for Newtonian mass: as the property of inertia and as the quantity of matter. While the same parameter m holds the two roles of mass in Newtonian physics, in relativity the rest mass m_0 holds the role of inertial mass, and relativistic mass m_R holds the role of quantity of matter. Thus there is truth to Field's assertion that two different parameters capture the significance of Newtonian mass in STR. Nevertheless, despite the overlap between the conclusions reached here and Field's conclusions, the inferences which led to these conclusions are quite different. There is nothing indeterminate or ambiguous about the reference of Newtonian mass. Given the centrality of the concept in both Newtonian and relativistic physics, it is not surprising that the concept fulfills various conceptual roles. Nevertheless, this doesn't imply that the reference of mass is indeterminate, or that during the replacement of Newtonian mass with relativistic mass the reference of the concept or our understanding of it has sharpened. Our understanding of *nature* has improved, not the coherence or determinateness of the concepts used to describe it. The idea that Newtonian mass somehow captures indeterminately the meaning of both rest mass and relativistic mass is based on viewing Newtonian mass as if it must have been a "precursor" to relativistic concepts. One should instead recognize the analogies between the Newtonian and relativistic concepts based on an internal examination of both theories.

To support his claim about the indeterminacy of reference, Field analyzes the semantic properties of the concept of mass. According to Field, the reference of two terms is the same if the replacement of one term by the other does not change the truth value of any statement in which they appear. To determine whether Newtonian mass m_N has the same reference as m_0 or m_R, one has to analyze the set of true statements containing the term m_N. If replacing m_N with another term leaves the class of true statements in tact, then the two terms have the same reference. But some statements remain true when m_N is replaced with m_0 and are made false when replaced with m_R. Yet other statements remain true when m_N is replaced with m_R and are made false when replaced with m_0. Thus, Field concludes that m_N partially refers to the same property that m_0 refers to and partially refers to the property that m_R refers to. Take for example the following list of statements:

S1. The mass m_N of an object is between 1.21 and 1.22 kg (measured in the rest frame).
S2. To accelerate a body uniformly between any pair of different velocities, more force is required if the mass m_N of the body is greater.
S3. $P = m_N v$
S4. m_N is invariant in all reference frames.

The statement **S1** and **S2** remain true when m_N is replaced with either m_0 or m_R. The statement **S3** remains true when m_N is replaced with m_R, but is made false when replaced with m_0. And finally, the statement **S4** remains true when m_N is replaced with m_0 and is made false when replaced with m_R.

Since all statements containing the "mass" term fall into one of three categories (determined by which substitution conserves the truth of the statement), Field concludes that the term m_N ambiguously refers to both properties referred to by m_0 and m_R. Field's semantic analysis supposedly sidesteps the problem of placing the terms of the theory within a particular interpretation. However, the notion that one can examine statements in which mass appears, and then consider whether replacing mass preserves the truth value of the statements, presupposes that mass is some monadic property instantiated by objects. According to this interpretation, there is a sharp division between spatiotemporal relations and other material properties. The description of material properties is made independently of the background spacetime in which these material properties are instantiated.

Given the interpretation of mass as an inherent property of objects, it seems appropriate to examine statements **S1–S4** as if they are true independently of the spacetime structure of the theory. The statement **S1** assumes that the mass of a body can be measured with the help of some measurement procedure. The statement is then qualified by the claim that measurements have to be made in the rest frame. Thus even the simplest procedure for measuring mass has to include an implicit reference to spacetime geometry, a reference Field assumes is irrelevant to the truth of the statement. The statements **S3** and **S4** are formulated in abstraction of the relevant spacetime geometries. However, the concept of momentum or invariance of mass is meaningless independently of the spacetime structure and the transformations between inertial reference frames. Field bases his semantic analysis by bracketing the spacetime geometry, and ignoring the possible relation between material properties and spacetime structure. It is exactly what Field excludes which gives us the precise meaning of mass.

The reconstruction introduced here implies a conceptual link between spacetime structure and the property of mass. The inertial role of mass receives a semi-geometric interpretation, which suggests that mass can be viewed as an expansion parameter necessary for the translation of measured accelerations to "standardized" reference frames. The notion that Newtonian mass indeterminately, or confusedly, refers to rest mass or relativistic mass is based on the assumption that mass is a primitive property, not analyzable in reference to other concepts of the theory. But we have seen that there is nothing confused or indeterminate about the reference of Newtonian mass. Once Galilean spacetime is reconstructed from a geometry of PUMs, it is made apparent that an expansion parameter μ_0 correlates between the acceleration of a body, measured in the frame $K_G^{\mu_0 111}$, and the standardized acceleration, measured in K_G^{1111}. The geometry of PUMs together with structural assumptions renders determinate the inertial role of mass. On the other hand we have seen that the Rule of Composition governing quantities of motion implies the additive nature of mass, or its role as quantity of matter. This renders determinate the conceptual role of mass as a quantity of matter. The same two roles have split into rest mass and relativistic mass in STR: but the splitting of conceptual roles is not a disambiguation or a sharpening of a concept.

It is not possible to bracket the spacetime geometry from true statements about mass. Just as much as one needs to relate our measurements to a particular inertial reference frame in order for them to be meaningful, so does one need to assign a mass parameter to a body whenever one replaces a measured acceleration with its "proper" representation. This explains why no theory presupposes that a particle's mass spontaneously changes throughout its life.

Earman and Fine (1977) argued similarly against Field that his semantic analysis does not take into account the spacetime context of each theory. They based their argument on a four-dimensional, intrinsic (i.e., coordinate-free) formulation of the laws of motion in each theory.

Earman and Fine's formulation creates a strong analogy between Newtonian mass and rest mass. However, this analogy does not vitiate the role of Newtonian mass as a quantity of matter. Assume one has measured the velocity of an object in the rest frame in Galilean spacetime or in relativistic spacetime, so that the velocity of an object is measured in K_G^{1111} or in K_R^{1111}. The measured velocities in the Galilean geometry of PUMs cannot distinguish between the various reference frames $K_G^{\mu_0 111}$. Similarly, in the relativistic geometry of PUMs one cannot distinguish between the various reference frames $K_R^{\mu_0 111}$. Thus, the four-velocity of an object can be represented as:

$$\begin{array}{ll} K_G^{\mu_0 111} & K_R^{\mu_0 111} \\ \mathbf{u} = (1, v_1, 0, 0) & \mathbf{u} = (1, v_1, 0, 0) \end{array} \tag{8.32}$$

In the standardized frames, the four-velocities of each individual body and of the composite body are represented as follows:

$$\begin{array}{ll} K_G^{1111} & K_R^{1111} \\ \mathbf{u}^0 = \mu_0(1, v_1, 0, 0) & \mathbf{u}^0 = m_R(1, v_1, 0, 0) \\ \mathbf{u}_\Pi^0 = \left(\sum_i \mu_{0i}, \sum_i \mu_{0i} v_{1i}, 0, 0\right) & \mathbf{u}_\Pi^0 = \left(\sum_i \mu_{0i}\gamma_i, \sum_i \mu_{0i}\gamma_i v_{1i}, 0, 0\right) \end{array} \tag{8.33}$$

Together with the Criterion of Isolation, the last equation in the left column implies the conservation of mass and momentum in the context of Newtonian physics.

In the context of relativity, in the right-hand column, the last equation implies the conservation of relativistic mass or energy and momentum.

From the conservation of energy and momentum one can derive the laws of motion governing a single body:

$$\begin{array}{ll} K_G^{1111} & K_R^{1111} \\ \mathbf{u}^0 = m_N(1, v_1, 0, 0) & \mathbf{u}^0 = m_R(1, v_1, 0, 0) \\ \mathbf{a}^0 = m_N(0, a_1, 0, 0) & \mathbf{a}^0 = \dot{m_R}(1, v_1, 0, 0) + m_R(1, a_1, 0, 0) \end{array} \tag{8.34}$$

There seems to be a disanalogy between relativist and Newtonian mass given the dissimilarity in form in (8.34) between the equation for the force involving relativistic mass, and the equation for the force involving Newtonian mass. But one can represent the relativistic four-momentum as follows:

$$\mathbf{u}^0 = m_0(\gamma, \gamma v_1, 0, 0) \tag{8.35}$$

Define $\tau = \frac{x_0}{\sqrt{1-\beta^2}} = \gamma x_0$, in the proper-time frame the force-law would look like the one in Newtonian physics:

$$\begin{aligned} \mathbf{u}_\tau^0 &= m_0(1, v_1, 0, 0) \\ \mathbf{a}_\tau^0 &= m_0(0, a_1, 0, 0) \end{aligned} \tag{8.36}$$

Unlike the momentum and force equations in (8.34), these latter formulas obtain the same form as the laws in Newtonian physics. Thus one is tempted to equate Newtonian mass with rest mass.

However, there are similarities between Newtonian and relativistic mass; the most important one is depicted in (8.33). The equations in (8.33) describe the fundamental relation between parts of a physical system and their composite, thus this relation is no less important than the force-law describing the trajectory of individual bodies.

Thus one may conclude that mass has two conceptual roles, both in Newtonian physics and STR; as the property of inertia and as the quantity of matter. The interpretive drive to view one of these roles as essential to the meaning of the concept stems in part from the attempt to reduce mass into an inherent property of bodies. A semantic test shows that Newtonian mass fulfills both conceptual roles in Newtonian physics, while rest mass fulfills the role of the property of inertia in relativity and relativistic mass fulfills the role of quantity of matter. Under the pressure of viewing mass as an inherent property, it then seems as if the quantity of matter is not an essential part of the meaning of the concept. Thus, commentators since Mach's time take quantity of matter to be a non-essential feature of mass, and they tend to take the similarity between Newtonian mass and relativistic mass as accidental. However, without the compulsion to reduce mass to an inherent property of matter, one may recognize that both roles of mass are derived from the geometry of PUMs and from structural assumptions, and neither role is more fundamental than the other. Moreover, thinking of inertial mass as an inherent property, which is causally responsible for the inertial tendencies of bodies, obscures the geometric origins of this concept and makes the relation between dynamic laws and geometry opaque. Thus we should view both roles of mass as derived aspects of the geometry of PUMs and structural assumptions.

8.2.3 Invariance as a Mark of Objectivity

There are some interpretive stumbling blocks on the way to attributing relativistic mass the role of quantity of matter. The first difficulty is the claim by some commentators that relativity simply does not have the correlate of the Newtonian quantity of matter. Another difficulty is the claim that only properties that are frame-invariant in relativity are objective properties. Thus while rest-mass is invariant and therefore objective, relativistic mass is frame-dependent and therefore observer-dependent.

The notion that invariance is a mark of objectivity has a long and venerable tradition, going back to neo-Kantian attempts in the late nineteenth century to separate the subjective from the objective components of scientific knowledge. In assimilating STR, many physicists – starting with Einstein himself – often differentiated between properties whose values depend on the inertial reference frame, from properties that are the same in all reference frames, and are therefore invariant. Thus, for example, while velocity and simultaneity are frame-dependent, the spacetime interval and rest mass are invariant. The frame-dependent properties are "subjective," and depend on the state of the observer, while invariant properties are frame-independent, and therefore "objective" and do not depend on the observer. This line of thinking is very much influenced by Minkowski's interpretation of spacetime. Minkowksi compared the Lorentz transformations to the Euclidean group. In Euclidean space, the particular breakdown into a coordinate system of a spatial figure has features that depend on the mode of representation; but the distance between points in Euclidean space is an invariant of the Euclidean group, and therefore an objective relation between any pair of points. Similarly, the particular inertial reference frame yields different representations of the temporal and spatial dimensions of physical processes, but the spacetime interval is an objective relation between pairs of spacetime points.

A recent example for this line of reasoning is Lange (2001). Lange relies on the distinction between frame-dependent and invariant properties to argue against the notion that energy and mass are different manifestations of the same substance. Since energy is frame-dependent, it must be non-objective and different in kind from mass. To substantiate this view Lange argues that:

> . . . under a standard interpretation, relativity theory denies the objective reality of various properties that we ordinarily assign to material bodies (such as their length and velocity) and to events (such as their separation in space and their separation in time). Each of these quantities is *frame-dependent*; none is "Lorentz invariant" – that is, the same in every inertial frame of reference. Only what is the same in every inertial frame is a genuine

> feature of reality. The value that any frame-dependent quantity assumes in a given inertial frame reflects not just reality, but also that reference frame's own particular perspective. The Lorentz invariant quantities are exactly those which depend only on how the universe really is, uncontaminated by any contribution from us in describing the universe. (Lange, 2001, p. 225)

The notion that the value of a property that is frame-dependent is somehow "infected" by the subjective nature of the inertial reference frame, is widespread. Most commentators take from this notion the idea that as much as the velocity of an object depends on the state of the object, it also depends on the velocity of the objects used as reference. Similarly, the amount of time elapsed while a process takes place is not the same for all observers, and is therefore long or short depending on the inertial reference frame. However, while most commentators are satisfied with emphasizing the non-objective nature of velocity, time and length, Lange takes the invariance view of objectivity further. He uses the distinction between frame-invariance and frame-dependence as a line distinguishing between "real" and "illusory," or mind-dependent properties. Since relativistic mass is frame-dependent, it is also not real.

As such, relativistic mass is not a proper candidate for mass, despite being an additive property:

> Because the so-called "relativistic mass" is not an invariant quantity (and the term "rest mass" refers back to "relativistic-mass") the best thing to do in order to avoid confusing frame-dependent properties with invariant ones is just to avoid the terms "relativistic mass" and "rest mass", and instead to stick solely with "mass" for the invariant quantity symbolized m. (Lange, 2001, p. 227)

Thus Lange's argument is that relativistic mass is a subjective property, tinged by the reference frame used to define it, and is therefore not a serious candidate for being mass, the objective property that characterizes substances. On the other hand, rest mass is not an additive quantity, and is therefore not a quantity of matter.

> In classical physics, a body's mass is often interpreted as the amount of some "stuff" (matter) of which the body is made. In relativity, however, this interpretation cannot be correct. That is because properties that represent the quantity of some "substance" must obey the following principle: the total quantity of some sort of stuff in a system is the sum of the various quantities of that stuff belonging to the system's parts (where those parts are finite in number, nonoverlapping, and together exhaust the system). For example, since the density of a whole is not the sum of the densities of the parts, density does not measure the amount of some stuff, and similarly for temperature and velocity. On the other hand, mass in classical physics is "additive" in this way. But in relativity, mass is not additive. (Lange, 2001, p. 229)

Thus for Lange, while Newtonian mass is an additive quantity, rest mass is not additive in relativity and thus there is no quantity of matter in relativity.

The concept of rest mass is indeed not additive. Relativistic mass is defined as follows:

$$m_{0\Pi}\gamma_{\Pi} = \sum_i m_{0i}\gamma_i \tag{8.37}$$

In the rest frame of the composite system $\gamma_\Pi = 1$ and under the assumption that $\beta_i \ll 1$ one may derive:

$$m_{0\Pi} \simeq \sum m_{0i} + \sum \frac{1}{2} m_{0i} \beta_i^2 \tag{8.38}$$

Rest mass is therefore not additive; the rest mass of the composite system is more than the sum of the rest masses of the components if they have kinetic energy. According to Lange, the meaning of mass should therefore be reduced to its inertial role:

> If a body's mass is not its total quantity of some sort of stuff of which it is made, what is a body's mass? A body's mass is the property it possesses which determines the acceleration it undergoes in response to a force: $F = p'$ (as in classical physics), where in relativity, $p = m\gamma v$ and p' is the rate at which p is changing. When referring to a body's "mass," then, we must be thinking of that body as a thing that can feel a force and respond to it (by moving) as a *unit*. (Lange, 2001, p. 231)

Thus for Lange the inertial role of mass constitutes its essential meaning, and there is no "objective" candidate for a quantity of matter.

But mass has a dual role in Newtonian physics, as the property of inertia and as the quantity of matter. The fact that rest mass does not capture the role of quantity of matter does not imply that relativity has no quantity of matter. One may stipulate that the meaning of mass should be identified with either inertial or quantity of matter, but there is no argument that can be given to prefer one meaning over another. What really seems to drive Lange's argument is the notion that both relativistic mass and energy are frame-dependent, and are therefore not objective properties.

However, the notion that invariance is a mark of objectivity is taken here to unwarranted conclusions. The distinction between frame-dependent and invariant properties should not be viewed as the line that demarcates between subjective and objective properties. Rather, the difference should indicate to us measurements that are *scale*-dependent, vs. measurements that are not. The use of different scales to represent the same property does not make it less objective, it only makes the number representing it sensitive to the coordinate system being used. In all reference frames the relativistic mass of the composite system is the sum of the relativistic masses of the non-overlapping parts, thus there is nothing subjective or observer-dependent about the role of relativistic mass as a quantity of matter. The relativistic mass describes the amount of "stuff" there is in a system. Relativistic mass is not "less real" than rest mass just because it is a frame-dependent property.

While relativistic mass is the quantity of matter in the relativistic context, one should keep in mind that the quantity of matter is a logical consequence of the Rule of Composition governing the momentum of a system. The conservation of relativistic mass and the conservation of momentum are not independent. If one assumes that momentum is conserved in various reference frames, then the conservation of relativistic mass follows. To see this consider the frame K_R^{1111} in which the total momentum of a system is conserved:

$$\mathbf{P}_{tot} = \sum_i \gamma_i m_{0i} v_i \tag{8.39}$$

In a frame of reference $K_R^{1\mu_1 11}$ which is moving relative to K_R^{1111} with a velocity $-v$, one may also articulate the conservation for the total momentum:

$$\mathbf{P}'_{tot} = \sum_i \gamma'_i m_{0i} v'_i \tag{8.40}$$

The transformed velocities can be calculated using the relativistic rule for adding velocities:

$$v'_i = \frac{v_i + v}{1 + v_i v} \tag{8.41}$$

The transformed γ's are:

$$\gamma'_i = \frac{1}{\sqrt{1 - (v'_i)^2}} = \frac{1}{\sqrt{1 - \frac{(v_i+v)^2}{(1+v_i v)^2}}} = \frac{1 + v_i v}{\sqrt{\left(1 - v_i^2\right)\left(1 - v^2\right)}} = \gamma_i \gamma \left(1 + v_i v\right) \tag{8.42}$$

Replacing (8.41) and (8.42) into (8.40), one gets:

$$\mathbf{P}'_{tot} = \sum_i m_{0i} \gamma_i \gamma (v_i + v) = \gamma \mathbf{P}_{tot} + \gamma v \sum_i \gamma_i m_{0i} \tag{8.43}$$

One may conclude that, if momentum is conserved in every reference frame, both $\mathbf{P}_{tot}$ and $\mathbf{P}'_{tot}$ are conserved. The conclusion is that the relativistic mass

$$m_{R\Pi} = \sum_i \gamma_i m_{0i} = \sum_i m_{Ri} \tag{8.44}$$

is conserved.

If momentum is conserved in all reference frames, it logically follows that relativistic mass is conserved. Thus, the conservation of quantity of motion implies the conservation of relativistic mass, and the two statements are not logically independent.

If the conservation of momentum and the conservation of relativistic mass are interdependent concepts, one cannot isolate the Rule of Composition that is valid for the dynamic laws (i.e., conservation of quantity of motion), from the Rule of Composition that is valid for material properties (i.e., conservation of mass). The two are conceptually interconnected. Thus, the idea that there is one rule for composing the dynamic evolutions of bodies and another for composing the properties of bodies is a metaphysical illusion. This supposed division governs Lange in his account:

> ...No macroscopic body is elementary; any macroscopic body is also a system of bodies. Its motion, then, is nothing but the motions of its constituents, and these motions are determined by *their* masses and the forces *they* feel. The remarkable fact is that the law of nature by which the constituents' motions are determined by their masses and the forces they feel is *the same as* the law by which the macroscopic body's motion is determined by its mass and the forces it feels: $F = p' = (m\gamma v)'$. In other words, the law "scales up." (Lange, 2001, p. 231)

According to this picture, for every physical system there are constituent parts. According to Lange, the remarkable thing about this system is that parts and wholes experience exactly the same laws of nature. This is remarkable since even though it seems as if the motion of the system is "nothing but" the motion of the parts, the same law holds at every level of description. Each part has its mass value, and each mass value connects the acceleration of the part with the force that operates it. Then, the macroscopic body has its mass value, and this macroscopic mass obeys the same laws as the ones obeyed by each of the parts. This is true of the system as a whole independently of the fact that the motion of the system is just the motions of the parts.

But this sense of wonder at the surprising fact that dynamic laws "scale up" in much the same way that physical properties "scale up" stems from imagining the properties and laws to be describing different ontological entities; dynamic laws that describe changes of state and parameters that describe "inherent" material properties. But both dynamic laws and material properties arise from the geometry of PUMs and presuppositions regarding the structure of physical systems. As the above reconstructions suggest, once a geometry of PUMs, a Criterion of Isolation, and a Rule of Composition are presupposed, the material property of mass is completely determined. The same presuppositions that determine the dynamics also determine the material properties. Thus the dynamic laws "scale up" in much the same way that material properties "scale up," because presuppositions about "scaling up" are in place before physical systems are analyzed.

One should disagree with Lange when he imagines different ways in which laws of motion and the quantity of matter apply. Lange claims that:

> ...from the fact that macroscopic bodies have constituents with masses *and* that macroscopic bodies have masses too, it does *not* follow that the law by which the elementary constituents' motions are determined by their masses and the forces they feel is *the same as* the law by which the macroscopic body's motion is determined by its mass and the forces it feels. If a body's "mass" is defined as the quantity with which it is associated that plugs into the particular law relating an *elementary* body's motion to the forces it feels, then once again, it is a remarkable fact that macroscopic bodies *have masses too*.
>
> In short, neither of these conceptions of what mass *is* requires that macroscopic bodies have masses, even considering that their elementary constituents do. Furthermore, given either of these conceptions of mass *and* that a macroscopic body has a mass, there is nothing inevitable about its mass being the sum of its elementary constituents' masses (as is the

> case classically but not relativistically). So these conceptions of mass should inoculate us against the temptation to think that a body's "mass" *is defined* as the quantity of matter composing it. (Lange, 2001, p. 232)

However, contrary to Lange's assertions, it is inevitable that one would find a quantity for scaling up from microscopic to macroscopic bodies, since in order to *define* the part in relation to the whole, one has to define a Rule of Composition governing physical systems. The process of isolating a physical system requires that one first assumes both a Criterion of Isolation and a Rule of Composition. Without these presuppositions it is not possible to analyze physical systems. Once a relation between the part and the whole is defined, the property which *differentiates* between the parts is also defined.

The notion of physical system and its role in understanding the fundamental properties of physical bodies seems to be reflected in the thoughts of David Bohm:

> The basic idea behind our procedure is that it is essential in physical theories to be able to analyze a whole system into parts or components. Thus in a theory of continuous medium, such as hydrodynamics, we regard the fluid as being constituted out of small elements of volume, and, in a theory which explains matter as having a discrete atomic structure, a whole system is likewise regarded as constituted out of small elements, now taken to be the atoms. In both kinds of theories we can treat the *total momentum* of a system as the sum of the momenta of its parts, likewise with the total mass and the total energy. Moreover, at least in the domain where Newtonian theory applies, such systems are known by experiment (as well as from the theory) to satisfy the laws of conservation of momentum, conservation of mass, and conservation of energy.
>
> Because of these conservation laws, the entire momentum and mass (and also the energy) of a system can be regarded not only as sums of the corresponding properties of the set of its parts but also as an *integral whole* with value of these total quantities that remain constant, as long as the system is isolated. Indeed, the total values are evidently independent of the changes that are going on in each of the parts, as they engage in very complex interactions. It is this fact that is at the basis of the possibility of treating a block of matter as a single macroscopic entity, ignoring the unknown and indescribably complicated details of the motions of the molecules.
>
> It is clear that the property possessed by bulk matter – being capable alternatively of analysis into parts or treatment as a single whole – is a general feature of the world. This feature must therefore be implied by any proposed set of laws of mechanics, if they are to be fully adequate to *all* the experimental facts that are available.
>
> The characteristics described above were first achieved in non-relativistic theories. But, according to the principle of relativity, the basic physical properties of a system do not depend on its speed relative to the observer. Therefore, it is necessary that a system should continue to be capable alternatively of being treated as a whole or by analysis into parts, with the same conservation laws applying, even it is is moving at a high speed relative to the laboratory. We shall see that this requirement, plus that of a Lorentz transformation between different frames, is sufficient to determine the proper relativistic formulas for momentum, mass and energy. (Bohm, 1965, p. 82)[1]

After suggesting that the analysis of systems to parts is essential to physical theorizing, Bohm proceeds to derive the laws of motion in STR from two

[1] I thank Arthur Fine for pointing out this passage to me.

assumptions. First, he presupposes that the total momentum of a system is $P = \sum_i^N m_i v_i$. Second, Bohm assumes that the total mass of the system is $\sum_i^N m_i$. Since the system is analyzable to parts in *all* reference frames, the transformed variables describing two particles would also conform with the above presupposition, so that $P' = m'_1 v'_1 + m'_2 v'_2$ and $M' = m'_1 + m'_2$. Using the Lorentz transformations and the law for adding velocities in STR, he derives the ratio between relativistic and rest mass $m(v) = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$. This derivation of relativistic mass treats the additive nature of mass as the additional assumption regarding part-whole relations that make it possible to reduce the quantity of motion of a system to its parts. The analysis here follows a similar path, but unlike Bohm it does not assume separate rules of composition for the quantity of motion and the quantity of matter. The additive nature of relativistic mass is implicit in the definition of the momentum and the degree of freedom implicit in the spacetime transformations. The additive nature of mass, as was shown above, logically follows from the additive nature of quantity of motion.

8.2.4 Einstein's Mass and Energy as Two Manifestations of Substance

Previously I disagreed with Lange's claim that STR does not include a quantity of matter. I also disagreed with the claim that relativistic mass (as well as energy and velocity) is not an objective property. However, I would like to express my agreement with one important point made by Lange in his paper. Lange has done much to clarify a common confusion in the interpretation of STR concerning the relation between mass and energy and the so called mass-energy conversions. Lange criticizes the view which believes it is possible to take mass and *convert* it into energy, in the sense that mass can *reappear* in the form of energy and vice versa:

> For example, after radioactive nucleus decays, there is often said to be a "mass-defect": the sum of the masses of the daughter bodies is less (by Δm) than the mass of the original nucleus. Some (Δm) of the original mass is said to have been "converted" into the kinetic energy of the daughter bodies, where $E = \Delta mc^2$. Since c (light's speed in a vacuum) is so large, a very small mass can be "turned into" a great deal of energy. For instance, when a tritium nucleus (one proton, two neutron) decays into a helium-3 nucleus (two protons, one neutron) along with an electron and an antineutrino, the tritium's mass exceeds the sum of the products' masses by a small quantity that is "equivalent" to about 0.0186 million electron volts of energy. Has mass turned into energy, or merely disappeared and been replaced by an "equivalent" quantity of energy? (Lange, 2001, p. 220)

The question is therefore whether the conversion of the mass of the tritium to the energy of the products that result from the tritium's decay is a physical process. On the face of things, it seems as if mass is converted into energy. If such conversions are possible, mass is simply another form of energy, or mass and energy are two forms of the same thing – substance. However, the presumed convertibility of mass into energy is constrained by additional physical restrictions such as the conservation of the baryon number. Not every portion of mass can be converted to energy and vice versa. If mass-energy conversions are not always possible, the notions of "mass" and "energy" cannot be different expressions of the same thing.

If mass and energy are interconvertible, one may still believe that mass and energy are different manifestations of the same underlying substance. It is often said that mass is simply another form of energy. However, as Lange points out, for mass and energy to be two manifestations of the same underlying substance, one needs to show that some of the mass has disappeared from the system, and some of the energy has appeared. But the fact of the matter is that no mass disappears and no energy appears. In the tritium example, the mass of the tritium is greater than the sum of the decaying products. But the composite system comprising of the decaying products still has the same mass as that of tritium:

> Return to the tritium nucleus (one proton, two neutrons) that decays into helium-3 nucleus (two protons, one neutron) and an electron and antineutrino that fly off at high speed. There is a "mass defect" in that the masses of a helium-3 nucleus, an electron, and an antineutrino add up to less than the mass of a tritium nucleus. The missing mass is said to have been "converted" to the kinetic energies of the resulting bodies. But this "conversion" of mass into energy is not real; it is an illusion produced by a subtle shift in our perspective. (The transformation of the tritium's nucleus neutron into a proton, an electron, and an antineutrino is, of course, a real occurrence.) We treated the system as initially forming a single body: a tritium nucleus. But we treated the system after the decay as consisting of three bodies, each with its own mass. The system's mass after the decay is the same as the system's mass before the decay. There is no "mass defect" here; mass is conserved. The "mass defect" appears to arise from the fact that the sum of the three masses after the decay is less than the system's mass before the decay (the difference reflecting the three bodies' kinetic energy in the $p = 0$ frame). But the sum of the three masses after the decay is less than the *system's* mass *after* the decay. Mass is not additive, and our expectation that it is additive (arising because we expect it to measure the amount of some stuff) leads us to refer to a "mass defect" – to ask where the "missing mass" has gone and to conclude that it has turned into energy. The "mass defect" results not from some physical transformation of matter-stuff into energy-stuff, but rather from our illicitly trying to view the system from two different "perspectives" at the same time. (Lange, 2001, p. 237)

As Lange demonstrates, the confusion regarding the supposed mass-energy conversions stems from viewing the system composed of the decaying products as non-existent after the decay. But the composite system is still "there," and it carries the same rest mass as the tritium nucleus. It might be that one has no *practical* uses for such a mode of description, but that does not imply that the rest mass of the composite system is destroyed or created in such interactions. The non-additivity of rest mass leads us to think that the sum of the products' decay is "less" than the composite system's rest mass, and so it misleads us to think that part of the mass of the composite system was "turned into" the energy of the components.

However, Lange's argument overlooks an important presupposition that must be present if one is to describe the composite system. If the composite system, which was the tritium nucleus before the decay, continues to exist *after* the decay, one has to consider the status of parameters that describe the trajectory of this composite system. One can no longer observe the composite system after the decay, so one must assume a rule for constructing the description of the composite system from the descriptions of the parts. This Rule of Composition amounts to a rule governing momentum. The momentum of the composite system is the sum of the momentums of the helium-3 nucleus, the electron, and the antineutrino. But if one assumes that the composite system is "there" both before and after the decay, one must recognize that description of the center of mass frame as the description of a real object. This description is not merely a calculational device designed to simplify the analysis of the interaction. It is a description of a composite entity, whose parts are located in different locations of spacetime.

The equation $\Delta E = \Delta mc^2$ does not describe the conversion of mass into energy, but the discrepancy between the rest mass of the composite system and the rest masses of the components. Nevertheless, the deeper illusion that needs to be dispelled is the notion that either "mass" or "energy" represents some material substrate underlying physical processes. The concepts of mass and energy are extensions of a geometry of PUMs, together with the Criterion of Isolation and the Rule of Composition. There is no primitive material essence inherent in physical objects. There are only moving parts and moving composite systems; material properties are aids in describing these motions and the relations between parts and wholes. One tends to think that mass and energy represent *matter*, the stuff which preserves its identity in interactions. This is in distinction from the velocity and momentum, which are thought of as *states* of matter. But the division between inherent properties (such as mass) and states of matter (such as velocity and momentum) is a product of metaphysical bias that divides material bodies into matter and form. This bias, a legacy of Aristotle's distinction between matter and form, provides the intuition that all properties (the form of bodies) are either essential or accidental. Thus, mass is an essential property while velocity is a state of the body, or an accidental property. Evidence for the confusion that stems from this metaphysical bias is the uncomfortable place that energy has in our thinking. Sometimes energy is likened to a state of a body, in much the same way that momentum is thought of as an accidental property of a body. But sometimes, as a result of the supposed mass-energy conversions, one thinks of energy as the essence of matter. But the whole confusion can be avoided if one ceases to think of properties as the form of bodies, either essential or accidental. Bodies are not material substrates with various properties instantiated in them. One only has states of motion, and laws describing the structure of physical systems. The permanence in bodies is the permanence of structure. Either it is the same structure of PUMs that governs a system throughout its life, or the same structure governing the relation between parts of a system and their composite.

There exists no substance that underlies physical processes in relativity. There is just the conservation of four-momentum which underlies these processes; the four-momentum is not underwritten by any substance, it is simply motion described from a particular kind of reference frame. The illusion that a material substance exists ultimately stems from the surface grammar of our language, which always predicates attributes to subjects. The fact that mass and energy are conserved lures us into thinking that they are the universal subject of predication, the essential property of matter, since they seem to function as the building blocks of matter. But the conservation of mass and energy are a logical consequence of the geometry of PUMs and the conservation of four-momentum, so these concepts are secondary in logical status to motion. Literally, mass and energy represent nothing, as they derive from coordinate reference frames required for a consistent practice of measuring motion.

The above reconstruction of relativity theory suggests that there is promise in tracing rest mass to its geometric origins. The suggestion is to think of the rest mass of bodies as analogous to the transformation between inertial reference frames, where different four-momentums $< \mu_0\gamma, \mu_0 v\gamma >$ represent the four-velocity of an object in different reference frames, $K_R^{\mu_0\mu_1}$[11]. Once presuppositions regarding the nature of physical systems are made explicit, different expansion parameters can be attributed to different particles depending on the rules for combining motions. This geometric/structural interpretation of the mass renders the notion of substance obsolete, since we can think of the mass parameter attributed to a particular body as a geometry parameter, one that correlates between different representations of motion.

Chapter 9
Conclusion

Despite its ubiquitous presence in physical discussions, the notion of physical systems has been under the philosophers' radar. The significance of the concept has been overlooked primarily because of certain metaphysical dispositions. The modern metaphysical presupposition is that reality is broken into three different layers: spacetime, material properties, and laws of nature that govern the behavior of bodies. This separation between levels of reality considers spacetime as a container within which bodies are located, and bodies to be centers of activity. Spacetime tells us where the body is (or how the field is "spread out"), material properties dictate what these bodies possess, and laws of nature predict how matter will behave over time. The standard interpretation of Newtonian physics takes space and time to be containers in which particles are placed. Each particle possesses a property of mass inherent to the object, and laws of motion determine how the motion states of the body change over time. While the Special Theory of Relativity revolutionized our thinking about matter, it did not change the metaphysical disposition that separates reality into a spacetime structure, a set of material properties, and laws of nature.

The disposition to separate reality into three distinct realms did not form at once. At first, Newton conceived of mass as impenetrable regions of space. Thus, for Newton, mass functioned both as a geometric concept and as a source of action, i.e., as the inherent property of matter that compels bodies to move uniformly in a straight line. Over time, prompted by strong empiricist tendencies such as Mach's, the clear separation of reality into the geometric laws and inherent material properties solidified, and the geometric origin of the concept of mass was forgotten. The quantity of matter was replaced with "the law of conservation of mass," and inertial mass was made to be the definition of mass. The discovery of relativity theory seemed to confirm the notion that mass is not a quantity of matter; rest mass was no longer "conserved" and a different, more general conservation law governed processes in which mass can be converted to energy and vice versa.

It is my contention that an alternative metaphysical approach better explains the fundamental physical concepts and laws governing Newtonian systems and systems in the Special Theory of Relativity. Reality should not be separated into three distinct realms. If one takes states of motion to be the fundamental elements of reality, and rules for constructing and isolating physical systems are given central place,

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a different picture emerges regarding the relationship between the three realms of reality. One can think of uniform rectilinear motion as providing the basic geometric elements of reality; one can conceive of spacetime as a geometry of PUMs and points of intersection between them. This approach then allows for an alternative interpretation of the Law of Inertia. Instead of thinking of bodies as being compelled by some inherent force to move uniformly in a straight line, or of spacetime as somehow "guiding" bodies along its geodesics, one can think of isolated systems, either simple or complex, as simply instantiating PUMs. The geometric form of PUMs and their relations therefore *shapes* or *structures* both free particles moving along the geodesic and the center of mass of isolated physical systems. One therefore does not distinguish between spacetime structure and the instantiation of uniform rectilinear motions, and the Law of Inertia is not assimilable to a law of nature that is only justified empirically. Finding the geometric description of inertial motion *amounts* to finding the correct geometry of the world. The conservation of momentum and energy thus is not separable from the foundations of spacetime, and there is no sharp separation between kinematics and dynamics.

Another way in which the metaphysical view here deviates from standard interpretations is the role of material properties in physical systems. One can see that mass can be given an interpretation that undermines the standard account that reduces material properties to inherent properties. Once spacetime is interpreted as a geometry of PUMs, one can think about mass as analogous to an inertial frame of reference for describing the motion of a particular body. On the one hand, there is the measured acceleration of a body. On the other hand, there is the frame of reference appropriate for describing the trajectory of the body. The ratio between the measured and the "standard" acceleration of a body is a product of the fact that each trajectory might reside in a different coordinate reference frame. One needs to recalibrate his or her measurement devices to correctly describe the trajectory of each part. Once a Rule of Composition governing the motions of each part of a system is articulated, the additive and conserved nature of mass follows as a logical consequence. The role of mass as a quantity of matter is a logical consequence of the geometry of PUMs, the Rule of Composition governing motions, and the Criterion of Isolation. The laws of motion are also derived from the same structure of physical systems, since the behavior of one body is related to the behavior of another body through the account one gives of the system comprising of the various parts. Thus both material properties and laws of motion are derived from the same underlying structure of physical systems, and neither "inherent" material properties nor "external" laws of nature determine the behavior of bodies. It is the geometry of PUMs and structure of physical systems that is determinative.

The approach developed here promises to shed light on various philosophical problems concerning the foundations of physical theory. I shall summarize some of the results reached in various parts of the book, to give a general overview of advancement that could be acquired once one adopts the physical systems approach.

9.1 Spacetime

I introduced a new approach to the foundations of spacetime called *Primitive Motion Relationalism*. This approach has two important aims. The first aim is to provide an account of spacetime that illuminates the nature of spacetime and its relation to material bodies. The second aim is to provide a new explanation for the symmetries of spacetime. This account takes uniform motions of isolated systems and their coincidences as the fundamental entities of spacetime, rather than spacetime points or relations between bodies.

To explain the nature of spacetime I considered the conceptual connection between the Law of Inertia and the Criterion of Isolation. According to the approach introduced here, the Law of Inertia is essentially a criterion physicists utilize to isolate systems, both conceptually and experimentally. A physical system is isolated if and only if it instantiates a PUM. This is the case for free particles, but it is also true for the center of mass of an isolated system of interacting bodies.

In relation to both Galilean spacetime and flat relavitistic spacetime, we were able to derive spacetime structure from a geometry of PUMs. This derivation explains the equivalence between inertial reference frames and the restricted Principle of Relativity, without supposing the principle as a postulate of the theory. This by itself should give pause to anyone thinking about the nature of spacetime. There is benefit to an account of spacetime in which the Principle of Relativity is not presupposed both at the kinematic and dynamic levels. Since spacetime consists of a geometry of PUMs, and since PUMs function as the content of a Criterion of Isolation, there is no gap between the structure of spacetime and the fundamental dynamic laws. Both structures are shaped by the same geometry of PUMs. Thus, to explain why a free particle moves along a geodesic of the spacetime one need not attribute inherent causal powers to bodies or a causal power to spacetime, which supposedly compels bodies to move along a geodesic. Nor does one need to "reduce" the kinematic behavior of bodies to dynamic theories about the forces governing rods and clocks. One simply asserts that a free particle (or an isolated system of interacting bodies), instantiates a PUM, and it immediately follows that both kinematic and dynamic entities should conform to the basic symmetries of a geometry of PUMs. The need to explain spacetime symmetries both at the kinematic and dynamic levels stems from the false metaphysical assumption that separates the motion of bodies into two distinct metaphysical layers, where spacetime provides a bedrock metaphysical structure onto which the dynamic physical processes are "written." But once one recognizes that PUMs are the fundamental entities of spacetime, and foregoes the tendency to separate spacetime from dynamic laws, the problem of explaining why dynamic processes obey the same implicit symmetries of spacetime becomes moot.

I introduced an axiomatic system describing flat, $\{1 + 1\}$ spacetimes constructed from PUMs. In these systems, events are defined as the intersections of PUMs. Various incidence axioms describe possible intersections between motions, or possible events in the spacetime. The main novelty of this approach to the foundation of spacetime is that spatial and temporal metrics are no longer defined separately, as if time and space were independent dimensions of spacetime. Rather, progression of a PUM (the motion interval of a PUM) is given relative to a set of parallel PUMs. The spatial aspect of this progression amounts to the "number" of parallel PUMs intersected by the PUMs, and the temporal aspect is the progression "along" the PUMs. An analogy between the geometry of PUMs and Euclidean geometry is established by comparing motion intervals to distances along straight lines, and relative motions to angles between straight lines. The "angle" between motions is the relation between a PUM and a set of parallel PUMs serving as reference.

Spacetime is therefore characterized by the decomposition of a PUM motion interval into its spatial and temporal components. For example, the Galilean spacetime is characterized by a Galilean PUM (GPUM), which defines a linear relation between the progression of the motion "across" the class of parallel motions and the progression "along" these parallel motions. The GPUM does not define separate metrics for the spatial and temporal dimension; the two metrics "arise together" from the metric governing motion intervals, and the metric governing the relation between PUMs. The decomposition of a motion interval into its spatial and temporal component gives rise to the symmetries of spacetime, or to the existence of various equivalent inertial reference frames, so that if K_G^{1111} is a coordinate system $\langle x_0, x_1, x_2, x_3 \rangle$, one may derive an equivalent coordinate reference frame $K_G^{1\mu_1 11}$ when the spatial unit is expanded or contracted by a factor μ_1 relative to the temporal unit. But the expansion parameter also enables us to define the relative velocity between two PUMs, since:

$$v = (1 - \mu_1)\frac{dx_1}{dx_0}$$

From this spacetime symmetry the Galilean transformations between inertial reference frames is derived.

The basic structure of the PUM determines the spacetime symmetry; the clocks and rods that constitute a coordinate system behave as they do because they are calibrated relative to the fundamental PUMs. The same calibration also dictates the equivalence between inertial reference frames. If states of isolated physical systems evolve according to the fundamental PUMs, the classical Principle of Relativity is shown to be a logical consequence of the geometry of PUMs.

An analogous story can be told for flat relativistic spacetime. One begins with a geometry of PUMs, but the motion interval now decomposes according to relativistic PUM decomposition (RPUM). The RPUM stems from a particular set of parallel wave motions that provides a unique standard for motion intervals. The wave-like motion leaves a degree of freedom where the period of the component waves or the component wavelengths can be expanded or contracted by factors μ_α and μ_β. Thus one may derive Λ, the spactime transformation from the standard reference frame K_R^{1111} to the "expanded" wave reference frame $K_R^{1\mu_1 11}$. One can then define the relative velocity between K_R^{1111} and $K_R^{1\mu_1 11}$ as:

$$\beta \equiv \left(\frac{\mu_\alpha + \mu_\beta}{\mu_\alpha - \mu_\beta}\right) = \frac{v}{c} \text{ and}$$
$$\gamma \equiv \sqrt{\left(1 - \beta^2\right)}$$

Together with the assumption that $\Lambda(v) = \Lambda^{-1}(-v)$ the expansion parameters lead to the Lorentz transformation:

As was the case for Galilean spacetime, the relativistic PUM and the presumption that a geometry of PUMs governs spatiotemporal events implies the equivalence between inertial reference frames. If one assumes that all complex motions are comprised of instantaneous sections of PUMs, the implication is that changes of states of physical systems must be Lorentz covariant. The upshot is that the restricted Principle of Relativity is a logical consequence of adopting a relativistic geometry of PUMs. If motion is more primitive than space and time, the Principle of Relativity does not need to be stipulated independently of the structure of spacetime, but is implied by it.

Primitive Motion Relationalism carries theoretical benefits over other approaches to spacetime. The advantage over other accounts is that the approach explains the Principle of Relativity rather than assuming it. The geometry of PUMs demonstrates that various inertial reference frames generate alternative representations of motions in spacetime. Because the structure of a PUM is given relative to a class of parallel PUMs, i.e., because the structure of one motion can only be decomposed into its spatial and temporal components relative to other motions, there is no unique "absolute" motion that can be defined independently of any other motion. Thus *Primitive Motion Relationalism* belongs to the family of relational theories. However, unlike other relationalist theories, one does not take spacetime points or potential events to be the fundamental entity of spacetime. Rather, it is uniform motions that are primitive. This suggests that the relation between events does not amount to four independent dimensions – one temporal and three spatial. Rather, the spatial and temporal dimensions are interdependent aspects of the progress of motions.

The approach developed here carries a clear advantage over conventionalist accounts of spacetime geometry. Conventionalist accounts presuppose that inertial reference frames produce equivalent empirical models. Each model is based on a coordinative definition, and thus stems from a conventional choice of measuring rods and clocks. But we have shown that inertial reference frames should be considered as introducing equivalent *theoretical* representations of a geometry of motions. Since the progression of a PUM is defined relative to a class of parallel PUMs, the definition of motion is essentially relative, and thus it naturally leads to an infinite number of models, all consistent with the same form of PUM progression.

One can also appreciate the advantage of this approach over the standard geometric approaches to spacetime, even though there is a sense in which a geometry of PUMs attempts to provide a unifying geometric account of spacetime. Traditional geometric interpretations of spacetime ordinarily tend to think of spacetime as an independent structure with causal powers that influence the motion of bodies. The idea is that the spacetime manifold is a set of real entities, i.e., spacetime points, is a physical structure analogous to material substances. One supposedly observes the causal roles of spacetime through the influence spacetime has over the structure of geodesic motion, and through its determination of the behavior of clocks and rods. However, one ought to replace this causal picture with the idea of spacetime as a set of possible trajectories. The influence spacetime exerts on physical processes is not through efficient causation, where the spacetime point acts as a causal agent in determining the evolution of a trajectory. Rather, spacetime acts as some formal cause, determining the shape of trajectories through the various motions spacetime structure makes possible. The behavior of clocks and rods is not determined by an underlying manifold, but is inferred from the hypothetical process of calibrating rods and clocks with the help of motions instantiating PUMs.

The main difficulty with *Primitive Motion Relationalism* is that one has yet to explain the status of geometric propositions describing relations between *potential* motions. One is accustomed to admitting the existence of potential properties or states of individual entities, but not accustomed to accept the existence of potential *relations* between non-existing entities. The geometry of PUMs asserts that two parallel PUMs hold a certain relation between them. The claim is that spacetime is a set of potential trajectories. Each instantaneous part of a trajectory must instantiate a potential PUM. However, it is not clear what underwrites this set of potential trajectories, since it is not reducible to either an underlying spacetime or to the relations between existing bodies. Perhaps in addition to a formal cause governing individual motions one also has to admit an overall formal cause governing the structure of *all* motions: a formal cause with no underlying substrate. This is indeed a difficult metaphysical assumption to swallow, and implies a certain holism about spacetime structure.

A related concern is the incomplete nature of our investigation into the geometry of PUMs. The above reconstruction of spacetime articulated a geometry for PUMs modeling a {1+1} Galilean and a flat {1+1} relativistic spacetime. One still needs to do the work of extending the analysis to a flat {3+1} relativistic spacetime. Moreover, an analysis of curved spacetime is also due.

There are important similarities between *Primitive Motion Relationalism* and dynamic interpretations of spacetime, however, there are also important differences. In spirit, the approach is very close to the idea that the notion of spacetime is implicit in the dynamic laws that govern the behavior of physical systems. However, the usual dynamic account does not separate the universal structures common to all dynamic laws from the specific details informing each and every type of interaction. But the dynamic laws have a general form, such as being Lorentz covariant and obeying the Principle of Relativity. The above geometry of PUMs seems able to separate the universal features from specific dynamic details by considering the composite motions of bodies as constructed from infinitesimal instantiations of PUMs. Thus the geometry of PUMs can be abstracted from the particular interactions, and there is no need to complete a neo-Lorentzian project of constructing rigid rods and clocks from dynamic principles governing particular laws. Our approach can separate the geometric component of physical processes from the intricate dynamic structures without thereby taking spacetime to be an independently existing substance.

Primitive Motion Relationalism also open the door to alternative accounts of the nature of time. In Chapter 4 I argued that the widespread distinction between Presentism and Eternalism carries with it two assumptions that may require reexamination. The first assumption is that time is more primitive than motion. The second assumption is that an event exists if and only if it is present at a temporal instant t. Both assumptions are undermined by the reconstruction of spacetime from PUMs. If motion is more primitive than time, and if motion is decomposed into spatial and temporal components only relative to other PUMs, one needs to give up the metaphysical priority of time over motion and over existence. Thus the approach to spacetime provided here opens the door to a "third alternative" to Presentism and Eternalism. The presentist quandaries about the evolution of the now are obviated when the existence of the motion as a whole takes priority over the existence of any temporal part. A PUM can be taken to represent time, and the evolution of other motions can be evaluated relative to the paradigm motion selected as reference. But if one examines the paradigm motion relative to another PUM, one can evaluate the evolution of "time." The priority of motion over time also undermines the eternalist claim that there is no evolution and no dyanmics, and that future events exist in much the same way as past events do. Relative to a particular motion, the distinction between future and past becomes the distinction between intersections that have actualized or have yet to be actualized.

The account here is therefore in its infancy compared with alternative accounts that have been around for a while. The hope is that an intuitive understanding of the restrictive Principle of Relativity given here will give credibility to the approach and enough of a push to give the direction indicated by it further attention.

9.2 Mass

One should clearly distinguish between two conceptual roles that the concept of mass plays both in Newtonian physics and in relativity; as the quantity of matter and as the property of inertia. Newton took the quantity of matter to be essential to the meaning of the concept. The property of inertia was implicit in the *Principia*'s Definitions 3 and 4 of inherent forces, according to which bodies have inherent forces that compel them to move in a straight line, or resist the forces that attempt to deflect them from such motions. The mass of the body provides it with the proportional force of inertia, which has more or less power to resist external forces.

The above reconstruction of spacetime and mass reinforces both conceptual roles of mass as a quantity of matter and as a force of inertia. Both conceptual roles are significant components in our physical theories, although neither conceptual role should be associated with the view that reduces mass to some inherent property that is instantiated in bodies and is the causal origin of inertial forces. The geometry of PUMs is the origin of the inertial role of mass, and our interpretation argues that this conceptual role does not describe a causal power, but an implicit spacetime symmetry leading to various types of accelerations made possible by the underlying spacetime structure. Both conceptual roles of mass – as the property of inertia and as a quantity of matter – are derived from the geometry of PUMs, the Criterion of Isolation and the Rule of Composition. The conservation of mass is therefore not an independent empirical law concerning the conservation of some material property. Rather, the conservation of mass is a logical consequence of the structure of physical systems.

The standard interpretation of inertial mass as the origin of inertial forces unnecessarily multiplies causes and effects. Instead of thinking of mass as some internal cause battling the external impressed forces, one should recognize that inertial mass functions as a parameter correlating between two different descriptions of the body's acceleration; there is the measured acceleration, and there is the acceleration of the body from the point of view of a "standardized" frame of reference. The measured acceleration is the acceleration one ordinarily attributes to the body. The force operating on a body is its standardized acceleration. The force and the acceleration are not two distinguishable physical entities, but two descriptions of a single geometric concept. Thus the distinction between inherent and external impressed forces become unnecessary, and the understanding of force and acceleration as cause and effect is redundant.

The above reconstruction of spacetime through the geometry of PUMs reveals an implicit, heretofore unrecognized conceptual connection between inertial reference frames and the concept of inertial mass. Implicit to Galilean spacetime, there is an infinite number of reference frames $K_G^{\mu_0 111}$ derived from the expansion parameter μ_0 applied to both the spatial and the temporal metrics. These reference frames are indistinguishable from the point of view of rigid rods and clocks that are used, since expanding length and duration does not affect the measured velocities, only the measured accelerations of bodies. Thus, when the acceleration of a body is first measured, one cannot distinguish between the various frames $K_G^{\mu_0 111}$ appropriate for describing the true trajectory of this body. But once we measure the acceleration of a body $\vec{a}$, one can calculate the standard acceleration for this body as:

$$\vec{a}_s = \mu_0 \vec{a}$$

One may therefore think of mass parameters as correcting for each individual body the measured acceleration and transforming it from $K_G^{\mu_0 111}$ to K_G^{1111}. Thus there is a *geometric* justification for the observed physical phenomena – fundamental particles have the same mass parameter throughout their life. There is no spontaneous or induced variation in the mass parameter of fundamental particles. The reason is not that inertial mass is an inherent property, unaffected by any external processes. Rather the reason for this is that once the appropriate transformation between the measured acceleration in $K_G^{\mu_0 111}$ and the standardized acceleration in K_G^{1111} is selected, the same standardized frame, and relation between measured accelerations and standardized frames should be used to describe the rest of the particle's life. It also explains why each fundamental particle has a potentially different mass parameter. For each individual particle there is an infinite number of frames that happen to be the ones associated with the measured acceleration; thus each individual particle can have its own mass-parameter associated with it. An analogous set of reasoning can be applied in the context of the Special Theory of Relativity, where we have shown that an infinite number of reference frames $K_R^{\mu_0 111}$ are implicit in relativistic descriptions of motions. The expansion parameters associated with each reference frame are essentially the rest masses in relativity.

One should resist the idea that force and acceleration are two different entities. Force, it is ordinarily believed, is a causal agent existing over and above individual bodies. The notion of "force" amounts to the counterfactual acceleration a body *would* experience were certain circumstances realized. One ordinarily combines various such counterfactual accelerations into one acceleration, but one does so using the Rule of Composition governing motions. It is possible to think of the instantaneous accelerations of individual bodies as "adding up" due to the participation of the body in various composite motions, the overall composite system comprises the various trajectories of the component bodies. It is thus the Rule of Composition governing composite systems that determines how to "add up" the various accelerations of a body that "participate" in the trajectory of a composite system. The upshot is that it is redundant to assume the existence of a force independently of the acceleration of a body. Various accelerations "add up" in various circumstances according to known rules, but there is no need to suppose that forces are causing the accelerations rather than merely providing a calculation from which the acceleration can be deduced.

It thus seems as if our approach to spacetime reveals a method for treating material and dynamic properties as reducible to geometric concepts together with rules for the construction of composite physical entities from their parts. The notions of mass, force, and momentum provide various means for *describing* motion. Such a viewpoint greatly simplifies the metaphysics required for analyzing physical concepts. One no longer needs to attribute bodies inherent properties such as mass, or to think of forces as some external causes flowing from the nature of bodies. Our physical theories simply provide systematic accounts of motions and changes in motions. The price one pays for such a simplified metaphysics is that one is compelled to accept the fundamental role of the structure of physical systems. This structure helps us compose the states of composite systems from states of the component trajectories. The Rule of Composition is not reducible to properties inherent in the bodies, nor does it consist of laws of motion governing the behavior of bodies from without. Rather, a Rule of Composition directly describes relations between parts and wholes.

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Index