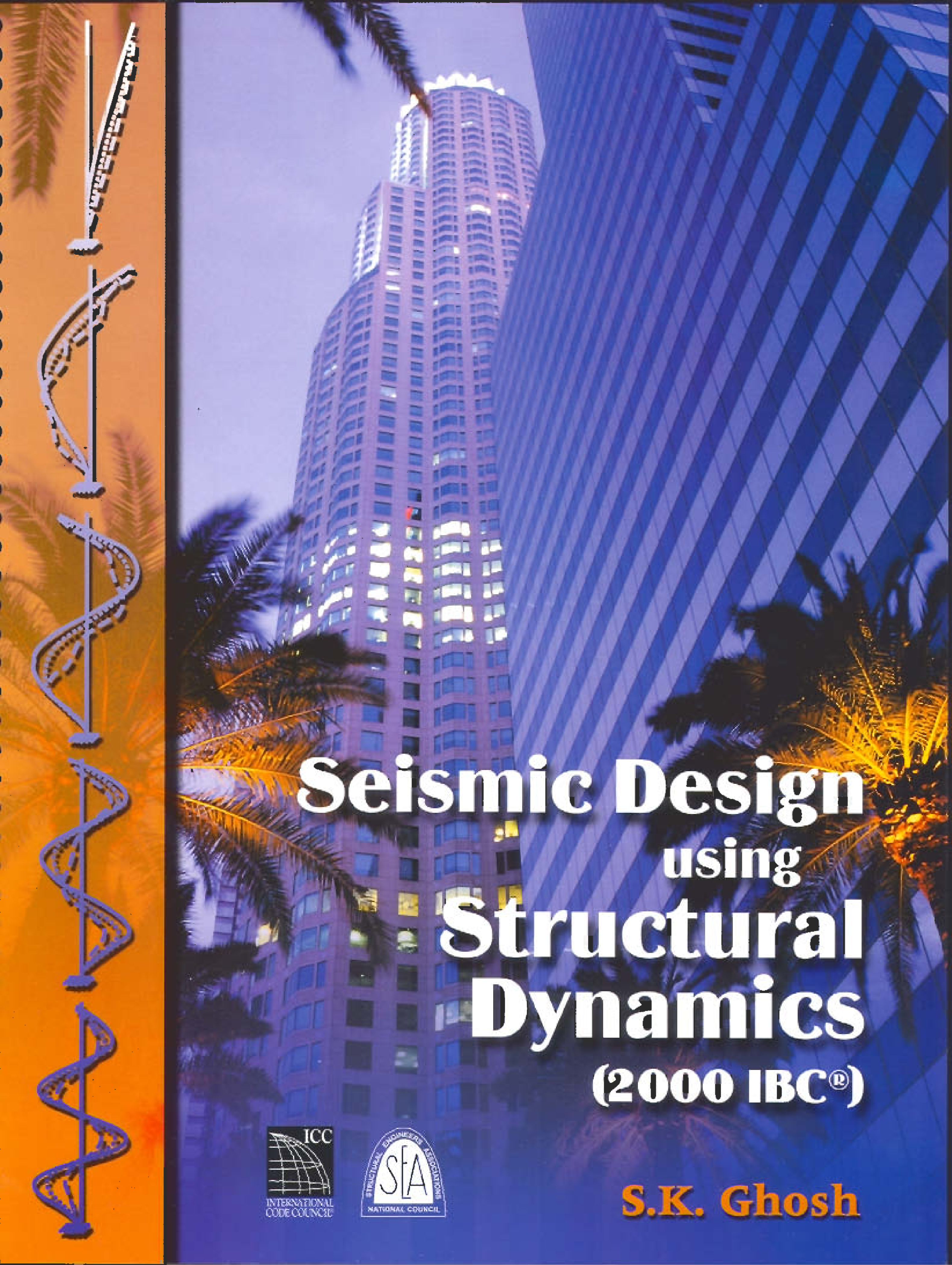

# Seismic Design using Structural Dynamics (2000 IBC®)

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STRUCTURAL ENGINEERS ASSOCIATIONS
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S.K. Ghosh

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# PREFACE

This publication addresses the two methods by which a designer may comply with the seismic design requirements of the 2000 *International Building Code*® (IBC®): Equivalent Lateral Force Procedure (IBC Section 1617.4) and Dynamic Analysis Procedure (IBC Section 1618). The Dynamic Analysis Procedure is more complicated and is required to be used under certain conditions of irregularity, occupancy, and height. Over the years, many questions have been asked about code provisions concerning the Dynamic Analysis Procedure, and this publication has been created to answer these questions and demystify the application of the code.

Although the 2000 IBC formally recognizes two dynamic analysis procedures: response spectrum analysis and time-history analysis, the response spectrum analysis is by far the more common and is the primary subject of this publication. The background and details are explained in the first half of this publication where a step-by-step analysis procedure is provided, and a three-story, one-bay frame example is solved manually to illustrate application of the procedure.

The second half of this publication is devoted exclusively to the detailed design of a 20-story reinforced concrete building that utilizes a dual shear wall-frame interactive system for earthquake resistance. Response spectrum analysis is used as the basis of design. Design utilizing the equivalent lateral force procedure is also illustrated because it is basically a prerequisite to design using the dynamic analysis procedure.

A key feature of this 20-story design example that will be of particular interest to users is the design of reinforced concrete shear walls utilizing the procedure in the 1999 edition of ACI 318, *Building Code Requirements for Structural Concrete*, which is different from that in the prior editions of ACI 318. Examples of reinforced concrete shear wall design using the ACI 318-99 procedure are not commonly available.

## ACKNOWLEDGEMENTS

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**Note:** Items shown in parentheses near right-hand margins refer to information contained in the 2000 IBC or ACI 318-99; i.e., section numbers, tables (T), and equations (Eq.)

Tables and figures exclusive to this document appear at the end of their appropriate chapter.

Chapter 1

# MODAL SPECTRUM ANALYSIS: BACKGROUND AND IMPLEMENTATION

## 1.1 The Nature of Earthquake Forces in a Structure

The forces that a structure subjected to earthquake motions must resist result directly from the distortions induced by the motion of the ground on which it rests. The response (i.e., the magnitude and distribution of forces and displacements) of a structure resulting from such a base motion is influenced by the properties of both the structure and the foundation, as well as the character of the exciting motion.

A simplified picture of the behavior of a building during an earthquake may be obtained by considering Figure 1-1. As the ground on which the building rests is displaced, the base of the building moves with it. However, the inertia of the building mass resists this motion and causes the building to suffer a distortion (greatly exaggerated in the figure). This distortion wave travels along the height of the structure in much the same manner as a stress wave in a bar with a free end.[1] The continued shaking of the base causes the building to undergo a complex series of oscillations.

It is important to draw a distinction between forces due to wind and those produced by earthquakes. Occasionally, even engineers tend to think of these forces as belonging to the same category just because codes specify design wind as well as earthquake forces in terms of equivalent static forces. Although both wind and earthquake forces are dynamic in character, a basic difference exists in the manner by which they are induced in a structure. Whereas wind loads are external loads applied and, therefore, proportional to the exposed surface of a structure, earthquake forces are essentially inertial forces that result from the distortion produced by both the earthquake motion and inertial resistance of the structure. Their magnitude is a function of the mass of the structure rather than its exposed surface. Also, in contrast to the structural response to essentially static gravity loading or even to wind loads, which can often be validly treated as static loads, the dynamic character of the response to earthquake excitation can seldom be ignored. Thus, while in designing for static loads one would feel greater assurance about the safety of a structure made up of members of heavy section, in the case of earthquake loading, the stiffer and heavier structure does not necessarily represent the safer design.

## 1.2 Earthquake Ground Motion

Data presently available to serve as a basis for estimating earthquake-induced ground motions at a site consist of observational and instrumental records of actual earthquakes, artificial earthquakes, and empirical scaling relationships based on past records.[2] Only records of actual earthquakes are discussed here.

Instrumental records of earthquake motions close to the epicenter are valuable in structural engineering. These usually consist of acceleration traces of motion along two perpendicular horizontal directions and in the vertical direction (the rotational components are usually unimportant). The records are obtained using strong-motion accelerographs (SMACs).

Although ground motions recorded at a site may not be repeated, strong-motion records, if available over a long period, reveal the general character of the ground motion and effect of geologic conditions at a particular location. Strong motion records from earthquakes in the United States (in corrected, digitized form) are available from several sources such as the United States Geological Survey (USGS), the California Division of Mines and Geology (CDMG), and the California Institute of Technology (Cal Tech). Where a number of accelerograms for a particular region are available, a set chosen by careful sampling can be used in dynamic response studies of proposed structures.

A set of acceleration traces that has often been used in dynamic response studies is that of the Imperial Valley (California) earthquake of 1940. The set was recorded at the El Centro instrument site, which rests on some 5,000 feet (1524 m) of alluvium about 4 miles (6.4 km) away from the causative fault break. The set represents one of the strongest earthquakes ever recorded, and exhibits high-frequency (frequency is a measure of how often the ground motion changes direction), large-amplitude (amplitude is a measure of how intense the ground motion is) pulses lasting over a long duration. A plot of the north-south component of horizontal ground accelerations during the first 30 seconds of the above earthquake is shown in Figure 1-2. Also shown are plots of the ground velocity and displacement, as obtained by successive integration. The maximum recorded ground acceleration in the N-S direction was about $0.33g$.

## 1.3 Response of Structures to Earthquakes

### 1.3.1 Dynamic versus static structural analysis

In a structural dynamics problem, the loading and all aspects of the structural response vary with time, so that a solution must be obtained for each instant during the history of response.

There is a more important distinction between a static and a dynamic problem.[3,4] When the simple column of Figure 1-3 is subjected to a static lateral load, the internal forces may be evaluated by simple statics. If the same load is applied dynamically, the time-varying deflections involve accelerations which in turn generate inertia forces resisting the motion (Fig. 1-3). The external loading, $p(t)$, that causes the motion and the inertia forces, $f_I(t)$, that resist its acceleration act simultaneously. The internal forces in the column must equilibrate this combined load system, so that it is necessary to know the inertia forces before the internal forces can be determined. The inertia forces depend on the rate of loading and on the flexibility and mass characteristics of the structure. The basic difficulty of dynamic analysis is that the deflections that lead to the development of inertia forces are themselves influenced by the inertia forces.

### 1.3.2 Degrees of freedom

The complete system of inertia forces acting in a structure can be determined only by evaluating the acceleration of every mass particle. The analysis can be greatly simplified if the deflections of the structure can be specified adequately by a limited number of displacement components or coordinates. This can be achieved through the *lumped mass* or the *generalized coordinate* approach.[3,4] In either case, the number of displacement components required to specify the positions of all significant mass particles in a structure is called the number of degrees of freedom of the structure. In the lumped-mass idealization, the mass of the structure is assumed to be concentrated at a number of discrete locations. An idea of the generalized coordinate approach is obtainable from Section 1.7.3.

## 1.4 Dynamics of Single-Degree-of-Freedom (SDOF) Systems

### 1.4.1 Response to earthquake ground motion

No external dynamic force is applied to the idealized one-story structure in Figure 1-4. The excitation in this case is the earthquake-induced motion of the base of the structure, presumed to be only a horizontal component of ground motion, with displacement, $x_g(t)$, velocity, $\dot{x}_g(t)$, and acceleration, $\ddot{x}_g(t)$. Under the influence of such an excitation, the base of the structure is displaced by an amount, $x_g(t)$, if the ground is rigid, and the structure undergoes displacement, $x(t)$, of roof relative to base. In the presence of viscous or velocity-proportional damping, the equation of motion is given by

$$m\ddot{x} + c\dot{x} + kx = -m\ddot{x}_g \qquad \text{(Eq. 1-1)}$$

where $m$, $c$, and $k$ are the mass, damping coefficient, and stiffness, respectively, as shown in Figure 1-4.

Equation 1-1 may be rewritten as

$$\ddot{x} + 2\beta\omega\dot{x} + \omega^2 x = -\ddot{x}_g \qquad \text{(Eq. 1-2)}$$

where $\omega^2 = (2\pi/T)^2 = k/m$, $T$ is the natural period of vibration as represented in Figure 1-5a, and $\beta = c/c_{cr} = c/2m\omega$, $\omega$ is the fraction of critical damping* (Figure 1-5b).

The solution to Equation 1-2 leads to the deformation response, $x(t)$, which depends on: a) the characteristics of the ground acceleration, $\ddot{x}_g(t)$, b) the natural circular frequency of vibration, $\omega = 2\pi/T$ (or equivalently the natural period of vibration, $T$) of the structure without damping, and c) the damping ratio, $\beta$, of the structure. The solution to Equation 1-2 is given by

$$x(t, \omega, \beta) = \frac{1}{\omega_d}\int_0^t \ddot{x}_g(\tau)\exp[-\beta\omega_d(t-\tau)]\sin\omega_d(t-\tau)d\tau = \frac{1}{\omega_d}R(t, \omega, \beta) \qquad \text{(Eq. 1-3)}$$

where $\omega_d = \omega\sqrt{1-\beta^2}$. For $\beta < 0.2$, $\omega_d$ $(=2\pi/T_D)$ is practically equal to $\omega$.

*Critical damping is defined as the least damping coefficient for which the free response of a system (i.e. in the absence of damping or an external exciting force) is nonvibratory; i.e., for which it returns to the static position without oscillation after any excitation.

While Equation 1-3 expresses deflection response, the effective earthquake force is

$$kx = m\,\omega^2 x = m\,\omega\,R \qquad \text{(Eq. 1-4)}$$

where $\omega^2 x = \omega R$ may be thought of as an effective acceleration.

### 1.4.2 Response spectrum

The earthquake accelerogram is digitized and appropriately processed to produce a corrected ground accelerogram.[5] In the CalTech strong motion data program, the corrected accelerograms are defined at 0.02-second time intervals. With the ground accelerations, $\ddot{x}_g(t)$, defined in this manner and substituting numerical values for $\omega$ and $\beta$, the response history can be determined by numerical integration of the Duhamel integral in Equation 1-3. The more common approach, however, is to solve the equation of motion (Eq. 1-2) by numerical procedures.

To obtain the entire history of seismic displacements and forces, as given by Equations 1-3 and 1-4, may be unnecessary in most practical situations; it may be sufficient to determine only the maximum response quantities. The maximum force as well as displacement response can be computed by introducing the maximum value of the response function $R$ into Equations 1-3 and 1-4. This maximum value of $R$ is called the *spectral pseudo-velocity:*

$$S_v = R_{max} = \left[\int_0^t \ddot{x}_g(\tau)\exp[-\beta\omega(t-\tau)\sin\omega(t-\tau)d\tau\right]_{max} \qquad \text{(Eq. 1-5)}$$

Maximum displacement equals the spectral pseudo-velocity divided by the circular frequency (Eq. 1-3). This quantity is called the *spectral displacement:*

$$S_d = S_v/\omega \qquad \text{(Eq. 1-6)}$$

Similarly, the maximum earthquake forces are seen from Equation 1-4 to equal the product of the mass, the circular frequency, and the spectral pseudo-velocity; leading to the following definition of the *spectral pseudo-acceleration*

$$S_a = \omega\,S_v = \omega^2 S_d \qquad \text{(Eq. 1-7)}$$

The physical significance of the spectral pseudo-velocity, $S_v$, can be explained as follows. The maximum displacement corresponds to a condition of zero kinetic energy and maximum strain energy, $\frac{1}{2}kS_d^2$. If this energy were in the form of kinetic energy, $\frac{1}{2}m(\dot{x})^2 = \frac{1}{2}kS_d^2$, the maximum relative velocity would be

$$\dot{x} = \sqrt{k/m}\,S_d = \omega S_d = S_v \qquad \text{(Eq. 1-8)}$$

If the subscript $_{max}$ is used to designate the maximum value, without regard to algebraic sign, of any response quantity, $r$, then

$$r_{max} = \max|r(t)|$$

A plot of the maximum value of a response quantity as a function of the natural vibration frequency of the structure or as a function of a quantity related to the frequency, such as natural period, constitutes the response spectrum for that quantity.[6] The displacement response spectrum is such a plot of the quantity $S_d$ defined as

$$S_d = x_{max} \quad \text{(Eq. 1-9)}$$

Figure 1-6 from Reference 6 shows the basic concept underlying computation of the displacement response spectrum. The time variations of displacement responses of three structures to a selected ground motion are presented. The damping ratio $\beta = 2\%$ is the same for the three structures, so that the differences in their displacement responses are associated with their natural periods of vibration. The time required for a structure to complete one cycle of vibration in response to typical earthquake ground motion is very close to the natural period of vibration of the structure. For each structure, the maximum value of the displacement, without regard to algebraic sign, during the earthquake is determined from its response history. The $x_{max}$ so determined for each structure provides one point on the displacement response spectrum. Repeating such computations for a range of values of $T$, while keeping the damping ratio, $\beta$, constant, produces the displacement response spectrum for the ground motion. Such spectral curves are typically produced for several values of damping for the same ground motion.

For the ground motion of Figure 1-6, the spectral pseudo-velocity, $S_v$, corresponding to any vibration period, $T$, can be determined from Equation 1-6 ($S_v = \omega S_d = 2\pi S_d/T$) and the $S_d$ value for the same $T$, computed as illustrated in Figure 1-6 and plotted in Figure 1-7a. The resulting values of $S_v$ are plotted in Figure 1-7b as a function of the vibration period $T$, for a fixed value of the damping ratio, to provide the pseudo-velocity response spectrum for the ground motion of Figure 1-6.

For the same ground motion, the spectral pseudo-acceleration, $S_a$, corresponding to any value of $T$ can be determined using Equation 1-7 and the $S_d$ value for the same $T$, computed as illustrated in Figure 1-6 and plotted in Figure 1-7a. The resulting values of $S_a$ are plotted in Figure 1-7c as a function of the vibration period, $T$, for a fixed value of the damping ratio, to provide the pseudo-acceleration response spectrum for the ground motion of Figure 1-6.

The displacement, pseudo-velocity, and pseudo-acceleration response spectra for an earthquake are interrelated through Equation 1-7. Any one of these spectra can be obtained from one of the other two, and each of the three spectra contains exactly the same information.

Because of the relationships indicated by Equation 1-7, a single plot can be constructed to show the variations of spectral pseudo-acceleration, pseudo-velocity, and displacement with frequency (or period). Figure 1-8 is such a re-plot of the information in Figure 1-7, with log scales on all axes, the spectral displacement and acceleration being read on diagonal scales.

The design ground motion(s) at a site is often specified in terms of a response spectrum. The relative (pseudo-) velocity response spectra for the N-S component of the 1940 El

Centro record, for different amounts of damping, are shown in Figure 1-9.[7] The plot is typical of earthquake response spectra and confirms certain intuitively obvious aspects of the dynamic response of simple systems. For low-frequency (long-period) systems, corresponding to a heavy mass supported by a light spring – the mass remains practically motionless when the base is seismically excited, the relative displacement of the mass with respect to the base being essentially equal to the base displacement. For high-frequency (short-period) systems, exemplified by a light mass supported by a very stiff spring (on the right side of the plot), the mass simply moves with the base. In the intermediate frequency or period range, some modification of the response parameters characterizing the motion of the mass relative to that of the base occurs. For linear systems with damping ratios of 5 to 10 percent subjected to the 1940 El Centro motion, the maximum amplification factors for displacement, velocity, and acceleration are about 1.5, 2.0, and 2.5, respectively. A typical response spectrum curve in Figure 1-9 can be approximately represented by three line segments shown as the dashed line a-b-c-d in the figure. Please note that while Figures 1-7 and 1-8 have period on the x-axes, Figure 1-9 has frequency on the same axis. Both practices are fairly common in spectral plotting.

### 1.4.3 Smoothed response spectra

The sharp peaks, conspicuous when damping is absent, reflect the resonant behavior of the system of Figure 1-4 in certain frequency ranges. Significantly, even a moderate amount of damping not only reduces the response but also smoothes out the jagged character of the spectral plot. Thus, the sharp peaks are not important in practice. Also, for design purposes, earthquake motions of varying frequency characteristics are customarily accounted for by using smoothed and averaged spectra based on a number of earthquake records, all reduced or normalized to a reference intensity. A comparison of Figures 1-10 and 1-11 should clarify the concept of the smoothing of response spectra. Smoothed averaged spectra for design purposes are discussed in Section 1.5.2.

## 1.5 Establishing Design Ground Motion(s) at a Site

A number of procedures for selecting design (earthquake) ground motions at a site are currently available. Werner[8] classified these as a) site-independent, or b) site-dependent using site-matched records, or site-response analysis. Site-response analysis is not discussed here.

### 1.5.1 Site-independent procedures

Site-independent procedures use standardized spectrum shapes developed from accelerograms that represent a variety of seismologic, geologic, and local soil conditions. The use of site-independent spectra was first introduced by Housner.[9] Since then, other shapes have been developed, including those of Newmark and Hall,[10] and Newmark, Blume, and Kapur.[11]

### 1.5.2 Site-dependent procedures

Seed, Ugas, and Lysmer[12] developed site-dependent spectra based specifically on local site conditions. Ensemble average and mean-plus-one standard deviation spectra were developed from 104 site-matched records corresponding to four broad site classifications: rock (28 records), stiff soil (31 records), deep cohesionless soil (30 records), and soft-to-medium soil deposits (15 records). The spectra developed for each site condition corresponded to a 5-percent damping ratio and were normalized to the zero-period acceleration or maximum ground acceleration.

A state-of-the-art recommendation for specifying earthquake ground shaking at a site in the United States was drawn up by the Applied Technology Council (ATC).[13] The ATC chose to represent the intensity of design ground shaking by the two parameters illustrated in Figure 1-12: effective peak acceleration (EPA) and effective peak velocity (EPV). EPA was expressed in terms of a dimensionless coefficient, $A_a$, which is equal to EPA expressed as a fraction of $g$ (e.g., if EPA $= 0.2g$, $A_a = 0.2$). EPV was expressed in terms of a dimensionless parameter, $A_v$, which is a velocity related acceleration coefficient [$A_v$ = EPV (in./sec) $\times$ 0.4/12]. The ATC document furnished detailed maps that divided the United States into seven areas. The $A_a$ and $A_v$ coefficients for each map area were given. The probability was estimated at 90 percent that the recommended EPA and EPV at a given location would not be exceeded during a 50-year period.

The Seed-Ugas-Lysmer[12] mean spectral shapes for different soil conditions as shown in Figure 1-13 were compared with the studies of spectral shapes conducted by Newmark et al,[14] Blume et al,[11] and Mohraj.[15] It was considered appropriate to simplify the form of the curves to a family of three by combining the spectra for rock and stiff soil conditions, leading to the normalized spectral curves shown in Figure 1-14a. Recommended ground motion spectra (for 5-percent damping) for the different map areas were to be obtained by multiplying the normalized spectral values shown in Figure 1-14a by the values of effective peak ground acceleration. ATC 3[13] included a correction factor of 0.8 for soft-to-medium stiff clay and sand type soils at locations with $A_a \geq 0.3$ (Fig. 1-14b). Where the $A_a$ and $A_v$ values for a map area differed, the portion of the response spectrum controlled by velocity (the descending parts in Fig. 1-14a) was to be increased in proportion to the EPV value, and the remainder of the response spectrum extended to maintain the same overall spectral form.

## 1.6 Design Spectra of the NEHRP Provisions and the UBC

A significant number of trial designs were carried out to assess the practicability and the economic impact of the seismic design requirements of the ATC 3 document.[13] The trial designs indicated the need for certain modifications in the document. The modifications were made and the resulting document became the first edition, dated 1985, of the NEHRP (National Earthquake Hazards Reduction Program) Recommended Provisions.[16] The NEHRP Provisions have been updated every three years since then.

The ATC 3 design spectra of Figure 1-14 remained unchanged in the 1985 NEHRP Provisions, which included contour maps for $A_a$ and $A_v$ in addition to the map areas of ATC 3 and the table that specifies values of $A_a$ and $A_v$ for the different map areas.

While the 1985 NEHRP Provisions included the three soil categories defined by ATC, the 1988 NEHRP Provisions[16] included a fourth soil category, $S_4$, based on experience from the Mexico earthquake of September 19, 1985. Much of the damage caused by that earthquake was in Mexico City where most of the underlying soil is very soft. The commentary to the 1988 NEHRP Provisions illustrated design spectra that are reproduced in Figure 1-15.

### 1.6.1 1994 NEHRP Provisions

Soil sites generally cause a higher amplification of rock spectral accelerations at long periods than at short periods and, for a severe level of shaking ($A_a \approx A_v \approx 0.4$), the short-period amplification or de-amplification is small. However, short-period accelerations including the peak acceleration can be amplified several times, especially at soft sites subject to low levels of shaking. The latter evidence suggested a two-factor approach sketched in Figure 1-16. In this approach, adopted in the 1994 NEHRP Provisions,[16] the short-period plateau, of a height proportional to $A_a$, is multiplied by a short-period site coefficient, $F_a$, and the curve proportional to $A_v/T$ is multiplied by a long-period site coefficient, $F_v$. Both $F_a$ and $F_v$ depend on the site conditions and on the level of shaking, defined by the $A_a$ and $A_v$ coefficients, respectively. The 1994 NEHRP Provisions introduced new seismic coefficients $C_a$ and $C_v$ such that

$$C_a = A_a F_a \text{ and } C_v = A_v F_v \qquad \text{(Eq. 1-10)}$$

Six soil categories (called Site Classes), designated as A through F, were introduced in the 1994 NEHRP Provisions. The first five are based primarily on the average shear wave velocity, $v_s$(ft/sec), in the upper 100 feet of the soil profile, and the sixth is based on a site-specific evaluation. The categories include: A) hard rock ($v_s > 5000$), B) rock ($5000 < v_s \leq 2500$), C) very dense soil and soft rock ($2500 < v_s \leq 1200$), D) stiff soil profile ($1200 < v_s \leq 600$), E) soft soil profile ($v_s < 600$), and F) soils requiring site-specific calculations such as liquefiable and collapsible soils, sensitive clays, peats and highly organic clays, very high-plasticity clays, and very thick soft/medium stiff clays.

In recognition of the fact that in many cases the shear wave velocities may not be available, alternative definitions of the site categories are also included in the 1994 NEHRP Provisions. They use the standard penetration resistance for cohesionless soil layers and the undrained shear strength for cohesive soil layers. These alternative definitions are rather conservative since the correlation between site amplification and these geotechnical parameters is less certain than that with $v_s$. There are cases where the values of $F_a$ and $F_v$ are smaller if the site category is based on $v_s$ rather than on geotechnical parameters. Also, the 1994 NEHRP Commentary cautions the reader not to interpret the site category definitions as implying any specific numerical correlation between shear wave velocity on the one hand and standard penetration resistance or shear strength on the other.

The short- and long-period amplification factors implied by the Loma Prieta strong-motion data and related calculations for the same earthquake by Joyner et al.[17] as well as modeling results at the $0.1g$ ground acceleration level provided the basis for the consensus values of site coefficients $Fa$ and $Fv$ provided in 1994 NEHRP Tables 1.4.2.3a and 1.4.2.3b.

### 1.6.2 1997 NEHRP Provisions and 2000 IBC

The Building Seismic Safety Council (BSSC) worked for several years to replace the 1994 NEHRP Provisions Maps 1 through 4, which provided the $A_a$ (effective peak acceleration coefficient) and $A_v$ (effective peak velocity-related acceleration coefficient) values on rock (Type S1 soil), corresponding to the 474-year average return period (90-percent probability of not being exceeded in 50 years) earthquake, for use in design.

These maps predated the 1985 NEHRP Provisions. The first significant changes were introduced in an appendix to Chapter 1 of the 1988 NEHRP Provisions. In the 1991 NEHRP Provisions, that appendix was revised to introduce new spectral maps and procedures. For the 1994 NEHRP Provisions, that appendix was again revised to introduce improved spectral maps. For the 1997 NEHRP Provisions, a seismic design procedure group was given the responsibility for replacing the existing effective peak acceleration and velocity-related acceleration design maps with new ground-motion spectral-response maps based on new USGS seismic hazard maps.

The seismic design provisions of the 2000 IBC are based on the 1997 NEHRP Provisions. Thus, in this section the two documents are grouped together for the purposes of discussion.

The design ground motions in all model codes preceding the 2000 IBC and in the NEHRP Provisions through its 1994 edition were based on an estimated 90-percent probability of not being exceeded in 50 years (about a 474-year mean recurrence interval or return period). This changed with the 1997 edition of the NEHRP Provisions and thus with the 2000 edition of the IBC.

Given the wide range in return periods for maximum-magnitude earthquakes in different parts of the United States (100 years in parts of California to 100,000 years or more in several other locations), the 1997 NEHRP Provisions focused on defining maximum considered earthquake ground motions for use in design. These ground motions may be determined in different ways depending on the seismicity of an individual region; however, they are uniformly defined as "the maximum level of earthquake ground shaking that is considered reasonable to design buildings to resist." This definition facilitates the development of a design approach that provides approximately uniform protection against collapse resulting from maximum considered earthquake ground motions throughout the United States.

It is widely recognized that the ground motion difference between 10-percent and 2-percent probabilities of being exceeded in 50 years in coastal California is typically smaller than the corresponding difference in inactive seismic areas such as the eastern and central United States. Figure 1-17, reproduced from the commentary to the 1997 NEHRP Provisions, plots the spectral acceleration at a period of 0.2 second, normalized at a 2-percent probability of being exceeded in 50 years (10 percent in 250 years), versus the annual frequency of being exceeded.

The figure shows that in coastal California, the ratio between the 0.2-second spectral accelerations for the 2- and 10-percent probabilities of being exceeded in 50 years is about 1.5, whereas the ratio varies between 2.0 and 5.0 in other parts of the United States.

The question therefore arose as to whether the definition of ground motion based on a constant probability for the entire United States would result in similar levels of seismic safety for all buildings.

In addressing the question, it was recognized that seismic safety is the result not only of the design earthquake ground motion definition, but also of such critical factors as proper site selection, structural design criteria, analysis procedures, adequacy of detailing and quality of construction.

The NEHRP 1997 seismic design provisions are based on the assessment that if a building experiences a level of ground motion 1.5 times the design level of the 1994 and prior Provisions, the building should have a low likelihood of collapse. Although quantification of this margin is dependent on the type of structure, detailing requirements, etc., the 1.5 factor was felt to be a conservative judgment.

As indicated above, in most U.S. locations, the 2-percent probability of ground motion values being exceeded in 50 years is more than 1.5 times those corresponding to a 10-percent probability within 50 years. This means that if the 10-percent probability of being exceeded in 50 years map were used as the design map and the ground motion corresponding to a 2-percent probability in 50 years were to occur, there would be a low confidence (particularly in the central and eastern U.S.) that buildings would not collapse because of these larger ground motions. Such a conclusion for most of the U.S. was not acceptable. The only location where the above results seemed to be acceptable was coastal California (ground motion corresponding to a 2-percent probability of being exceeded in 50 years is about 1.5 times that corresponding to a 10-percent probability in 50 years) where buildings have experienced levels of ground shaking equal to and above the design value.

Probabilistic seismic hazard maps from the U.S. Geological Survey for Coastal California indicate that the ground motion corresponding to a 10-percent probability of being exceeded in 50 years is significantly different (in most cases larger) than the design ground motion values contained in the 1994 Provisions and in recent editions of the Uniform Building Code.[18] One unique issue for coastal California is that the recurrence interval for the estimated maximum-magnitude earthquake is less than the recurrence interval represented by a 10-percent probability of being exceeded in 50 years. In other words, the recurrence interval for a maximum magnitude earthquake is 100 to 200 years versus 500 years.

Given that the maximum earthquake for many seismic faults in coastal California is fairly well known, a decision was made to develop a procedure that would use the best estimate of ground motion from maximum-magnitude earthquakes on seismic faults with higher probabilities of occurrence. For the purpose of the 1997 Provisions, these earthquakes are defined as "deterministic earthquakes." Following this approach and recognizing the inherent margin of 1.5 contained in the Provisions, it was determined that the level of seismic safety achieved in coastal California would be approximately equivalent to that associated with a 2- to 5-percent probability of being exceeded in 50 years for areas outside of coastal California. The use of the deterministic earthquakes to establish the maximum considered earthquake ground motions for use in design in coastal California results in a level of protection close to that implied in the 1994 NEHRP Provisions. Additionally, this approach results in less drastic changes to ground motion values for coastal California than the alternative approach of using probabilistic maps.

Based on the inherent margin contained in the NEHRP Provisions, the ground motion corresponding to a 2-percent probability of being exceeded in 50 years was selected as the maximum considered earthquake ground motion for use in design where the deterministic earthquake approach discussed above is not used.

The 1997 NEHRP Provisions include two sets of maps for the mapped maximum considered earthquake spectral response accelerations at short periods ($S_s$) and at 1-second period ($S_1$). Each set consists of 12 maps. In seismic design complying with the 2000 IBC, $S_s$- and $S_1$- values are to be determined from Figures 1615(1) through 1615(6). Where a site is between contours, straight-line interpolation or the value of the higher contour may be used. Figure 1615 is adapted from Maps 1 through 24 of the 1997 NEHRP Provisions.

The short- and long-period site coefficients, $F_a$ and $F_v$ respectively, of the 1997 NEHRP Provisions and the 2000 IBC are the same as those of the 1994 NEHRP Provisions, except that $F_a$ is a function of $S_s$, rather than $A_a$, and $F_v$ is a function of $S_1$, rather than $Av$ (Table 1-1). The conversion is based on $S_s = 2.5A_a$ and $S_1 = A_v$. $F_aS_s$ is $S_{MS}$, the maximum considered earthquake (MCE) spectral acceleration in the short-period range adjusted for site class effects. $F_vS_1$ is $S_{M1}$, the MCE spectral acceleration at 1-second period adjusted for site class effects.

Five-percent damped design spectral response accelerations at short-periods, $S_{DS}$, and at 1-second period, $S_{D1}$, are equal to two-thirds $S_{MS}$ and two-thirds $S_{M1}$, respectively. In other words, the design ground motion is 1/1.5 or two-thirds times the soil-modified MCE ground motion. This is in recognition of the inherent margin contained in the NEHRP Provisions that would make collapse unlikely under one and one-half times the design level ground motion. Table 1-2 summarizes the derivation of the design quantities $S_{DS}$ and $S_{D1}$.

Section 1615.1.4 of the 2000 IBC provides a general method for obtaining 5-percent damped response spectrum form the site design acceleration response parameters $S_{DS}$ and $S_{D1}$. This spectrum is based on that proposed by Newmark and Hall[10] as a series of three curves representing the short-period range, a region of constant spectral response acceleration; the long-period range, a region of constant spectral response velocity; and the very long-period range, a region of constant spectral response displacement. Figure 1-18 shows that response acceleration at any period in the short-period range is equal to the design spectral response acceleration at short periods, $S_{DS}$:

$$S_a = S_{DS} \qquad \text{(Eq. 1-11)}$$

The spectral response acceleration at any point in the constant velocity range (Fig. 1-18) can be obtained from the relationship:

$$S_a = \frac{S_{D1}}{T} \qquad \text{(Eq. 1-12)}$$

The constant displacement domain of the response spectrum is not included in the generalized response spectrum because relatively few structures have a period long enough to fall into this range.

The ramp building up to the flat top of the design spectrum (Fig. 1-18) is defined by specifying that the spectral response acceleration at zero period is equal to 40 percent of the spectral response acceleration corresponding to the flat top, $S_{DS}$, and that the period $T_0$, at which the ramp ends is 20 percent of the period, $T_s$, at which the constant acceleration and the constant velocity portions of the spectra meet. That period,

$$T_s = S_{D1}/S_{DS} \qquad \text{(Eq. 1-13)}$$

is solely a function of the seismicity and the soil characteristics at the site of the structure. It also serves as the dividing line between short- and long-period structures.

## 1.7 Dynamics of Multi-Degree-of-Freedom Systems

### 1.7.1 Equations of motion

The building in Figure 1-19 is used to illustrate multi-degree-of-freedom analysis.[3,4] The mass of the structure is assumed to be concentrated at the floor levels (lumped-mass idealization) and subject to lateral displacements only. The equations of dynamic equilibrium of the system may be written as

$$[m]\{\ddot{x}\}+[c]\{\dot{x}\}+[k]\{x\} = \{p(t)\} \qquad \text{(Eq. 1-14)}$$

where $\{\ddot{x}\}$, $\{\dot{x}\}$, and $\{x\}$ are the acceleration, velocity, and displacement vectors, respectively; $\{p(t)\}$ is the load vector; and $[m]$, $[c]$, and $[k]$ are the mass, damping, and stiffness matrices, respectively. The mass matrix for a lumped mass system is diagonal; i.e., the inertia force corresponding to any degree of freedom depends only on the acceleration in that degree of freedom. In general, it is not practical to evaluate the damping coefficients in matrix $[c]$, and damping is expressed in fractions of critical damping. The stiffness coefficient, $k_{ij}$, in matrix $[k]$ is the force corresponding to displacement coordinate $i$, resulting from a unit displacement of coordinate $j$.

### 1.7.2 Vibration mode shapes and frequencies

The free vibration behavior of a structure corresponds to no damping ($[c] = [0]$) and no applied loading ($\{p\} = \{0\}$), so that Equation 1-14 becomes

$$[m]\{\ddot{x}\}+[k]\{x\} = \{0\} \qquad \text{(Eq. 1-15)}$$

The motions of a system in free vibration are simple harmonic. Thus,

$$\{x\} = \{A\} \sin \omega t \qquad \text{(Eq. 1-16)}$$

where $\{A\}$ represents the amplitude of motion and $\omega$ is the circular frequency.

Introducing Equation 1-16 and its second derivative into Equation 1-15,

$$-\omega^2[m]\{A\}+[k]\{A\} = \{0\} \qquad \text{(Eq. 1-17)}$$

Equation 1-17 is a form of eigenvalue equation. Computer programs are available for the solution of very large eigenvalue equation systems. Figure 1-20 shows the solution to Equation 1-17 for an $n$-degree-of-freedom system consists of a frequency, $\omega_m$, and a mode shape, $\{\varphi_m\}$, for each of its $n$ modes of vibration. The distinguishing feature of a mode of vibration is that a dynamic system can, under certain circumstances, vibrate in that mode alone; during such vibration, the ratio of the displacements of any two masses remains constant with time. These ratios define the characteristic shape of the mode; the absolute amplitude of motion is arbitrary.

### 1.7.3 Modal equations of motion

An important simplification in Equation 1-14 is possible because the vibration mode shapes of any multi-degree system are orthogonal with respect to the mass and stiffness matrices. The same type of orthogonality condition may be assumed to apply to the damping matrix as well:

$$\{\varphi_m\}^T[m]\{\varphi_n\} = 0,\ \{\varphi_m\}^T[c]\{\varphi_n\} = 0,\ \{\varphi_m\}^T[k]\ \{\varphi_n\} = 0 \text{ for } m \neq n \qquad \text{(Eq. 1-18)}$$

Figure 1-20 shows that any arbitrary displaced shape of the structure may be expressed in terms of the amplitudes of mode shapes, treating them as generalized displacement coordinates:

$$\{x\} = [\varphi]\{X\} \qquad \text{(Eq. 1-19)}$$

in which $\{X\}$, the vector of the so-called normal coordinates of the system, represents the vibration mode amplitudes.

Substituting Equation 1-19 and its derivatives into Equation 1-14 and multiplying the resulting equation by the transposition of any mode shape vector yields

$$\{\varphi_m\}^T[m]\{\varphi_m\}\ddot{X}_m + \{\varphi_m\}^T[c]\ \{\varphi_m\}\dot{X}_m+\{\varphi_m\}^T[k]\{\varphi_m\}X_m = \{\varphi_m\}\{p(t)\} \qquad \text{(Eq. 1-20)}$$

by virtue of the orthogonality properties of Equation 1-18. Introducing

| | | | |
|---|---|---|---|
| Generalized mass | $M_m$ | $= \{\varphi_m\}^T[m]\{\varphi_m\}$ | (Eq. 1-21) |
| Generalized damping | $C_m$ | $= \{\varphi_m\}^T[c]\{\varphi_m\}$ | |
| Generalized stiffness | $K_m$ | $= \{\varphi_m\}^T[k]\ \{\varphi_m\}$ | |
| Generalized loading | $P_m(t)$ | $= \{\varphi_m\}^T\{p(t)\}$ | |

and recognizing that the generalized damping, stiffness, and mass are related:

$$C_m = 2\beta_m\ \omega_m M_m \quad \text{and} \quad K_m = \omega^2{}_m M_m \qquad \text{(Eq. 1-22)}$$

where $\beta_m$ is the fraction of critical damping in mode $m$, Equation 1-20 becomes

$$\ddot{X}_m + 2\beta_m \omega_m \dot{X}_m + \omega^2{}_m X_m = P_m(t)/M_m \qquad \text{(Eq. 1-23)}$$

Equation 1-23 shows that the equation of motion of any mode, $m$, of a multi-degree system is equivalent to the equation for a single-degree system (Eq. 1-1). Thus, the mode shapes or normal coordinates of a multi-degree system reduce its equations of motion to a set of independent or decoupled equations (as against the coupled equations of motion, Eq. 1-14).

### 1.7.4 Modal superposition analysis of earthquake response

The dynamic analysis of a multi-degree system by modal superposition requires the solution of Equation 1-23 for each mode to obtain its contribution to response. The total response is obtained by superposing the modal effects (Eq. 1-16).

Figure 1-21 shows that in the case of earthquake excitation the effective load is[6]

$$\{p_{eff}(t)\} = -[m]\{1\}\ddot{x}_g(t) \qquad \text{(Eq. 1-24)}$$

where $\{1\}$ represents a unit vector of dimension $n$ (the total number of degrees of freedom). Substituting Equation 1-24 into Equation 1-21 gives

$$P_{meff}(t) = -\{\varphi_m\}^T[m]\{1\}\ddot{x}_g(t) = -L_m\ddot{x}_g(t) \qquad \text{(Eq. 1-25)}$$

where: $$L_m = \{\varphi_m\}^T[m]\{1\} \qquad \text{(Eq. 1-26)}$$

represents the *earthquake participation factor* for mode $m$. Introducing Equation 1-26 into Equation 1-23, the equation of motion for mode $m$ of a multi-degree system subject to earthquake excitation becomes

$$\ddot{X}_m + 2\beta_m\omega_m\dot{X}_m + \omega_m{}^2X_m = -\frac{L_m}{M_m}\ddot{x}_g(t) \qquad \text{(Eq. 1-27)}$$

The response of the $m$th mode at any time, $t$, may be obtained, by analogy with Equation 1-3 from

$$X_m(t) = -\frac{L_m}{M_m}\frac{1}{\omega_m}\int_0^t \ddot{x}_g(\tau)\exp[-\beta_m\omega_m(t-\tau)]\sin\omega_m(t-\tau)d\tau \qquad \text{(Eq. 1-28)}$$

or, using the symbol $R_m(t)$ to represent the value of the integral at time $t$,

$$X_m(t) = -\frac{L_m}{M_m}\frac{R_m(t)}{\omega_m} \qquad \text{(Eq. 1-29)}$$

The total response of the $n$-degree system may be obtained as follows:

*Displacements* From Equation 1-16

$$\{x\} = [\varphi]\{X\} = -[\varphi]\left\{\frac{L_m}{M_m}\frac{R_m(t)}{\omega_m}\right\} \qquad \text{(Eq. 1-30)}$$

*Elastic forces*

$$\{f_s\} = [k]\{x\} = [k][\varphi]\{X\} = [m][\varphi][\Omega^2]\{X\} = -[m][\varphi]\left\{\frac{L_m}{M_m}\omega_m R_m(t)\right\} \quad \text{(Eq. 1-31)}$$

where $[\Omega^2]$ is a diagonal matrix of the squared modal frequencies $\omega^2{}_m$. It should be noted that the elastic force vector associated with the *m*th mode is

$$\{f_{sm}\} = -[m][\varphi_m]\left\{\frac{L_m}{M_m}\omega_m R_m(t)\right\} \quad \text{(Eq. 1-31a)}$$

*Internal forces*

These can be found from $\{f_s\}$ by statics.

*Base shear*

The base shear associated with the *m*th mode is the summation, over *n* stories, of the elastic forces associated with that mode:

$$V_m(t) = \{1\}^T\{f_{sm}(t)\} = -\{1\}^T[m][\varphi_m]\left\{\frac{L_m}{M_m}\omega_m R_m(t)\right\} = -\frac{L^2{}_m}{M_m}\omega_m R_m(t) \quad \text{(Eq. 1-32)}$$

from Equation 1-23. The total base shear is

$$V(t) = -\sum_{m=1}^{n}\frac{L_m^2}{M_m}\omega_m R_m(t) = -\sum_{m=1}^{n}\frac{W_m}{g}\omega_m R_m(t) \quad \text{(Eq.1-32a)}$$

where:

$$W_m = \frac{L_m^2}{M_m}g \quad \text{(Eq. 1-33)}$$

$W_m$ is the effective weight for mode *m*, and represents the portion of the total structural weight that is effective in developing base shear in the *m*th mode.

An important advantage of the mode superposition procedure is that an approximate solution can frequently be obtained by considering only the first few modes (sometimes, just the first or fundamental mode) in analysis.

An important limitation is that the mode superposition procedure is not applicable to any structure that is stressed beyond the elastic limit.

### 1.7.5 Response spectrum analysis

The entire response history of a multi-degree system is defined by Equations 1-30 and 1-31 once the modal response amplitudes are determined (Eq. 1-29). The maximum response of any mode can be obtained from the earthquake response spectra by procedures used earlier for single-degree systems.

On this basis, introducing $S_{vm}$, the spectral pseudo-velocity for mode $m$, into Equation 1-29 leads to an expression for the maximum response of mode $m$

$$|X_{m_{\max}}| = \frac{L_m}{M_m}\frac{S_{vm}}{\omega_m} = \frac{L_m}{M_m}S_{dm} \qquad \text{(Eq. 1-34)}$$

where $S_{dm}$ is the spectral displacement for mode $m$. The distribution of maximum displacement in mode $m$ is given (from Eq. 1-27) by

$$\{|x_{m_{\max}}|\} = \{\varphi_m\}|X_{m_{\max}}| = \{\varphi_m\}\frac{L_m}{M_m}S_{dm} \qquad \text{(Eq. 1-35)}$$

Similarly, the distribution of maximum effective earthquake forces in the $m$th mode (from Eq. 1-31) becomes

$$\{|f_{sm_{\max}}|\} = [m]\{\varphi_m\}\frac{L_m}{M_m}\omega_m S_{vm} = [m]\{\varphi_m\}\frac{L_m}{M_m}S_{am} \qquad \text{(Eq. 1-36)}$$

where $S_{am}$ is the spectral pseudo-acceleration for mode $m$. From Equation 1-32a, the maximum base shear in mode $m$ is

$$V_{m_{\max}} = \frac{W_m}{g}S_{am} = \frac{L_m^2}{M_m}S_{am} \qquad \text{(Eq. 1-37)}$$

Because, in general, the modal maxima (e.g., $V_{m_{\max}}$) do not occur simultaneously, they cannot be directly superimposed to obtain the maximum of the total response (e.g., $V_{\max}$). The direct superimposition of modal maxima provides an upper bound to the maximum of total response. A satisfactory estimate of the total response can usually be obtained from

$$V_{\max} \approx \sqrt{\Sigma V^2_{m_{\max}}} \qquad \text{(Eq. 1-38)}$$

As discussed, the summation needs to include only the lower few modes.

### 1.7.6 First-mode analysis of multi-degree-of-freedom systems

This is illustrated through an example adapted from Clough.[3]

A typical five-story building is shown in Figure 1-22a; it is assumed to have a period $T_1 = 0.5$ second and a damping ratio $\beta_1 = 10\%$ in the first mode. For these values, Figure 1-22b gives the following spectral values: $S_{d1} = 0.48$ in. (12 mm), $S_{v1} = 6.0$ in./sec (152 mm/sec), $S_{a1} = 76.0$ in./sec$^2$ ( = $0.2g$, 1.96 m/sec$^2$).

As is customary, the mass is assumed concentrated in the floor slabs. In a complete analysis, the lateral motion of each floor slab would be an independent degree of freedom. An approximate single-degree-of-freedom analysis for the building can be made by assuming that the lateral displacements are of a specified form. A reasonable assumption is that the displacements corresponding to the first mode increase linearly with height, i.e., $\varphi_{i1} = h_i/h_n$.

The generalized mass and earthquake participation factors corresponding to the first mode are then given by

$$M_1 = \{\varphi_1\}^T[m]\{\varphi_1\} = \sum_i m_i\varphi_{i1}^2 \qquad \text{(Eq. 1-21a)}$$

$$= \frac{500}{g}(0.2^2 + 0.4^2 + 0.6^2 + 0.8^2 + 1.0^2) = \frac{1,100 \text{ kips}}{g}$$

$$L_1 = \{\varphi_1\}^T[m]\{1\} = \sum_i m_i\varphi_{i1} \qquad \text{(Eq. 1-26a)}$$

$$= \frac{500}{g}(0.2 + 0.4 + 0.6 + 0.8 + 1.0) = \frac{1,500 \text{ kips}}{g}$$

The maximum earthquake deflection is given by Equation 1-35

$$\{|x_{1_{\max}}|\} = \{\varphi_1\}\frac{L_1}{M_1}S_{d1} = \{\varphi_1\}\frac{1,500}{1,100} \times 0.48 \text{ in.} = \{\varphi_1\} \times 0.65 \text{ in.} \qquad \text{(Eq. 1-35a)}$$

For example,

$$x_{31_{\max}} = 0.6 \times 0.65 = 0.39 \text{ in., etc.} \qquad \text{(Fig. 1-22a)}$$

The maximum base shear force is given by Equation 1-37

$$V_{1_{\max}} = \frac{W_1}{g}S_{a1} = \frac{L_1^2}{M_1}S_{a1} = \frac{(1,500)^2}{1,000} \times \frac{76.0}{386} = 403 \text{ kips} \qquad \text{(Eq. 1-34a)}$$

The forces at the various story levels may be obtained by distributing the base shear force according to Equations 1-36 and 1-37:

$$\{f_{s1_{\max}}\} = [m]\{\varphi_1\}\frac{V_{1\max}}{L_1} \qquad \text{(Eq. 1-36a)}$$

or

$$f_{s1_{\max}} = \frac{m_i\varphi_{i1}V_{1_{\max}}}{L_1} = \frac{500}{1,500}\varphi_{i1} \times 403 \text{ kips} = \varphi_{i1} \times 134.3 \text{ kips}$$

For example,

$$f_{s31_{\max}} = 0.6 \times 134.3 = 80.6 \text{ kips, etc.} \qquad \text{(Fig. 1-22c)}$$

### 1.7.7 Comparison of static-force procedure of 2000 IBC with first-mode analysis of multi-degree systems

The design base shear and the distribution of that shear along the height of a building, as prescribed in the 2000 IBC, and as obtained from the first-mode analysis of a multi-degree system (Section 1.7.6), are compared in Table 1-3. It can be seen that the code-prescribed distribution of base shear corresponds essentially to the fundamental mode of vibration response: the higher modes are accounted for through increases in the coefficient $k$ for structures with elastic fundamental period exceeding 0.5 second. As to magnitude, the base shear coefficient, $C_s$, of the code is compared with the base shear coefficient, $S_a/g$, from the first mode analysis in Figure 1-23. It is clear that the code forces, which are assumed to be elastically resisted by a structure, are substantially smaller than those that would develop if the structure were to respond elastically to earthquake ground motions taken in the SAC steel project[19] to be representative of the Los Angeles area. Thus, code-designed buildings would be expected to undergo fairly large inelastic deformations when subjected to an earthquake of such intensity. The realization that it is economically unwarranted to design buildings to resist major earthquakes elastically, and the recognition of the capacity of structures possessing adequate strength and deformability to withstand major earthquakes inelastically, lie behind the relatively low forces specified by the codes, coupled with the special requirements for ductility particularly at and near member connections.

## 1.8 Code Design Criteria

The procedures and limitations for the design of structures by the 2000 IBC are determined considering zoning, site characteristics, occupancy, configuration, structural system, and height. Two of the major parameters in the selection of design criteria are occupancy and structural configuration.

Four categories of occupancy are defined in Table 1604.5 of the 2000 IBC: I – standard occupancy, II – high-occupancy and hazardous facilities (buildings and other structures that represent a substantial hazard to human life in the event of failure), III – essential facilities, and IV – low-hazard facilities (buildings and other structures that represent a low hazard to human life in the event of failure). Occupancy categories I and II are equivalent to Seismic Use Groups II and III, respectively.

Structural configuration is addressed by defining two categories of structural irregularities in Table 1616.5.1 (plan structural irregularities) and 1616.5.2 (vertical structural irregularities). Five different types of plan irregularity are defined in Table 1616.5.1 and illustrated in Figure 1-24: Torsional irregularity (to be considered when diaphragms are not flexible), reentrant corners, diaphragm discontinuity, out-of-plane offsets, and nonparallel lateral-force-resisting systems. Torsional irregularity is subdivided into torsional irregularity and extreme torsional irregularity. Five different types of vertical structural irregularities are defined in Table 1616.5.2 and illustrated in Figure 1-25: Stiffness irregularity – soft story, weight (mass) irregularity, vertical geometric irregularity, in-plane discontinuity in vertical lateral-force-resisting elements, and discontinuity in capacity – weak story. Stiffness irregularity is subdivided into stiffness irregularity – soft story and stiffness irregularity – extreme soft story. Exceptions are provided to the definition of stiffness irregularity and mass irregularity (1616.5.2). Where no story drift ratio under design lateral forces is greater than 1.3 times the story drift ratio of the story above, a structure may be deemed to not have stiffness or mass irregularity. Torsional effects need not be considered in the calculation of story drifts for the purpose of this determination.

The story drift ratio relationships for the top two stories of the building are not required to be evaluated. Also, stiffness and mass irregularities are not required to be considered for one-story buildings in any seismic design category or for two-story buildings in Seismic Design Category A, B, C or D (see below for discussion of seismic design category). Regular structures are defined as having no significant physical discontinuities in plan or vertical configuration or in their lateral-force-resisting systems, such as those identified for irregular structures.

Static as well as dynamic analysis procedures are recognized in the 2000 IBC for the determination of seismic effects on structures. The dynamic analysis procedures of Section 1618 are always acceptable for design. The equivalent lateral-force procedure of Section 1617.4 is allowed only for certain given combinations of seismic design category, regularity, and height.

The Seismic Design Category (SDC) of the IBC is a function of occupancy (Seismic Use Group or SUG) and of soil-modified seismic risk at the site of the structure in the form of the design spectral response acceleration at short periods, $S_{DS}$, and the design spectral response acceleration at 1-second period, $S_{D1}$. For a structure, the SDC needs to be determined twice – first as a function of $S_{DS}$ by the 2000 IBC Table 1616.3 (1) (reproduced here as Table 1-4) and a second time as a function of $S_{D1}$ by the 2000 IBC Table 1616.3(2) (reproduced here as Table 1-5). The more severe category governs.

The IBC has chosen to include a simplified analysis procedure (1617.5), which represents slight modifications of a procedure first introduced in the 1997 UBC. It was developed by the SEAOC Seismology Committee in response to a need to provide conservative, simple methods to determine design forces for certain simple buildings. The procedure is limited to buildings of light frame construction not exceeding three stories in height (excluding basements), and to buildings of any construction other than light frames, not exceeding two stories in height. Table 1-6 summarizes the applicability of various analysis procedures per the 2000 IBC.

A summary of the above discussion is provided in Figure 1-26.

Structures with a vertical discontinuity in capacity (weak story) are not permitted to be over two stories or 30 feet in height where the weak story has a calculated strength of less than 65 percent of the story above (1620.1.3), except where the weak story is capable of resisting a total lateral seismic force of $\Omega_0$ times the design force prescribed in 1617.4.

Irregular structures are beyond the scope of this publication. The dynamic analysis procedure – specifically the modal response spectra analysis procedure – is used and illustrated in this publication. The equivalent lateral-force procedure is also illustrated because it is basically a prerequisite to the dynamic analysis procedure, as discussed later.

## 1.9 Analysis Procedures

The design load combinations of Section 1605 involve the terms $E$ (the combined effect of horizontal and vertical earthquake-induced forces) and $E_m$ (the maximum seismic load effect). Both terms are defined in part by $Q_E$ (the effect of horizontal seismic forces), the determination of which requires structural analysis in accordance with the requirements of this section. Such analysis also leads to the determination (1617.4.6) of the design story drift, Δ, which must be kept within the limits prescribed in 1617.3. Certain types of structures are exempt from seismic design requirements while structures assigned to SDC A must satisfy the provision of 1616.4 (1614.1). Also the design story drift can be evaluated per 1617.5.4 when the simplified analysis procedure of 1617.5 is used.

The commentary to the 1997 NEHRP Provisions lists the standard procedures for the analysis of forces and deformations in structures subjected to earthquake ground motion, in the order of expected rigor and accuracy, as follows:

1. Equivalent lateral-force procedure (IBC 1617.4)
2. Modal analysis procedure (response spectrum analysis) (IBC 1618.1-1618.9)
3. Inelastic static procedure, involving incremental application of a pattern of lateral forces and adjustment of the structural model to account for progressive yielding under load application (push-over analysis), and
4. Inelastic response history analysis involving step-by-step integration of the coupled equations of motion (IBC 1618.10.3).

The IBC chose to include a simplified analysis procedure (1617.5) that, in the order of expected rigor and accuracy, would precede Item 1 above. Push-over analysis is not formally recognized in the IBC, although it is high among the recognized analysis procedures in the ATC 33/FEMA 273 *Seismic Rehabilitation Guidelines*.[20]

The IBC also recognizes an elastic or linear time-history analysis (1618.10.2) that, in the order of expected rigor and accuracy, would probably rank the same as Item 2 above. It should be recognized that Items 1 and 3 above and the simplified analysis procedure of Section 1617.5 are static procedures, while the other analysis procedures mentioned are dynamic procedures. As indicated in the preceding section, dynamic analysis is always acceptable for design. Static procedures are allowed only for certain combinations of seismic risk, structural regularity, occupancy, and height.

The equivalent lateral-force procedure (1617.4) and the modal analysis procedure (Sections 1618.1 through 1618.9) are both based on the approximation that inelastic seismic structural response can be adequately represented by linear analysis of the lateral-force-resisting system using the design spectrum, which is the elastic acceleration response spectrum amplified by the importance factor, $I_E$, and reduced by the response modification factor, $R$. The effects of the horizontal component of ground motion perpendicular to the direction of analysis, the vertical component of ground motion and torsional motions of the structure are all considered in the same approximate manner in both cases, if only two-dimensional analysis is used. The main difference between the two procedures lies in the distribution of the seismic lateral forces over the building. In the modal analysis procedure, the distribution is based on the deformed shapes of the natural modes of vibration, which are determined from the distribution of the masses and the stiffnesses of the structure. In the equivalent lateral-force procedure, the distribution is based on simplified formulas that are appropriate for regular structures (1617.4.3). Otherwise, the two procedures are subject to the same limitations. The total design forces used in the two procedures are also similar (see Section 1618.7).

According to the 1997 NEHRP Commentary, the equivalent lateral-force procedure and the modal analysis procedure "are all likely to err systematically on the unsafe side if story strengths are distributed irregularly over height. This feature is likely to lead to concentration of ductility demand in a few stories of the building. The inelastic static (or the so-called push-over) procedure is a method to more accurately account for irregular strength distribution. However, it also has limitations and is not particularly applicable to tall structures or structures with relatively long fundamental periods of vibration."

Current professional practice and computational capabilities may lead to the choice of a three-dimensional model for both static and dynamic analyses. Although three-dimensional models are not specifically required for static-force procedures, nor for dynamic analysis procedures for regular structures with independent orthogonal seismic force-resisting systems, they often have important advantages over two-dimensional models.

A three-dimensional model is appropriate for the analysis of torsional effects (actual plus accidental), diaphragm deformability, and systems having nonrectangular plan configurations. According to the commentary to the 1999 SEAOC Blue Book,[21] when a three-dimensional model is needed for any purpose, "it can also serve for all required loading conditions, including seismic loading in each principal direction, other selected directions, and for orthogonal effects." Three-dimensional analysis is beyond the scope of this publication.

The actual strength and other properties of the various components of a structure can be explicitly considered only by a nonlinear analysis of dynamic response by direct integration of the coupled equations of motion. If the two translational motions and the torsional motion are expected to be essentially uncoupled, it is sufficient to include only one degree of freedom per floor in the direction of analysis; otherwise at least three degrees of freedom per floor, two translational and one torsional, need to be included. The 1997 NEHRP Commentary points out, and it cannot be overemphasized, that the results of nonlinear response history analysis of mathematical structural models are only as good as the models chosen to represent the structure vibrating at amplitudes of motion large enough to cause significant yielding at several locations. Proper modeling and proper interpretation of results require background and experience. Also, reliable results can be achieved only by calculating the response to several ground motions – recorded accelerograms and/or simulated motions – and examining the statistics of response.

The least rigorous analytical procedure that may be used in determining the design seismic forces and deformations in a structure depend on the seismic zone in which the structure is located and the structural characteristics (in particular, regularity and height). See discussion in the preceding section and Table 1-6.

### 1.9.1 Dynamic analysis procedures

It should be obvious by now that the IBC formally recognizes three dynamic analysis procedures: modal analysis, elastic time-history analysis, and inelastic time-history analysis. Only modal analysis is considered in this publication.

***Ground motion representation.*** As mentioned and discussed in Section 1.6.2 of this publication, 1615.1.4 of the 2000 IBC provides a general method for obtaining a 5-percent damped response spectrum from the site design acceleration response parameters $S_{DS}$ and $S_{D1}$.

The 2000 IBC 1615.2 details a site-specific procedure for determining ground motion accelerations. It specifically enumerates the five significant aspects that must be accounted for in an investigation undertaken to determine site-specific ground motion: 1) regional seismicity and geology; 2) the expected recurrence rates and maximum magnitudes of events on known source zones; 3) the location of the site with respect to these source zones; 4) near-source effects, if any; and 5) subsurface site characteristics and conditions.

A probabilistic, site-specific maximum considered earthquake (MCE) acceleration response spectrum is defined in 1615.2.1 . Section 1615.2.3 defines a deterministic, site-specific MCE response spectrum. Section 1615.2.2 defines an acceleration response spectrum that represents a deterministic limit on MCE ground motion. The site-specific MCE ground-motion spectrum is required to be taken as the lesser of the probabilistic MCE ground motion of 1615.2.1 or the deterministic MCE ground motion of 1615.2.3, subject to a minimum of the deterministic limit ground motion of 1615.2.2 (see exception to 1615.2.1 and Figure 1-27).

Section 1615.2.1 defines the probabilistic MCE acceleration-response spectrum as corresponding to a 2-percent probability of being exceeded within 50 years. The probabilistic MCE spectral response acceleration at any period, $S_{am}$, is to be taken from this spectrum. It is worthwhile to point out that the same probability of being exceeded over the same period of time is used in the generalized procedure for determining MCE spectral response accelerations.

In 1615.2.2, the deterministic limit on MCE ground motion is represented by the acceleration-response spectrum of Figure 1615.2.2, where the coefficients, $F_a$ and $F_v$ are as given in Tables 1615.1.2 (1) and 1615.1.2 (2), respectively (see Table 1-1). For this spectrum, the value of the mapped short-period spectral-response acceleration, $S_S$, is taken as $1.5g$ and the value of the mapped spectral response acceleration at 1-second period, $S_1$, is taken as $0.6g$.

According to 1615.2.3, the median spectral response accelerations, $S_{am}$, at all periods resulting from a characteristic earthquake on any known active fault within the region and amplified by a factor of 1.5, define the deterministic MCE acceleration-response spectrum. The median values are increased by 50 percent to represent MCE ground motion values. The deterministic MCE spectral response acceleration, $S_{am}$, is to be taken from this spectrum.

Section 1615.2.4 requires that the spectral response acceleration, $S_a$, at any period is to be taken as two thirds of the MCE response spectral acceleration, $S_{am}$, at that period, subject to a minimum of 80 percent of the design spectral response acceleration, $S_a$, at the same time period determined from the general response spectrum of Figure 1615.1.4 (reproduced here as Fig. 1-18). The two-thirds factor is in recognition of the inherent margin contained in the NEHRP Provisions that would make collapse unlikely at less than 1.5 times the design level ground motion.

Section 1615.2.5 requires that the $S_a$, as defined in Section 1615.2.4, at a period of 0.2 second divided by $g$, is to be taken as the design spectral-response acceleration coefficient at short periods, $S_{DS}$. The value of $S_a$, as defined in 1615.2.4, at a period of 1 second divided by $g$, is to be taken as the design spectral-response acceleration coefficient at a period of 1 second, $S_{D1}$. Neither value is to be taken as less than 80 percent of the corresponding value obtained from Figure 1-18.

Either the general procedure response spectrum of 1615.1.4 or the site-specific response spectrum of 1615.2 may be used for modal response spectra analysis.

***Mathematical model.*** The 1997 edition of the UBC introduced a set of modeling requirements for structural analysis – static as well as dynamic. As pointed out in the 1999 SEAOC Blue Book,[21] certain key assumptions are common to most analysis models: the structure is assumed to be linearly elastic; small deformation theory applies; structural mass is commonly lumped at a few selected joints and nodes; and energy dissipation (damping) is assumed to be viscous or velocity proportional. The 1997 UBC specifically requires that the mathematical model of a physical structure must include:

- all elements of the lateral-force-resisting system
- stiffnesses and strengths of all elements that are significant to the distribution of forces
- representation of spatial distribution of mass and stiffness of the structure
- effects of cracked sections in concrete and masonry structures
- contribution of panel zone deformations to story drift for steel moment frame structures.

IBC 2000 Section 1618.1 specifically includes the last three bullet items. Two-dimensional vs. three-dimensional analysis is addressed. This topic has been discussed earlier in this section. Diaphragm flexibility is also addressed.

## 1.10 Scaling of Results of Dynamic Analysis

The results of spectrum analysis are required to be scaled up to and are permitted to be scaled down to the base shear calculated with the Equivalent Lateral Force (ELF) approach (Table 1-7) and using a period taken as 1.2 times the upper limit coefficient for period calculation, $C_u$, times the period calculated using approximate period formulas. This scaling is primarily done to ensure that the design forces are not under-estimated through the use of a structural model that is excessively flexible. The rather large period estimate that is permitted when calculating the ELF base shear is an arbitrarily selected approach to providing some incentive for use of dynamic analysis through limited reductions in base shear.

For buildings with $T$ (rationally determined, subject to a maximum of $C_u T_a$) > 0.7 second, located on Site Class E or F sites where $S_{D1} > 0.2g$, scaling must be done on the basis of the ELF base shear calculated using the same period.

The scaling provisions have been changed in the 2000 NEHRP Provisions to require that when the base shear obtained from a dynamic analysis is less than 85 percent of the base shear obtained by the ELF procedure, the dynamic analysis results be scaled to no less than 85 percent of the ELF values (Table 1-7). However, when the response spectrum analysis produces results that are larger than the ELF values, no scaling is permitted. The 85-percent rule is felt to be a more direct way of providing an incentive for performing a dynamic analysis.

The 1997 UBC required scaling up and permits scaling down of the results of dynamic analysis to 90 percent of the ELF design base shear for regular buildings and 100 percent of the ELF design base shear for irregular buildings (Table 1-7). This distinction between regular and irregular buildings, always made in the UBC, is not a consideration in the 1997 NEHRP Provisions or the 2000 IBC. The distinction was supposed to act as a disincentive against the design of irregular structures, but was not effective in that role.

The scaling rules of the 1994 UBC were somewhat different from those of the 1997 UBC. These are included in Table 1-7 as being of possible interest to some readers.

The deletion of the scale-down feature in the 2000 NEHRP Provisions is justified by pointing out that the ELF method may result in an under prediction of response for structures with significant higher mode participation. However, with the deletion, there will be no scale-down to static force levels even when a site-specific response spectrum is used in dynamic analysis. This probably places too much confidence in the geotechnical input. The confidence is now felt justified in view of the controls placed on the geotechnical input by the provisions in 1615.2.

## 1.11 Response Spectrum Analysis

In accordance with the discussion in Section 1.6, the following steps are involved in response spectrum analysis.

Step 1– Develop mathematical model of structure…to represent proper spatial distribution of mass and stiffness of structure. (1618.1.1)

Step 2 –Determine mode shapes, $\{\varphi_m\}$, and corresponding periods, $T_m$, of structure…by eigenvalue analysis.

Step 3 –For each mode $m$, determine:

Earthquake participation

Factor, $$L_m = \sum_{i=1}^{n} w_i \varphi_{im}/g$$

Modal mass, $M_m = \sum_{i=1}^{n} w_i \varphi_{im}^2 / g$

Effective weight, $W_m = \frac{L_m^2}{M_m} g$

Participating mass, $PM = \frac{L_m^2 g}{M_m W} = \frac{W_m}{W}$

where: $W = \sum_{i=1}^{n} w_i$

$w_i$ = weight at floor level $i$ as defined in 1618.4

$\varphi_{im}$ = displacement at floor level $i$ for mode $m$.

Step 4 – Determine number of modes to be considered...to represent at least 90 percent of participating mass of structure. (1618.2)

$$\Sigma PM = \Sigma(W_m/W) \geq 0.90$$

Step 5 – Determine spectral acceleration and seismic design coefficient for each mode.

a. From design response spectrum (1615.4 or 1615.2), determine $S_{am}$ for $T_m$
b. Determine modal seismic design coefficient

$$C_{sm} = S_{am} \times I_E / R$$

where: $I_E$ = Importance factor from Table 1604.5

and $R$ = Response modification factor from Table 1617.6

Step 6 – Determine modal base shears, $V_m$, and total dynamic base shear, $V_d$.

$$V_m = \frac{C_{sm}}{g} W_m$$

$$V_d = (V_1^2 + V_2^2 + ... + V_n^2)^{1/2}$$

Note: the 2000 IBC specifically requires that where closely spaced periods in the translational or torsional modes result in cross correlation of the modes, the complete quadratic combination (CQC) technique must be used, instead of taking the square root of the sum of the squares of the modal values, as shown above.

Step 7 – Determine design base shear from equivalent lateral-force procedure (1617.4.1, using $T' = 1.2C_uT_a$,) and compare with base shear from dynamic analysis.

$$V_s = \frac{S_{D1} I_E}{RT'} W$$

$$\le \frac{S_{DS} I_E}{R} W$$

$$\ge 0.044 S_{DS} I_E W$$

$$\ge \frac{0.5 S_1 I_E}{R} W$$ (for structures assigned to SDC E or F, or located where $S_1 \ge 0.6g$)

$T' = 1.2 C_u T_a$

$C_u$ is given in Table 1617.4.2

$T_a = C_T h_n^{3/4}$, or as permitted to be computed alternatively by 1617.4.2.1

Step 8 – Scale dynamic analysis results. (1618.7)

Scale factor $= V_s / V_d$

Adjusted $V_m$ $= (V_s/V_d)$ (Original $V_m$)

Adjusted $V_d$ = mean root square of adjusted $V_m$ values

$$V_d = (V_1^2 + V_2^2 + \ldots + V_n^2)^{1/2}$$

Step 9 – Distribute base shear for each mode over height of structure.

$$F_{im} = \frac{w_i \varphi_{im}}{\Sigma w_i \varphi_{im}} V_m$$

where: $F_{im}$ = lateral force at level $i$ for mode $m$

$V_m$ = base shear for mode $m$

Step 10 – Perform lateral analysis for each mode...to determine member forces for each mode of vibration being considered. (1618.6)

For rigid diaphragms...include accidental torsion in the distribution of story shears to lateral-force-resisting systems. (1618.8)

Step 11 – Combine dynamic analysis results (moments, shears, axial forces, and displacements) for all considered modes using root mean square combination (SRSS) or by the complete quadratic combination (CQC). . . to approximate total structure response or resultant design values. (1618.7)

## 1.12 Response Spectrum Analysis Example

### 1.12.1 General

A three-story reinforced concrete building is designed following the requirements of the 2000 IBC. The building is located in Los Angeles (on Site Class D). The dynamic analysis procedure is used as the basis of design.

### 1.12.2 Design criteria

A typical elevation of the building is shown in Figure 1-28(a). The member sizes for the structure are chosen as follows:

| | |
|---|---|
| Beams | 16.67 × 12 in. |
| Columns | 16.67 × 16.67 in. |

Material properties:

Concrete (all members) $f'_c = 4{,}000$ psi

All members are constructed of normal weight concrete ($w_c = 145$ pcf)

Reinforcement: $f_y = 60{,}000$ psi

Service Loads:

Assumed floor load = 386.4 kips/floor

Total weight $W$ = 386.4 × 3 = 1159.2 kips

*Seismic Design Data:*

The maximum considered earthquake spectral response acceleration at short period, $S_S = 1.5g$, and that at 1-second period, $S_1 = 0.6g$.

Assume standard occupancy or Seismic Use Group = I
and seismic importance factor, $I_E = 1.0$ (Table 1604.5)

Site Class = D
Site coefficient $F_a = 1.0$ [Table 1615.1.2(1)]
Site coefficient $F_v = 1.5$ [Table 1615.1.2(2)]

Adjusted $S_S = S_{MS} = F_a S_S = 1.0 \times 1.5g = 1.5g$ (Eq. 16-16)

Adjusted $S_1 = S_{M1} = F_v S_1 = 1.5 \times 0.6 = 0.9g$ (Eq. 16-17)

Design Spectral Response Acceleration Parameters (at 5-percent damping)

At short periods: $S_{DS} = 2/3(S_{MS}/g) = 2/3 \times 1.5 = 1.0$ (Eq. 16-18)

At 1-second period: $S_{D1} = 2/3(S_{M1}/g) = 2/3 \times 0.9 = 0.6$ (Eq. 16-19)

Special moment-resisting frame (SMRF) system gives $R = 8$ (Table 1617.6)

Seismic Design Category: based on both $S_{DS}$ [Table 1616.3(1)] and $S_{D1}$ [Table 1616.3(2)], the SDC for the example building is D.

**Design Basis**

Calculation of the design base shear and distribution of that shear along the height of the building using the equivalent lateral-force procedure (which is used in a majority of designs) is not appropriate and is not allowed by the International Building Code for buildings exceeding 240 feet in height in SDC D and above. In these cases, the Dynamic Analysis Procedure (1631) must be used. In this example, the Equivalent Lateral Force procedure could be used because the height of the building is 30 feet in SDC D (less than 240 feet). However, for illustration purposes, Modal Response Spectra Analysis (1618.1 through 1618.9) has been used.

**Given:**

$h_s$ = 10 feet
$w$ = 386.4 kips/floor [See Fig. 1-28(a)]
$E$ = 4,000 ksi
$I_{col}$ = 4,500 in.$^4$ each column (taken equal to $0.7I_g$)

**Determine mass matrix**

$$m = w/g = 386.4/386.4 = 1.0 \text{ kip-sec}^2\text{/in.}$$

$$[m]=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

**Determine stiffness matrix**

$$k = 12EI/h_s^3 = 12 \times 4{,}000 \times 9{,}000/(12 \times 10)^3 = 250 \text{ kips/in.}$$ [Fig. 1-28(b)]

$k_{ij}$ = force corresponding to displacement of coordinate $i$ resulting from a unit displacement of coordinate $j$

$$[k]=250\begin{bmatrix}2 & -1 & 0\\-1 & 2 & -1\\0 & -1 & 1\end{bmatrix}$$

**Determine determinant for matrix $[k] - \omega^2[m]$**

$$[k]-\omega^2[m]=\begin{bmatrix}500-\omega^2 & -250 & 0\\-250 & 500-\omega^2 & -250\\0 & -250 & 250-\omega^2\end{bmatrix}$$

Setting the determinant of the above matrix equal to zero yields the following frequencies:

$\omega_1$ = 7.036 radians/sec
$\omega_2$ = 19.685 radians/sec
$\omega_3$ = 28.491 radians/sec

The period is equal to $2\pi/\omega$

$T_1$ = 0.893 sec
$T_2$ = 0.319 sec
$T_3$ = 0.221 sec

**Find modal shapes**

*First mode:*

$$\begin{bmatrix} 500-(7.036)^2 & -250 & 0 \\ -250 & 500-(7.036)^2 & -250 \\ 0 & -250 & 250-(7.036)^2 \end{bmatrix} \begin{bmatrix} \varphi_{31} \\ \varphi_{21} \\ \varphi_{11} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\varphi_{31} = 1.0$ $\varphi_{21} = 0.802$ $\varphi_{11} = 0.445$ [Fig. 1-28c(i)]

*Second mode:*

$$\begin{bmatrix} 500-(19.685)^2 & -250 & 0 \\ -250 & 500-(19.685)^2 & -250 \\ 0 & -250 & 250-(19.685)^2 \end{bmatrix} \begin{bmatrix} \varphi_{32} \\ \varphi_{22} \\ \varphi_{12} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\varphi_{32} = 1.0$ $\varphi_{22} = -0.55$ $\varphi_{12} = -1.22$ [Fig. 1-28c(ii)]

*Third mode:*

$$\begin{bmatrix} 500-(28.491)^2 & -250 & 0 \\ -250 & 500-(28.491)^2 & -250 \\ 0 & -250 & 250-(28.491)^2 \end{bmatrix} \begin{bmatrix} \varphi_{33} \\ \varphi_{23} \\ \varphi_{13} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\varphi_{33} = 1.0$ $\varphi_{23} = -2.25$ $\varphi_{13} = 1.802$ [Fig. 1-28c(iii)]

**Determine modal mass and participation factors for each mode**

$$L_1 = \frac{\sum_{i=1}^{3} w_i \varphi_{i1}}{g} = 1.0 \text{ kip-sec}^2\text{/in.} (\varphi_{11} + \varphi_{21} + \varphi_{31})$$
$$= 1.0\,(0.445 + 0.802 + 1.0) = 2.247 \text{ kip-sec}^2\text{/in.}$$

$$M_1 = \frac{\sum_{i=1}^{3} w_i \varphi_{i1}^2}{g} = 1.0 \text{ kip-sec}^2\text{/in.} (\varphi_{11}^2 + \varphi_{21}^2 + \varphi_{31}^2)$$
$$= 1.0\,(0.445^2 + 0.802^2 + 1.0^2) = 1.841 \text{ kip-sec}^2\text{/in.}$$

$$L_2 = \frac{\sum_{i=1}^{3} w_i \varphi_{i2}}{g} = 1.0 \text{ kip-sec}^2\text{/in.} (\varphi_{12} + \varphi_{22} + \varphi_{32})$$
$$= 1.0\,(-1.22 - 0.55 + 1.0) = -0.77 \text{ kip-sec}^2\text{/in.}$$

$$M_2 = \frac{\sum_{i=1}^{3} w_i \varphi_{i2}^2}{g} = 1.0 \text{ kip-sec}^2\text{/in. } (\varphi_{12}^2 + \varphi_{22}^2 + \varphi_{32}^2)$$
$$= 1.0\,(1.22^2 + 0.55^2 + 1.0^2) = 2.791 \text{ kip-sec}^2\text{/in.}$$

$$L_3 = \frac{\sum_{i=1}^{3} w_i \varphi_{i3}}{g} = 1.0 \text{ kip-sec}^2\text{/in. } (\varphi_{13} + \varphi_{23} + \varphi_{33})$$
$$= 1.0\,(1.802 - 2.25 + 1.0) = 0.552 \text{ kip-sec}^2\text{/in.}$$

$$M_3 = \frac{\sum_{i=1}^{3} w_i \varphi_{i3}^2}{g} = 1.0 \text{ kip-sec}^2\text{/in. } (\varphi_{13}^2 + \varphi_{23}^2 + \varphi_{33}^2)$$
$$= 1.0\,(1.802^2 + 2.25^2 + 1.0^2) = 9.310 \text{ kip-sec}^2\text{/in.}$$

**Determine effective weight and participating mass for each mode**

$$W_1 = \frac{L_1^2}{M_1} g = \frac{2.247^2}{1.841} \times 386.4 \frac{\text{kip-sec}^2}{\text{in.}} \times \frac{\text{in.}}{\text{sec}^2} = 1,059.72 \text{ kips}$$
$$W_2 = \frac{L_2^2}{M_2} g = \frac{(-0.77)^2}{2.791} \times 386.4 = 82.08 \text{ kips}$$
$$W_3 = \frac{L_3^2}{M_3} g = \frac{(0.552)^2}{9.310} \times 386.4 = 12.65 \text{ kips}$$

$$\Sigma W_i = 1{,}154.45 \text{ kips}$$

$$PM_1 = \frac{W_1}{W} = \frac{1,059.72}{3 \times 386.4} = 0.914$$

$$PM_2 = \frac{W_2}{W} = \frac{82.08}{3 \times 386.4} = 0.071$$

$$PM_3 = \frac{W_3}{W} = \frac{12.65}{3 \times 386.4} = 0.011$$

$$\Sigma PM = 0.996$$

Therefore, consideration of the above three modes (modes 1, 2, 3) is sufficient per 1631.5.2. Indeed, the consideration of just the first mode would have been sufficient (as $PM_1 \geq 0.90$).

**Determine (spectral acceleration and) seismic design coefficient, $C_{sm}$, for each mode**

For the example building considered

$$C_{sm} = S_{am} I_E / R$$

$$= \frac{S_{D1} I_E}{RT} g = \frac{0.6 \times 1g}{8\,T} g = \frac{0.075}{T} g$$

$$\leq \frac{S_{DS}I_E}{R}g = \frac{1.0 \times 1g}{8\,T}g = 0.125\,g$$

Mode 1: $T_1 = 0.893$ sec $\quad C_{s1} = \frac{0.075}{0.893} = 0.0840g$

Mode 2: $T_2 = 0.319$ sec $\quad C_{s2} = \frac{0.075}{0.319} = 0.2351g \leq 0.125g$ $\quad$ Use $0.125g$

Mode 3: $T_3 = 0.221$ sec $\quad C_{s3} = \frac{0.075}{0.221} = 0.3394g \leq 0.125g$ $\quad$ Use $0.125g$

**Determine modal base shears**

$$V_m = \frac{C_{sm}}{g}W_m$$

Mode 1: $\quad V_1 = \frac{C_{s1}W_1}{g} = 0.0840 \times 1059.72 = 89.02$ kips

Mode 2: $\quad V_2 = \frac{C_{s2}W_2}{g} = 0.125 \times 82.08 = 10.26$ kips

Mode 3: $\quad V_3 = \frac{C_{s3}W_3}{g} = 0.125 \times 12.65 = 1.58$ kips

$$V_d = [89.02^2 + 10.26^2 + 1.58^2]^{1/2} = 89.6 \text{ kips}$$

**Determine design base shear from static-force procedure and compare with base shear from dynamic analysis**

$$V = \frac{S_{D1}I_E}{RT}W$$

$$\leq \frac{S_{DS}I_E}{R}W$$

$$\geq 0.044S_{DS}I_EW$$

$$\geq \frac{0.5S_I\,I_E}{R}$$

Period using Approximate Period formula

$T_a \quad = C_T\,(h_n)^{3/4}$
$C_T \quad = 0.03$ (MRF system)
$h_n \quad =$ total height $= 30$ ft
$T_a \quad = 0.03 \times (30)^{3/4} = 0.385$ sec

Period of 1.2 $C_u T_a$

$$T' = 1.2 \times 1.2 \times 0.385 = 0.554 \text{ sec}$$

$$V = \frac{S_{D1} I_E}{RT'} W = \frac{0.60 \times 1 \times W}{8 \times 0.554} = 0.136W$$

$$\leq \frac{S_{DS} I_E}{R} W = \frac{1.0 \times 1 \times W}{8} = 0.125W \quad \text{. . . governs}$$

$$\geq 0.044\, S_{DS} I_E W = 0.044 \times 1.0 \times 1 \times W = 0.044W$$

$$\geq \frac{0.5 S_1 I_E}{R} W = \frac{0.5 \times 0.6 \times 1 \times W}{8} = 0.0375W$$

Use $V = 0.125 \times 3 \times 386.4 = 144.9$ kips

The base shear $V$ using modal analysis should not be less than that using the static procedure based on a period = $1.2C_u T_a$

$V_s = 144.9$ kips > 89.6 kips

So, the modal forces must be scaled up.

Scale factor = 144.9/89.6 = 1.617

**Scale up modal results**

$V_1 = 1.617 \times 89.02 = 144.0$ kips

$V_2 = 1.617 \times 10.26 = 16.6$ kips

$V_3 = 1.617 \times 1.58 = 2.6$ kips

$V = [144.0^2 + 16.6^2 + 2.6^2]^{1/2} = 145.0$ kips

**Distribute base shear for each mode over height of structure**

Lateral force at level $i$ for mode $m$,

$$F_{im} = \frac{w_i \varphi_{im}}{\Sigma w_i \varphi_{im}} V_m$$

The distribution of the modal base shear for each mode is shown in the table below and also in Figure 1-28d.

**Determine story shears and moments**

See Fig 1-28d.

Mode 1 — $V_m = 144.0$ kips

| Level, $i$ | Weight, $w_i$ | $\varphi_{i1}$ | $w_i\varphi_{i1}$ | $F_{i1}$ |
|---|---|---|---|---|
| 3 | 386.4 | 1 | 386.4 | 64.1 |
| 2 | 386.4 | 0.802 | 309.9 | 51.4 |
| 1 | 386.4 | 0.445 | 171.9 | 28.5 |
| | | $\Sigma =$ | 868.2 | 144.0 |

Mode 2 — $V_m = 16.6$ kips

| Level, $i$ | Weight, $w_i$ | $\varphi_{i2}$ | $w_i\varphi_{i2}$ | $F_{i2}$ |
|---|---|---|---|---|
| 3 | 386.4 | 1 | 386.4 | -21.6 |
| 2 | 386.4 | -0.55 | -212.5 | 11.9 |
| 1 | 386.4 | -1.22 | -471.4 | 26.3 |
| | | $\Sigma =$ | -297.5 | 16.6 |

Mode 3 — $V_m = 2.6$ kips

| Level, $i$ | Weight, $w_i$ | $\varphi_{i3}$ | $w_i\varphi_{i3}$ | $F_{i3}$ |
|---|---|---|---|---|
| 3 | 386.4 | 1 | 386.4 | 4.7 |
| 2 | 386.4 | -2.25 | -869.4 | -10.6 |
| 1 | 386.4 | 1.802 | 696.3 | 8.5 |
| | | $\Sigma =$ | 213.3 | 2.6 |

**Table 1-1. Site coefficient parameters of the 2000 IBC**

**(a) Site coefficient $F_a$**

| SITE CLASS | MAPPED SPECTRAL RESPONSE ACCELERATION AT SHORT PERIODS | | | | |
|---|---|---|---|---|---|
| | $S_s \le 0.25$ | $S_s = 0.50$ | $S_s = 0.75$ | $S_s = 1.00$ | $S_s \ge 1.25$ |
| A | 0.8 | 0.8 | 0.8 | 0.8 | 0.8 |
| B | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| C | 1.2 | 1.2 | 1.1 | 1.0 | 1.0 |
| D | 1.6 | 1.4 | 1.2 | 1.1 | 1.0 |
| E | 2.5 | 1.7 | 1.2 | 0.9 | Note b |
| F | Note b | Note b | Note b | Note b | Note b |

a. Use straight line interpolation for intermediate values of mapped spectral acceleration at short period, $S_s$.
b. Site-specific geotechnical investigation and dynamic site response analyses shall be performed to determine appropriate values.

**(b) Site coefficient $F_v$**

| SITE CLASS | MAPPED SPECTRAL RESPONSE ACCELERATION AT 1 SECOND PERIOD | | | | |
|---|---|---|---|---|---|
| | $S_1 \le 0.1$ | $S_1 = 0.2$ | $S_1 = 0.3$ | $S_1 = 0.4$ | $S_1 \ge 0.5$ |
| A | 0.8 | 0.8 | 0.8 | 0.8 | 0.8 |
| B | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| C | 1.7 | 1.6 | 1.5 | 1.4 | 1.3 |
| D | 2.4 | 2.0 | 1.8 | 1.6 | 1.5 |
| E | 3.5 | 3.2 | 2.8 | 2.4 | Note b |
| F | Note b | Note b | Note b | Note b | Note b |

a. Use straight line interpolation for intermediate values of mapped spectral acceleration at 1-second period, $S_1$.
b. Site-specific geotechnical investigation and dynamic site response analyses shall be performed to determine appropriate values.

**Table 1-2. Design ground motion of the 1997 NEHRP Provisions and the 2000 IBC**

| | | |
|---|---|---|
| $S_S$ | = | MCE spectral response acceleration in the short-period range for Site Class B. |
| $S_1$ | = | MCE spectral response acceleration at 1-second period for Site Class B. |
| $S_{MS}$ | = | $F_a S_S$, MCE spectral response acceleration in the short-period range adjusted for site class effects. |
| $S_{M1}$ | = | $F_v S_1$, MCE spectral response acceleration at 1-second period adjusted for site class effects. |
| $S_{DS}$ | = | $\frac{2}{3} S_{MS}$, spectral acceleration in the short-period range for design ground motion. |
| $S_{D1}$ | = | $\frac{2}{3} S_{M1}$, spectral acceleration at 1-second period for design ground motion. |

**Table 1-3. First mode analysis of MDOF systems compared with static force procedure of IBC 2000**

| | IBC 2000 | First-mode analysis | Comparison |
|---|---|---|---|
| Base shear | $V = C_s W$<br>$C_s = \frac{S_{D1}}{(R/I_E)T}$<br>$\leq \frac{S_{DS}}{(R/I_E)}$<br>$\geq 0.044 S_{DS} I_E$<br>$\geq \frac{0.5 S_1}{(R/I_E)}$ | $V = \frac{S_{a1}}{g} W_1$ from Eq. (1-35)<br>$W_1 = \frac{L_1^2 g}{M_1} = \frac{\left(\sum_{i=1}^{n} m_i \phi_{i1}\right)^2 g}{\sum_{i=1}^{n} m_i \phi_{i1}^2}$ from<br>Eqs. (1-33), (1-26a) and (1-21a) | $W_1 < W = (0.6 - 0.8)W$, typically<br>$C_s$ versus $S_{a1}/g$: see Fig. 1-23 |
| Distribution along height | $F_x = \frac{w_x h_x^k}{\sum_{i=1}^{n} w_i h_i^k}$<br>k = 1.0 for T ≤ 0.5 sec<br>k = 2.0 for T ≥ 2.5 sec<br>k = 1.0 + (T - 0.5)/2, or<br>k = 2.0 for 0.5 sec < T < 2.5 sec | $f_{sj,1max} = \frac{m_j \phi_{j1}}{L_1} V_{1max}$ Eq. (1-36a)<br>$= \frac{m_j \phi_{j1}}{\sum_{i=1}^{n} m_i \phi_{i1}} V_{1max}$ Eq. (1-26a)<br>$= \frac{w_j \phi_{j1}}{\sum_{i=1}^{n} w_i \phi_{i1}} V_{1max}$ | For $\phi_{j1} = h_j/h_n$, as assumed in Section 1.7.6 (Fig. 1-27),<br>$f_{sj\,1max} = \frac{w_j h_j^k}{\sum_{i=1}^{n} w_j h_i^k} V_{1max}$<br>Thus, IBC 2000 distribution of base shear along height for short-period structures corresponds essentially to the fundamental mode of vibration response. Increase in k-value beyond 1 represents the effects of higher modes of vibration. |

**Table 1-4. Seismic design category based on short period response accelerations**

| VALUE OF $S_{DS}$ | SEISMIC USE GROUP | | |
|---|---|---|---|
| | I | II | III |
| $S_{DS} < 0.167g$ | A | A | A |
| $0.167g \leq S_{DS} < 0.33g$ | B | B | C |
| $0.33g \leq S_{DS} < 0.50g$ | C | C | D |
| $0.50g \leq S_{DS}$ | D[a] | D[a] | D[a] |

a. Seismic Use Groups I and II structures located on sites with mapped maximum considered earthquake spectral response acceleration at 1-second period, $S_1$, equal to or greater than $0.75g$, shall be assigned to Seismic Design Category E, and Seismic Use Group III structures located on such sites shall be assigned to Seismic Design Category F.

**Table 1-5. Seismic design category based on 1 second period response accelerations**

| VALUE OF $S_{D1}$ | SEISMIC USE GROUP I | II | III |
|---|---|---|---|
| $S_{D1} < 0.067g$ | A | A | A |
| $0.067g \leq S_{D1} < 0.133g$ | B | B | C |
| $0.133g \leq S_{D1} < 0.20g$ | C | C | D |
| $0.20g \leq S_{D1}$ | D[a] | D[a] | D[a] |

a. Seismic Use Groups I and II structures located on sites with mapped maximum considered earthquake spectral response acceleration at 1-second period, $S_1$, equal to or greater than $0.75g$, shall be assigned to Seismic Design Category E, and Seismic Use Group III structures located on such sites shall be assigned to Seismic Design Category F.

**Table 1-6. Analysis procedures for Seismic Design Category D, E, or F**

| STRUCTURE DESCRIPTION | MINIMUM ALLOWABLE ANALYSIS PROCEDURE FOR SEISMIC DESIGN |
|---|---|
| 1. Seismic Use Group I buildings of light-framed construction three stories or less in height and of other construction, two stories or less in height with flexible diaphragms at every level. | Simplified procedure of Section 1617.5. |
| 2. Regular structures, other than those in Item 1 above, up to 240 feet in height. | Equivalent lateral-force procedure (Section 1617.4). |
| 3. Structures that have vertical irregularities of Type 1a, 1b, 2 or 3 in Table 1616.5.2, or plan irregularities of Type 1a or 1b of Table 1616.5.1, and have a height exceeding 240 feet in height. | Modal Analysis Procedure (Section 1618). |
| 4. Other structures designated as having plan or vertical irregularities. | Equivalent lateral-force procedure (Section 1617.4) with dynamic characteristics included in the analytical model. |
| 5. Structures with all of the following characteristics :<br>- located in an area with assigned $S_{D1}$ of 0.2 or greater, as determined in Section 1615.1.3<br>- located in an area assigned to Site Class E or F, in accordance with Section 1615.1.1 and;<br>- with a natural period $T$ of 0.7 second or greater, as determined in Section 1617.4.2 | Modal Analysis Procedure (Section 1618). A site specific response spectrum shall be used but the design base shear shall not be less than that determined from Section 1617.4.1. |

**Table 1-7. Scaling of results of dynamic analysis**

| Section 1618.7, 2000 IBC<br>Section 5.4.8, 1997 NEHRP | Sec. 5.4.8, 2000 NEHRP |
|---|---|
| **Regular or Irregular Structures** | **Regular or Irregular Structures** |
| $V_{dyn} \geq V_{static}$ based on $T = 1.2\ C_u T_a$<br>$C_u$ per IBC Table 1617.4.2<br>$T_a$ per IBC Section 1617.4.2.1 | $V_{dyn} \geq 0.85 V_{static}$<br>No scale-down |
| **Section 1631.5.4, 1997 UBC<br>Scaling of Results** | **Sec. 1629.5.3, 1994 UBC<br>Scaling of Results** |
| **Regular Structures** | **Regular Structures** |
| $V_{dyn} \geq 0.9 V_{static}$ Using Figure 16-3<br>$V_{dyn} \geq 0.8 V_{static}$ Using site-specific response spectrum | $V_{dyn} \geq 0.8 V_{static}$ based on $T$ by Method A<br>$V_{dyn} \geq 0.9 V_{static}$ based on $T$ by Method B |
| **Irregular Structures** | **Irregular Structures** |
| $V_{dyn} \geq V_{static}$ | $V_{dyn} \geq V_{static}$ |
| Figure 16-3 is the generalized design spectrum printed as part of the 1997 UBC | Method A – approximate period formulas<br>Method B – rational analysis |
| Story shears and displacements and other response quantities are to be adjusted proportionately | |

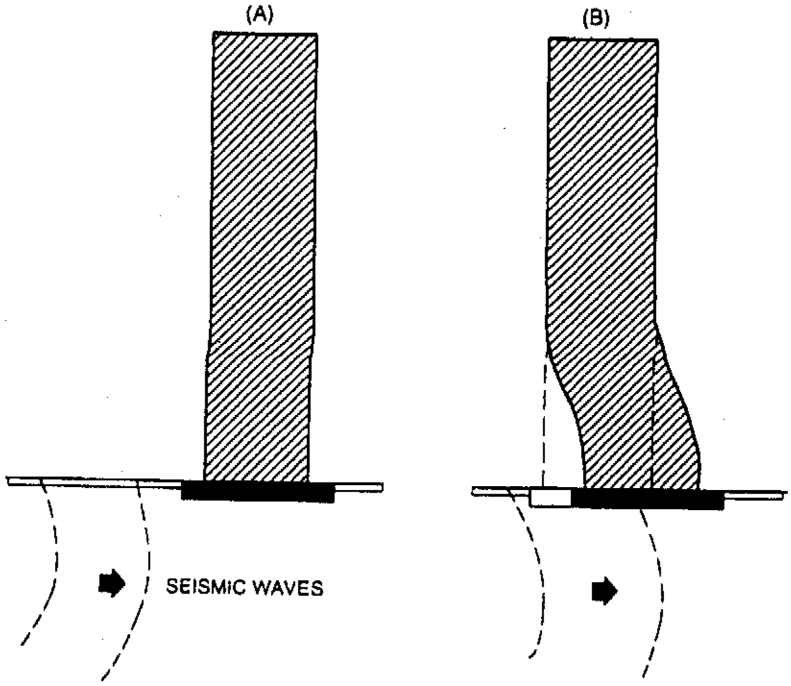



**Figure 1-1. Behavior of buildings during an earthquake**

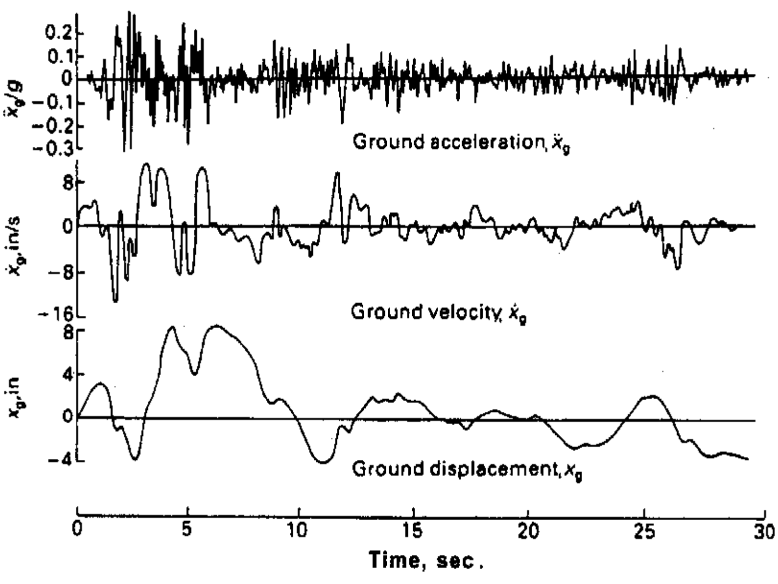



**Figure 1-2. 1940 El Centro (California) earthquake accelerogram: N-S component and corresponding derived velocity and displacement plots**

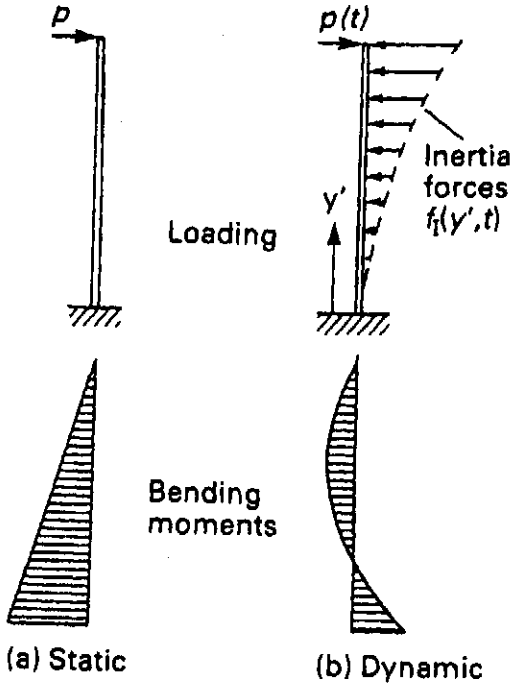



**Figure 1-3. Essential difference between static and dynamic loading**

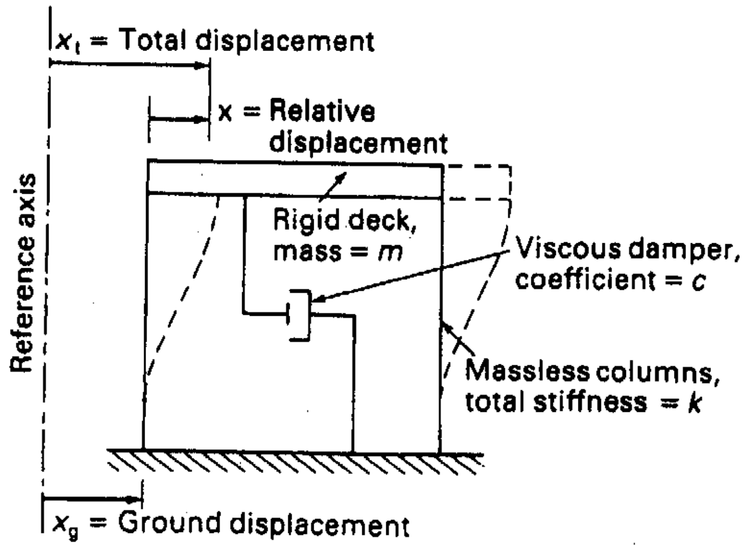



**Figure 1-4. Lumped SDOF system subject to base translation**

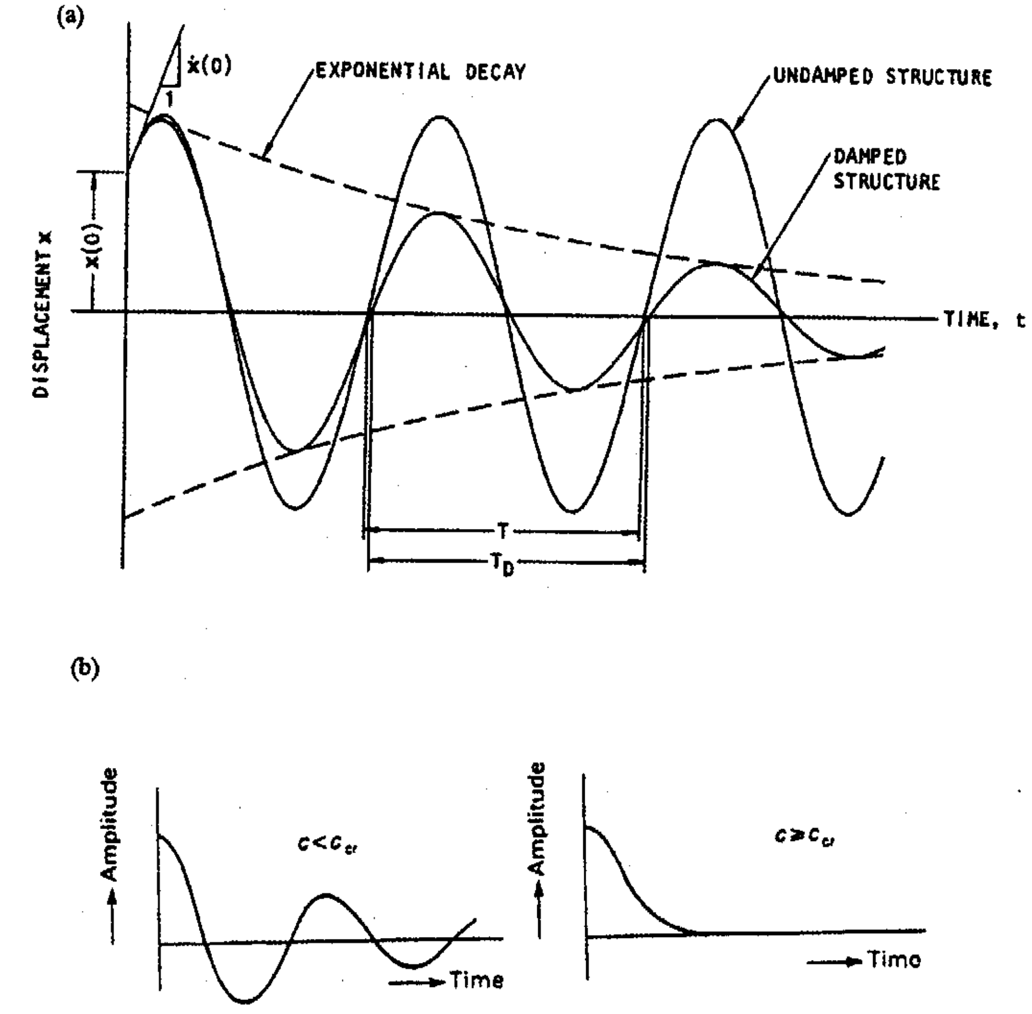



**Figure 1-5. (a) Effect of damping on free vibration, (b) Non-periodic dying-out of vibration**

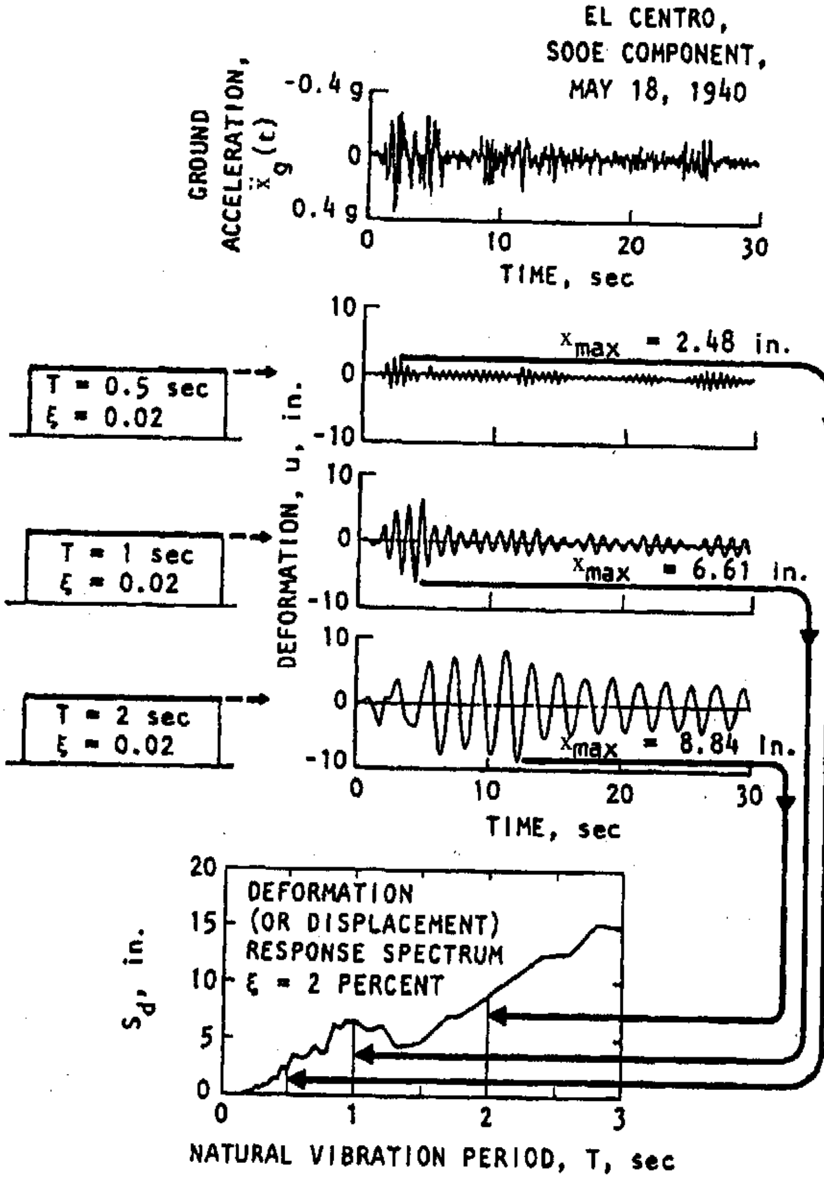



**Figure 1-6. Computation of displacement response spectrum[6]**

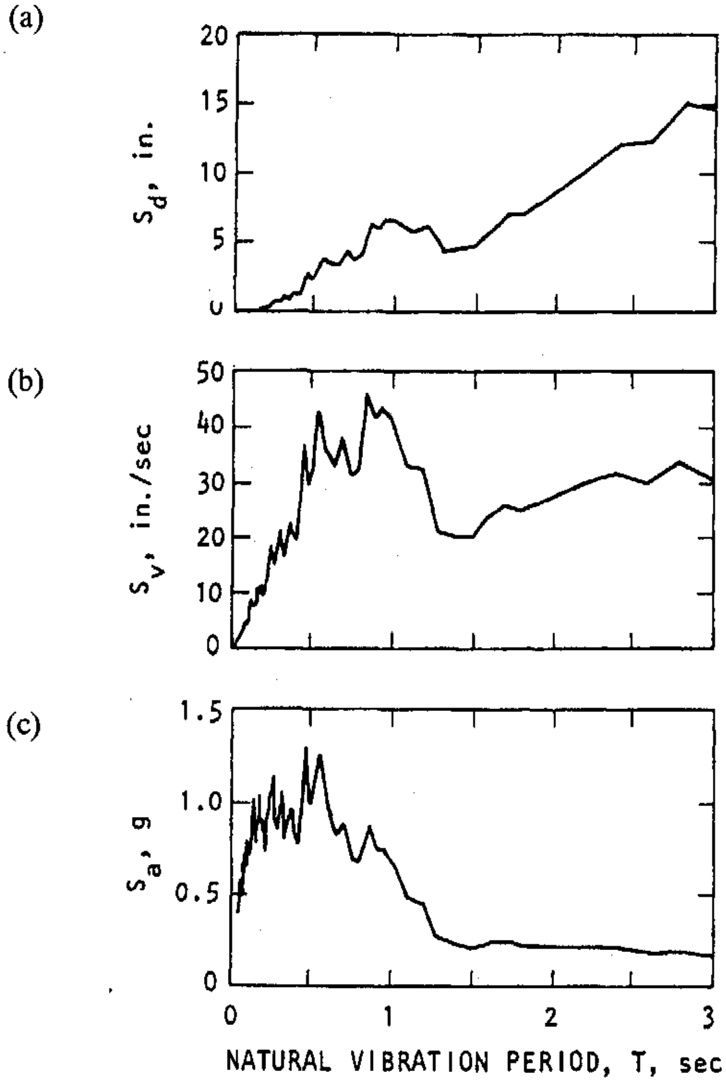



**Figure 1-7. (a) Displacement, (b) Pseudo-velocity, and (c) Pseudo-acceleration response spectra. 1940 El Centro ground motion, N-S component. Damping ratio = 2 percent[6]**

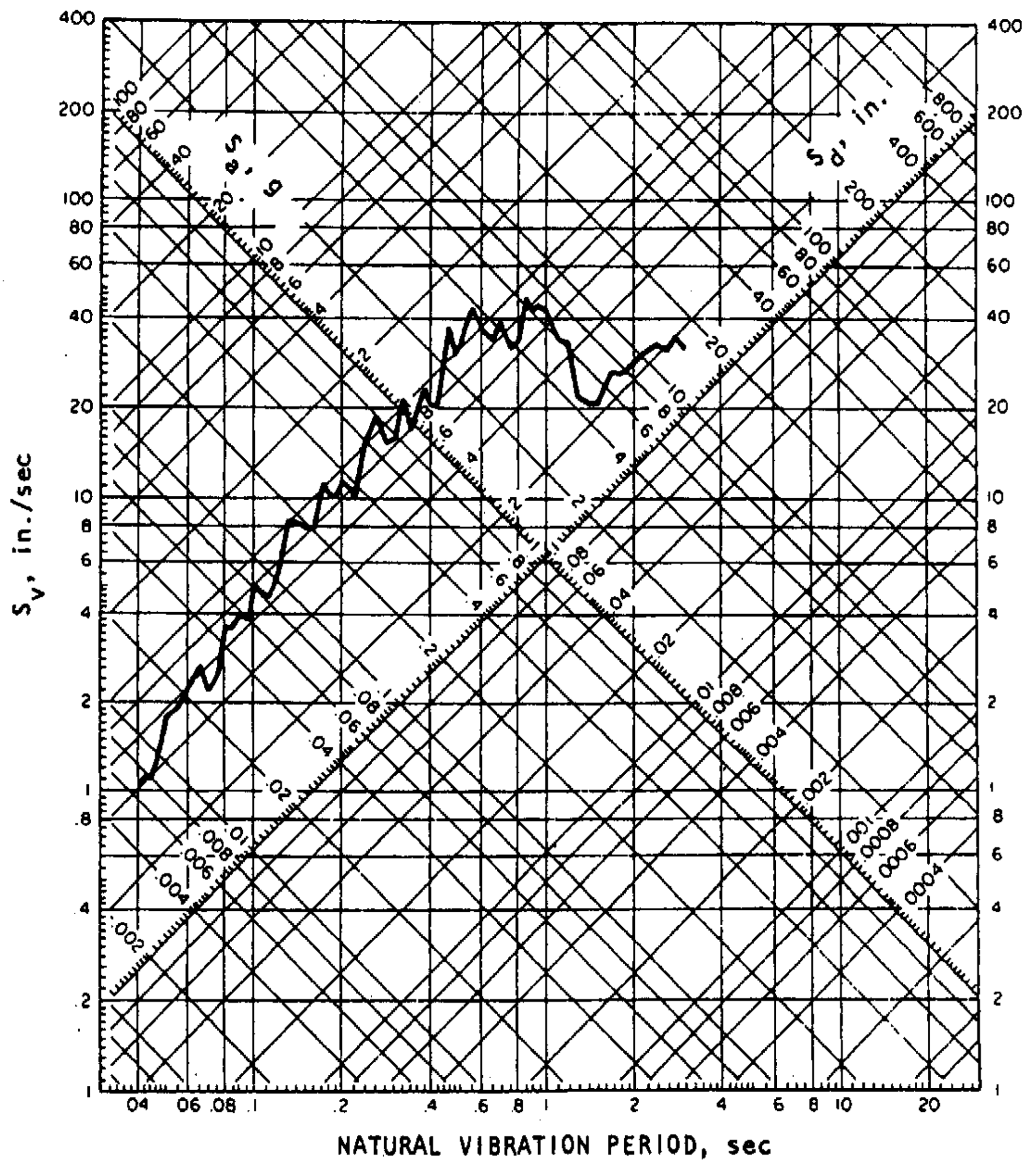



**Figure 1-8. Four-way logarithmic plot of response spectrum. 1940 El Centro ground motion, N-S component. Damping ratio = 2 percent[6]**

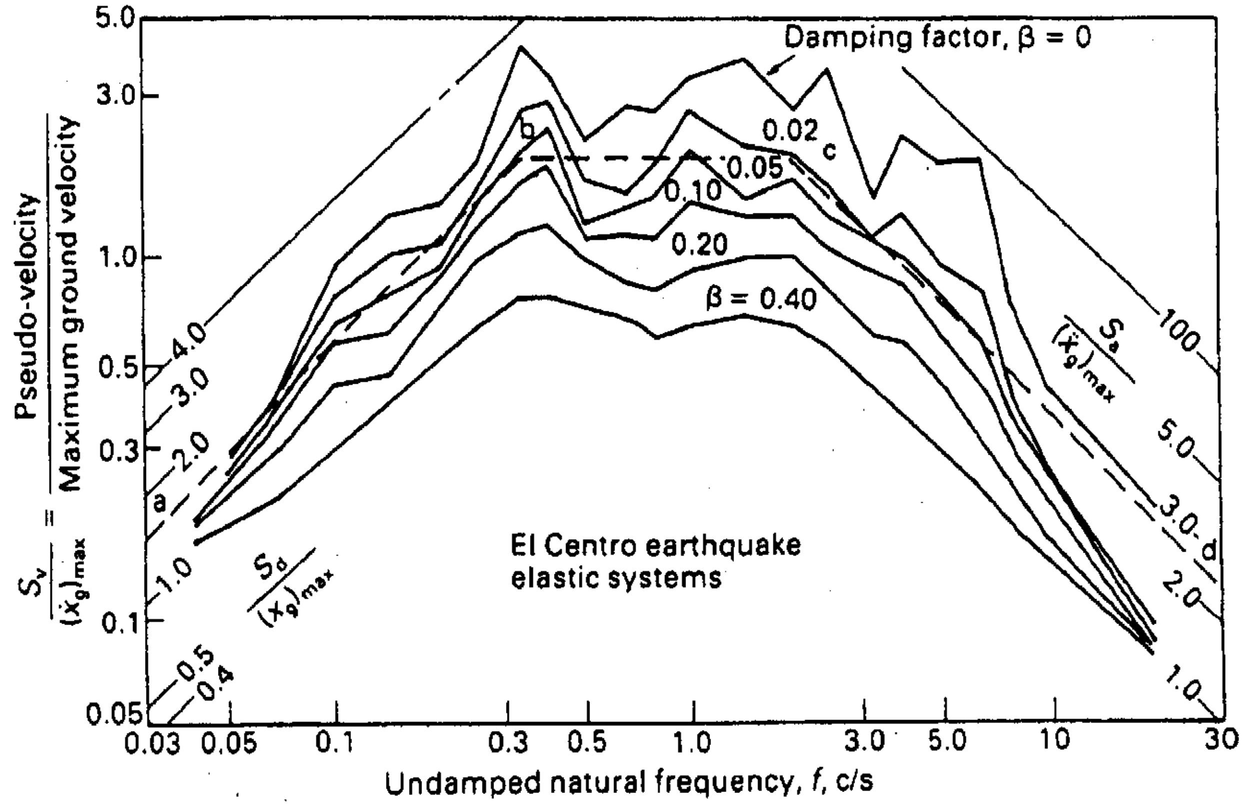



**Figure 1-9. Response spectra for elastic SDOF systems subjected to 1940 El Centro ground motion, N-S component[7]**

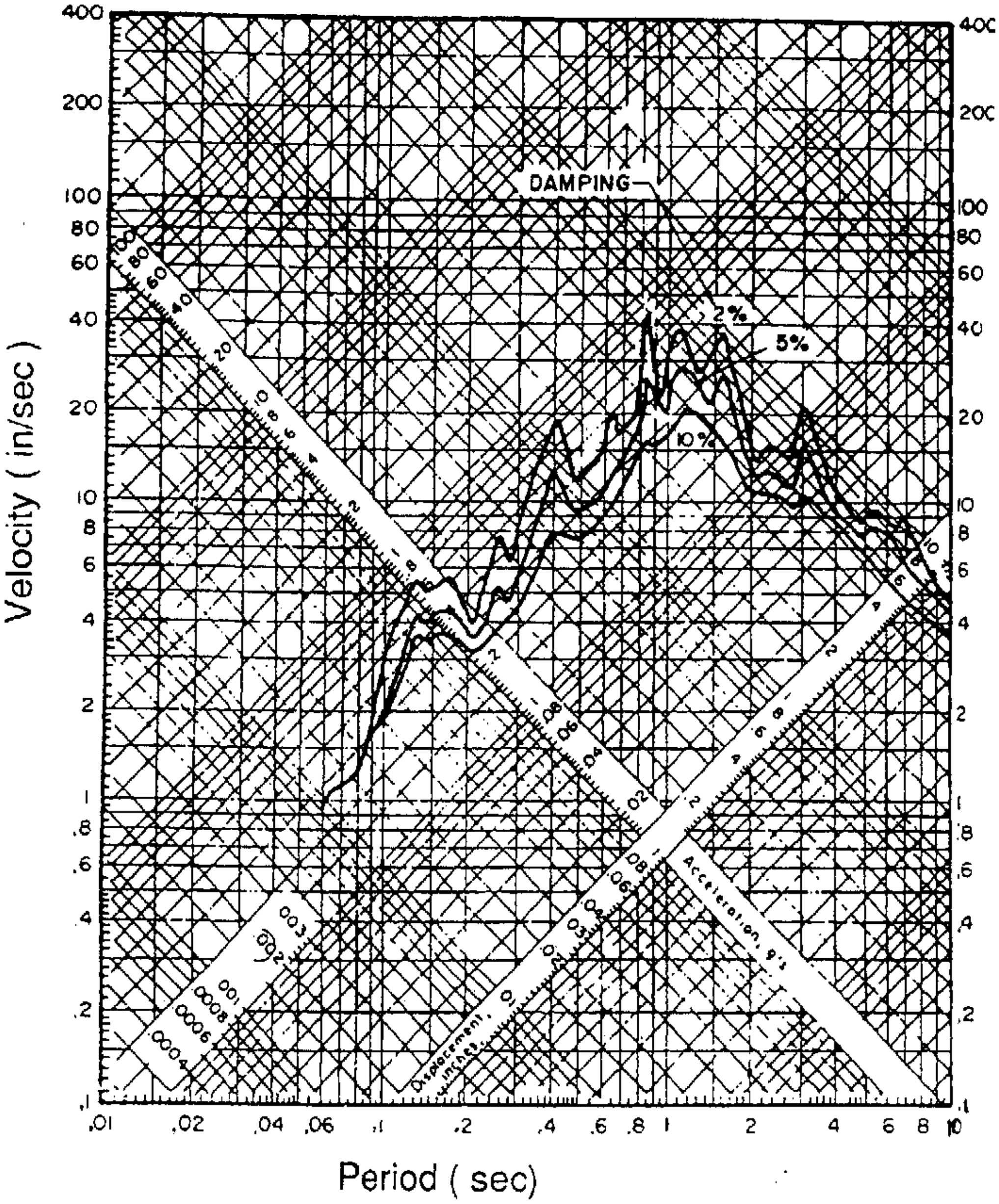



**Figure 1-10. Site-specific response spectra**

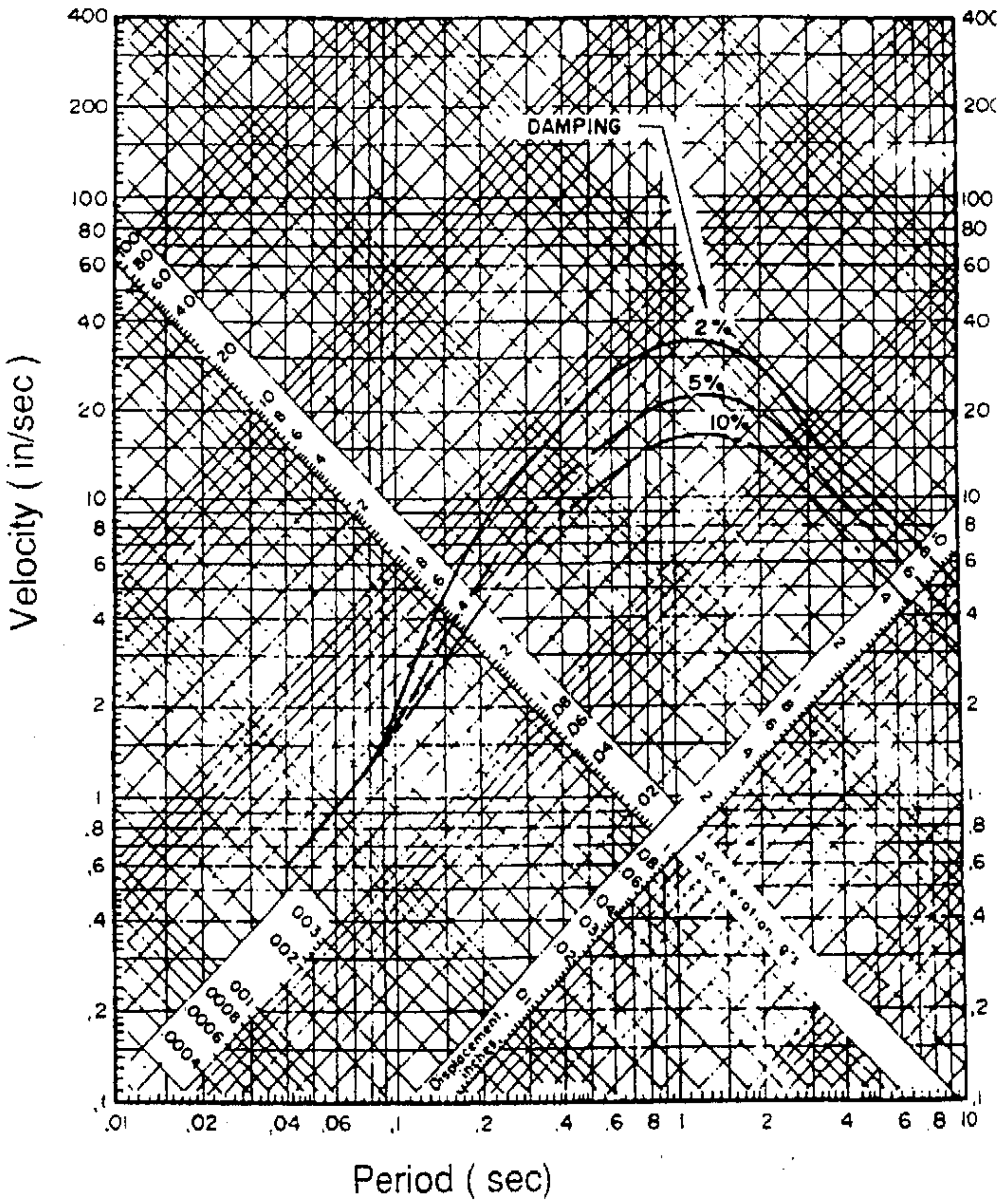



**Figure 1-11. Smoothed site-specific response spectra**

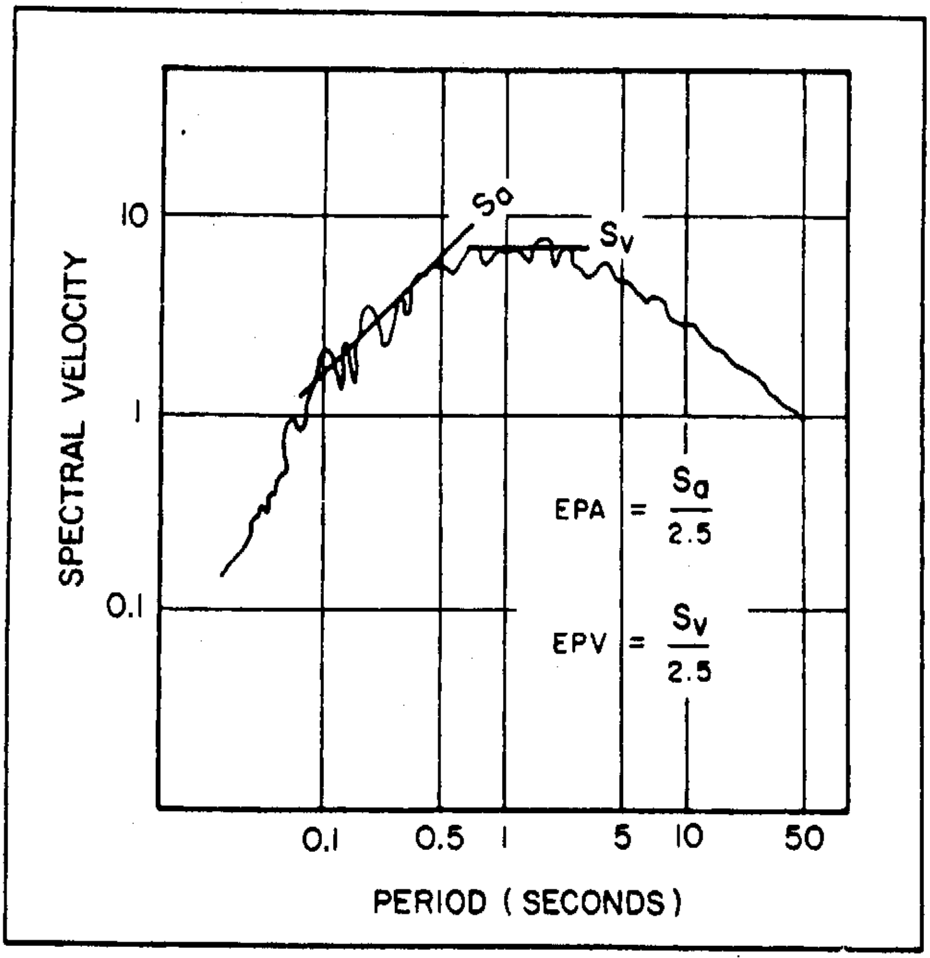



**Figure 1-12. Schematic representation showing determination of EPA and EPV from a response spectrum**

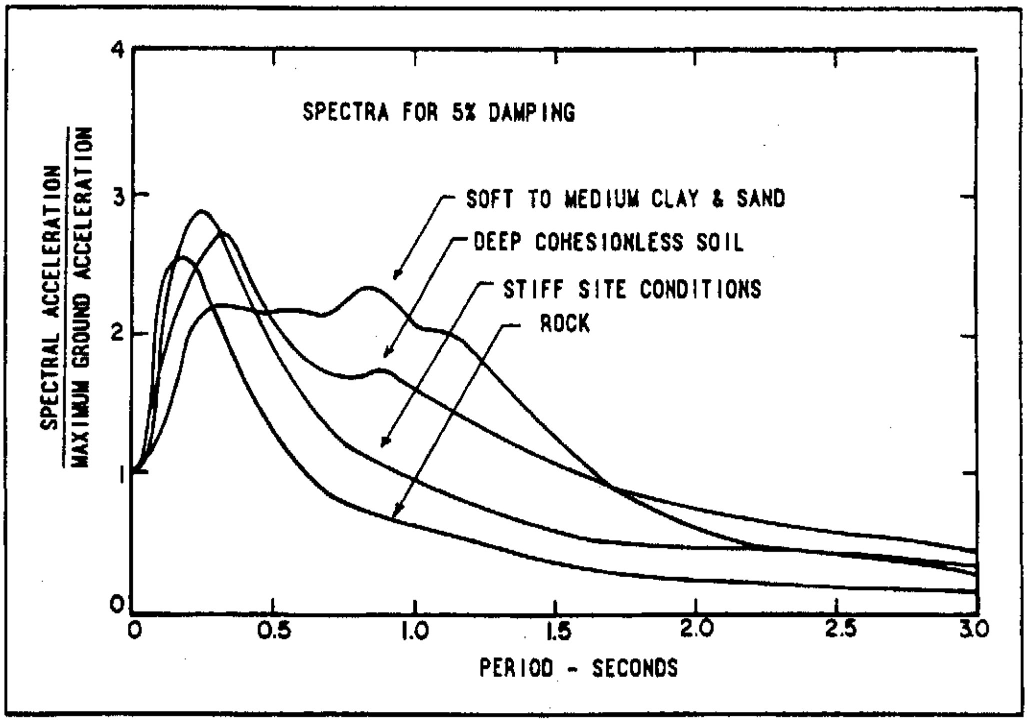
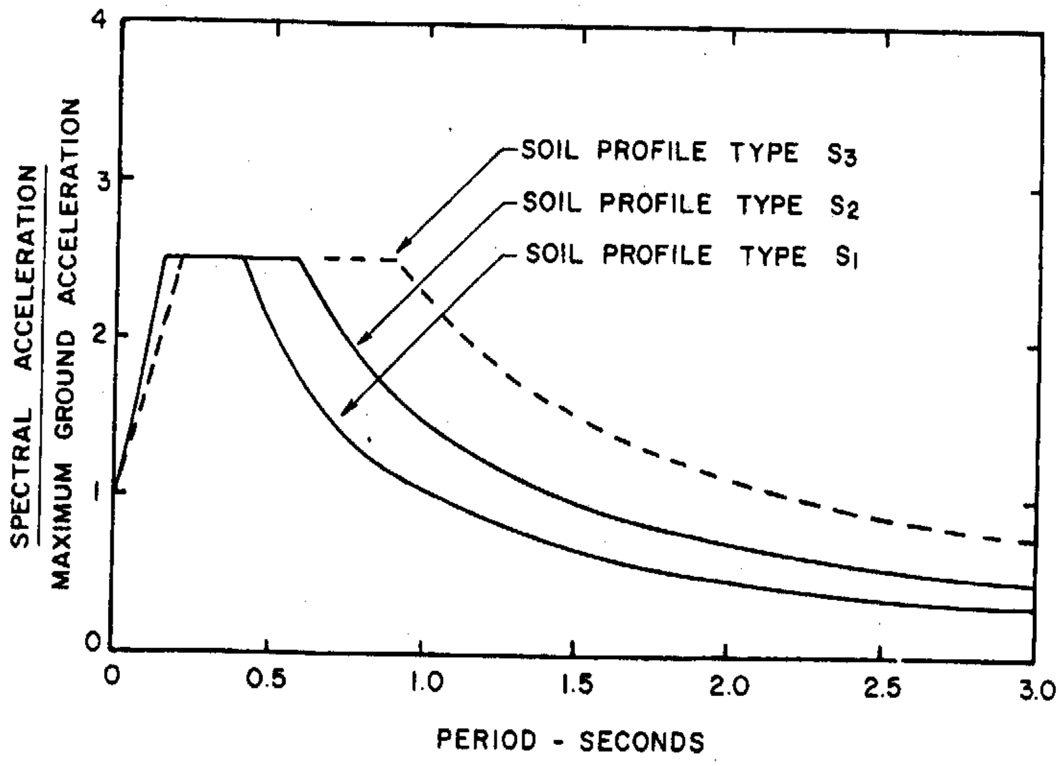



**Figure 1-13. Average acceleration spectra for different site conditions[12]**

(a)




(b)

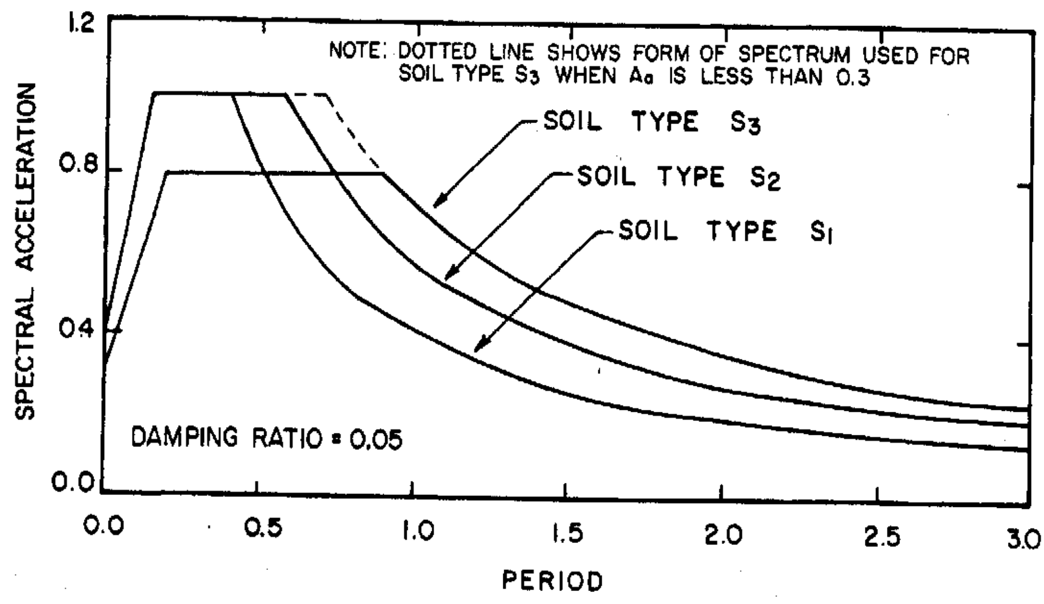
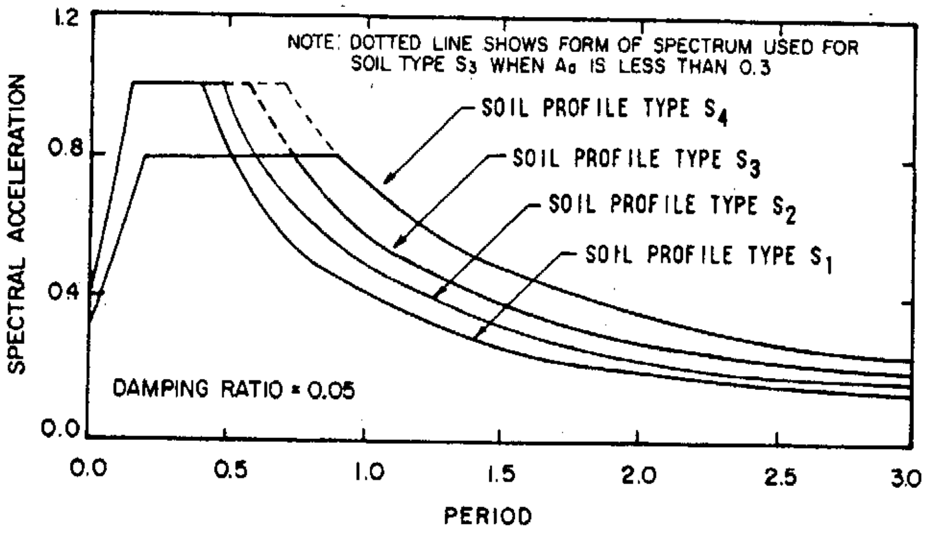



**Figure 1-14. (a) Normalized response spectra recommended for use in building code, (b) Ground motion spectra for ATC/NEHRP Map Area 7 ($A_a$ = 0.4)**

(a)




(b)

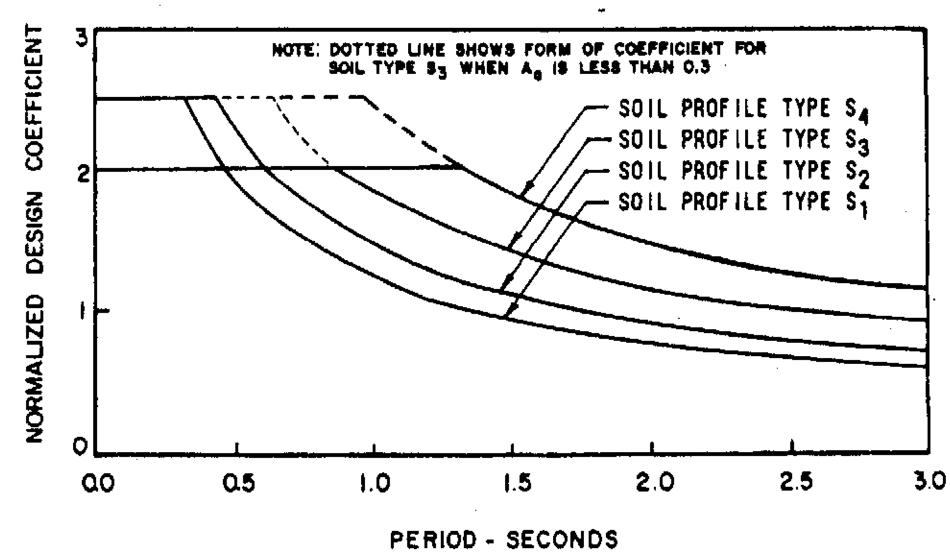



**Figure 1-15. (a) Ground motion spectra for ATC/NEHRP Map Area 7 ($A_a$ = 0.4), (b) Normalized lateral design force coefficients ($A_a$ = $A_v$ = 1.0)**

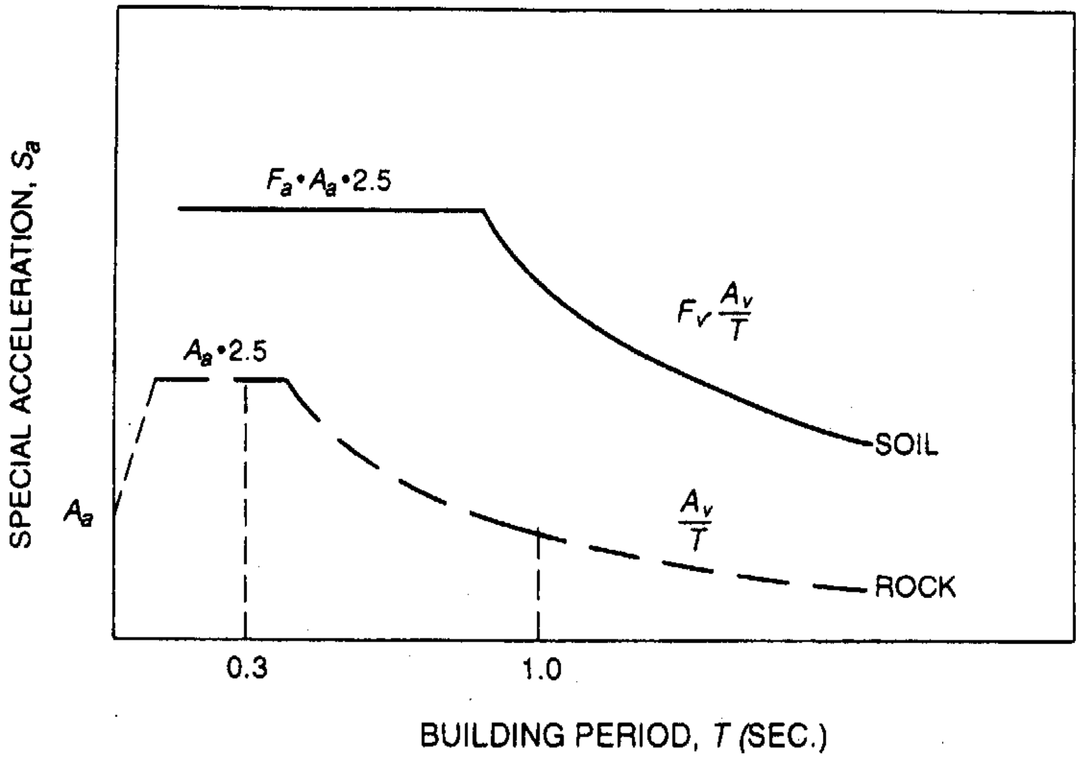



**Figure 1-16. Two-factor approach to local site response**

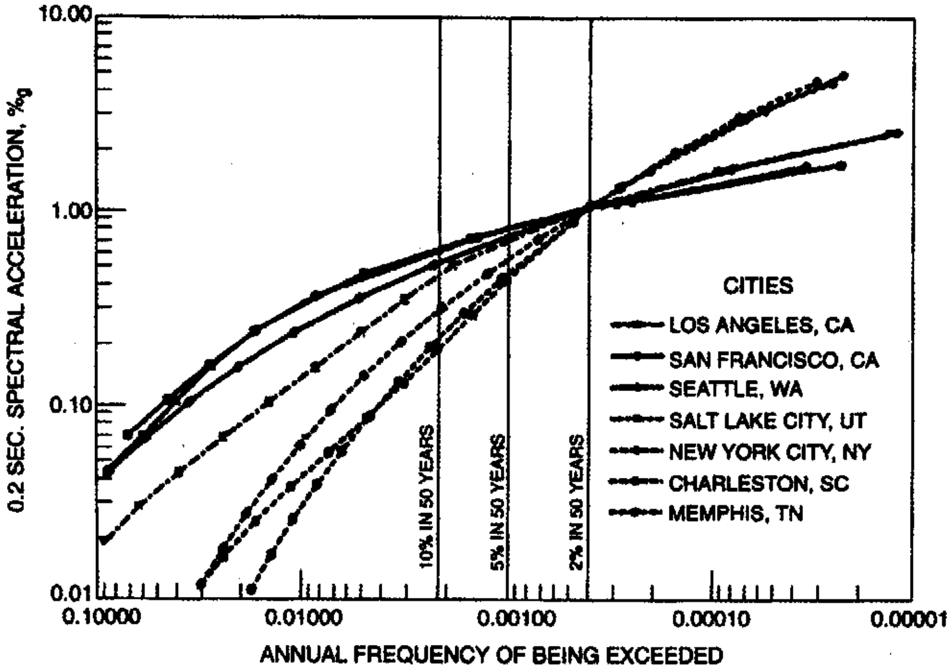



**Figure 1-17. Hazard curves for selected cities (1997 NEHRP)**

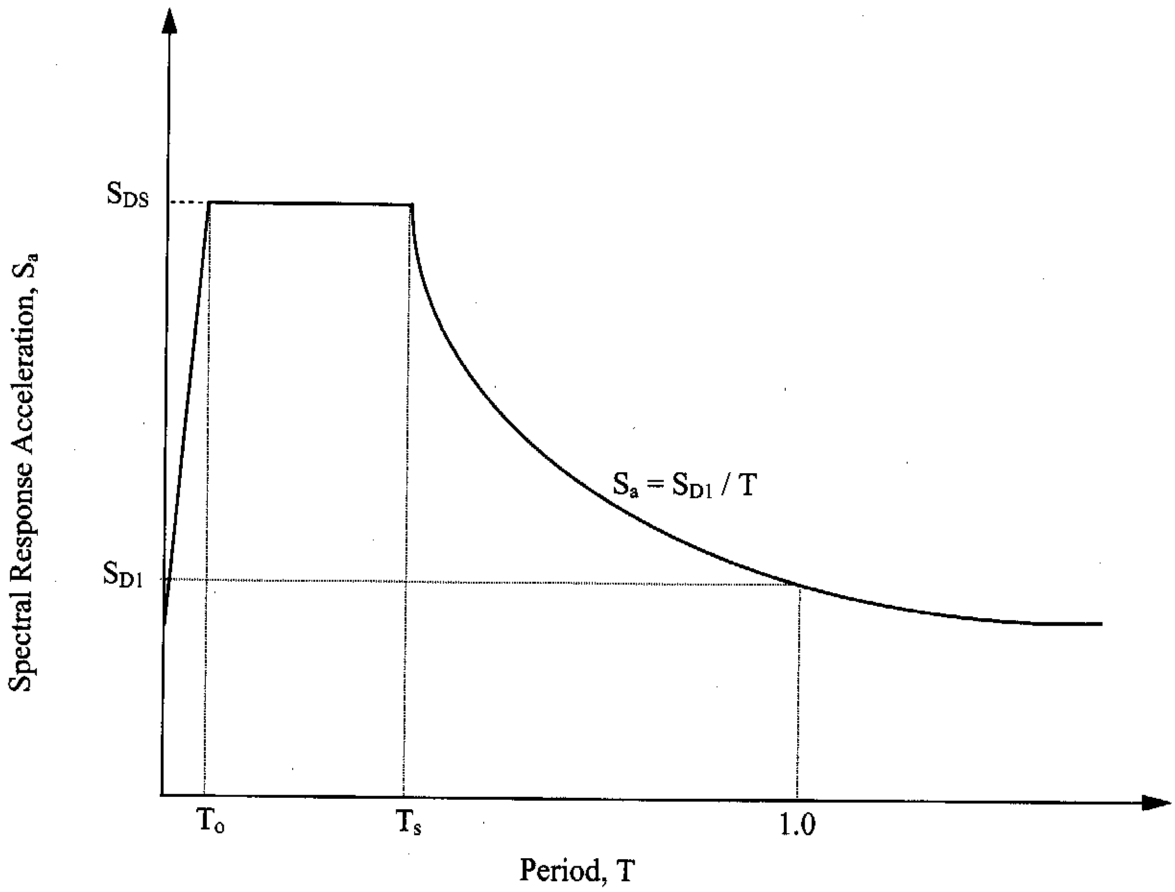



**Figure 1-18. Design response spectra of the 2000 IBC**

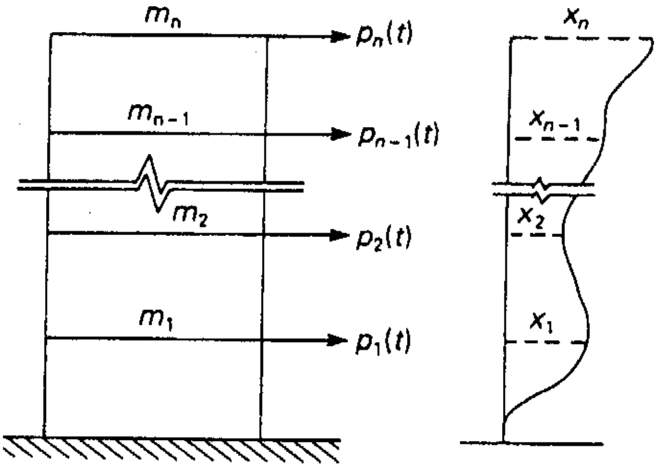
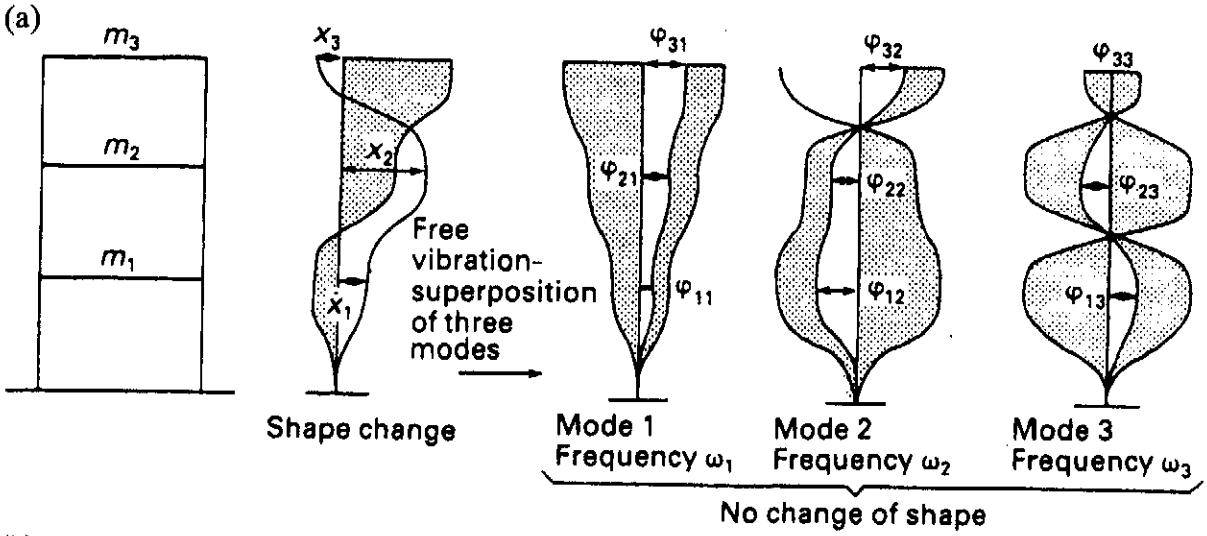



**Figure 1-19. Lumped MDOF system with applied loading and resulting displacements**

(a)




(b)

$$x_1 = \varphi_{11}X_1 + \varphi_{12}X_2 + \varphi_{13}X_3$$
$$x_2 = \varphi_{21}X_1 + \varphi_{22}X_2 + \varphi_{23}X_3$$
$$x_3 = \varphi_{31}X_1 + \varphi_{32}X_2 + \varphi_{33}X_3$$

or,
$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{bmatrix} \varphi_{11} & \varphi_{12} & \varphi_{13} \\ \varphi_{21} & \varphi_{22} & \varphi_{23} \\ \varphi_{31} & \varphi_{32} & \varphi_{33} \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix}$$

or,
$$\{x\} = [\{\varphi_1\}\{\varphi_2\}\{\varphi_3\}]\{X\}$$
$$= [\varphi]\{X\}$$

(c)

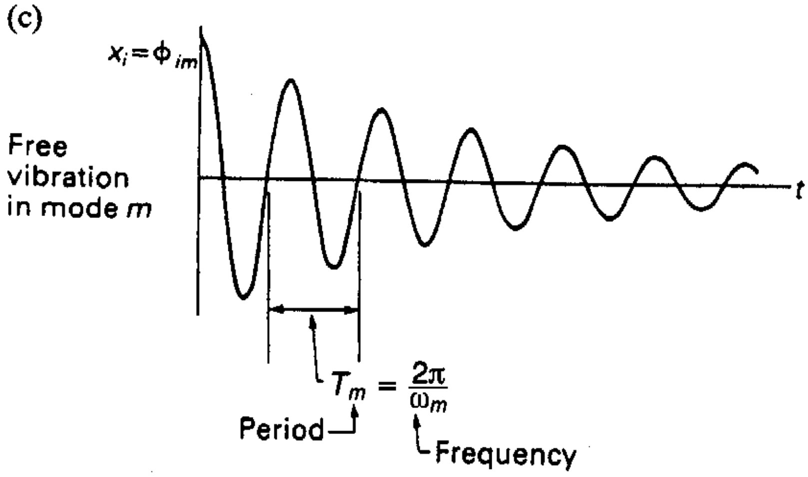



**Figure 1-20. Mode superposition analysis of earthquake response**

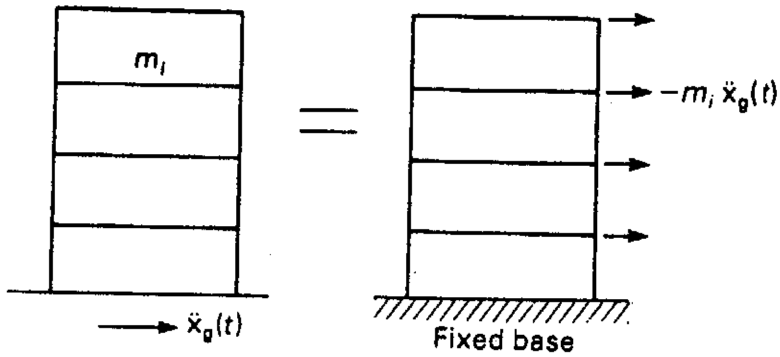



**Figure 1-21. Effective load caused by earthquake excitation**

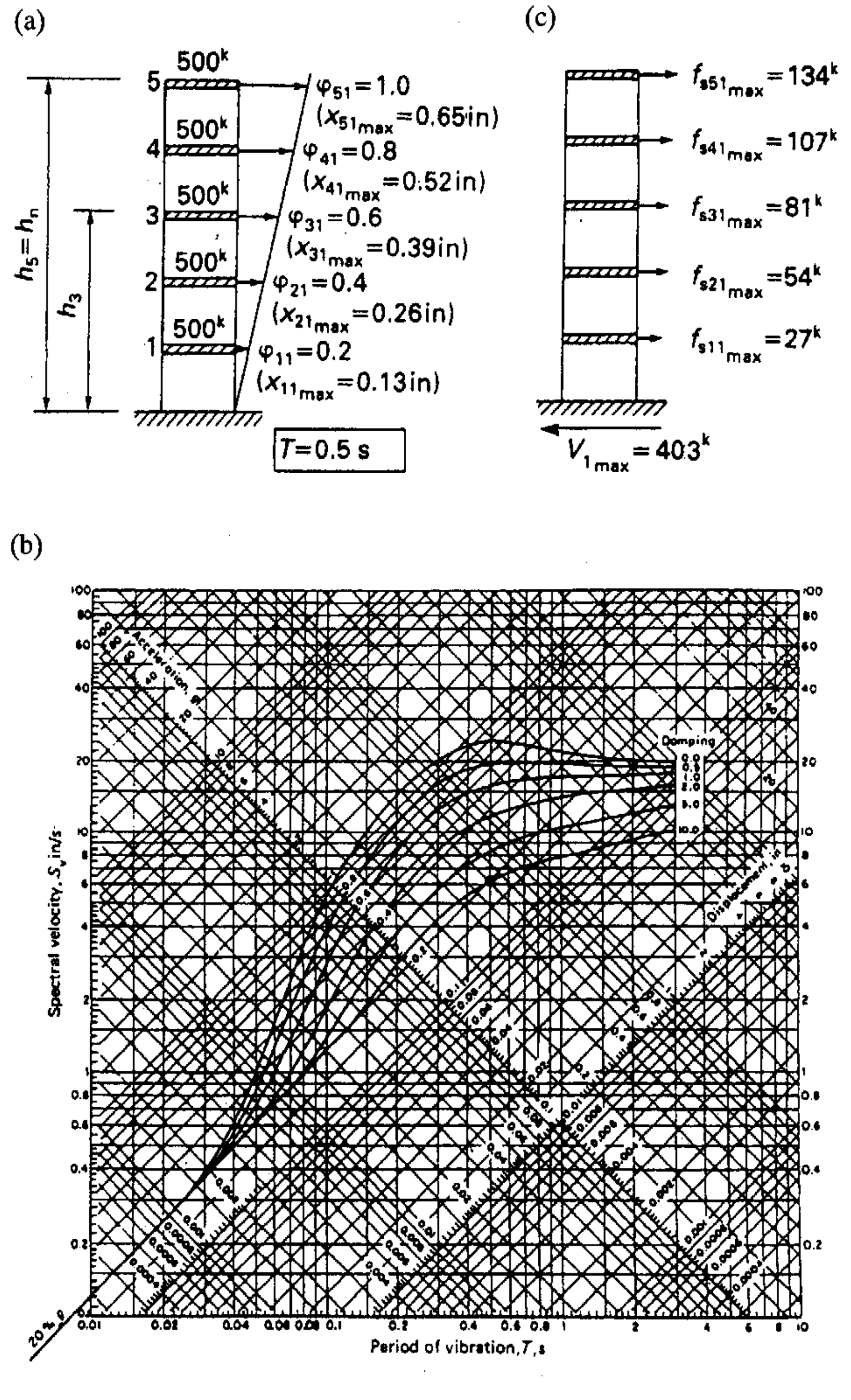
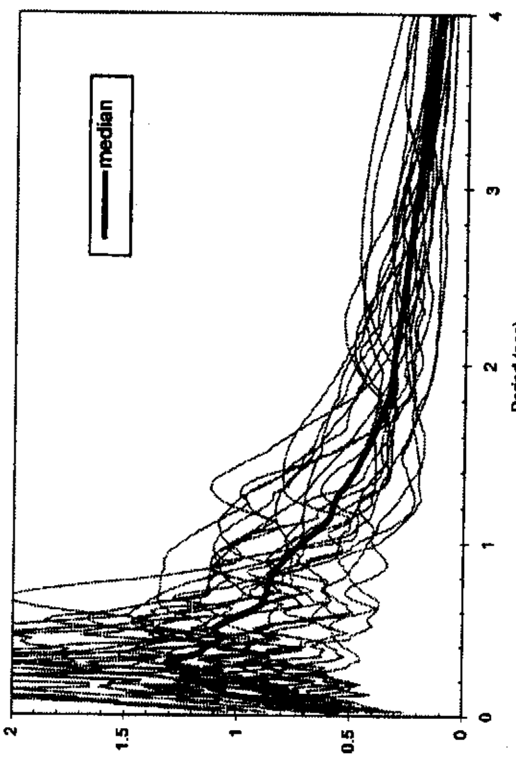
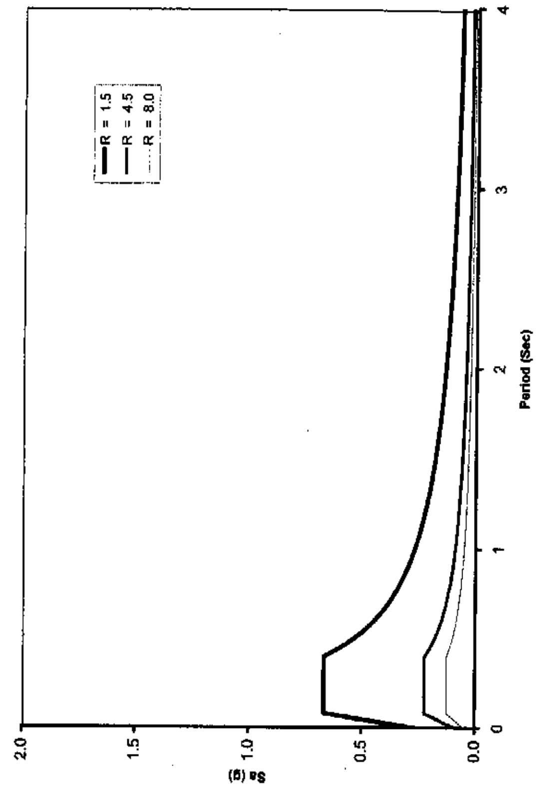



**Figure 1-22. First-mode analysis fo MDOF systems: (a) System analyzed and maximum displacement response, (b) Response spectra used in analysis, (c) Maximum earthquake forces (first subscript refers to floor level, second subscript to mode number)**




20 Acceleration Response Spectra for SAC LA ground motions with 10% in 50 years hazard level




Design Response Spectra from the 2000 IBC

**Figure 1-23. Comparison of acceleration response spectra (SAC Los Angeles) and 2000 IBC design response spectra**

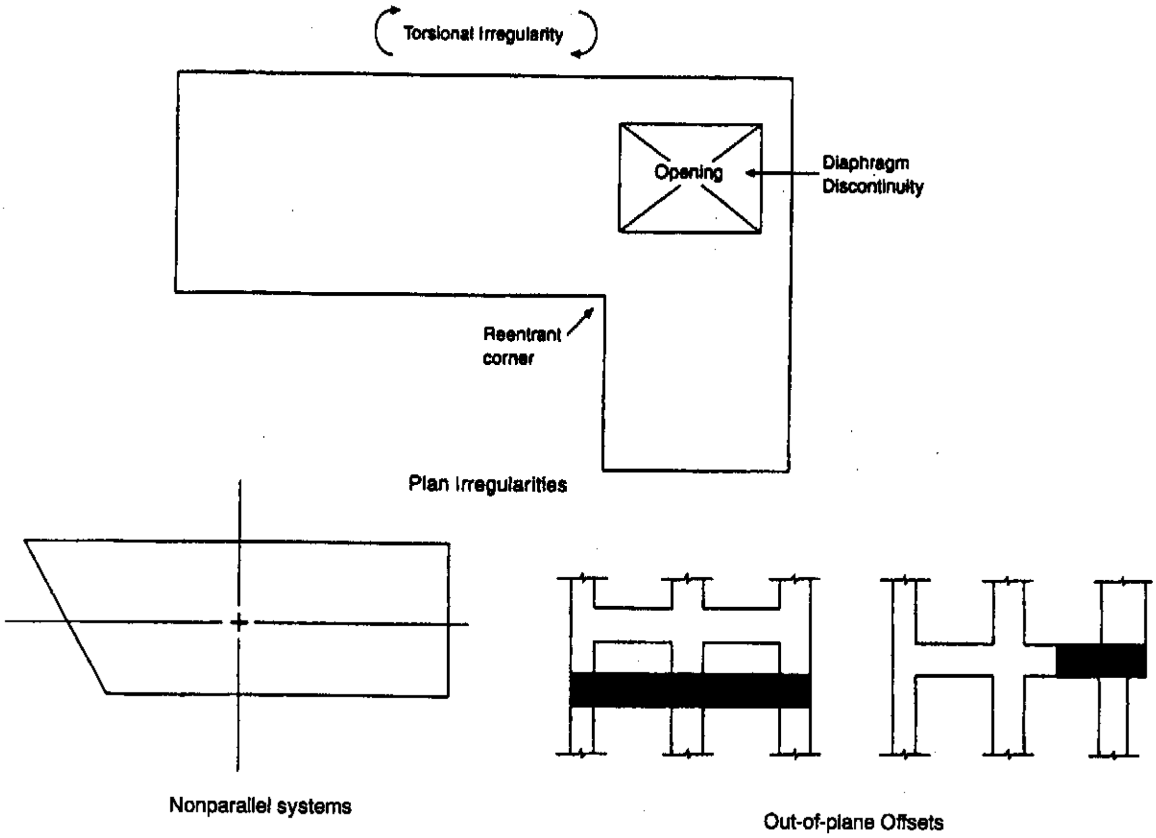



**Figure 1-24. Plan structural irregularities**

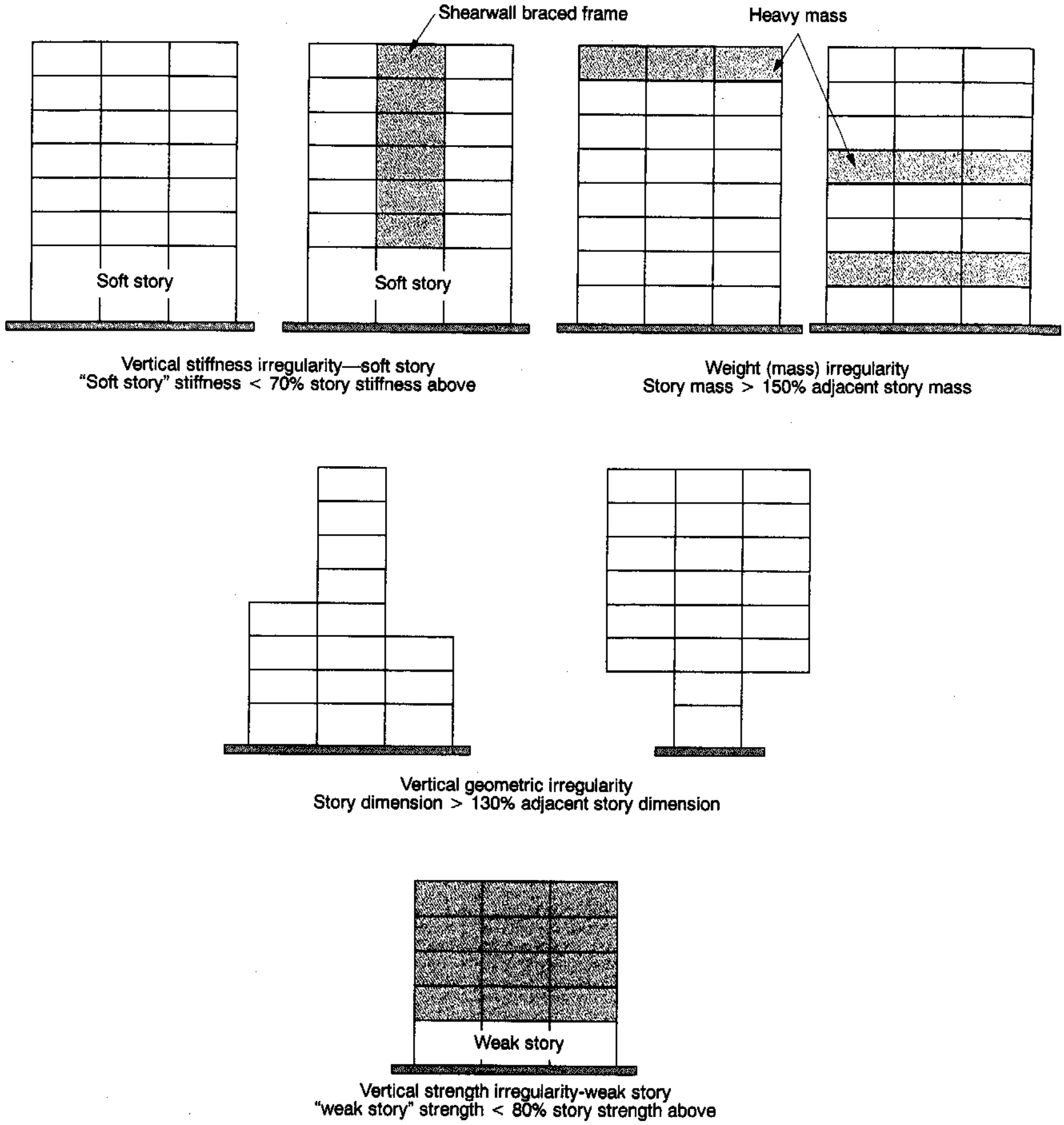



**Figure 1-25. Vertical structural irregularities**

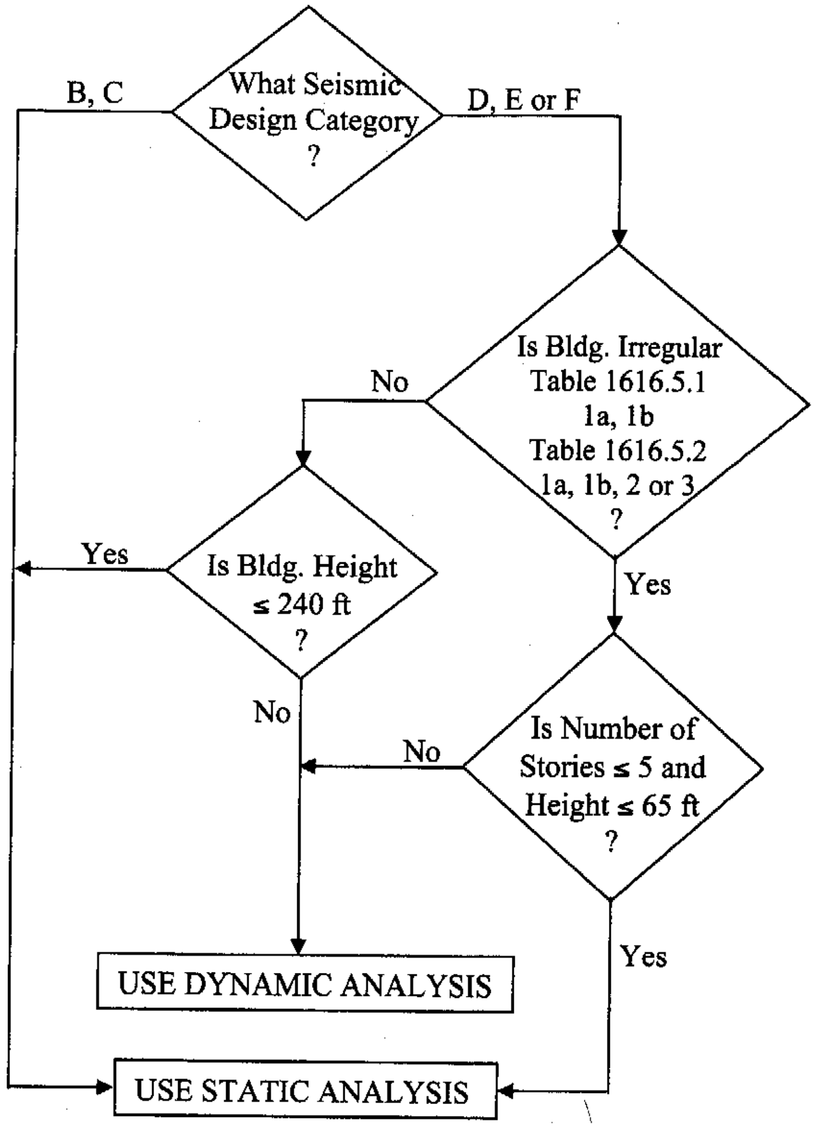



**Figure 1-26. Static versus dynamic analysis—2000 IBC**

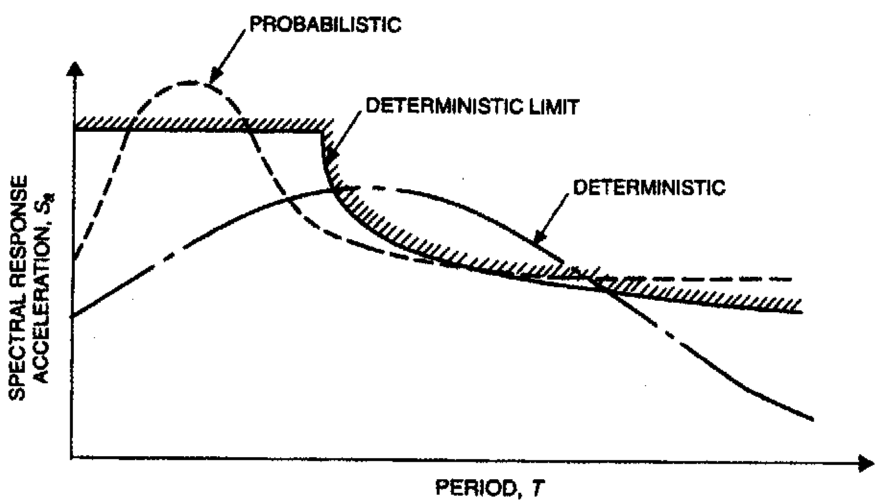
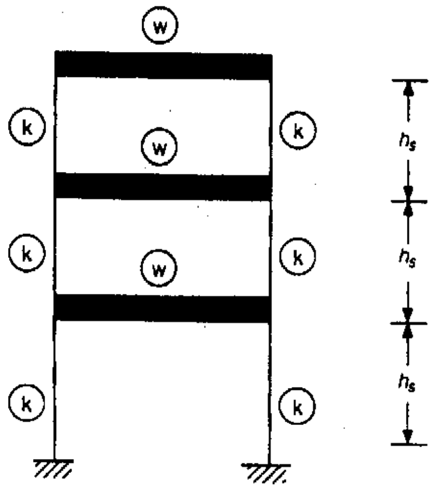
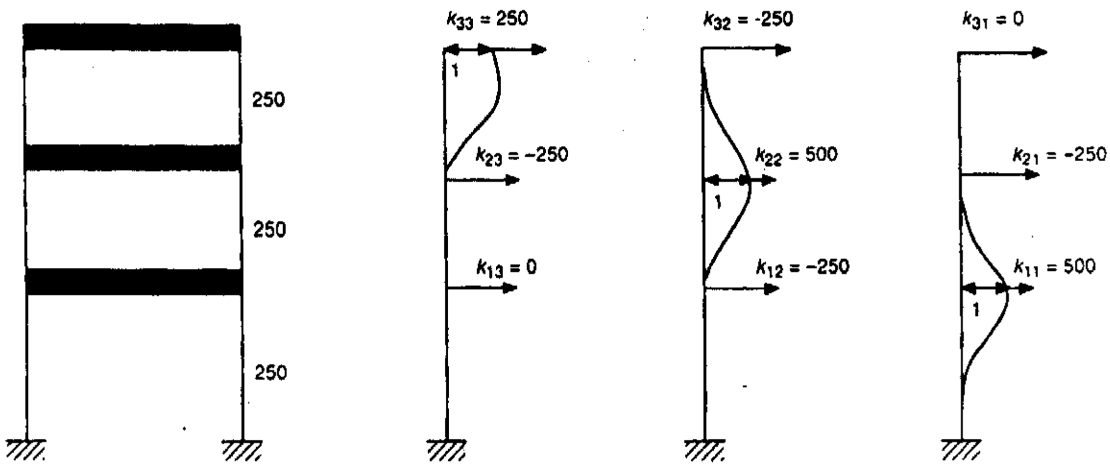



**Figure 1-27. Site-specific maximum considered earthquake response spectrum**

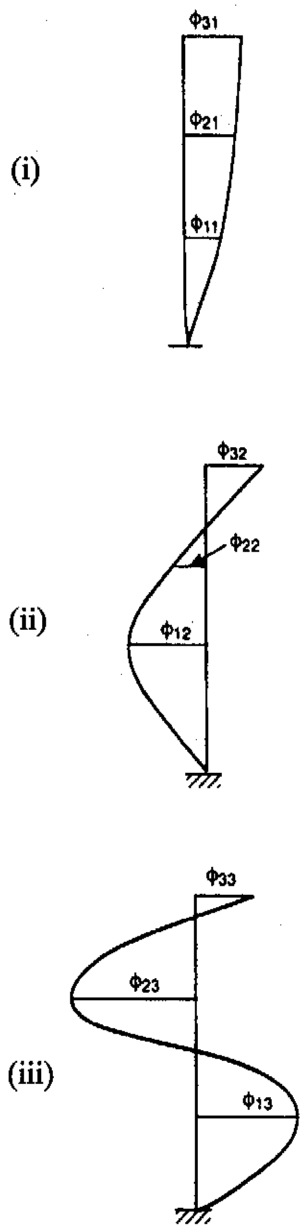



(a)



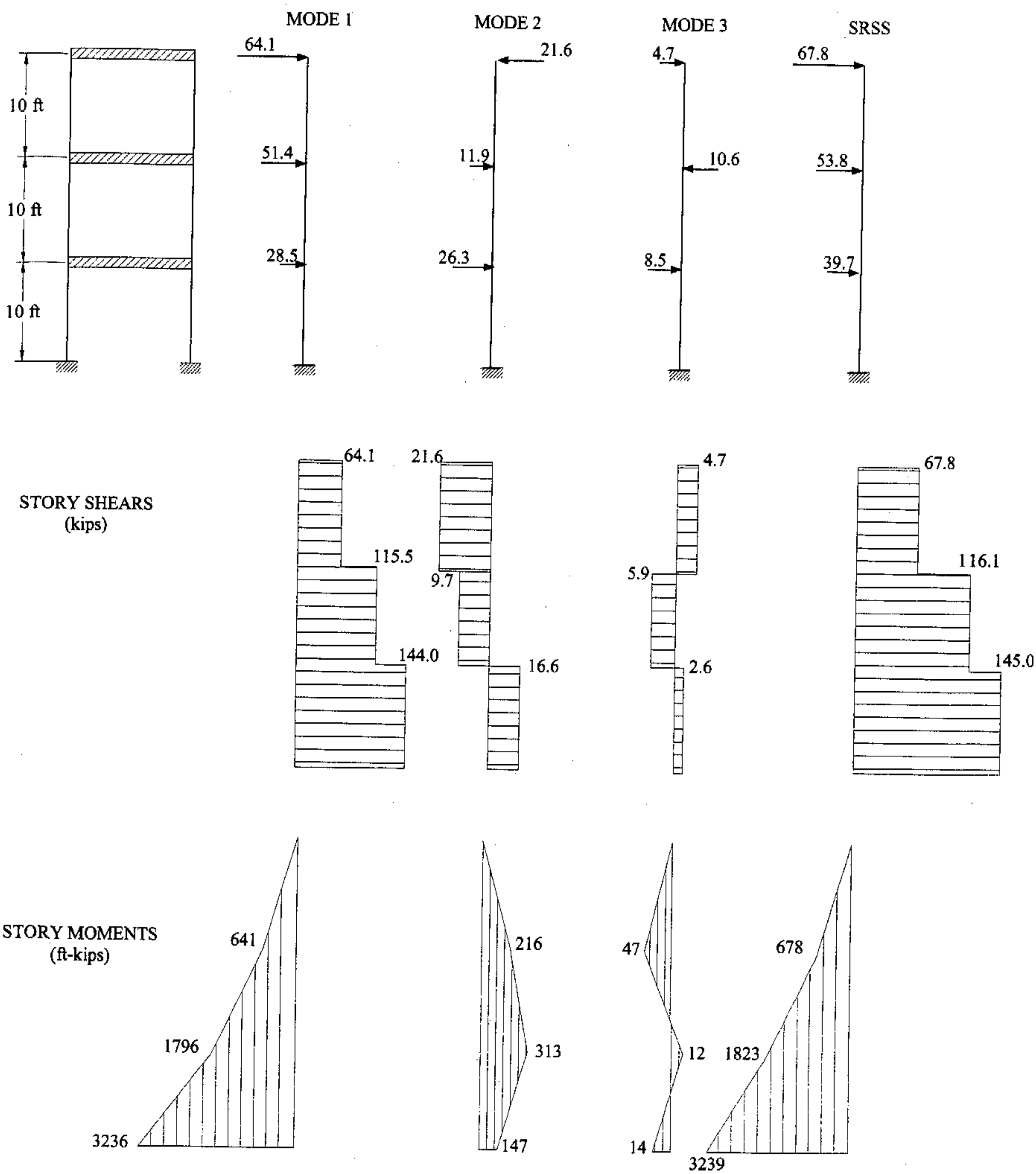

(b)

**Figure 1-28. Response spectrum analysis example**




(c)

**Figure 1-28. Response spectrum analysis example (continued)**




(d)

**Figure 1-28. Response spectrum analysis example (continued)**

# Chapter 2

# DESIGN OF REINFORCED CONCRETE STRUCTURES FOR EARTHQUAKE FORCES

## 2.1. Introduction

### 2.1.1. General

A 20-story reinforced concrete building is designed following the requirements of the International Building Code (IBC), 2000 edition. The building is located on IBC Site Class D. Both dynamic and static lateral-force procedures are used as the basis of design.

The building is symmetrical about both principal plan axes. Along each axis a dual system (concrete shear walls with special moment-resisting frames or SMRF) is utilized for resistance to lateral forces.

A dual system is defined as a structural system with the following features: (IBC 1617.6.1)

> **1617.6.1 Dual systems.** For a dual system, the moment frames shall be capable of resisting at least 25 percent of the design forces. The total seismic force resistance is to be provided by the combination of the moment frame and the shear walls or braced frames in proportion to their stiffness.

### 2.1.2. Design criteria

A typical plan and elevation of the building are shown in Figures 2-1 and 2-2, respectively. The member sizes for the structure are chosen as follows:

| | |
|---|---|
| Spandrel beams (width = 34 in.) | 34x24 in. |
| Interior beams | 34x24 in. |
| Columns | 34x34 in. |
| Shear walls: | |
| Grade to 9th floor | 16 in. thick |
| 10th floor to 16th floor | 14 in. thick |
| 17th floor to roof | 12 in. thick |
| Shear wall boundary elements | 34x34 in. |

Other relevant design data are as follows:

*Material properties*

Concrete $f'_c$ = 4,000 psi (all members)

All members are constructed of normal weight concrete ($w_c$ = 145 pcf)

Reinforcement $f_y$ = 60,000 psi

*Service Loads*

| | | |
|---|---|---|
| Superimposed dead load | 20 psf | . . . partition and equipment |
| Live load | 80 psf | . . . per practice, minimum 50 psf (T 1607.1) |
| Loads on roof | 10 psf SDL | . . . roofing + 200 kips for penthouse and equipment |
| | 20 psf LL | (1607.11.2.1) |
| Joists and topping | 86 psf | |
| Cladding | 8 psf | |

*Seismic Design Data*

It is assumed that, at the site of the structure, the maximum considered earthquake spectral response acceleration at short periods, $S_S = 1.5g$, and that at 1-second period, $S_1 = 0.6g$.

Assume standard occupancy or Seismic Use Group = I
. . . seismic importance factor, I = 1.0 (T 1604.5)

Use default Site Class . . . *D*
Site coefficient $F_a$ = 1.0 [T 1615.1.2(1)]
Site coefficient $F_v$ = 1.5 [T 1615.1.2(2)]

Soil-modified $S_S$ $= S_{MS} = F_a S_S$ (Eq. 16-16)
$= 1.0 \times 1.5g = 1.5g$

Soil-modified $S_1$ $= S_{M1} = F_v S_1$ (Eq. 16-17)
$= 1.5 \times 0.6g = 0.9g$

Design Spectral Response Acceleration Parameters (at 5% damping):
At short periods: $S_{DS} = 2/3\, S_{MS}$ (Eq. 16-18)
$= 2/3 \times 1.5 = 1.0g$

At 1-second period: $S_{D1} = 2/3\, S_{M1}$ (Eq. 16-19)
$= 2/3 \times 0.9 = 0.6g$

Dual system (RC shear walls with SMRF) . . . $R = 8$; $C_d = 6.5$ (Table 1617.6)
where: $R$ and $C_d$ are the response modification factor and deflection amplification factor, respectively.

Seismic Design Category: Based on both $S_{DS}$ [Table 1616.3(1)] and $S_{D1}$ [Table 1616.3(2)], the seismic design category (SDC) for the example building is D.

### 2.1.3. Design basis

Calculation of the design base shear and distribution of that shear along the height of the building using the equivalent lateral-force procedure (which is used in a majority of designs) is not appropriate and is not allowed by the International Building Code for buildings exceeding 240 feet in height in SDC D (T 1616.6.3, Item 3). In these cases, the dynamic lateral-force procedure (1618) must be used. In this example, the dynamic procedure will be used as the height of the building is 255 feet in SDC D (more than 240 feet). However, for comparison purposes, the equivalent lateral-force procedure (1617.4) will also be illustrated.

## 2.2. Gravity Loads and Load Combinations

### 2.2.1. Weights at each floor level

Table 2-1 shows the weights (self weight + SDL) at each floor level. The weights are calculated as follows:

$$
\begin{aligned}
w_i = \; & (86 + 20\{10 \text{ psf for roof}\}) \times 130^2 \{+200 \text{ kips for roof}\} && \ldots \text{SDL + Joists} \\
& + 8 \times 12.5^* \times 130 \times 4 && \ldots \text{Cladding} \\
& + 150 \times 12.5^* \times (34/12)^2 \times 36 && \ldots \text{Column selfweight} \\
& + 150 \times 23.1 \times 34/12 \times 17/12 \times 56 && \ldots \text{Beam selfweight} \\
& + 150 \times 12.5^* \times 23.1 \times (h/12) \times 4 && \ldots \text{Shear wall selfweight} \\
& \quad {}^*\{15.0 \text{ for } 2^{nd} \text{ floor \& } 6.25 \text{ for roof}\} \; (h = \text{wall thickness})
\end{aligned}
$$

$$
\begin{aligned}
& = 3{,}559 \text{ kips for floor 2} \\
& = 3{,}394 \text{ kips for floors 3 to 9} \\
& = 3{,}380 \text{ kips for floor 10} \\
& = 3{,}366 \text{ kips for floors 11 to 16} \\
& = 3{,}352 \text{ kips for floor 17} \\
& = 3{,}338 \text{ kips for floors 18 to 20} \\
& = 2{,}987 \text{ kips for roof (floor 21)}
\end{aligned}
$$

Total weight of the building: $W = \sum_{i=1}^{20} w_i = 67{,}246$ kips

### 2.2.2. Gravity load analysis

Service-level axial forces due to dead and live loads for shear wall, edge column, and interior column at different floor levels are given in Table 2-2. Live load reduction factors were used as follows: (1607.9.2)

$$
\begin{aligned}
R & = r(A - 150) \\
& \le 0.6
\end{aligned}
$$

$\leq 23.1\ (1 + D/L)$ for floors other than the roof
$= 0.6$ (1607.11.2) for flat roof with tributary area $A_t \geq 600$ ft$^2$
$= 1.2 - 0.001A_t$ for flat roof with tributary area 200 ft$^2$ $< A_t <$ 600 ft$^2$

where: $R$ = 0.08 for floors other than the roof
$A$ = 676 (= 26 × 26) ft$^2$ for roof (interior column)
= 2 × 676 ft$^2$ for floor 20 & so on (interior column)
= 375 (= 26 × 14.42) ft$^2$ for roof (edge column)
= 750 ft$^2$ for floor 20 & so on (edge column)
= 1352 (= 26 × 52) ft$^2$ for roof (shear wall)
= 2 × 1352 ft$^2$ for floor 20 & so on (shear wall)

In Table 2-2, the reduced live load ($RLL$) is calculated as $RLL = L\ (1 - R)$

### 2.2.3. Load combinations for design

The following load combinations are used in the strength design method for concrete:

(1) $U = 1.4D + 1.7L$ (ACI 318-99 Eq. 9-1)
(2) $U = 1.2D + f_1 L + 1.0E$ (Formula 16-5)
(3) $U = 0.9D \pm 1.0E$ (Formula 16-6)

where: $D$ = dead load effect
$L$ = live load effect
$f_1$ = 0.5 (1605.2)
$E = \rho Q_E + 0.2 S_{DS} D$ when the effects of gravity and seismic loads are additive (Eq. 16-28)
$E = \rho Q_E - 0.2 S_{DS} D$ when the effects of gravity and seismic loads are counteractive (Eq. 16-29)

$Q_E$ = the effect of horizontal seismic forces
$\rho$ = a reliability factor based on system redundancy

## 2.3. Equivalent Lateral Force Procedure (1617.4)

### 2.3.1. Design base shear (1617.4.1)

$$V = C_s W \qquad \text{(Eq. 16-34)}$$

where:

$$C_s = \frac{S_{D1} I_E}{RT} \qquad \text{(Eq. 16-36)}$$

$$\leq \frac{S_{DS} I_E}{R} \qquad \text{(Eq. 16-35)}$$

$$\geq 0.044\ S_{DS} I_E \qquad \text{(Eq. 16-37)}$$

$$\geq \frac{0.5 S_1 I_E}{R} \text{ \{for SDC E and F or where } S_1 \geq 0.6g\text{\}} \qquad \text{(Eq. 16-38)}$$

For the example building considered

$$S_{DS} = 1.0g$$
$$S_{D1} = 0.6g$$
$$S_1 = 0.6g$$
$$R = 8$$
$$I_E = 1.0$$

Approximate fundamental period $T_a$ (1617.4.2.1)

$$T_a = C_T(h_n)^{3/4} \quad \text{(Eq. 16-39)}$$
$$C_T = 0.02 \text{ for a dual system}$$
$$h_n = \text{total height} = 255 \text{ ft}$$
$$T_a = 0.02 \times (255)^{3/4} = 1.28 \text{ sec}$$

$$\frac{S_{D1}I_E}{RT}W = \frac{0.6 \times 1 \times 67,246}{8 \times 1.28} = 3,940 \text{ kips} \quad \text{. . . governs}$$

$$\frac{S_{DS}I_E}{R}W = \frac{1.0 \times 1 \times 67,246}{8} = 8,406 \text{ kips}$$

$$0.044 S_{DS} I_E W = 0.044 \times 1.0 \times 1 \times 67,246 = 2,959 \text{ kips}$$

Since $S_1 = 0.6g$, Equation 16-38 is applicable for the example building in SDC D.

$$\frac{0.5 S_1 I_E}{R}W = \frac{(0.5)(0.6)(1.0)}{8}(67,246) = 2,522 \text{ kips}$$

Use $V = 3,940$ kips.

### 2.3.2. Vertical distribution of base shear (1617.4.3)

Distribute the base shear as follows:

$$F_x = C_{vx} V \quad \text{(Eq. 16-41)}$$

$$C_{vx} = \frac{w_x h_x^k}{\sum_{i=1}^{20} w_i h_i^k} \quad \text{(Eq. 16-42)}$$

$$T = 1.28 \text{ sec}$$

$$k = 1 \le 1 + 0.5\,(T - 0.5) \le 2 = 1.39 \quad \text{(1617.4.3)}$$

Distribution of the design base shear along the height of the building is shown in Table 2-1.

### 2.3.3. Lateral analysis

A three-dimensional analysis of the structure was performed under the lateral forces shown in Table 2-1 using the SAP 2000 computer program. To account for accidental torsion, the mass at each level was assumed to be displaced from the center of mass by a distance equal to 5 percent of the building dimension perpendicular to the direction of force (1617.4.4.4). In the model, rigid diaphragms were assigned at each level, and rigid-end offsets were defined at the ends of each member so that results were automatically obtained at the faces of each support.

According to 1618.1, the mathematical model must consider cracked section properties.

The stiffnesses of members used in the analyses were as follows:

For columns and shear walls, $I_{eff} = I_g$
For beams, $I_{eff} = 0.5I_g$ (considering slab contribution)

$P$-$\Delta$ effects are considered in the lateral analysis. It may be noted that this effect is allowed to be neglected in many situations as explained later. (1617.4.6.2)

Lateral displacements of the example building, computed elastically under the distributed lateral forces of Table 2-1, are shown in Table 2-3.

### 2.3.4. Modification of approximate period

The use of period by the approximate method (1617.4.2.1) often results in a conservative design. It is appropriate to use a more rational method for computation of period to reduce the design forces. However, the modified period must not exceed the approximate period by a factor (referred to as coefficient $C_u$) shown in Table 1617.4.2. The Rayleigh-Ritz procedure, given by the following equation, is used as a rational method.

$$T = 2\pi\sqrt{\Sigma w_i\delta_i^2/g\Sigma F_i\delta_i}$$

where the values of $F_i$ represent any lateral force distributed approximately in accordance with the principles of Equations 16-41 and 16-42 or any other rational distribution. The elastic deflections, $\delta_i$, shall be calculated using the applied lateral forces, $F_i$.

Table 2-3 shows values of $F_i$ and the corresponding $\delta_i$ based on the approximate period. The modified period can be found as follows:

$$\begin{aligned} T &= 2\pi\sqrt{\Sigma w_i\delta_i^2/g\Sigma F_i\delta_i} \\ &= 2\pi\sqrt{(2,378,842/386 \times 28,202)} \qquad \text{(see Table 2-3)} \\ &= 2.937 \text{ sec} \\ &\le 1.2 \times T \text{ from approximate method} \qquad \text{(T 1617.4.2 for } S_{D1} > 0.4\text{)} \\ &\le 1.2 \times 1.28 = 1.536 \text{ sec} \qquad \ldots \text{governs} \end{aligned}$$

### 2.3.5. Revised design base shear

Using the modified period of $T = 1.536$ seconds, the design base shear is recalculated as:

$$V = \frac{S_{D1}I_E}{RT}W = \frac{0.6 \times 1 \times 67,246}{8 \times 1.536} = 3,284 \text{ kips} \qquad \ldots \text{governs}$$

$$\leq \frac{S_{DS} I_E}{R} W = \frac{1.0 \times 1 \times 67{,}246}{8} \qquad = 8{,}406 \text{ kips}$$

$$\geq 0.044 S_{DS} I_E W = 0.044 \times 1.0 \times 1 \times 67{,}246 \qquad = 2{,}959 \text{ kips}$$

Since $S_1 = 0.6g$, Equation 16-38 is applicable for the example building in SDC D.

$$V \geq \frac{0.5 S_1 I_E}{R} W = \frac{(0.5)(0.6)(1.0)}{8}(67{,}246) \qquad = 2{,}522 \text{ kips}$$

Use $V = 3{,}284$ kips

Figure 2-3 shows the graphical representation of the above four expressions in non-dimensionalized form.

Distribute the base shear as follows:

$$F_x = C_{vx} V \qquad \text{(Eq. 16-41)}$$

$$C_{vx} = \frac{w_x h_x^k}{\sum_{i=1}^{20} w_i h_i^k} \qquad \text{(Eq. 16-42)}$$

$$T = 1.536 \text{ sec}$$

$$k = 1 \leq 1 + 0.5\,(T - 0.5) \leq 2 = 1.52 \qquad \text{(1617.4.3)}$$

The distribution of the design base shear along the height of the building is shown in Table 2-4.

### 2.3.6. Results of analysis

The maximum shear force and bending moment at the base of each shear wall (between ground and 2nd floor) were found to be 1,571 kips and 118,596 ft-kips, respectively (Table 2-7).

Because of the location of the shear walls within the plan of the building, the earthquake-induced axial force in each shear wall is equal to zero.

The lateral displacements at every floor level ($\delta_{xe}$) are shown in Table 2-5. The maximum inelastic response displacements ($\delta_x$) and story drifts are computed and shown in Table 2-5.

$\delta_x$ is calculated per 1617.4.6.1

$$\delta_x = \frac{C_d \delta_{xe}}{I_E} \qquad \text{(Eq. 16-46)}$$

### 2.3.7. Story drift limitation

According to 1617.3, the calculated story drift, $\Delta$, as shown in Table 2-5, shall not exceed 0.020 times the story height (Table 1617.3 for Seismic Use Group I and all other buildings).

| Floor | Maximum allowable drift | | Largest drift : | (Table 2-5) |
|---|---|---|---|---|
| 1st | $0.02 \times 17.5$ ft = 4.2 in. | > | 0.85 in. | . . . o.k. |
| Others | $0.02 \times 12.5$ ft = 3.0 in. | > | 2.60 in. | . . . o.k. |

### 2.3.8. *P*-Δ effects

According to 1617.4.6.2, *P*-Δ effects on story shears and moments, the resulting member forces and moments, and story drifts induced by these effects need not be considered when the stability coefficient, θ, as determined by the following formula, is equal to or less than 0.1

$$\theta = \frac{P_x \Delta}{V_x h_{sx} C_d} \qquad \text{(Eq. 16-47)}$$

where: $P_x$ = the total unfactored vertical force
$\Delta$ = the design story drift
$V_x$ = the seismic shear force acting between level $x$ and $x$-1
$h_{sx}$ = the story height below level $x$
$C_d$ = the deflection amplification factor

In the lateral analysis performed using the SAP 2000 computer program, the *P*-Δ effects are included. However, for illustration purposes, the stability coefficient is calculated as shown in Table 2-6. As the maximum stability coefficient θ (= 0.044) is less than 0.1, the *P*-Δ effect could have been neglected.

### 2.3.9. Redundancy factor, ρ (1617.2)

Typically, in a dual system, the shear walls will carry the largest proportion of the story shear at the base of the structure. The redundancy factor is expressed as follows:

$$1 \le \rho = \rho_1 = 2 - \frac{20}{r_{max_1}\sqrt{A_1}} \le 1.5 \qquad \text{(Eq. 16-32)}$$

where: $A_1$ = floor area of diaphragm immediately above first story, ft$^2$

$$r_{\max_1} = \frac{\text{maximum design shear in any of the walls at base}}{\text{total design base shear}} \times \frac{10}{\ell_w}$$

$$= \frac{1{,}571}{3{,}284} \times \frac{10}{28.83} = 0.166$$

$$A_1 = 132.83 \times 132.83 \text{ ft}^2$$

$$\rho = 2 - \frac{20}{0.166 \times \sqrt{132.83 \times 132.83}} = 1.09$$

For dual systems, the value of ρ need not exceed 80 percent of the value calculated above. (1617.2.2)

$80\%$ of above $= 0.8 \times 1.09 = 0.87 < 1$
Use $\rho = 1$

## 2.4. Design of Shear Walls

The design of one of the shear walls at the base of the structure is illustrated in this example. Similar procedures may be followed to design the shear wall at the other floor levels. The systematic procedure for designing the shear wall is shown in a flowchart in Figure 2-4. The design of shear walls by the 2000 IBC follows the procedure in ACI 318-99.

### 2.4.1. Design loads

Table 2-7 shows a summary of the axial force, shear force, and bending moment at the base of the example shear wall based on different load combinations.

Required axial load strength, $P_u = 7{,}952$ kips
Required shear strength, $V_u = 1{,}571$ kips
Required flexural strength, $M_u = 118{,}596$ kips

### 2.4.2. Check strength under flexure and axial load

Determine the P-M interaction diagram for the shear wall with assumed dimensions of the wall and assumed longitudinal reinforcement in boundary elements and web. Check to see that all the points representing strength demand (from the three load combinations shown in Table 2-7) are within the design strength interaction diagram.

In this example, the shear wall dimensions and reinforcement, as shown in Figure 2-5, are considered. Note that the shear wall boundary element size has been increased from the preliminary estimate of 34x34 in. to 42x42 in.

Using 36 #11 bars in each boundary element, the reinforcement ratio in the element is $(36 \times 1.56)/(42 \times 42) = 3.18\%$. This is high, but not excessive, and was deemed acceptable.

Figure 2-6 shows the P-M interaction diagrams for the example shear wall. As can be seen, all the points representing required strength are within the design strength curve.

One other quantity needs to be determined at this stage. That is the neutral axis depth, $c$, corrsponding to the maximum axial force (given by the lateral force combinations):

$P_u = 7{,}952$ kips
$c = 104$ in.

### 2.4.3. Design for shear

Height of the shear wall, $h_w = 255$ ft

Length of the shear wall, $\ell_w = 26 + 42/12 = 29.5$ ft

$$h_w/\ell_w = 255/29.5 = 8.64$$

ACI 318-99 (hereafter just ACI) 21.6.4.4.

$V_u$ must not exceed $\phi 8 A_{cv}\sqrt{f_c'}$
$A_{cv} = 16 \times (26 \times 12 + 42) = 5{,}664 \text{ in.}^2$

Take $\phi = 0.85$, since a wall with $h_w/\ell_w = 8.64$ is not going to be governed by shear in its failure mode (Note: when shear failure may govern, $\phi = 0.6$ must be used).

$$\phi 8 A_{cv}\sqrt{f_c'} = 0.85 \times 8 \times 5{,}664\sqrt{4{,}000}/1{,}000$$
$$= 2{,}436 \text{ kips} > 1{,}571 \text{ kips } (V_u) \quad \ldots \text{o.k.}$$

ACI 21.6.2.2

At least two curtains of reinforcement shall be used if $V_u > 2A_{cv}\sqrt{f_c'}$

$$2A_{cv}\sqrt{f_c'} = 2 \times 5{,}664\sqrt{4{,}000}/1{,}000$$
$$= 716 \text{ kips} < 1{,}571 \text{ kips}$$

So provide two curtains of reinforcement.

ACI 21.6.2.1

For two #5 horizontal bars @ 11 in. o/c.

$$\rho_n = \frac{2 \times 0.31}{16 \times 11} = 0.0035 > 0.0025 \quad \ldots \text{o.k.}$$
$$s = 11 \text{ in.} < 18 \text{ in.} \quad \ldots \text{o.k.}$$

Use two #5 horizontal bars @ 11 in. o/c.

ACI 21.6.4.3

The vertical reinforcement ratio, $\rho_v$, shall not be less than the horizontal reinforcement ratio, $\rho_n$, if the ratio $h_w/\ell_w < 2.0$. As $h_w/\ell_w = 8.64 > 2.0$, this clause is not applicable.

Provide two #5 vertical bars @ 11 in. o/c

$$\rho_v = 0.0035 > 0.0025 \quad \text{(ACI 21.6.2.1)} \quad \ldots \text{o.k.}$$

ACI 21.6.4.1.

For $h_w/\ell_w = 8.64 > 2.0$

$$\alpha_c = 2$$
$$V_n = A_{cv}(\alpha_c\sqrt{f_c'} + \rho_n f_y) \quad \text{(ACI Eq. 21-7)}$$
$$\phi V_n = 0.85 \times 5{,}664\,[2\sqrt{4{,}000} + 0.0035 \times 60{,}000]/1{,}000$$
$$= 1{,}620 \text{ kips} > 1{,}571 \text{ kips} \quad \ldots \text{o.k.}$$

A 12-inch vertical spacing of the horizontal bars would probably have been more desirable than the 11-inch spacing used. However, that would not have provided sufficient shear strength.

### 2.4.4. Design for flexure and axial loads (ACI 21.6.5)

ACI 21.6.5.1

Shear walls and portions of such walls subject to combined flexural and axial loads are to be designed in accordance with Sections 10.2 and 10.3, i.e., the provisions applicable to columns. Boundary elements as well as the wall web are to be considered effective.

### 2.4.5. Boundary elements fo special reinforced concrete shear walls (ACI 21.6.6)

ACI 21.6.6.1

The need for special boundary elements at the edges of shear walls is to be evaluated in accordance with Section 21.6.6.2 (displacement-based approach) or 21.6.6.3 (stress-based approach). In this example, the displacement-based approach is used.

ACI 21.6.6.2(a): Displacement-based approach

Compression zones are to be reinforced with special confinement reinforcement where:

$$c \geq c_{cr} = \frac{I_w}{600(\delta_u/h_w)} \qquad \text{(ACI Eq. 21-8)}$$

As computed earlier,

$c$ = 104 in.

$\ell_w$ = 29.5 ft

$h_w$ = 255 ft

$\delta_u$ = 40.43 in. along the wall line ($\delta_x$ at roof from Table 2-5).

$\delta_u/h_w$ = 40.43/255 × 12 = 0.0132 > 0.007 . . . Use $\delta_u/h_w$ = 0.0132

$$C_{cr} = \frac{29.5 \times 12}{600(0.0132)} = 44.7 \text{ in.}$$

ACI 21.6.6.2(b): Height of boundary element

The special boundary element reinforcement is to extend vertically from the critical section a distance not less than the larger of $\ell_w$ and $M_u/4V_u$.

$\ell_w$ = 29.5 ft . . . governs

$$\frac{M_u}{4V_u} = \frac{118{,}596}{4 \times 1{,}571} = 18.9 \text{ ft}$$

#### 2.4.5.1. Shear wall boundary zone details (ACI 21.6.6.4)

ACI 21.6.6.4(a): Length of zone

Confined boundary zone shall extend horizontally from the extreme compression fiber a distance not less than the larger of $c - 0.1\ell_w$ and $c/2$.

$c - 0.1\ell_w = 104 - 0.1 \times 29.5 \times 12 = 68.6$ in. . . .governs

$c/2 = 104/2 = 52.0$ in.

Since the length of the needed boundary zone (= 69 in.) exceeds the depth of the physical boundary element or column (= 42 in.), a portion of web (69 − 42 = 27 in.) must be confined.

<u>ACI 21.6.6.4(c):</u> Transverse reinforcement

Special boundary zone transverse reinforcement shall satisfy the requirements of 21.4.4.1 through 21.4.4.3, except that Equation 21-3 need not be satisfied.

*Boundary column confinement*

Minimum area of rectangular hoop reinforcement (ACI 21.4.4.1b)

$$A_{sh} = 0.09 s h_c f'_c / f_y h \qquad \text{(ACI Eq. 21-4)}$$

Because there are ten layers of longitudinal reinforcement in the boundary column, minimum number of legs (hoops and ties) needed to support alternate bars is six.

Maximum horizontal spacing of hoop or crosstie legs,

$$h_x = 2\left[\frac{42 - 2(1.5 + 0.625) - 1.41}{9}\right] + 1.41 + 0.625 = 10.11 \text{ in.}$$

According to ACI 21.4.4.2, the transverse reinforcement shall be spaced at a distance not exceeding: a) one-quarter of the minimum member dimension, b) six times the diameter of the longitudinal reinforcement, and c) $s_x$, as defined by ACI Equation 21-5.

$4 \text{ in.} \le s_x = 4 + (14 - h_x)/3 \le 6 \text{ in.}$ (ACI Eq. 21-5)
$4 \text{ in.} \le s_x = 5.30 \text{ in.} \le 6 \text{ in.}$

Use $s_x = 5.30$ in.

$s \le 5.30$ in. . . . governs
$\le 6d_b = 6 \times 1.41 = 8.46$ in.
$\le$ minimum member dimension / 4 = 42 / 4 = 10.5 in.

$h_c = 42 - 2 \times 1.5 - 5/8 = 38.4$ in.

$A_{sh} \ge 0.09 \times 5 \times 38.4 \times 4/60 = 1.15 \text{ in}^2$

With one tie all around the longitudinal reinforcement and four crossties in either direction (as shown in Fig. 2-5),

$A_{sh}$ provided $= 6 \times 0.31 = 1.86 \text{ in}^2 > 1.15 \text{ in}^2$ . . . o.k.

Confinement is to be provided at both ends over a length of 42 inches. In addition, a portion of the web is to be confined as follows.

*Web confinement*

Wall portion of length 69 − 42 = 27 in. ($= \ell_{wbz}$) must be confined.
Maximum horizontal spacing of hoop or crosstie legs, $h_x = 11$ in.

According to ACI 21.4.4.2, the transverse reinforcement shall be spaced at a distance not exceeding: a) one-quarter of the minimum member dimension, b) six times the diameter of the longitudinal reinforcement, and c) $s_x$, as defined by ACI Equation 21-5.

$$4 \text{ in.} \leq s_x = 4 + (14 - h_x)/3 \leq 6 \text{ in.} \qquad \text{(ACI Eq. 21-5)}$$

$$4 \text{ in.} \leq s_x = 5.0 \text{ in.} \leq 6 \text{ in.}$$

Use $s_x = 5.0$ in.

$$s \leq 5.0 \text{ in.}$$

$$\leq 6d_b = 6 \times 0.625 = 3.75 \text{ in.} \qquad \text{... governs}$$

$$\leq \min(\ell_{wbz}, h)/4 = 4.0 \text{ in.}$$

Use $s = 3$ in.

Confinement in direction perpendicular to the wall

$$h_c = 27 \text{ in.}$$

$$A_{sh} \geq 0.09 \times 3 \times 27 \times 4/60 = 0.486 \text{ in}^2$$

With three crossties (as shown in Fig. 2-5)

$$A_{sh} \text{ provided} = 3 \times 0.31 = 0.93 \text{ in}^2 > 0.486 \text{ in}^2 \qquad \text{... o.k.}$$

Confinement in direction parallel to the wall

$$h_c = 16 - 2 \times 1.5 - 5/8 = 12.4 \text{ in.}$$

$$A_{sh} \geq 0.09 \times 3 \times 12.4 \times 4/60 = 0.22 \text{ in}^2$$

With two layers of reinforcement in the horizontal direction (2 #5 @ 3 in.), as shown in Figure 2-5, the confining steel area:

$$A_{sh} \text{ provided} = 2 \times 0.31 = 0.62 \text{ in}^2 > 0.22 \text{ in}^2 \qquad \text{... o.k.}$$

### 2.4.6. Design of shear wall by spreadsheet

Figure 2-7 shows the design of the shear wall in a spreadsheet format.

## 2.5. Dynamic Analysis Procedure (Response Spectrum Analysis)

As explained earlier, a dynamic analysis procedure is required for this example building (having height > 240 feet in Seismic Design Category D). The response spectrum analysis method (1618.1) was used, utilizing the SAP 2000 computer program.

The following items are worth mentioning in conjunction with the analyses carried out.

Self weight is automatically considered by SAP 2000. The superimposed dead load (SDL) needs to be computed and assigned to relevant joints as masses.

SDL on each floor

$= (86 + 20\{10 \text{ for roof}\}) \times 130^2$ ............................ [Joists + SDL]
$+ 8 \times 12.5\{15 \text{ for } 2^{nd} \text{ floor and } 6.25 \text{ for roof}\} \times 130 \times 4$ ......... [Cladding]
$- 150 \times (26\text{-}34/12) \times 34/12 \times 7^*/12 \times 56$ .... [Equivalent self weight for joists]
$= 1{,}521$ kips {1,532 for $2^{nd}$ floor and 1,526 for roof including 200 kips}
*{86 psf for joists gives 7 inches of equivalent concrete slab thickness}

Masses of magnitude 0.475, 0.95, and 1.90 kip-sec$^2$/ft are assigned to each corner, edge, and interior joint, respectively, based on the above loads. These values are obtained as follows:

On each floor, there are 4 corner joints, each with a tributary area X = 13x13 ft (mass assigned to each = $m$), 16 edge joints, each with a tributary area of 2X (mass assigned to each = $2m$), and 16 interior joints, each with a tributary area of 4X (mass assigned to each = $4m$). Thus, the total mass on each floor becomes

$$(m \times 4) + (2m \times 16) + (4m \times 16) = [\text{SDL (kips)}]/32 \text{ ft/sec}^2$$

or, $m$ = SDL/3,200 (kip-sec$^2$/ft)

Considering the magnitude of SDL = 1,521 kips on each floor (approximately),
$m$ = 0.475 kip-sec$^2$/ft

The magnitude of mass to be assigned to each corner joint is equal to $m$ or 0.475 kip-sec$^2$/ft. Similarly, the masses assigned to each edge and interior joint would be 0.95 kip-sec$^2$/ft ($2m$) and 1.90 kip-sec$^2$/ft ($4m$), respectively.

### 2.5.1. Mode shapes

The 3-D analysis by SAP 2000 yielded the following periods for the first four modes

| Mode | Period (sec) | Participating Mass (%) |
|---|---|---|
| 1 | 2.485 | 71.2 |
| 2 | 0.659 | 14.8 |
| 3 | 0.300 | 6.1 |
| 4 | 0.178 | 3.1 |

As seen from the above, consideration of modes 1 (period = 2.485 sec), 2 (period = 0.659 sec), and 3 (period = 0.3 sec) should be adequate for lateral load analysis, as they account for about 92.1 percent (more than 90 percent) of the participating mass (1618.2). The periods and mode shapes of these three modes are given in Table 2-8.

The three modes considered in modal analysis have periods of 2.485, 0.659, and 0.300 seconds.

### 2.5.2. Verification of results from SAP 2000

To check the accuracy of results obtained from the SAP 2000 computer program, the example building was analyzed using STAAD-III. Both three-dimensional and equivalent two-dimensional analyses, as shown in Figure 2-8, were performed to compute the modal periods. Five cases were considered, each with different stiffnesses assigned to beams, columns, and shear walls. Table 2-8 shows the comparison between the results obtained from both the computer programs. It shows a good correlation between the results from SAP 2000 and STAAD-III. In addition, the table shows that considering an equivalent two-dimensional model is quite reasonable. It may be noted that the command for rigid diaphragm was not available in the version of STAAD-III used and it was necessary to assign rigid diagonal truss elements at each floor level.

### 2.5.3. $L_m$ and $M_m$ for each mode shape

According to 1618.4, the portion of base shear contributed by the $m^{th}$ mode, $V_m$, shall be determined from the following equations:

$$V_m = C_{sm}\overline{W}_m \qquad \text{(Eq. 16-51)}$$

$$\overline{W}_m = L_m^2/M_m \qquad \text{(Eq. 16-21)}$$

$$L_m = \sum_{i=1}^{n} w_i\phi_{im}$$

$$M_m = \sum_{i=1}^{n} w_i\phi_{im}^2$$

where: $C_m$ = the modal seismic response coefficient determined in Equation 16-53
$\overline{W}_m$ = the effective modal gravity load
$w_i$ = the portion of total gravity load, $W$, of the building at Level $i$
$\phi_{im}$ = the displacement amplitude at the $i^{th}$ level of the building when vibrating in its $m^{th}$ mode.

From Table 2-9,

$L_1$ = 33,440 kips/$g$ $\quad$ $L_2$ = −14,622 kips/$g$ $\quad$ $L_3$ = 10,202 kips/$g$
$M_1$ = 23,347 kips/$g$ $\quad$ $M_2$ = 21,480 kips/$g$ $\quad$ $M_3$ = 24,445 kips/$g$

### 2.5.4. Modal seismic design coefficients, $C_{sm}$

$$C_{sm} = \frac{S_{am}}{(R/I_E)} \qquad \text{(Eq. 16-53)}$$

where: $S_{am}$ = the modal design spectral response acceleration at period $T_m$ determined from either the general design response spectrum of 1615.1 or a site-specific response spectrum per 1615.2.

In the example considered here, the general procedure of 1615.1 will be followed. Under this procedure, the spectral response acceleration, $S_a$, can be expressed by the following equations (Fig. 1615.1.4):

for $T > T_s$ $\quad S_a = S_{D1}/T$

for $T_o \le T \le T_s$ $\quad S_a = S_{DS}$

for $T \le T_o$ $\quad S_a = 0.6S_{DS}T/T_o + 0.4S_{DS}$

where: $T_s = S_{D1}/S_{DS}$, and $T_o = 0.2T_s$

According to 1618.4 (Exception), when the general response spectrum of 1615.1 is used for buildings on Site Class D, E, or F sites, the modal seismic design coefficients for modes other than the fundamental mode that have periods less than 0.3 second are permitted to be determined by the following equation:

$$C_{sm} = \frac{0.4S_{DS}}{(R/I_E)}(1.0 + 5.0\,T_m) \qquad \text{(Eq. 16-54)}$$

For the example building considered, the periods from the second and the third modes are greater than or equal to 0.3 second. Equation 16-53 is therefore used for the following calculations.

For the example building, $T_s = 0.6/1.0 = 0.60$ sec
$T_o = 0.6/5 = 0.12$ sec

Mode 1: $T_1 = 2.485$ sec $> 0.60$ sec $\quad C_{s1} = \dfrac{0.6g}{2.485 \times (8/1)} = 0.0302g$

Mode 2: $T_2 = 0.659$ sec $> 0.60$ sec $\quad C_{s2} = \dfrac{0.6g}{0.659 \times (8/1)} = 0.1138g$

Mode 3: $T_3 = 0.300$ sec $> 0.12$ sec $< 0.60$ sec $\quad C_{s3} = \dfrac{1.0g}{(8/1)} = 0.1250g$

### 2.5.5. Base shear using modal analysis

$$V_m = C_{sm}\overline{W}_m = \frac{L_m^2}{M_m}C_{sm}$$

Mode 1: $V_1 = \dfrac{33,440^2}{23,347g} \times 0.0302g = 1,446$ kips

Mode 2: $V_2 = \frac{(-14,622)^2}{21,480g} \times 0.1138g = 1,133$ kips

Mode 3: $V_3 = \frac{10,202^2}{24,445g} \times 0.1250g = 532$ kips

The modal base shears are combined by the SRSS method to give the resultant base shear (1618.7)

$$V = [1,446^2 + 1,133^2 + 532^2]^{1/2} = 1,912 \text{ kips}$$

The participating mass (*PM*) for each of the above three modes is determined as:

$$PM = \frac{L_m^2 g}{M_m W}$$

Mode 1: $PM_1 = \frac{33,440^2}{23,347 \times 67,246} = 0.712$

Mode 2: $PM_2 = \frac{(-14,622)^2}{21,480 \times 67,246} = 0.148$

Mode 3: $PM_3 = \frac{10,202^2}{24,445 \times 67,246} = 0.063$

Please note that these participating masses are equal to or very close to the values obtained directly from SAP 2000 (See 2.5.1 of this publication).

$\Sigma PM = 0.712 + 0.148 + 0.063 = 0.923 > 0.90$ . . . o.k.

Therefore, consideration of the above three modes (1, 2, and 3) is sufficient per 1618.2.

### 2.5.6. Design base shear using static procedure

The design base shear using the static lateral-force procedure was computed in the previous section using a fundamental period of 1.536 seconds (i.e., $= C_u \times T_a$) and was found to be 3,284 kips.

### 2.5.7. Scaling of elastic response parameters for design

Section 1618.7 stipulates that the base shear using modal analysis must be scaled up when the base shear calculated using the equivalent lateral-force procedure is greater. However, it is permitted to use a fundamental period of $T = 1.2\, C_u T_a$ instead of $C_u T_a$ in the calculation of base shear by the equivalent lateral-force procedure.

Based on the new period $T = 1.2 \times 1.536 = 1.843$ sec, the design base shear is recalculated as follows:

$$V = \frac{S_{D1}I_E}{RT}W = \frac{0.6 \times 1 \times 67{,}246}{8 \times 1.843} = 2{,}736 \text{ kips}$$

$$\leq \frac{S_{DS}I_E}{R}W = \frac{1.0 \times 1 \times 67{,}246}{8} = 8{,}406 \text{ kips}$$

$$\geq 0.044 S_{DS} I_E W = 0.044 \times 1.0 \times 67{,}246 = 2{,}959 \text{ kips} \quad \text{. . . governs}$$

Since $S_1 = 0.6g$, Equation 16-38 is applicable for the example building in SDC D

$$V \geq \frac{0.5 S_1 I_E}{R}W = \frac{(0.5)(0.6)(1.0)}{8}(67{,}246) = 2{,}522 \text{ kips}$$

Use $V = 2{,}959$ kips

2,959 kips (equivalent base shear) > 1,912 kips (modal base shear)

Therefore, the modal forces must be scaled up per Equation 16-59

Scale factor = 2,959/1,912 = 1.548

The modified modal base shears are as follows:

$$V_1 = 1.548 \times 1{,}446 = 2{,}238 \text{ kips}$$
$$V_2 = 1.548 \times 1{,}133 = 1{,}754 \text{ kips}$$
$$V_3 = 1.548 \times 532 = 824 \text{ kips}$$
$$V = [2{,}238^2 + 1{,}754^2 + 824^2]^{1/2} = 2{,}960 \text{ kips}$$

### 2.5.8. Distribution of base shear

Lateral force at level $i$ (1 to 20) for mode $m$ (1 to 3) is to be calculated as (see Equations 16-55 and 16-56):

$$F_{im} = \frac{w_i \Phi_{im}}{\Sigma w_i \Phi_{im}} V_m$$

The distribution of the modal base shear for each mode is shown in Table 2-9.

### 2.5.9. Lateral analysis

Three-dimensional analysis of the structure was performed for each set of modal forces using the SAP 2000 computer program. To account for accidental torsion, the mass at each level was assumed to be displaced from the center of mass by a distance equal to 5 percent of the building dimension perpendicular to the direction of force (1617.4.4.4). In the model, rigid diaphragms were assigned at each level, and rigid-end offsets were defined at the ends of each member so that results were automatically obtained at the faces of each support.

The stiffnesses of members used in the analyses were as follows:

For columns and shear walls, $I_{eff} = I_g$
For beams, $I_{eff} = 0.5I_g$

Table 2-10 shows the shear force and bending moment at each floor level of each shear wall (the four shear walls are identical in every respect and are subject to the same forces) due to each considered mode and the resultant load effects. Because of the location of the shear walls within the plan of the building, the earthquake-induced axial force in each shear wall is equal to zero.

The above values at the base level from the dynamic procedure can be compared with the corresponding values from the static procedure as follows: (subscripts *d* and *s* represent results from dynamic and static procedures, respectively).

$$\frac{V_d}{V_s} = \frac{1,419}{1,571} = 0.90$$

$$\frac{M_d}{M_s} = \frac{88,920}{118,596} = 0.75$$

The lower ratio of dynamic-to-static moments reflects the different distribution of lateral forces along the height of the building obtained from dynamic analysis. This also shows the possible advantage of doing dynamic analysis.

Resultant lateral displacements (square root of the sum of the squares of modal displacements) at every floor level, $\delta_{xe}$, are shown in Table 2-11. The maximum inelastic response displacements, $\delta_x$, and story drifts are computed and shown in Table 2-11.

### 2.5.10. Story drift limitation

According to 1617.3, the calculated story drifts, $\Delta$, as shown in Table 2-11 shall not exceed 0.020 times the story height (Table 1617.3 for Seismic Use Group I and all other buildings).

| Floor | Maximum allowable drift | | Largest drift | (Table 2-11) |
|---|---|---|---|---|
| 1st | $0.02 \times 17.5$ ft $= 4.2$ in. | > | 0.64 in. | . . . o.k. |
| Others | $0.02 \times 12.5$ ft $= 3.0$ in. | > | 1.64 in. | . . . o.k. |

### 2.5.11. *P*-Δ effects

According to Section 1617.4.6.2, *P*-Δ effects on story shears and moments, the resulting member forces and moments, and story drifts induced by these effects need not be considered when the stability coefficient, $\theta$, as determined by the following formula is equal to or less than 0.1.

$$\theta = \frac{P_x \Delta}{V_x h_{sx} C_d} \qquad \text{(Eq. 16-47)}$$

where: $P_x$ = the total unfactored vertical force
$\Delta$ = the design story drift
$V_x$ = the seismic shear force acting between level $x$ and $x$-1
$h_{sx}$ = the story height below level $x$
$C_d$ = the deflection amplification factor.

In the lateral analysis performed using SAP 2000, the $P$-$\Delta$ effects are included.

### 2.5.12. Redundancy factor, ρ

At the base, $r_{max}$ = (1,419/2,960) × 10/29.17 = 0.164 (assuming 38x38-inch boundary elements)

ρ = 1 (as computed under equivalent lateral-force procedure).

## 2.6. Design of Shear Walls

The design of one of the shear walls at the base of the structure is illustrated in this example. Similar procedures can be followed to design the shear wall at the other floor levels. The systematic procedure for designing the shear wall is shown in a flowchart in Figure 2-4. The design of shear walls by the 2000 IBC follows the procedure in ACI 318-99.

### 2.6.1. Design loads

Table 2-12 shows a summary of the axial force, shear force, and bending moment at the base of the example shear wall based on different load combinations.

Required axial load strength, $P_u$ = 7,952 kips
Required shear strength, $V_u$ = 1,419 kips
Required flexural strength, $M_u$ = 88,920 ft-kips

### 2.6.2. Check strength under flexural and axial loads

Determine the P-M interaction diagram for the shear wall with assumed dimensions of wall and assumed longitudinal reinforcement in boundary elements and web. Check to see that all the points representing strength demand (from the three load combinations shown in Table 2-12) are within the design strength interaction diagram.

In this example, the shear wall dimensions and reinforcement as shown in Figure 2-9 are considered.

Using 36 #10 bars in each boundary element, the reinforcement ratio is

$$\frac{36 \times 1.27}{38 \times 38} = 3.17\%$$

This is high but not excessive and was deemed acceptable.

Figure 2-10 shows the P-M interaction diagrams for the example shear wall. As can be seen, all the points representing required strength are within the design strength curve.

One other quantity needs to be determined at this stage. That is the neutral axis depth, c, corresponding to the maximum axial force (in the presence of lateral force).

$$P_u = 7{,}952 \text{ kips}$$
$$c = 118 \text{ in.}$$

### 2.6.3. Design for shear

Height of the shear wall, $h_w$ = 255 ft

Length of the shear wall, $\ell_w$ = 26 + 38/12 = 29.17 ft

$$h_w/\ell_w = 255/29.17 = 8.74$$

ACI 318-99 (hereafter just ACI) 21.6.4.4

$V_u$ must not exceed $\phi 8 A_{cv}\sqrt{f_c'}$
$A_{cv} = 16 \times (26 \times 12 + 38) = 5{,}600 \text{ in.}^2$

Take $\phi = 0.85$, since a wall with $h_w/\ell_w = 8.74$ is not going to be governed by shear in its failure mode.

$$\phi 8 A_{cv}\sqrt{f_c'} = 0.85 \times 8 \times 5{,}600\sqrt{4{,}000}/1{,}000$$
$$= 2{,}408 \text{ kips} > 1{,}419 \text{ kips } (V_u) \quad \ldots \text{o.k.}$$

ACI 21.6.2.2

At least two curtains of reinforcement shall be used if $V_u > 2 A_{cv}\sqrt{f_c'}$

$$2 A_{cv}\sqrt{f_c'} = 2 \times 5{,}600\sqrt{4{,}000}/1{,}000$$
$$= 708 \text{ kips} < 1{,}419 \text{ kips}$$

Provide two curtains of reinforcement.

ACI 21.6.2.1

For two #5 horizontal bars @ 11 in. o/c

$$\rho_n = \frac{2 \times 0.31}{16 \times 11} = 0.0035 > 0.0025 \quad \ldots \text{o.k.}$$

$$s = 11 \text{ in.} < 18 \text{ in.} \quad \ldots \text{o.k.}$$

Use two #5 horizontal bars @ 11 in. o/c.

ACI 21.6.4.3

The vertical reinforcement ratio, $\rho_v$, shall not be less than the horizontal reinforcement ratio, $\rho_n$, if the ratio $h_w/\ell_w < 2.0$. As $h_w/\ell_w = 8.74 > 2.0$, this clause is not applicable.

Provide two #5 vertical bars @ 11 in. o/c.

$\rho_v = 0.0035 > 0.0025$ (ACI 21.6.2.1) . . . o.k.

ACI 21.6.4.1

$$\begin{aligned} \text{For } h_w/\ell_w &= 8.74 > 2.0 \\ \alpha_c &= 2 \\ V_n &= A_{cv}(\alpha_c\sqrt{f_c'} + \rho_n f_y) \quad \text{(ACI Eq. 21-7)} \\ \phi V_n &= 0.85 \times 5{,}600\,[2\sqrt{4{,}000} + 0.0035 \times 60{,}000]/1{,}000 \\ &= 1{,}602 \text{ kips} > 1{,}419 \text{ kips} \quad \ldots \text{o.k.} \end{aligned}$$

A 12-inch vertical spacing of the horiontal bars would probably have been more desirable than the 11-inch spacing used. However, that would not have provided sufficient shear strength.

### 2.6.4. Design for flexure and axial loads (ACI 21.6.5)

ACI 21.6.5.1

Shear walls and portions of such walls subject to combined flexural and axial loads are to be designed in accordance with 10.2 and 10.3 (i.e., the provisions applicable to columns). Boundary elements as well as the wall web are to be considered effective.

### 2.6.5. Boundary elements of special reinforced concrete shear walls (ACI 21.6.6)

ACI 21.6.6.1

The need for special boundary elements at the edges of shear walls is to be evaluated in accordance with ACI 21.6.6.2 (displacement-based approach) or 21.6.6.3 (stress-based approach). In this example, the displacement-based approach is used.

ACI 21.6.6.2(a): Displacement-based approach

Compression zones are to be reinforced with special boundary elements where:

$$c \geq c_{cr} = \frac{\ell_w}{600(\delta_u/h_w)} \quad \text{(ACI Eq. 21-8)}$$

As computed earlier

$$\begin{aligned} c &= 118 \text{ in.} \\ \ell_w &= 29.17 \text{ ft} \end{aligned}$$

$h_w$ = 225 ft
$\delta_u$ = 26.7 in. along the shear wall line ($\delta_x$ at roof level; see Table 2-11)
$\delta_u/h_w = 26.7/255 \times 12 = 0.0087 > 0.007$ . . . Use $\delta_u/h_w = 0.0087$

$$c_{cr} = \frac{29.17 \times 12}{600(0.0087)} = 66.9 \text{ in.}$$

ACI 21.6.6.2 (b): Height of boundary element

The special boundary element reinforcement shall extend vertically from the critical section a distance not less than the larger of $\ell_w$ and $M_u/4V_u$.

$\ell_w$ = 29.17 ft . . . governs

$$\frac{M_u}{4V_u} = \frac{88,920}{4 \times 1,419} = 15.67 \text{ ft}$$

### 2.6.5.1. Shear wall boundary zone details (ACI 21.6.6.4)

ACI 21.6.6.4(a): Length of boundary zone

Confined boundary zone shall extend horizontally from the extreme compression fiber a distance not less than the larger of $c - 0.1\ell_w$ and $c/2$.

$c - 0.1\ell_w = 118 - 0.1 \times 29.17 \times 12 = 83.0$ in. . . .governs
$c/2 = 118/2 = 59.0$ in.

Since the length of the needed boundary zone (= 83 in.) exceeds the depth of the physical boundary element or column (= 38 in.), a portion of the web (83 − 38 = 45 in.) must be confined.

ACI Sec 21.6.6.4 (c): Transverse reinforcement

Special boundary zone transverse reinforcement shall satisfy the requirements of ACI 21.4.4.1 through 21.4.4.3, except that ACI Equation 21-3 need not be satisfied.

*Boundary column confinement*

Minimum area of rectangular hoop reinforcement (ACI 21.4.4.1b)

$$A_{sh} = 0.09\, sh_c f_c'/f_{yh} \qquad \text{(ACI Eq. 21-4)}$$

Because there are ten layers of longitudinal reinforcement in the boundary column, the minimum number of legs (hoops and ties) needed to support alternate bars is six.

Maximum horizontal spacing of hoop or crosstie legs,

$$h_x = 2\left[\frac{38 - 2(1.5 + 0.625) - 1.27}{9}\right] + 1.27 + 0.625 = 9.11 \text{ in.}$$

According to ACI 21.4.4.2, the transverse reinforcement shall be spaced at a distance not exceeding: a) one-quarter of the minimum member dimension, b) six times the diameter of the longitudinal reinforcement, and c) $s_x$, as defined by ACI Equation 21-5.

$4 \text{ in.} \leq s_x = 4 + (14 - h_x)/3 \leq 6 \text{ in.}$ (ACI Eq. 21-5)
$4 \text{ in.} \leq s_x = 5.6 \text{ in.} \leq 6 \text{ in.}$
Use $s_x = 5.6$ in.
$s \leq 5.6$ in. . . . governs
$\leq 6d_b = 6 \times 1.27 = 7.6$ in.
$\leq$ minimum member dimension/4 = 9.5 in.
$h_c = 38 - 2 \times 1.5 - 5/8 = 34.4$ in.
$A_{sh} \geq 0.09 \times 5 \times 34.4 \times 4/60 = 1.03 \text{ in.}^2$

With one tie all around the longitudinal reinforcement and four crossties in either direction (as shown in Fig. 2-9)

$A_{sh}$ provided $= 6 \times 0.31 = 1.86 \text{ in.}^2 > 1.03 \text{ in.}^2$ . . . o.k.

Confinement is to be provided at both ends over a length of 38 inches. In addition, a portion of the web is also to be confined as follows.

*Web confinement*

Wall portion of length 45 in. ($= \ell_{wbz}$) must be confined.
Maximum horizontal spacing of hoop or crosstie legs, $h_x = 11$ in.

According to ACI 21.4.4.2, the transverse reinforcement shall be spaced at a distance not exceeding: a) one-quarter of the minimum member dimension, b) six times the diameter of the longitudinal reinforcement, and c) $s_x$, as defined by ACI Equation 21-5

$4 \text{ in.} \leq s_x = 4 + (14 - h_x)/3 \leq 6 \text{ in.}$ (ACI Eq. 21-5)
$4 \text{ in.} \leq s_x = 5.0 \text{ in.} \leq 6 \text{ in.}$
Use $s_x = 5.0$ in.
$s \leq 5.0$ in.
$\leq 6d_b = 6 \times 0.625 = 3.75$ in. . . . governs
$\leq \min(\ell_{wbz}, h)/4 = 4.0$ in.
Use $s = 3.0$ in.

Confinement in direction perpendicular to the wall

$h_c = 45$ in.
$A_{sh} \geq 0.09 \times 3 \times 45 \times 4/60 = 0.81 \text{ in.}^2$

With five crossties (as shown in Fig. 2-9)

$A_{sh}$ provided $= 5 \times 0.31 = 1.55 \text{ in.}^2 > 0.81 \text{ in.}^2$ . . . o.k.

Confinement in direction parallel to the wall

$h_c \quad = 16 - 2 \times 1.5 - 5/8 = 12.4$ in.
$A_{sh} \quad \geq 0.09 \times 3 \times 12.4 \times 4/60 = 0.22$ in.$^2$

With two layers of reinforcement in the horizontal direction (2 #5 @ 3 in.), as shown in Figure 2-9, the confining steel area is:

$A_{sh}$ provided $= 2 \times 0.31 = 0.62$ in.$^2 > 0.22$ in.$^2$ . . . o.k.

### 2.6.6. Design of shear wall by spreadsheet

Figure 2-11 shows the design of the shear wall in a spreadsheet format.

## 2.7. Design of Flexural Members

The design of two beams will be illustrated in this example. These are:

| | | | | |
|---|---|---|---|---|
| Beam 1: | A2-B2 | Exterior | At level 2 | |
| Beam 2: | B2-C2 | Interior | At level 2 | (See Fig. 2-1) |

### 2.7.1. Design loads

Bending moments in beams due to lateral loads over the entire building height are shown in Table 2-13. The SRSS method was used to calculate the resultant moments. Table 2-13 also shows the internal forces caused by 25 percent of the design base shear acting on the frames alone. The shear forces induced in Beams A2-B2 and B2-C2 are also given in Table 2-13.

Table 2-14 gives the internal forces in beams due to gravity load analysis with different load patterns as shown in Figure 2-12.

In addition, the following simplified equations were used to obtain the internal forces based on the approximate analysis procedure of ACI 8.3.3: (Table 2-15)

Interior span (beam B2-C2 of the example building)

Maximum negative moment $= w\ell_n^2/11$
Maximum positive moment $= w\ell_n^2/16$
Maximum shear $= w\ell_n/2$

Exterior span (beam A2-B2 of the example building)

Maximum negative moment $= w\ell_n^2/16$ (exterior)
Maximum negative moment $= w\ell_n^2/10$ (interior)
Maximum positive moment $= w\ell_n^2/14$
Maximum shear $= 1.15w\ell_n/2$

where: $w = w_D$ for dead load and $w_L$ for live load.

The gravity loads for the beams are as follows:

$w_D = (86 + 20) \times 26 + 34 \times 24/144 \times 150 = 3.61$ k/ft {Beam section is 34x24}

$w_L = 32 \times 26 = 832$ lb/ft $= 0.832$ k/ft {Reduced live load is 32 psf}

Clear span, $\ell_n = 26 - 34/12 = 23.17$ ft

Results from the simplified method (shown in Table 2-15) were compared with those obtained by using pattern loading (Table 2-14) in Table 2-16. The results show that the simplified coefficient method is generally conservative and reasonable. In this example, the values obtained using ACI 8.3.3 were utilized for design.

Table 2-17 shows the summary of design axial forces, shear forces, and bending moments in beams (obtained from Tables 2-13 and 2-15).

### 2.7.2. Design of beam A2-B2 in interior frame

#### 2.7.2.1. General requirements (ACI 21.3.1)

According to ACI 21.3.1, flexural members shall satisfy the following conditions:

ACI 21.3.1.1

$P_u \le 0.1 A_g f_c'$

$A_g = 34 \times 24 = 816$ in.$^2$

$0.1 A_g f_c' = 0.1 \times 816 \times 4 = 326.4$ kips $\ge 0$ kip (Table 2-17) . . . o.k.

ACI 21.3.1.2

$\ell_n \ge 4d$

$\ell_n = 26 - 34/12 = 23.17$ ft

$d = 24 - 2.5 = 21.5$ in.

$4d = 7.17$ ft $\le 23.17$ ft . . . o.k.

ACI 21.3.1.3

Width/depth $\ge 0.3$

Width/depth $= 34/24 = 1.4 \ge 0.3$ . . . o.k.

ACI 21.3.1.4

i) Width $\ge$ 10 in.
Width = 34 in. $\ge$ 10 in. . . . o.k.

ii) Beam width $\le$ width of supporting member (column) + 1.5 beam depth
34 in. $\le 34 + 1.5 \times 24 = 70$ in. . . . o.k.

### 2.7.2.2. Longitudinal reinforcement (ACI 21.3.2)

ACI 21.3.2.1

$$A_{top} \text{ or } A_{bottom} \geq \frac{3\sqrt{f'_c}}{f_y} bd = 2.31 \text{ in.}^2$$
$$\geq \frac{200}{f_y} bd = 2.44 \text{ in.}^2$$
$$\leq 0.025\, bd = 18.3 \text{ in.}^2$$

At least two bars should be continuous.

Try the following reinforcements:

| | |
|---|---|
| 3 #8 bars at bottom near support | ($A_s$ provided = 2.37 in.$^2$) |
| 6 #8 bars at top near support | ($A_s$ provided = 4.74 in.$^2$) |
| 4 #8 bars at bottom at midspan | ($A_s$ provided = 3.16 in.$^2$) |
| 3 #8 bars at top at midspan | ($A_s$ provided = 2.37 in.$^2$) |

Minimum $A_s$ provided = 2.37 in.$^2$ ≅ the required minimum of 2.44 in.$^2$ . . . o.k.
Maximum $A_s$ provided = 4.74 in.$^2$ ≤ the required maximum of 18.3 in.$^2$ . . . o.k.

ACI 21.3.2.2

Positive design moment strength at support (i.e., $\phi M_n^+$ with 3 #8 bars)
= 223 ft-kips ≥ 48 ft-kips (Table 2-17) . . . o.k.
Negative design moment strength at support (i.e., $\phi M_n^-$ with 6 #8 bars)
= 432 ft-kips ≥ 400 ft-kips (Table 2-17) . . . o.k.
Positive design moment strength at midspan (i.e., $\phi M_n^+$ with 4 #8 bars)
= 294 ft-kips > 1.4 × 138 + 1.7 × 31 = 240 ft-kips . . . o.k.

At the joint face, the positive moment strength must be at least half the negative moment strength

223 ft-kips ≥ 0.5 × 432 = 216 ft-kips . . . o.k.

Providing two #8 bars at all sections throughout (where more bars are not required for strength), design moment strength $\phi M_n$ = 150 ft-kips ≥ 0.25 × 432 = 108 ft-kips, which would be acceptable. However, to satisfy minimum reinforcement requirements, provide three #8 bars throughout the span.

This gives more than two continuous bars as required by ACI 21.3.2.1 . . . o.k.

### 2.7.2.3. Shear strength (ACI 21.3.4)

ACI 21.3.4.1

$$V_e = \frac{M_{pr}^- + M_{pr}^+}{\ell_n} \pm \frac{w_u \ell_n}{2}$$

For calculating the probable flexural strength, the tensile stress in steel should be taken as 1.25 times the specified yield strength and the strength reduction factor, $\phi$, is to be taken as 1.0.

With six #8 bars at top and three #8 bars at bottom at the joint face

$M_{pr}^{-}$ = 591 ft-kips

$M_{pr}^{+}$ = 307 ft-kips

Factored gravity load from second load combination

$w_u = 1.4w_D + 0.5w_L = 5.5$ k/ft

$\ell_n = 23.17$ ft

$$V_e = \frac{591 + 307}{23.17} + 5.5 \times \frac{23.17}{2}$$

$$= 38.8 + 63.7$$

$$= 102.5 \text{ kips}$$

From analysis, maximum shear force = 85 kips (Table 2-17)

Use design shear force, $V_u$ = 102.5 kips (as $V_e$ > 85 kips found from analysis).

<u>ACI 21.3.4.2</u>

Transverse reinforcement (per ACI 21.3.3.1) to resist shear, $V_u$, must be determined assuming $V_c = 0$ if the following two conditions are met:

i) Earthquake-induced shear force $\geq 0.5\,V_e$
   Here, earthquake-induced shear = 38.8 < 102.5/2 = 51.3 kips (Not satisfied)
ii) $P_u = 0 \text{ kips} \leq 0.05A_g f_c' = 0.05 \times 34 \times 24 \times 4 = 163$ kips (Satisfied)

Since the first condition is not satisfied, $V_c$ need not be taken equal to zero.

However, recent research[22, 23] has indicated that in plastic hinge regions the concrete contribution, $V_c$, degrades with ductility level and should be taken as zero for displacement ductility of more than 4 (which is expected for special moment frames).

Conservatively, take $V_c = 0$ for potential plastic hinge regions. The shear reinforcement can be determined as follows:

$$V_s = \frac{V_u}{\phi} - V_c$$

$$= 102.5/0.85 - 0 = 120.6 \text{ kips}$$

Required spacing of #4 stirrups (with 4 legs for 6 main reinforcing bars)

$$s = \frac{A_v f_y d}{V_s} = \frac{4 \times 0.2 \times 60 \times 21.5}{120.6} = 8.6 \text{ in.}$$

ACI 21.3.3.2

| | | | | |
|---|---|---|---|---|
| $s$ | $\leq \frac{d}{4} = \frac{21.5}{4}$ | | $= 5.4$ in. | . . . governs |
| | $\leq 8$ times diameter of main bar | $= 8 \times 1$ | $= 8$ in. | |
| | $\leq 24$ times diameter of hoop | $= 24 \times 1/2$ | $= 12$ in. | |
| | $\leq 12$ in. | | $= 12$ in. | |

Use $s = 5$ in.

Place first hoop @ 2 inches from support.

The shear force carried by web reinforcement, $V_s$, should not exceed $8\sqrt{f'_c}\, bd$ (ACI 11.5.6.8)

$$8\sqrt{f'_c}\, bd = 8\sqrt{4,000} \times 34 \times 21.5/1,000$$
$$= 370 \text{ kips} > 120.6 \text{ kips} \qquad \text{. . . o.k.}$$

ACI 21.3.3.1

Hoops shall be provided over a length of:

i) 2 times the total depth, $2h = 2 \times 24 = 48$ in. from support faces.
ii) $2h$ on either side of a critical section where there is a possibility of flexural yielding. Assume no flexural yielding away from the above regions of potential plastic hinging.

Provide 11 hoops over 4 feet, 4 inches from faces of joints.

Shear force at 4.33 feet from joint face $= 102.5 - 5.50 \times 4.33 = 78.7$ kips

Take $V_c = 2\sqrt{f'_c}\, bd$ at sections 4.33 feet from the joint face because there is no possibility of flexural yielding and thus no degradation of $V_c$.

$$V_c = 2\sqrt{4,000} \times 34 \times 21.5 = 92.5 \text{ kips},$$
$$\phi V_c = 0.85 \times 92.5 = 78.6 \text{ kips} > 78.7 \text{ kips} \qquad \text{. . . o.k.}$$

Thus, there is no need of shear reinforcement beyond 4.33 feet from the joint faces. Provide minimum shear reinforcement.

Provide stirrups with seismic hooks at both ends at a spacing not to exceed $d/2$ (i.e., 10.75 in.) throughout. Use two-legged #4 bars at 10-inch spacing, as shown in Figure 2-13.

### 2.7.2.4. Reinforcing bar cut-off points

#### 2.7.2.4.1 *Negative bar cutoff:*

The negative reinforcement at the joint face is six #8 bars. The location where three of the six bars can be terminated will be determined. Note that three #8 bars must be continuous throughout the length of the beam to satisfy the minimum reinforcement requirements of ACI 21.3.2.1.

The loading used to find the cut-off point of the three #8 bars is 0.7 times the dead load in combination with the probable flexural strengths, $M_{pr}$, at the ends of the members (third load combination), because this combination will produce the longest bar cut-off lengths.

The design flexural strength, $\phi M_n$, provided by three #8 bars is 223 ft-kips. Therefore, the three reinforcing bars can be terminated after the factored moment, $M_u$, has been reduced to 223 ft-kips.

With $\phi = 1.0$ and $f_s = 75$ ksi $(= 1.25 \times 60)$

$M_{pr}$ = 591 ft-kips at one end (negative) and
= 307 ft-kips at the other (positive)

$w = 0.7w_D = 0.7 \times 3.61 = 2.53$ kips/ft

Referring to Figure 2-14(a)

$$2.53x^2/2 + 591 - 68.1x = 223 \qquad \{\text{Reaction at left support} = 68.1 \text{ kips}\}$$

or, $x = 6.1$ ft

Three #8 bars (to be cut off) must extend a distance $\geq d = 21.5$ in. (ACI 12.10.3)
$\geq 12d_b = 12$ in. beyond $x$.

Thus, from the face of support, the total bar length must be at least
$6.1 + 21.5/12 = 7.9$ ft

The cut-off points should be beyond the confinement zone of 4 feet.

Provide cut-off point at 7.9 feet from the joint face (for top bars) $\geq$ 4 feet. . . . o.k.

### 2.7.2.4.2 *Positive bar cutoff*

The positive reinforcement at midspan is four #8 bars. The location where one of the four bars can be terminated will be determined. Note that three #8 bars must be continuous throughout the length of the beam to satisfy the minimum reinforcement requirements of ACI 21.3.2.1.

The loading used to find the cut-off point of the one #8 bar is the factored gravity load ($w_u = 1.4\,w_D + 1.7w_L = 1.4 \times 3.61 + 1.7 \times 0.832 = 6.47$ klf) in combination with the probable flexural strengths, $M_{pr}^{+}$, at midspan and $M_u$ corresponding to the $1.4D + 1.7L$ load combination at the exterior end of the end span, as shown in Figure 2-14(b).

The design flexural strength, $\phi M_n$, provided by three #8 bars is 223 ft-kips. Therefore, the one reinforcing bar can be terminated after the factored moment, $M_u$, has been reduced to 223 ft-kips.

With $\phi = 1.0$ and $f_s = 75$ ksi $(= 1.25 \times 60)$

$M_{pr}$ = 404 ft-kips at midspan (positive) and

$M_u$ = 211 ft-kips at the end (negative) (see Table 2-17)
$w_u$ = 6.47 kips/ft

Referring to Figure 2-14(b)

$$6.47x_1^2/2 - 404 + 15.6\,x_1 = -223$$
or
$$x_1 = 5.45 \text{ ft}$$

where $15.6 = \dfrac{404 + 211}{23.17/2} - \dfrac{6.47 \times 23.17/2}{2} = 53.98 - 37.48$ (downward shear at midspan).

One #8 bar (to be cut off) must extend a distance $\geq d$ = 21.5 in. (ACI 12.10.3)
$\geq 12d_b$ = 12 in. beyond $x_1$

Thus, from the center of span, the total bar length must be at least $5.45 + 21.5/12 = 7.2$ ft

Provide cut-off point at 7.2 feet from center of span (for bottom bar). . . . o.k.

### 2.7.2.5. Development of main reinforcement

The #8 bars being terminated must be properly developed at the support or midspan.

ACI 12.2

Bars in tension (bottom bars in positive bending and top bars in negative bending).

ACI 12.2.1

Compute development length, $\ell_d$, from ACI 12.2.2 or 12.2.3
But $\ell_d$ must be more than 12 in.

ACI 12.2.3

$$\frac{\ell_d}{d_b} = \frac{3}{40}\frac{f_y}{\sqrt{f_c'}}\frac{\alpha\beta\gamma\lambda}{\left(\frac{c+K_{tr}}{d_b}\right)}$$

where $\alpha$ = reinforcement location factor = 1.3 for top bars and 1.0 for bottom bars
$\beta$ = epoxy coating factor = 1.0 ($\alpha\beta \leq 1.7$)
$\gamma$ = reinforcement size factor = 1.0 ($\geq$ #7 bars)
$\lambda$ = lightweight aggregate concrete factor = 1.0

$c$ = spacing and cover index
$\leq \{1.5 + 0.5$ (#4 bars) + 1/2 (#8 bars)$\}$ = 2.5 in. . . . governs

$$\leq \frac{34 - 2(1.5 + 0.5) - 1}{5 \times 2} = 2.9 \text{ in.}$$

$c + K_{tr} \leq 2.5\, d_b = 2.5$ in.
$K_{tr}$ = transverse reinforcement index, need not be determined in this case.

For bottom bars

$$\frac{\ell_d}{d_b} = \frac{3}{40} \times \frac{60,000}{\sqrt{4,000}} \times \frac{1.0 \times 1.0 \times 1.0 \times 1.0}{2.5} = 28.5$$

$\ell_d = 28.5 \text{ in.} = 2.4 \text{ ft} \le 7.2 \text{ ft}$ . . . o.k.

For top bars

$$\frac{\ell_d}{d_b} = \frac{3}{40} \times \frac{60,000}{\sqrt{4,000}} \times \frac{1.3 \times 1.0 \times 1.0 \times 1.0}{2.5} = 37.0$$

$\ell_d = 37.0 \text{ in.} = 3.1 \text{ ft} \le 7.9 \text{ ft}$ . . . o.k.

ACI 12.10.5

Flexural reinforcement shall not be terminated in a tension zone unless one of the conditions set forth in ACI 12.10.5 is satisfied. In lieu of increasing the amount of shear reinforcement (ACI 12.10.5.2) or flexural reinforcement (ACI 12.10.5.3), determine the location where factored shear force, $V_u$, is equal to two thirds of that permitted, $2\phi V_n/3$, and extend the flexural reinforcement to at least that location (ACI 12.10.5.1).

ACI 12.10.5.1

$V_{ux} \le 2/3\ \phi V_n \quad (x = 7.9 \text{ ft})$

At 7.9 ft [Fig. 2-14(a)] $V_u = 68.1 - 2.53 \times 7.9 = 48.1$ kips
Spacing of two-legged #4 hoops = 10 in.

$$\phi V_n = 0.85 \times \left[\frac{0.2 \times 2 \times 60 \times 21.5}{10} + 92.5\right] \{V_c = 92.5 \text{ kips}\}$$

$$= 122.5 \text{ kips}$$

$$2/3 \times 122.5 = 81.6 \text{ kips} > 48.1 \text{ kips}$$

Since $2/3\ \phi V_n \ge V_u$, the cut-off points for three #8 bars can be 7.9 feet beyond the face of both exterior and interior joints.

The reinforcement details for the interior beam A2-B2 are shown in Figure 2-13.

### 2.7.2.6. Development of #8 bars at exterior columns

Reinforcing bars that terminate at exterior columns must be properly developed by providing 90-degree hooks embedded in the confined core of the column (ACI 21.5.1.3). The development length of a hooked bar, $\ell_{dh}$, is the largest of the following: (ACI 21.5.4.1)

$\ell_{dh} \ge 8d_b = 8 \times 1.0 = 8.0$ in.

$\ge 6$ in.

$$> \frac{f_y d_b}{65\sqrt{f'_c}} = \frac{60,000 \times 1.0}{65\sqrt{4,000}} = 14.6 \text{ in.}$$ . . . governs

Therefore, the hooks must extend at least 15 inches into the column with a $12d_b$ $(=12 \times 1.0 = 12.0$ in.) extension (ACI 7.1.2) beyond the hook.

#### 2.7.2.7. Design of beam by spreadsheet

The design of beam A2-B2 in the exterior frame is shown in Figure 2-15 in a spreadsheet format.

### 2.7.3. Design of beam B2-C2 in interior frame

#### 2.7.3.1. General requirements (ACI 21.3.1)

According to ACI 21.3.1, flexural members shall satisfy the following conditions:

ACI 21.3.1.1

$$P_u \le 0.1 A_g f'_c$$
$$A_g = 34 \times 24 = 816 \text{ in.}^2$$
$$0.1 A_g f'_c = 0.1 \times 816 \times 4 = 326.4 \text{ kips} \ge 0 \text{ kip} \quad \text{(Table 2-17)} \quad \ldots \text{o.k.}$$

ACI 21.3.1.2

$$\ell_n \ge 4d$$
$$\ell_n = 26 - \frac{40}{2 \times 12} - \frac{38}{2 \times 12} = 22.75 \text{ ft}$$
$$d = 24 - 2.5 = 21.5 \text{ in.}$$
$$4d = 7.17 \text{ ft} \le 22.67 \text{ ft} \quad \ldots \text{o.k.}$$

ACI 21.3.1.3

Width/depth $\ge$ 0.3
Width/depth = 34/24 = 1.4 $\ge$ 0.3 . . . o.k.

ACI 21.3.1.4

iii) Width $\ge$ 10 in.
Width = 34 in. $\ge$ 10 in. . . . o.k.

iv) Beam width $\le$ width of supporting member (column) + 1.5 beam depth
34 in. $\le 38 + 1.5 \times 24 = 74$ in. . . . o.k.

#### 2.7.3.2. Longitudinal reinforcement (ACI 21.3.2)

ACI 21.3.2.1

$$A_{top} \text{ or } A_{bottom} \ge \frac{3\sqrt{f'_c}}{f_y} bd = 2.31 \text{ in.}^2$$
$$\ge \frac{200}{f_y} bd = 2.44 \text{ in.}^2$$
$$\le 0.025\, bd = 18.3 \text{ in.}^2$$

At least two bars should be continuous.

Try the following reinforcements:

| | |
|---|---|
| 3 #8 bars at bottom near support | ($A_s$ provided = 2.37 in.$^2$) |
| 6 #8 bars at top near support | ($A_s$ provided = 4.74 in.$^2$) |
| 4 #8 bars at bottom at midspan | ($A_s$ provided = 3.16 in.$^2$) |
| 3 #8 bars at top at midspan | ($A_s$ provided = 2.37 in.$^2$) |

Minimum $A_s$ provided = 2.37 in.$^2$ ≅ the required minimum of 2.44 in.$^2$ . . . o.k.
Maximum $A_s$ provided = 4.74 in.$^2$ ≤ the required maximum of 18.3 in.$^2$ . . . o.k.

ACI 21.3.2.2

Positive design moment strength at support (i.e., $\phi M_n^+$ with 3 #8 bars) (Table 2-17)
= 223 ft-kips ≥ 7 ft-kips . . . o.k.
Negative design moment strength at support (i.e., $\phi M_n^-$ with 6 #8 bars) (Table 2-17)
= 432 ft-kips ≥ 386 ft-kips . . . o.k.
Positive design moment strength at midspan (i.e., $\phi M_n^+$ with 4 #8 bars) (Table 2-16)
= 294 ft-kips = $1.4 \times 118 + 1.7 \times 27 = 211$ ft-kips, . . . o.k.

At the joint face, the positive moment strength must be at least half the negative moment strength.

$$223 \text{ ft-kips} \geq 0.5 \times 432 = 216 \text{ ft-kips}$$ . . . o.k.

Providing two #8 bars at all sections throughout (where more bars are not required for strength), design moment strength $\phi M_n = 150$ ft-kips $\geq 0.25 \times 432$ ft-kips, which should be acceptable. However, to satisfy minimum reinforcement requirements per ACI 21.3.2.1, provide three #8 bars throughout the span.

This gives more than two continuous bars. . . . o.k.

### 2.7.3.3. Shear strength (ACI 21.3.4)

ACI 21.3.4.1

$$V_e = \frac{M_{pr}^- + M_{pr}^+}{\ell_n} \pm \frac{w_u \ell_n}{2}$$

For calculating the probable flexural strength, the tensile stress in steel should be taken as 1.25 times the specified yield strength and the strength reduction factor, $\phi$, is to be taken as 1.0.

With six #8 bars at top and three #8 bars at bottom at the joint face

$$M_{pr}^- = 591 \text{ ft-kips}$$
$$M_{pr}^+ = 307 \text{ ft-kips}$$
$$w = w_D + w_L$$
$$= 3.61 + 0.83 = 4.44 \text{ k/ft}$$

Factored gravity load from second load combination

$$w_u = 1.4w_D + 0.5w_L = 5.5 \text{ k/ft}$$

$$\ell_n = 23.0 \text{ ft}$$

$$V_e = \frac{591 + 307}{22.67} + 5.5 \times \frac{22.67}{2}$$

$$= 39.5 + 62.6$$

$$= 102.1 \text{ kips}$$

From analysis, maximum shear force = 73 kips (Table 2-17)

Use design shear force, $V_u$ = 102.1 kips (as $V_e$ > 73 kips found from analysis).

ACI 21.3.4.2

Transverse reinforcement (per ACI 21.3.3.1) to resist shear, $V_u$, must be determined assuming $V_c = 0$ if the following two conditions are met:

i) Earthquake-induced shear force $\geq 0.5\ V_e$
Here, earthquake-induced shear = 39.5 kips < 102.1/2 = 51.1 kips (Not satisfied)

ii) $P_u = 0 \text{ kips} \leq 0.05 A_g f'_c = 0.05 \times 34 \times 24 \times 4 = 163 \text{ kips}$ (Satisfied)

Since the first condition is not satisfied, $V_c$ need not be taken equal to zero.

However, recent research[22, 23] has indicated that in plastic hinge regions the concrete contribution, $V_c$, degrades with ductility level and should be taken to be zero for displacement ductility of more than 4 (which is expected for special moment frames).

Conservatively, take $V_c = 0$ for potential plastic hinge regions. The shear reinforcement can be computed as:

$$V_s = \frac{V_u}{\phi} - V_c$$

$$= 102.1/0.85 - 0 = 120.1 \text{ kips}$$

Required spacing of #4 stirrups (with 4 legs for 6 main reinforcing bars)

$$s = \frac{A_v f_y d}{V_s} = \frac{4 \times 0.2 \times 60 \times 21.5}{102.1} = 8.6 \text{ in.}$$

ACI 21.3.3.2

| | | | |
|---|---|---|---|
| $s \leq \frac{d}{4}$ | $= \frac{21.5}{4}$ | = 5.4 in. | . . . governs |
| $\leq 8$ times diameter of main bar, $d_b$ | $= 8 \times 1$ | = 8 in. | |
| $\leq 24$ times diameter of hoop, $d_h$ | $= 24 \times 1/2$ | = 12 in. | |
| $\leq 12$ in. | | = 12 in. | |

Use $s$ = 5 in.

Place first hoop 2 inches from support.

The shear force carried by web reinforcement, $V_s$, should not exceed $8\sqrt{f_c'}\,bd$ (ACI 11.5.6.9)

$$8\sqrt{f_c'}\,bd = 8\sqrt{4{,}000} \times 34 \times 21.5/1{,}000$$
$$= 370 \text{ kips} > 120.1 \text{ kips} \qquad \ldots \text{o.k.}$$

ACI 21.3.3.1

Hoops shall be provided over a length of

i) 2 times the total depth ($2h$) $= 2 \times 24 = 48$ in. from support faces.
ii) $2h$ on either side of a critical section where there is a possibility of flexural yielding. Assume no flexural yielding away from the above regions of potential plastic hinging.

Provide 11 hoops over 4 feet, 4 inches from faces of joints.

Shear force at 4.33 feet from joint face $= 102.1 - 5.5 \times 4.33 = 78.3$ kips

Take $V_c = 2\sqrt{f_c'}\,bd$ at sections 4.33 feet from the joint face because there is no possibility of flexural yielding and thus no degradation of $V_c$.

$$V_c = 2\sqrt{4{,}000} \times 34 \times 21.5 = 92.5 \text{ kips}$$
$$\phi V_c = 0.85 \times 92.5 = 78.6 \text{ kips} > 78.3 \text{ kips} \qquad \ldots \text{o.k.}$$

Thus, there is no need of shear reinforcement beyond 4.33 feet from joint faces. Provide minimum shear reinforcement.

Provide stirrups with seismic hooks at both ends at a spacing not to exceed $d/2$ (i.e., 10.75 in.) throughout. Use two-legged #4 bars at 10-inch spacing, as shown in Figure 2-16.

### 2.7.3.4. Reinforcing bar cut-off points

#### 2.7.3.4.1 *Negative bar cutoff:*

The negative reinforcement at the joint face is six #8 bars. The location where three of the six bars can be terminated will be determined. Note that three #8 bars must be continuous throughout the length of the beam to satisfy the minimum reinforcement requirements of ACI 21.3.2.1.

The loading used to find the cut-off point of the three #8 bars is 0.7 times the dead load in combination with the probable flexural strengths, $M_{pr}$, at the ends of the members, as this combination will produce the longest bar cut-off lengths.

The design flexural strength, $\phi M_n$, provided by three #8 bars is 223 ft-kips. Therefore, the three reinforcing bars can be terminated after the factored moment, $M_u$, has been reduced to 223 ft-kips.

With $\phi = 1.0$ and $f_s = 75$ ksi ($= 1.25 \times 60$)

$M_{pr}$ = 591 ft-kips at one end (negative) and
= 307 ft-kips at the other (positive)

$w_u = 0.7\, w_D = 0.7 \times 3.61 = 2.53$ kips/ft

Referring to Figure 2-14(a)

$$2.53x^2/2 + 591 - 68.3x = 223$$

or $x = 6.1$ ft

Three #8 bars (to be cut off) must extend a distance $\geq d = 21.5$ in. (ACI 12.10.3)
$\geq 12d_b = 12$ in. beyond $x$.

Thus, from the face of support, the total bar length must be at least $6.1 + 21.5/12 = 7.9$ ft

The cut-off points should be beyond the confinement zone of 4 feet.

Provide cut-off point at 8.0 feet from joint face (for top bars) $\geq$ 4 feet. . . . o.k.

### 2.7.3.4.2 *Positive bar cutoff:*

The positive reinforcement at midspan is four #8 bars. The location where one of the four bars can be terminated will be determined. Note that three #8 bars must be continuous throughout the length of the beam to satisfy the minimum reinforcement requirements of ACI 21.3.2.1.

The loading used to find the cut-off point of the one #8 bar is the factored gravity load ($w_u = 1.4w_D + 1.7w_L = 1.4 \times 3.61 + 1.7 \times 0.832 = 6.47$ klf) in combination with the probable flexural strengths, $M_{pr}^{+}$, at midspan and $M_u$ corresponding to the $1.4D + 1.7L$ load combination at the ends of the interior span, as shown in Figure 2-14(b).

The design flexural strength, $\phi M_n$, provided by three #8 bars is 223 ft-kips. Therefore, the one reinforcing bar can be terminated after the factored moment, $M_u$, has been reduced to 223 ft-kips.

With $\phi = 1.0$ and $f_s = 75$ ksi ($= 1.25 \times 60$)

$M_{pr}$ = 404 ft-kips at midspan (positive)

$M_u$ = 306 ft-kips at the end (negative)

$w_u$ = 6.47 kips/ft

Referring to Figure 2-14(b)

$$6.47x_1^2/2 - 404 + 25.6\,x_1 = -223$$

or $x_1 = 4.5$ ft

$$\text{where } 25.6 = \frac{404 + 306}{22.75/2} - \frac{6.47 \times 22.75/2}{2}$$

$= 62.4 - 36.8$ (downward shear at midspan).

One #8 bar (to be cut off) must extend a distance $\geq d = 21.5$ in. (ACI 12.10.3)
$\geq 12d_b = 12$ in. beyond $x_1$.

Thus, from the center of span, the total bar length must be at least 4.5 + 21.5/12 = 6.3 ft

Provide cut-off point at 6.3 feet from center of span (for bottom bar). . . . o.k.

### 2.7.3.5. Development of main reinforcement

The #8 bars being terminated must be properly developed at the support and at midspan.

<u>ACI 12.2</u>

Bars in tension (bottom bars in positive bending and top bars in negative bending).

<u>ACI 12.2.1</u>

Compute development length, $\ell_d$, from ACI 12.2.2 or ACI 12.2.3
but $\ell_d$ must be more than 12 in.

<u>ACI 12.2.3</u>

$$\frac{\ell_d}{d_b} = \frac{3}{40}\frac{f_y}{\sqrt{f'_c}}\frac{\alpha\beta\gamma\lambda}{\frac{(c+K_{tr})}{d_b}}$$

where: $\alpha$ = reinforcement location factor = 1.3 for top bars and 1.0 for bottom bars

$\beta$ = epoxy coating factor = 1.0

$\gamma$ = reinforcement size factor = 1.0 ($\geq$ #7 bars)

$\lambda$ = lightweight aggregate concrete factor = 1.0

c = spacing or cover dimension

$\leq \{1.5 + 0.5 \text{ (\#4 bars)} + 1/2 \text{ (\#8 bars)}\} = 2.5$ in. . . . governs

$\leq \dfrac{34 - 2(1.5 + 0.5) - 1}{5 \times 2} = 2.9$ in.

$c + K_{tr} \leq 2.5\, d_b = 2.5$ in.

$K_{tr}$ = transverse reinforcement index, need not be determined in this case.

For bottom bars,

$$\frac{\ell_d}{d_b} = \frac{3}{40} \times \frac{60{,}000}{\sqrt{4{,}000}} \times \frac{1.0 \times 1.0 \times 1.0 \times 1.0}{2.5} = 28.5$$

$\ell_d$ = 28.5 in. = 2.4 ft $\leq$ 6.3 ft

. . . o.k.

For top bars,

$$\frac{\ell_d}{d_b} = \frac{3}{40} \times \frac{60{,}000}{\sqrt{4{,}000}} \times \frac{1.3 \times 1.0 \times 1.0 \times 1.0}{2.5} = 37.0$$

$\ell_d$ = 37.0 in. = 3.1 ft $\leq$ 7.9 ft . . . o.k.

ACI 12.10.5

Flexural reinforcement shall not be terminated in a tension zone unless one of the conditions set forth in ACI 12.10.5 is satisfied. In lieu of increasing the amount of shear reinforcement (12.10.5.2) or flexural reinforcement (12.10.5.3), determine the location where factored shear force, $V_u$, is equal to two thirds of that permitted, $2\phi V_n/3$, and extend the flexural reinforcement to at least that location (12.10.5.1).

ACI 12.10.5.1

$$V_{ux} \leq 2/3\ \phi V_{nx}\ (x = 7.9 \text{ ft})$$

At 7.9 ft (Fig. 2-16) $V_u = 68.3 - 2.53 \times 7.9 = 48.3$ kips

Spacing of two-legged #4 hoops = 10 in.

$$\phi V_n = 0.85\left(0.2 \times 2 \times 60 \times \frac{21.5}{10} + 92.5\right) \quad \{V_c = 92.5 \text{ kips}\}$$
$$= 122.5 \text{ kips}$$

$$2/3 \times 122.5 = 81.6 \text{ kips} > 48.3 \text{ kips}$$

Since $2/3\ \phi V_n \geq V_u$, the cut-off points for three #8 bars can be 7.9 feet beyond the face of both exterior and interior joints.

Reinforcement details for the interior beam B2-C2 are shown in Figure 2-16.

### 2.7.3.6. Design of beam by spreadsheet

The design of beam B2-C2 in the interior frame is shown in Figure 2-17 in a spreadsheet format.

## 2.8. Design of Columns and Joints

### 2.8.1. General

The design of four columns will be illustrated in this example. These are:

| | | | |
|---|---|---|---|
| Column 1: | Location B2 | Interior | Between ground level and level 2 |
| Column 2: | Location B2 | Interior | Between level 2 and level 3 |
| Column 3: | Location A2 | Exterior | Between ground level and level 2 |
| Column 4: | Location A2 | Exterior | Between level 2 and level 3 |

The axial forces due to service DL and LL in the exterior column (not the corner) and interior column are shown in Table 2-2, which gives $P_D$ and $P_L$ values for interior and exterior columns.

The axial forces, shear forces, and bending moments in columns due to lateral loads are shown in Table 2-18. The SRSS method was used to calculate the resultant forces. Table

2-18 also shows the internal forces caused by 25 percent of the design base shear acting on the frames alone.

Figure 2-18 gives the forces in columns due to gravity load analysis of connected beams.

Table 2-19 shows the summary of axial forces, shear forces, and bending moments for columns (obtained from Table 2-2, Table 2-18, and Figure 2-18).

### 2.8.2. Proportioning and detailing of interior columns (Columns 1 and 2)

#### 2.8.2.1. Check strength

Figure 2-19 shows the column dimensions and reinforcement details (40x40-in. cross-section and 36 #10 bars) considered.

$\phi \times 0.8 P_o = 4{,}495$ kips $> 4{,}478$ kips for column 1
$> 4{,}244$ kips for column 2

{Note: Using 34x34 in. and 28 #10 bars ($\rho = 3.80\%$)
$\phi \times 0.8 P_o = 3{,}328$ kips $< 4{,}478$ kips . . . Not o.k.}

Figure 2-20 shows the P-M interaction diagram for the interior column.

#### 2.8.2.2. General

ACI 21.4 applies to frame members:

i) resisting earthquake forces, and
ii) having a factored axial load $> 0.1 A_g f_c'$

For column 1: $P_u = 4{,}478$ kips (Table 2-19) {4,244 kips for column 2}
$A_g = 40 \text{ in.} \times 40 \text{ in.} = 1{,}600 \text{ in.}^2$
$0.1 A_g f_c' = 0.1 \times 1{,}600 \times 4 = 640 \text{ kips} < P_u$

So ACI 21.4 applies for both columns.

ACI 21.4.1.1
Shortest dimension $\geq 12$ in.
40 in. $\geq 12$ in. . . . o.k.

ACI 21.4.1.2
Ratio of shortest dimension to the perpendicular dimension $\geq 0.4$
$40/40 = 1 \geq 0.4$ . . . o.k.

ACI 21.4.3.1
Longitudinal reinforcement ratio

$$\rho_g = \frac{36 \times 1.27}{40 \times 40} = 2.86\% \; \frac{\geq 1\%}{\leq 6\%}$$ . . . o.k.

ACI 21.4.2 (Minimum flexural strength of columns)

Since $P_u \geq 0.1 A_g f_c'$, satisfy ACI 21.4.2.2 or 21.4.2.3
ACI 21.4.2.2 is satisfied in this example.

ACI 21.4.2.2

$\Sigma M_c > 1.2\ \Sigma M_g$

where: $M_g$ is the sum of nominal flexural strengths of girders framing into the joint.
$M_c$ is the sum of nominal flexural strengths of columns framing into the joint (lowest column flexural strength, calculated based on factored axial force, consistent with the direction of the lateral forces considered).

Girders (See Sections 2.6.2.2 and 2.6.3.2)

$\phi M_{n1}^- = 432$ ft-kips (with 6 #8 bars) (Fig. 2-16)
$M_{n1}^- = 480$ ft-kips
$\phi M_{n2}^+ = 223$ ft-kips (with 3 #8 bars) (Fig. 2-16)
$M_{n2}^+ = 248$ ft-kips
$\Sigma M_g = 728$ ft-kips

Columns

$M_n$ (lowest flexural strength corresponding to the axial force consistent with the direction of lateral forces considered) ≅ 4,300 ft-kips for Column 1 and 4,500 ft-kips for Column 2 (Figure 2-20).

Upper end of column 1, Lower end of column 2

$\Sigma M_c = 4{,}300 + 4{,}500 = 8{,}800$ ft-kips

$\Sigma M_c/\Sigma M_g = \dfrac{8{,}800}{728} = 12.09 > 1.2$ . . . o.k.

Upper end of column 2

$\Sigma M_c \cong 4{,}500 \times 2 = 9{,}000$ ft-kips

$\Sigma M_c/\Sigma M_g > \dfrac{9{,}000}{728} = 12.36 > 1.2$ . . . o.k.

### 2.8.2.3. Transverse reinforcement (ACI 21.4.4)

ACI 21.4.4.1

The minimum required cross-sectional area of hoop reinforcement, $A_{sh}$, is the larger value obtained from the following two equations:

$$A_{sh} = \frac{0.3\, s h_c f_c'}{f_{yh}} [(A_g/A_{ch}) - 1] \quad \text{(ACI Eq. 21-3)}$$

$$A_{sh} = 0.09\, s h_c f_c' / f_{yh} \quad \text{(ACI Eq. 21-4)}$$

ACI 21.4.4.2

Spacing $s \leq \dfrac{\text{least member dimension}}{4} = 40/4 = 10$ in.

$\leq 6 \times$ longitudinal bar diameter $= 6 \times 1.27 = 7.62$ in. ... governs
$\leq s_x$

where: 4 in. $\leq s_x = 4 + (14 - h_x)/3 \leq 6$ in.
and $h_x$ = maximum horizontal spacing of hoop or crosstie legs on all faces of the column

$$h_x = \frac{40 - 2 \times 1.5 - 2 \times 0.5 - 1.27}{9} \times 2 + 1.27 + 0.5 = 9.48 \text{ in.}$$

$s_x = 4 + (14 - 9.48)/3 = 5.5$ in. ... governs

Assuming a clear cover of 1.5 in. and using #4 bars as hoops,

$A_{ch} = (40 - 1.5 \times 2)^2 = 37 \times 37$ in.$^2$
$h_c = 40 - 1.5 \times 2 - 0.5 = 36.5$ in.
$A_{sh} \geq 0.3 \times 5.5 \times 36.5 \times 4 \times [(40^2/37^2 - 1)]/60 = 0.667$ in.$^2$
$\geq 0.09 \times 5.5 \times 36.5 \times 4/60 = 1.205$ in.$^2$ ... governs

Using #4 hoops with four crossties (i.e., total number of 6 legs)
$A_{sh}$ provided $= 6 \times 0.2 = 1.2$ in.$^2 \approx 1.205$ in.$^2$ ... o.k.

ACI 21.4.4.4

Special transverse reinforcement for confinement is required over a distance, $\ell_o$, at the column ends and on both sides of any section with flexural yielding (i.e., if not at column ends), where

$\ell_o \geq$ depth of member $= 40$ in. ... governs
$\geq 1/6 \times$ clear height $= 33$ in. for column 1 (clear height $= 198$ in.)
{21 in. for column 2 with a clear height of 126 in.}
$\geq 18$ in.

Use $\ell_o = 40$ in.

### 2.8.2.4. Shear strength requirements (ACI 21.4.5)

ACI 21.4.5.1

The design shear force, $V_e$, shall be determined based on maximum probable moment strengths, $M_{pr}$, of the member associated with factored axial loads.

The largest probable moment strength can be conservatively assumed to be the nominal moment strength corresponding to the balanced point $\times 1.25$ (with $f_s = 1.25 f_y$ and $\phi = 1$) $= 5{,}000 \times 1.25 = 6{,}250$ ft-kips (Fig. 2-20).

As explained earlier, the probable positive and negative moment strengths at beam ends meeting at the interior joint are 307 and 591 ft-kips respectively (see 2.7.2.3 and 2.7.3.3).

The largest moment that can develop from the beams is
$591 + 307 = 898$ ft-kips $< 6{,}250 \times 2 = 12{,}500$ ft-kips

Therefore, the columns need only be designed to resist the maximum shear that can be transferred through the beams.

It is assumed that the beam moments resisted by the columns above and below the joint are inversely proportional to their lengths (applicable for the case where moments of inertia are equal). The bending moments and shear forces, as shown in Figure 2-21, can be computed as follows:

$$M_{u2} = \frac{\Sigma M_g}{\ell_1 + \ell_2} \times \ell_1 = \frac{898}{(16.5 + 10.5)} \times 16.5 = 549 \text{ ft-kips}$$

$$M_{u1} = \frac{\Sigma M_g}{\ell_1 + \ell_2} \times \ell_2 = \frac{898}{(16.5 + 10.5)} \times 10.5 = 349 \text{ ft-kips}$$

$$V_{u2} = \frac{M_{u2} \times 2}{\ell_2} = \frac{549}{10.5} \times 2 = 104.6 \text{ kips}$$

$$V_{u1} = \frac{M_{u1} \times 2}{\ell_1} = \frac{349}{16.5} \times 2 = 42.3 \text{ kips}$$

Since the factored axial load, $P_u$, (minimum), i.e., 1,662 kips for Column 2 (or 1,761 kips for Column 1) $\geq 0.05\ A_g f'_c$ $(= 0.05 \times 1{,}600 \times 4 = 320$ kips), the shear strength of concrete may be used. (ACI 21.4.5.2)

Column 2:

$$\begin{aligned} V_c &= 2\,[1 + N_u/2{,}000A_g]\sqrt{f'_c}\,bd \\ &= 2\,[1 + (1{,}662 \times 10^3)/(2{,}000 \times 40^2)]\,\sqrt{4{,}000} \times 40 \times 37.4 \\ &\quad \text{where } 37.4 = 40 - 1.5 - 0.5 - 1.27/2 \\ &= 287.5 \text{ kips} \end{aligned}$$

For Column 1, $N_u = 1{,}761$ kips and $V_c = 293.4$ kips

Column 2: $\phi V_c = 0.85 \times 287.5 = 244.4 \text{ kips} \geq 104.6 \text{ kips } (= V_{u2})$
Column 1: $\phi V_c = 249.4 \text{ kips} \geq 42.3 \text{ kips } (= V_{u1})$ . . . o.k.

Theoretically, no shear reinforcement is needed.

Thus, use 5.5-inch spacing over the distance $\ell_o = 40$ inches near the column ends. The remainder of the column length must contain hoop reinforcement with center-to-center spacing not to exceed 6 inches or $6d_b$ (= 7.6 in.). Use 6-inch spacing or, to simplify detailing, use 5.5-inch spacing throughout column height.

Figures 2-19 and 2-16 show the reinforcement details for the interior columns.

### 2.8.2.5. Splice length for column vertical bars

The lap splice length of the #10 bars in Column 1 can be determined according to ACI 12.2.3

$$\frac{\ell_d}{d_b} = \frac{3}{40}\frac{f_y}{\sqrt{f'_c}}\frac{\alpha\beta\gamma\lambda}{\frac{(c+K_{tr})}{d_b}}$$

$\alpha = 1.0$
$\beta = 1.0$
$\gamma = 1.0$
$\lambda = 1.0$

$$K_{tr} = \frac{A_{tr} f_{yt}}{1{,}500\, sn} = \frac{6 \times 0.2 \times 60{,}000}{1{,}500 \times 5.5 \times 5} = 1.8$$

In calculation of $K_{tr}$, it is assumed that one half of the bars are spliced, so that $n = 10/2 = 5$, the number of bars spliced along the plane of splitting. Also, note that the spacing at splice location is 5.5 inches and 6-legged hoop/ties are used.

$$c = \frac{40 - 2 \times 2.635}{9 \times 2} = 1.93 \text{ in.} < 2.6 \text{ in.} \qquad \ldots \text{use } c = 1.93 \text{ in.}$$

$$\frac{(c + K_{tr})}{d_b} = \frac{1.93 + 1.8}{1.27} = 2.9 > 2.5 \qquad \ldots \text{use } 2.5 \qquad \ldots \text{o.k.}$$

$$\frac{\ell_d}{d_b} = \frac{3}{40} \times \frac{60{,}000}{\sqrt{4{,}000}} \times \frac{1.0 \times 1.0 \times 1.0 \times 1.0}{2.5} = 28.5$$

$$\ell_d = 28.5 \times 1.27 = 36.2 \text{ in.}$$

According to ACI 21.4.3.2, lap splices must be located within the center half of the column length and must be proportioned as tension splices.

Length of Class B splice $= 1.3 \times 36.2 = 47.1$ in. $= 3.9$ ft. Use 4-ft splice length.

Figures 2-19 and 2-16 show the reinforcement details in the interior columns.

### 2.8.2.6. Design of columns by spreadsheet

Design of the interior columns (Columns 1 and 2) was also performed using a spreadsheet and is shown in Figure 2-22.

## 2.8.3. Design of interior beam-column joint (Between Columns 1 and 2)

### 2.8.3.1. Transverse reinforcement (ACI 21.5.2)

ACI 21.5.2.2

Beam width = 34 in.
Column width = 40 in.
Beam width/column width = 34/40 = 0.85 > 0.75

So, within the joint depth, transverse reinforcement equal to at least one half the amount required by ACI 21.4.4.1 shall be provided, and the spacing is permitted to be $0.25 \times 40 = 10$ inches or 6 inches (6 in. governs). This relaxation of 6-inch spacing is permitted only in cases of joints confined on four sides. (ACI 21.5.2.2)

Bending moments and shear forces acting on the joint are shown in Figure 2-21.

The net top shear at section $x$-$x$ (Fig. 2-21) is

$$T_1 + C_2 - V_u(\text{top}) = 355.5 + 178 - 104.6 = 428.9 \text{ kips}$$

The net bottom shear at section $x$-$x$ is

$$T_2 + C_1 - V_u(\text{bottom}) = 178 + 355.5 - 42.3 = 491.2 \text{ kips}$$

For a joint confined on all 4 faces, the nominal shear strength (ACI 21.5.3.1)

$V_c = 20\sqrt{f_c'}\,A_j$

$A_j = b_j d_j$

$d_j$ = overall depth of column = 40 in.

$b_j$ = width of the joint

$\leq$ beam width + joint depth = 34 + 40 = 74 in.

$\leq 2 \times (17 + 3) = 40$ in. . . . governs

$A_j = 40 \times 40 = 1{,}600 \text{ in.}^2$

$\phi V_c = 0.85 \times 20\sqrt{4{,}000} \times 1{,}600 = 1{,}720 \text{ kips} > 491.2 \text{ kips}$ . . . o.k.

Use shear reinforcement with spacing of 5.5 inches inside the interior joint, because a relaxation of spacing from 5.5 inches to 6 inches is not worth taking advantage of.

### 2.8.3.2. Design of joint by spreadsheet

Design of the interior beam-column joint was also performed using a spreadsheet and is shown in Figure 2-22.

### 2.8.4. Proportioning and detailing of exterior columns (Columns 3 and 4)

### 2.8.4.1. Check strength

Figure 2-23 shows the column dimensions and reinforcement details (34x34-inch cross-section and 12 #10 bars) considered.

$\phi \times 0.8\,\{0.85 f_c'\,(A_g - A_{st}) + A_{st} f_y\}$

$0.7 \times 0.8\,\{0.85 \times 4(34^2 - 12 \times 1.27) + 12 \times 1.27 \times 60\}$

$= 2{,}684 \text{ kips} > 2{,}495$ kips for Column 3 (Table 2-19)

$> 2{,}364$ kips for Column 4

Figure 2-24 shows the P-M interaction diagram for the exterior columns.

### 2.8.4.2. General

Section ACI 21.4 applies to frame members:

i) resisting earthquake forces and

ii) having a factored axial load $> 0.1 A_g f_c'$

For Column 3 (Table 2-19)

$P_u$ = 2,495 kips {2,364 kips for Column 4}

$A_g$ = 34 in. × 34 in. = 1,156 in.$^2$

$0.1 A_g f'_c$ = 0.1 × 1,156 × 4 = 462.4 kips < $P_u$

So ACI 21.4 applies to both columns.

ACI 21.4.1.1

Shortest dimension ≥ 12 in.

34 in. ≥ 12 in. . . . o.k.

ACI 21.4.1.2

Ratio of shortest dimension to the perpendicular dimension ≥ 0.4

34/34 = 1 ≥ 0.4 . . . o.k.

ACI 21.4.3.1

Longitudinal reinforcement ratio

$$\rho_g = \frac{12 \times 1.27}{34 \times 34} = 1.32\% \begin{matrix} \geq 1\% \\ \leq 6\% \end{matrix}$$ . . . o.k.

ACI 21.4.2 (Minimum flexural strength of columns)

Since $P_u \geq 0.1\ A_g f'_c$, satisfy ACI 21.4.2.2 or ACI 21.4.2.3

Section ACI 21.4.2.2 is satisfied in this example.

ACI 21.4.2.2

$\Sigma M_c > 1.2\ \Sigma M_g$

where: $\Sigma M_g$ is the sum of nominal flexural strengths of girders framing into the joint. $\Sigma M_c$ is the sum of nominal flexural strengths of columns framing into the joint (lowest column flexural strength, calculated based on factored axial force, consistent with the direction of the lateral forces considered).

Girders (See Sections 2.6.2.2 and 2.6.3.2)

$\phi M_{n1}^-$ = 432 ft-kips (with 6 #8) (Fig. 2-13)

$M_{n1}^-$ = 480 ft-kips

$\Sigma M_g$ = 480 ft-kips considering only one girder for edge column.

Columns

$M_n$ (lowest flexural strength corresponding to the axial force consistent with the direction of lateral forces considered) ≅ 1,800 ft-kips for Column 3 and 1,700 ft-kips for Column 4 (Figure 2-24).

Upper end of column 3, Lower end of column 4

$\Sigma M_e$ = 1,800 + 1,700 = 3,500 ft-kips

$\Sigma M_c / \Sigma M_g = \frac{3,500}{480}$ = 7.29 > 1.2 . . . o.k.

Upper end of column 4

$\Sigma M_c \cong 1{,}700 \times 2 = 3{,}400$ ft-kips

$\Sigma M_c/\Sigma M_g \cong \dfrac{3{,}400}{480} = 7.08 > 1.2$ . . . o.k.

### 2.8.4.3. Transverse reinforcement (ACI 21.4.4)

ACI 21.4.4.1

The minimum required cross-sectional area of hoop reinforcement, $A_{sh}$, is the larger value obtained from the following two equations:

$$A_{sh} = \frac{0.3\, s h_c f'_c}{f_{yh}} [(A_g/A_{ch}) - 1] \qquad \text{(ACI Eq. 21-3)}$$

$$A_{sh} = 0.09\, s h_c f'_c / f_{yh} \qquad \text{(ACI Eq. 21-4)}$$

ACI 21.4.4.2

Spacing $s \le \dfrac{\text{least member dimension}}{4} = 34/4 = 8.5$ in.

$\le 6 \times$ longitudinal bar diameter $= 6 \times 1.27 = 7.62$ in. . . . governs

$\le s_x$

where $4 \text{ in.} \le s_x = 4 + (14 - h_x)/3 \le 6$ in.

and $h_x$ = maximum horizontal spacing of hoop or crosstie legs on all faces of the column

$h_x = \dfrac{34 - 2 \times 1.5 - 2 \times 0.5 - 1.27}{3} + 1.27 + 0.5 = 11.35$ in.

$s_x = 4 + (14 - 11.35)/3 = 4.9$ in. . . . governs

Provide four-legged #4 bars @ 4.5-inch spacing.

Assuming a clear cover of 1.5 inches and using #4 bars as hoops

$A_{ch} = (34 - 1.5 \times 2)^2 = 31 \times 31 \text{ in.}^2$

$h_c = 34 - 1.5 \times 2 - 0.5 = 30.5$ in.

$A_{sh} \ge 0.3 \times 4.5 \times 30.5 \times 4/60\, [34^2/31^2 - 1] = 0.56 \text{ in.}^2$

$\ge 0.09 \times 4.5 \times 30.5 \times 4/60 = 0.82 \text{ in.}^2$ . . . governs

Using #4 hoops with two crossties (i.e., total number of 4 legs)

$A_{sh}$ provided $= 4 \times 0.2 = 0.8 \text{ in.}^2 \cong 0.82 \text{ in.}^2$ . . . o.k.

ACI 21.4.4.4

Special transverse reinforcement for confinement is required over a distance, $\ell_o$, at the column ends and on both sides of any section with flexural yielding (i.e., if not at column ends),

where:

$\ell_o \ge$ depth of member $= 34$ in. . . . governs

$\ge 1/6 \times$ clear height $= 33$ in. for Column 3 (clear height $= 198$ in.)
{21 in. for Column 4 with a clear height of 126 in.}

$\geq$ 18 in.

Use $\ell_o$ = 34 in.

### 2.8.4.4. Shear strength requirements (ACI 21.4.5)

ACI 21.4.5.1

The design shear force, $V_e$, shall be determined based on maximum probable moment strengths, $M_{pr}$, of the member associated with factored axial loads.

The largest probable moment strength can be conservatively assumed to be that corresponding to the balanced point $\times$ 1.25 (with $f_s = 1.25 f_y$ and $\phi = 1$) = 2,160 $\times$ 1.25 = 2,700 ft-kips. (Fig. 2-24)

As explained before, the probable negative moment strength at beam ends meeting at exterior joint is 591 ft-kips.

The largest moment that can develop from the beam is

$$591 \text{ ft-kips} < 2{,}700 \times 2 = 5{,}400 \text{ ft-kips}$$

Therefore, the columns need only be designed to resist the maximum shear that can be transferred through the beam.

It is assumed that the beam moments resisted by the columns above and below the joint are inversely proportional to their lengths. The bending moments and shear forces, as shown in Figure 2-25, can be computed as:

$$M_{u4} = \frac{\Sigma M_g}{\ell_3 + \ell_4} \times \ell_3 = \frac{591}{(16.5 + 10.5)} \times 16.5 = 361 \text{ ft-kips}$$

$$M_{u3} = \frac{\Sigma M_g}{\ell_3 + \ell_4} \times \ell_4 = \frac{591}{(16.5 + 10.5)} \times 10.5 = 230 \text{ ft-kips}$$

$$V_{u4} = \frac{M_{u4} \times 2}{\ell_4} = \frac{361}{10.5} \times 2 = 69 \text{ kips}$$

$$V_{u3} = \frac{M_{u3} \times 2}{\ell_3} = \frac{230}{16.5} \times 2 = 28 \text{ kips}$$

Since the factored axial load $P_u$ (minimum), i.e., 721 kips for Column 4 (or 770 kips for Column 3) $\geq 0.05\ A_g f'_c$ (= 0.05 $\times$ 1,156 $\times$ 4 = 231 kips), the shear strength of concrete may be used. (ACI 21.4.5.2)

For Column 4,

$$\begin{aligned} V_c &= 2\,[1 + N_u/2{,}000A_g]\sqrt{f'_c}\,bd \\ &= 2\,[1 + (721 \times 10^3)/(2{,}000 \times 34^2)]\sqrt{4{,}000} \times 34 \times 31.4 \\ &= 177.7 \text{ kips} \end{aligned}$$

For Column 3,

$$N_u = 770 \text{ kips and } V_c = 180.6 \text{ kips}$$

| | | | | |
|---|---|---|---|---|
| Column 4: | $\phi V_c = 0.85 \times 177.7 = 151.1$ kips | $\geq 69$ kips | $(= V_{u4})$ | |
| Column 3: | $\phi V_c = 153.5$ kips | $\geq 28$ kips | $(= V_{u3})$ | . . . o.k. |

Theoretically, no shear reinforcement is needed.

Thus, use 4.5-inch spacing over the distance $\ell_o = 34$ inches near the column ends.

The remainder of the column length must contain hoop reinforcement with center-to-center spacing not to exceed 6 inches or $6d_b$ ($= 7.62$ in). Use 6-inch spacing.

Figures 2-23 and 2-13 show the reinforcement details in the exterior columns.

### 2.8.4.5. Splice length for column vertical bars

The lap splice length of the #10 bars in Column 3 can be determined according to ACI 12.2.3 as

$$\frac{\ell_d}{d_b} = \frac{3}{40} \frac{f_y}{\sqrt{f_c'}} \frac{\alpha\beta\gamma\lambda}{\frac{(c+K_{tr})}{d_b}}$$

$$\alpha = 1.0$$

$$\beta = 1.0$$

$$\gamma = 1.0$$

$$\lambda = 1.0$$

$$K_{tr} = \frac{A_{tr} f_{yt}}{1{,}500\, sn} = \frac{4 \times 0.2 \times 60{,}000}{1{,}500 \times 6 \times 2} = 2.7 \text{ (assuming one half of the bars are spliced)}$$

$$c = \frac{34 - 2 \times 2.6}{3 \times 2} = 4.8 \text{ in.} > 2.6 \text{ in.} \quad \text{. . . use } c = 2.6 \text{ in.}$$

$$\frac{(c + K_{tr})}{d_b} = \frac{2.6 + 2.7}{1.27} = 4.2 > 2.5 \quad \text{. . . use } (c + K_{tr})/d_b = 2.5 \quad \text{. . . o.k.}$$

$$\frac{\ell_d}{d_b} = \frac{3}{40} \times \frac{60{,}000}{\sqrt{4{,}000}} \times \frac{1.0 \times 1.0 \times 1.0 \times 1.0}{2.5} = 28.5$$

$$\ell_d = 28.5 \times 1.27 = 36.2 \text{ in.}$$

According to ACI 21.4.3.2, the lap splices must be located within the center half of the column length and must be proportioned as tension splices.

Length of Class B splice $= 1.3 \times 36.2 = 47.1$ in. $= 3.9$ ft

Figures 2-13 and 2-23 show the exterior column details.

### 2.8.4.6. Design of columns by spreadsheet

The design of the exterior columns (Columns 3 and 4) was also performed using a spreadsheet and is shown in Figure 2-26.

## 2.8.5. Design of exterior beam-column joint (Between Columns 3 and 4)

### 2.8.5.1. Transverse reinforcement (ACI 21.5.2)

ACI 21.5.2.2

| | |
|---|---|
| Beam width | = 34 in. |
| Column width | = 34 in. |
| Beam width/column width | = 34/34 = 1.0 > 0.75 |

So, within the joint depth, transverse reinforcement equal to at least one half the amount required by ACI 21.4.4.1 shall be provided, and the spacing is permitted to be $0.25 \times 34 = 8.5$ or 6 inches (6 in. governs). This relaxation to 6-inch spacing is permitted only in case of joints confined on four sides (ACI 21.5.2.2). For the exterior joint considered in this example, this section is not applicable.

The bending moments and shear forces acting on the joint are shown in Figure 2-25.

The net top shear at section *x-x* (Fig. 2-25) is

$$T - V_u(\text{top}) = 355.5 - 69 = 286.5 \text{ kips}$$

The net bottom shear at section *x-x* is

$$C - V_u(\text{bottom}) = 355.5 - 28 = 327.5 \text{ kips}$$

For a joint confined on all three faces, the nominal shear strength is (ACI 21.5.3.1)

$$V_c = 15\sqrt{f_c'}\,A_j$$

$$A_j = b_j d_j$$

$d_j$ = overall depth of column = 34 in.

$b_j$ = width of the joint

$\leq$ beam width + joint depth = 34 + 34 = 68 in.

$\leq 2 \times (17 + 0) = 34$ in. . . . governs

$$A_j = 34 \times 34 = 1{,}156 \text{ in.}^2$$

$$\phi V_c = 0.85 \times 15\sqrt{4{,}000} \times 1{,}156 = 932 \text{ kips} > 327.5 \text{ kips}$$ . . . o.k.

Use shear reinforcement with spacing of 4 inches inside the exterior joint.

### 2.8.5.2. Design of joint by spreadsheet

The design of the exterior beam-column joint was also performed using a spreadsheet and is shown in Figure 2-26.

**Table 2-1. Lateral forces by equivalent force procedure using approximate period**

$V$ = 3940 kips, $T$ (sec.) = 1.28
$k$ = 1.39

| Floor Level $x$ | Weight $w_x$, kips | Height $h_x$, ft | $w_x h_x^k$, ft-kips | Lateral Force $F_x$, kips | Story Shear $V_x$, kips |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 |
| 21 | 2987 | 255.0 | 6,611,897 | 392 | 392 |
| 20 | 3338 | 242.5 | 6,890,261 | 409 | 801 |
| 19 | 3338 | 230.0 | 6,401,592 | 380 | 1181 |
| 18 | 3338 | 217.5 | 5,923,177 | 351 | 1532 |
| 17 | 3352 | 205.0 | 5,478,250 | 325 | 1857 |
| 16 | 3366 | 192.5 | 5,040,491 | 299 | 2157 |
| 15 | 3366 | 180.0 | 4,591,376 | 272 | 2429 |
| 14 | 3366 | 167.5 | 4,154,270 | 246 | 2675 |
| 13 | 3366 | 155.0 | 3,729,711 | 221 | 2897 |
| 12 | 3366 | 142.5 | 3,318,309 | 197 | 3094 |
| 11 | 3366 | 130.0 | 2,920,757 | 173 | 3267 |
| 10 | 3380 | 117.5 | 2,548,410 | 151 | 3418 |
| 9 | 3394 | 105.0 | 2,188,593 | 130 | 3548 |
| 8 | 3394 | 92.5 | 1,835,054 | 109 | 3657 |
| 7 | 3394 | 80.0 | 1,499,709 | 89 | 3746 |
| 6 | 3394 | 67.5 | 1,184,252 | 70 | 3816 |
| 5 | 3394 | 55.0 | 890,873 | 53 | 3869 |
| 4 | 3394 | 42.5 | 622,547 | 37 | 3906 |
| 3 | 3394 | 30.0 | 383,628 | 23 | 3929 |
| 2 | 3559 | 17.5 | 190,174 | 11 | 3940 |
| Σ | 67,246 | Σ | 66403331 | 3940 | |

**Table 2-2(a). Service-level axial forces due to DL and LL in an interior column**

| Floor Level | DL psf | DL kips | Cum.DL kips | LL psf | Supported Area - psf | RLL psf | RLL kips | Cum.LL kips |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 21 | 177 | 119.7 | 120 | 20 | 676 | 12 | 8.1 | 8.1 |
| 20 | 198 | 133.8 | 253 | 80 | 1352 | 32 | 21.6 | 29.7 |
| 19 | 198 | 133.8 | 387 | 80 | 2028 | 32 | 21.6 | 51.3 |
| 18 | 198 | 133.8 | 521 | 80 | 2704 | 32 | 21.6 | 72.9 |
| 17 | 198 | 133.8 | 655 | 80 | 3380 | 32 | 21.6 | 94.5 |
| 16 | 199 | 134.5 | 790 | 80 | 4056 | 32 | 21.6 | 116.1 |
| 15 | 199 | 134.5 | 924 | 80 | 4732 | 32 | 21.6 | 137.7 |
| 14 | 199 | 134.5 | 1059 | 80 | 5408 | 32 | 21.6 | 159.3 |
| 13 | 199 | 134.5 | 1193 | 80 | 6084 | 32 | 21.6 | 180.9 |
| 12 | 199 | 134.5 | 1328 | 80 | 6760 | 32 | 21.6 | 202.5 |
| 11 | 199 | 134.5 | 1462 | 80 | 7436 | 32 | 21.6 | 224.1 |
| 10 | 200 | 135.2 | 1597 | 80 | 8112 | 32 | 21.6 | 245.7 |
| 9 | 201 | 135.9 | 1733 | 80 | 8788 | 32 | 21.6 | 267.3 |
| 8 | 201 | 135.9 | 1869 | 80 | 9464 | 32 | 21.6 | 288.9 |
| 7 | 201 | 135.9 | 2005 | 80 | 10140 | 32 | 21.6 | 310.5 |
| 6 | 201 | 135.9 | 2141 | 80 | 10816 | 32 | 21.6 | 332.1 |
| 5 | 201 | 135.9 | 2277 | 80 | 11492 | 32 | 21.6 | 353.7 |
| 4 | 201 | 135.9 | 2413 | 80 | 12168 | 32 | 21.6 | 375.3 |
| 3 | 201 | 135.9 | **2549** | 80 | 12844 | 32 | 21.6 | **396.9** |
| 2 | 210 | 142.0 | **2690** | 80 | 13520 | 32 | 21.6 | **418.5** |

DL = Dead Load, LL = Live Load, RLL = Reduced Live Load

**Table 2-2(b). Service-level axial forces due to DL and LL in an edge column**

| Floor Level | DL psf | DL kips | Cum.DL kips | LL psf | Supported Area - psf | RLL psf | RLL kips | Cum.LL kips |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 21 | 177 | 66.4 | 66 | 20 | 375 | 16.5[a] | 6.2 | 6.2 |
| 20 | 198 | 74.3 | 141 | 80 | 750 | 32 | 15.6 | 21.8 |
| 19 | 198 | 74.3 | 215 | 80 | 1125 | 32 | 12 | 33.8 |
| 18 | 198 | 74.3 | 289 | 80 | 1500 | 32 | 12 | 45.8 |
| 17 | 198 | 74.3 | 363 | 80 | 1875 | 32 | 12 | 57.8 |
| 16 | 199 | 74.6 | 438 | 80 | 2250 | 32 | 12 | 69.8 |
| 15 | 199 | 74.6 | 513 | 80 | 2625 | 32 | 12 | 81.8 |
| 14 | 199 | 74.6 | 587 | 80 | 3000 | 32 | 12 | 93.8 |
| 13 | 199 | 74.6 | 662 | 80 | 3375 | 32 | 12 | 105.8 |
| 12 | 199 | 74.6 | 737 | 80 | 3750 | 32 | 12 | 117.8 |
| 11 | 199 | 74.6 | 811 | 80 | 4125 | 32 | 12 | 129.8 |
| 10 | 200 | 75.0 | 886 | 80 | 4500 | 32 | 12 | 141.8 |
| 9 | 201 | 75.4 | 962 | 80 | 4875 | 32 | 12 | 153.8 |
| 8 | 201 | 75.4 | 1037 | 80 | 5250 | 32 | 12 | 165.8 |
| 7 | 201 | 75.4 | 1112 | 80 | 5625 | 32 | 12 | 177.8 |
| 6 | 201 | 75.4 | 1188 | 80 | 6000 | 32 | 12 | 189.8 |
| 5 | 201 | 75.4 | 1263 | 80 | 6375 | 32 | 12 | 201.8 |
| 4 | 201 | 75.4 | 1338 | 80 | 6750 | 32 | 12 | 213.8 |
| 3 | 201 | 75.4 | **1414** | 80 | 7125 | 32 | 12 | **225.8** |
| 2 | 210 | 78.8 | **1493** | 80 | 7500 | 32 | 12 | **237.8** |

DL = Dead Load, LL = Live Load, RLL = Reduced Live Load

[a] Based on the expression $R = 1.2 - 0.001\ A_t$ (1607.11.2.1)

**Table 2-2(c). Service-level axial forces due to DL and LL in a shear wall**

| Floor Level | DL psf | DL kips | Cum.DL kips | LL psf | Supported Area - psf | RLL psf | RLL kips | Cum.LL kips |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 21 | 177 | 239.3 | 239 | 20 | 1352 | 12 | 16.2 | 16 |
| 20 | 198 | 267.7 | 507 | 80 | 2704 | 32 | 43.2 | 59 |
| 19 | 198 | 267.7 | 775 | 80 | 4056 | 32 | 43.2 | 103 |
| 18 | 198 | 267.7 | 1042 | 80 | 5408 | 32 | 43.2 | 146 |
| 17 | 198 | 267.7 | 1310 | 80 | 6760 | 32 | 43.2 | 189 |
| 16 | 199 | 269.0 | 1579 | 80 | 8112 | 32 | 43.2 | 232 |
| 15 | 199 | 269.0 | 1848 | 80 | 9464 | 32 | 43.2 | 275 |
| 14 | 199 | 269.0 | 2117 | 80 | 10816 | 32 | 43.2 | 319 |
| 13 | 199 | 269.0 | 2386 | 80 | 12168 | 32 | 43.2 | 362 |
| 12 | 199 | 269.0 | 2655 | 80 | 13520 | 32 | 43.2 | 405 |
| 11 | 199 | 269.0 | 2924 | 80 | 14872 | 32 | 43.2 | 448 |
| 10 | 200 | 270.4 | 3195 | 80 | 16224 | 32 | 43.2 | 491 |
| 9 | 201 | 271.8 | 3467 | 80 | 17576 | 32 | 43.2 | 535 |
| 8 | 201 | 271.8 | 3738 | 80 | 18928 | 32 | 43.2 | 578 |
| 7 | 201 | 271.8 | 4010 | 80 | 20280 | 32 | 43.2 | 621 |
| 6 | 201 | 271.8 | 4282 | 80 | 21632 | 32 | 43.2 | 664 |
| 5 | 201 | 271.8 | 4554 | 80 | 22984 | 32 | 43.2 | 707 |
| 4 | 201 | 271.8 | 4825 | 80 | 24336 | 32 | 43.2 | 751 |
| 3 | 201 | 271.8 | 5097 | 80 | 25688 | 32 | 43.2 | 794 |
| 2 | 210 | 283.9 | **5381** | 80 | 27040 | 32 | 43.2 | **837** |

DL = Dead Load, LL = Live Load, RLL = Reduced Live Load

**Table 2-3. Calculation of period by rational method (equivalent lateral procedure)**

| Floor Level $x$ | Weight $w_x$, kips | Lateral Force $F_x$, kips | Displacement $\delta_x$, in | $w_x \delta_x^2$,kip-in.$^2$ | $F_x \delta_x$, kip-in. |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 |
| 21 | 2987 | 392 | 10.20 | 310601 | 4001 |
| 20 | 3338 | 409 | 9.75 | 317097 | 3985 |
| 19 | 3338 | 380 | 9.27 | 286794 | 3521 |
| 18 | 3338 | 351 | 8.77 | 256932 | 3083 |
| 17 | 3352 | 325 | 8.26 | 228441 | 2683 |
| 16 | 3366 | 299 | 7.72 | 200388 | 2308 |
| 15 | 3366 | 272 | 7.16 | 172326 | 1949 |
| 14 | 3366 | 246 | 6.58 | 145520 | 1621 |
| 13 | 3366 | 221 | 5.98 | 120307 | 1323 |
| 12 | 3366 | 197 | 5.37 | 97028 | 1057 |
| 11 | 3366 | 173 | 4.75 | 75993 | 823 |
| 10 | 3380 | 151 | 4.13 | 57715 | 625 |
| 9 | 3394 | 130 | 3.52 | 42055 | 457 |
| 8 | 3394 | 109 | 2.92 | 28955 | 318 |
| 7 | 3394 | 89 | 2.34 | 18636 | 209 |
| 6 | 3394 | 70 | 1.80 | 10969 | 126 |
| 5 | 3394 | 53 | 1.30 | 5701 | 69 |
| 4 | 3394 | 37 | 0.85 | 2462 | 31 |
| 3 | 3394 | 23 | 0.48 | 782 | 11 |
| 2 | 3559 | 11 | 0.20 | 140 | 2 |
| Σ | 67,246 | | Σ | 2,378,842 | 28,202 |

**Table 2-4. Lateral forces by equivalent lateral force procedure using period from rational analysis**

V = 3284 kips, T (sec)= [illegible]

k = 1.52

| Floor Level<br>x | Weight<br>$w_x$, kips | Height<br>$h_x$, ft | $w_x h_x^k$, ft-kips | Lateral Force<br>$F_x$, kips | Story Shear<br>$V_x$, kips |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 |
| 21 | 2987 | 255.0 | 13,438,884 | 344 | 344 |
| 20 | 3338 | 242.5 | 13,914,857 | 356 | 700 |
| 19 | 3338 | 230.0 | 12,840,714 | 328 | 1028 |
| 18 | 3338 | 217.5 | 11,796,400 | 302 | 1330 |
| 17 | 3352 | 205.0 | 10,827,953 | 277 | 1607 |
| 16 | 3366 | 192.5 | 9,882,799 | 253 | 1859 |
| 15 | 3366 | 180.0 | 8,925,196 | 228 | 2088 |
| 14 | 3366 | 167.5 | 8,001,448 | 205 | 2292 |
| 13 | 3366 | 155.0 | 7,112,751 | 182 | 2474 |
| 12 | 3366 | 142.5 | 6,260,443 | 160 | 2634 |
| 11 | 3366 | 130.0 | 5,446,031 | 139 | 2774 |
| 10 | 3380 | 117.5 | 4,690,662 | 120 | 2894 |
| 9 | 3394 | 105.0 | 3,970,793 | 102 | 2995 |
| 8 | 3394 | 92.5 | 3,275,782 | 84 | 3079 |
| 7 | 3394 | 80.0 | 2,627,861 | 67 | 3146 |
| 6 | 3394 | 67.5 | 2,030,462 | 52 | 3198 |
| 5 | 3394 | 55.0 | 1,487,930 | 38 | 3236 |
| 4 | 3394 | 42.5 | 1,006,019 | 26 | 3262 |
| 3 | 3394 | 30.0 | 592,901 | 15 | 3277 |
| 2 | 3559 | 17.5 | 274,321 | 7 | 3284 |
| Σ | 67,246 | | 128,404,210 | 3284 | |

**Table 2-5. Lateral displacements and drifts (with revised T) of example building by equivalent lateral force procedure (in.)**

**(along outer frame line F)**

| Floor Level | $\delta_{xe}$ | $C_d$ | $\delta_x$ | Drift, $\Delta$ |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 |
| 21 | 6.47 | 6.50 | 42.06 | 1.76 |
| 20 | 6.20 | 6.50 | 40.30 | 1.89 |
| 19 | 5.91 | 6.50 | 38.42 | 1.95 |
| 18 | 5.61 | 6.50 | 36.47 | 2.15 |
| 17 | 5.28 | 6.50 | 34.32 | 2.21 |
| 16 | 4.94 | 6.50 | 32.11 | 2.28 |
| 15 | 4.59 | 6.50 | 29.84 | 2.41 |
| 14 | 4.22 | 6.50 | 27.43 | 2.47 |
| 13 | 3.84 | 6.50 | 24.96 | 2.54 |
| 12 | 3.45 | 6.50 | 22.43 | **2.60** |
| 11 | 3.05 | 6.50 | 19.83 | 2.60 |
| 10 | 2.65 | 6.50 | 17.23 | 2.60 |
| 9 | 2.25 | 6.50 | 14.63 | 2.47 |
| 8 | 1.87 | 6.50 | 12.16 | 2.41 |
| 7 | 1.50 | 6.50 | 9.75 | 2.28 |
| 6 | 1.15 | 6.50 | 7.48 | 2.15 |
| 5 | 0.82 | 6.50 | 5.33 | 1.82 |
| 4 | 0.54 | 6.50 | 3.51 | 1.56 |
| 3 | 0.30 | 6.50 | 1.95 | 1.11 |
| 2 | 0.13 | 6.50 | 0.85 | 0.85 |
| 1 | 0.00 | 0.00 | 0.00 | 0.00 |

$\delta_x = C_d\delta_{xe}/I$ $\quad\Delta = \delta_{x,i}\text{-}\delta_{x,i\text{-}1}$

**Table 2-5. (continued)**

**(along shearwall line E)**

| Floor Level | $\delta_{xe}$ | $C_d$ | $\delta_x$ | Drift, $\Delta$ |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 |
| 21 | 6.22 | 6.50 | 40.43 | 1.69 |
| 20 | 5.96 | 6.50 | 38.74 | 1.82 |
| 19 | 5.68 | 6.50 | 36.92 | 1.95 |
| 18 | 5.38 | 6.50 | 34.97 | 2.02 |
| 17 | 5.07 | 6.50 | 32.96 | 2.08 |
| 16 | 4.75 | 6.50 | 30.88 | 2.21 |
| 15 | 4.41 | 6.50 | 28.67 | 2.34 |
| 14 | 4.05 | 6.50 | 26.33 | 2.41 |
| 13 | 3.68 | 6.50 | 23.92 | 2.41 |
| 12 | 3.31 | 6.50 | 21.52 | **2.54** |
| 11 | 2.92 | 6.50 | 18.98 | 2.47 |
| 10 | 2.54 | 6.50 | 16.51 | 2.47 |
| 9 | 2.16 | 6.50 | 14.04 | 2.41 |
| 8 | 1.79 | 6.50 | 11.64 | 2.34 |
| 7 | 1.43 | 6.50 | 9.30 | 2.15 |
| 6 | 1.10 | 6.50 | 7.15 | 2.02 |
| 5 | 0.79 | 6.50 | 5.14 | 1.76 |
| 4 | 0.52 | 6.50 | 3.38 | 1.50 |
| 3 | 0.29 | 6.50 | 1.89 | 1.11 |
| 2 | 0.12 | 6.50 | 0.78 | 0.78 |
| 1 | 0.00 | 0.00 | 0.00 | 0.00 |

$\delta_x = C_d\delta_{xe}/I$ $\quad\Delta = \delta_{x,i}-\delta_{x,i-1}$

**Table 2-6. Calculation of stability coefficient**

| Story Level | DL psf | LL psf | Area sq.ft | $P_x$ kips | $V_x$ kips | $h_{sx}$ ft | Drift, Δ in. | θ |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| [redacted] | [redacted] | [redacted] | [redacted] | 3194 | [redacted] | 12.5 | 1.76 | 0.017 |
| [redacted] | [redacted] | [redacted] | 33800 | 7774 | [redacted] | 12.5 | 1.89 | 0.021 |
| [redacted] | [redacted] | [redacted] | 50700 | 11661 | [redacted] | 12.5 | 1.95 | 0.023 |
| [redacted] | [redacted] | [redacted] | 67600 | 15548 | [redacted] | 12.5 | 2.15 | 0.026 |
| [redacted] | [redacted] | [redacted] | 84500 | 19435 | [redacted] | 12.5 | 2.21 | 0.027 |
| [redacted] | [redacted] | [redacted] | 101400 | 23423 | [redacted] | 12.5 | 2.28 | 0.029 |
| [redacted] | [redacted] | [redacted] | 118300 | 27327 | [redacted] | 12.5 | 2.41 | 0.032 |
| [redacted] | [redacted] | [redacted] | 135200 | 31231 | [redacted] | 12.5 | 2.47 | 0.035 |
| [redacted] | [redacted] | [redacted] | 152100 | 35135 | [redacted] | 12.5 | 2.54 | 0.037 |
| [redacted] | [redacted] | [redacted] | 169000 | 39039 | [redacted] | 12.5 | 2.60 | 0.040 |
| [redacted] | [redacted] | [redacted] | 185900 | 42943 | [redacted] | 12.5 | 2.60 | 0.041 |
| [redacted] | [redacted] | [redacted] | 202800 | 47050 | [redacted] | 12.5 | 2.60 | 0.043 |
| [redacted] | [redacted] | [redacted] | 219700 | 51190 | [redacted] | 12.5 | 2.47 | 0.043 |
| [redacted] | [redacted] | [redacted] | 236600 | 55128 | [redacted] | 12.5 | 2.41 | 0.044 |
| [redacted] | [redacted] | [redacted] | 253500 | 59066 | [redacted] | 12.5 | 2.28 | 0.044 |
| [redacted] | [redacted] | [redacted] | 270400 | 63003 | [redacted] | 12.5 | 2.15 | 0.043 |
| [redacted] | [redacted] | [redacted] | 287300 | 66941 | [redacted] | 12.5 | 1.82 | 0.039 |
| [redacted] | [redacted] | [redacted] | 304200 | 70879 | [redacted] | 12.5 | 1.56 | 0.035 |
| [redacted] | [redacted] | [redacted] | 321100 | 74816 | [redacted] | 12.5 | 1.11 | 0.026 |
| [redacted] | [redacted] | [redacted] | 354900 | 85886 | [redacted] | 17.5 | 0.85 | 0.016 |

**Table 2-7. Summary of design axial force, shear force and bending moment for shearwall between grade and level 2**

| Loads | Symbol | Axial Force (kips) | Shear Force (kips) | Bending Moment (ft-kips) |
|---|---|---|---|---|
| Dead Load | *D* | [redacted] | | |
| Live Load | *L* | | | |
| Lateral Load | *EQ* | | | |

| Load Combinations | | | | |
|---|---|---|---|---|
| 1 | $1.4D+1.7L$ | 8956 | 0 | 0 |
| 2 | $1.2D+(\rho E_h+0.2DS_{DS})+0.5L$ | 7952 | 1571 | 118,596 |
| 3 | $0.9D-(\rho E_h+0.2DS_{DS})$ | 3767 | -1571 | -118,596 |
| | Max. axial force with lateral force | 7952 | | |

**Table 2-8. Comparison of periods from SAP 2000 and STAAD analysis**

| Case | Period (sec.) | SAP2000 3D | STAAD 3D | STAAD 2D | STAAD-3D/ STAAD-2D | STAAD-3D/ SAP2000 | STAAD-2D/ SAP2000 |
|---|---|---|---|---|---|---|---|
| 1 | $T_1$ | [redacted] | [redacted] | [redacted] | 0.93 | 1.05 | 1.13 |
| | $T_2$ | [redacted] | [redacted] | [redacted] | 0.94 | 1.03 | 1.09 |
| | $T_3$ | [redacted] | [redacted] | [redacted] | 0.92 | 1.01 | 1.10 |
| 2 | $T_1$ | [redacted] | [redacted] | [redacted] | 0.93 | 1.04 | 1.11 |
| | $T_2$ | [redacted] | [redacted] | [redacted] | 0.95 | 1.02 | 1.07 |
| | $T_3$ | [redacted] | [redacted] | [redacted] | 0.92 | 1.01 | 1.10 |
| 3 | $T_1$ | [redacted] | [redacted] | [redacted] | 0.94 | 1.05 | 1.12 |
| | $T_2$ | [redacted] | [redacted] | [redacted] | 0.96 | 1.02 | 1.06 |
| | $T_3$ | [redacted] | [redacted] | [redacted] | 0.93 | 1.01 | 1.08 |
| 4 | $T_1$ | [redacted] | [redacted] | [redacted] | 0.94 | 1.06 | 1.13 |
| | $T_2$ | [redacted] | [redacted] | [redacted] | 0.96 | 1.04 | 1.08 |
| | $T_3$ | [redacted] | [redacted] | [redacted] | 0.95 | 1.03 | 1.09 |
| 5 | $T_1$ | [redacted] | [redacted] | [redacted] | 0.93 | 1.06 | 1.14 |
| | $T_2$ | [redacted] | [redacted] | [redacted] | 0.95 | 1.04 | 1.09 |
| | $T_3$ | [redacted] | [redacted] | [redacted] | 0.94 | 0.98 | 1.05 |

**Note 1: Definition of different Cases**

| Case | Column | Beam | Wall |
|---|---|---|---|
| 1 | 1.0 $I_g$ | 1.0 $I_g$ | 1.0 $I_g$ |
| 2 | 1.0 $I_g$ | 0.5 $I_g$ | 0.5 $I_g$ |
| 3 | 0.7 $I_g$ | 0.35 $I_g$ | 0.35 $I_g$ |
| 4 | 0.7 $I_g$ | 0.35 $I_g$ | 0.35 $I_g$ |
| 5 | 0.5 $I_g$ | 0.5 $I_g$ | 0.5 $I_g$ |

**Note 2:** Only the first three modes (not necessarily in sequence) contributing more than 90% mass participation are taken into consideration.

**Table 2-9. Calculation of $L_m$ and $M_m$ and distribution of modal base shear for example building**

Mode 1, $T_1$ = 2.485 sec

$V_1$ = 2238 kips

| Floor Level $i$ | $w_i$ kips | $\phi_{i1}$ | $w_i\phi_{i1}$ kips | $w_i\phi_{i1}^2$ kips | $w_i\phi_{i1}/\Sigma w_i\phi_{i1}$ | $F_{i1}$ $V_1$ x col.6 kips |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 21 | 2987 | 1 | 2987.0 | 2987.0 | 0.0893 | 200 |
| 20 | 3338 | 0.9585 | 3199.6 | 3066.9 | 0.0957 | 214 |
| 19 | 3338 | 0.9140 | 3051.0 | 2788.7 | 0.0912 | 204 |
| 18 | 3338 | 0.8676 | 2895.9 | 2512.4 | 0.0866 | 194 |
| 17 | 3352 | 0.8184 | 2743.1 | 2244.9 | 0.0820 | 184 |
| 16 | 3366 | 0.7664 | 2579.6 | 1977.0 | 0.0771 | 173 |
| 15 | 3366 | 0.7122 | 2397.2 | 1707.3 | 0.0717 | 160 |
| 14 | 3366 | 0.6558 | 2207.4 | 1447.6 | 0.0660 | 148 |
| 13 | 3366 | 0.5972 | 2010.1 | 1200.4 | 0.0601 | 135 |
| 12 | 3366 | 0.5372 | 1808.2 | 971.3 | 0.0541 | 121 |
| 11 | 3366 | 0.4758 | 1601.6 | 762.0 | 0.0479 | 107 |
| 10 | 3380 | 0.4142 | 1399.8 | 579.8 | 0.0419 | 94 |
| 9 | 3394 | 0.3531 | 1198.3 | 423.1 | 0.0358 | 80 |
| 8 | 3394 | 0.2928 | 993.7 | 290.9 | 0.0297 | 67 |
| 7 | 3394 | 0.2347 | 796.7 | 187.0 | 0.0238 | 53 |
| 6 | 3394 | 0.1800 | 610.9 | 109.9 | 0.0183 | 41 |
| 5 | 3394 | 0.1294 | 439.1 | 56.8 | 0.0131 | 29 |
| 4 | 3394 | 0.0849 | 288.1 | 24.5 | 0.0086 | 19 |
| 3 | 3394 | 0.0478 | 162.3 | 7.8 | 0.0049 | 11 |
| 2 | 3559 | 0.0196 | 69.9 | 1.4 | 0.0021 | 5 |
| Σ | 67,246 | | 33,440 | 23,347 | 1.0000 | 2238 |
| | $W$ = | | $L_1 \times g$ = | $M_1 \times g$ = | | $V_1$ = |

**Table 2-9. (continued)**

Mode 2, $T_2$ = 0.659 sec

$V_2$ = 1754 kips

| Floor Level $i$ | $w_i$ kips | $\phi_{i2}$ | $w_i\phi_{i2}$ kips | $w_i\phi_{i2}^2$ kips | $w_i\phi_{i2}/\Sigma w_i\phi_{i2}$ | $F_{i2}$ $V_2$ x col.13 kips |
|---|---|---|---|---|---|---|
| 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 21 | 2987 | 1 | 2987.0 | 2987.0 | -0.2043 | -358 |
| 20 | 3338 | 0.8065 | 2692.0 | 2171.1 | -0.1841 | -323 |
| 19 | 3338 | 0.5981 | 1996.4 | 1194.0 | -0.1365 | -239 |
| 18 | 3338 | 0.3836 | 1280.4 | 491.1 | -0.0876 | -154 |
| 17 | 3352 | 0.1683 | 564.0 | 94.9 | -0.0386 | -68 |
| 16 | 3366 | -0.0381 | -128.1 | 4.9 | 0.0088 | 15 |
| 15 | 3366 | -0.2308 | -777.0 | 179.4 | 0.0531 | 93 |
| 14 | 3366 | -0.4033 | -1357.7 | 547.6 | 0.0928 | 163 |
| 13 | 3366 | -0.5488 | -1847.1 | 1013.6 | 0.1263 | 222 |
| 12 | 3366 | -0.6614 | -2226.4 | 1472.6 | 0.1523 | 267 |
| 11 | 3366 | -0.7372 | -2481.3 | 1829.1 | 0.1697 | 298 |
| 10 | 3380 | -0.7736 | -2614.8 | 2022.8 | 0.1788 | 314 |
| 9 | 3394 | -0.7706 | -2615.5 | 2015.6 | 0.1789 | 314 |
| 8 | 3394 | -0.7302 | -2478.3 | 1809.6 | 0.1695 | 297 |
| 7 | 3394 | -0.6566 | -2228.7 | 1463.4 | 0.1524 | 267 |
| 6 | 3394 | -0.5565 | -1888.7 | 1051.0 | 0.1292 | 227 |
| 5 | 3394 | -0.4382 | -1487.3 | 651.8 | 0.1017 | 178 |
| 4 | 3394 | -0.3123 | -1059.8 | 330.9 | 0.0725 | 127 |
| 3 | 3394 | -0.1905 | -646.5 | 123.1 | 0.0442 | 78 |
| 2 | 3559 | -0.0857 | -305.1 | 26.2 | 0.0209 | 37 |
| Σ | 67,246 | | -14,622 | 21,480 | 1.0000 | 1754 |
| | $W$ = | | $L_2 \times g$ = | $M_2 \times g$ = | | $V_2$ = |

**Table 2-9. (continued)**

Mode 3, $T_3$ = 0.300 sec

$V_3$ = 824 kips

| Floor Level $i$ | $w_i$ kips | $\phi_{i3}$ | $w_i\phi_{i3}$ kips | $w_i\phi_{i3}^2$ kips | $w_i\phi_{i3}/\Sigma w_i\phi_{i3}$ | $F_{i3}$ $V_3$ x col.20 kips |
|---|---|---|---|---|---|---|
| 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| 21 | 2987 | 1.0000 | 2987.0 | 2987.0 | 0.2928 | 241 |
| 20 | 3338 | 0.6386 | 2131.6 | 1361.3 | 0.2089 | 172 |
| 19 | 3338 | 0.2501 | 834.9 | 208.8 | 0.0818 | 67 |
| 18 | 3338 | -0.1231 | -410.8 | 50.6 | -0.0403 | -33 |
| 17 | 3352 | -0.4388 | -1470.9 | 645.5 | -0.1442 | -119 |
| 16 | 3366 | -0.6562 | -2208.9 | 1449.6 | -0.2165 | -178 |
| 15 | 3366 | -0.7654 | -2576.3 | 1971.8 | -0.2525 | -208 |
| 14 | 3366 | -0.7571 | -2548.5 | 1929.6 | -0.2498 | -206 |
| 13 | 3366 | -0.6355 | -2139.1 | 1359.4 | -0.2097 | -173 |
| 12 | 3366 | -0.4186 | -1409.2 | 589.9 | -0.1381 | -114 |
| 11 | 3366 | -0.1381 | -464.9 | 64.2 | -0.0456 | -38 |
| 10 | 3380 | 0.1668 | 563.9 | 94.1 | 0.0553 | 46 |
| 9 | 3394 | 0.4543 | 1541.8 | 700.4 | 0.1511 | 125 |
| 8 | 3394 | 0.6854 | 2326.2 | 1594.3 | 0.2280 | 188 |
| 7 | 3394 | 0.8300 | 2816.9 | 2337.9 | 0.2761 | 228 |
| 6 | 3394 | 0.8711 | 2956.5 | 2575.3 | 0.2898 | 239 |
| 5 | 3394 | 0.8078 | 2741.6 | 2214.7 | 0.2687 | 221 |
| 4 | 3394 | 0.6558 | 2225.8 | 1459.6 | 0.2182 | 180 |
| 3 | 3394 | 0.4465 | 1515.4 | 676.6 | 0.1485 | 122 |
| 2 | 3559 | 0.2217 | 789.0 | 174.9 | 0.0773 | 64 |
| Σ | 67,246 | | 10,202 | 24,445 | 1.0000 | 824 |
| | $W$ = | | $L_3$ x $g$ = | $M_3$ x g = | | $V_3$ = |

**Table 2-10. Internal forces in a shear wall due to lateral forces given in Table 2-9**

| Story Level 1 | Shear Force (kips) | | | |
|---|---|---|---|---|
| | Mode 1 2 | Mode 2 3 | Mode 3 4 | Resultant 5 |
| 20-21 | -348 | 93 | 54 | **364** |
| 19-20 | -98 | -152 | 158 | **240** |
| 18-19 | -33 | -258 | 193 | **324** |
| 17-18 | 47 | -334 | 183 | **384** |
| 16-17 | 120 | -387 | 140 | **429** |
| 15-16 | 188 | -388 | 61 | **435** |
| 14-15 | 250 | -362 | -29 | **441** |
| 13-14 | 308 | -306 | -119 | **450** |
| 12-13 | 363 | -226 | -194 | **469** |
| 11-12 | 415 | -126 | -244 | **498** |
| 10-11 | 460 | -16 | -257 | **527** |
| 9-10 | 521 | 107 | -244 | **585** |
| 8-9 | 565 | 234 | -184 | **639** |
| 7-8 | 617 | 356 | -100 | **719** |
| 6-7 | 670 | 469 | 4 | **817** |
| 5-6 | 726 | 569 | 113 | **930** |
| 4-5 | 789 | 656 | 216 | **1048** |
| 3-4 | 859 | 728 | 301 | **1166** |
| 2-3 | 942 | 795 | 368 | **1287** |
| 1-2 | 1070 | 843 | 396 | **1419** |

Table 2-10. (continued)

| Story Level 1 | Section 2 | Bending Moment (ft-kips) Mode 1 3 | Mode 2 4 | Mode 3 5 | Resultant 6 |
|---|---|---|---|---|---|
| 20-21 | top | -1208 | 751 | -196 | **1436** |
| | bottom | 5556 | -1911 | -476 | **5895** |
| 19-20 | top | -6863 | 2724 | 265 | **7389** |
| | bottom | 8094 | -823 | -2235 | **8437** |
| 18-19 | top | -9449 | 1651 | 2030 | **9805** |
| | bottom | 9865 | 1572 | -4440 | **10932** |
| 17-18 | top | -11298 | -734 | 4256 | **12096** |
| | bottom | 10712 | 4909 | -6541 | **13477** |
| 16-17 | top | -12231 | -4086 | 6398 | **14395** |
| | bottom | 10730 | 8927 | -8151 | **16163** |
| 15-16 | top | -12332 | -8145 | 8060 | **16834** |
| | bottom | 9982 | 12993 | -8817 | **18606** |
| 14-15 | top | -11665 | -12280 | 8787 | **19081** |
| | bottom | 8540 | 16803 | -8423 | **20645** |
| 13-14 | top | -10298 | -16187 | 8455 | **20966** |
| | bottom | 6446 | 20010 | -6967 | **22147** |
| 12-13 | top | -8269 | -19516 | 7055 | **22338** |
| | bottom | 3737 | 22338 | -4625 | **23116** |
| 11-12 | top | -5609 | -21984 | 4755 | **23181** |
| | bottom | 421 | 23559 | -1700 | **23624** |
| 10-11 | top | -2323 | -23360 | 1854 | **23548** |
| | bottom | -3426 | 23558 | 1365 | **23845** |
| 9-10 | top | 1518 | -23518 | -1211 | **23598** |
| | bottom | -8030 | 22175 | 4258 | **23966** |
| 8-9 | top | 6142 | -22282 | -4125 | **23478** |
| | bottom | -13206 | 19351 | 6425 | **24293** |
| 7-8 | top | 11367 | -19593 | -6329 | **23519** |
| | bottom | -19076 | 15147 | 7573 | **25508** |
| 6-7 | top | 17319 | -15500 | -7526 | **24430** |
| | bottom | -25689 | 9643 | 7476 | **28440** |
| 5-6 | top | 24054 | -10078 | -7482 | **27132** |
| | bottom | -33131 | 2964 | 6069 | **33813** |
| 4-5 | top | 31660 | -3442 | -6123 | **32430** |
| | bottom | -41522 | -4755 | 3428 | **41934** |
| 3-4 | top | 40265 | 4279 | -3518 | **40644** |
| | bottom | -51007 | -13375 | -240 | **52732** |
| 2-3 | top | 50019 | 12954 | 137 | **51670** |
| | bottom | -61793 | -22897 | -4735 | **66069** |
| 1-2 | top | 61141 | 22586 | 4642 | **65344** |
| | bottom | -79869 | -37335 | -11567 | **88920** |

**Table 2-11. Lateral displacements and drifts of example building from dynamic analysis (in.)**

**(along outer frame line F)**

| Floor Level | Mode 1 $\delta_{xe\,1}$ | Mode 2 $\delta_{xe\,2}$ | Mode 3 $\delta_{xe\,3}$ | Resultant $\delta_{xe}$ | $C_d$ | $\delta_x$ | Drift $\Delta$ |
|---|---|---|---|---|---|---|---|
| 21 | 4.24 | -0.54 | 0.08 | 4.27 | 6.50 | 27.78 | 1.20 |
| 20 | 4.07 | -0.43 | 0.06 | 4.09 | 6.50 | 26.58 | 1.27 |
| 19 | 3.88 | -0.32 | 0.03 | 3.89 | 6.50 | 25.31 | 1.31 |
| 18 | 3.69 | -0.20 | 0.00 | 3.69 | 6.50 | 24.00 | 1.37 |
| 17 | 3.48 | -0.09 | -0.03 | 3.48 | 6.50 | 22.63 | 1.42 |
| 16 | 3.26 | 0.03 | -0.04 | 3.26 | 6.50 | 21.21 | 1.47 |
| 15 | 3.03 | 0.13 | -0.05 | 3.04 | 6.50 | 19.74 | 1.51 |
| 14 | 2.80 | 0.23 | -0.05 | 2.81 | 6.50 | 18.23 | 1.55 |
| 13 | 2.55 | 0.31 | -0.04 | 2.57 | 6.50 | 16.68 | 1.59 |
| 12 | 2.29 | 0.37 | -0.03 | 2.32 | 6.50 | 15.09 | 1.62 |
| 11 | 2.03 | 0.41 | -0.01 | 2.07 | 6.50 | 13.46 | 1.64 |
| 10 | 1.77 | 0.43 | 0.02 | 1.82 | 6.50 | 11.82 | 1.64 |
| 9 | 1.51 | 0.42 | 0.04 | 1.57 | 6.50 | 10.18 | 1.63 |
| 8 | 1.25 | 0.40 | 0.06 | 1.31 | 6.50 | 8.55 | 1.60 |
| 7 | 1.00 | 0.36 | 0.07 | 1.07 | 6.50 | 6.94 | 1.54 |
| 6 | 0.77 | 0.30 | 0.07 | 0.83 | 6.50 | 5.40 | 1.45 |
| 5 | 0.55 | 0.24 | 0.06 | 0.61 | 6.50 | 3.95 | 1.31 |
| 4 | 0.36 | 0.17 | 0.05 | 0.41 | 6.50 | 2.64 | 1.12 |
| 3 | 0.21 | 0.10 | 0.03 | 0.23 | 6.50 | 1.51 | 0.87 |
| 2 | 0.08 | 0.05 | 0.02 | 0.10 | 6.50 | 0.64 | 0.64 |

Table 2-11. (continued)

(along shearwall line E)

| Floor Level | Mode 1 $\delta_{xe\,1}$ | Mode 2 $\delta_{xe\,2}$ | Mode 3 $\delta_{xe\,3}$ | Resultant $\delta_{xe}$ | $C_d$ | $\delta_x$ | Drift $\Delta$ |
|---|---|---|---|---|---|---|---|
| 21 | 4.07 | -0.52 | 0.08 | 4.11 | 6.50 | 26.70 | 1.16 |
| 20 | 3.91 | -0.42 | 0.05 | 3.93 | 6.50 | 25.54 | 1.22 |
| 19 | 3.73 | -0.31 | 0.03 | 3.74 | 6.50 | 24.32 | 1.27 |
| 18 | 3.54 | -0.20 | 0.00 | 3.55 | 6.50 | 23.05 | 1.32 |
| 17 | 3.34 | -0.09 | -0.03 | 3.34 | 6.50 | 21.73 | 1.37 |
| 16 | 3.13 | 0.02 | -0.04 | 3.13 | 6.50 | 20.36 | 1.41 |
| 15 | 2.91 | 0.12 | -0.05 | 2.92 | 6.50 | 18.95 | 1.46 |
| 14 | 2.68 | 0.21 | -0.05 | 2.69 | 6.50 | 17.49 | 1.50 |
| 13 | 2.44 | 0.29 | -0.04 | 2.46 | 6.50 | 16.00 | 1.53 |
| 12 | 2.20 | 0.35 | -0.03 | 2.23 | 6.50 | 14.47 | 1.56 |
| 11 | 1.95 | 0.39 | -0.01 | 1.99 | 6.50 | 12.91 | 1.58 |
| 10 | 1.70 | 0.41 | 0.02 | 1.74 | 6.50 | 11.33 | 1.58 |
| 9 | 1.44 | 0.40 | 0.04 | 1.50 | 6.50 | 9.75 | 1.57 |
| 8 | 1.20 | 0.38 | 0.05 | 1.26 | 6.50 | 8.18 | 1.54 |
| 7 | 0.96 | 0.34 | 0.06 | 1.02 | 6.50 | 6.65 | 1.48 |
| 6 | 0.74 | 0.29 | 0.07 | 0.80 | 6.50 | 5.17 | 1.39 |
| 5 | 0.53 | 0.23 | 0.06 | 0.58 | 6.50 | 3.78 | 1.26 |
| 4 | 0.35 | 0.16 | 0.05 | 0.39 | 6.50 | 2.52 | 1.08 |
| 3 | 0.20 | 0.10 | 0.03 | 0.22 | 6.50 | 1.45 | 0.84 |
| 2 | 0.08 | 0.04 | 0.02 | 0.09 | 6.50 | 0.61 | 0.61 |

**Table 2-12. Summary of design axial force, shear force and bending moment for shear wall between grade and level 2**

( dynamic-modal analysis)

| Loads | Symbol | Axial Force (kips) | Shear Force (kips) | Bending Moment (ft-kips) |
|---|---|---|---|---|
| Dead Load | $D$ | 5381 | 0 | 0 |
| Live Load | $L$ | 837 | 0 | 0 |
| Lateral Load | $EQ$ | 0 | 1419 | 88,920 |

| Load Combinations | | | | |
|---|---|---|---|---|
| 1 | $1.4D + 1.7L$ | 8956 | 0 | 0 |
| 2 | $1.2D$□ $+(\rho_h E + 0.2DS_{DS}) + 0.5L$□ | 7952 | 1419 | 88,920 |
| 3 | $0.9D$□ $\rho(E_h$□$+0.2DS_{DS}$□$)$ | 3767 | -1419 | -88,920 |
| | Max. axial force with lateral force | 7952 | | |

## Table 2-13. Bending moments in beams (in ft-kips) due to lateral forces of Table 2-8

| | | Bending Moments in Beam A2-B2 (Exterior Span) | | | | | Bendings Moment in Beam B2-C2 (Interior Span) | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Floor | Location | Mode 1 | Mode 2 | Mode 3 | Resultant | w/o Shearwall with 25% V | Mode 1 | Mode 2 | Mode 3 | Resultant | w/o Shearwall with 25% V |
| 21 | Left | 109.8 | -71.9 | 19.9 | 132.8 | 6.8 | 194.7 | -120.1 | 31.4 | 230.9 | 9.4 |
| | Right | -103.2 | 67.9 | -18.9 | 125.0 | 10.1 | -204.9 | 126.1 | -32.9 | 242.8 | 11.1 |
| 20 | Left | 132.0 | -86.7 | 24.3 | 159.8 | 14.4 | 220.8 | -136.5 | 35.8 | 262.0 | 18.7 |
| | Right | -130.4 | 85.7 | -24.0 | 157.8 | 17.8 | -223.8 | 138.0 | -35.9 | 265.4 | 20.4 |
| 19 | Left | 137.7 | -90.0 | 24.5 | 166.3 | 27.1 | 227.8 | -139.2 | 35.1 | 269.2 | 31.3 |
| | Right | -134.5 | 88.0 | -24.0 | 162.5 | 27.6 | -232.4 | 141.4 | -35.3 | 274.3 | 30.6 |
| 18 | Left | 147.4 | -92.6 | 22.4 | 175.5 | 39 | 241.3 | -141.7 | 31.8 | 281.6 | 42.9 |
| | Right | -154.6 | 90.8 | -22.0 | 180.7 | 35.2 | -245.5 | 143.3 | -31.7 | 286.0 | 38.9 |
| 17 | Left | 157.4 | -91.3 | 17.5 | 182.9 | 49.8 | 255.5 | -139.1 | 24.8 | 292.0 | 53.5 |
| | Right | -154.2 | 89.6 | -17.2 | 179.2 | 41.3 | -259.9 | 140.7 | -24.7 | 296.5 | 45.7 |
| 16 | Left | 167.4 | -86.8 | 10.8 | 188.8 | 59.4 | 269.5 | -132.2 | 15.6 | 300.6 | 62.9 |
| | Right | -164.0 | 85.2 | -10.6 | 185.1 | 47.4 | -273.9 | 133.6 | -15.7 | 305.2 | 52.2 |
| 15 | Left | 177.1 | -79.5 | 3.2 | 194.1 | 67.8 | 283.2 | -120.8 | 5.2 | 307.9 | 71.1 |
| | Right | -173.5 | 78.0 | -3.2 | 190.3 | 54.7 | -287.7 | 122.0 | -5.2 | 312.6 | 59.6 |
| 14 | Left | 186.1 | -68.7 | -4.5 | 198.4 | 75.4 | 301.7 | -104.6 | -5.6 | 319.4 | 78.5 |
| | Right | -182.4 | 67.3 | 4.5 | 194.5 | 63.4 | -300.3 | 105.6 | 5.5 | 318.4 | 68 |
| 13 | Left | 194.0 | -54.9 | -11.6 | 202.0 | 82.3 | 306.4 | -84.2 | -15.3 | 318.1 | 85.2 |
| | Right | -190.2 | 64.0 | 11.4 | 201.0 | 72.7 | -311.0 | 85.0 | 15.1 | 322.8 | 76.9 |
| 12 | Left | 200.2 | -38.8 | -16.9 | 204.6 | 88.9 | 322.4 | -60.4 | -22.6 | 328.8 | 91.5 |
| | Right | -196.2 | 38.0 | 16.6 | 200.6 | 81.4 | -309.0 | 61.0 | 22.4 | 315.8 | 85.1 |
| 11 | Left | 204.4 | -20.9 | -19.8 | 206.5 | 95.4 | 309.3 | -34.2 | -26.6 | 312.3 | 97.5 |
| | Right | -200.4 | 20.5 | 19.4 | 202.4 | 88.6 | -324.1 | 34.6 | 26.4 | 327.0 | 92 |
| 10 | Left | 205.6 | -2.7 | -19.6 | 206.5 | 101.7 | 320.1 | -7.4 | -26.6 | 321.3 | 103.4 |
| | Right | -201.5 | 2.6 | 19.2 | 202.4 | 93.9 | -324.8 | 7.5 | 26.4 | 326.0 | 97.1 |
| 9 | Left | 203.8 | 14.5 | -16.7 | 205.0 | 108 | 316.5 | 17.6 | -22.9 | 317.8 | 109.1 |
| | Right | -199.9 | -14.2 | 16.4 | 201.1 | 97.5 | -321.2 | -17.5 | 22.8 | 322.4 | 100.5 |
| 8 | Left | 199.8 | 30.0 | -12.0 | 202.4 | 114.1 | 308.5 | 40.3 | -16.5 | 311.6 | 114.5 |
| | Right | -195.8 | -29.5 | 11.8 | 198.4 | 100.2 | -312.9 | -40.4 | 16.5 | 315.9 | 103 |
| 7 | Left | 191.9 | 43.2 | -5.7 | 196.8 | 119.8 | 294.8 | 59.5 | -8.0 | 300.9 | 109.1 |
| | Right | -188.1 | -42.4 | 5.6 | 192.9 | 102.9 | -298.8 | -59.6 | 8.0 | 304.8 | 106.3 |
| 6 | Left | 180.0 | 53.1 | 1.2 | 187.7 | 124.9 | 274.8 | 73.5 | 1.3 | 284.4 | 124 |
| | Right | -176.5 | -52.1 | -1.2 | 184.0 | 106.2 | -278.2 | -73.7 | -1.0 | 287.8 | 108.3 |
| 5 | Left | 163.5 | 58.5 | 7.6 | 173.8 | 129.3 | 247.5 | 81.2 | 9.7 | 260.7 | 12707 |
| | Right | -160.4 | -57.5 | -7.5 | 170.5 | 110.2 | -250.3 | -81.3 | -9.5 | 263.4 | 111.7 |
| 4 | Left | 10.1 | 59.0 | 12.5 | 61.2 | 132.7 | 212.1 | 81.3 | 16.0 | 227.8 | 130.2 |
| | Right | -139.1 | -58.0 | -12.2 | 151.2 | 114.3 | -214.3 | -81.3 | -15.7 | 229.7 | 115.1 |
| 3 | Left | 114.2 | 53.7 | 14.7 | 127.0 | 133.8 | 168.0 | 72.8 | 18.8 | 184.0 | 131.1 |
| | Right | -112.3 | -52.9 | -14.5 | 124.9 | 117.1 | -168.9 | -72.4 | -16.3 | 184.5 | 116.9 |
| 2 | Left | 80.5 | 43.1 | 14.5 | 92.5 | 131.2 | 112.7 | 55.2 | 17.3 | 126.6 | 126.6 |
| | Right | -78.5 | -42.1 | -14.2 | 90.2 | 114.6 | -112.9 | -54.6 | -16.8 | 126.5 | 115.4 |

<u>Note:</u> Shear Forces (kips) in Beams are found to be as follows:

| Beam | Due to 3 Modes | Due to 25% of V |
|---|---|---|
| A2-B2 | 8 | 10 |
| B2-C2 | 11 | 10 |

**Table 2-14. Shear forces and bending moments in beams A2-B2 and B2-C2 at level 2 due to gravity loading (using pattern live loads)**

**Exterior Beam A2-B2**

| Load Type | Bending Moment (ft-kips) | | | Shear Force (kips) | | |
|---|---|---|---|---|---|---|
| | Left | Middle | Right | Left | Middle | Right |
| Dead Load | **-147** | **86** | **-169** | **-43** | **1** | **43** |
| Pattern LL1 | -34 | 20 | -39 | -10 | 0 | 10 |
| Pattern LL2 | -35 | 21 | -36 | -10 | 0 | 10 |
| Pattern LL3 | 2 | -1 | -4 | 0 | 0 | 0 |
| Pattern LL4 | -34 | 20 | -39 | -10 | 0 | 10 |
| Governing LL | **-35** | **21** | **-39** | **-10** | **0** | **10** |

**Interior Beam B2-C2**

| Load Type | Bending Moment (ft-kips) | | | Shear Force (kips) | | |
|---|---|---|---|---|---|---|
| | Left | Middle | Right | Left | Middle | Right |
| Dead Load | **-160** | **82** | **-160** | **-43** | **0** | **43** |
| Pattern LL1 | -37 | 19 | -37 | -10 | 0 | 10 |
| Pattern LL2 | -4 | -2 | 1 | 0 | 0 | 0 |
| Pattern LL3 | -34 | 20 | -39 | -10 | 0 | 10 |
| Pattern LL4 | -38 | 19 | -37 | -10 | 0 | 10 |
| Governing LL | **-38** | **20** | **-39** | **-10** | **0** | **10** |

Note: For definitions of different pattern loadings, Refer to Fig. 2-12.

**Table 2-15. Shear forces and bending moments in beams A2-B2 and B2-C2 at level 2 due to gravity loading based on ACI Sec. 8.3.3**

**Exterior Beam A2-B2**

| Clear Span $l_n$ | Load Type | (in klf) | $M$- (ext.) ft-kips | $M$+ ft-kips | $M$- (int.) ft-kips | $V$ kips |
|---|---|---|---|---|---|---|
| ft | Dead Load, $w_D$ | 3.61 | -118 | 134 | -188 | 47 |
| 22.83 | Live Load, $w_L$ | 0.832 | -27 | 31 | -43 | 11 |

**Interior Beam B2-C2**

| Clear Span $l_n$ | Load Type | (in klf) | $M$- (ext.) ft-kips | $M$+ ft-kips | $M$- (int.) ft-kips | $V$ kips |
|---|---|---|---|---|---|---|
| ft | Dead Load, $w_D$ | 3.61 | -171 | 118 | -171 | 41 |
| 22.83 | Live Load, $w_L$ | 0.832 | -39 | 27 | -39 | 9 |

**Table 2-16. Comparison of internal forces between ACI 318-99 Sec. 8.3.3 and pattern loading**

**Exterior Beam A2-B2**

| Load Type | $M$- (ext.) | | $M$+ | | $M$- (int.) | | $V$ (kips) | |
|---|---|---|---|---|---|---|---|---|
| | ACI* | Pattern | ACI | Pattern | ACI | Pattern | ACI | Pattern |
| Dead Load, $w_d$ | -118 | -147 | 134 | 86 | -188 | -169 | 45 | 43 |
| Live Load, $w_L$ | -27 | -35 | 31 | 21 | -43 | -39 | 11 | 10 |

**Interior Beam B2-C2**

| Load Type | $M$- (ext.) | | $M$+ | | $M$- (int.) | | $V$ (kips) | |
|---|---|---|---|---|---|---|---|---|
| | ACI | Pattern | ACI | Pattern | ACI | Pattern | ACI | Pattern |
| Dead Load, $w_d$ | -171 | -160 | 118 | 82 | -171 | -160 | 39 | 43 |
| Live Load, $w_L$ | -39 | -38 | 27 | 20 | -39 | -39 | 10 | 9 |

Bending moments are in ft-kips.
* Results obtained using ACI Sec. 8.3.3.

**Table 2-17(a). Summary of design axial force, shear force, and bending moment for support sections of beams at level 2**

**Exterior Beam A2-B2 (interior end)**

| Loads | Symbol | Axial Force (kips) | Shear Force (kips) | Bending Moment (ft-kips) |
|---|---|---|---|---|
| Dead Load | $D$ | 0 | 47 | 188 |
| Live Load | $L$ | 0 | 11 | 43 |
| Lateral Load | $E_h$ | 0 | 10 | 115 |
| **Load Combinations** | | | | |
| 1 | $1.4D+1.7L$ | 0 | 85 | 336 |
| 2 | $1.2D+(\rho E_h+0.2DS_{DS})+0.5L$ | 0 | 81 | 400 |
| 3 | $0.9D-(\rho E_h+0.2DS_{DS})$ | 0 | 23 | 17 |

**Exterior Beam A2-B2 (exterior end)**

| Loads | Symbol | Axial Force (kips) | Shear Force (kips) | Bending Moment (ft-kips) |
|---|---|---|---|---|
| Dead Load | $D$ | 0 | 47 | 118 |
| Live Load | $L$ | 0 | 11 | 27 |
| Lateral Load | $E_h$ | 0 | 10 | 131 |
| **Load Combinations** | | | | |
| 1 | $1.4D+1.7L$ | 0 | 85 | 211 |
| 2 | $1.2D+(\rho E_h+0.2DS_{DS})+0.5L$ | 0 | 81 | 310 |
| 3 | $0.9D-(\rho E_h+0.2DS_{DS})$ | 0 | 23 | -48 |

**Interior Beam B2-C2 (both ends)**

| Loads | Symbol | Axial Force (kips) | Shear Force (kips) | Bending Moment (ft-kips) |
|---|---|---|---|---|
| Dead Load | $D$ | 0 | 41 | 171 |
| Live Load | $L$ | 0 | 9 | 39 |
| Lateral Load | $E_h$ | 0 | 11 | 127 |
| **Load Combinations** | | | | |
| 1 | $1.4D+1.7L$ | 0 | 73 | 306 |
| 2 | $1.2D+(\rho E_h+0.2DS_{DS})+0.5L$ | 0 | 73 | 386 |
| 3 | $0.9D-(\rho E_h+0.2DS_{DS})$ | 0 | 18 | -7 |

**Table 2-17(b). Summary of design axial force, shear force, and bending moment for midspan sections of beams at level 2**

**Exterior Beam A2-B2**

| Loads | Symbol | Axial Force (kips) | Shear Force (kips) | Bending Moment (ft-kips) |
|---|---|---|---|---|
| Dead Load | $D$ | 0 | 0 | 134 |
| Live Load | $L$ | 0 | 0 | 31 |
| Lateral Load | $E_h$ | 0 | 10 | 0 |

| Load Combinations | | | | |
|---|---|---|---|---|
| 1 | $1.4D+1.7L$ | 0 | 0 | 240 |
| 2 | $1.2D+(\rho E_h+0.2DS_{DS})+0.5L$ | 0 | 10 | 203 |
| 3 | $0.9D-(\rho E_h+0.2DS_{DS})$ | 0 | -10 | 94 |

**Interior Beam B2-C2**

| Loads | Symbol | Axial Force (kips) | Shear Force (kips) | Bending Moment (ft-kips) |
|---|---|---|---|---|
| Dead Load | $D$ | 0 | 0 | 118 |
| Live Load | $L$ | 0 | 0 | 27 |
| Lateral Load | $E_h$ | 0 | 11 | 0 |

| Load Combinations | | | | |
|---|---|---|---|---|
| 1 | $1.4D$□ $+1.7L$□ | 0 | 0 | 211 |
| 2 | $1.2D$□ $+(\rho E_h+0.2DS_{DS})+0.5L$□ | 0 | 11 | 179 |
| 3 | $0.9D-(\rho E_h+0.2DS_{DS})$ | 0 | -11 | 83 |

**Exterior Beam A2-B2 (Interior end)**

| Loads | Symbol | Axial Force (kips) | Shear Force (kips) | Bending Moment (ft-kips) |
|---|---|---|---|---|
| Dead Load | $D$ | 0 | 0 | 129 |
| Live Load | $L$□ | 0 | 0 | 32 |
| Lateral Load | $E_h$□ | 0 | 11 | 0 |

| Load Combinations | | | | |
|---|---|---|---|---|
| 1 | $1.4D$□ $+1.7L$□ | 0 | 0 | 235 |
| 2 | $1.2D$□ $+(\rho E_h+0.2DS_{DS})+0.5L$□ | 0 | 11 | 197 |
| 3 | $0.9D$□ $\rho E_h+0.2DS_{DS})$ | 0 | -11 | 90 |

## Table 2-18. Axial forces, shear forces, and bending moments in columns due to lateral forces of Table 2-9

**Axial Forces in Exterior Column A2**

| Floor | Dual system with 100% V | | | | No shearwall |
|---|---|---|---|---|---|
| | Mode 1 | Mode 2 | Mode 3 | Resultant | with 25% V |
| 21 | 8.7 | -5.9 | 1.7 | 10.6 | 1.4 |
| 20 | 19.4 | -13.1 | 3.7 | 23.7 | 3.9 |
| 19 | 30.6 | -20.6 | 5.8 | 37.3 | 7.8 |
| 18 | 42.6 | -28.4 | 7.7 | 51.8 | 12.7 |
| 17 | 55.6 | -36.1 | 9.2 | 66.9 | 18.3 |
| 16 | 69.3 | -43.3 | 10.1 | 82.3 | 24.2 |
| 15 | 83.8 | -50.0 | 10.4 | 98.2 | 30.2 |
| 14 | 99.3 | -55.7 | 10.0 | 114.3 | 36.2 |
| 13 | 115.3 | -60.3 | 8.9 | 130.5 | 42.1 |
| 12 | 131.9 | -63.4 | 7.4 | 146.6 | 47.9 |
| 11 | 149.0 | -65.1 | 5.8 | 162.7 | 53.7 |
| 10 | 166.1 | -65.2 | 4.1 | 178.5 | 59.6 |
| 9 | 183.1 | -63.9 | 2.6 | 194.0 | 65.7 |
| 8 | 199.9 | -61.3 | 1.7 | 209.1 | 72 |
| 7 | 215.9 | -57.6 | 1.2 | 223.5 | 78.9 |
| 6 | 231.1 | -52.9 | 1.3 | 237.1 | 86.3 |
| 5 | 244.8 | -48.0 | 1.9 | 249.5 | 94.5 |
| 4 | 256.7 | -42.8 | 2.9 | 260.3 | 103.4 |
| 3 | 266.4 | -38.3 | 4.2 | **269.2** | **113.1** |
| 2 | 273.2 | -34.5 | 5.5 | **275.4** | **123** |

**Shear Forces in Exterior Column A2**

| Floor | Dual system with 100% V | | | | No shearwall |
|---|---|---|---|---|---|
| | Mode 1 | Mode 2 | Mode 3 | Resultant | with 25% V |
| 21 | 14.6 | -9.4 | 2.5 | 17.6 | 1.4 |
| 20 | 11.7 | -8.2 | 2.6 | 14.5 | 4 |
| 19 | 12.3 | -8.5 | 2.5 | 15.2 | 5.3 |
| 18 | 13.7 | -9.1 | 2.4 | 16.6 | 6.3 |
| 17 | 14.5 | -8.3 | 1.4 | 16.8 | 6.8 |
| 16 | 15.6 | -8.1 | 0.8 | 17.5 | 7.1 |
| 15 | 16.5 | -7.2 | -0.1 | 18.0 | 7.3 |
| 14 | 17.3 | -6.1 | -1.0 | 18.3 | 7.6 |
| 13 | 18.0 | -4.5 | -1.7 | 18.6 | 7.9 |
| 12 | 18.5 | -2.9 | -2.1 | 18.8 | 8.3 |
| 11 | 19.1 | -0.9 | -2.5 | 19.3 | 8.7 |
| 10 | 18.5 | 0.8 | -1.9 | 18.6 | 9.2 |
| 9 | 18.7 | 2.3 | -1.7 | 18.9 | 9.7 |
| 8 | 18.2 | 3.8 | -1.0 | 18.6 | 10.4 |
| 7 | 17.4 | 4.9 | -0.2 | 18.1 | 11.3 |
| 6 | 16.3 | 5.9 | 0.6 | 17.3 | 12.2 |
| 5 | 14.5 | 6.2 | 1.3 | 15.8 | 12.9 |
| 4 | 12.7 | 6.3 | 1.8 | 14.3 | 14.2 |
| 3 | 9.3 | 4.7 | 1.6 | **10.6** | **13.2** |
| 2 | 7.3 | 5.7 | 2.6 | **9.6** | **19.6** |

**Axial Forces in Interior Column B2**

| Floor | Dual system with 100% V | | | | No shearwall |
|---|---|---|---|---|---|
| | Mode 1 | Mode 2 | Mode 3 | Resultant | with 25% V |
| 21 | 5.2 | -3.5 | 0.9 | 6.3 | 0.8 |
| 20 | 10.0 | -6.8 | 1.8 | 12.2 | 1.6 |
| 19 | 15.3 | -10.1 | 2.5 | 18.5 | 2.4 |
| 18 | 20.7 | -13.4 | 3.2 | 24.9 | 3.2 |
| 17 | 26.6 | -16.7 | 3.7 | 31.6 | 3.9 |
| 16 | 32.7 | -19.8 | 4.1 | 38.5 | 4.6 |
| 15 | 39.3 | -22.5 | 4.2 | 45.5 | 5.3 |
| 14 | 46.3 | -24.9 | 4.0 | 52.7 | 5.9 |
| 13 | 53.4 | -26.8 | 3.7 | 59.9 | 6.5 |
| 12 | 60.9 | -28.1 | 3.2 | 67.1 | 7 |
| 11 | 68.6 | -28.8 | 2.6 | 74.4 | 7.5 |
| 10 | 76.3 | -28.8 | 2.1 | 81.5 | 7.9 |
| 9 | 83.9 | -28.3 | 1.6 | 88.6 | 8.3 |
| 8 | 91.4 | -27.2 | 1.2 | 95.4 | 8.7 |
| 7 | 98.6 | -25.6 | 1.0 | 101.8 | 9 |
| 6 | 105.2 | -23.7 | 1.0 | 107.9 | 9.2 |
| 5 | 111.2 | -21.8 | 1.2 | 113.3 | 9.3 |
| 4 | 116.3 | -19.9 | 1.4 | 117.9 | 9.4 |
| 3 | 120.1 | -18.3 | 1.8 | **121.5** | **9.4** |
| 2 | 122.4 | -17.3 | 2.0 | **123.6** | **9.4** |

**Shear Forces in Interior Column B2**

| Floor | Dual system with 100% V | | | | No shearwall |
|---|---|---|---|---|---|
| | Mode 1 | Mode 2 | Mode 3 | Resultant | with 25% V |
| 21 | 43.3 | -27.2 | 7.3 | 51.7 | 3.9 |
| 20 | 28.6 | -18.7 | 5.4 | 34.6 | 7.4 |
| 19 | 33.9 | -21.4 | 5.7 | 40.5 | 10.2 |
| 18 | 35.5 | -21.5 | 4.8 | 41.7 | 12.1 |
| 17 | 37.8 | -20.4 | 3.2 | 43.1 | 13.2 |
| 16 | 39.9 | -19.3 | 1.8 | 44.3 | 13.9 |
| 15 | 42.0 | -17.2 | -0.1 | 45.4 | 14.4 |
| 14 | 43.8 | -14.4 | -1.9 | 46.2 | 14.9 |
| 13 | 45.5 | -11.0 | -3.3 | 46.9 | 15.5 |
| 12 | 46.5 | -7.1 | -4.3 | 47.2 | 16.1 |
| 11 | 47.4 | -2.7 | -4.8 | 47.7 | 16.9 |
| 10 | 46.7 | 1.2 | -4.1 | 46.9 | 17.8 |
| 9 | 46.4 | 4.8 | -3.4 | 46.7 | 18.9 |
| 8 | 44.8 | 8.3 | -2.1 | 45.6 | 20.1 |
| 7 | 42.6 | 10.9 | -0.5 | 44.0 | 21.6 |
| 6 | 39.4 | 12.7 | 1.1 | 41.4 | 23.2 |
| 5 | 35.1 | 13.5 | 2.5 | 37.7 | 24.8 |
| 4 | 29.2 | 12.9 | 3.3 | 32.1 | 25.7 |
| 3 | 23.2 | 11.3 | 3.4 | **26.1** | **28** |
| 2 | 10.3 | 7.2 | 3.1 | **12.9** | **23.6** |

Note:
1. Resultant is obtained using root-mean-square values of the three modes.
2. Axial Forces in kips; Shear Forces in kips; Bending Moments in ft-kips.
3. The larger of forces due to (a) the presence of shear wall with 100% base shear V and (b) absence of shear wall with 25% of V is considered in design.

## Table 2-18. (continued)

| | | Bending Moments in Exterior Column A2 | | | | | Bending Moments in Interior Column B2 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Dual system with 100% V | | | | No shearwall | Dual system with 100% V | | | | No shearwall |
| Floor | Location | Mode 1 | Mode 2 | Mode 3 | Resultant | with 25% V | Mode 1 | Mode 2 | Mode 3 | Resultant | with 25% V |
| 21 | Bottom | 45.6 | -28 | 7.3 | 54.0 | 11.6 | 164.6 | -101.6 | 27.2 | 195.3 | 9 |
| | Top | -106.5 | 70 | -20.1 | 129.0 | 17.6 | -286 | 180.6 | -50 | 341.9 | 33.7 |
| 20 | Bottom | 47.3 | -34.8 | 13.3 | 60.2 | 8.2 | 144.7 | -94.9 | 29.3 | 175.5 | 24.7 |
| | Top | -74.4 | 50.5 | -15.51 | 91.2 | 38 | -152.8 | 99.2 | -28.2 | 184.3 | 53.7 |
| 19 | Bottom | 47.2 | -40.1 | 17.8 | 64.4 | 17.5 | 157.4 | -106.1 | 34 | 192.8 | 42.7 |
| | Top | -81.1 | 48.2 | -9.3 | 94.8 | 39.9 | -194.2 | 116.9 | -26.8 | 228.2 | 65.2 |
| 18 | Bottom | 52.8 | -50.1 | 22.1 | 76.1 | 26.8 | 167.5 | -114.5 | 35.1 | 205.9 | 56.5 |
| | Top | -89.9 | 44.9 | -2.8 | 100.5 | 41.6 | -200.9 | 108.8 | -16.7 | 229.1 | 71.2 |
| 17 | Bottom | 57.4 | -52.2 | 20.74 | 80.3 | 33.3 | 179.1 | -114.4 | 29.7 | 214.6 | 65.9 |
| | Top | -93.3 | 33.7 | 5.9 | 99.4 | 41.5 | -213.2 | 97.2 | -4.7 | 234.4 | 74.4 |
| 16 | Bottom | 63.3 | -56.7 | 19.1 | 87.1 | 37.3 | 191.1 | -114.3 | 23.2 | 223.9 | 71.5 |
| | Top | -97.9 | 27.6 | 10.6 | 102.3 | 41.8 | -223.5 | 85.8 | 4.8 | 239.5 | 76.5 |
| 15 | Bottom | 69.6 | -58.7 | 14.9 | 92.3 | 39.4 | 203.5 | -109.5 | 13.9 | 231.5 | 74.9 |
| | Top | -101 | 16.1 | 16 | 103.5 | 42.8 | -233 | 69.5 | 15.3 | 243.6 | 78.5 |
| 14 | Bottom | 76.4 | -58.3 | 8.8 | 96.5 | 40.6 | 215.5 | -100.3 | 2.9 | 237.7 | 77.4 |
| | Top | -102.9 | 4.3 | 19 | 104.7 | 44.5 | -240.3 | 49.5 | 23.3 | 246.4 | 81.3 |
| 13 | Bottom | 83.6 | -55.2 | 1.6 | 100.2 | 41.7 | 226.8 | -86.8 | -8.1 | 243.0 | 79.9 |
| | Top | -103.2 | -8 | 19.4 | 105.3 | 46.6 | -245.2 | 27.2 | 28 | 248.3 | 84.8 |
| 12 | Bottom | 89.9 | -49.9 | -5.4 | 103.0 | 43 | 236 | -69.7 | -17.7 | 246.7 | 83 |
| | Top | -101.8 | -19.7 | 17.1 | 105.1 | 48.9 | 247 | 4.1 | 28.7 | 248.7 | 88.7 |
| 11 | Bottom | 99.8 | -40.4 | -13.3 | 108.5 | 44.6 | 246.8 | -47.8 | -26.2 | 252.7 | 86.6 |
| | Top | -99 | -30.6 | 12.6 | 104.4 | 51.5 | -246 | -19 | 25.6 | 248.1 | 93.3 |
| 10 | Bottom | 102.2 | -29.6 | -15.8 | 107.6 | 46.4 | 248.2 | -25.2 | -27.4 | 251.0 | 90.6 |
| | Top | -90.3 | -37.6 | 4.6 | 97.9 | 54.7 | -236.9 | -39.2 | 16.9 | 240.7 | 98.6 |
| 9 | Bottom | 108.1 | -18.6 | -18 | 111.2 | 48.3 | 251.2 | -3.3 | -27 | 252.7 | 95.2 |
| | Top | -86.4 | -72.6 | -0.3 | 112.9 | 58.6 | -230.7 | -54.2 | 9.7 | 237.2 | 105.1 |
| 8 | Bottom | 112.7 | -6.3 | -17.5 | 114.2 | 50.4 | 250.4 | 18.4 | -22.8 | 252.1 | 100.8 |
| | Top | -76.3 | -45.8 | -7.1 | 89.3 | 63.1 | -216.1 | -67.5 | -0.4 | 226.4 | 112.7 |
| 7 | Bottom | 116.6 | 6.4 | -14.5 | 117.7 | 53.2 | 246.2 | 38.5 | -15.3 | 249.7 | 107.9 |
| | Top | -64.3 | -45.4 | -12.3 | 79.7 | 67.6 | -197 | -75.3 | -10 | 211.1 | 121 |
| 6 | Bottom | 119.5 | 19 | -9.3 | 121.4 | 57.4 | 237.5 | 55.5 | -5.9 | 244.0 | 115.8 |
| | Top | -49.3 | -411.6 | -15.5 | 414.8 | 71.5 | -171.6 | -77 | -17.5 | 188.9 | 128.9 |
| 5 | Bottom | 120.8 | 30.3 | -2.8 | 124.6 | 61.5 | 225.1 | 68.8 | 4.3 | 235.4 | 125.8 |
| | Top | -30.8 | -33.9 | -16.1 | 48.5 | 73.7 | -139.9 | -72 | -22 | 158.9 | 135.6 |
| 4 | Bottom | 123.3 | 42.2 | 5.3 | 130.4 | 72.1 | 204.3 | 75.7 | 13.3 | 218.3 | 131.7 |
| | Top | -9.1 | -22.8 | -13.9 | 28.2 | 74.9 | -99.6 | -59 | -22 | 117.8 | 138 |
| 3 | Bottom | 114.9 | 43.3 | 8.4 | **123.1** | **68.1** | 187.7 | 78.6 | 19.5 | **204.4** | **154.9** |
| | Top | 18.7 | -6.3 | -8.1 | **21.3** | **67.6** | -53.6 | -39.3 | -17.7 | **68.8** | **138.7** |
| 2 | Bottom | 161.2 | 97.8 | 40.5 | **192.8** | **255.6** | 177.8 | 106.2 | 43.4 | **211.6** | **278.2** |
| | Top | 42.2 | 5.9 | -3.9 | **42.8** | **64.7** | 9.8 | -10.6 | -9.6 | **17.3** | **110** |

**Table 2-19. Summary of design axial force and bending moment for columns**

**Interior Column B2: Column 1 (between Ground and Level 2)**

| Loads | Symbol | Axial Force (kips) | Shear Force (kips) | Bending Moment (ft-kips) |
|---|---|---|---|---|
| Dead Load | $D$ | 2690 | 3 | 22 |
| Live Load | $L$ | 419 | 1 | 5 |
| Lateral Load | $E_h$ | 122 | 24 | 279 |

| Load Combinations | | | | |
|---|---|---|---|---|
| 1 | $1.4D+1.7L$ | 4478 | 6 | 39 |
| 2 | $1.2D+(\rho E_h+0.2DS_{DS})+0.5L$ | 4098 | 29 | 312 |
| 3 | $0.9D-(\rho E_h+0.2DS_{DS})$ | 1761 | -22 | -264 |

**Interior Column B2: Column 2 (between Level 2 and Level 3)**

| Loads | Symbol | Axial Force (kips) | Shear Force (kips) | Bending Moment (ft-kips) |
|---|---|---|---|---|
| Dead Load | $D$ | 2549 | 7 | 34 |
| Live Load | $L$ | 397 | 2 | 8 |
| Lateral Load | $E_h$ | 122 | 28 | 204 |

| Load Combinations | | | | |
|---|---|---|---|---|
| 1 | $1.4D+1.7L$ | 4244 | 13 | 61 |
| 2 | $1.2D+(\rho E_h+0.2DS_{DS})+0.5L+B26$ | 3889 | 39 | 256 |
| 3 | $0.9D-(\rho E_h+0.2DS_{DS})$ | 1662 | -23 | -180 |

**Exterior Column A2: Column 3 (between Ground and Level 2)**

| Loads | Symbol | Axial Force (kips) | Shear Force (kips) | Bending Moment (ft-kips) |
|---|---|---|---|---|
| Dead Load | $D$ | 1493 | 10 | 76 |
| Live Load | $L$ | 238 | 3 | 18 |
| Lateral Load | $E_h$ | 275 | 20 | 256 |

| Load Combinations | | | | |
|---|---|---|---|---|
| 1 | $1.4D+1.7L$ | 2495 | 19 | 137 |
| 2 | $1.2D+(\rho E_h+0.2DS_{DS})+0.5L$ | 2484 | 36 | 371 |
| 3 | $0.9D-(\rho E_h+0.2DS_{DS})$ | 770 | -13 | -203 |

**Exterior Column A2: Column 4 (between Level 2 and Level 3)**

| Loads | Symbol | Axial Force (kips) | Shear Force (kips) | Bending Moment (ft-kips) |
|---|---|---|---|---|
| Dead Load | $D$ | 1414 | 23 | 119 |
| Live Load | $L$ | 226 | 6 | 28 |
| Lateral Load | $E_h$ | 269 | 13 | 123 |

| Load Combinations | | | | |
|---|---|---|---|---|
| 1 | $1.4D+1.7L$ | 2364 | 42 | 214 |
| 2 | $1.2D+(\rho E_h+0.2DS_{DS})+0.5L$ | 2362 | 48 | 304 |
| 3 | $0.9D-(\rho E_h+0.2DS_{DS})$ | 721 | 3 | -40 |

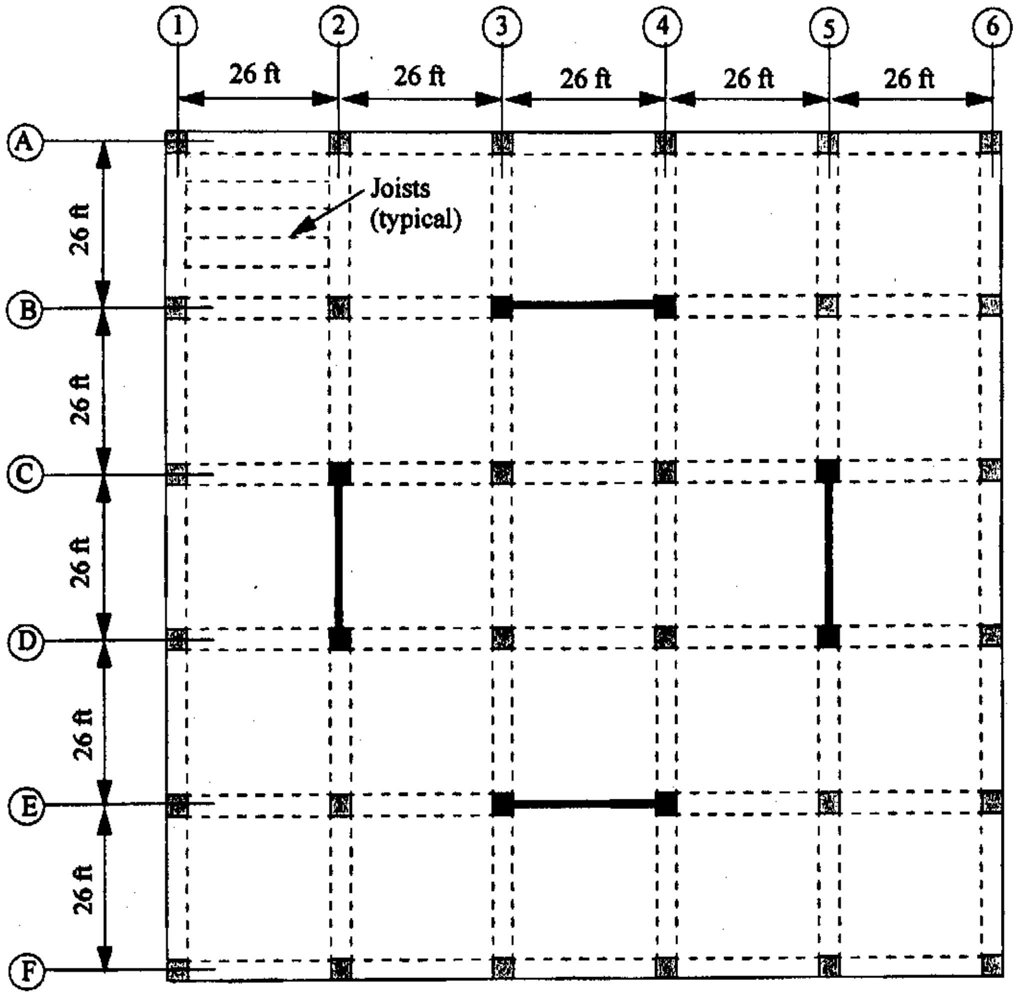



**Figure 2-1. Typical floor plan of example building**

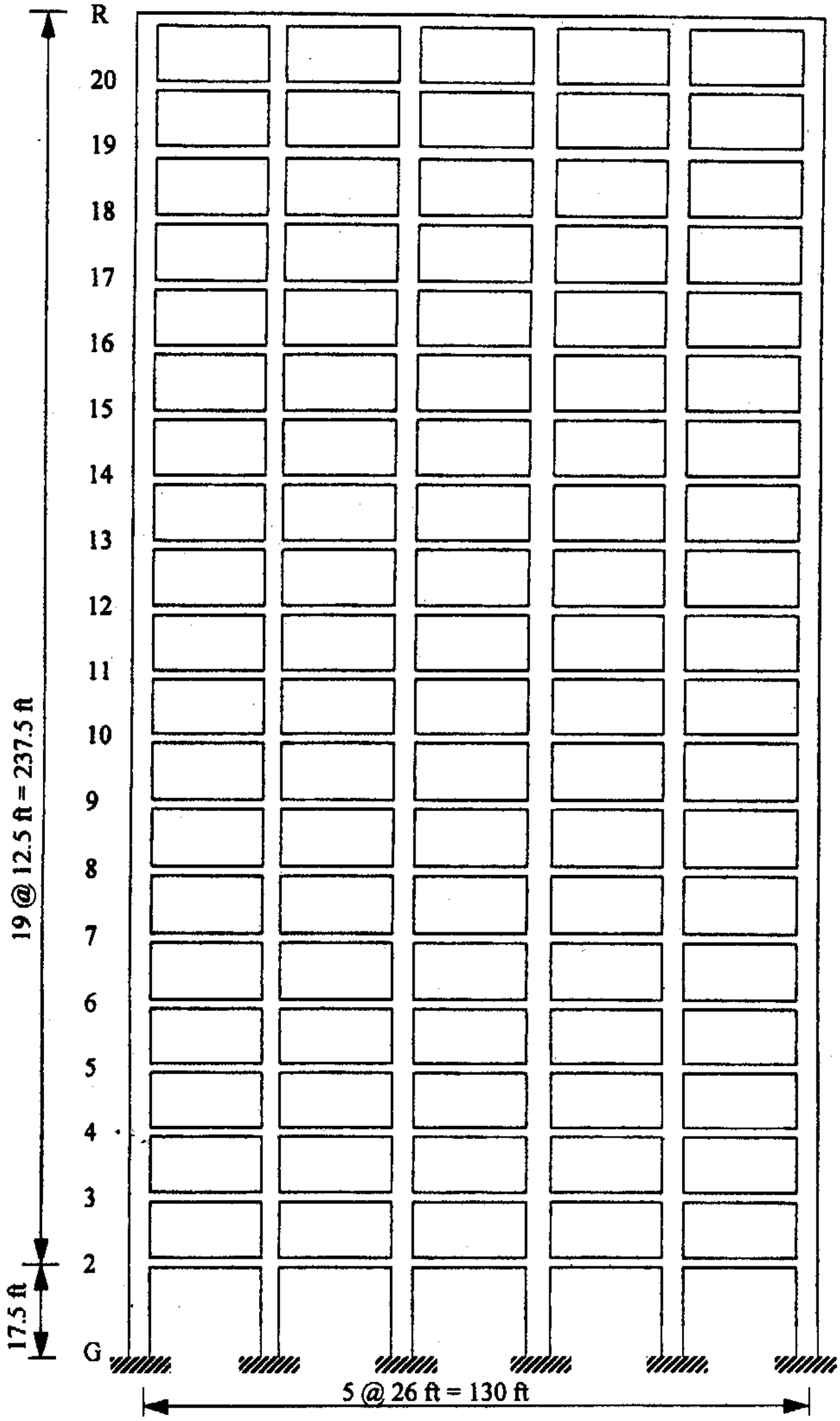



**Figure 2-2. Elevation of example building**

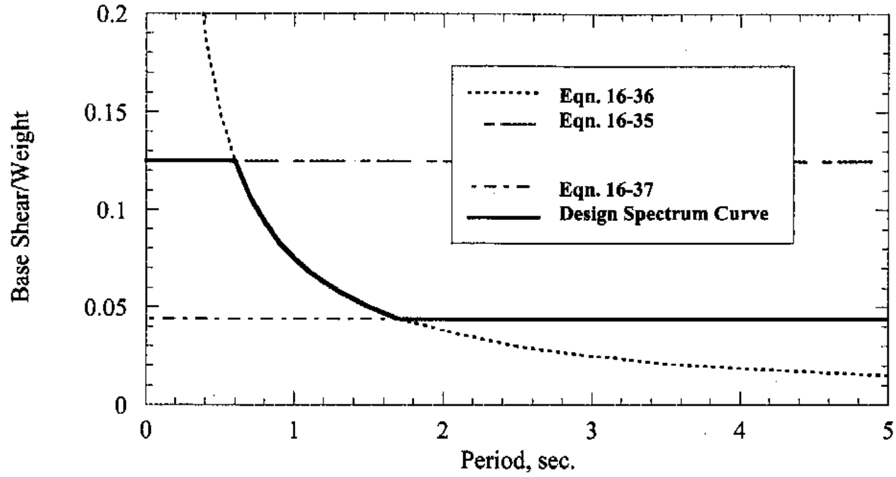



**Figure 2-3. Design response spectrum**

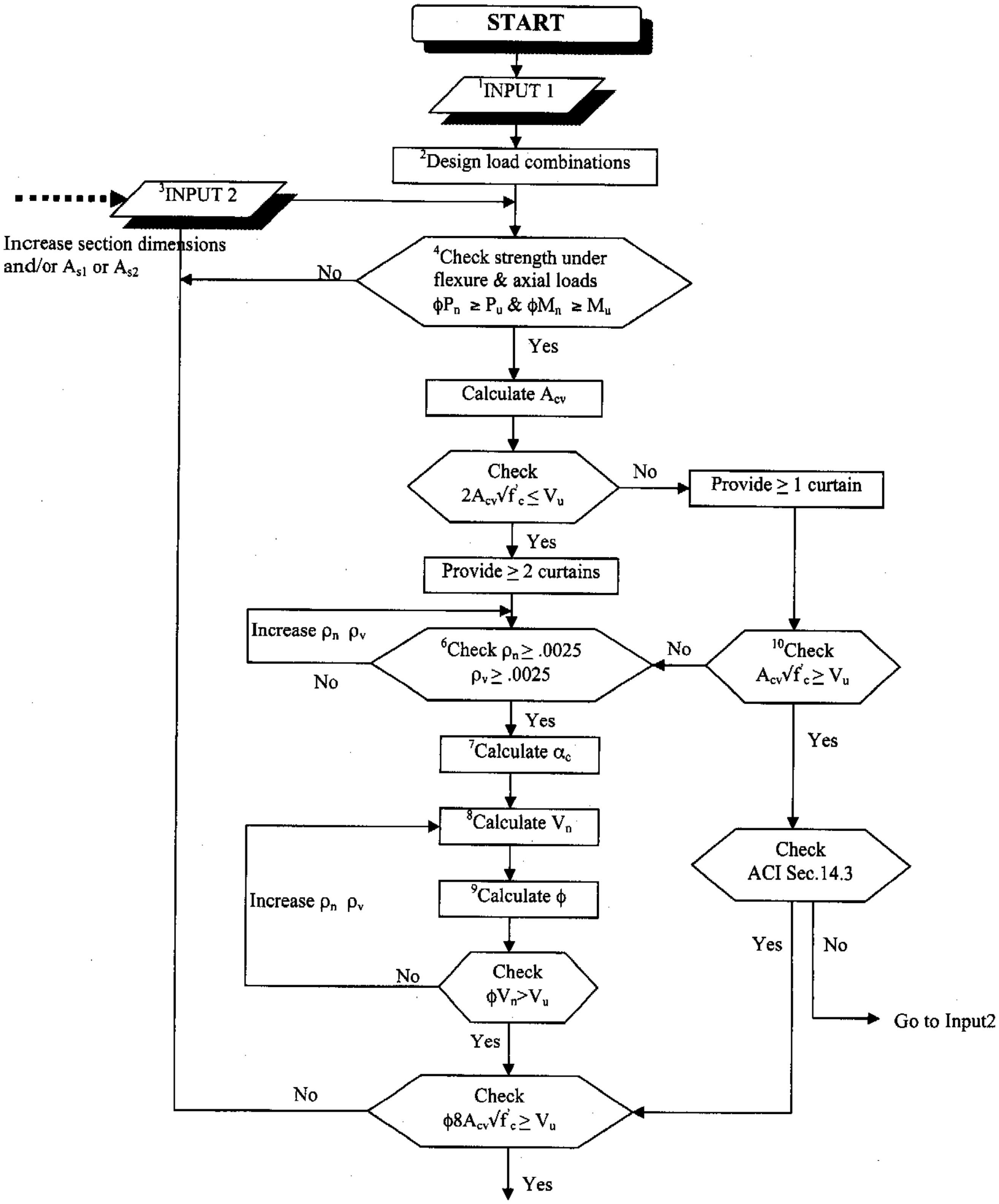



**Figure 2-4. Flow chart of shear wall design per IBC 2000 (Part 1)**

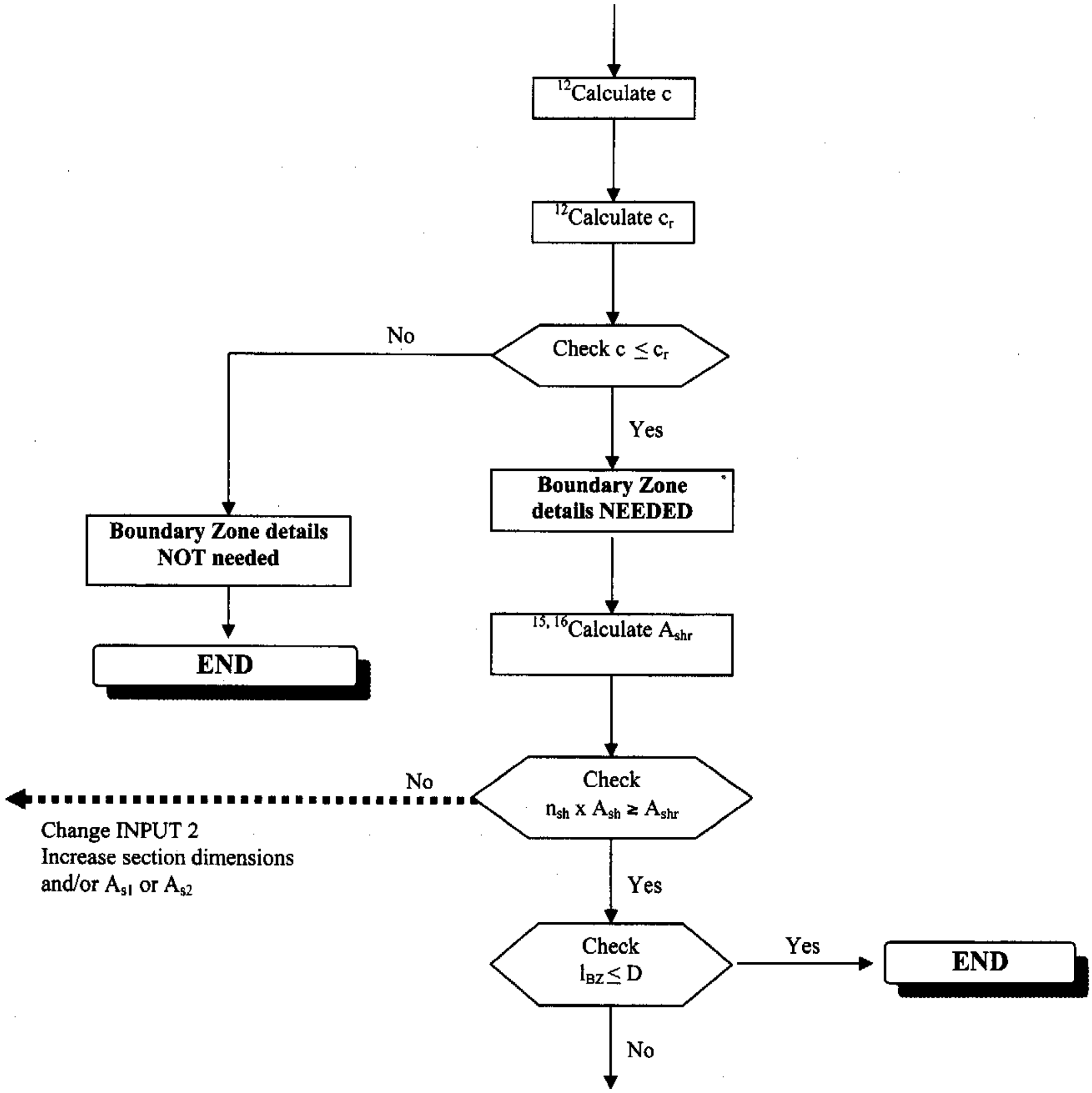



**Figure 2-4. Flow chart of shear wall design (Part 2)**

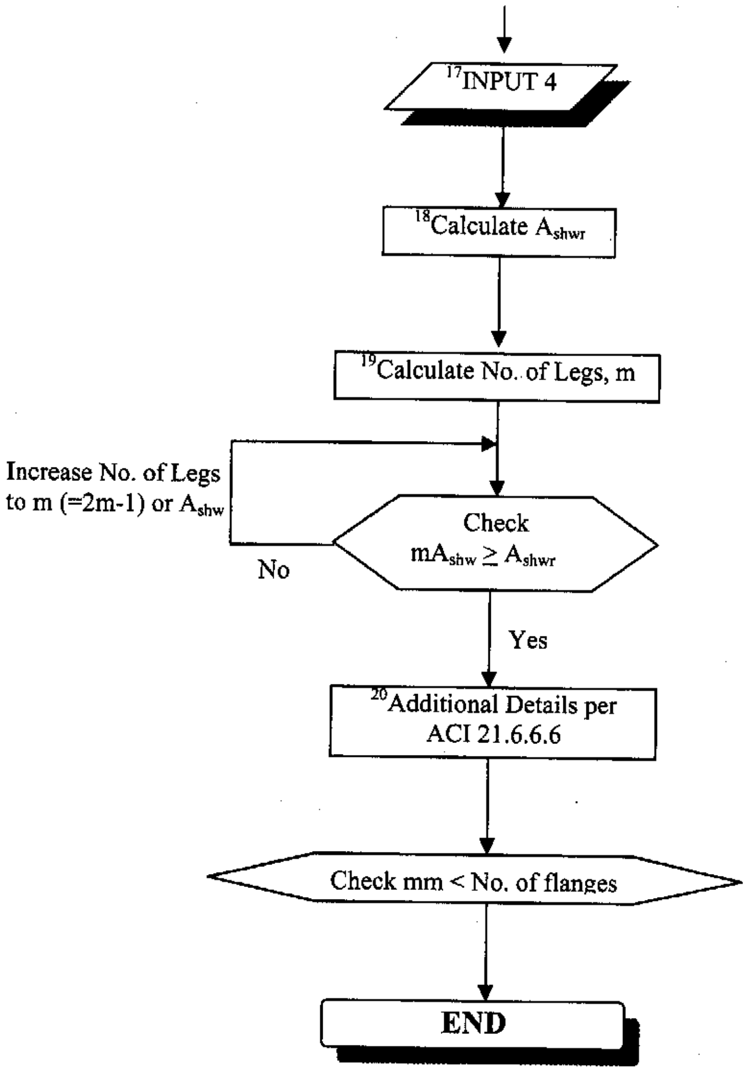



**Figure 2-4. Flow chart of shear wall design (Part 3)**

**1. Input 1:**

| Axial | Shear Force | Bending Moment | Due to |
|---|---|---|---|
| $P_D$ | $V_D$ | $M_D$ | DL |
| $P_L$ | $V_L$ | $M_L$ | LL |
| $P_{EQ}$ | $V_{EQ}$ | $M_{EQ}$ | Earthquake |
| $P_S$ | $V_S$ | $M_S$ | Snow load |

Base shear V (story shear)
Length of wall, $l_w$
Ground floor area, $A_B$
Seismic coefficient, $C_a$
Importance factor, I
Total height of wall, $h_w$

**2. Design load combinations**

i) $U = 1.4D + 1.7L$ (ACI Eq. 9-1)
ii) $U = 1.2D + f_1L + 1.0E$ (Formula 16-5)
iii) $U = 0.9D \pm 1.0E$ (Formula 16-6)

where: D = dead load effect
L = live load effect
$f_1 = 0.5$ (Sec. 1605.2)
$E = \rho Q_E + 0.2S_{DS}D$ when the effects of gravity and seismic loads are additive (Eqn. 16-28)
$E = \rho Q_E - 0.2S_{DS}D$ when the effects of gravity and seismic loads counteract (Eqn. 16-29)

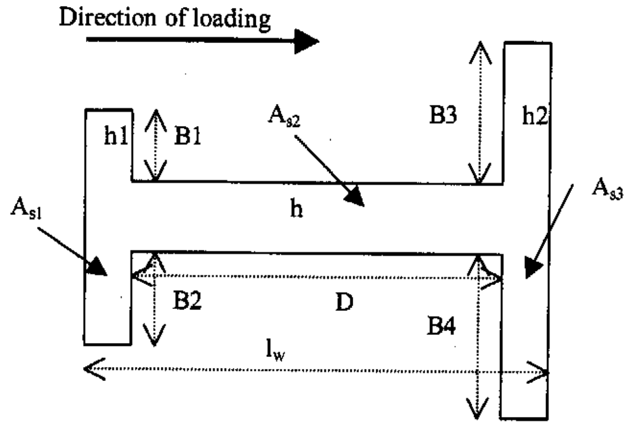
$Q_E$ = the effect of horizontal seismic forces
$\rho$ = a reliability factor based on system redundancy

**3. Input 2:** (see Figure below)

B1, B2, B3, B4 = widths of overhanging flanges as shown, in.
h = thickness of shear wall web, in.
h1, h2 = thickness of flanges, in.
$l_w$ = total length of shear wall, in.
$f'_c$ = compressive strength of concrete, ksi
$A_{s1}$ = cross-sectional area of vertical bars in flange 1, sq. in.
$A_{s2}$ = cross-sectional area of vertical bars in flange 2, sq. in.
$A_v$ = cross-sectional area of one vertical bar in shear wall web, sq. in.
$s_v$ = horizontal spacing of vertical bars in shear wall web, in. (should be less than 18 in.)
$A_n$ = cross-sectional area of one horizontal bar, sq. in.
$s_n$ = horizontal spacing of horizontal bars, in. (should be less than 18 in.)
$L_d$ = development length of horizontal bars in shear wall web, sq. in.
$h_w$ = height of shear wall, ft

{Note 1: Make sure that $\frac{A_{s1}}{h1 \times (B1 + B2 + h)}$ and $\frac{A_{s2}}{h2 \times (B3 + B4 + h)}$ do not exceed 0.03 from a practical viewpoint}

**Figure 2-4. Flow chart of shear wall design (Part 4)**




**Shear wall cross-section considered**

**4. Check strength under flexure and axial load**

Determine bending moment-axial load interaction diagram and check that all load combination points representing required strength are within the design strength envelope. Also, determine c corresponding largest neutral axis depth consistent with the design displacement $\delta_u$.

**5. Shear wall sectional properties**

$A_{cv}$ = area of shear wall resisting shear force = h x $l_w$
$b_{b1}$ = width of flange 1 = B1 + B2 + h
$b_{b2}$ = width of flange 2 = B3 + B4 + h
D = clear distance between the columns/walls = $l_w$ – h1 – h2

**6. Calculation of reinforcement ratio $\rho_n$, $\rho_v$**

$\rho_n = (A_n \text{ x no. of curtains})/(s_n \text{ x } h) \geq 0.0025$
$\rho_v = (A_v \text{ x no. of curtains})/(s_v \text{ x } h) \geq 0.0025$

{Note 2: If $h_w/l_w \leq 2$, provide $A_v s_v \geq A_n/s_n$}

**7. Calculation of coefficient $\alpha_c$**

$2 \leq \alpha_c = 6 - 2\, h_w/l_w \leq 3$

**8. Calculation of nominal shear strength**

$V_n = A_{cv}\,(\alpha_c \sqrt{f'_c} + \rho_n f_y)$

**9. Calculation of shear strength reduction factor $\phi$**

If $h_w/l_w \geq 2.0$ use $\phi = 0.85$ else
If $M_n/(2h_w/3) < V_n$ use $\phi = 0.85$ else
Use $\phi = 0.60$

**10. Satisfy the following requirements when $V_u \leq A_{cv}\sqrt{f'_c}$**

(i) Provide $\rho_v \geq 0.0012$ if using #5 bar or smaller, else $\rho_v \geq 0.0015$
(ii) Provide $\rho_n \geq 0.0012$ if using #5 bar or smaller, else $\rho_n \geq 0.0015$
(iii) Satisfy ACI Sec. 14.3.4
(iv) Provide $s_n$ and $s_v$ < 18 in. or 3h, whichever is smaller
(v) Lateral ties are not needed if $\rho_v \leq 0.010$

**11. Calculation of axial load capacity at zero eccentricity, $P_o$**

Calculate gross area, $A_g = l_w * h + h1*(B1+B2) + h2*(B3+B4)$

**Figure 2-4. Flow chart of shear wall design (Part 5)**

Steel area in web, $A_{s3} = A_v * (D+2L_d)/s_v$ * Number of curtains
Total steel area, $A_{st} = A_{s1} + A_{s2} + A_{s3}$
$P_o = 0.85f'_c (A_g - A_{st}) + f_y A_{st}$

**12. Calculation of c (distance from the extreme compression fiber to N.A.)**

$$c \geq c_r = \frac{l_w}{600(\delta_u / h_w)}$$

where $\frac{\delta_u}{h_w} \geq 0.007$

**13. Length of boundary zone by Displacement-based approach**

$h_{BZ} \geq l_w$
$\geq M_u/4V_u$

$L_{BZ} \geq c/2$
$\geq c - 0.1 l_w$

**14. Input 3: Flange/BZ Detailing**

$A_{sh1}$ = area of one leg of confining hoop (provided) in flange 1, sq. in.
$n_{h1}$ = number of legs of confining hoops in flange 1, in.
(considering lateral support for each alternate main rebars)
$s_1$ = vertical spacing of confining hoops in flange 1, in.
where $s \leq 6d_b$
$\leq$ Min. (D, B)/4
$\leq s_x$ where $s_x = 4 \leq 4+(14-h_x)/3 \leq 6$
$d_{h1}$ = diameter of confining hoop (provided) in flange 1, sq. in.
$A_{sh2}$ = area of one leg of confining hoop (provided) in flange 2, sq. in.
$n_{h2}$ = number of legs of confining hoops in flange 2, in.
(considering lateral support for each alternate main rebars)
$s_2$ = vertical spacing of confining hoops in flange 2, in.
where $s \leq 6d_b$
$\leq$ Min. (D, B)/4
$\leq s_x$ where $s_x = 4 \leq 4+(14-h_x)/3 \leq 6$
$d_{h2}$ = diameter of confining hoop (provided) in flange 2, sq. in.
d' = clear cover to hoops in both flanges, in.
$l_u$ = unsupported height of shear wall, in.
mm = an integer to check number of iterations

**15. Calculation of area of confining hoops in flange 1, $A_{shr1}$**

$A_{shr1} = 0.09\, s_1 h_{c1} f'_c/f_{yh}$
Where $h_{c1} = h1 - 2d' - 2d_{h1}$

**16. Calculation of area of confining hoops in flange 2, $A_{shr2}$**

$A_{shr2} = 0.09\, s_2 h_{c2} f'_c/f_{yh}$
Where $h_{c2} = h2 - 2d' - 2d_{h2}$

**17. Input 4: Wall Detailing**

$A_{shw}$ = area of one cross-ties in web, sq. in.
s = vertical spacing of cross-ties in web, in.
$s \leq 6d_b$
$\leq$ Min. (D, B)/4
$\leq s_x$ where $s_x = 4 \leq 4+(14-h_x)/3 \leq 6$

$d_h$ = diameter of horizontal bars in web, in.

**Figure 2-4. Flow chart of shear wall design (Part 6)**

d' = clear cover to horizontal bars, in.

**18. Calculation of area of cross-ties in web, $A_{shwr}$**

$A_{shwr} = 0.09\, sh_c\, f'_c/f_y$

Where $h_c = h - 2d' - 2d_h$

**19. Calculation of number of cross-ties in web, m (as an integer)**

$m = L_{BZW}/2s_v + 1$

**20. Additional details as per ACI Sec. 21.6.6.6**

**Figure 2-4. Flow chart of shear wall design (Part 7)**

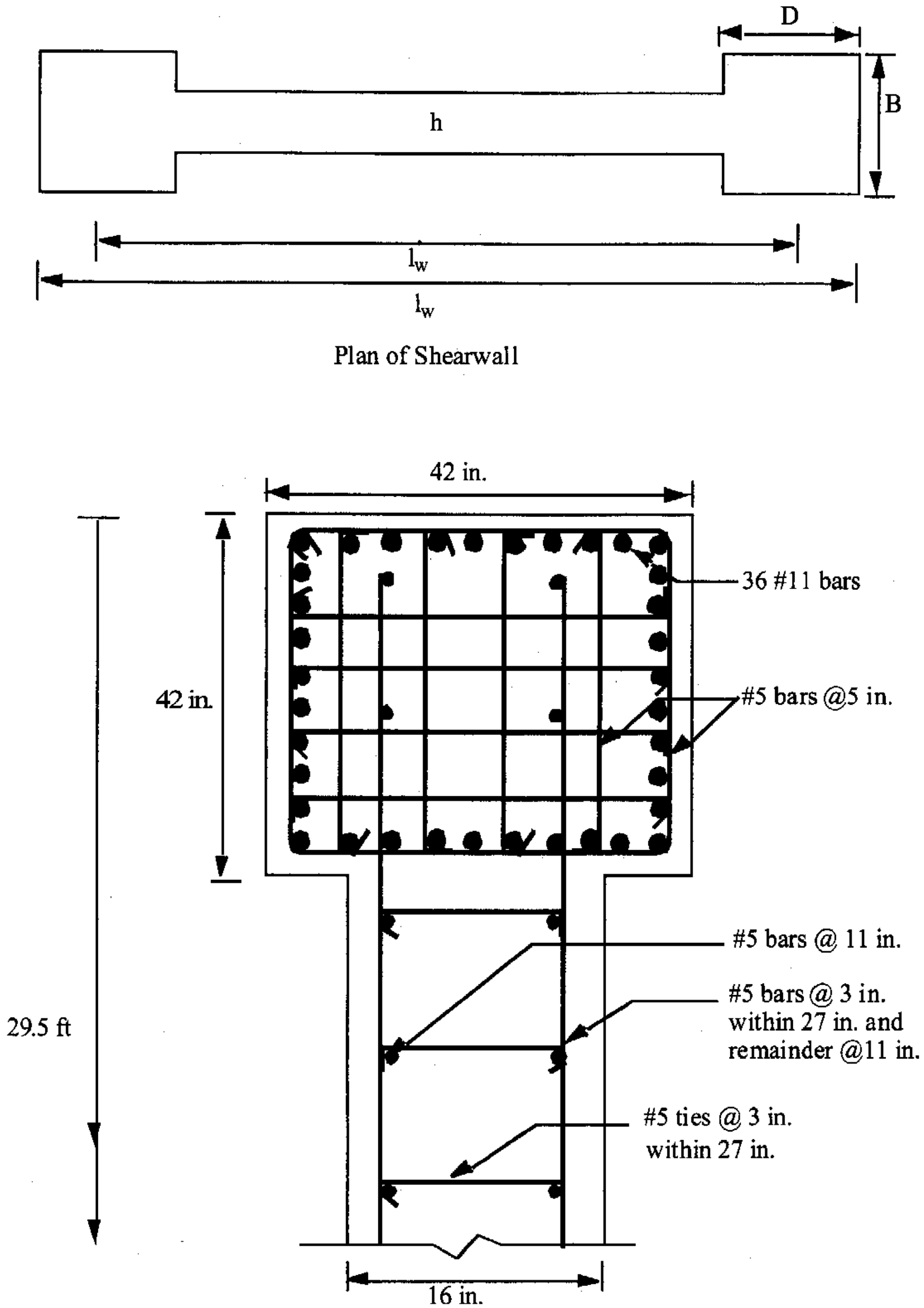



**Figure 2-5. Reinforcement details for shear wall C2-D2 (between grade and level 2) by static force procedure**

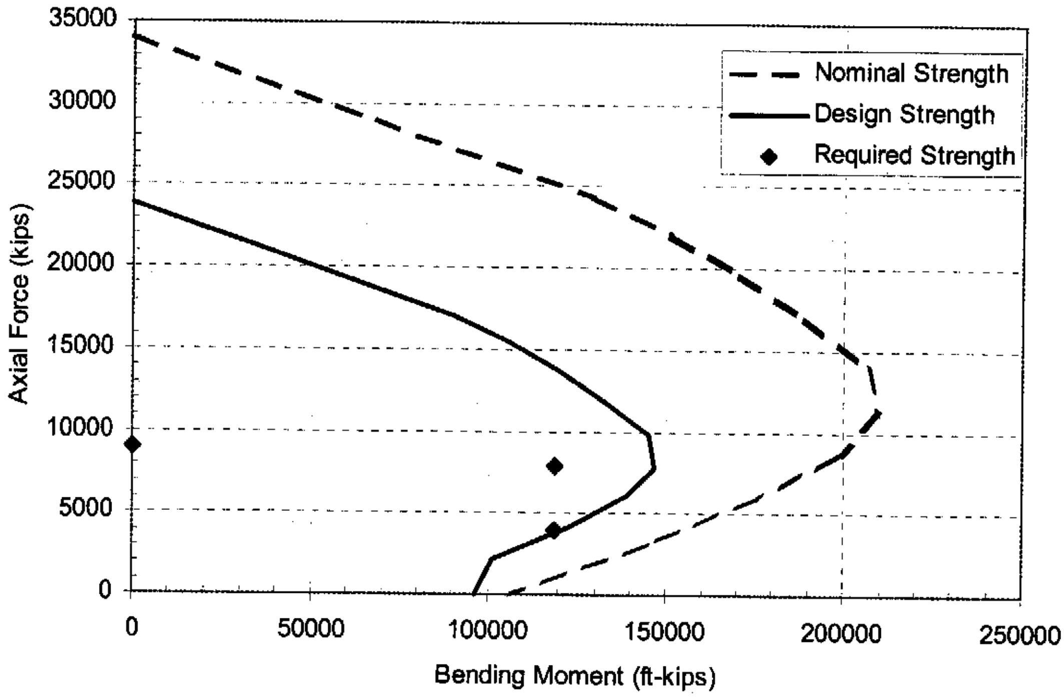



**Figure 2-6. Design strength interaction diagram for shear wall (equivalent lateral force procedure)**

**<u>Building</u>** **<u>Office</u>** **Seismic Design Category** **D**

[redacted]

| | | | | | |
|---|---|---|---|---|---|
| See Fig. | **6** | **O.K.** | $\delta_u$ = | 56.6 in. along SW line | same as 1.4 $\delta_x$ |
| Pu' | **7952** kips | | Pu | 7952 kips | max. in conjunction with lateral load |
| Mn' | NA ft-kips | | Vu | 1571 kips | |
| c | **104** in. | | Mu | 118,596 ft-kips | |
| Note: | Neutral axia depth c & Nominal Moment Mn' correspond to Max. Axial Force Pu' | | | | |
| Note: | c should correspond to maximum axial force (preferably in presence of lateral force) | | | | |

[redacted]

| | | | | | |
|---|---|---|---|---|---|
| | | | $\delta_{xe}$ = | 6.22 in. along SW line | from Drift Table |
| hw | 255 ft | | fc' | 4 ksi | |
| D | 42 in. | | fy | 60 ksi | |
| B | 42 in. | | Layers | 10 | in boundary element |
| L=lw' | 26 ft | | # bars | 36 | in boundary element |
| h | 16 in. | | Bar size # | 11 | in boundary element |
| lw | 29.50 ft | | 1 Bar area | 1.56 sq.in. | |
| hw/lw | 8.64 | | cover = | 2.5 in. | |
| $\phi$ | 0.85 | | As-total | 56.16 sq.in. | in boundary element |
| Acv | 5664 sq.in. | | $\rho$ = | 3.18 % | in boundary element |
| $\phi$8Acvfc'^.5 | 2436 kips | > | 1571 kips (Vu) | | O.K. |
| 2Acvfc'^.5 | 716 kips | < | 1571 kips (Vu) | Provide minimum | 2 curtains |
| Acvfc'^.5 | 358 kips | < | 1571 kips (Vu) | Wall Design by 21.6.2.1 | |

[redacted]

| | | | | | |
|---|---|---|---|---|---|
| **Wall reinforcement** | | | | | |
| Layers | 2 in horizontal direction | with area of one bar = | 0.31 sq.in. | | #5 |
| Layers | 2 in vertical direction | with area of one bar = | 0.31 sq.in. | | #5 |
| Spacing (in.)= | 11 in horizontal direction | | **Spacing must be ≤ 18 in.** | | |
| Spacing (in.)= | 11 in vertical direction | | **Spacing must be ≤ 18 in.** | | |
| $\rho_n$ = | 0.0035 reinforcement ratio in hor. direction | | > | **0.0025** | O.K. |
| $\rho_v$ = | 0.0035 reinforcement ratio in vertical direction | | > | **0.0025** | O.K. |
| | | | **If hw/lw < 2, ρv > ρn** | | |
| $\alpha_c$ = | 2 | | **$\alpha_c$ ≤ 3** | | |
| Vn = Vc+Vs = | 1914 kips | Vc = | 716 kips | Vs = | 1197 |
| $\phi$Vn = | 1627 kips | > | 1571 kips (Vu) | | O.K. |

| | | | | | |
|---|---|---|---|---|---|
| [redacted]**ure and Axial Load** | **by Displacement-Based Approach** | | (21.6.6.2) | | |
| $\delta_u$/hw = | 0.0185 | **Must be ≥ 0.007** | So, use | $\delta_u$/hw = | 0.0185 |
| $c_r$ = | **31.9** in. | < | **104** in. (=c), So | BZ Details Needed | |
| $c_r$ = lw/600($\delta$u/hw) | | | | | |

**Figure 2-7. Shear wall design by IBC 2000 (High Seismic Zones) (equivalent lateral force procedure)**

| | | | | | |
|---|---|---|---|---|---|
| **BZ DETAILS** | | ***Disregard this section if BZ Details are not needed*** | | | |
| Ht. of BE > | 354 in. | | | | |
| > | 226 in. | **Height of Boundary Element =** | | **354 in.** | |
| Lbz > | 52 in. | | | | |
| > | 68.6 in. | **Length of Boundary Element =** | | **68.6 in.** | |
| | | Confine BE & Part of Wall of Length | | **26.6 in.** | |
| | | **Provide wall confinement over (in.)** | | **27.0** | |
| **Transverse reinforcemnt** | | | | | |
| hoop# | 5 | dh= | 0.625 in. | | |
| hoop/ties area | 0.31 sq.in. | | | | |
| # of legs = | 6 | 1 hoop + | 4 cross-ties | **Should be ≥** | 6 |
| cover = | 1.5 in. | | | | |
| hc = | 38.4 in. | Hor. spacing of ties/hoops (in.)= | | 4.11 < 14 in. | O.K |
| s1=min(B,D)/4 | **10.5 in.** | & < 6db= | **8.25 in. & ≤ sx=** | **7.30** should be **≤ 6 in.** | |
| s= | 6 in. | | | | |
| Ash (min) = | 1.382 sq.in. | | | | |
| Ash(provided)= | **1.86** sq.in. | > | **1.382** sq.in. (needed) | | O.K. |

| | | | | | |
|---|---|---|---|---|---|
| **WEB CONFINEMENT** | | ***Disregard this section if web Details are not needed*** | | | |
| hc1= | 12.4 in. | | | | |
| hc2= | 27.0 in. | Hor. spacing of ties/hoops (in.)= | | 11 | |
| s1=min(h,hc2)/4 | **4.0** in. | & < 6db= | **3.75** in. & ≤ sx= | **5.00** should **≤ 6 in.** | |
| s= | 3 in. | Provide # of ties = | 2 perpendicular to wall | | |
| Ash1 (min) = | 0.223 sq.in. | Ash2 (min) = | 0.486 sq.in. | | |
| Ash1(provided)= | 0.62 sq.in. | > | **0.223** sq.in. (needed) | | O.K. |
| Ash2(provided)= | 0.62 sq.in. | > | **0.486** sq.in. (needed) | | O.K. |

| | | | | | |
|---|---|---|---|---|---|
| **Minimum Confinement** | ***Disregard this section if BZ Details are needed*** | | | | IGNORE |
| Longitudinal reinforcement | **0.0318** greater than 300/fy | **0.0067** | **YES** | | IGNORE |
| Lbz (min.)= | **68.6 in.** | : is maximum of c/2 & c - 0.1lw | | | IGNORE |
| | Confine only Boundary Element | | Although 21.6.6.4(a) => Longer Length | | IGNORE |
| Provide Transverse reinforcement @ | **8** in. spacing | | | | IGNORE |
| Cross-ties @ | 14 in. spacing Use bar# | 5 | horizontally | | IGNORE |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| FINAL RESULT: | Use Boundary Zone BxD= | 42 | 42 in. with | 36 | # 11 | ρ = | 3.18 % |
| | Use confining hoops of # | 5 | @ a spacing of | | 6 in. | | |
| Wall: | thickness= | 16 in. with | #5 | 2 layers | @ | 11 in. horizontally | |
| | | | #5 | 2 layers | @ | 11 in. vertically | |
| | Provide confinement in web: | | | 2 #5 | cross-ties | | |

**Figure 2-7. Shear wall design by IBC 2000 (High Seismic Zones)–Continued**

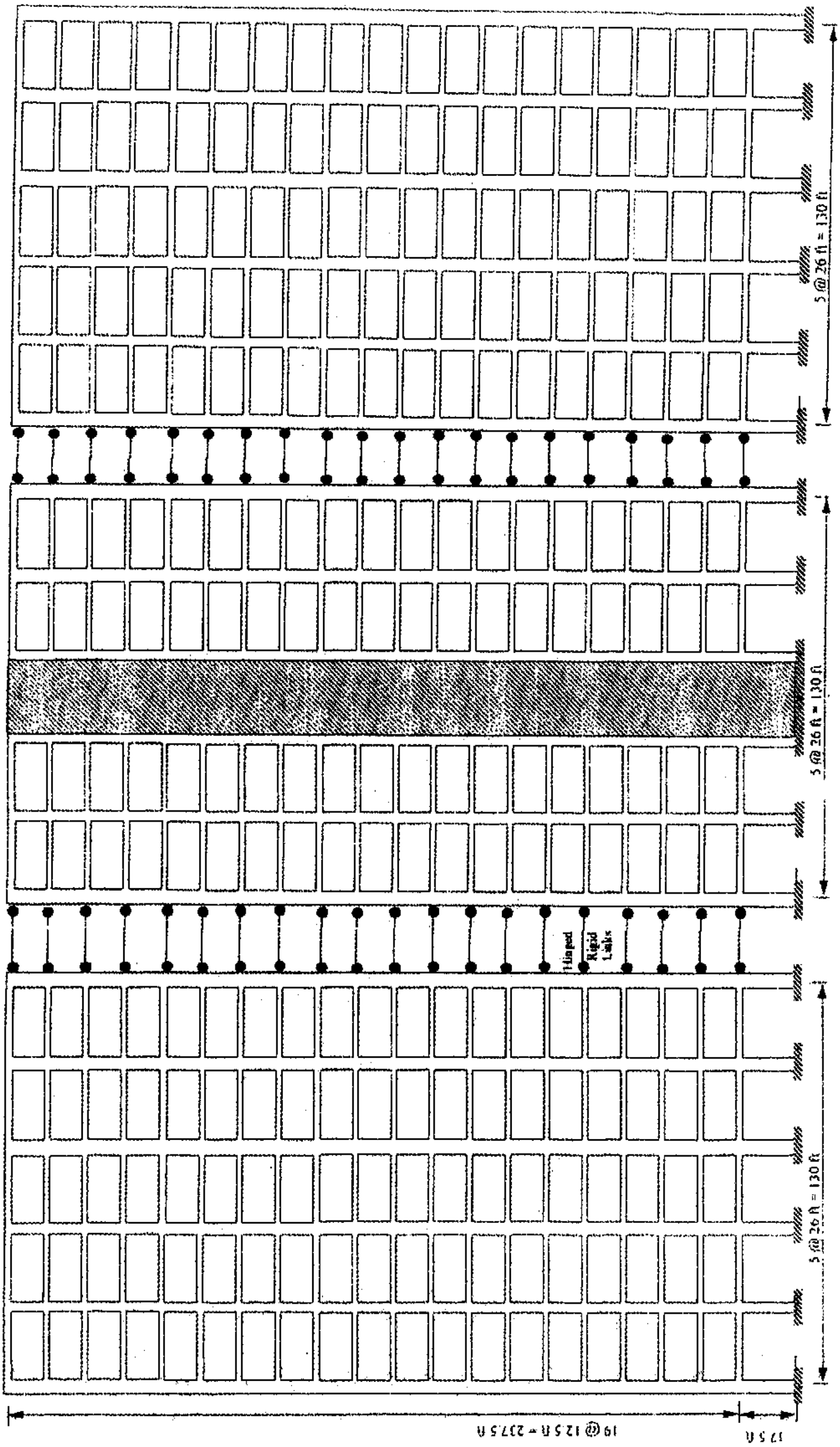



**Figure 2-8. Two-dimensional modeling of example building**

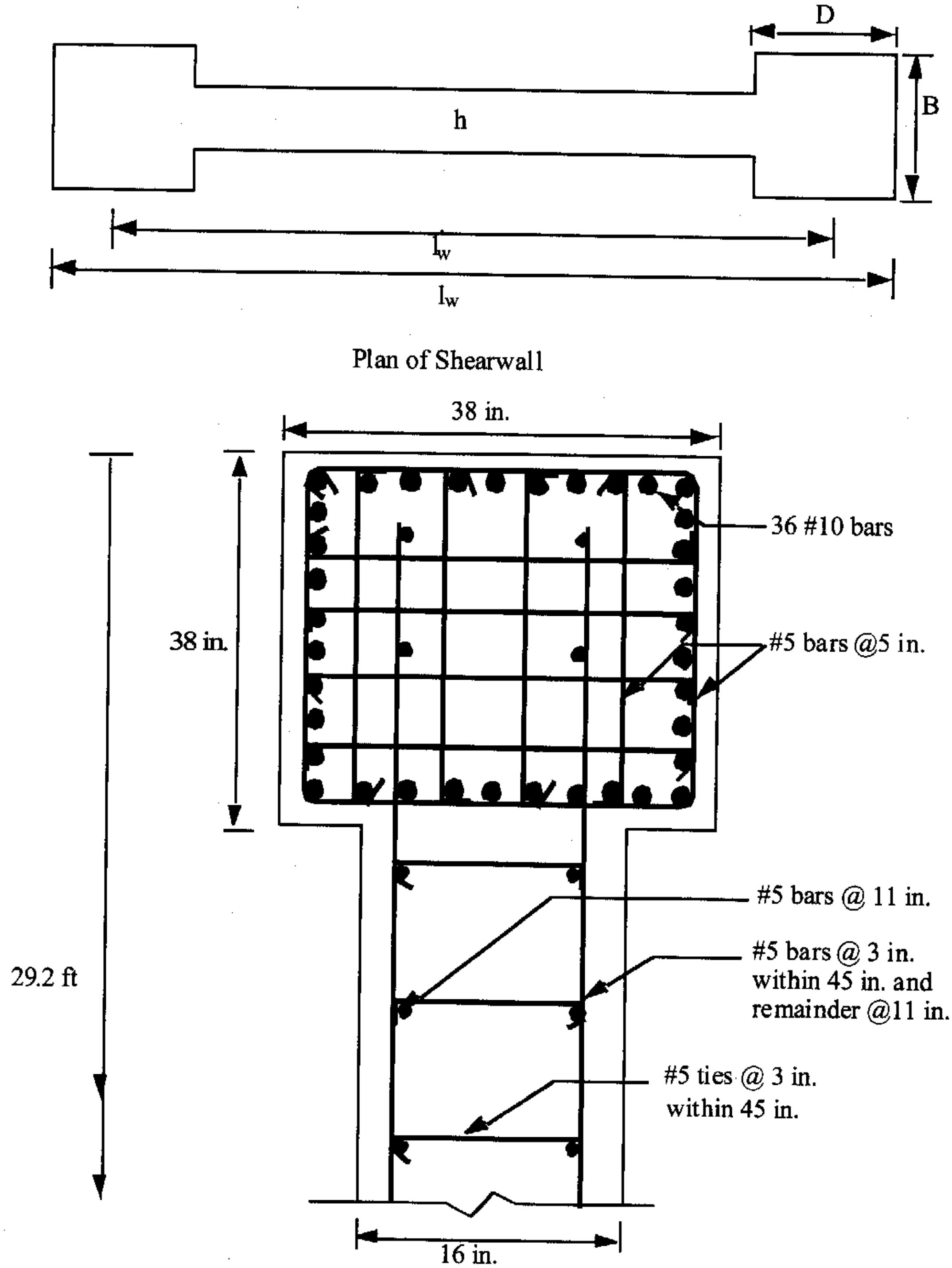



**Figure 2-9. Reinforcement details for shear wall C2-D2 (between grade and level 2) by dynamic procedure**

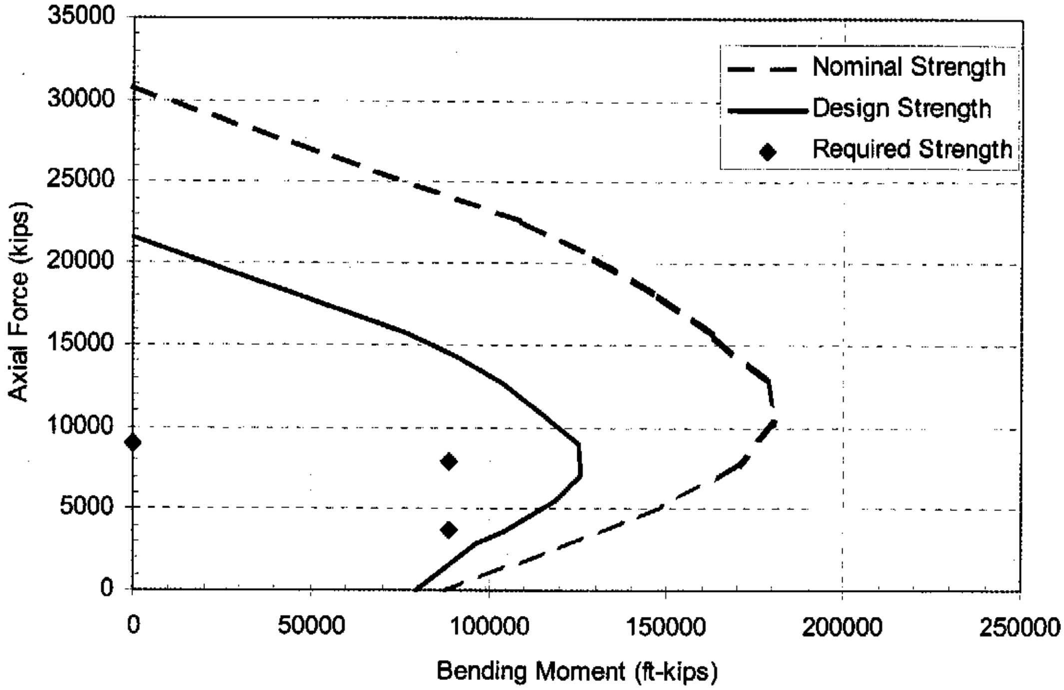



**Figure 2-10. Design strength interaction diagram for shear wall (dynamic procedure)**

| **Building** | **Office** | **Seismic Design Category** | **4D** |
|---|---|---|---|

## Check Strength for Mu and Pu

| | | | | | | |
|---|---|---|---|---|---|---|
| See Fig. | **11** | **O.K.** | $\delta_u$ = | 26.7 | in. along SW line | same as 1.4 $\delta_x$ |
| Pu' | **7952** | kips | Pu | 7952 | kips | max. in conjunction with lateral load |
| Mn' | NA | ft-kips | Vu | 1419 | kips | |
| c | **118** | in. | Mu | 88,920 | ft-kips | |

Note: Neutral axia depth c & Nominal Moment Mn' correspond to Max. Axial Force Pu'

Note: c should correspond to maximum axial force (preferably in presence of lateral force)

## Shear Design for Vu

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | $\delta_{xe}$ = | 4.56 | in. along SW line | from Drift Table |
| hw | 255 | ft | fc' | 4 | ksi | |
| D | 38 | in. | fy | 60 | ksi | |
| B | 38 | in. | Layers | 10 | | in boundary element |
| L=lw' | 26 | ft | # bars | 36 | | in boundary element |
| h | 16 | in. | Bar size # | 10 | | in boundary element |
| lw | 29.17 | ft | 1 Bar area | 1.27 | sq.in. | |
| hw/lw | 8.74 | | cover = | 2.5 | in. | |
| $\phi$ | 0.85 | | As-total | 45.72 | sq.in. | in boundary element |
| Acv | 5600 | sq.in. | $\rho$ = | 3.17 | % | in boundary element |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\phi$8Acvfc'^.5 | 2408 | kips | > | 1419 | kips (Vu) | | O.K. |
| 2Acvfc'^.5 | 708 | kips | < | 1419 | kips (Vu) | Provide minimum | 2 curtains |
| Acvfc'^.5 | 354 | kips | < | 1419 | kips (Vu) | Wall Design by 21.6.2.1 | |

**Wall reinforcement**

| | | | | | | |
|---|---|---|---|---|---|---|
| Layers | 2 | in horizontal direction | with area of one bar = | 0.31 | sq.in. | #5 |
| Layers | 2 | in vertical direction | with area of one bar = | 0.31 | sq.in. | #5 |
| Spacing (in.)= | 11 | in horizontal direction | **Spacing must be ≤ 18 in.** | | | |
| Spacing (in.)= | 11 | in vertical direction | **Spacing must be ≤ 18 in.** | | | |
| $\rho_n$ = | 0.0035 | reinforcement ratio in hor. direction | > | **0.0025** | O.K. | |
| $\rho_v$ = | 0.0035 | reinforcement ratio in vertical direction | > | **0.0025** | O.K. | |
| | | | **If hw/lw < 2, ρv > ρn** | | | |
| $\alpha_c$ = | 2 | | **$\alpha_c$ ≤ 3** | | | |
| Vn = Vc+Vs = | 1892 | kips | Vc = 708 kips | Vs = | 1184 | |
| $\phi$Vn = | 1608 | kips | > 1419 kips (Vu) | | | O.K. |

## [redacted]ure and Axial Load

**by Displacement-Based Approach** (21.6.6.2)

| | | | | | |
|---|---|---|---|---|---|
| $\delta_u$/hw = | 0.0087 | **Must be ≥ 0.007** | So, use | $\delta_u$/hw = | 0.0087 |
| $c_r$ = | **66.9** in. | < | **118** in. (=c), So | BZ Details Needed | |
| $c_r$ = lw/600($\delta$u/hw) | | | | | |

**Figure 2-11. Shear wall design by IBC 2000 (High Seismic Zones)**

| BZ DETAILS | | | Disregard this section if BZ Details are not needed | | | |
|---|---|---|---|---|---|---|
| Ht. of BE > | 350 | in. | | | | |
| > | 188 | in. | Height of Boundary Element = | | 350 in. | |
| Lbz > | 59 | in. | | | | |
| > | 83.0 | in. | Length of Boundary Element = | | 83.0 in. | |
| | | | Confine BE & Part of Wall of Length | | 45.0 in. | |
| | | | Provide wall confinement over (in.) | | 45.0 | 12*4+2 |
| **Transverse reinforcemnt** | | | | | | |
| hoop# | 5 | | dh= | 0.625 in. | | |
| hoop/ties area | 0.31 | sq.in. | | | | |
| # of legs = | 6 | | 1 hoop + | 4 cross-ties | Should be ≥ | 6 |
| cover = | 1.5 | in. | | | | |
| hc = | 34.4 | in. | Hor. spacing of ties/hoops (in.)= | | 3.67 < 14 in. | O.K. |
| s1=min(B,D)/4 | **9.5** | in. | & < 6db= | **7.5** in. & ≤ sx= | **7.44** | should ≤ **6 in.** |
| s= | 6 | in. | | | | |
| Ash (min) = | 1.238 | sq.in. | | | | |
| Ash(provided)= | **1.86** | sq.in. | > | **1.238** sq.in. (needed) | | O.K. |

| WEB CONFINEMENT | | | Disregard this section if web Details are not needed | | | |
|---|---|---|---|---|---|---|
| hc1= | 12.4 | in. | | | | |
| hc2= | 45.0 | in. | Hor. spacing of ties/hoops (in.)= | | 11 | |
| s1=min(h,hc2)/4 | **4.0** | in. | & < 6db= | **3.75** in. & ≤ sx= | **5.00** | should ≤ **6 in.** |
| s= | 3 | in. | Provide # of ties = | | 3 | perpendicular to wall |
| Ash1 (min) = | 0.223 | sq.in. | Ash2 (min) = | 0.810 sq.in. | | |
| Ash1(provided)= | 0.62 | sq.in. | > | **0.223** sq.in. (needed) | | O.K. |
| Ash2(provided)= | 0.93 | sq.in. | > | **0.810** sq.in. (needed) | | O.K. |

| Minimum Confinement | | Disregard this section if BZ Details are needed | | | | |
|---|---|---|---|---|---|---|
| Longitudinal reinforcement | **0.0317** | greater than 300/fy | **0.0067** | YES | | IGNORE |
| Lbz (min.)= | **83** in. | : is maximum of c/2 & c - 0.1lw | | | | IGNORE |
| | Confine only Boundary Element | | Although 21.6.6.4(a) => Longer Length | | | IGNORE |
| Provide Transverse reinforcement @ | | **8** in. spacing | | | | IGNORE |
| Cross-ties @ | | 14 in. spacing | Use bar# 5 | horizontally | | IGNORE |

| FINAL RESULT: | | | | | | |
|---|---|---|---|---|---|---|
| Use Boundary Zone BxD= | 38 | 38 in. with | 36 | # 10 | ρ = | 3.17 % |
| | Use confining hoops of # | 5 | @ a spacing of | 6 in. | | |
| Wall: | thickness= | 16 in. with #5 | 2 layers | @ | 11 in. horizontally | |
| | | #5 | 2 layers | @ | 11 in. vertically | |
| | | Provide confinement in web: | 3 #5 | cross-ties | | |

**Figure 2-11. Shear wall design by IBC 2000 (High Seismic Zones) (Continued)**

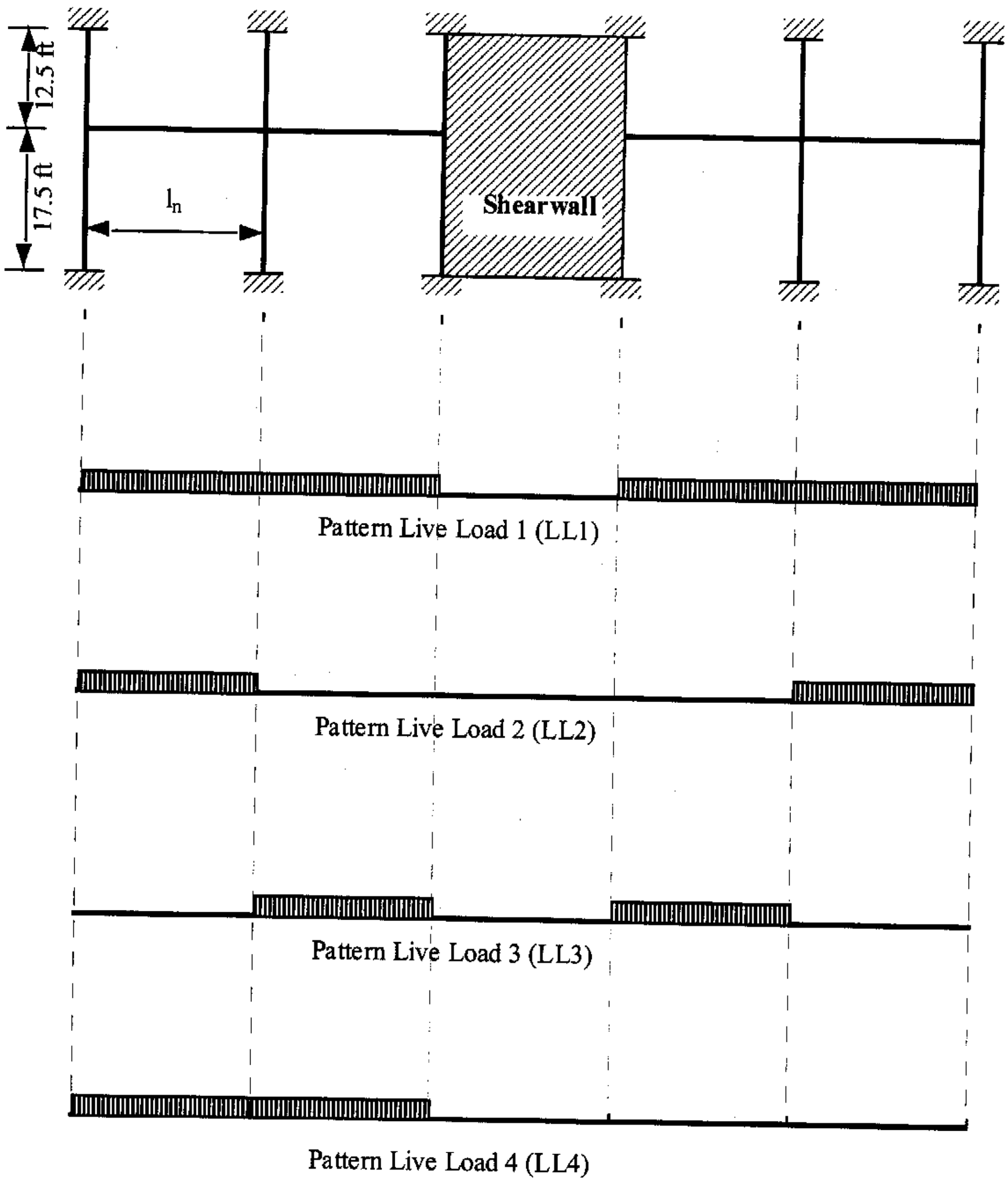



**Figure 2-12. Calculation of forces due to pattern live loads**

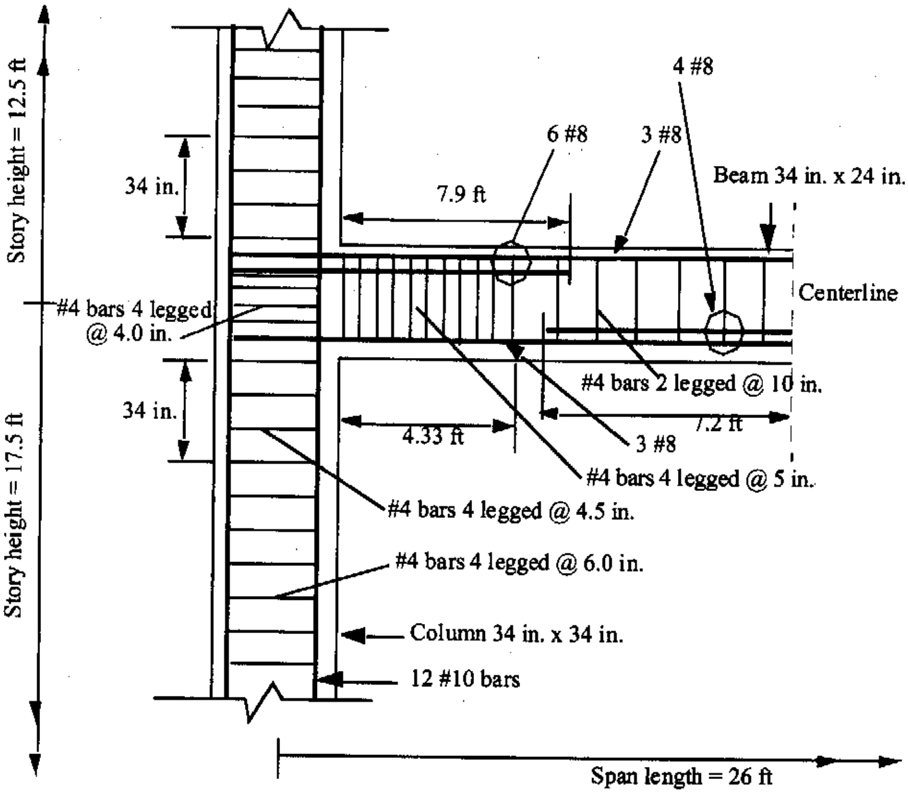



**Figure 2-13. Reinforcement arrangements in exterior beam A2-B2, column 3 (between grade and floor 2) and column 4 (between floor 2 and floor 3)**

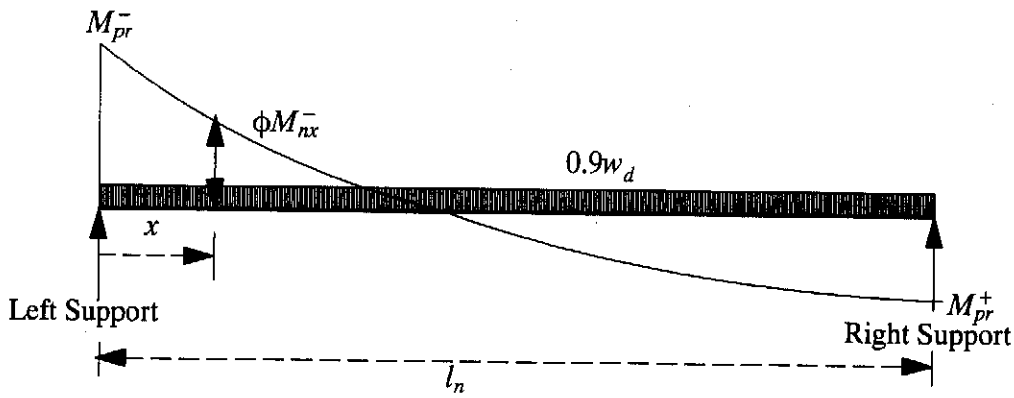



**Figure 2-14(a). Moment diagram for cut-off location of negative bars at support section of beams**

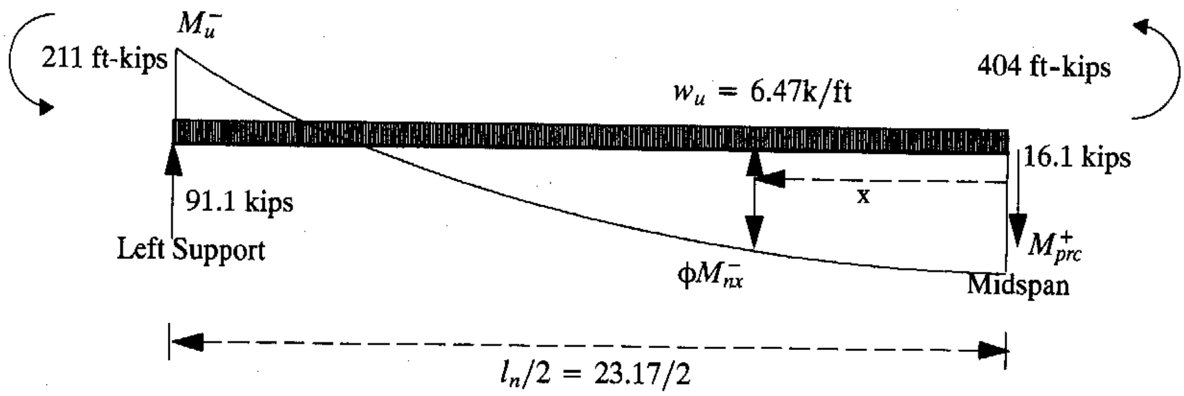



**Figure 2-14(b). Moment diagram for cut-off location of positive bars at support section of beams**

## BEAM DESIGN Exterior IBC 2000 Site Class D

### Design Data:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Span | (l) | = | 26 ft. | | | | |
| Column width | (Bc) | = | 34 in. | | | | |
| Column depth | (Dc) | = | 34 in. | | | | |
| Beam width | (Bb) | = | 34 in. | | | | |
| Beam depth | (Db) | = | 24 in. | | | | |
| Net concrete cover | (c') | = | 2.5 in. | | | | |
| Effective beam depth | (d) | = | **21.5** in. | | | | |
| Strength of concrete | (f'c) | = | 4000 psi | | | | |
| Strength of steel | (fy) | = | 60000 psi | | | | |
| Clear span | (ln) | = | **23.17** ft. | | | | |
| Unif. dist. dead load | (wd) | = | 3.61 kips/ft | | | | |
| Unif. dist. live load | (wl) | = | 0.832 kips/ft | | | | |
| Total unif. dist. load | (w) | = | **4.44** kips/ft | | | | |
| Req. flexural strength at support (Mu1-) | | = | 400 ft-kips | <= Table | 2.17 | | |
| Req. flexural strength at support (Mu1+) | | = | 48 ft-kips | <= Table | 2.17 | | |
| Req. flexural strength at Midspan (Mu2+) | | = | 240 ft-kips | <= Table | 2.17 | | |
| Req. shear strength at support (Vu) | | = | 82 kips | <= Table | 2.17 | | |
| Req. shear strength at midspan (Vu) | | = | 10 kips | <= Table | 2.17 | | |
| Axial force (Pu) | | = | 0 kips | <= Table | 2.17 | | |
| Try reinf. at bottom near support | (As1+) | = | **2.37** sq.in. | 3 # | | 8 bars, each of | 0.79 sq.in. |
| Try reinf. at top near support | (As1-) | = | **4.74** sq.in. | 6 # | | 8 bars, each of | 0.79 sq.in. |
| Try reinf. at bottom at midspan | (As2+) | = | **3.16** sq.in. | 4 # | | 8 bars, each of | 0.79 sq.in. |
| Try reinf. at top at midspan | (As2-) | = | **2.37** sq.in. | 3 # | | 8 bars, each of | 0.79 sq.in. |

### Solution:

**1. According to ACI Sec. 21.3.1., flexural members shall satisfy the following conditions:**

| | | | | | | |
|---|---|---|---|---|---|---|
| Sec. 21.3.1.1. | Pu ≤ 0.1*Ag f'c | (Ag = gross cross-sectional area) | | | | |
| | Pu = | 0 | ≤ | **326.4** kips | O.K. | |
| Sec. 21.3.1.2. | ln ≥ 4d | 23.17 | ≥ | **7.17** ft. | O.K. | |
| Sec. 21.3.1.3. | Bb/Db ≥ 0.3 | 1.42 | between | 0.3 and 3.33 | O.K. | |
| Sec. 21.3.1.4. | Bb ≥ 10 | 34 | ≥ | **10** in. | O.K. | |
| | Bb ≤ Bc + 1.5xDb | 34 | ≤ | **70** in. | O.K. | |

**2. ACI Sec. 21.3.2. (Longitudinal reinforcement)**

Sec. 21.3.2.1.

| | | | | | | |
|---|---|---|---|---|---|---|
| As(top) or As (bot) ≥ 3sqrt.f'cxBbxd/fy | **2.37** | ≥ | **2.31** | sq.in. | O.K. | |
| As(top) or As (bot) > 200Bbxd/fy | **2.37** | ≥ | **2.44** | sq.in. | REVISE | O.K. as close |
| As(top) or As (bot) < 0.025Bbxd | **4.74** | ≤ | **18.275** | sq.in. | O.K. | |

**Figure 2-15. Design of exterior beam (A2-B2 at level 2) per IBC 2000 (Part 1)**

Provide No.of continuous bars 2

Sec. 21.3.2.2.

\+ ve moment strength at support

phi Mn = phi Asfy(d - Asfy/1.7f'cBb) | phi Mn1+ = | 223 | ≥ | 48 | ft-kips | O.K.

\+ ve moment strength at midspan

phi Mn = phi Asfy(d - Asfy/1.7f'cBb) | phi Mn2+ = | 294 | ≥ | 240 | ft-kips | O.K.

\- ve moment strength at support

phi Mn = phi Asfy(d - Asfy/1.7f'cBb) | phi Mn1- = | 432 | ≥ | 400 | ft-kips | O.K.

Make sure phiMn1+ at support≥ 50% of phiMn1- | 223 | ≥ | 50% of | 432 | ft-kips | O.K.

Min. phiMn (anywhere) > phiMn(-)/4 = | 108 ft-kips

Min. phi Mn = phi Asfy(d - Asfy/1.7f'cBb) = | 223 | ≥ | 108 | ft-kips | O.K.

**Shear strength (ACI Sec. 21.3.4)**

Sec. 21.3.4.1.

Mpr1- = 1.25Asfy(d - 1.25Asfy/1.7f'cBb). Mpr1 = | 591 ft-kips | with | 6 | # | 8 | bars

Mpr1+ = 1.25Asfy(d - 1.25Asfy/1.7f'cBb). Mpr2 = | 307 ft-kips | with | 3 | # | 8 | bars

**w=wd+wl =** | 4.44 kips/ft

ln = | 23.17 ft.

Ve = {(Mpr1-) + (Mpr1+)}/ln + wln/2 | Ve = | 90 kips

Sec. 21.3.4.2.

i. Shear caused by earthquake forces {(Mpr1-)+(Mpr1+)}/ln

39 | ≥ | Ve/2= | 45 | kips | No

ii. Factored axial force (taken as 0) ≤ 0.05Agf'c, So | can take Vc = | 92 | kips. Conservatively use Vc = | 0 | kips

<u>All the shear must be resisted by transverse reinforcement</u>

Shear to be resisted by steel, Vs = Vu/ϕ – Vc, | Vs = | 106 kips

Vu is taken as Ve | Ve = | 90 kips | ≥ | 82 kips | Found from analysis

Required spacing of # | 4 | stirrups | 4 legged | # of legs should be ≥ | 4

Av = | 0.8 sq.in. for | 4 legs

Spacing of hoops, s = Avfyd/Vs | s = | 9.72 in.

Each alternate bar needs to be supported (ACI Sec. 21.3.3.3.)

6 | # | 8 | at top

4 | legs of # | 4 | stirrups/ties

Sec. 21.3.3.2.

Spacing of hoops, s = Avfyd/Vs | 9.72

s ≤ d/4, | ≤ | 5.4 | in.

s ≤ 8 x diameter of smallest long bar | ≤ | 8.0 | in.

s ≤ 24 x diameter of hoop bars | ≤ | 12.0 | in.

s ≤ 12 in. | ≤ | 12.0 | in.

With first hoop at 2 in. from face of col. | use s= | 5 | in. o/c

Sec. 11.5.6.8.

Vs < 8sqrtf'c*Bbd | Vs = | 106 | ≤ | 370 | kips | O.K.

Sec. 21.3.3.1. Hoops shall be provided over a length of

i) 2Db = | 4.00 | ft. from face of support

ii) 2Db @ midspan if there is possibility of flexural yielding there

( in this case no flexural yilding at midspan as | ϕ Mn >> Mu )

Provide hoops near the face of the joint for length, Lob = | 4.0 ft.

S.F. there, Vx = Ve - w x Lob | Vx = | 72 kips

Taking | Vc = | 92 kips

Vs = Vx/0.85-Vc | Vs = | -7 kips

**Figure 2-15. Design of exterior beam per IBC 2000 (Part 2)**

**Cut off length**

| | | | | | |
|---|---|---|---|---|---|
| Prob. -ve flexural strength at support | Mpr1- = | **591** ft-kips | **6** | # | **8** bars |
| Prob. +ve flexural strength at support | Mpr1+ = | **307** ft-kips | **3** | # | **8** bars |
| -ve flexural strength at cut-off point | ϕ Mnx- = | **223** ft-kips | **3** | # | **8** bars |
| Prob. +ve flexural strength at midspan | Mpr2+ = | **404** ft-kips | **4** | # | **8** bars |
| +ve flexural strength at cut-off point | ϕ Mnx+ = | **223** ft-kips | **3** | # | **8** bars |

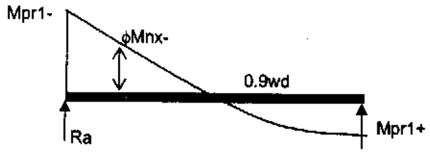
Note that no less than **3** # **8** bars are to be provided to satisfy Sec. 21.3.2.1

**Negative bar cut-off:**

Length of cut-off to be calculated by using a quadratic equation:

$ax^2 + b\,x + c = 0$

| | |
|---|---|
| 0.9wd= | 3.249 kips/ft |
| Ra= | 76.4 kips |
| a= | 1.62 |
| b= | -76.4 |
| c= | 369 |
| x-= | **5.46** ft. |




For Top Bars Out of **6** **3** bars can be cut off at (leaving remaining **3** bars)

x+d = **7.2** ft.

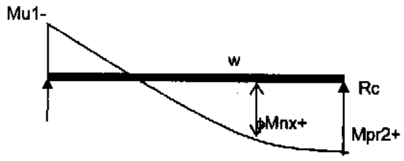
x+12db = **6.5** ft  Cut the bars @ **7.5** ft. from the support

**Positive bar cut-off:**

Length of cut-off to be calculated by using a quadratic equation:

$ax^2 + b\,x + c = 0$

| | |
|---|---|
| w = | 6.47 kips/ft |
| Rc= | -16.7 kips |
| a= | 3.235 |
| b= | 16.7 |
| c= | -182 |
| x+= | **5.35** ft. |




For Bottom Bars Out of **4** **1** bars can be cut off at (leaving remaining **3** bars)

x+d= **7.1** ft.

x+12db= **6.3** ft  Cut the bars @ **7.2** ft. from midspan

The cut off points should be beyond the confinement zone, i.e. **4.0** ft.

Provide cut off point at **7.2** ft.

**Shear reinforcement beyond** **4.0** ft.

Vs = **-7** kips conservatively

Number of legs 2 # **4** bars Av = 0.4 sq.in # of legs ≥ **2**

s ≤ Avfyd/Vs  s ≤ **NA** in.

s ≤ d/2  s ≤ **10.75** in.  Max. s = 10.75 **in.**

Provide **2** legged # **4** stirrups with seismic hooks @ **10** in. beyond the confinement zone  O.K

Spacing of stirrups (s) = **5** in. the confinement zone

**Development length**

Sec. 12.2.1.  (ld = development length; db = dia. of main bar)  Take ktr=0 (conservatively)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| c = | 2.5 in. | | | alpha = | 1 bottom bar | | |
| db = | 1 in. | | | alpha = | 1.3 top bar | | |
| Bottom Bars: ld = | 28.5 | db = | **2.37** ft. | < | **7.2** | ft. | O K |
| Top Bars: ld = | 37.0 | db = | **3.08** ft. | < | **7.5** | ft. | O.K. |

**Figure 2-15. Design of exterior beam per IBC 2000 (Part 3)**

Sec. 12.10.5
Flexural reinforcement shall not be terminated in a tension zone, unless following condition is satisfied.
Sec. 12.10.5.1 $V_{ux} \le 2/3\phi V_{nx}$
Vux=Ra-0.9wd*cutoff distance, Vux = 53 kips
φ Vnx= 0.85(Av*fy*d/s + 2sqrt(fc') Bbxd) => φ Vnx = 122 kips
2/3 φ Vnx = 82 kips ≥ 53 kips O.K.
Check Column Dimension: Interior column dimension (Bc) must be > 20 x dia. of main bar in beam (Sec. 21.5.1.4)
Bc = 34.0 in. > 20 in. O.K.

RESULT

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Beam Size:** | | **34 in. width** | **24 in. depth** | | | | |
| **Main Rebars at Top:** | | **6 #** | **8** | **As =** | **4.74 sq.in.** | | |
| | **Use** | **3 #** | **8** | **As =** | **2.37 sq.in. after cut-off @** | | **7.5 ft.** |
| **Main Rebars at Bottom:** | | **3 #** | **8** | **As =** | **2.37 sq.in.** | | |
| | **Use** | **4 #** | **8** | **As =** | **3.16 sq.in. after cut-off @** | | **7.2 ft.** |
| **Hoops & Ties:** | | **4 legs of #** | **4** | **@ spacing of** | | **5 in. within a length of** | **4.0 ft. of confinement zone** |
| | | **2 legs of #** | **4** | **@ spacing of** | | **10 in. within a length of** | **15.2 ft. beyond confinement** |

**Figure 2-15. Design of exterior beam per IBC 2000 (Part 4)**

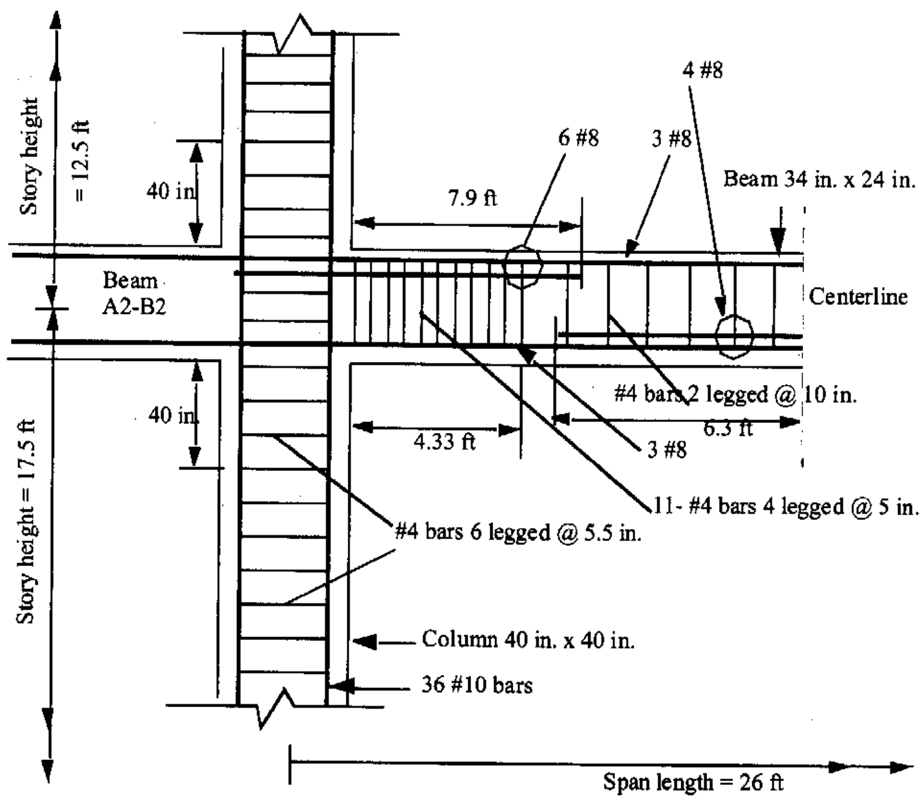



**Figure 2-16. Reinforcement arrangements in interior beam B2-C2, column 1 (between grade and floor 2) and column 2 (between floor 2 and floor 3)**

| BEAM DESIGN | Interior | | | IBC 2000 | | Site Class: | D |
|---|---|---|---|---|---|---|---|
| **Design Data:** | | | | | | | |
| Span | (l) | = | 26 ft. | | | | |
| Column width | (Bc) | = | 40 in. | | | | |
| Column depth | (Dc) | = | 40 in. | | | | |
| Beam width | (Bb) | = | 34 in. | | | | |
| Beam depth | (Db) | = | 24 in. | | | | |
| Net concrete cover | (c') | = | 2.5 in. | | | | |
| Effective beam depth | (d) | = | 21.5 in. | | | | |
| Strength of concrete | (f'c) | = | 4000 psi | | | | |
| Strength of steel | (fy) | = | 60000 psi | | | | |
| Clear span | (ln) | = | 22.67 ft. | | | | |
| Unif. dist. dead load | (wd) | = | 3.61 kips/ft | | | | |
| Unif. dist. live load | (wl) | = | 0.83 kips/ft | | | | |
| Total unif. dist. load | (w) | = | 4.44 kips/ft | | | | |
| Req. flexural strength at support (Mu1-) | | = | 397 ft-kips | <= Table | 2.17 | | |
| Req. flexural strength at support (Mu1+) | | = | 7 ft-kips | <= Table | 2.17 | | |
| Req. flexural strength at Midspan (Mu2+) | | = | 217 ft-kips | <= Table | 2.17 | | |
| Req. shear strength at support (Vu) | | = | 72 kips | <= Table | 2.17 | | |
| Req. shear strength at midspan (Vu) | | = | 11 kips | <= Table | 2.17 | | |
| Axial force (Pu) | | = | 0 kips | <= Table | 2.17 | | |
| Try reinf. at bottom near support | (As1+) | = | 2.37 sq.in. | 3 # | 8 bars, each of | 0.79 sq.in. | |
| Try reinf. at top near support | (As1-) | = | 4.74 sq.in. | 6 # | 8 bars, each of | 0.79 sq.in. | |
| Try reinf. at bottom at midspan | (As2+) | = | 3.16 sq.in. | 4 # | 8 bars, each of | 0.79 sq.in. | |
| Try reinf. at top at midspan | (As2-) | = | 2.37 sq.in. | 3 # | 8 bars, each of | 0.79 sq.in. | |

**Solution:**

**1. According to ACI Sec. 21.3.1., flexural members shall satisfy the following conditions:**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Sec. 21.3.1.1. | Pu ≤ 0.1*Ag f'c | | (Ag = gross cross-sectional area) | | | | |
| | Pu | = | 0 | ≤ | 326.4 kips | | O.K. |
| Sec.21.3.1.2. | ln ≥ 4d | | 22.67 | ≥ | 7.17 ft. | | O.K. |
| Sec. 21.3.1.3. | Bb/Db ≥ 0.3 | | 1.42 | between | 0.3 and 3.33 | | O.K. |
| Sec. 21.3.1.4. | Bb ≥ 10 | | 34 | ≥ | 10 in. | | O.K. |
| | Bb ≤ Bc + 1.5xDb | | 34 | ≤ | 76 in. | | O.K. |

**2. ACI Sec. 21.3.2. (Longitudinal reinforcement)**

Sec. 21.3.2.1.

| | | | | | | |
|---|---|---|---|---|---|---|
| As(top) or As (bot) ≥ 3sqrt.f'cxBbxd/fy | 2.37 | ≥ | 2.31 | sq.in. | O.K. | |
| As(top) or As (bot) > 200Bbxd/fy | 2.37 | ≥ | 2.44 | sq.in. | REVISE | O.K. as close |
| As(top) or As (bot) < 0.025Bbxd | 4.74 | ≤ | 18.28 | sq.in. | O.K. | |

**Figure 2-17. Design of interior beam (B2-C2 at level 2) per IBC 2000 (Part 1)**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Provide No.of continuous bars | 2 | | | | | | | | |
| Sec. 21.3.2.2. | | | | | | | | | |
| + ve moment strength at support | | | | | | | | | |
| phi Mn = phi Asfy(d - Asfy/1.7f'cBb) | phi Mn1+ = | 223 | ≥ | 7 | ft-kips | | | | O.K. |
| + ve moment strength at midspan | | | | | | | | | |
| phi Mn = phi Asfy(d - Asfy/1.7f'cBb) | phi Mn2+ = | 294 | ≥ | 217 | ft-kips | | | | O.K. |
| - ve moment strength at support | | | | | | | | | |
| phi Mn = phi Asfy(d - Asfy/1.7f'cBb) | phi Mn1- = | 432 | ≥ | 397 | ft-kips | | | | O.K. |
| Make sure phiMn1+ at support≥ 50% of phiMn1- | | 223 | ≥ | 50% of | 432 | ft-kips | | | O.K. |
| Min. phiMn (anywhere) > phiMn(-)/4 = | | 108 ft-kips | | | | | | | |
| Min. phi Mn = phi Asfy(d - Asfy/1.7f'cBb) = | | 223 | ≥ | 108 | ft-kips | | | | O.K. |
| **Shear strength (ACI Sec. 21.3.4)** | | | | | | | | | |
| Sec. 21.3.4.1. | | | | | | | | | |
| Mpr1- = 1.25Asfy(d - 1.25Asfy/1.7f'cBb). Mpr1 = | | 591 ft-kips | | with | 6 | # | 8 | bars | |
| Mpr1+ = 1.25Asfy(d - 1.25Asfy/1.7f'cBb). Mpr2 = | | 307 ft-kips | | with | 3 | # | 8 | bars | |
| w=wd+wl = | | 4.44 kips/ft | | | | | | | |
| | ln = | 22.67 ft. | | | | | | | |
| Ve = {(Mpr1-) + (Mpr1+)}/ln + wln/2 | Ve = | 90 kips | | | | | | | |
| Sec. 21.3.4.2. | | | | | | | | | |
| i. Shear caused by earthquake forces {(Mpr1-)+(Mpr1+)}/ln | | 40 | ≥ | Ve/2= | 45 | kips | No | | |
| ii. Factored axial force (taken as 0) ≤ 0.05Agf'c, So | can take Vc = | 92 | | kips. Conservatively use Vc = | | | 0 | kips | |
| <u>All the shear must be resisted by transverse reinforcement</u> | | | | | | | | | |
| Shear to be resisted by steel, Vs = Vu/ϕ - Vc, | Vs = | 106 kips | | | | | | | |
| Vu is taken as Ve | Ve = | 90 kips | | ≥ | 72 kips | | Found from analysis | | |
| Required spacing of # 4 | stirrups | 4 legged | | # of legs should be ≥ | | 4 | | | |
| | Av = | 0.8 sq.in. for | | 4 legs | | | | | |
| Spacing of hoops, s = Avfyd/Vs | s = | 9.75 in. | | | | | | | |
| Each alternate bar needs to be supported (Sec. 21.3.3.3.) | | | | | | | | | |
| 6 | # | 8 | at top | | | | | | |
| 4 | legs of # | 4 | stirrups/ties | | | | | | |
| Sec. 21.3.3.2. | | | | | | | | | |
| Spacing of hoops, s = Avfyd/Vs | | 9.75 | | | | | | | |
| s ≤ d/4, | | ≤ | 5.4 | in. | | | | | |
| s ≤ 8 x diameter of smallest long bar | | ≤ | 8.0 | in. | | | | | |
| s ≤ 24 x diameter of hoop bars | | ≤ | 12.0 | in. | | | | | |
| s ≤ 12 in. | | ≤ | 12.0 | in. | | | | | |
| With first hoop at 2 in. from face of col. | use s= | 5 | in. o/c | | | | | | |
| Sec. 11.5.6.8. | | | | | | | | | |
| Vs < 8sqrtf'c*Bbd | Vs = | 106 | ≤ | 370 | kips | | | | O.K. |
| Sec. 21.3.3.1. Hoops shall be provided over a length of | | | | | | | | | |
| i) 2Db = | | 4.00 | ft. from face of support | | | | | | |
| ii) 2Db @ midspan if there is possibility of flexural yielding there | | | | | | | | | |
| ( in this case no flexural yilding at midspan as | | ϕ Mn ≥≥ Mu ) | | | | | | | |
| Provide hoops near the face of the joint for length, Lob = | | 4.0 ft. | | | | | | | |
| S.F. there, Vx = Ve - w x Lob | Vx = | 72 kips | | | | | | | |
| Taking | Vc = | 92 kips | | | | | | | |
| Vs = Vx/0.85-Vc | Vs = | -8 kips | | | | | | | |

**Figure 2-17. Design of interior beam (Part 2)**

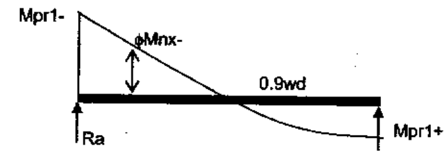
**Cut off length**

| | | | | | | |
|---|---|---|---|---|---|---|
| Prob. -ve flexural strength at support | Mpr1- = | 591 ft-kips | 6 | # | 8 bars | |
| Prob. +ve flexural strength at support | Mpr1+ = | 307 ft-kips | 3 | # | 8 bars | |
| -ve flexural strength at cut-off point | ϕ Mnx- = | 223 ft-kips | 3 | # | 8 bars | |
| Prob. +ve flexural strength at midspan | Mpr2+ = | 404 ft-kips | 4 | # | 8 bars | |
| +ve flexural strength at cut-off point | ϕ Mnx+ = | 223 ft-kips | 3 | # | 8 bars | |

Note that no less than 3 # 8 bars are to be provided to satisfy Sec. 21.3.2.1

**Negative bar cut-off:**

| | | |
|---|---|---|
| | 0.9wd= | 3.249 kips/ft |
| | Ra= | 76.5 kips |
| Length of cut-off to be calculated by using | a= | 1.62 |
| a quadratic equation: | b= | -76.5 |
| $ax^2 + b x + c = 0$ | c= | 369 |
| | x-= | 5.45 ft. |



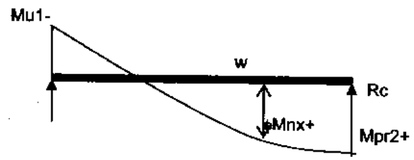

For Top Bars Out of **6** — **3** bars can be cut off at (leaving remaining 3 bars)

x+d = **7.2** ft.

x+12db = **6.5** ft — Cut the bars @ **7.5** ft. from the support

**Positive bar cut-off:**

| | | |
|---|---|---|
| | w = | 6.47 kips/ft |
| Length of cut-off to be calculated by using | Rc= | -26.0 kips |
| a quadratic equation: | a= | 3.235 |
| $ax^2 + b x + c = 0$ | b= | 26.0 |
| | c= | -182 |
| | x+= | 4.48 ft. |




For Bottom Bars Out of **4** — **1** bars can be cut off at (leaving remaining 3 bars)

x+d= **6.3** ft.

x+12db= **5.5** ft — Cut the bars @ **6.3** ft. from midspan

The cut off points should be beyond the confinement zone, i.e. **4.0** ft.

Provide cut off point at **6.3** ft.

**Shear reinforcement beyond** **4.0** ft.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Vs = | **-8** kips | conservatively | | | | | |
| Number of legs | 2 | # | 4 bars | Av = | 0.4 | sq.in | # of legs ≥ | 2 | |
| s ≤ Avfyd/Vs | | s ≤ | **NA** in. | | | | | | |
| s ≤ d/2 | | s ≤ | **10.75** in. | | Max. s = | 10.75 | in. | | |
| Provide | **2** | legged # | **4** | stirrups with seismic hooks @ | | **10** | in. beyond the confinement zone | | O.K. |
| | | | | Spacing of stirrups (s) = | | **5** | in. the confinement zone | | |

**Development length**

Sec. 12.2.1.

(ld = development length; db = dia. of main bar) Take ktr=0 (conservatively)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| c = | 2.5 in. | | alpha = | 1 bottom bar | | | |
| db = | 1 in. | | alpha = | 1.3 top bar | | | |
| Bottom Bars: ld = | 28.5 | db = | **2.37** ft. < | **6.3** | ft. | O.K. |
| Top Bars: ld = | 37.0 | db = | **3.08** ft. < | **7.5** | ft. | O.K. |

**Figure 2-17. Design of interior beam (Part 3)**

Sec. 12.10.5

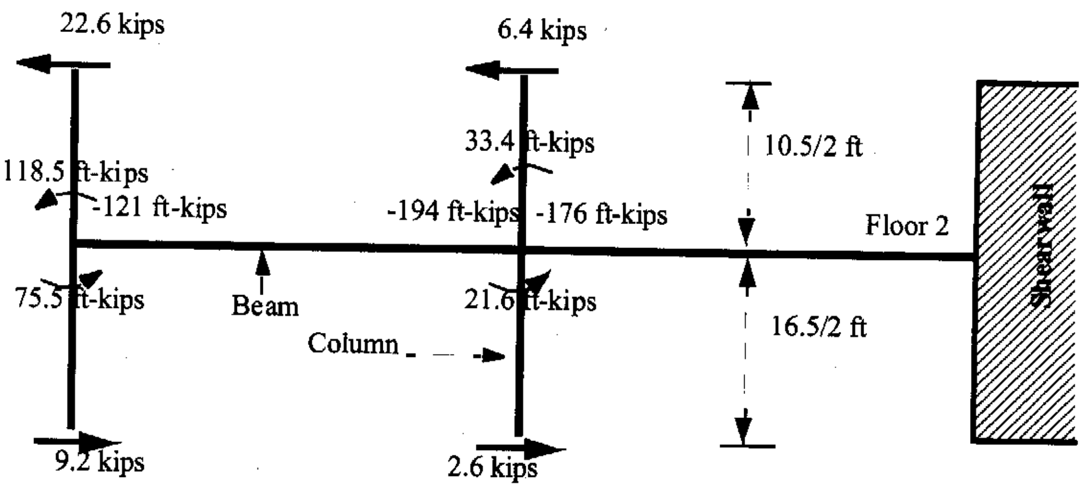
Flexural reinforcement shall not be terminated in a tension zone, unless following condition is satisfied.

| | | | | | | |
|---|---|---|---|---|---|---|
| Sec. 12.10.5.1 | $V_{ux} \le 2/3\phi V_{nx}$ | | | | | |
| Vux=Ra-0.9wd*cutoff distance, | Vux = | 56 kips | | | | |
| $\phi$ Vnx= 0.85(Av*fy*d/s + 2sqrt(fc') Bbxd) => $\phi$ Vnx = | | 122 kips | | | | |
| | 2/3 $\phi$ Vnx = | 82 kips ≥ | 56 kips | | O.K | |
| Check Column Dimension: | Interior column dimension (Bc) must be > 20 x dia. of main bar in beam (1921.5.1.4) | | | | | |
| | Bc = | 40.0 | in. > | 20 | in. | O.K. |

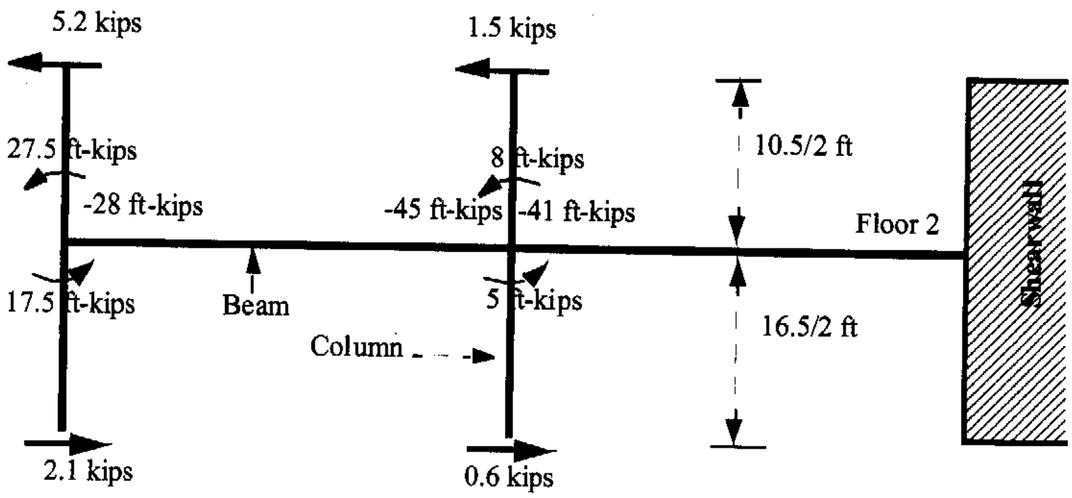
**RESULT**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Beam Size:** | | **34 in. width** | **24 in. depth** | | | | |
| **Main Rebars at Top:** | | **6 #** | **8** | **As =** | **4.74 sq.in.** | | |
| | **Use** | **3 #** | **8** | **As =** | **2.37 sq.in. after cut-off @** | **7.5** | **ft.** |
| **Main Rebars at Bottom:** | | **3 #** | **8** | **As =** | **2.37 sq.in.** | | |
| | **Use** | **4 #** | **8** | **As =** | **3.16 sq.in. after cut-off @** | **6.3** | **ft.** |
| **Hoops & Ties:** | | **4 legs of #** | **4 @ spacing of** | | **5 in. within a length of** | **4.0** | **ft. of confinement zone** |
| | | **2 legs of #** | **4 @ spacing of** | | **10 in. within a length of** | **14.7** | **ft. beyond confinement** |

**Figure 2-17. Design of interior beam (Part 4)**




(a) Due to Dead Load




(b) Due to Live Load

**Figure 2-18. Forces in columns due to gravity loads (based on bending moments at beam ends)**

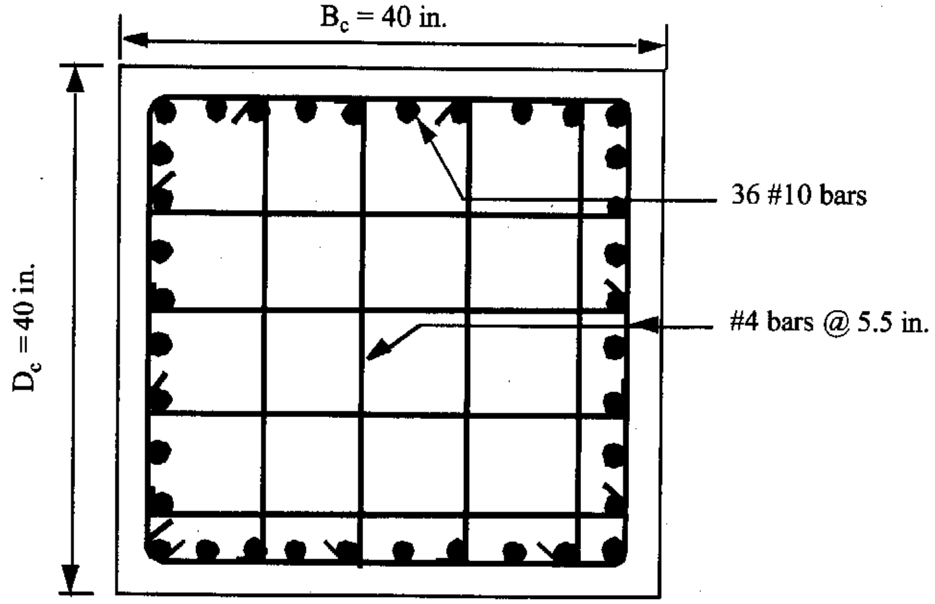



**Figure 2-19. Reinforcement details for interior column 1 (between grade and floor 2) and column 2 (between floor 2 and floor 3)**

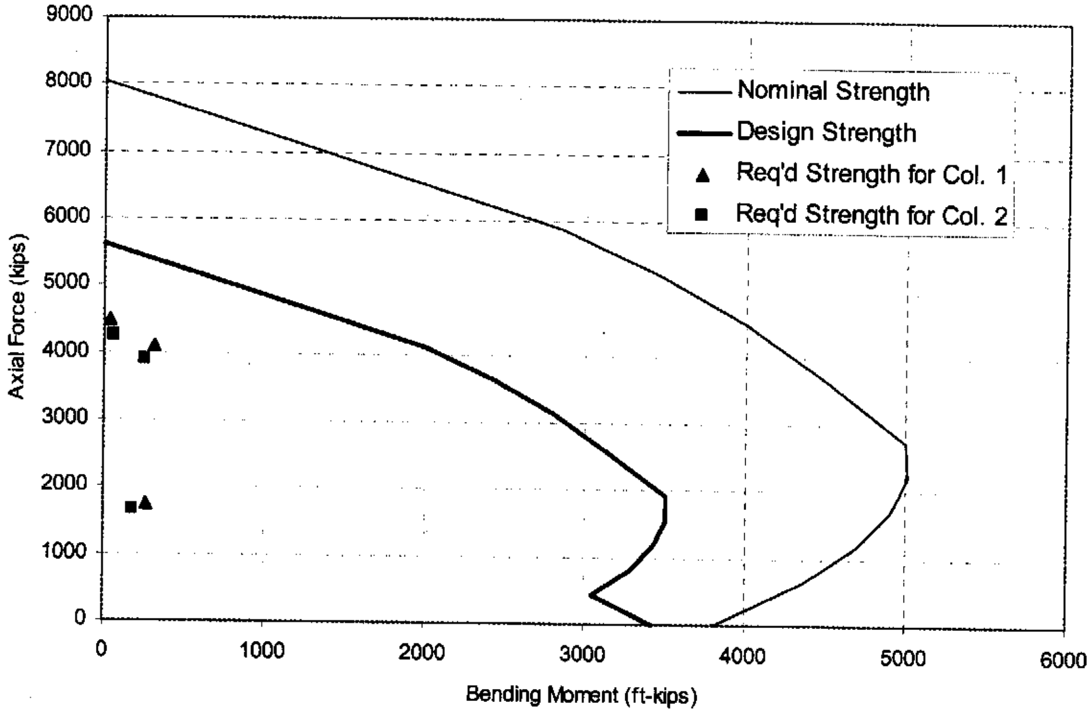



**Figure 2-20. Design strength interaction diagram for interior column B2**

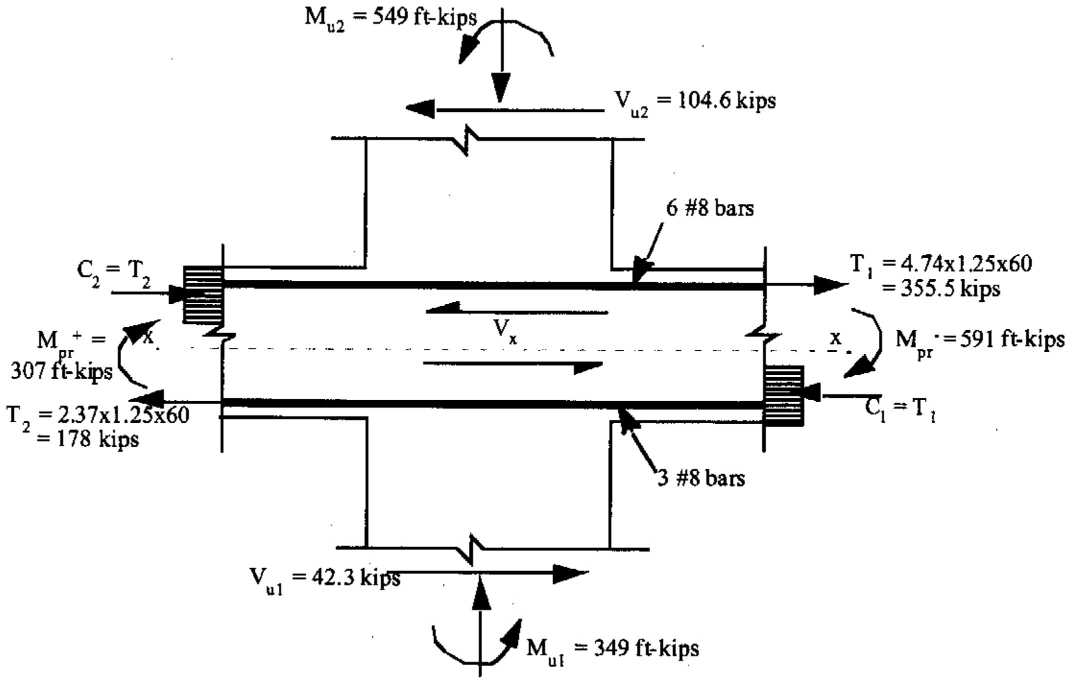


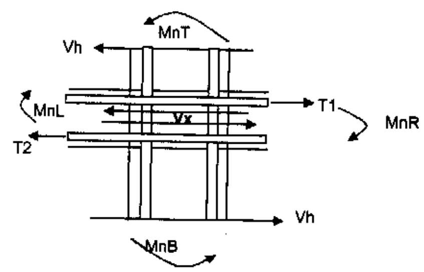

**Figure 2-21. Shear analysis of interior beamcolumn joint (B2 at floor 2)**

## COLUMN DESIGN — IBC 2000 — Interior Column 2

| | | | | |
|---|---|---|---|---|
| Design Axial Force, Pu = | 4244 kips | Max. Mn-Top Col, MnT= | 5000 ft.kips | |
| Design Moment, Mu = | 257 ft.kips | Max.Mn-Bot. Col, MnB= | 5000 ft.kips | |
| Design Shear, Vu = | 39 kips | As-(Beam Top Steel)= | 4.74 sq.in. | |
| Col. Width, Bc | 40 in. | As+(Beam Bot. Steel)= | 2.37 sq.in. | |
| Col. Depth, Dc = | 40 in. | Beam Depth Db | 24 in. | |
| Conc. Strength, fc' = | 4000 psi | Beam Width Bb | 34 in. | |
| Min. Axial Force, Pu-min | 1612 kips | Story Height ht | 12.5 ft | |
| Net Cover | 2.5 in. | Yield Strength fy | 60 ksi | |
| effective d | 37.5 | Main Bars in col. are # | 10 | |
| Steel area in Col. As | 45.72 sq.in. | # of bars 36 As of 1bar | 1.27 sq.in. | |
| 21.4.3 ρ | 0.0286 | should be > 1% and < 6% | | O.K. |
| 21.4.1 0.1fc'xAg | 640 kips | < Pu-max (kips) = | 4244 | O.K. |
| 21.4.1.1 min (Bc or Dc) | 40 in. | should be ≥ 12 in. | | O.K. |
| 21.4.1.2 Bc/Dc | 1.00 | should be ≥ 0.4 and ≤ 2.5 | | O.K. |
| 21.4.2 φMn-Top Col | 1600 ft.kips | the lowest corresponding to different Pu | | |
| Check φMn-Bot. Col | 1600 ft.kips | the lowest corresponding to different Pu | | |
| strong col.- φMn-beam-Right | 432 ft.kips | Mpr-beam-Right=MnR | 591 ft.kips | |
| weak beam φMn-beam-Left | 223 ft.kips | Mpr-beam-Left=MnL | 307 ft.kips | |
| ΣφMn-Beam | 655 ft.kips | | | |
| ΣφMn-Column | 3200 ft.kips | | | |
| 21.4.2.2 1.2xΣφMn-Beam | 786 ft.kips | **should be less than Σφ Mn-Column** | | O.K. |

**21.4.4 Shear Design**

| | | | | |
|---|---|---|---|---|
| 21.4.4.2(a) max. spacing, s1 | 10 in. | | | |
| 21.4.4.2(b) max. spacing, s2 | 4 in. | | | |
| provide spacing, s= | **4 in.** | | | |
| Clear Cover, cc | 1.5 in. | | | |
| Hoop size, # | 4 | dh (in.)= 0.5 | | |
| Yield strength, fyh | 60 ksi | | | |
| hc=Bc-2xcc-dh | 36.5 in. | | | |
| Bch=Bc-2xcc | 37 in. | | | |
| Dch=Dc-2xcc | 37 in. | | | |
| Eqn. 21-3 Req. conf. Steel, Ash1 | 0.493 sq.in. | | | |
| Eqn. 21-4 Req. conf. Steel, Ash2 | 0.876 sq.in. | | | |
| 21.4.4.1 Min. conf. Steel, Ash | 0.876 sq.in. | | | |
| Area of 1 leg | 0.2 sq.in. | of confining steel | | |
| # of layers of main bars | 10 | # of legs (hoops+ties) | 6 ≥ 6 | O.K. |
| Ash Provided = | 1.2 sq.in. | > | 0.876 sq.in. needed | O.K. |




**21.4.4.4 Special Transverse reinforcement over length lo, where:**

| | | | |
|---|---|---|---|
| 21.4.4.4a lo1=Dc | 40 in. | | |
| 21.4.4.4b lo2=ln/6 | 21.0 in. | | |
| 21.4.4.4c lo3 | 18 in. | | |
| lo=max(lo1,lo2,lo3) | **40 in.** | | |
| Mprc=1.25(MnT+MnB) | 12500 ft.kips | Probable moment in columns | |
| Mprb=(MnR+MnL) | 898 ft.kips | Probable moment in beams | **TAKE Mn-Beam for Shear Design** |
| | | Governing moment, Mprd (min. Mprc, Mprb) = | 898 ft.kips |
| Clear Height-Col, L1= | 10.50 ft | Clear height , L2= | 16.5 ft |
| Vh=Mprd*2*L2/L1/(L1+L2) | 104.6 kips | Shear force in column | |
| 21.4.5.2 0.05xAgxfc' | 320 kips | Consider Concrete contribution Vc for Shear Design as Agfc'/20<Pu-min | |
| Eqn. 11-4 Vc | 285.3 kips | Use Vc = | 285.3 kips |
| Vs | 0.0 kips | | |
| Reqd. spacing s1= | . in. | **IGNORE** | |
| 21.4.4.6 max. spacing, s2= | 6 in. | | |
| 21.4.4.6 max. spacing s3=6db= | 7.62 in. | Main bar Dia. | 1.27 in. |
| spacing,s=min(s1,s2,s3 | 6 in. | **Use s =** | **5.5 in.** |

### Final Results:

| | | | | |
|---|---|---|---|---|
| **Use Column Size Bc x Dc of** | **40 in. by** | **40 in. with** | **36 bars of # 10** | **ρ (%) = 2.86** |
| **Use hoop/ties #** | **4 bars** | **6 legged @** | **4 in. over** | **40 in. and** |
| **a spacing of** | **5.5 in. over remainder of height** | | **1st hoop @ 2 in. from support** | |

## JOINT DESIGN

| | | | | |
|---|---|---|---|---|
| joint width, Bj | 40 in. | | | |
| joint depth, Dj | 40 in. | | | |
| shear strength factor | **20** | 20 if interior confined on all sides, 15 if 3 sides and 12 if 2 sides | | |
| T1=1.25xAsxfy | 356 kips | tension force in top steel (see Fig. Above) | | |
| T2=1.25xAsxfy | 178 kips | tension force in bottom steel (see Fig. Above) | | |
| Vh | 105 kips | | | |
| Vx | 429 kips | | | |
| 21.5.3 **Joint Strength, φVc** | **1720** kips | ≥ | **429** kips | O.K. |
| 21.5.2.2 Bb/Bc | 0.85 > 0.75 | & < 1.33 | | O.K. |

Within joint depth, transverse reinforcement equal to at least half of the amount required by 1921.4.4.1 shall be provided, and the spacing is:

| | | | |
|---|---|---|---|
| 21.4.4.2a | < Bj/4= | 10 in. | |
| 21.4.4.2b | < | 6 in. Use spacing of | 5.5 in. inside the joint |

**Figure 2-22. Design of interior column and joint per IBC 2000**

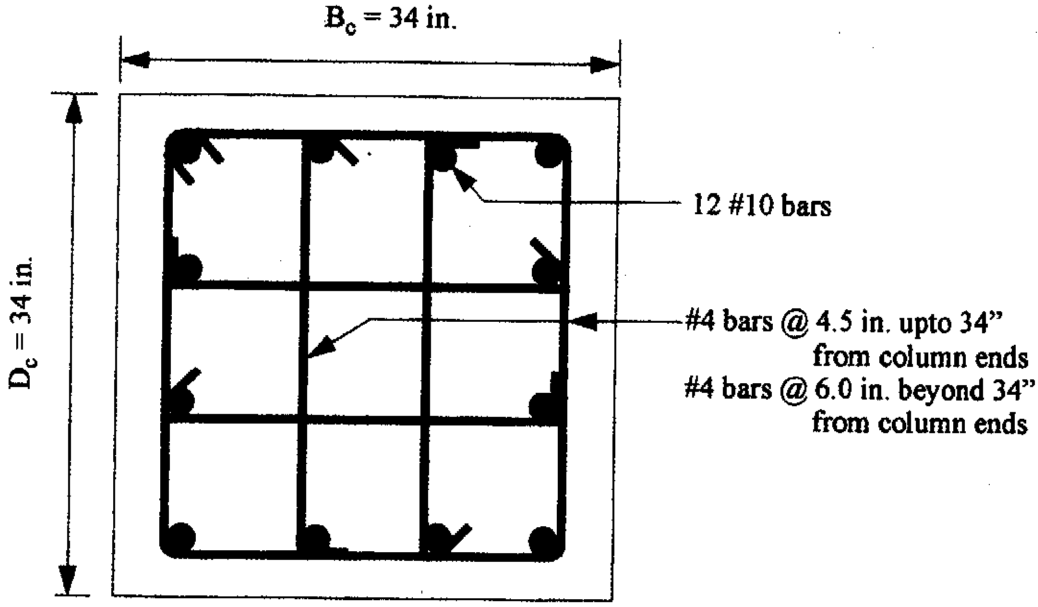



**Figure 2-23. Reinforcement details for column 3 (between grade and floor 2) and column 4 (between floor 2 and floor 3)**

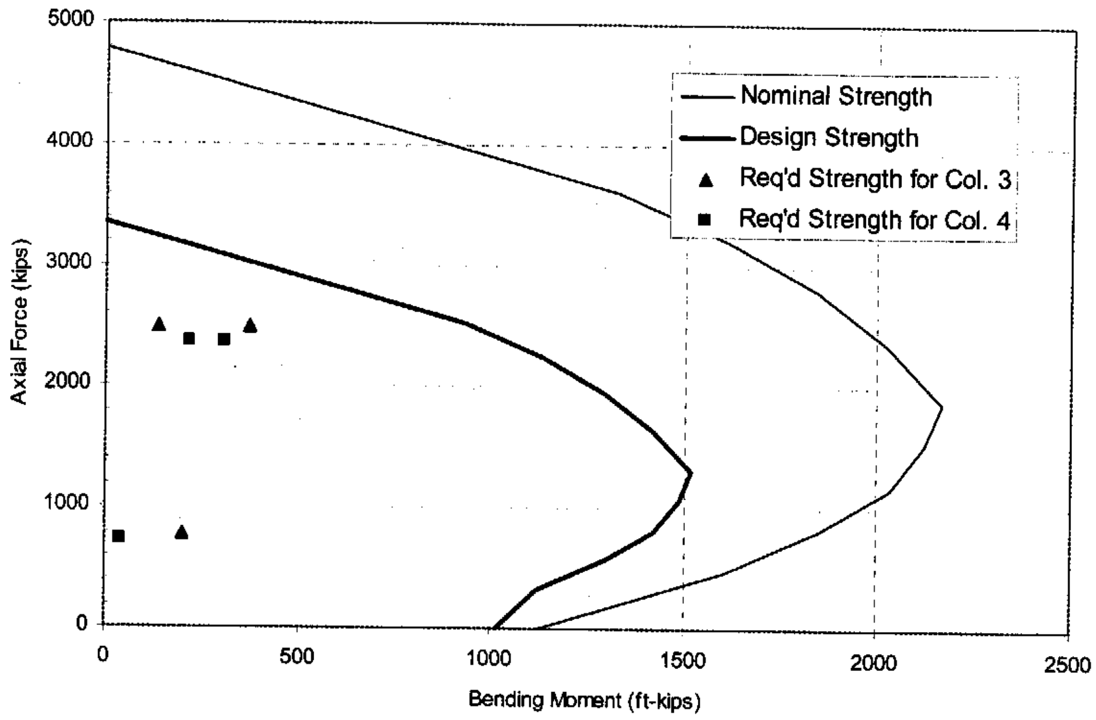



**Figure 2-24. Design strength interaction diagram for exterior column A2**

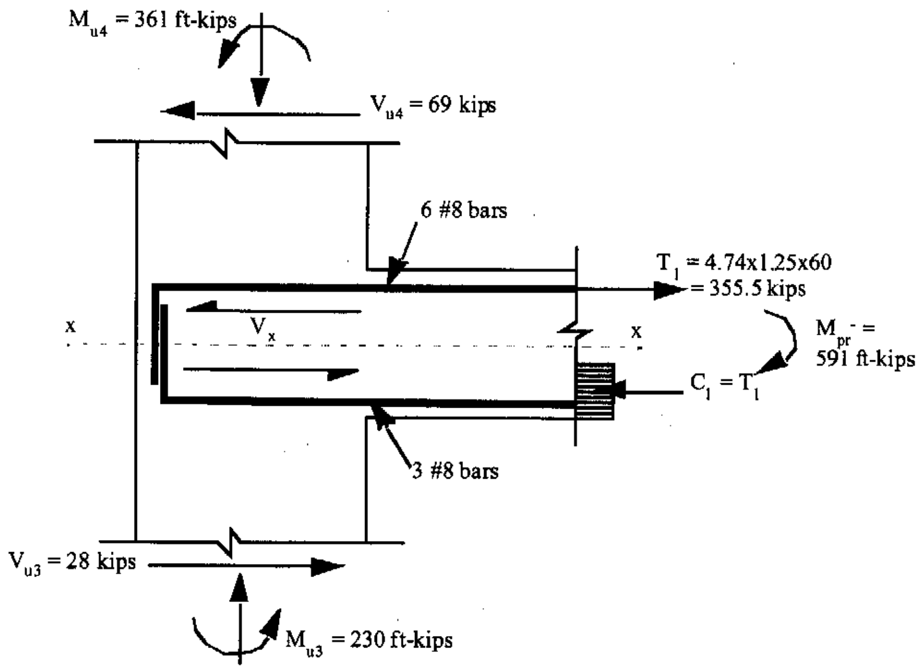
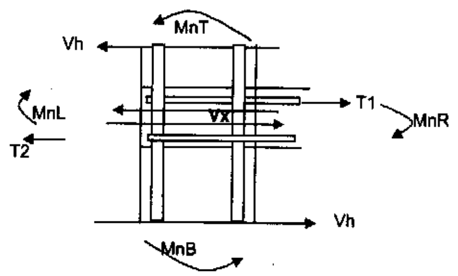



**Figure 2-25. Shear analysis of exterior beam-column joint (A2 at floor 2)**

## COLUMN DESIGN — IBC 2000 — Exterior Column 3

| | | | | | |
|---|---|---|---|---|---|
| Design Axial Force, Pu = | 2512 kips | Max. Mn-Top Col, MnT= | 2160 ft.kips | | |
| Design Moment. Mu = | 373 ft.kips | Max.Mn-Bot. Col, MnB= | 2160 ft.kips | | |
| Design Shear, Vu = | 36 kips | As-(Beam Top Steel)= | 4.74 sq.in. | | |
| Col. Width, Bc | 34 in. | As+(Beam Bot. Steel)= | 2.37 sq.in. | | |
| Col. Depth, Dc = | 34 in. | Beam Depth Db | 24 in. | | |
| Conc. Strength, fc' = | 4000 psi | Beam Width Bb | 34 in. | | |
| Min. Axial Force, Pu-min | 770 kips | Story Height ht | 17.5 ft | | |
| Net Cover | 2.5 in. | Yield Strength fy | 60 ksi | | |
| effective d | 31.5 | Main Bars in col. are # | 10 | | |
| Steel area in Col. As | 15.24 sq.in. | # of bars | 12 As of 1bar | 1.27 sq.in. | |
| 21.4.3 ρ | 0.0132 | should be > 1% and < 6% | | | O.K. |
| 21.4.1 0.1fc'xAg | 462 kips | < Pu-max (kips) = | 2512 | | O.K. |
| 21.4.1.1 min (Bc or Dc) | 34 in. | should be ≥ 12 in. | | | O.K. |
| 21.4.1.2 Bc/Dc | 1.00 | should be ≥ 0.4 and ≤ 2.5 | | | O.K. |
| 21.4.2 φMn-Top Col | 900 ft.kips | the lowest corresponding to different Pu | | | |
| Check φMn-Bot. Col | 900 ft.kips | the lowest corresponding to different Pu | | | |
| strong col.- φMn-beam-Right | 432 ft.kips | Mpr-beam-Right=MnR | 591 ft.kips | | |
| weak beam φMn-beam-Left | 223 ft.kips | Mpr-beam-Left=MnL | 307 ft.kips | | |
| ΣφMn-Beam | 655 ft.kips | | | | |
| ΣφMn-Column | 1800 ft.kips | | | | |
| 21.4.2.2 1.2x ΣφMn-Beam | 786 ft.kips | **should be less than Σφ Mn-Column** | | | O.K |

| | | | |
|---|---|---|---|
| 21.4.4 **Shear Design** | | | |
| 21.4.4.2(a) max. spacing, s1 | 8.5 in. | | |
| 21.4.4.2(b) max. spacing, s2 | 5.5 in. | | |
| provide spacing, s= | **5.5 in.** | | |
| Clear Cover, cc | 1.5 in. | | |
| Hoop size, # | 4 dh (in.)= | 0.5 | |
| Yield strength, fyh | 60 ksi | | |
| hc=Bc-2xcc-dh | 30.5 in. | | |
| Bch=Bc-2xcc | 31 in. | | |
| Dch=Dc-2xcc | 31 in. | | |
| Eqn. 21-3 Req. conf. Steel, Ash1 | 0.681 sq.in. | | |
| Eqn. 21-4 Req. conf. Steel, Ash2 | 1.007 sq.in. | | |
| 21.4.4.1 Min. conf. Steel, Ash | 1.007 sq.in. | | |
| Area of 1 leg | 0.2 sq.in. | of confining steel | |
| # of layers of main bars | 4 | # of legs (hoops+ties) 4 > 3 | O.K |
| Ash Provided = | 0.8 sq.in. | < 1.007 sq.in. needed | REVISE |




| | | | |
|---|---|---|---|
| 21.4.4.4 **Special Transverse reinforcement over length loc, where:** | | | |
| 21.4.4.4a lo1=Dc | 34 in. | | |
| 21.4.4.4b lo2=ln/6 | 31.0 in. | | |
| 21.4.4.4c lo3 | 18 in. | | |
| loc=max(lo1,lo2,lo3) | **34 in.** | | |
| Mprc=1.25(MnT+MnB) | 5400 ft.kips | Probable moment in columns | |
| Mprb=(MnR+MnL) | 898 ft.kips | Probable moment in beams | **TAKE Mn-Beam for Shear Design** |
| | | Governing moment, Mprd (min. Mprc, Mprb) = | 898 ft.kips |
| Clear Height-Col, L1= | 16.50 ft | Clear height , L2= | 10.5 ft |
| Vh=Mprd*2*L2/L1/(L1+L2) | 42.4 kips | Shear force in column | |
| 21.4.5.2 0.05xAgxfc' | 231.2 kips | Consider Concrete contribution Vc for Shear Design as Agfc'/20<Pu-m | |
| Eqn. 11-4 Vc | 180.6 kips | Use Vc = | 180.6 kips |
| Vs | 0.0 kips | | |
| Reqd. spacing s1= | . in. | | |
| 21.4.4.6 max. spacing, s2= | 6 in. | | |
| 21.4.4.6 max. spacing s3=6db= | 7.62 in. | Main bar Dia. | 1.27 in. |
| spacing,s=min(s1,s2,s3) | 6 in. | **Use s =** | **5.5 in.** |

### Final Results:

**Use Column Size Bc x Dc of 34 in. by 34 in. with 12 bars of # 10 ρ (%) = 1.32**
**Use # 4 bars, 4 legged @ 5.5 in. over 34 in. and**
**a spacing of 5.5 in. over remainder of height 1st hoop @ 2 in. from support**

## JOINT DESIGN

| | | | |
|---|---|---|---|
| joint width, Bj | 34 in. | | |
| joint depth, Dj | 34 in. | | |
| shear strength factor | **15** | 20 if interior confined on all sides, 15 if 3 sides and 12 if 2 sides | |
| T1=1.25xAsxfy | 356 kips | tension force in top steel (see Fig. Above) | |
| T2=1.25xAsxfy | 178 kips | tension force in bottom steel (see Fig. Above) | |
| Vh | 42 kips | | |
| Vx | 491 kips | | |
| 21.5.3 **Joint Strength, φVc** | **932 kips** | **≥** **491 kips** | O.K. |
| 21.5.2.2 Bb/Bc | 1.00 > 0.75 | & < 1.33 | O.K. |
| | Within joint depth, transverse reinforcement equal to at least half of the amount required by 1921.4.4.1 shall be provided, and the spacing is: | | |
| 21.4.4.2a | < Bj/4= | 8.5 in. | |
| 21.4.4.2b | < | 4 in. Use spacing of | 4 in. inside the joint |

**Figure 2-26. Design of exterior column and joint per IBC 2000**

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## Seismic Design Using Structural Dynamics (2000 IBC®)

This publication addresses the two methods by which a designer may comply with the seismic design requirements of the 2000 IBC®: Equivalent Lateral Force Procedure and Dynamic Analysis Procedure. The focus is on response spectrum analysis, the most common dynamic analysis procedure, which is used as the basis of design. The purpose of this publication is to demystify design based on dynamic analysis and to provide answers to questions that have risen over the years.

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