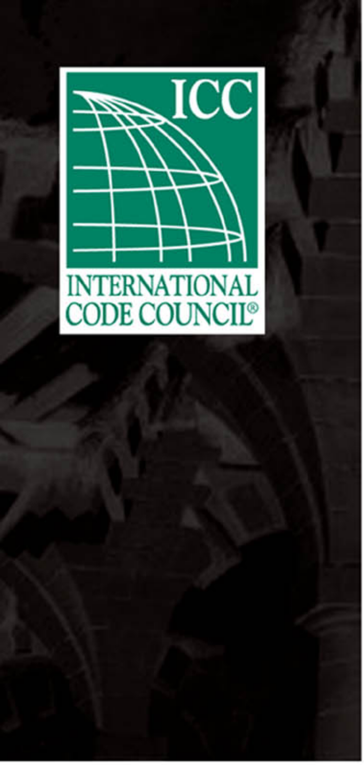

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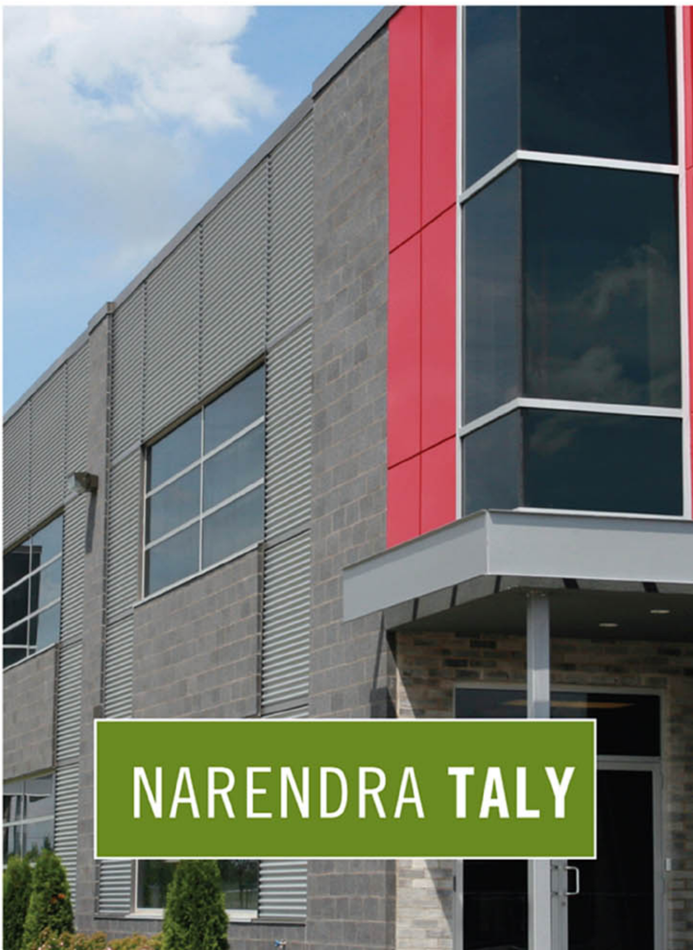

NARENDRA **TALY**

DESIGN OF REINFORCED MASONRY STRUCTURES

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DESIGN OF REINFORCED MASONRY STRUCTURES

Narendra Taly, Ph.D., P.E., F.ASCE

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To my wife, Trish, for her high-limit state of endurance,
to my daughters, Neena and Beena, for their love of teaching,
and to the memory of my parents, Sundar Bai and Bhagwan Das Taly,
this book is dedicated.

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ABOUT THE AUTHOR

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CONTENTS

Preface to the Second Edition **xiii**
Preface to the First Edition **xvii**
Acknowledgments **xix**
Notation **xxi**
Acronyms **xxvii**

Chapter 1. Introduction **1.1**

1.1 What Is Masonry? / *1.1*
1.2 Plain and Reinforced Masonry / *1.1*
1.3 A Brief History of Masonry Construction / *1.2*
1.4 Evolution of Reinforced Masonry / *1.3*
1.5 Unreinforced and Reinforced Masonry / *1.5*
1.6 Historical Development of Building Codes and Standards for Masonry Construction / *1.6*
1.7 Design Methods / *1.9*
1.8 Load Combinations / *1.11*
References / *1.14*

Chapter 2. Masonry Units: Applications, Types, Sizes, and Classification **2.1**

2.1 Introduction / *2.1*
2.2 Application of Masonry Units in Construction / *2.1*
2.3 General Description of Masonry Units / *2.2*
2.4 Clay Building Brick / *2.4*
2.5 Functional Aspects / *2.15*
2.6 Concrete Masonry Units / *2.23*
2.7 Bonds and Patterns in Masonry Work / *2.35*
2.8 Structural Requirements for Masonry in Stack Bond / *2.41*
2.9 Mortar Joints / *2.42*
2.10 Types of Wall Construction / *2.43*
2.11 Glass Unit Masonry / *2.46*
2.12 Mortarless Block Systems / *2.51*
2.13 Prefabricated Masonry / *2.51*
2.14 Autoclaved Aerated Concrete / *2.54*
References / *2.55*

Chapter 3. Materials of Masonry Construction **3.1**

3.1 Introduction / *3.1*
3.2 Mortar / *3.1*
3.3 Grout / *3.6*
3.4 Differences between Mortar, Grout, and Concrete / *3.11*
3.5 Compressive Strength of Masonry / *3.12*
3.6 Steel Reinforcement / *3.15*
3.7 Modulus of Elasticity of Masonry Materials / *3.22*

3.8 Thermal Effects on Masonry / *3.23*
3.9 Influence of Moisture on Masonry: Shrinkage / *3.25*
3.10 Creep of Masonry / *3.27*
References / *3.28*

Chapter 4. Design of Reinforced Masonry Beams 4.1

4.1 Introduction / *4.1*
4.2 Historical Development / *4.2*
4.3 Strength Design Philosophy / *4.2*
4.4 Assumptions in Strength Design Philosophy / *4.5*
4.5 Analysis of Rectangular Sections in Flexure / *4.7*
4.6 Modulus of Rupture and Nominal Cracking Moment of a Masonry Beam / *4.26*
4.7 Design of Masonry Beams / *4.31*
4.8 Procedure for Flexural Design of Beams / *4.41*
4.9 Overreinforced Beams / *4.53*
4.10 Design for Shear in Reinforced Masonry Beams / *4.56*
4.11 Lateral Support of Masonry Beams / *4.69*
4.12 Analysis of Doubly Reinforced Masonry Beams / *4.69*
4.13 Lintels / *4.74*
4.14 Masonry Wall Beams (Deep Wall Beams) / *4.101*
4.15 Bond Beams / *4.109*
4.16 Diaphragm Action / *4.111*
4.17 Flexural Strength of a Wall due to In-Plane Loads / *4.115*
4.18 Development Lengths for Reinforcing Bars / *4.117*
4.19 Serviceability Criteria for Beams / *4.119*
4.20 Service Load Analysis of Reinforced Masonry Beams / *4.120*
4.21 Deflections of Reinforced Masonry Beams / *4.126*
References / *4.139*

Chapter 5. Columns 5.1

5.1 Introduction / *5.1*
5.2 Behavior of Axially Loaded Columns / *5.4*
5.3 Axial Strength of Reinforced Masonry Columns / *5.7*
5.4 MSJC *Code* Provisions for Reinforced Masonry Columns / *5.10*
5.5 Analysis of Reinforced Masonry Columns / *5.16*
5.6 Design Procedure for Reinforced Masonry Columns / *5.21*
5.7 Columns under Combined Axial Load and Bending / *5.28*
5.8 Discussion and Interpretation of the Axial Load-Bending Moment Interaction Diagrams / *5.57*
5.9 Interaction Diagram for a Wall under Combined Loading (Axial Load and Bending) / *5.58*
5.10 Shear Strength of Masonry Columns / *5.60*
5.11 Masonry Piers / *5.64*
References / *5.68*

Chapter 6. Walls under Gravity and Transverse Loads 6.1

6.1 Introduction / *6.1*
6.2 Types of Masonry Walls / *6.1*
6.3 Bond Patterns in Masonry Walls / *6.16*
6.4 Analysis of Walls under Gravity and Transverse Loads / *6.23*
6.5 Out-of-Plane Loads on Walls / *6.25*

6.6 Analysis of Masonry Walls for Out-of-Plane Loads / *6.38*
6.7 Design of Walls for Gravity and Transverse Loads / *6.44*
6.8 Axial Loads on Walls Subjected to Out-of-Plane Loads / *6.69*
6.9 Pilasters / *6.69*
6.10 Nonload-Bearing Walls / *6.77*
References / *6.86*

Chapter 7. Shear Walls 7.1

7.1 Introduction / *7.1*
7.2 Fundamental Concepts / *7.2*
7.3 Types of Shear Walls / *7.6*
7.4 Rigidity and Relative Rigidity of a Shear Wall / *7.10*
7.5 Rigidity of a Shear Wall with Openings / *7.17*
7.6 Determination of Seismic Lateral Forces in Shear Walls / *7.39*
7.7 Horizontal Diaphragms / *7.50*
7.8 Influence of Building Configuration on Lateral Force Distribution in Shear Walls / *7.57*
7.9 Analysis of Shear Walls and Diaphragms under Direct Shear and Torsional Moments / *7.69*
7.10 Design Considerations for Shear Walls / *7.81*
7.11 Analysis of Shear Walls under Flexure and Axial Loads / *7.95*
7.12 Design of Multistory Shear Walls / *7.108*
7.13 Failure Modes of Shear Walls / *7.110*
References / *7.121*

Chapter 8. Retaining and Subterranean Walls 8.1

8.1 Introduction / *8.1*
8.2 Principal Types of Retaining Walls / *8.2*
8.3 Lateral Pressures on Retaining Walls / *8.9*
8.4 External Stability of a Retaining Wall / *8.25*
8.5 Design Procedure for Masonry Retaining Walls / *8.29*
8.6 Subterranean or Basement Walls / *8.35*
8.7 Construction Considerations / *8.42*
References / *8.48*

Chapter 9. Construction Aspects 9.1

9.1 Introduction / *9.1*
9.2 Placement of Steel Reinforcement / *9.2*
9.3 Grouting / *9.7*
9.4 Movements of Construction Materials, Their Causes and Effects / *9.23*
9.5 Control of Cracking and Movement Joints / *9.33*
9.6 Quality Assurance / *9.42*
9.7 Flashing for Masonry Construction / *9.43*
References / *9.46*

Chapter 10. Anchorage to Masonry 10.1

10.1 Introduction / *10.1*
10.2 Types of Anchor Bolts / *10.1*
10.3 Placement and Embedment of Anchor Bolts in Masonry Grout / *10.2*
10.4 Nominal Strength of Anchor Bolts / *10.3*

10.5 Nominal Axial Strength of Anchor Bolts Loaded in Tension and in Combined Tension and Shear / *10.5*
10.6 Nominal Shear Strength of Headed and Bent-Bar Anchor Bolts in Shear / *10.14*
10.7 Headed and Bent-Bar Anchor Bolts in Combined Axial Tension and Shear / *10.15*
10.8 Structural Walls and Their Anchorage Requirements / *10.16*
References / *10.27*

Appendix Design Aids: Tables A.1
Glossary G.1
Index I.1

PREFACE TO THE SECOND EDITION

Why write?

"I hear, I forget;
I see, I remember;
I write, I understand."

A Chinese proverb

The writing of this book was motivated by a professional need to update changes in the reinforced masonry design philosophy that have occurred as a result of incorporation of *strength design* philosophy in the 2008 Building Code Requirements for Masonry Structures reported by the Masonry Standards Joint Committee (referred to in this book as the MSJC-08 *Code*) and corresponding requirements of the *2009 International Building Code* (2009 IBC), and to update changes brought out by the ASCE/SEI 7-05 Standard, *Minimum Design Loads for Buildings and Other Structures* (referred to in this book as ASCE 7-05 Standard). While the fundamental principles of designing reinforced masonry structures discussed in the first edition (2001) of this book remain valid, revisions in codes, specifications, and reference standards applicable to design and construction of masonry structures that have since occurred required updating that book in the form of this second edition.

The allowable stress design (ASD) method of designing reinforced masonry structures presented in the first edition of this book is still acceptable, and is expected to remain so for the foreseeable future. However, the general trend in the structural engineering profession is to move toward using the strength design philosophy for the design of concrete structures, and load and resistance factor design (LRFD) for the design of steel structures. Readers of the first edition of this book will note that the topic of *strength design of reinforced masonry* was briefly covered in App. D. This second edition is a natural, follow-up publication that focuses exclusively on strength design philosophy for reinforced masonry structures. In addition, a new chapter on anchorage to masonry (Chap. 10) has been introduced.

Consistent with the first edition, this edition of the book is written in a stand-alone format and independent of the ASD philosophy. While knowledge of and familiarity with the strength design principles for design of reinforced concrete structures would enable readers to quickly grasp the fundamentals of strength design of reinforced masonry, neither that knowledge nor that of allowable stress design of masonry are considered prerequisites for understanding the discussion presented herein. Each chapter of the book presents the theory based on first principles and is supported by references and followed by numerous examples that illustrate its application.

Like the first edition of this book, this edition is written for use by students and professionals of reinforced masonry design and construction. It is written in a simple, practical, and logical manner, and is styled to suit as a text for teaching reinforced masonry design and construction in a classroom environment at senior/graduate level. Frequent references to the MSJC-08 *Code* and ASCE/SEI 7-05 Standard are made throughout all discussions and examples in this book to acquaint readers with the design and specification requirements that must be followed; readers will find it helpful to keep copies of these two references handy while reading this book.

Chapter 1 introduces the topic of masonry design and construction—from ancient times to modern times—a practice that began as the art of construction and evolved into the modern engineered construction. Also presented in the chapter are brief discussions of the governing building codes and specifications for masonry structures, and governing provisions of ASCE/SEI 7-05 Standard that form the basis of load calculations for analysis and design.

Masonry structures are built from units that are fabricated in production plants from clay and concrete, and hand-laid by skilled masons, one unit at a time. Chapter 2 is devoted to a detailed discussion of both clay and concrete units with respect to industry standards, product availability, modular sizes, design properties, and applicable ASTM Standards.

Chapter 3 presents a discussion on materials of masonry construction: masonry units, mortar, grout, and steel reinforcing bars. Reinforced masonry structures are built from placing masonry units with mortar between them, placing horizontal and vertical reinforcements, and grouting the cells of masonry units to accomplish the desired design objectives. Adherence to the specifications of these materials is the key to acceptable performance of as-built structures, hence the importance of this chapter.

Chapters 4 through 10 present analysis and design of masonry structures subjected to flexure, shear, compression, and combined axial compression and flexure; walls subjected to out-of-plane loads; shear walls (walls subjected to in-plane loads); retaining walls; and anchorage to masonry.

Chapter 4 presents an exhaustive discussion of fundamentals of strength design philosophy and their application to flexural analysis and design of masonry structures. This is the longest and also the most important chapter in the book for it embodies principles of strain compatibility and ductility, and requirements of the MSJC-08 *Code* pertaining to design for flexure, shear, deflection, and cracking moment, concepts which are used in later chapters of the book. The author has provided in-depth explanation of fundamental principles of strength design in this chapter, followed by numerous examples designed to satisfy the many "what if" questions and curiosities of readers, particularly students. The purpose of this chapter is to encourage discussion and to develop confidence in understanding the ramifications of improper designs.

Chapter 5 is devoted to design of compression members—reinforced masonry columns—loaded axially or in combination with bending. Many examples are presented to illustrate the design concepts and alternatives. An in-depth discussion of interaction diagrams for columns subjected to combined axial load and bending, including detailed, step-by-step calculations for developing such diagrams, forms the highlight of this chapter.

Chapter 6 presents analysis and design of reinforced masonry walls subjected to out-of-plane loads due to wind or earthquakes. The chapter presents a discussion and calculation of these forces based on ASCE/SEI 7-05 Standard. Also presented in this chapter are many different types of masonry walls and their uses.

Chapter 7 deals with an all-important topic of analysis and design of reinforced masonry shear walls which are used as systems for resisting lateral forces in building structures—either as the main wind force–resisting systems (MWFRS) or as the seismic force–resisting systems (SFRS). Because of the extreme importance of this topic, this chapter provides an in-depth discussion of seismic load provisions of ASCE 7-05 Standard and design requirements pertaining to the many different types of shear walls as classified and permitted by the standard for use as lateral force–resisting systems.

Chapter 8 describes analysis and design of reinforced masonry earth-retaining walls and basement walls which are commonly used in practice.

Chapter 9 provides a discussion of masonry construction practices, with an emphasis on grouting practices. Masonry construction involves hand placement of brick or concrete masonry units interfaced with mortar, and then providing reinforcing bars as specified and followed by grouting. Following recommended procedures for all of these facets of construction is important to ensure intended performance of the as-built masonry structures.

Connection between masonry and other structural components, such as ledger beams or other load-carrying elements that are required to transfer forces through connections, is accomplished by anchorage. Chapter 10 is devoted to analysis and design of anchorage to masonry. The discussion in this chapter presents the various limit states that govern design of bolted connections to masonry.

The examples in each chapter are presented in a comprehensive, step-by-step manner that is easy to understand. Every step is worked out from first principles. Typical problems are provided at the ends of Chaps. 4 to 8 and 10 for readers' practice to develop confidence in understanding the subject matter.

The appendix provides many helpful tables that make analysis and design of masonry quick, efficient, and interesting, thus avoiding the drudgery of longhand calculations. Use of these tables is explained in the many examples presented in this book.

As with any professional book, readers will find many new terms introduced. A *glossary of terms* used in this book is provided following the appendix.

Narendra Taly, Ph.D., P.E., F.ASCE

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PREFACE TO THE FIRST EDITION

Reinforced masonry design and construction is both an art and science. Truly recognized as the oldest building material known to man, masonry has been used in one form or another since the dawn of history. The Sphinx, Coliseum (the Flavian Amphitheater in Rome), Parthenon, Roman aqueducts, the Great Wall of China, and many castles, cathedrals, temples, mosques, and dams and reservoirs all over the world stand as a testimony of enduring and aesthetic quality of masonry. Masonry construction continues to be used for many types of buildings, ranging from multistory high-rises to low-income apartment buildings. This book is intended for engineers, architects, builders, manufacturers of masonry products, and students who wish to engage in planning, design, construction, and acquire knowledge of masonry. It can be used as a useful reference volume by engineering professionals as well as a suitable textbook for students of masonry design and construction.

The book is the outgrowth of author's combined professional and teaching experience of 40 years. Its development began with a set of notes prepared for an senior/graduate course at the California State University, Los Angeles, beginning 1987 when the author developed a new course titled "Timber and Masonry Design," which he has been teaching ever since. These notes were expanded and periodically updated with the *Uniform Building Code,* which was revised every 3 years. No originality is claimed in writing this book, however. I acknowledge my debt to the numerous authors and organizations whose work I have quoted.

The book presents a comprehensive discussion on both theory and design of masonry structures built from clay and concrete masonry. Each chapter begins with *introduction* followed by discussion of theory of structural design using masonry as a structural material, which is quite general and not code-specific. The discussion is supplemented by several examples to illustrate the application of the principles involved. Engineering practice requires that structures be universally built according to building codes, which continue to change. Theory and examples presented in this book are referenced to building codes used in the United States. Because of the heavy use of the *Uniform Building Code* (UBC 1997) and its strong emphasis on reinforced masonry, it has been referenced in detail in this book. Considerable space has been given to discussion of earthquake loads and computational methods as specified in the UBC 1997. However, in text as well as in examples, pertinent references to the *International Building Code* (IBC 2000, the new code that will soon be adopted nationally), and the *Masonry Standards Joint Committee (MSJC) Code* (ACI 530-99/ASCE 5-99/TMS 402-99) have also been given. Wherever possible, all three codes have been referenced in the book. Equations and formulas have also been identified with proper references to the UBC, IBC, and MSJC *Code* for the use and convenience of a broad spectrum of readers.

Written for use by professionals of reinforced masonry design and construction, this book is written in a simple, practical and logical manner, and is formatted to suit as a text for teaching masonry design and construction in a classroom environment. Because of the practical nature of the subject, the first three chapters are devoted to a comprehensive discussion on masonry products, materials of construction, and building codes and ASTM Standards. Chapters 4 (flexural analysis and design of masonry beams), 5 (columns), 6 (walls subjected to axial and out-of-plane loads), 7 (shear walls), and 8 (retaining and subterranean walls) cover theory of design, followed by code requirements, and detailed examples. Chapters 6 and 7 introduce design for wind and earthquake loads with a comprehensive discussion of the seismic design provisions of 97-UBC. The examples in each chapter are presented in a comprehensive, step-by-step manner that is easy to understand. Every step is worked out from first principles.

Reinforced masonry consists of four different materials: masonry units, mortar, reinforcement, and grout. Masonry derives its strength from the assemblage of these four elements when they are laid together carefully by skilled masons. Therefore, construction procedures used in masonry work are just as important as design. In recognition, Chap. 9 is devoted to a comprehensive discussion on various aspects of masonry construction that include placement of reinforcement, mortar joints, grouting, curing, movement joints, and water-penetration resistance. Chapter 10 presents brief case studies of many masonry high-rise buildings to inform readers of the potential of masonry as versatile building material. This is followed by a discussion on planning and layout of masonry load-bearing building systems, and design example of a four-story concrete masonry shear wall building. Component design for masonry buildings is covered in Chaps. 4 through 8.

An extensive glossary of terms related to masonry has been provided following Chap. 10 for readers' quick reference.

The appendices in the book provide rich information. Appendix A presents 24 design tables referred to throughout the book. These are gathered together for easy reference, which makes it possible to use the book in design offices or teaching courses without the need for a handbook.

This book makes frequent references to Chaps. 16 (Structural Design Requirements) and 21 (Masonry) of the 97-UBC. These two chapters are provided in Apps. B1 and C, respectively, for ready reference. Appendix B2 presents a comprehensive discussion and examples of load combinations as specified in IBC 2000 and 1999-ACI *Code*. These load combinations are referred to throughout many examples in the book.

The design of masonry structures presented in this book is based on the *allowable stress design* (ASD) principles. Appendix D presents a comprehensive discussion on the strength design philosophy for masonry structures. Concepts of load factors, strength reduction factors, and slender wall, and the strength design provisions of the 97-UBC for masonry structures have been introduced. Detailed examples, including design of slender wall, based on the strength design principles have been presented in this appendix.

ACKNOWLEDGMENTS

The author is grateful for the help and assistance provided by many individuals with whom he had numerous discussions about the interpretation and the intent of the MSJC-08 *Code* during the writing of this book. Notably, these include Dr. Richard Bennett, Department of Civil Engineering, University of Tennessee, Knoxville, Tenn.; Dr. Dave McLean, Department of Civil Engineering, Washington State University, Pullman, Wash.; and Mr. Jason Thompson, Director of Engineering, National Concrete Masonry Association (NCMA), Herndon, Va.

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Great care has been exercised in organizing and presenting the material in this book, including giving due credit for permission. However, in spite of numerous proofreadings, it is inevitable that some errors and omissions will still be found. The author would be grateful to readers for conveying to him any errors they might find, and for any suggestions or comments offered.

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NOTATION

The following notation has been used in this book. Much of this notation is in conformance with IBC 2009, Chap. 21, Sec. 2102.2, and MSJC-08, Chap. 1, Sec. 1.5.

A_b = cross-sectional area of anchor bolt, in.^{2} (mm^2)

A_e = effective cross-sectional area of masonry, in.^{2} (mm^2)

A_g = gross cross-sectional area of masonry, in.^{2} (mm^2)

A_n = net cross-sectional area of masonry, in.^{2} (mm^2)

A_p = projected area on the masonry surface of a right circular cone for anchor bolt allowable shear and tension calculations, in.^{2} (mm^2)

A_{ps} = area of prestressing steel, in.^{2} (mm^2)

A_{pt} = projected area on masonry surface of right circular cone for calculating tensile breakout capacity of anchor bolts, in.^{2} (mm^2)

A_{pv} = projected area on masonry surface of one-half of a right circular cone for calculating tensile breakout capacity of anchor bolts, in.^{2} (mm^2)

A_s = effective cross-sectional area of reinforcement, in.^{2} (mm^2)

A'_s = effective cross-sectional area of compression reinforcement in a flexural member, in.^{2} (mm^2)

A_v = area of steel required for shear reinforcement perpendicular to the longitudinal reinforcement, in.^{2} (mm^2)

A_1 = bearing area, in.^{2} (mm^2)

A_2 = effective bearing area, in^2 (mm^2)

A_{st} = total area of laterally tied longitudinal reinforcing steel in a reinforced masonry column or pilaster, in.^{2} (mm^2)

a = depth of an equivalent compression zone (rectangular stress block) at nominal strength, in. (mm)

B_a = allowable axial force on an anchor bolt, lb (N)

B_{ab} = allowable axial force on an anchor bolt when governed by masonry breakout, lb (N)

B_{an} = nominal axial strength of an anchor bolt, lb (N)

B_{anb} = nominal axial force on an anchor bolt when governed by masonry breakout, lb (N)

B_{anp} = nominal axial force on an anchor bolt when governed by anchor pullout, lb (N)

B_{ans} = nominal axial force on an anchor bolt when governed by steel yielding, lb (N)

B_{ap} = allowable axial force on an anchor bolt when governed by anchor pullout, lb (N)

B_{as} = allowable axial force on an anchor bolt when governed by steel yielding, lb (N)

B_v = allowable shear force on an anchor bolt, lb (N)

B_{vb} = allowable shear load on an anchor bolt when governed by masonry breakout, lb (N)

B_{vc} = allowable shear load on an anchor bolt when governed by masonry crushing, lb (N)

B_{vn} = nominal shear load on an anchor bolt, lb (N)

B_{vnb} = nominal shear strength of an anchor bolt when governed by masonry breakout, lb (N)

B_{vnc} = nominal shear strength of an anchor bolt when governed by masonry crushing, lb (N)

B_{vns} = nominal shear strength of an anchor bolt when governed by steel yielding, lb (N)

B_{vpry} = allowable shear load on an anchor bolt when governed by anchor pryout, lb (N)

B_{vnpry} = nominal shear strength of an anchor bolt when governed by anchor pryout, lb (N)

B_{vs} = allowable shear load on anchor bolt when governed by steel yielding

b = width of section, in. (mm)

b_a = total applied design axial force on an anchor bolt, lb (N)

b_{af} = factored axial force in an anchor bolt, lb (N)

b_v = total applied design shear force on an anchor bolt, lb (N)

b_{vf} = factored shear force in an anchor bolt, lb (N)

b_w = width of wall beam

C_d = deflection amplification factor

c = distance from the neutral axis to extreme compression fiber, in. (mm)

D = dead load or related internal forces and moments

d = distance from the extreme fibers of a flexural member to the centroid of longitudinal tension reinforcement, in. (mm)

d_b = diameter of the reinforcing bar or anchor bolt, in. (mm)

d_v = actual depth of masonry in the direction of shear considered, in. (mm)

E = load effects of earthquake or related internal forces and moments

E_{AAC} = modulus of elasticity of AAC masonry in compression, psi (MPa)

E_m = modulus of elasticity of masonry in compression, psi (MPa)

E_s = modulus of elasticity of steel, psi (MPa)

E_v = modulus of rigidity (shear modulus) of masonry, psi (MPa)

e = eccentricity of axial load, in. (mm)

e_b = projected leg extension of bent bar anchor, measured from inside edge of anchor at bend to farthest point of anchor in the plane of the hook

e_u = eccentricity of P_{uf}, in. (mm)

F = lateral pressure of liquids or related internal forces and moments

F_a = allowable compressive stress due to axial load only, psi (MPa)

F_b = allowable compressive stress due to flexure only, psi (MPa)

F_s = allowable tensile or compressive stress in reinforcement, psi (MPa)

F_v = allowable shear stress in masonry, psi (MPa)
f_a = calculated compressive stress in masonry due to axial load only, psi (MPa)
f_b = calculated compressive stress in masonry due to flexure only, psi (MPa)
f'_{AAC} = specified compressive strength of AAC, the minimum compressive strength for a class of AAC as specified in ASTM C1386, psi (MPa)
f'_g = specified compressive strength of grout, psi (MPa)
f'_m = specified compressive strength of masonry, psi (MPa)
f'_{mi} = specified compressive strength of masonry at the time of prestress transfer, psi (MPa)
f_{ps} = stress in prestressing tendon at nominal strength, psi (MPa)
f_{pu} = specified tensile strength of prestressing tendon, psi (MPa)
f_{py} = specified yield strength of prestressing tendon, psi (MPa)
f_r = modulus of rupture, psi (MPa)
f_{rAAC} = modulus of rupture of AAC, psi (MPa)
f_s = calculated tensile or compressive stress in reinforcement, psi (MPa)
f_{se} = effective stress in prestressing tendon after all prestress losses have occurred, psi (MPa)
f_{tAAC} = splitting tensile strength of AAC as determined in accordance with ASTM C1006
f_v = calculated shear stress in masonry, psi (MPa)
f_y = specified yield strength of steel for reinforcement and anchors, psi (MPa)
H = lateral pressure of soil or related forces and moments
h = effective height of column, wall, or pilaster, in. (mm)
h_w = height of entire wall or segment of wall considered, in. (mm)
I_{cr} = moment of inertia of cracked cross-sectional area of member, in.^{4} (mm^4)
I_{eff} = effective moment of inertia of member, in.^{4} (mm^4)
I_g = moment of inertia of gross (or uncracked) cross-sectional area of member, in.^{4} (mm^4)
j = ratio of distance between centroid of flexural compressive forces and centroid of tensile forces to depth, d
K = dimension used to calculate reinforcement development length, in. (mm)
K_{AAC} = dimension used to calculate reinforcement development length for AAC masonry, in. (mm)
K_c = coefficient of creep of masonry, per psi (MPa)
k_e = coefficient of irreversible moisture expansion of clay masonry
k_t = coefficient of thermal expansion of masonry, per degree Fahrenheit (degree Celcius)
L = live load or related forces and moments
l = clear span between supports, in. (mm)
l_b = effective embedment length of plate, headed or bent anchor bolts, in. (mm)
l_{be} = anchor bolt edge distance, measured in the direction of load, from edge of masonry center of cross section of anchor bolt, in. (mm)
l_d = development length or lap length of straight reinforcement, in. (mm)

l_e = equivalent embedment length provided by standard hooks measured from the start of the hook (point of tangency), in. (mm)

l_p = clear span of prestressed member in the direction of prestressing tendon, in. (mm)

l_w = length of the entire wall or of segment of wall considered in the direction of shear force

M = maximum moment in section under consideration, in.-lb (N-mm)

M_a = maximum moment in member due to the applied loading for which deflection is considered, in.-lb (N-mm)

M_{cr} = nominal cracking moment strength, in.-lb (N-mm)

M_n = nominal moment strength, in.-lb (N-mm)

M_{ser} = service moment at midheight of a member, including P-delta effects, in.-lb (N-mm)

M_u = factored moment, in.-lb (N-mm)

n = modular ratio = E_s/E_m

N_u = factored compressive force acting normal to shear force that is associated with V_u loading combination case under consideration, in.-lb (N-mm)

N_v = compressive force acting normal to shear surface, lb (N)

P = axial load, lb (N)

P_a = allowable axial compressive force in reinforced member, lb (N)

P_e = Euler's buckling load, lb (N)

P_n = nominal axial strength, lb (N)

P_{ps} = prestressing tendon force at time and location relevant for design, lb (N)

P_u = factored axial load

P_{uf} = factored axial load from tributary floor or roof areas under consideration, lb (N)

P_{uw} = factored weight of wall area tributary to wall section under consideration, lb (N)

Q = first moment about the neutral axis of an area between the extreme fibers and the plane at which the shear stress is being calculated, in.^{3} (mm^3)

Q_E = the effect of horizontal seismic (earthquake) forces

R = seismic response modification factor

r = radius of gyration, in. (mm)

S = section modulus of the gross cross-sectional area of a member, in.^{3} (mm^3)

S_n = section modulus of the net cross-sectional are of a member, in.^{3} (mm^3)

s = spacing of reinforcement, in. (mm)

T = forces and moments caused by restraint of temperature, creep, and shrinkage, or differential settlement

t = nominal thickness of a member, in. (mm)

U = required strength to resist factored loads, or related internal moments and forces

v = shear stress, psi (MPa)

V = shear force, lb (N)

V_{AAC} = shear strength provided by AAC masonry, lb (N)

$V_{m'}$ = shear strength provided by masonry, lb (N)
V_n = nominal shear strength, lb (N)
V_{nm} = nominal shear strength provided by masonry, lb (N)
V_{ns} = nominal shear strength provided by shear reinforcement, lb (N)
V_u = factored shear force, lb (N)
W = wind load or related internal forces and moments
w_{strut} = horizontal projection of the width of the diagonal strut, in. (mm)
w_u = out-of-plane distributed load, lb/in. (N/mm)
α = tension reinforcement strain factor
β = 0.25 for fully grouted masonry or 0.15 for other than fully grouted masonry
β_b = ratio of area of reinforcement cut off to total area of tension reinforcement at a section
γ = reinforcement size factor
Δ = calculated story drift, in. (mm)
Δ_a = allowable story drift, in. (mm)
δ = moment magnification factor
δ_{ne} = displacements computed using code prescribed seismic forces and assuming elastic behavior
δ_s = horizontal deflection at midheight under service loads, in. (mm)
δ_u = deflection due to factored loads, in. (mm)
ε_{cs} = drying shrinkage of AAC
ε_{mu} = maximum usable compressive strain of masonry
ε_s = strain in steel reinforcement
ε_y = yield strain in steel reinforcement
μ_{AAC} = coefficient of friction of AAC
ϕ = strength reduction factor
ρ = reinforcement ratio
ρ_b = reinforcement ratio producing balanced strain conditions
$\rho_{\max}$ = maximum flexural reinforcement ratio

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ACRONYMS

The following is a list of acronyms/abbreviations frequently used in this text:

AASHTO	American Association of State Highway and Transportation Officials
ACI	American Concrete Institute
AISC	American Institute of Steel Construction
AISCM	American Institute of Steel Construction Manual
AISCS	American Institute of Steel Construction Specifications
AISI	American Iron and Steel Institute
AITC	American Institute of Timber Construction
ANSI	American National Standards Institute
APA	American Plywood Association
ASCE	American Society of Civil Engineers
ASD	allowable stress design
ASTM	American Society for Testing and Materials
B&S	beams and stringers
BIA	Brick Institute of America, also Brick Industry Association
BOCA	Building Officials and Code Administrators International
CABO	Council of American Building Officials
CMACN	Concrete Masonry Association of California and Nevada
CRC	Column Research Council
CRSI	Concrete Reinforcing Steel Institute
FEMA	Federal Emergency Management Agency
IBC	International Building Code
ICBO	International Conference of Building Officials
IMI	International Masonry Institute
LF	light framing
LFD	load factor design
LRFD	load and resistance factor design
MIA	Masonry Institute of America
MSJC	Masonry Standards Joint Committee
NCMA	National Concrete Masonry Association
NDS	National Design Specifications for Wood Construction
NEHRP	National Earthquake Hazard Reduction Program
PCA	Portland Cement Association
PCI	Prestressed Concrete Institute

PD	plastic design
P&T	posts and timbers
SBCCI	Southern Building Code Congress International
SCPI	Structural Clay Products Institute (now BIA)
SCR	structural clay (trademark of Structural Clay Products Institute, now BIA)
SEI	Structural Engineering Institute
SSRC	Structural Stability Research Council
TCM	Timber Construction Manual
TMS	The Masonry Society
UBC	Uniform Building Code
W	wide flange shape
WSD	working stress design
WWPA	Western Wood Products Association

DESIGN OF REINFORCED MASONRY STRUCTURES

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CHAPTER 1
INTRODUCTION

1.1 WHAT IS MASONRY?

Masonry is one of the oldest forms of construction known to humans. The term *masonry* refers generally to brick, tile, stone, concrete-block etc., or combination there of, bonded with mortar. However, many different definitions of masonry are in vogue. The *International Building Code* (IBC 2009) [1.1] defines masonry as “a built-up construction or combination of building units or materials of clay, shale, concrete, glass, gypsum, stone or other approved units bonded together with or without mortar or grout or other accepted methods of joining.” ASTM E631 defines *masonry* as “construction usually in mortar, of natural building stone or manufactured units such as brick, concrete block, adobe, glass, block tile, manufacture stone, or gypsum block.” The *McGraw-Hill Dictionary of Scientific and Technical Terms* defines masonry as “construction of stone or similar materials such as concrete or brick.” A commonality in these various definitions is that masonry essentially is an *assemblage of individual units* which may be of the same or different kind, and which have been bonded together in some way to perform intended function. An interesting discussion on the various definitions can be found in Ref. [1.2].

1.2 PLAIN AND REINFORCED MASONRY

From a structural engineering perspective, masonry is classified as *plain* or *unreinforced* masonry and *reinforced* masonry. Plain masonry refers to construction from natural or manufactured building units of burned clay, concrete, stone, glass, gypsum, or other similar building units or combination thereof, made to be bonded together by a cementitious agent. The strength of plain masonry depends primarily on the high compressive strength of masonry units. Plain masonry, like plain concrete, possesses little tensile strength. Therefore, it cannot be used as an efficient building material for structures or structural elements that must resist tensile forces. The poor tensile strength of plain masonry makes it unsuitable for horizontal spanning structural elements such as beams and slabs, which resist loads in flexure and, thereby, are subjected to tensile stresses. Similarly, plain masonry also cannot be used for columns subjected to eccentric loads that will produce tensile stresses in them. To overcome this drawback, plain masonry is strengthened with reinforcing materials such as steel bars, which greatly enhance both its tensile as well as compressive strength. This later form of masonry construction is referred to as *reinforced* masonry. Stated simply, reinforced masonry construction is “masonry construction in which reinforcement acting in conjunction with the masonry is used to resist forces” [1.1]. Reinforced *grouted masonry* and reinforced *hollow unit masonry* are subheads that are used in the building codes to characterize different forms of reinforced masonry construction.

Masonry construction is accomplished by laying masonry units by hand. As such, they can be laid in a variety of arrangements. Units of different sizes can be used in the same construction. For example, various-sized rectangular units having sawed, dressed, or squared surfaces, properly bonded and laid in mortar, can be used for wall construction, an arrangement of units referred to as *ashlar* masonry. The same masonry can be laid in courses of equal or different heights and is then referred to as *coursed ashlar* masonry. The term *random ashlar* is used to describe ashlar masonry laid in courses of stones without continuous joints and laid up without drawn patterns. Masonry units can be solid or hollow. *Solid* masonry consists of masonry units that are solid (i.e., without any voids) and laid contiguously with the joints between the units filled with mortar. Various types of units and their arrangements are described in Chap. 2.

1.3 A BRIEF HISTORY OF MASONRY CONSTRUCTION

The history of masonry construction can be considered as the beginning of the history of civil engineering. Naturally availability of stones has been responsible for masonry being the oldest building material known to humans. The first use of stones for any form of construction was *random rubble dry masonry*, a form of construction in which stones of various sizes were randomly stacked on top of each other to build a wall without using any mortar (hence, the term *dry masonry*). Smaller stones were used to fill the voids between the larger stones; mud was used sometimes to bond the stones together. This form of construction is still in use today in some third world countries, used mainly for building temporary wall fences for rural farm areas and land, and for retaining walls. A variation of random rubble dry masonry uses horizontal and vertical bands of lime or cement mortar at regular intervals in otherwise dry wall. Unreinforced masonry has been used for centuries throughout the world, and is still in use today for construction of buildings and small dams. These structures typically use masonry fully grouted in cement or lime mortar, with stones of approved geologic classification, size, and quality. Voids between the large grouted stones are packed with small stones and mortar. The exterior surfaces of these structures may have finish with coursed or uncoursed masonry with *flush* or *pointed* joints.

Soon to follow the use of natural stone as a building material was the man-made building material called *brick*. The art of brick building is reported to be some 10 to 12 millennia old. The earliest type of brick, called “adobe,” evolved as sun-dried lumps of mud or clay, which later developed into a preformed modular masonry unit of sun-dried mud. Sun-dried bricks are known to have been widely used in Babylon, Egypt, Spain, and South America. These bricks were made by pressing mud or clay into small lumps [1.3]. An excellent documentation of adobe brick wall construction has been given by McHenry [1.4].

The earliest molded brick was developed, supposedly in Mesopotamia, about 5000 B.C. However, it was the invention of the *fired* brick about 3500 B.C. that revolutionized structural construction, and gave birth to permanent structures all over the world. Firing gave the brick the quality of resilience which the mud bricks lacked–its most significant aspect was the simplicity with which bricks could be easily shaped and used for potentially endless exact repetitions of decorative patterns. Glazing, a subsequent development, made it possible to provide rich ornament in brick as well to produce brick in vivid colors.

Brick continues to be used today as one of the most versatile building materials, with a rich, enduring, and illustrious history. Some of the most magnificent and remarkable brick structures built in the past stand as testimony to the architectural elegance and potential of brick as a structural material. These include the Hanging Gardens, one of the Seven Wonders of the World; the Great Wall of China, the largest man-made object on the earth; the Hagia Sophia, one of the most beautiful churches ever built; the great medieval castle of Malbork, Poland, which is the size of a small town; the 2000 temples in Pagan, Burma, which have survived intact for 900 years; the structure of the Taj Mahal, India; and the 1200 miles of sewers which the Victorians built under the city of London.

The earliest evidence of masonry construction is the arches found in the excavations at Ur in the Middle East. These ruins have been dated at 4000 B.C. Arch structures dating to 3000 B.C. have been found in Egypt [1.5]. The oldest surviving stone masonry structure is said to be an arch bridge over Meles River at Smyrna, Turkey [1.6]. The advanced civilizations in Mesopotamia and Egypt used stone masonry for building arches and vaults. Sumerians, living in the Tigris-Euphrates Valley in Mesopotamia, are known to have used bricks and stones as early as 4000 B.C. Their permanent building material was brick, at first only sun-baked; burned bricks were rare because of lack of fuel. Stone was used sparingly because it had to be imported by way of water from Persia or else carried by humans down from the eastern hills [1.7]. The pyramid of Khufu in Egypt, built about 2700 B.C., is still one of the largest single stone masonry structures built by humans even though its original height of 147 m (482 ft) is now reduced to 137 m* [1.8]. Romans of the pre-Christian period were master builders of the earlier civilizations; they built monumental buildings, bridges, and aqueducts, and used solid squared-stone masonry exhaustively. The earliest specimen of a stone arch, a *voussoir* arch, still extant in Rome, is over a drain in front of the Temple of Saturn, built between the sixth and the fourth centuries B.C. The Romans preferred use of stones over the sun-dried bricks, apparently for reasons of durability. Augustus boasted that he "found Rome in brick and left it of marble." Although baked bricks were also used by Romans for building arches, it was in stone that they made their most significant contribution. The description of the building skills of early Romans can be found in the writings of Marcus Vitruvius Pollio, the famous mason and architect who lived in the first century B.C. [1.9] and whose *Ten Books on Architecture* [1.10] comprise the earliest building manual to survive from the earliest times (all earlier works have been lost). The Great Wall of China built more than 600 years ago is made from stones and bricks† [1.11]. A historical description of masonry construction and the many magnificent ancient stone buildings can be found in the literature [1.8, 1.12]; a brief summary can be found in Ref. [1.3]. An interesting and pictorial description of the historical development of brick construction through ages can be found in Ref. [1.13] which cites 171 references on the subject matter. One of the best introductions to history of brick construction all over the world is a collection of essays in Ref. [1.14].

1.4 EVOLUTION OF REINFORCED MASONRY

In unreinforced masonry structures, the lateral stability is provided by gravity. Because masonry is weak in tension, no tension can be allowed to develop at the base of the structure. This requires unreinforced masonry structures to be sufficiently massive (meaning large base width) that the resultant of all forces acting on the structure does not fall outside the middle third of the base. This requirement imposes an economic limit on the height of the masonry structures that can be built. Furthermore, slender structures proved incapable of withstanding lateral loads due to earthquakes as demonstrated by damage during seismic events in many countries throughout the world, such as India, China, Iran, Mexico, the former U.S.S.R., and Turkey, to name a few. Extensive damage and collapse of masonry structures during earthquakes continue to demonstrate the need for a better engineered construction. Reinforced masonry provided the required answer, and thus began the

*The height of this pyramid was not surpassed by any Gothic cathedral, except the Beauvias cathedral in the north of France whose tower collapsed in the year 1284, 12 years after its completion. The spire of Ulm Cathedral, built in the nineteenth century, is slightly taller (159 m). The first multistory structure to exceed it in height was the Singer Building in New York (206 m or 675 ft) erected in 1907 [1.8].

†The Great Wall of China is reported to have first appeared in the seventh century B.C. and was strengthened or expanded in the succeeding 2300 years virtually by every dynasty in China [1.11].

present-day engineered-masonry construction, which uses methods completely different from the empirical methods of the past. What once evolved as merely masons' creations came to be designed and built as engineered structures.

Reinforced masonry construction as we know it today is rather recent. The principles of reinforced masonry construction are said to have been discovered by Marc Isambard Brunel, once a chief engineer for New York City, a great innovator, and one of the greatest engineers of his time. In 1813, he first proposed the use of reinforced brick masonry as a means of strengthening a chimney then under construction. However, his first major application of reinforced masonry was in connection with the building of the Thames Tunnel in 1825. As a part of this construction project, two brick shafts were built, each 30 in. thick, 50 ft in diameter, and 70 ft deep. These shafts were reinforced vertically with 1-in.-diameter wrought iron rods, built into the brickwork. Iron hoops, 9 in. wide and ½ in. thick, were laid in the brickwork as the construction progressed [1.15]. Continuing his work with reinforced brick masonry, Brunel, in 1836, constructed test structures in an effort to determine the additional strength contributed to the masonry by the reinforcement.

The credit for the modern development of reinforced brick masonry is generally given to A. Brebner, once an Under Secretary in the Public Works Department, Government of India, who conducted pioneering research on reinforced brick. In his report of extensive tests on reinforced brick masonry conducted over a two-year period and published in 1923 [1.16], Brebner stated that "nearly 3,000,000 ft^2 have been laid in the last three years." Thus began the era of *reinforced* brick construction. Reinforced brickwork was quickly followed in Japan. Skigeyuki Kanamori, Civil Engineer, Department of Home Affairs, Imperial Japanese Government, is reported to have stated [1.17]:

> There is no question that reinforced brickwork should be used instead of (unreinforced) brickwork when any tensile stress would be incurred in the structure. We can make them safer and stronger, saving much cost. Further I have found that reinforced brickwork is more convenient and economical in building than reinforced concrete and, what is still more important, there is always a very appreciable saving in time.

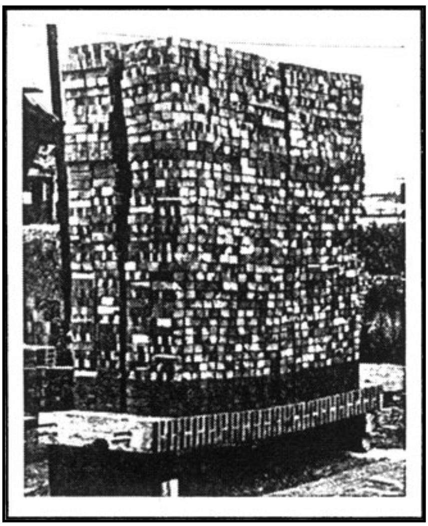

FIGURE 1.1 Early test of reinforced concrete brick masonry element [1.10].

Structures designed by Kanamori include sea walls, culverts, and railways retaining walls, as well as buildings [1.18].

Research on brick construction in the United States is credited to the work undertaken by the Brick Manufacturers Association of America and continued by the Structural Clay Products Institute and the Structural Clay Products Research Foundation (SCR). This research effort generated much valuable information on various aspects of reinforced brick masonry. Since 1924, numerous field and laboratory tests have been made on reinforced brick beams, slabs, columns, and on full-size structures. Figure 1.1 shows an example of a 1936 test to demonstrate the structural capabilities of reinforced brick elements [1.18].

Concrete block masonry units (often referred to as CMU) were developed by the construction industry in the 1930s. Use of steel reinforcement for concrete masonry began between 1930 and 1940. In the ensuing years, reinforced concrete masonry became a viable construction practice for single and multistory buildings such as schools, hospitals, hotels, apartment complexes, commercial, shopping centers, and industrial and office buildings. One of the tallest modern reinforced concrete masonry structures is the 28-story Excalibur Hotel in Las Vegas, Nevada. The load-bearing walls of this four-building complex are built from concrete masonry of 4000 psi compressive strength [1.19]

Research on masonry continues in the United States in an organized way. In 1984, the Technical Coordinating Committee for Masonry Research (TCCMR) was formed for the purpose of defining and performing both experimental and analytical research and development necessary to improve structural masonry technology [1.20]. Masonry construction, which evolved as masons' creations, turned into engineered construction based first on *empirical design* and later on engineering principles. From 1984 to 1994, 19 researchers conducted the most extensive research program ever into the development of a *limit states design standard* for the design of masonry buildings in seismic areas [1.21].

1.5 UNREINFORCED AND REINFORCED MASONRY

Unreinforced masonry has been in use in the United States as in the rest of the world for many centuries. The early masonry structures were unreinforced and built to support only the gravity loads; lateral forces from wind and earthquakes were ignored (for lack of basic knowledge of dynamic forces). The massiveness of these structures provided stability against lateral loads. Stone masonry dams and reservoirs are examples of unrfeinforced masonry structures that resisted water pressure through their massiveness. However, the lateral load resisting capability of ordinary masonry structures had always been questionable. In the western United States, the inherent weakness of unreinforced masonry structures to resist lateral loads was clearly exposed during the 1933 Long Beach earthquake (M 6.3).* Although strong enough to resist gravity loads, these structures proved incapable of providing the required lateral resistance to seismic forces. Thus, in the ensuing period, reinforcing of masonry construction was codified, resulting in the modern form of engineered reinforced masonry construction. A significant advantage of reinforced masonry was dramatic reduction in the thickness of walls that were designed to resist dynamic lateral loads due to wind and earthquakes.

Poor performance of unreinforced masonry was evident during the October 1, 1987 Whittier Narrows earthquake (M 6.3) and the October 17, 1989 Loma Prieta earthquake (M 7.1) [1.21] in the United States, and during many earthquakes in other parts of the world. In the January 17, 1994 Northridge earthquake ($M_w = 6.7$), hundreds of unreinforced masonry structures were severely damaged and some simply collapsed. Many engineered reinforced masonry structures and retrofitted unreinforced masonry structures also were severely damaged during this earthquake, due presumably to poor engineering design, lack of proper detailing, or as a result of poor workmanship and quality control. Extensive destruction of unreinforced masonry structures during these earthquakes again called attention to, among other factors, poor tension and shear resistance of unreinforced masonry.

*The devastating effects of this earthquake served as wake-up call to the rapidly growing Southern California region from which evolved a new profession: earthquake engineering. The 1933 earthquake led to the passage of two laws in California—one outlawing the construction of unreinforced brick buildings in the State of California and the other requiring that all school buildings be designed to meet specific earthquake resistance standards.

Wind loads constitute a major lateral force that masonry structures, like all other structures, must be able to resist. Ability of masonry structures to resist high wind loads has been demonstrated by their performance in the regions of hurricanes and tornadoes. Reinforced masonry buildings in the coastal region of North Carolina that were subjected to wind gusts up to 115 mph due to Hurricane Fran on September 6, 1996, performed well [1.22].

1.6 HISTORICAL DEVELOPMENT OF BUILDING CODES AND STANDARDS FOR MASONRY CONSTRUCTION

Codes, standards, and *specifications* are documents that embody available professional and technical knowledge required for completing a project. Structural engineering is a broad and multifaceted discipline that involves knowledge of many fields, and no one person can be an expert in all these fields. Furthermore, the accumulated and the newly found knowledge, and the complex research developments need to be translated into simple procedures suitable for routine design purposes. This goal is accomplished with the help and guidance of many experts who are well versed in the many subdisciplines of structural engineering, resulting in documented standards and procedures (codification) to be followed for successful completion of a structure that would be safe. *Codes* and *standards*, which are the resulting documents, thus become authoritative source of information for designers and builders; they represent a unifying order of engineering practice.

Concern for the safety of occupants in buildings has been evident in the recorded laws of some of the most ancient civilizations. Figure 1.2 shows a portion of the *Code of Hammurabi,* written during circa 1780–1727 B.C. (and predating the Hebrew "Ten Commandments" by some 500 years) by King Hammurabi, the most famous Mesopotamian king who wrote some 282 laws that were depicted on stelae [1.23].

As a general practice, the regulation of building construction in the United States is accomplished through building codes. The purpose of a building code is to establish minimum acceptable requirements considered necessary for preserving public health, safety, and welfare in the built environment. Building codes provide a legal basis to accomplish this objective as best expressed by the *International Building Code* [1.1]:

> This code is founded on principles intended to establish provisions consistent with the scope of a building code that adequately protects public health, safety and welfare; provisions that do not necessarily increase construction costs; provisions that do not restrict the use of new materials, products or methods of construction; provisions that do not give preferential treatment to particular types of classes of materials, products or methods of construction.

The primary application of a building code is to regulate new or proposed construction. However, they are also used to enforce safety criteria for the existing structures. While the concerns of life and fire safety and structural adequacy have traditionally remained as the main preoccupation of building codes, they also deal with other issues such as the type of construction materials used, lighting and ventilation, sanitation, and noise control.

In the context of design and construction, a *code* may be defined as a systematically arranged and comprehensive collection of laws, or rules and regulations, especially one given statutory status. A *building code* generally covers all facets related to a structure's safety, such as design loads, structural design using various kinds of materials (steel, concrete, timber, aluminum, etc.), architectural details, fire protection, plumbing, heating and air conditioning, lighting, sanitation, etc. *Specifications* comprise a detailed statement of

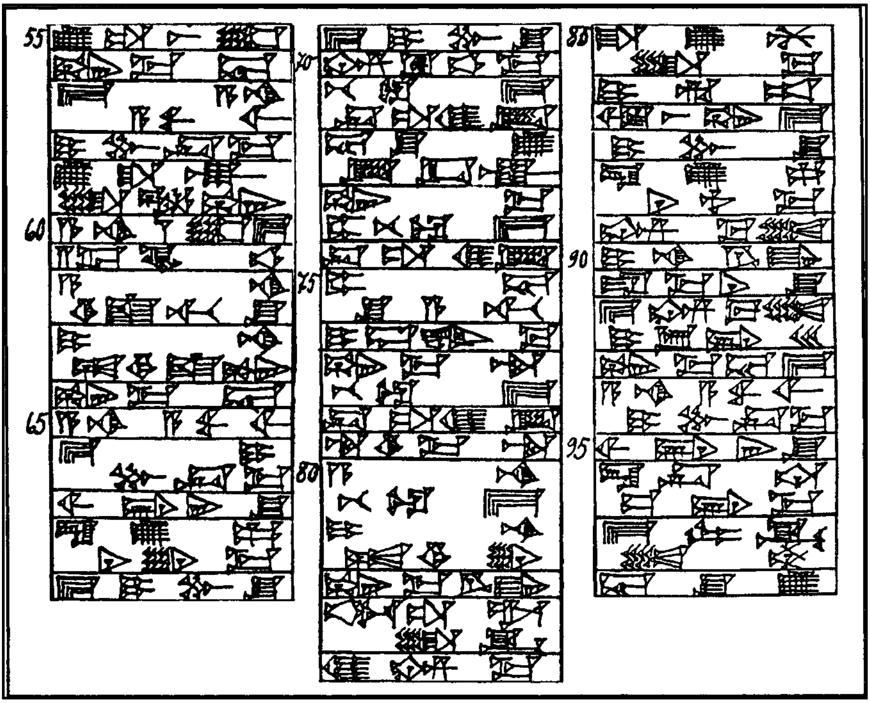

FIGURE 1.2 *Code of Hammurabi* (circa 1780–1727 B.C.). The six laws addressing the construction industry, covering the prices of construction and contractor liability, read (translation) [1.24]: If a builder build a house for some one and complete it, he shall give him a fee two shekels for each sar of surface. If a builder build a house for some one and does not construct it properly, and the house which be built fall in and kill its owner, then that builder shall be put to death. If it kill the son of the owner, the son of that builder shall be put to death. If it kill a slave of the owner, then he shall pay slave for slave to the owner of the house. If it ruin goods, he shall make compensation for all that has been ruined, and in as much as he did not construct this house properly this house which he built and it fell, he shall re-erect the house from his own means. If a builder build a house for some one, even though he has not yet completed it, if then walls seem toppling, the builder must make the wall solid from his own means.

particulars and procedures for design work, often for one project at hand; they are complied by the interested group or individuals.

Building codes are legal documents that comprise systematic collections of rules and regulations; many of which are adopted from model building codes. They advance minimum requirements that will ensure adequate levels of public safety under most conditions. *Model codes* are consensus documents that are written in a language that can be adopted by governmental agencies (city, county, and state) as legal documents. Codes contain statements such as "*shall be*" or "*may be*." The language in the first statement conveys that the specified requirement is mandatory; by contrast, the latter statement conveys that the specified requirement is discretionary.

In the United States, up until the year 2000, with the exception of some large cities and several states, there were three model codes used.

1. *Uniform Building Code (UBC) [1.25]:* It was published by the International Conference of Building Officials (ICBO), Whittier, California, and widely used in the West, in the area extending from the Mississippi River to the West Coast. The ICBO was formed in 1922 and was the first to publish a building code; its first edition was published in 1927 and the last in 1997. The UBC was best known for its seismic design provisions.

2. *BOCA National Building Code (NBC) [1.26]:* Earlier known as the *Basic Building Code*, it was published by the Building Officials and Code Administrators International (BOCA), Country Club Hills, Illinois and widely used in Eastern and North-Central states, in the area extending from the East Coast to the Mississippi River. Founded in 1915, BOCA published the first edition of the code in 1950.
3. *The Standard Building Code (SBC) [1.27]:* It was published by the Southern Building Code Congress International (SBCCI), Birmingham, Alabama, and widely used in the South and the Southeast. The SBCCI was founded in 1940, with the first edition of the code published in 1945. The SBCCI code was best known for its high wind load provisions.

Although the three model codes had been in use for many years, they suffered from duplication of work related to various provisions and format, and lacked uniformity until recently. Because of the vast geographical area of the United States, some differences in the building codes were obviously justifiable. These included geographical, climatic, and environmental differences, differences due to soil conditions, and region's susceptibility to natural hazards such as earthquakes, tornadoes, hurricanes, and floods. But many other differences, such as determining the built-up area and building height, could hardly be justified. The duplication and nonuniformity of codes became a growing concern of the building officials throughout the country. In order to address these concerns at the national level, the Council of American Building Officials (CABO) was formed in 1972. A major success of this organization was the adoption of a common format* by all the three model code organizations, and recognition of the need for, and importance of, a single code to replace the three existing model building codes. The result was the formation of a new organization in 1994, the International Code Council, to develop a single set of regulatory documents covering building, mechanical, plumbing, fire, and related regulations. The result of this joint effort was the *International Building Code* (2000 IBC) the first edition of which was published in the year 2000. Following the practice of the earlier model codes, IBC is updated on a three-year cycle, the current edition being 2009 IBC. With the advent of the *International Building Code*, the separate codes put forth by BOCA, ICBO, and the SBCCI have been phased out, and are no longer published.

The material presented in this book is referenced to two codes because of their present and future uses in the United States: *2009 International Building Code* (2009 IBC) [1.1] and the Masonry Standards Joint Committee *Code* (hereinafter referred to as the MCJC-08 *Code*) [1.28]. The MSJC *Code* has been incorporated in the IBC as a *reference code* and is briefly described in the following paragraphs.

The masonry industry has long needed a unified standard for all segments of related work and materials. The American Concrete Institute (ACI), American Society of Civil Engineers (ASCE), and The Masonry Society (TMS) promulgate a national standard for the structural design of masonry elements and standard specification for masonry construction. The development of a single standard for design and construction of masonry structures began in 1977. At that time, there were several design standards for masonry, all of which did not have consistent requirements. Therefore, it was difficult for engineers

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*The common code format means that all model codes are organized into identical chapter headings, chapter sequence, and chapter contents. For example, Chap. 16 in all three model codes deals with the "structural design requirements."

and architects to select appropriate design criteria for masonry construction. Concerned professionals in the masonry industry recognized the need for a single, national consensus standard for the design and construction of all types of masonry. In 1977, the ACI and ASCE agreed jointly to develop such a standard with the help and support of the masonry industry. The MSJC was formed with balanced membership of building officials, contractors, university professors, consultants, material producers, and designers who were members of the ACI and ASCE. TMS joined as sponsoring organization in 1991. Through this effort evolved the two documents (published as a set in one document) titled the *Building Code Requirements for Masonry Structures* (ACI 530/ASCE 5/TMS 402) and the *Specification for Masonry Structures* (ACI 530.1/ASCE 6/TMS 602)*, aimed at consolidating and advancing existing standards for the design and construction of masonry. Approval of the first edition of the MSJC *Code* and *Specification* occurred in June 1986. Public review began in 1988 with the final approval of the 1988 MSJC *Code* and *Specification* in August 1988 [1.29].

The MSJC *Code* covers the design and construction of masonry structures while the MSJC *Specification* is concerned with minimum construction requirements. As a source of valuable information, *commentaries* for the MSJC *Code* and *Specification* were also developed. These documents provide background information on the design and specification provisions. They contain considerations of the MSJC members in determining requirements and references to research papers and articles.

The MSJC *Code*, *Specification,* and *Commentaries* are revised on a three-year cycle. The first revision was issued in 1992, the second in 1995, the third in 1999, the fourth in 2002, the fifth in 2005, and the current edition in 2008. The 1995 edition included significant changes from its 1992 version, with addition of a new chapter on glass unit masonry, masonry veneers, seismic design, and a total reformat of the MSJC *Specification*. Thus, for the first time in the history of masonry standards, brick, concrete, glass block, composite construction, and veneers appeared in the same documents. The 2008 MSJC *Code* incorporated complete revisions pertaining to anchor bolts, seismic design requirements, and several others design related revisions.

Topics covered in the 2008 MSJC *Code* include definitions, contract documents, quality assurance, materials, placement of embedded items, analysis and design, strength and serviceability, flexural and axial loads, shear, details and development of reinforcement, walls, columns, pilasters, beams and lintels, seismic design requirements, prestressed masonry, veneers glass unit masonry, veneers, and autoclaved aerated masonry. An empirical design method and a prescriptive method applicable to buildings meeting specific location and construction criteria are included. The *Specification* covers topics such as quality assurance requirements for materials, the placing, bonding and anchoring of masonry, and the placement of grout and of reinforcement. An important provision in the 2008 MSJC *Code* is Section 1.17 which deals with *seismic design requirements*.

1.7 DESIGN METHODS

For many years, masonry structures have been and continue to be designed based on the traditional *allowable stress design* method (also called service load method or working stress design). In this method, a structure is proportioned (designed) to resist code-specified service loads, which are assumed to be loads that a structure might be subjected to during its service life. The allowable (or working) stresses used in design are a fraction of the accepted failure strengths of materials (viz., compressive strength of masonry and yield

*Together, the two documents are referred to as the MSJC *Code* and *Specification*.

strength of steel reinforcement) used in design. The structure is so proportioned that actual (i.e., calculated) stresses do not exceed the allowable stresses. Following the trend in design methodology being used for concrete structures, design codes such as 2009 IBC and MSJC-08 *Code* now permit design of masonry structures based on the strength design concept. The basic premise of the strength method is that it results in more economical structures and provides more realistic consideration of safety.

In the strength design method, the code-specified service loads (such as dead load, live load, wind load, earth pressure, fluid pressure, etc.) assumed to act on a structure are augmented by multiplying with certain factors called *load factors* (which are different for different loads, also called γ factors in some design codes), resulting in what are called *factored loads*. The structure is then so proportioned that its design strength, ϕR_n, is equal to or greater than the required strength (i.e., effects of factored loads, also sometimes referred to as *demand*). This simple relationship can be expressed by Eq. (1.1):

$$\text{Design strength} \geq \text{required strength}$$

$$\phi R_n \geq U \tag{1.1}$$

where ϕ = strength reduction factor
R_n = resistance offered by structure (or nominal strength)
U = the required strength (i.e., effects of factored loads on the structure)

Equation (1.1) is a generic equation that represents the relationship between the design strength of a structure and the required strength (load effects). On the right-hand side of this equation, U represents the required strength or the effects due to loads, such as bending moment, shear force, axial force, etc., obtained from structural analysis. In specific terms of member strengths, such as moments (M), shear (V), and axial load (P), Eq. (1.1) can be written as follows:

$$\phi M_n \geq M_u \tag{1.2}$$

$$\phi V_n \geq V_u \tag{1.3}$$

$$\phi P_n \geq P_u \tag{1.4}$$

In Eqs. (1.2) to (1.4), subscripts n and u denote, respectively, nominal strength and the factored load effects (or demand). The strength reduction factor ϕ associated with different types of nominal strengths (moment, shear, axial load, etc.) is specified for each type of loading condition (see Chap. 4). Nominal strengths are calculated from principles of applied mechanics using factored loads, which are based on code-specified service loads. There are many good reasons for applying strength reduction factors to nominal strengths:

1. Actual loads might be different from those assumed in design calculations.
2. Actual load distribution in a structure might be different from that assumed in design.
3. Actual member dimensions might be different than assumed in design calculations.
4. Actual material strengths might be different from those assumed/specified in design.
5. The assumptions and simplifications assumed in design might result in load effects (moment, shear, axial loads, etc.) different from those actually acting on the structure.
6. Actual structural behavior might be different (usually is) owing to the presence of redundancies (influence of rigidity provided by nonstructural members is ignored in structural analysis).
7. Reinforcement might not be in the same exact position as specified by the designer.

To account for these uncertainties, the nominal strength of a member is reduced by multiplying it with the strength reduction factor ϕ; its value depends on member loads, such as moment, shear, compression, etc.

1.8 LOAD COMBINATIONS

1.8.1 General

All structures and their load-carrying elements/components should be analyzed and/or designed for one or more loads acting simultaneously. A structure at some point in time may be subjected to only one type of load, for example, a dead load, whereas at other times it might be subjected to several different loads acting simultaneously, for example, dead and live loads, or dead, live, and lateral loads such as wind- and earthquake-induced loads. In order to maintain uniformity in application of load combinations by design professionals, the many loads which are assumed to act singly or simultaneously are specified in the ASCE 7-05 Standard [1.30] for both strength design (SD) and allowable stress design (ASD) philosophies.

Reinforced masonry structures can be designed based on either strength design philosophy or allowable stress design philosophy. Allowable stress design has been in use for many years for designing masonry structures, but the trend is shifting toward using strength design. Design loads and their combinations are used differently when using these two different design philosophies. Although discussion in this book is based on only strength design load combinations, both strength and allowable stress design load combinations are presented in this section for completeness. A discussion of load combinations has been provided in *Commentary* to ASCE 7-05 Standard.

1.8.2 Load Combinations for Strength Design

The following are the basic load combinations for which structures, components, and their foundations are to be designed so that their strength equals or exceeds the effects of factored loads in the following combinations:

1. $1.4(D + F)$

2. $1.2(D + F + T) + 1.6(L + H) + 0.5(L_r \text{ or } S \text{ or } R)$

3. $1.2D + 1.6(L_r \text{ or } S \text{ or } R) + 0.5(L \text{ or } 0.8W)$

4. $1.2D + 1.6W + L + 0.5(L_r \text{ or } S \text{ or } R)$

5. $1.2D + 1.0E + L + 0.2S$

6. $0.9D + 1.6W + 1.6H$

7. $0.9D + 1.0E + 1.6H$

where D = dead load
E = earthquake load
F = loads due to fluid with well-defined pressures and maximum heights
H = loads due to lateral earth pressure, ground water pressure, or pressure of bulk materials
L = live load
L_r = roof live load
R = rain load
S = snow load
T = self-straining force
W = wind load

The following are exceptions to the above load combinations:

1. The load factor on L in combinations (3), (4), and (5) is permitted to equal 0.5 for all occupancies in which floor live load L_o in ASCE 7-05 Table 4-1 is less than or equal to 100 psf, with the exception of garages or areas occupied as places of public assembly.
2. The load factor on H shall be set equal to zero in combinations (6) and (7) if the structural action due to H counteracts that due to W or E. Where lateral earth pressure provides resistance to structural actions from other forces, it shall not be included in H but shall be included in the design resistance.
3. In combinations (2), (4), and (5), the companion load S shall be taken as either the flat roof snow load (p_f) or the sloped roof snow load (p_s).

A designer should investigate each limit state, including the effects of one or more loads not acting on the structure or its components. In case of wind and earthquake loads, the most unfavorable effects from each should be investigated and accounted for in design, but they need not be considered acting simultaneously.

In addition to the above load combinations, there are additional load combinations which should be investigated for structures subjected to floods (ASCE 7-05 Section 2.3.3) and to atmospheric ice loads (ASCE 7-05 Section 2.3.4).

Attention should be paid to load factors associated with the earthquake load, E (load factor = 1.0), and the wind load, W (load factor = 1.6), even though both are lateral loads acting on the structure. The reason for this difference in load factors is the fact that W represents the allowable stress-level wind load (hence load factor 1.6 to convert it into a factored load), whereas E represents strength-level load (hence load factor = 1.0).

1.8.3 Load Combinations for Allowable Stress Design

The following are the load combination which should be investigated when using allowable stress design.

1. $D + F$
2. $D + H + F + L + T$
3. $D + H + F + (L_r \text{ or } S \text{ or } R)$
4. $D + H + F + 0.75\,(L + T) + 0.75(L_r \text{ or } S \text{ or } R)$
5. $D + H + F + (W \text{ or } 0.7E)$
6. $D + H + F + 0.75(W \text{ or } 0.7E) + 0.75L + 0.75(L_r \text{ or } S \text{ or } R)$
7. $0.6D + W + H$
8. $0.6D + 0.7E + H$

In combinations (4) and (6), the companion load S shall be taken as either the flat roof snow load (p_f) or the sloped roof snow load (p_s).

In addition to the above load combinations, there are additional load combinations which should be investigated for structures subjected to floods (ASCE 7-05 Section 2.4.2) and to atmospheric ice loads (ASCE 7-05 Section 2.4.3).

As with strength design load combinations, a designer should investigate each load combination, including the effects of one or more loads not acting on the structure or its components. In case of wind and earthquake loads, the most unfavorable effects from each should be investigated and accounted for in design, but they need not be considered acting simultaneously.

Attention should be paid to the factor 0.7 associated with the earthquake load (E) in the above load combinations and a factor 1.0 to the wind load (W) even though both are lateral loads acting on the structure. The reason for this is the fact that W represents the allowable stress-level wind load, whereas E represents strength-level load (hence load factor = 0.7 to convert it to allowable stress-level load).

The earthquake load included in both the strength and allowable stress design combinations is defined in ASCE 7-05 Section 12.4 as explained in the next section.

1.8.4 Earthquake Load Effects and Combinations

Earthquake load effects, denoted as E, include axial, shear, and flexural member forces resulting from horizontal and vertical earthquake forces, both of which are caused by ground motion. These effects are determined somewhat differently for strength design and allowable stress design as follows.

1. For use in load combination (5) for strength design, and load combinations (5) and (6) for allowable stress design:

$$E = E_h + E_v \qquad \text{[ASCE 7-05 Eq. (12.4-1)]}$$

2. For use in load combination (7) in strength design and load combination (8) for allowable stress design:

$$E = E_h - E_v \qquad \text{[ASCE 7-05 Eq. (12.4-2)]}$$

where E = seismic load effect
E_h = effect of horizontal seismic forces
E_v = effect of vertical seismic forces

The horizontal load effect is determined from the following equation:

$$E_h = \rho Q_E \qquad \text{[ASCE 7-05 Eq. (12.4-3)]}$$

where ρ = redundancy factor, as defined in ASCE-05 Sec. 12.3.4
Q_E = effects of horizontal forces resulting from the base shear, V, or seismic force acting on a component in a structure, F_p

The vertical seismic load effect is determined from the following equation:

$$E_v = 0.2S_{\mathrm{DS}}D \qquad \text{[ASCE 7-05 Eq. (12.4-4)]}$$

where S_{DS} = design spectral response acceleration parameter at short periods (ASC 7-05 Sec. 11.4.4)
D = effects of dead load

The vertical seismic load effect, E_v, can be taken to be equal to zero when using ASCE 7-05 Eqs. (12.4-1), (12.4-2), (12.4-5), and (12.4-6) (excerpted later in this section) where $S_{\mathrm{DS}} \le 0.125$.

1.8.5 Seismic Load Combinations

When a structure is subjected to earthquake forces, the seismic load effect, E, in both the strength and allowable stress design load combination equations presented earlier can be rewritten by substituting its value from ASCE 7-05 Eqs. (12.4-2), (12.4-3), and (12.4-4) as follows (only those equations are presented which contain the seismic load effect, E):

1. Strength Design

5. $(1.2 + 0.2S_{DS})D + \rho Q_E + L + 0.2S$

7. $(0.9 - 0.2S_{DS})D + \rho Q_E + 1.6H$

2. Allowable Stress Design

5. $(1.0 + 0.14S_{DS})D + H + F + 0.7\rho Q_E$

6. $(1.0 + 0.105S_{DS})D + H + F + 0.525\rho Q_E + 0.75L + 0.75(L_r \text{ or } S \text{ or } R)$

7. $(0.6 - 0.14S_{DS})D + 0.7\rho Q_E + H$

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CHAPTER 2

MASONRY UNITS: APPLICATIONS, TYPES, SIZES, AND CLASSIFICATION

2.1 INTRODUCTION

Whereas masonry construction can be truly claimed as the oldest building art practiced by humans since the dawn of history, both stones and bricks can be considered as two of the oldest building materials—stone the oldest *natural* material, and the brick the oldest *man-made* material. A brief description of the historical development of masonry construction was presented in Chap. 1.

Masonry is typically laid of prefabricated units of different materials, shapes, and sizes. The common types of masonry units, typically rectangular, are clay bricks, clay tiles, concrete blocks, lightweight cellular concrete blocks, sand-lime bricks, and natural building stones. Units of shapes other than rectangular, particularly bricks, are also available. Both clay and concrete masonry units (referred to as CMU hereafter) are available in several colors, textures, and profiles to suit practically every conceivable need and convenience, visual effect, and aesthetic appeal as desired by engineers and architects. There is a world of masonry units out there.

This chapter discusses various types of clay units (bricks) and CMU which are referred to throughout this book. Masonry construction typically involves placement of masonry units one by one which, like any other type of construction such as steel, concrete, and wood, is a time-consuming and weather-dependent process. To reduce construction time, the concept of *prefabricated masonry* was developed, which is becoming increasingly popular. Research and advancement in technology of masonry construction led to the development in the 1980s of *mortarless block systems* in the United States. Both the prefabricated masonry and mortarless block systems are discussed briefly in this chapter.

2.2 APPLICATION OF MASONRY UNITS IN CONSTRUCTION

Both clay units and CMU offer themselves as versatile construction materials to be used for load-carrying elements of a structure. Bricks in particular can be used for both the building's exterior skin as wall and load-carrying elements as in a single-wythe bearing wall system. A major advantage of using bricks is that, with their many hues and colors, they can be used to create many attractive patterns and designs for architectural treatments of wall and floor surfaces, an advantage not offered by other construction materials such as concrete and steel. Some of the common uses of masonry units are [2.1, 2.2]:

1. Exterior load-bearing walls (below and above grade)
2. Interior load-bearing and nonload-bearing walls
3. Firewalls, party walls, curtain walls
4. Partitions, panel walls, solar screens
5. Piers, pilasters, columns
6. Bond beams, lintels, sills
7. Chimneys and fireplaces (indoors and outdoors)
8. Retaining walls, slope protection, ornamental garden walls, and highway sound barriers
9. Backing for screens
10. Backing for brick, stone, stucco, and exterior insulation and finishing systems
11. Veneer or nonstructural facing for steel, wood, concrete, or masonry
12. Fire protection for steel structural members
13. Fire-safe enclosures of stairwells, elevator shafts, storage vaults, or fire-hazardous areas
14. Catch basins, manholes, and valve vaults
15. Paving for walkways and landscaping

While brick and CMU are the most commonly used construction materials for structural and nonstructural purposes, there are others (e.g., glass units) that are less common. Recent research and advances in building materials technology have led to the development of prefabricated brick panels, autoclaved aerated concrete (AAC) blocks, and mortarless blocks. These are also briefly described in this chapter.

2.3 GENERAL DESCRIPTION OF MASONRY UNITS

Masonry units are manufactured in the United States to conform to the requirements of the American Society for Testing and Materials (ASTM), which publishes the most widely accepted standards on all kinds of materials. These standards are voluntary consensus standards, which have gone through a review process by various segments of construction industry—producers, users, and general interest members. Specifications for various kinds of masonry units—clay or shale, concrete, glass block, etc., are provided by various ASTM Standards, which are also referenced in most model building codes and specifications, such as the Masonry Standards Joint Committee (MSJC) *Code* [2.3], MSJC *Specification* [2.4], and the *International Building Code* (IBC) [2.5]. A compilation of ASTM Standards pertinent to concrete masonry can be found in Ref. [2.6]. Various *reference standards* applicable to masonry construction can be found in Refs. [2.6, 2.7].

Considerable information about both clay units and CMU is available in the literature. Two important and exhaustive references for brick and concrete masonry, respectively, are the Brick Institute of America's (BIC) *Technical Notes on Brick Construction* [2.8] and National Concrete Masonry Association's (NCMA) *TEK Manual for Concrete Masonry Design and Construction* [2.9]. They contain valuable information, updated periodically, on various aspects of masonry design and construction. These include production and manufacturing of masonry units and their uses, design considerations, and industry practices in masonry construction. Both have been referenced throughout this book in various chapters.

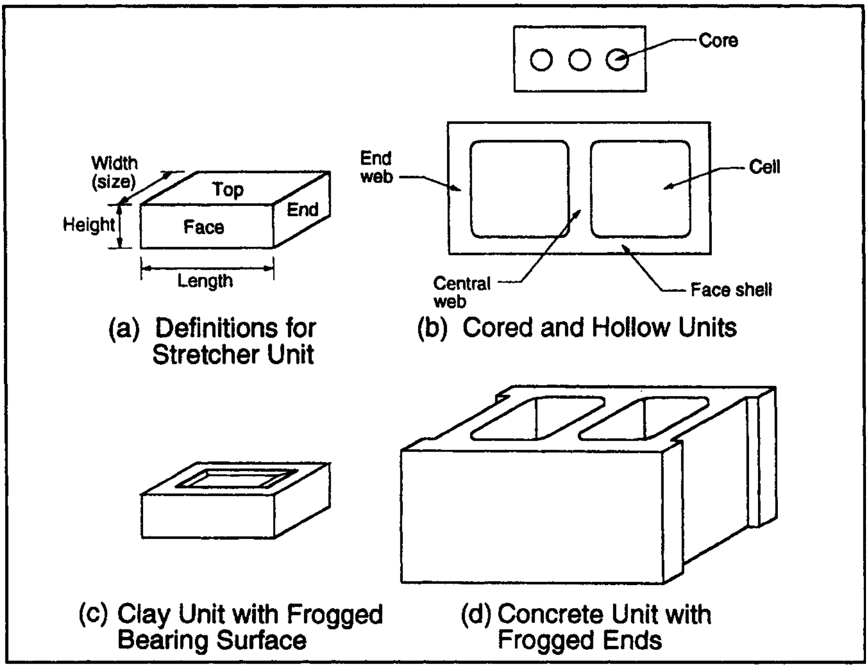
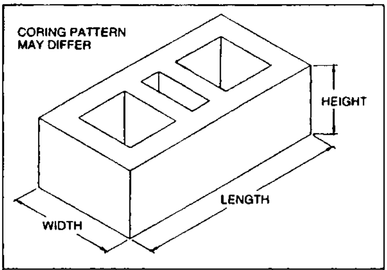


FIGURE 2.1 Common definitions used to describe masonry units. (*Courtesy*: *BIA*.)

Materials from which masonry units can be made include clay or shale, concrete, calcium silicate (or sand lime), stone, and glass. The two major types of masonry units used in general masonry construction are (1) *clay building brick* and (2) *concrete building brick and blocks*. Both types of units can be either solid or hollow. Units are made hollow by providing voids that extend through the full height of the units. These voids are commonly referred to as *cores* or *cells*, which are distinguished by their cross-sectional areas. Cores are void spaces having cross-sectional areas less than 1½ in.^{2}, whereas a cell is a void space having a cross-sectional area larger than that of a core. Voids that extend through only part of the height are called *frogs*. Essentially, frogs are indentations in one bed surface of bricks manufactured by molding or pressing. Frogs may be in the form of shallow troughs or a series of conical shapes. Panel frogs are limited to a specified depth and a specified distance from a face. *Deep frogs* are frogs that are deeper than ⅜ in., and must conform to the requirements for coring, hollow spaces, and void area of the applicable standard. Indentations over the full height of the ends of units are also referred to as frogs, and the units are said to have *frogged ends*. Concrete masonry units with frogged ends are produced by extending the face shells on one or both sides as required [2.10]. Various definitions are illustrated in Fig. 2.1.

In construction, masonry units are assembled together with mortar joints in between them. Mortar is applied to the top and bottom horizontal surfaces of units, which are referred to as *bedding areas*. The horizontal layer of mortar on which a masonry unit is laid is called the *bed joint*. Most masonry units are rectangular in shape, and defined by their three dimensions: width, height, and length, in that order (Fig. 2.2). Masonry units used in wall construction are rectangular in shape, which are laid with their long dimensions oriented horizontally in the plane of the wall, and height perpendicular in the plane of the



Width, in. (mm)	Height, in. (mm)	Length, in. (mm)
4 (100)	3.2 or 4 (81 or 100)	12 (300)
4 (100)	4 or 8 (100 or 200)	16 (400)
4 (100)	8 (200)	8 (200)
5 (125)	3.2 (81)	10 (250)
6 (150)	4 (100)	12 (300)
6 (150)	4 or 8 (100 or 200)	16 (400)
8 (200)	3.2 or 8 (81 or 200)	12 (300)
8 (200)	4 or 8 (100 or 200)	16 (400)
12 (300)	4 (100)	16 (400)

FIGURE 2.2 General dimensions of a masonry unit. (*Courtesy*: *BIA*.)

wall. A unit placed in this specific position is called a *stretcher*; its exposed vertical surface is called the *face* of the unit. Vertical faces of the unit perpendicular to the length of the wall are referred to as the *ends* of the unit.

A *solid* masonry unit may actually be less than 100 percent solid. Accordingly, it is defined as a masonry unit whose net cross-sectional area in every plane parallel to the bearing surface is 75 percent or more of the gross-sectional area in the same plane. That is, a solid unit can be 100 percent solid, or it can be a cored unit, that is, a unit from which up to 25 percent of material has been removed by providing voids or holes through the full height of the unit. A *hollow* masonry unit is defined as a masonry unit whose net cross-sectional area in every plane parallel to the bearing surface is less than 75 percent of the gross cross-sectional area in the same plane.

These definitions are significant from the standpoint of structural calculations and therefore defined in ASTM Standards as well as in building codes. Design with solid units can be based on the properties of the dimensions of the gross area (i.e., based on the area delineated by the out-to-out dimensions of the masonry unit). This is because the removal of small areas from the 100 percent solid units does not make an appreciable difference in its section properties, particularly in the value of the moment of inertia. Design with hollow units should be based on the net area section properties. The net cross-sectional area is defined as the area of masonry units, grout, and mortar crossed by the plane under consideration based on out-to-out dimensions.

2.4 CLAY BUILDING BRICK

2.4.1 General Description

Second only to stone, bricks and tiles are the oldest masonry units used in construction. Clay masonry units (bricks) are formed from clay through molding, pressing, or extrusion process. Their physical properties depend on raw materials, method of forming, and firing temperature. The latter is important because it must cause incipient fusion, a melting and joining of clay particles that is necessary for developing strength and durability of clay masonry units. They are available in a wide variety of shapes, sizes, colors, and strengths.

Bricks may be solid or hollow. Solid bricks may be 100 percent solid or cored as previously defined. A wide variety of solid and hollow bricks are produced throughout the United States. It is best to contact the manufacturers for their availability with regard to size, shape, and color. Figure 2.3 shows examples of solid building brick shapes used in the western United States.

In wall construction, individual bricks can be placed in various positions to create many different architectural patterns and pleasing visual effects. Figure 2.4 shows the many different positions in which bricks might be placed in a wall. They are called *stretcher, header,*

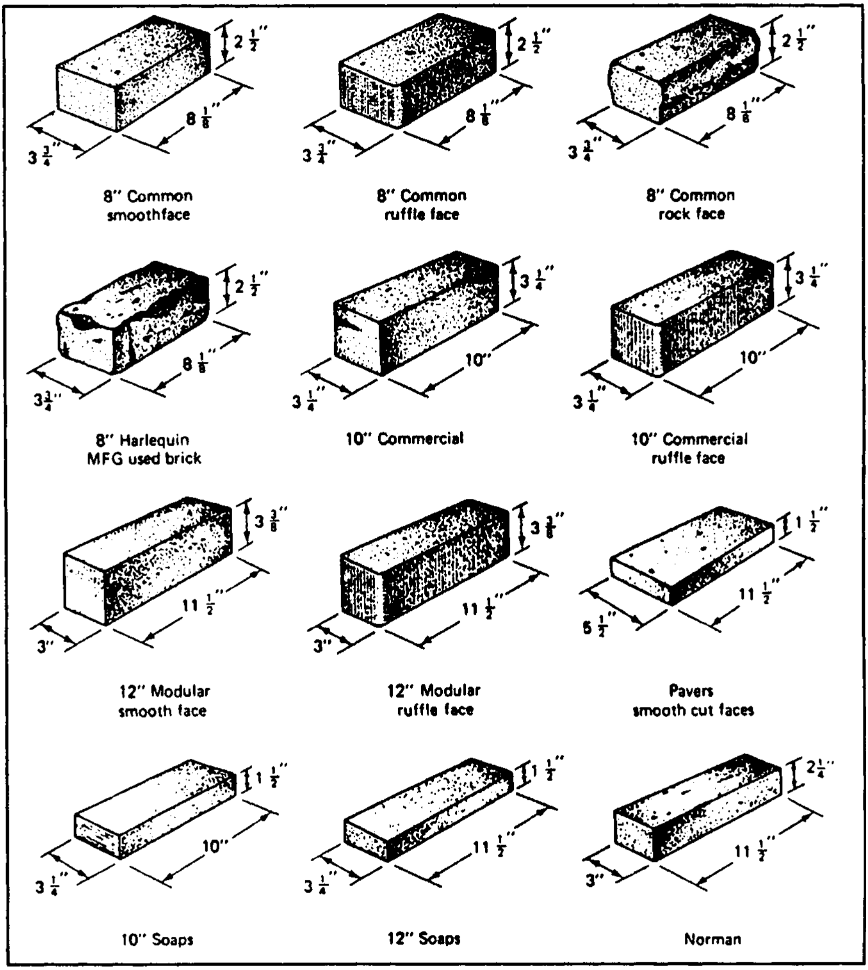


FIGURE 2.3 Examples of solid building brick shapes used in the western United States. (*Courtesy*: *BIA*.)

rowlock, rowlock stretcher, soldier, and *sailor*; these designations characterize brick positions with respect to their wide or narrow faces or the ends visible in the face of a wall. Each of the three faces of the brick (wide face, narrow face, and the end) can be either horizontal or vertical in the plane of the wall, thus creating a total of six positions.

A vertical layer of masonry that is one unit thick is called *wythe*. In one-wythe wall construction (i.e., a wall having a width equal to the width of the brick width), which is the simplest to build, bricks are placed as *stretchers*—with their narrow face laid horizontally in the plane of the wall. Other positions are obtained by turning a stretcher at 90° in the plane of the wall in various ways. For example, a header is a unit placed with its end showing in the plane of the wall, with its height (thickness) oriented vertically, the width oriented horizontally, and length oriented perpendicular to the wall. A *rowlock* is a header turned 90° in the plane of the wall. By virtue of their positioning, the headers and rowlocks effectively

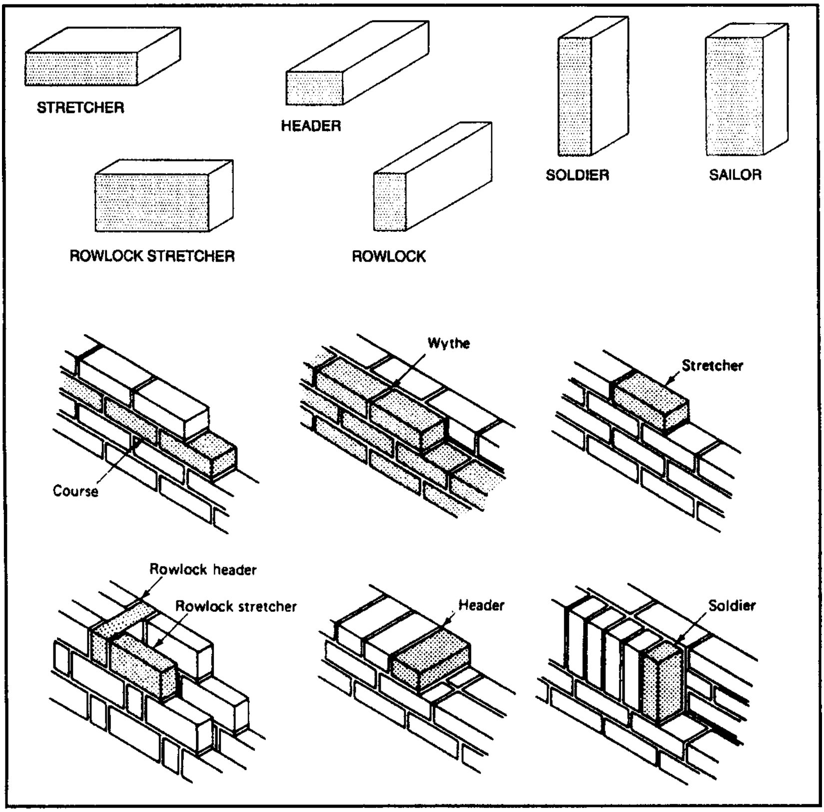


FIGURE 2.4 Brick positions in wall construction. (*Courtesy*: *BIA*.)

tie two or more wythes of a wall. A *soldier* is a stretcher turned 90° in the vertical plane of the wall so the length is oriented vertically (narrow face is vertical). A *rowlock stretcher* (also called *shiner*) is a stretcher laid on its narrow face so that its wider face is in the plane of the wall and its ends perpendicular to the plane of the wall (same as a rowlock turned 90° in a horizontal plane). A sailor is a soldier turned 90° in a horizontal plane [2.11].

While a stretcher is the most common position of a brick in a wall, other forms are invariably used when walls are two or three wythes thick. Headers are used to bond wythes in multiwythe walls to form a structural unit. Some of the brick positions are used specifically to create pleasing visual effects. For example, soldier courses are placed for visual emphasis in such locations as tops of walls or window lintels. Rowlock courses are frequently used as sloping sills under windows and as caps for garden walls.

2.4.2 Hollow Brick Configurations

Figure 2.5 shows configurations of hollow bricks. They may have only cores (i.e., small circular or rectangular voids) or both cores and cells. Cells are similar to cores except that a cell is larger in cross-section than a core. Attention should be paid to the definitions of various parts of units shown in Fig. 2.5. A hollow brick unit may have two or three cells, referred to as two- or three-core (or two- or three-cell) units, respectively. The partition between the cells is called the *web*. The two parts of the block perpendicular to the web are called the *face shells* (oriented along the length of a wall); those parallel to the web (or webs) are called the *ends* (oriented perpendicular to the wall). The cells are used for the placement of reinforcement and/or grouting as necessary. Figure 2.6 shows positioning of reinforcement in conjunction with solid and hollow brick units. A stretcher may be produced with depressed webs and end shells, called a *bond beam stretcher*, to permit placement of horizontal reinforcement in bond beams (Fig. 2.7).

The coring patterns shown in Fig. 2.5 are for illustrative purposes only. Coring practices vary with manufacturers who should be consulted for actual sizes and patterns. ASTM C62-01: Specification for Building Brick (Solid Masonry Units made from Clay or Shale) [2.12] requires that the

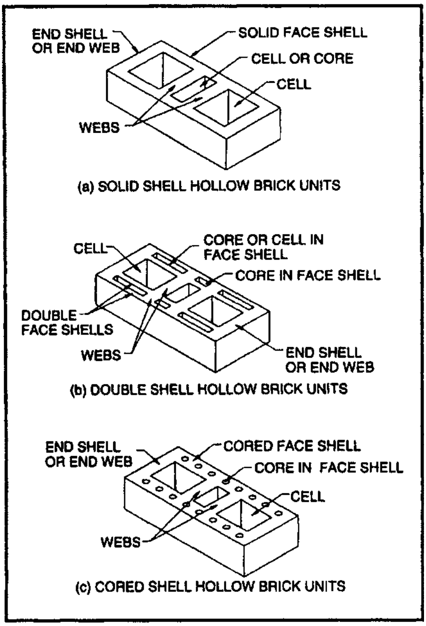


FIGURE 2.5 Configurations of hollow bricks. (*Courtesy*: *BIA*.)

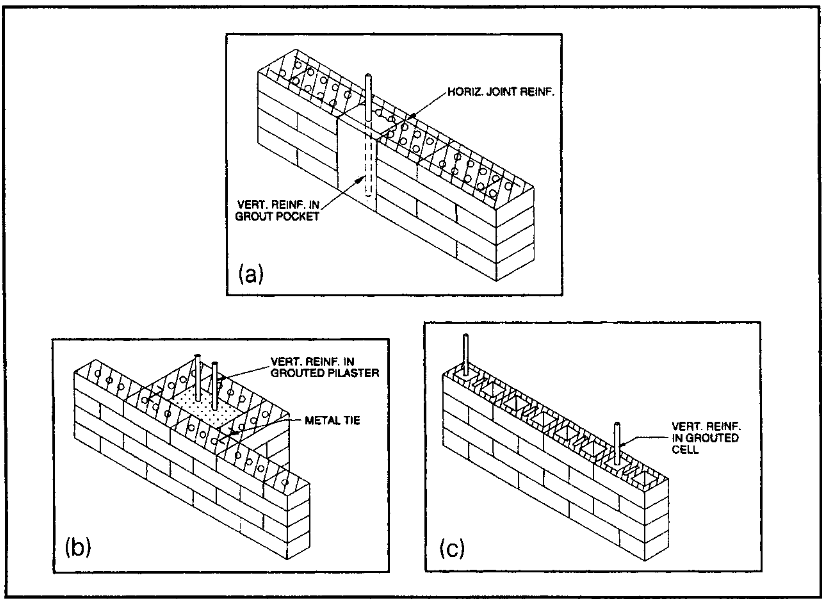


FIGURE 2.6 Reinforced masonry construction with solid and hollow brick units: (*a*) solid brick wall with reinforcement in grouted pocket; (*b*) pilaster with solid units; (*c*) reinforced hollow brick wall. (*Courtesy*: *BIA*.)

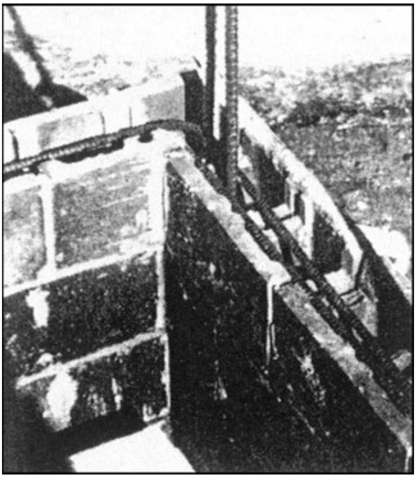


FIGURE 2.7 Placement of reinforcement in brick bond beam units. (*Courtesy*: *BIA*.)

net cross-sectional area of the cored brick in any plane parallel to the bearing surface be not less than 75 percent of the gross cross-sectional area measured in the same plane. It is also required that no part of any hole be less than ¾ in. from any edge of the brick unit. Cores are only found in brick manufactured by extrusion process. Limits to the amount of coring allowed in brick, the distance from a core to face, and the web thickness where applicable, are specified by the applicable ASTM Standard; a summary can be found in Ref. [2.10].

Cored bricks offer many advantages. They aid in manufacturing process and shipping of brick. Coring results in lighter bricks as compared to the solid bricks; consequently, they are easier to handle during placement. The reduced weight of cored bricks results in reduced shipping costs. Cores reduce the amount of fuel necessary to fire the units. Since bricks have to be laid by hand, brick by brick, holes in bricks allow easy handling (lifting) and holding of the brick by a bricklayer. Contrary to the perception that cored brick may be relatively weaker (due to the reduced area) than the solid brick, the opposite is true. Cored bricks demonstrate higher strengths due to more uniform drying and burning made possible by the presence of holes. Cores also help develop better mechanical mortar bond as the mortar pressing into the holes provides a good mechanical "key" [2.10, 2.13].

2.4.3 ASTM Classification of Brick Units

Clay building bricks are manufactured from clay or shale, and hardened by heat. Brick units can be used to bear loads, or they can be used simply as nonload-bearing veneer to give a weather-resistant skin or for architectural/aesthetic enhancements, the latter being more common these days as load-bearing brick construction is not cost-effective. They are commonly available in rectangular shapes of various sizes and colors. Building bricks must conform to ASTM C62-01: Specification for Building Brick (Solid Masonry Units Made from Clay or Shale) [2.12]. Hollow bricks, which are very similar in size and shape to hollow concrete units, must conform to the following ASTM Standards:

ASTM C216-02: Specification for Building Brick (Solid Masonry Units Made from Clay or Shale) [2.14].

ASTM C652-01a: Specification for Hollow Brick (Solid Masonry Units Made from Clay or Shale) [2.15].

In addition to building bricks and hollow bricks, there are other kinds of bricks used in brick masonry construction for various applications. See Table 2.1 for a summary of various types of bricks and the applicable ASTM Standards. A discussion on manufacturing, classification, and selection of brick units can be found in Ref. 2.10.

As the names imply, the uses of bricks are similar to their respective ASTM designations. Various types of brick units listed in Table 2.1 are defined in the *Glossary* and are not

TABLE 2.1 ASTM Brick Classification [2.10]

Types of brick units	ASTM designation*
Building brick (solid)	ASTM C62 [2.12]
Facing brick (solid)	ASTM C216 [2.14]
Hollow brick	ASTM C652 [2.15]
Paving brick	ASTM C902 [2.16]
Ceramic glazed brick	ASTM C126 [2.17]
Thin brick veneer units	ASTM C1088 [2.18]
Sewer and manhole brick	ASTM C32 [2.19]
Chemical resistant brick	ASTM C279 [2.20]
Industrial floor brick	ASTM C410 [2.21]
Load-bearing wall tile	ASTM C34 [2.22]
Nonload-bearing wall tile	ASTM C56 [2.23]

*American Society for Testing and Materials, West Conshohocken, PA.

repeated here. There are several terms in each standard used for classification, which may include exposure, appearance, physical properties, efflorescence, dimensional tolerances, distortion, chippage, cores, and frogs. Classification of bricks is determined by the usage of brick in specific applications. Bricks are classified in most specifications by use, type, and/or class. All options should be specified as each ASTM brick standard has requirements for grade and type that apply automatically if an option is omitted. By not specifying the desired requirements, a delivery may contain bricks not suitable for the intended use. Bricks used in the wrong application can lead to failure or an unpleasing appearance [2.10].

2.4.4 Brick Sizes and Nomenclature

A size characteristic of brick units is their smallness. Brick is a building element with a human scale. Although brick sizes have varied over the centuries, essentially they have always been similar to the present-day sizes. The size of a brick has historically been small enough to be held in the hand, and most bricks have remained small. A discussion on brick sizes and related information can be found in Ref. [2.11].

Advances in brick construction and needs for specific designs led to the development of new brick sizes. These include hollow units for reinforced construction, and larger units for faster construction and increased economy. Hollow units have varying coring patterns but are typically larger than standard or modular size, and have larger cells to allow placement of vertical reinforcement. Obviously, the brick size has influence on completion time required for a job. For example, units with larger face dimensions allow a brick layer to lay more square foot of wall per day. Such units, compared to standard or modular size units, may increase the number of bricks laid per day by as much as 50 percent. However, as units get larger, they also become heavier, require more effort in lifting and placement, and reduce productivity [2.11].

Interestingly, efforts have been made in the past to develop brick laying machines in order to increase productivity in brick masonry construction. According to literature [2.24–2.28], the bricklaying machines have been around since 1902 when Knight, an Englishman, invented a model with which “any one could lay 500 to 600 bricks per hour” [2.24]. Over the years, several efforts were made in different parts of the United States, with claims ranging from laying 1200 bricks per hour to 10,000 bricks in 8 hours, but none proved to be commercially successful. A discussion on mason productivity has been provided by Grimm [2.28].

TABLE 2.2 Standard Nomenclature for Brick Sizes [2.11]

Unit designation	Nominal dimensions, in. w	h	l	Joint thickness* in.	Specified dimensions† in. w	h	l	Vertical coursing
Modular brick sizes								
Modular	4	2⅔	8	⅜	3⅝	2¼	7⅝	3C = 8 in.
				½	3½	2¼	7½	
Engineer modular	4	3⅕	8	⅜	3⅝	2¾	7⅝	5C = 16 in.
				½	3½	2 13/16	7½	
Closure modular	4	4	8	⅜	3⅝	3⅝	7⅝	1C = 4 in.
				½	3½	3½	7½	
Roman	4	2	12	⅜	3⅝	1⅝	11⅝	2C = 4 in.
				½	3½	1½	11½	
Norman	4	2⅔	12	⅜	3⅝	2¼	11⅝	3C = 8 in.
				½	3½	2¼	11½	
Engineer norman	4	⅓	12	⅜	3⅝	2¾	11⅝	5C = 16 in.
				½	3½	2 13/16	11½	
Utility	4	4	12	⅜	3⅝	3⅝	11⅝	1C = 4 in.
				½	3½	3½	11½	
Nonmodular brick sizes								
Standard				⅜	3⅝	2¼	8	3C = 8 in.
				½	3½	2¼	8	
Standard engineer				⅜	3⅝	2¾	8	5C = 16 in.
				½	3½	2 13/16	8	
Closure standard				⅜	3⅝	3⅝	8	1C = 4 in.
				½	3½	3½	8	
King				⅜	3	2¾	9⅝	5C = 16 in.
					3	2⅝	9⅝	
Queen				⅜	3	2¾	8	5C = 16 in.

*Common joint sizes used with length and width dimensions. Joint thicknesses of bed joints vary based on vertical coursing and specified unit height.

†Specified dimensions may vary within this range from manufacturer to manufacturer.

The standard nomenclature of brick sizes is presented in Table 2.2. These terms were developed by a consensus process involving companies across the country, led by a joint committee of the Brick Institute of America and the National Association of Brick Distributors. In the past, a given brick may have been known by several names due to regional variations.

Since clay is a flexible medium, manufacturers can make bricks of many different sizes depending on their demand. Table 2.3 presents other brick sizes that are produced by a limited number of manufacturers. Figures 2.8 and 2.9 illustrate, respectively, modular and nonmodular sizes of cored and hollow brick units.

Hollow brick units are manufactured to conform to ASTM C652-01a: Specification for Hollow Brick (Hollow Masonry Units Made from Clay or Shale) [2.15], and should be specified as such. They evolved as a natural development in brick masonry construction. Hollow bricks contain voided areas or cells that permit easy placement of reinforcement and grouting; the latter are required for masonry to resist loads in high wind and seismic regions. These requirements could not be achieved with solid or cored bricks. Large size bricks also resulted in economical construction. These advantages led to the establishment

TABLE 2.3 Other Brick Sizes [2.11]

Modular brick sizes							
Nominal dimensions in.			Joint thickness* in.	Specified dimensions† in.			Vertical coursing
w	*h*	*l*		*w*	*h*	*l*	
4	6	8	$\frac{3}{8}$	$3\frac{5}{8}$	$5\frac{5}{8}$	$7\frac{5}{8}$	2C = 12 in.
			$\frac{1}{2}$	$3\frac{1}{2}$	$5\frac{1}{2}$	$7\frac{1}{2}$	
4	8	8	$\frac{3}{8}$	$3\frac{5}{8}$	$7\frac{5}{8}$	$7\frac{5}{8}$	1C = 8 in.
			$\frac{1}{2}$	$3\frac{1}{2}$	$7\frac{1}{2}$	$7\frac{1}{2}$	
6	$3\frac{1}{5}$	12	$\frac{3}{8}$	$5\frac{5}{8}$	$2\frac{3}{4}$	$11\frac{5}{8}$	5C = 16 in.
			$\frac{1}{2}$	$5\frac{1}{2}$	$2\frac{13}{16}$	$11\frac{1}{2}$	
6	4	12	$\frac{3}{8}$	$5\frac{5}{8}$	$3\frac{5}{8}$	$11\frac{5}{8}$	1C = 4 in.
			$\frac{1}{2}$	$5\frac{1}{2}$	$3\frac{1}{2}$	$11\frac{1}{2}$	
8	4	12	$\frac{3}{8}$	$7\frac{5}{8}$	$3\frac{5}{8}$	$11\frac{5}{8}$	1C = 4 in.
			$\frac{1}{2}$	$7\frac{1}{2}$	$3\frac{1}{2}$	$11\frac{1}{2}$	
8	4	16	$\frac{3}{8}$	$7\frac{5}{8}$	$3\frac{5}{8}$	$15\frac{5}{8}$	1C = 4 in.
			$\frac{1}{2}$	$7\frac{1}{2}$	$3\frac{1}{2}$	$15\frac{1}{2}$	
			$\frac{3}{8}$	3	$2\frac{3}{4}$	$8\frac{5}{8}$	5C = 16 in.
				3	$2\frac{5}{8}$	$8\frac{5}{8}$	

*Common joint sizes used with length and width dimensions. Thickness of bed joints vary based on vertical coursing and specified unit height.

†Specified dimensions may vary within this range from manufacturer to manufacturer.

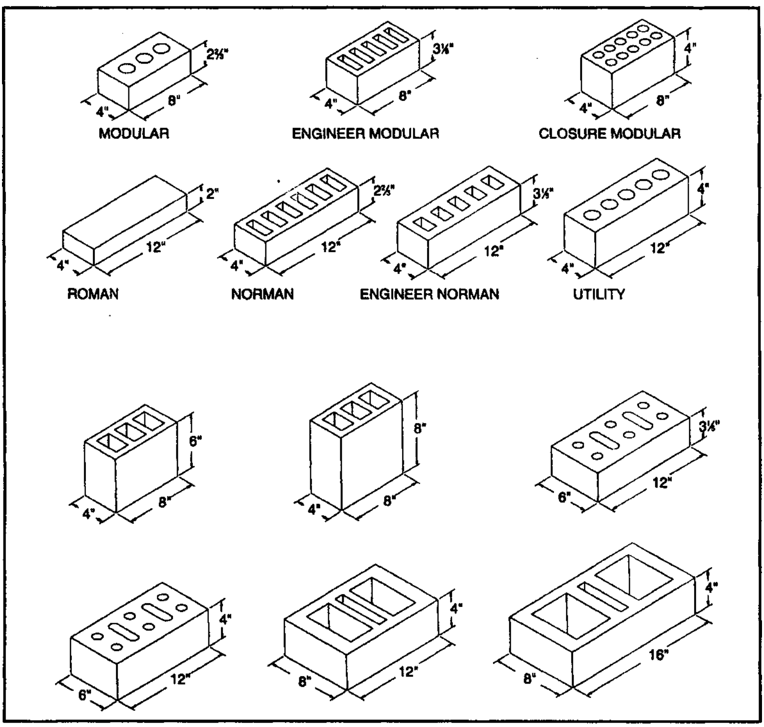


FIGURE 2.8 Modular brick sizes. (*Courtesy*: *BIA*.)

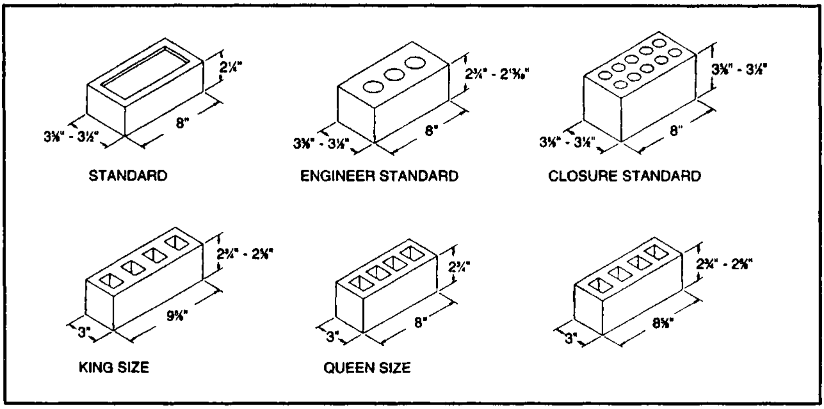


FIGURE 2.9 Nonmodular brick sizes. (*Courtesy*: *BIA*.)

of a new brick classification—the *hollow* brick, whose void area is greater than 25 percent of its gross area. The development of hollow bricks began with 4 × 4 × 12 in. oversized solid units followed by 6 × 4 × 12 and 8 × 8 × 16 in. hollow brick units. These large hollow bricks are often called *through-the-wall* (TTW) units because the wall consists of a single wythe of masonry. They offer a considerable advantage in both speed and economy of construction. A discussion on hollow brick is presented in Ref. [2.29]. Configurations of hollow bricks are shown in Fig. 2.5; their section properties are listed in Table 2.4.

TABLE 2.4 ASTM C652-01a Section Properties of Hollow Brick Units [2.15]

Nominal width of units, in.	Minimum solid face shell thickness, in.	Minimum cored or double face shell thickness*, in.	Minimum end shell or end web thickness†, in.
3 and 4	¾	–	¾
6	1	1½	1
8	1¼	1½	1
10	1⅜	1⅝	1⅛
12	1½	2	1⅛

*Cores greater than 1 in.^{2} (650 mm^2) in cored shells shall be not less than ½ in. (13 mm) from any edge. Cores not greater than 1 in.^{2} (650 mm^2) in shells cored not more than 35 percent shall be not less than ⅜ in. (10 mm) from any edge.

†The thickness of webs shall not be less than ½ in. (13 mm) between cells, ⅜ in. (10 mm) between cells and cores or ¼ in. (6 mm) between cores.

Hollow bricks are also made in a variety of special shapes to meet practical, architectural, or aesthetic requirements. These special shapes include bullnose, radial, interior and exterior angled units, and others.

2.4.5 Brick Dimensions

Bricks are identified by three dimensions: width, height, and length (Fig. 2.2). Height and length are the face dimensions as seen when a brick is laid as a *stretcher*. When specifying and ordering bricks, it is common practice to list the brick dimensions in the standard order of width, height, and length.

Three different sets of dimensions should be recognized when specifying or designing with brick: nominal dimensions, specified dimensions, and actual dimensions. Nominal dimensions are used most often by the architects to plan *modular construction* so that all dimensions of the brick and the elements of buildings appear as multiples of a given module.

The nominal dimension for brick masonry equals the specified dimension plus one intended mortar joint thickness, which is the thickness required so that the thickness of the unit and the joint thickness together match the coursing module. Nominal brick dimensions are based on multiples (or fractions) of 4 in. Specified dimensions are shown in Tables 2.2 and 2.3. Details of modular construction can be found in Ref. [2.11].

Specified dimension is defined as the anticipated manufactured dimension. They are typically $\frac{3}{8}$ to $\frac{1}{2}$ in. smaller than the nominal dimensions, based on the joint thicknesses of $\frac{3}{8}$ and $\frac{1}{2}$ in., respectively. The specified dimensions are used by engineers for structural design purposes. Therefore, these dimensions should be stated in the project specifications and purchase orders. See Tables 2.2 and 2.3 for details.

The *actual* dimension is the dimension of the unit as manufactured. Actual dimensions of a brick unit may vary slightly from the specified dimensions; however, the variations must be within the range of acceptable tolerances. Dimensional tolerances are defined by ASTM C216-05a: Specifications for Hollow Brick (Hollow Masonry Units Made from Clay or Shale) [2.14].

Although nominal dimensions are given only for the modular bricks, the *heights* for modular and nonmodular brick units are the same, that is, all bricks are modular in height. See Tables 2.2 and 2.3. This is because when modular bricks were first introduced, they were faced with the problem of supplying bricks to match the existing nonmodular construction, an important criterion for preserving the architectural and aesthetic characteristics of a project.

2.4.6 Durability Grades

Durability of material used in engineering construction is an important consideration. For this purpose, bricks are specified by the *durability grades* according to ASTM C652-01a [2.15] as follows:

1. *SW (Severe weathering):* These bricks are to be used where a high degree of resistance to disintegrating weathering and frost action is required while saturated with water. They also provide higher compressive strength than that of other category bricks.
2. *MW (Moderate weathering):* These bricks are to be used where only moderate degree of resistance to weathering action is required, and where frost action is not very likely. They are intended for applications that are unlikely to be saturated with water when exposed to freezing temperatures.

Reference [2.30] provides an additional durability classification:

3. *NW (No weathering):* Bricks intended for use as backup or interior masonry; or if exposed, for use where no frost action occurs; or if frost action occurs, where the annual precipitation is less than 20 in. The reason for these restrictions is that these bricks would disintegrate rapidly when subjected to freeze-thaw cycles.

The selection of specific grade of brick for face exposure of vertical or horizontal surfaces is related to weathering index which is defined as the product of the average annual

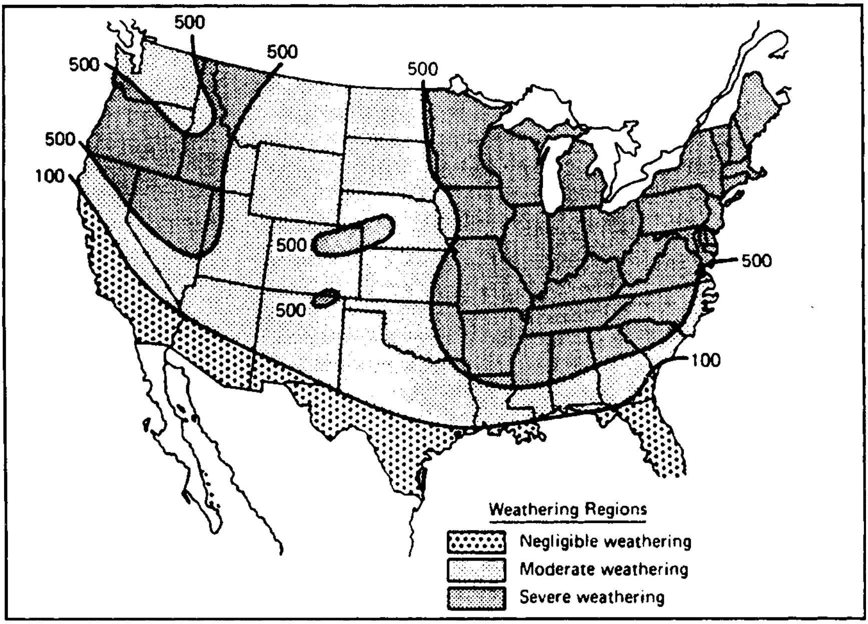


FIGURE 2.10 Weathering index map of the United States. (*Courtesy*: *BIA*.)

number of freezing cycle days and the average annual winter rainfall in inches. A freezing cycle day is defined to occur on any day when the air temperature passes either above or below 32°F. Brick must meet a grade of SW, MW, or NW based on the weathering index and the exposure of the brick, and whether the brick is in contact with the earth. Figure 2.10 shows areas of the United States with differing weather conditions. Table 2.5 shows where each grade is required for face exposure according to ASTM C62-05 [2.12]. It will be observed that a higher weathering index requires a face brick to meet the SW requirements. Most manufacturers make bricks to meet the severe weathering (SW) grade so they may ship brick to all parts of the country. Some manufacturers produce bricks to meet only the moderate weathering (MW) grade. Brick manufacturers can furnish certification that their product will meet a certain grade requirement.

TABLE 2.5 Grade Requirements for Face Exposure [2.20]

	Weathering index		
Exposure	Less than 50	50–500	500 and greater
In vertical surfaces:			
In contact with earth	MW	SW	SW
Not in contact with earth	MW	SW	SW
In other than vertical surfaces:			
In contact with earth	SW	SW	SW
Not in contact with earth	MW	SW	SW

2.4.7 Visual Inspection

Building bricks are subject to visual inspection requirements according to the ASTM Standards. They should be free of defects, deficiencies, and surface treatments, including coatings, which would interfere with the proper setting of the brick, or significantly impair the strength or performance of the structure. However, all bricks have some minor indentations and surface cracks inherent in the manufacturing process. They may be chipped at corners or edges, or otherwise damaged during packaging, handling, and shipping, or on the jobsite. These imperfections should not be considered as grounds for rejecting the bricks. According to Ref. 2.10, there are no chippage requirements for bricks conforming to ASTM C62-05 [2.12], ASTM C126-99: Specification for Ceramic Glazed Structural Clay Facing Tile, Facing Brick, and Solid Masonry Units [2.17], or ASTM C32-05: Specification for Sewer and Manhole Brick [2.19].

2.5 FUNCTIONAL ASPECTS

2.5.1 Facing Brick

Facing bricks are intended for use in both structural and nonstructural masonry where appearance is a requirement. They are produced for the express purpose of forming the exposed face of a wall, and covered by ASTM C216 [2.14]. They are available in a wide variety of colors and textures. In California, facing brick units are available in a full range of colors, from chalk or paper white to charcoal to jet black—blues, reds, purples, oranges, yellows, browns, and grays, with intermediate shades. Terminology describing various colors and textures may vary regionally and manufacture-wise; designers should check with local suppliers for availability of selected color and texture.

Facing bricks come in two grades: Grade SW or MW; Grade SW may be used in lieu of Grade MW. There is no Grade NW, since these bricks are not permitted to be used as backup. Suppliers will normally supply Grade MW bricks, unless specifically told otherwise.

Brick *types* are related to the appearance of the unit and specifically to limits on dimensional tolerances, distortion tolerances, and chippage of the units. There are three types of *facing bricks*:

- *Type FBS:* These bricks are meant for general use for exposed exterior walls and interior masonry walls and partitions, where wider color ranges and greater variation in sizes are permitted than are specified for Type FBX.
- *Type FBX:* These are to be used for the exposed exterior and interior masonry walls where a high degree of mechanical perfection, narrow color range, and minimum permissible variation in size are required.
- *Type FBA:* Bricks manufactured and selected to produce characteristic architectural effects resulting from nonuniformity in size, texture, and color of individual units.

2.5.2 Hollow Brick

2.5.2.1 General Description. Solid and hollow bricks were defined in Section 2.3. Solid bricks are also cored bricks. *Cores* are void spaces having a gross cross-sectional area equal to or less than $1\frac{1}{2}$ in.^{2}. A *cell* is a void space having a gross cross-sectional area greater than $1\frac{1}{2}$ in.^{2} A deep frog is an indentation in the bed surface of the brick which is deeper than 3⁄8 in. Void areas may be cores, cells, deep frogs, or combinations of these. ASTM C652 [2.15] defines *hollow brick* as a clay unit masonry whose net cross-sectional area (solid area) in any plane parallel to the bearing surface is less than 75 percent of its gross cross-sectional area measured in the same plane. The extent of void area of a hollow brick is separated into two classes: H40V and H60V. Bricks with void areas greater than 25 percent and less than 40 percent of the gross cross-sectional area are classified as H40V. Bricks with void areas greater than 40 percent, but less than 60 percent of the gross cross-sectional area are classified as H60V.

Strict edge distance requirements are imposed by ASTM Standards for creating hollow spaces within the bricks. Core holes may not be less than 5⁄8 in. from any edge of the brick except for the cored-shell hollow brick. The minimum shell thickness required for a cored-shell hollow brick is 1½ in. The minimum edge distance for cores greater than 1 in.^{2} in cored shells is ½ in. Cores not greater than 1 in.^{2} in shells cored more than 35 percent may not be 3⁄8 in. from any edge. Cells may not be less than ¾ in. from any edge of the brick, except for double shell hollow brick. The thickness of webs between the cells should not be less than ½ in., ¾ in. between cells and cores, ¼ in. between the cores.

Hollow bricks are identical to facing bricks except that they occupy a larger core area. Most hollow bricks are used in the same application as the facing brick. Hollow bricks with very large cores are used in walls that are reinforced with steel and grouted solid. The two most common types of hollow brick units used in reinforced masonry construction are the stretchers and bond beam units which are easily reinforced. The stretcher units, typically used in wall construction, are placed so that cells align vertically to permit placement of vertical reinforcement and grouting (Fig. 2.6). Bond beam units are placed to align horizontally, which permits placement of horizontal reinforcement and grouting (Fig. 2.7).

2.5.2.2 Types of Hollow Bricks. As discussed in Refs. 2.29 and 2.30, four types of hollow bricks are specified by ASTM C652 [2.15], which are described as follows. Each of these types relate to the appearance of the unit. Dimensional variation, chippage, warping, and other imperfections are the qualifying conditions of the type. The most common type, Type HBS, is considered standard and specified for most applications.

- *Type HBS:* Hollow bricks for general use in exposed exterior and interior masonry walls and partitions, where wider color ranges and greater variation in sizes are permitted than are specified for Type HBX.
- *Type HBX:* These are to be used for the exposed exterior and interior masonry walls where a high degree of mechanical perfection, narrow color range, and minimum permissible variation in size are required.
- *Type HBA:* Bricks are manufactured and selected to produce characteristic architectural effects resulting from nonuniformity in size, texture, and color of individual units.
- *Type HBB:* Hollow brick for general use in masonry walls and partitions where texture and color are not a consideration, and a greater variation in size is permitted than is specified for Type HBX.

2.5.3 Relationship Between Unit Strength of Clay Masonry Units and Specified Compressive Strength of Clay Masonry, f'_m

The specified compressive f'_m of a masonry is greatly influenced by the variability in mortar strength, grout strength, and their interaction in generating compressive strength. As such, the relationship between the masonry unit strength and the compressive strength (f'_m) of the masonry assemblage varies considerably, depending on the properties of mortar and grout used in the preparation of the masonry assemblage. The relationship between the compressive strength of clay masonry units and the type of mortar is given in Table A4 (App. A).

2.5.4 Visual Inspection

Requirements for visual inspection of the clay building bricks are also applicable to hollow bricks. Additionally, bricks used in exposed wall surfaces should have faces free of unsightly cracks or other imperfections detracting from the appearance of a sample wall when viewed from a distance of 15 ft for Type HBX and a distance of 20 ft for Types HBS and HBA [2.30].

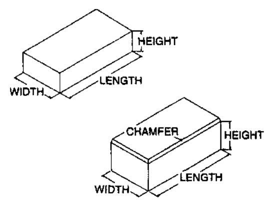
2.5.5 Paving Bricks

Bricks used for paving are produced both from clay as well as concrete. A variety of shapes, sizes, and colors are available to conform to the desired theme of architecture and visual effects. Clay bricks are generally in terra-cotta red. Availability of particular kind of paving bricks should be checked with the local suppliers/manufacturers.

Paving bricks, also called *pavers*, are high strength, durable, and dense. Their manufacture and usage should conform to ASTM C62 [2.12], ASTM C216 [2.14], and ASTM C902: Specification for Pedestrian and Light Traffic Paving Brick [2.16] for pedestrian and light traffic usage and ASTM C 410: Specification for Industrial Floor Brick [2.21] for industrial floors. Brick pavers are produced in many sizes; the most common, summarized from Ref. [2.32], are shown in Table 2.6.

TABLE 2.6 Typical Brick Pavers Sizes* [2.32]

Width, in.	Length, in.	Height, in.
4	8	Varies according to manufacturer and application, usually $1\frac{1}{4}$, $2\frac{1}{4}$, $2\frac{5}{8}$, or $2\frac{3}{4}$ in.
$3\frac{5}{8}$	$7\frac{5}{8}$	
$3\frac{1}{2}$	$7\frac{1}{2}$	
$7\frac{5}{8}$	$7\frac{5}{8}$	
8	8	



*Check with manufacturers for availability of chamfers.

In the past, paving bricks were used to pave roads that carried iron-wheeled vehicular traffic. In modern times, paving bricks are used for pedestrian traffic on the ground floors of industrial, commercial, and residential buildings, walkways in shopping centers and malls. Pavers can be arranged in many imaginative ways and colors, resulting in aesthetically appealing patterns, several of which are shown in Fig. 2.11. Alternative sizes and shapes can also be manufactured or cut from standard units. Figure 2.12 shows examples of radial brick units which are often used to create curves or circles in the pavement.

Brick paving surfaces can be classified in two basic systems: mortarless and mortared. Mortarless brick paving contains sand between the units which are laid on a variety of materials. By contrast, mortared paving consists of units with mortar between the units and always laid in a mortar setting bed. Bases of paving surfaces are classified as flexible, semirigid, rigid, and suspended diaphragm [2.33]. Flexible brick pavements consist of

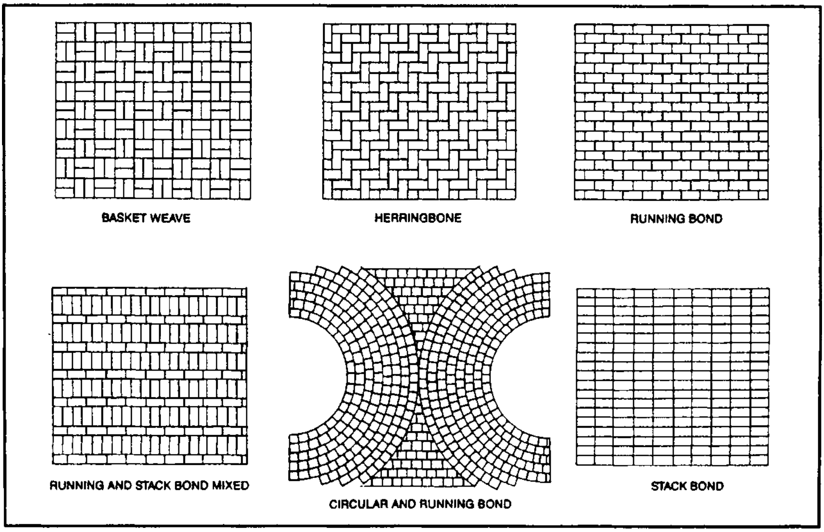


FIGURE 2.11 Clay paving bricks and paving patterns. (*Courtesy*: *BIA*.)

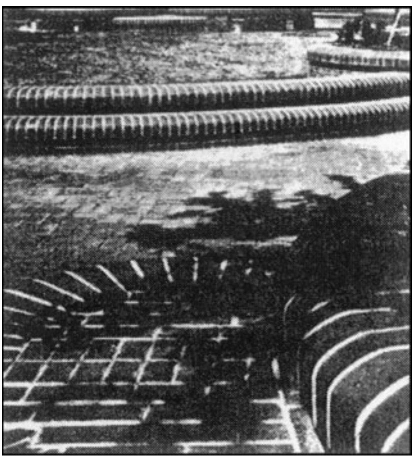


FIGURE 2.12 Radial brick shapes to create curves and circles. (*Courtesy*: *BIA*.)

mortarless brick paving over a base that consists of compacted crushed stone, gravel, or coarse sand (and hence flexible). Mortarless brick system can be used over any base. By contrast, rigid brick pavements consist of mortared bricks over a concrete slab. The slab should be sufficiently rigid to prevent pavers and/or the mortar joints from cracking. The semirigid base consists of asphalt concrete (commonly referred to as asphalt). Only mortarless brick paving is suitable over this type base. A suspended diaphragm base consists of structural roof or floor assemblies of concrete, steel, or wood. Suitability of mortarless or mortared brick paving for this type base depends on the stiffness of the diaphragm. Details of various brick pavement systems can be found in several references [2.33– 2.35].

2.5.6 Thin Brick Veneer

Thin bricks, commonly referred to as "thin brick veneers" are thin fired clay units which are used for interior and exterior wall coverings. They have become extremely popular in commercial, residential, and do-it-yourself markets. Thin bricks are produced from shale and/or clay, and kiln-fired. These units are produced to conform to ASTM C1088: Thin Veneer Brick Units Made from Clay or Shale [2.18], and similar to facing brick units [ASTM C216 (2.14)] except that they are approximately ½ to 1 in. thick. ASTM C1088 covers two grades for exposure conditions: *Exterior* and *Interior*, and three types of thin veneer brick based on appearance: *TBS*, *TBX*, and *TBA*. Minimum compressive strength is not required in ASTM C1088 because there is no proper way to test thin bricks in compression.

Thin bricks are available in many different sizes, colors, and textures, which vary with producers. The most commonly found size is standard modular with nominal dimensions of $2\frac{2}{3} \times 8$ in. The actual face dimensions vary, typically $\frac{3}{8}$ to $\frac{1}{2}$ in. smaller than the nominal dimensions. The economy size is 50 percent longer and wider. The economy modular size—4×12 in.—is popular for use in large buildings. Various types of thin units are shown in Fig. 2.13.

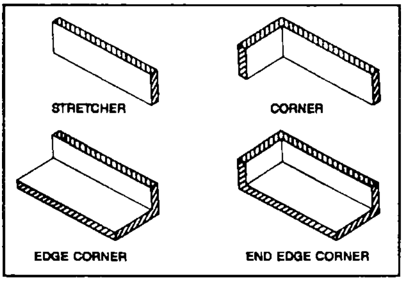


FIGURE 2.13 Thin brick units. (*Courtesy*: *BIA*.)

A variety of procedures are in use for the installation of thin brick units. Ceramic tile installation techniques are used to install the brick units, either at the jobsite or on prefabricated panels. In both United States and Japan, thin bricks have been placed into forms and cast integrally with concrete to produce architecturally attractive precast panels. Another procedure involves bonding thin bricks to a 16×48 in. substrate, resulting in small, lightweight, and modular panels which can be easily installed. A discussion on recommended installation practices can be found in Ref. [2.36].

Veneers can be used in masonry work in one of the three ways:

1. Adhered veneers, veneers that are secured to and supported by the backing through adhesion.
2. Anchored veneers, those that are secured to and supported laterally by the backing through anchors and supported vertically by the foundation or other structural elements.
3. Masonry veneers, veneers installed in the form of a wythe that provides the exterior finish of a wall system and transfers out-of-plane load directly to a backing, but is not considered adding load resisting capacity to the wall system.

Design provisions for veneers are covered in Chap. 6 of the MSJC-08 *Code* [2.3] and not discussed further in this book.

2.5.7 Ceramic Glazed Brick and Facing Tiles

Ceramic glazed brick facing tiles are governed by ASTM C126*:* Ceramic Glazed Structural Clay Facing Tile, Facing Brick, and Solid Masonry Units [2.17]. For exposed exterior applications, these tiles should also meet the durability requirements of ASTM C652 [2.15]. These units are produced from shale, fire clay, or a mixture thereof, and are characterized by their glazed finish. Glazing results from a highly specialized and carefully controlled process that involves spraying a number of ingredients on the units and burning at above 1500°F. Essentially, the glaze is a blend of clay, ceramic frit, fluxes, and base metals sprayed on the units before burning, and then subjected to normal firing temperatures. The sprayed ingredients fuse together to form a glasslike coating, rendering them inseparable. This coating may be produced in the form a clear glasslike finish, or in a number of solid colors and multicolor arrangements. The glaze is required to be impervious to liquids, and resistant to stain, crack, or craze. Since these tiles are not dependent on the color of their clay for their final color, they are available in a complete color gradation from pure whites to pastels of varying hue to deep solid colors and jet-blacks.

Ceramic glazed tiles are primarily used for decorative and sanitary effects. Since these tiles are required to be stain resistant, they can be easily cleaned; a major advantage. The tiles are usually flat, but vary in size from ½ in.^{2} to more than 6 in. They are available in a variety of shapes; rectangular, square, and hexagonal being the more predominant forms. Ceramic tiles are used for both wall and floor coverings. They are laid on a solid backing by means of mortar or adhesive. They are usually applied with the thinnest possible mortar joints; consequently, dimensional accuracy of tiles is of greatest importance. Classification of glazed tiles includes *grade* and *type*. Grade *S* (select) units are used with comparatively narrow mortar joints, whereas Grade *SS* (selected size) are used where variation in face dimensions must be very small. Both grades may be produced in either *Type I* or *Type II*. Type I units have only one face glazed; Type II have both opposite faces glazed.

Ceramic glazed tiles are also available in cellular forms (Fig. 2.14) that are similar to hollow bricks and can be reinforced and used as load-bearing elements. ASTM C212-00:

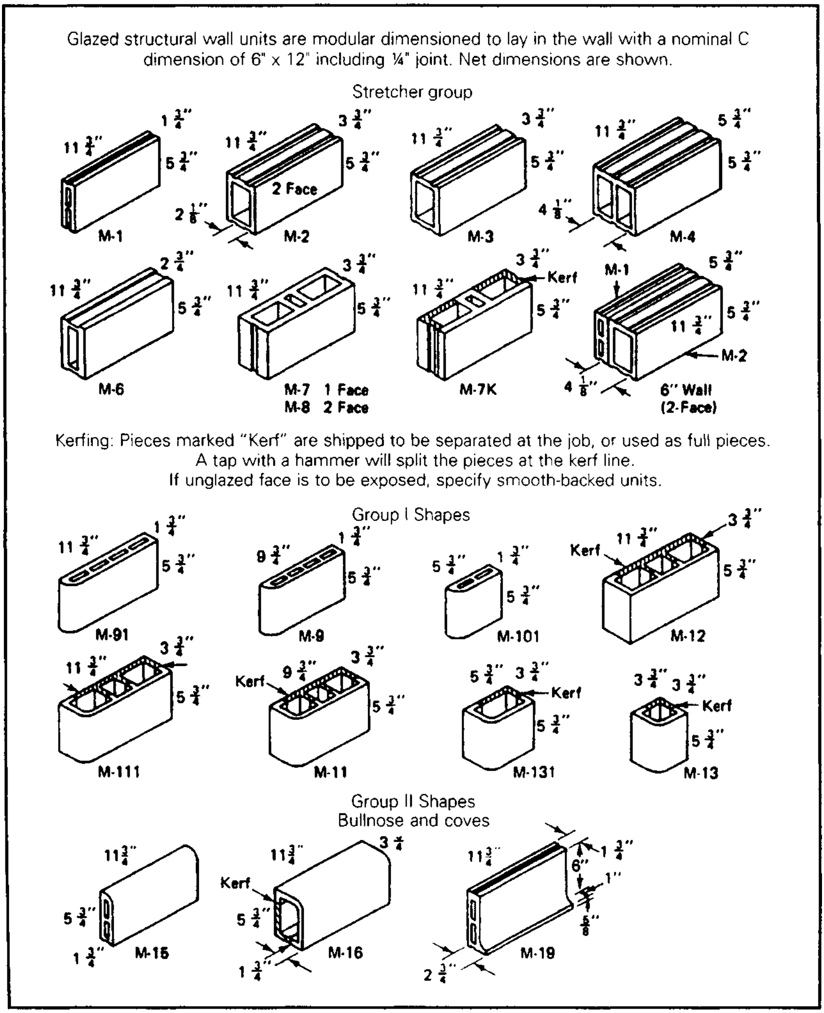


FIGURE 2.14 Glazed structural units. (*Courtesy*: *BIA*.)

Specification for Structural Clay Tile [2.37] places requirements on size and number of cells (different from ASTM C34) as well as web and shell thicknesses (e.g., one cell for 4-in.-thick, two for 6 and 8-in.-thickness, and three for 10 and 12-in. units). Other uses include nonload-bearing partitions, facings over other supports, and components of composite walls. A discussion on production, usage, and application of glazed ceramic tiles can be found in several references [2.7, 2.13, 2.38].

2.5.8 Architectural Terra-Cotta

Terra-cotta is a fired clay product used for ornamental work. The term "terra-cotta," which means "baked earth," has been applied for centuries to describe the molded and fired clay objects whose properties are similar to those of brick. Architectural terra-cotta was originally used as a load-bearing element in multiwythe walls. In the late nineteenth and early twentieth century, it gained popularity as a cladding material, particularly for structural frame buildings. Today, terra-cotta is used essentially as a ceramic veneer or ornamental facing tile for both exterior and interior walls [2.7].

Terra-cotta can be either hand molded or extruded. Hand-molded slabs are produced in the traditional manner with either plain or sculptured surfaces. Extruded units are mechanically fabricated with smooth-ground, beveled, scored, scratched, or fluted surfaces. Both may be glazed in clear monochrome, or polychrome colors and in matte (i.e., dull or lusterless), satin, or gloss finishes. Individual pieces are usually larger in face dimensions than brick, and are usually produced to order.

Terra-cotta also falls into the ceramic glazed classification, although it has only nonstructural uses. There are no governing ASTM Specifications for terra-cotta, but the units should meet the minimum requirements of the Standard Specifications for Ceramic Veneer and Standards for Sampling and Testing Ceramic Veneer [2.39].

2.5.9 Second-Hand Clay Masonry Units

Second-hand (also referred to as *reclaimed*, *used*, or *salvaged*) bricks are occasionally used in construction due to their "rugged appearance" and/or low initial cost. Architects may specify their use because of their weathered appearance, and broad color range that varies from dark-red to the whites and grays of units still partially covered with mortar. Most salvaged bricks are obtained from demolished buildings that may be 40 or 50 years old, or even older. Generally speaking, walls using salvaged bricks are weaker and less durable than those built from new brick units.

Several arguments are advanced in favor of using the salvaged bricks [2.40]:

1. Because bricks are extremely durable, they can be salvaged and used again.
2. If the bricks were satisfactory at the time they were first used, they are satisfactory at present.

However, both arguments are weak. This is because bricks, when initially placed in contact with mortar, absorb some particles of cementitious material. These absorbed particles cannot be cleaned completely from the surfaces of the brick units, resulting in a weakened bond between brick and mortar when reused. Therefore, salvaged units require attention to their condition at the time of use. Building code requirements vary regarding the use of salvaged bricks, and should be consulted prior to their selection and specification. There may be serious ramifications for using the salvaged bricks. 2009 IBC [2.5] which states: "Second-hand units shall not be reused unless they conform to the requirements of new units. The Units shall be of whole, sound materials and free from cracks and other defects that will interfere with proper laying or use. Old mortar shall be cleaned from the units before reuse." The MSJC-08 *Code* [2.43] apparently does not permit the use of salvaged brick units. MSJC *Specification for Masonry Structures* [2.4, Section 2.3B] requires clay or shale masonry units to conform to following ASTM Standards as specified:

1. ASTM C34-03: Specification for Structural Clay Load-Bearing Wall Tile [2.22].
2. ASTM C56-05: Specification for Structural Clay Nonload-Bearing Structural Wall Tile [2.23].
3. ASTM C62-05: Specification for Building Brick (Solid Masonry Units Made from Clay or Shale [2.12].
4. ASTM C126-99 (2005): Specification for Ceramic Glazed Structural Clay Facing Tile, Facing Brick, and Solid Masonry Units [2.17].
5. ASTM C212-00: Specification for Structural Clay Facing Tile [2.37].
6. ASTM C216-05a: Specification for Building Brick (Solid Masonry Units Made from Clay or Shale [2.14].
7. ASTM C652-05a: Specification for Hollow Brick (Hollow Masonry Units Made from Clay or Shale).
8. ASTM C1088-02: Specification for Thin Veneer Brick Units Made form Clay or Shale [2.18].
9. ASTM C1405-05a: Specification for Glazed Brick (single fired brick).

Or to

ANSI A137.1-88: Standard Specification for Ceramic Tile [2.41].

Bricks salvaged from old demolished buildings are generally inferior compared to their modern counterparts. This is because the early manufacturing methods were markedly different from today's methods. The old manufacturing methods were such that large volumes of brick were fired under greater kiln-temperature variations than could be permitted today. This condition resulted in a wide variance in finished products. The temperature variations also resulted in a wide range of absorption properties and color. The under-burned bricks were more porous, slightly larger, and lighter colored than the harder-burned bricks*. Their usual pinkish-orange color resulted in the name *salmon brick* [2.40]. However, when existing walls are demolished, hard-burned bricks and salmons are hopelessly mixed (Fig. 2.15) so that it is virtually impossible to distinguish between durable and nondurable

FIGURE 2.15 Brick salvaged from demolition of existing walls. (*Courtesy*: *BIA*.)

*It is the nature of ceramic products to shrink during firing. Generally, for a given raw clay, the greater the firing temperature, the greater the shrinkage and darker the color.

brick units. Inadvertent use of salmon brick for the exterior exposure would subject it to rapid and excessive deterioration.

2.5.10 Customized Masonry

Conventional masonry construction usually involves masonry units that are produced in modular sizes and rectangular shapes for economical reasons. However, brick may be formed in other shapes and sizes to suit specific job requirements. These include square and hexagonal pavers, bullnose and stair tread units, caps, sills, special corner brick, and wedges for arch construction. Unique shapes may be available from producers upon request. However, these units may be expensive depending on the size of the order.

Examples of customized masonry are sculptured pieces that are handcrafted from the green clayware before firing. Artists can create their artwork and carvings on the unburned clay units that are sufficiently stiff yet soft for carving, scrapping, and cutting. Upon completion of the artwork, the units are fired in the plant, and the result is permanent artwork on the clay panels (Fig. 2.16).

FIGURE 2.16 Sculptured brick panels at Loew's Anatole Hotel, Dallas, Texas. Beran and Shelmire, architects. Mara Smith, sculptor. (*Courtesy*: *BIA*).

2.6 CONCRETE MASONRY UNITS

2.6.1 General Description

Concrete masonry units (commonly referred to as CMUs) are made from a mixture of portland cement, aggregate (normal weight or lightweight), and water. They are available in a variety of shapes, sizes, configuration, strength, and colors.

Because the properties of concrete vary with the aggregate type and mix proportion, a wide range of physical properties and weights is available in concrete masonry units. Concretes containing various aggregates range in unit weight as shown in Table 2.7. The weight class of a concrete masonry unit is based on the density or oven-dry weight per cubic of the concrete it contains.

TABLE 2.7 Aggregate Type-Concrete Unit Weight Relationships [2.1]

Concrete	Unit weight, lb/ft^3
Sand and gravel concrete	130–145
Crushed stone and sand concrete	120–140
Air-cooled slag concrete	100–125
Coal cinder concrete	80–105
Expanded slag concrete	80–105
Pelletized-fly ash concrete	75–125
Scoria concrete	75–100
Expanded clay, shale, and sintered fly ash concrete	75–90
Pumice concrete	60–85
Cellular concrete	25–44

Concrete masonry units may be classified by their unit weights or applications. A concrete masonry unit is considered as follows (see Table 2.8):

1. Lightweight if it has a density of 105 lb/ft^3 or less.
2. Medium weight if it has a density between 105 and 125 lb/ft^3.
3. Normal weight if it has a density of more than 125 lb/ft^3.

Approximate weights of various hollow units may be determined from Fig. 2.17.

TABLE 2.8 Weight Classification for Load-Bearing Concrete Masonry Units

Weight classification	Oven-dry weight of concrete	Maximum water absorption
Lightweight	105 lb/ft^3 max.	18 lb/ft^3
Medium weight	105–125 lb/ft^3	15 lb/ft^3
Normal weight	≥125 lb/ft^3	13 lb/ft^3

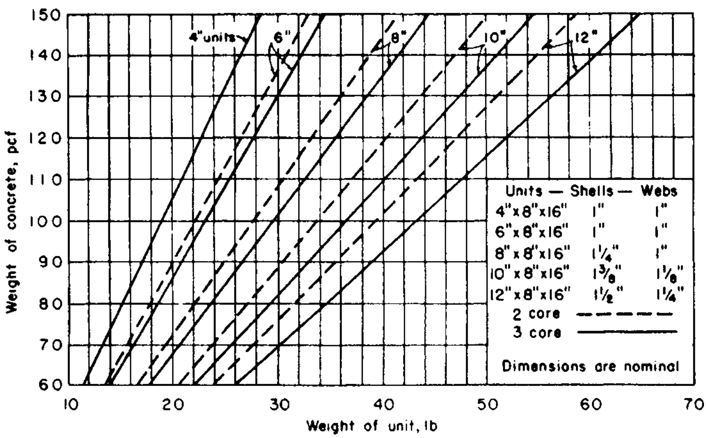


FIGURE 2.17 Weights of some hollow units for various concrete densities [2.1].

All units have to meet a minimum compressive strength calculated on average net area as follows:

Individual unit	minimum 1700 lb/in^2
Average of 3 units	minimum 1900 lb/in^2

Permissible variations in the dimensions are set at ± ⅛ in. For special units, different standards may be applied.

2.6.2 ASTM Standards for Concrete Masonry Units

Concrete masonry units are available in various sizes of rectangular shapes, which are manufactured according to the following ASTM Standards:

1. Load-bearing concrete masonry units (hollow load-bearing units conforming to ASTM C90-06: Specification for Load-Bearing Concrete Masonry Units, commonly referred to as concrete block [2.42].
2. Nonload-bearing concrete masonry units conforming to ASTM C129-05: Specification for Concrete Brick [2.43].
3. Concrete brick conforming to ASTM C55-03: Specification for Concrete Brick [2.44].
4. Prefaced concrete and calcium silicate masonry units conforming to ASTM C744-05: Specification for Prefaced Concrete and Calcium Silicate Masonry Units [2.45].

The focus of discussion in this book is load-bearing hollow concrete masonry units; therefore, these are discussed in considerable detail in this section. ASTM C90 [2.42] is the most frequently referenced standard for load-bearing concrete masonry units. A compilation of various ASTM Standards applicable to concrete masonry units can be found in Ref. [2.6].

2.6.3 Solid and Hollow Load-bearing Concrete Masonry Units

Solid masonry units are defined as those units whose net cross-sectional area is not less than 75 percent of the gross cross-sectional area; units not conforming to this requirement are classified as hollow units. These units are made from portland cement, water and mineral aggregates, with or without other aggregates, in two grades:

1. *Grade N:* For general use in walls above and below grade which may or may not be exposed to moisture or weather.
2. *Grade S:* For use in above-grade exterior walls with weather-resistant protective coatings, or walls not exposed to weather.

2.6.4 Relationship Between Unit Strength of Concrete Masonry Units and Specified Compressive Strength of Concrete Masonry, f'_m

The specified compressive strength, f'_m, of a masonry is greatly influenced by the variability in mortar strength, grout strength, and their interaction in generating compressive strength. As such, the relationship between the masonry unit strength and the strength of the masonry assemblage (f'_m) varies considerably, depending on the properties of mortar and grout used in the preparation of the masonry assemblage. The relationship between the compressive strength of concrete masonry units and the type of mortar is given in Table A5 (App. A).

2.6.5 Concrete Building Brick

Concrete building bricks conforming to ASTM C55 [2.43] are made from portland cement, water and suitable mineral aggregates with or without the inclusion of other materials. Their shapes are similar to those used for the clay bricks. Concrete brick has two grades:

1. *Grade N:* For use as architectural veneer and facing units in exterior walls and for use where high strength and resistance to moisture penetration and severe frost is required.
2. *Grade S:* For general use where moderate strength and resistance to frost are desired.

2.6.6 Nonload-Bearing Concrete Masonry Units

These units may be solid or hollow, and are made from portland cement, water, mineral aggregates with or without the inclusion of other ingredients. These units are generally used for nonload-bearing partitions or nonload-bearing exterior walls above grade, which are effectively protected from weather. All concrete masonry units are classified further according to their unit weights. The weight classification for the nonload-bearing units is same as shown in Table 2.7.

2.6.7 Shapes of Concrete Masonry Units and Their Applications

Concrete masonry units are produced in many different shapes to suit the many structural, architectural and functional needs for a wide variety of applications (Fig. 2.18). A discussion on shapes and sizes of concrete masonry units can be found in Refs. [2.1, 2.46], which is summarized here. Some of these shapes are commonly used for most wall applications (Fig. 2.19) while others are used for special applications. Of these the most commonly used in masonry construction is the hollow concrete block conforming to ASTM C90 [2.42], a *two-cell* hollow concrete masonry unit, which is specified as nominal $8 \times 8 \times 16$ in. but actually measures as $7\frac{5}{8} \times 7\frac{5}{8} \times 15\frac{5}{8}$ in. A more complete guide to masonry units is the *Shapes and Sizes Directory* [2.47].

Typically, the face shells and webs are tapered on the concrete masonry units so that their one end is slightly wider than the other. Depending on the type of molds used in the manufacture of the units, the face shells and webs may be tapered with a flare at one end, or may have a straight taper from top to bottom. The units are placed with the wider ends facing up, which provide a wider surface for mortar, and afford easier handling.

Figure 2.20 shows several shapes of concrete masonry units that have been developed to accommodate vertical and horizontal reinforcement required in structural masonry construction. These include open-ended or “A” shaped units, double open-ended or “H” shaped units, and pilaster units. All three are used where vertical reinforcement is to be provided (e.g., in walls, columns and pilasters). The double open-ended unit is the biggest and has the most voids. This unit should be specified when maximum grout flow is preferred. While ungrouted open-ended cells are permitted by the code, it is preferable to grout them. This is because an ungrouted open-ended cell is vulnerable to impact load on its face shell. Bond beam units and lintel units are used where horizontal reinforcement is to be provided.

While the two cell units shown in Fig. 2.19 can also be used where vertical reinforcement is to be provided, the open-ended units shown in Fig. 2.20 are special in that

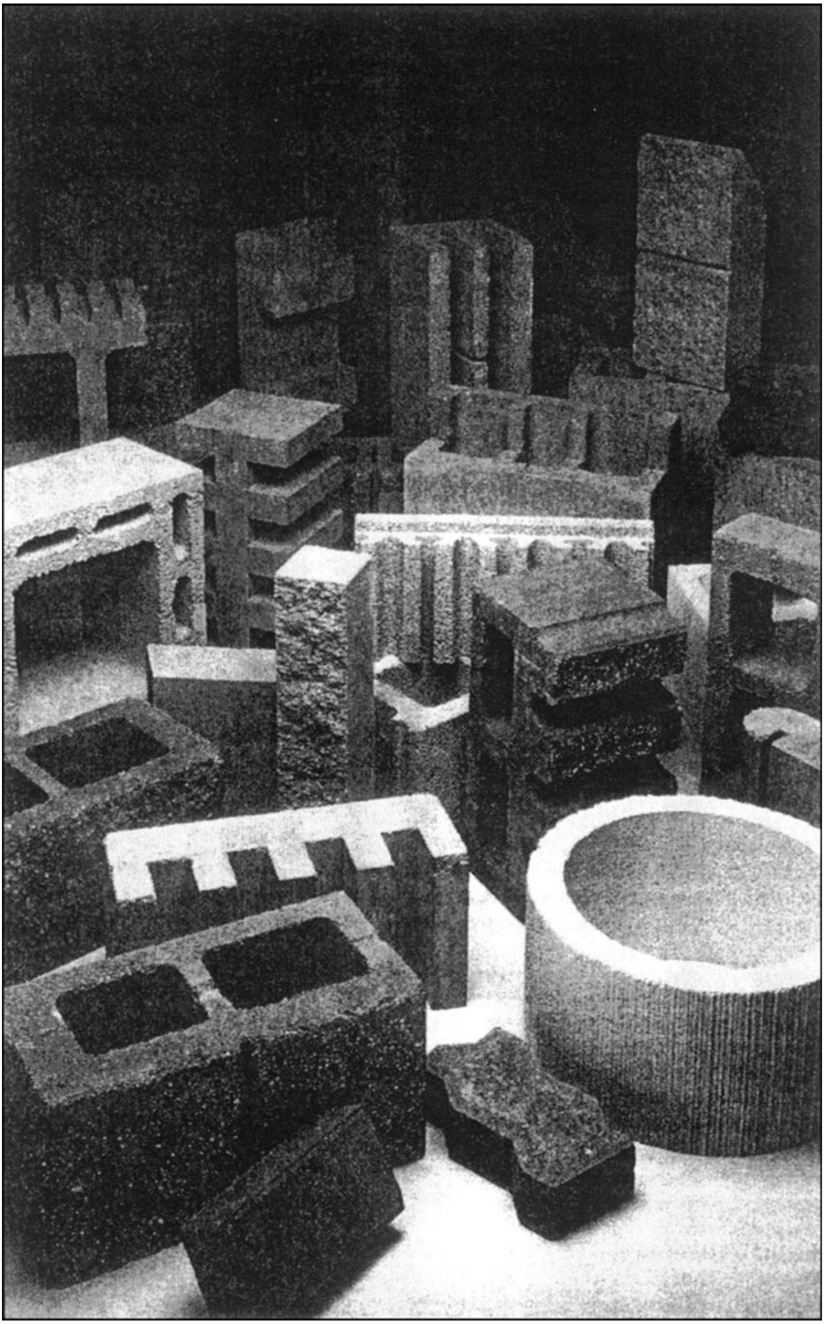


FIGURE 2.18 Some sizes and shapes of concrete masonry units for sills, copings, bond beams, and lintels. (*Courtesy*: *PCA*.)

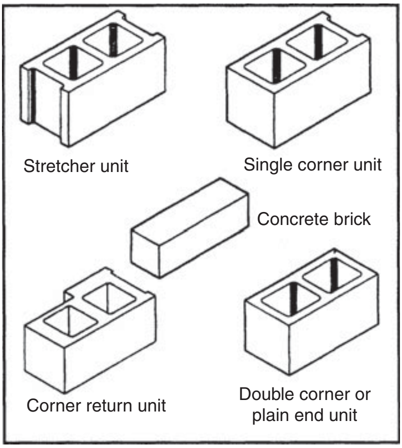


FIGURE 2.19 Typical concrete masonry units. (*Courtesy*: *PCA*.)

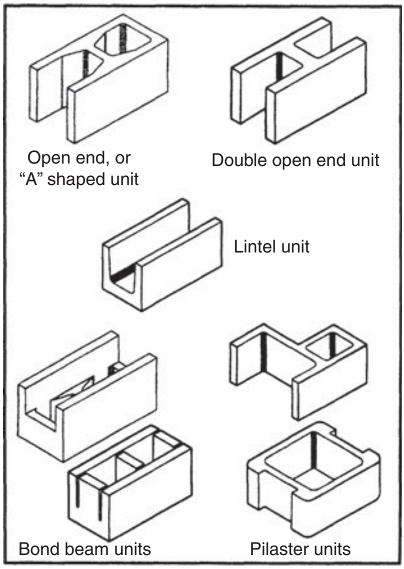


FIGURE 2.20 Shapes to accommodate vertical or horizontal reinforcement. (*Courtesy*: *PCA*.)

FIGURE 2.21 Threading a reinforcing bar through a hollow concrete block. (*Courtesy: Author.*)

they can be placed conveniently around the reinforcing bars already in place. The major advantage of this is the elimination of the need to lift the units over the tops of the reinforcing bar or to thread the reinforcement through the cores after the wall is constructed (Fig. 2.21). Bond beam and lintel units are essentially U- or channel-shaped units that are used to accommodate horizontal reinforcement. Bond beam units are characterized by depressed webs to accommodate horizontal reinforcing bars. They are produced either with reduced webs, or with “knock-out” webs which are removed prior to placement in the wall. Lintel units are produced in various depths that may be required to carry desired lintel loads over the door and window openings. The solid bottoms in both bond beam and lintel units confine the grout. Pilaster and column units are used to accommodate wall-column or wall-pilaster interface, and allow space for load-carrying reinforcement.

Figure 2.22 shows examples of special shape units, typically used in wall applications. Sash blocks are produced with a vertical groove molded on one end to accommodate a window sash. Sash blocks can be laid adjacent to one another to accommodate a preformed control joint gasket. Control joint units have one male and one female end to provide lateral load transfer across the control joints. An all-purpose or kerf unit is produced with two closely spaced webs in the center instead of typical one web. This permits the unit to be split into two at the jobsite into two 8-in. blocks, which are typically used at the ends or corner of a wall, or adjacent to an opening. Bull-nosed units are manufactured with single or double bullnose, to soften corners. Screen units are produced in a variety of shapes, sizes, and patterns. Beveled-end units are produced with one end oriented at 45°, which can be used to form walls intersecting at 135°. It is common to employ mechanical cutters at jobsites where masonry units are used for construction, to cut modular units to the desired size and shape, particularly for joints where walls meet at angles other than right angles (Fig. 2.23).

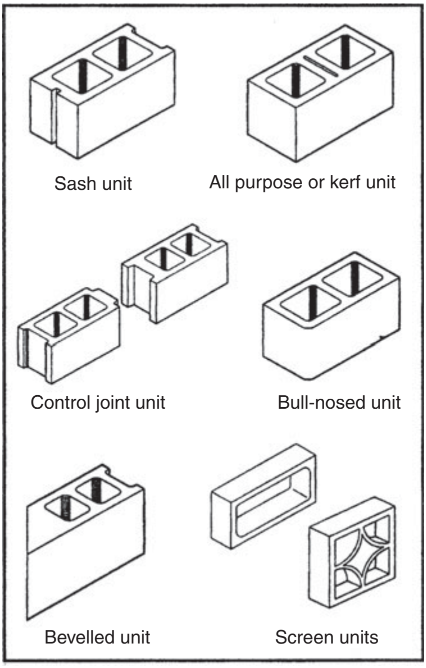


FIGURE 2.22 Special shape units. (*Courtesy*: *PCA*.)

Figure 2.24 shows examples of special units designed to increase energy efficiency. These units may have reduced webs to reduce heat loss through the webs. Reduction in web areas can be accomplished by reducing web thickness or height, reducing the number of webs, or both. In addition, the interior face shell of the unit can be made thicker than a typical face shell for increased thermal storage.

Figure 2.25 shows examples of acoustical units designed for sound mitigation and improving internal sound acoustics. These units are often used in churches, industrial plants, and schools.

Figure 2.26 shows radial units that can be used for manholes, catch basins, valve vaults, and underground structures. Figure 2.27 shows examples of architectural units. Customized column and chimney units and chimney caps are shown in Fig. 2.28.

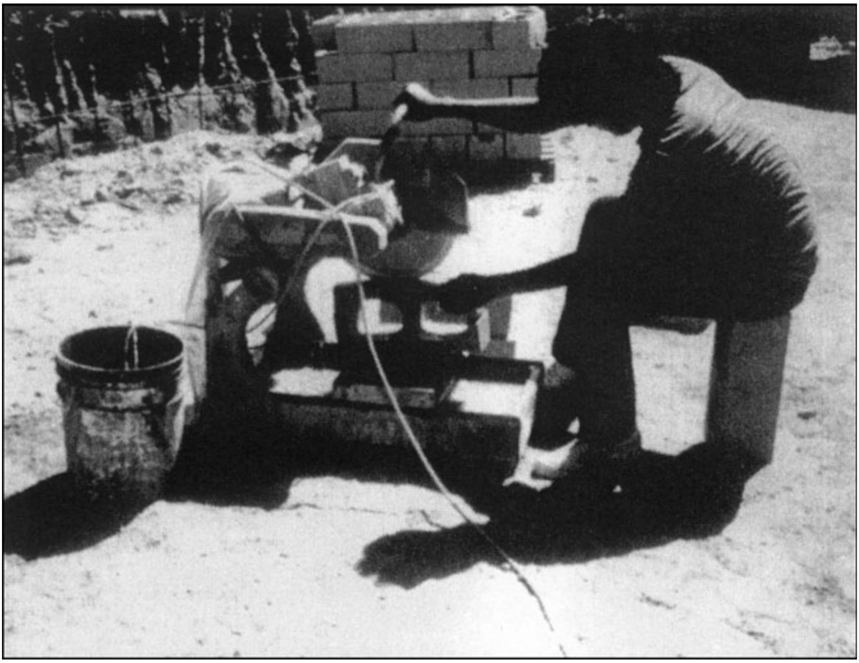


FIGURE 2.23 A mechanical cutter in use at a jobsite. (*Courtesy*: *Author.*)

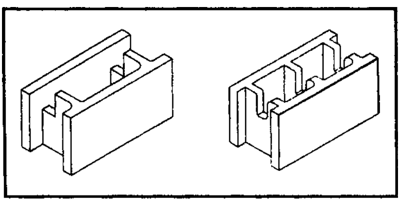


FIGURE 2.24 Examples of concrete masonry units designed for energy efficiency. (*Courtesy*: *PCA.*)

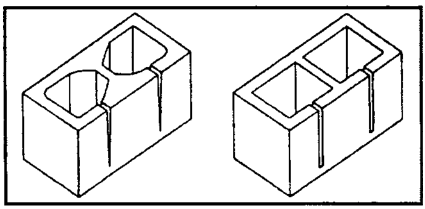


FIGURE 2.25 Examples of acoustical concrete masonry units. (*Courtesy*: *PCA.*)

2.6.8 Sizes of Concrete Masonry Units

Concrete masonry units, produced in a variety of sizes are specified by their sizes: width (thickness), height, and length, in that order. It is common practice to specify units by their nominal sizes, although the actual sizes are shorter by $\frac{3}{8}$ in., to allow for the thickness ($\frac{3}{8}$ in.) of typical mortar joint. For example, an $8 \times 8 \times 16$ standard concrete masonry unit has its nominal dimensions as 8 in. wide ($7\frac{5}{8}$ in. actual), 8 in. high ($8\frac{5}{8}$ in. actual) and 16 in. long ($15\frac{5}{8}$ in. actual). Figure 2.29 presents the nomenclature for various parts of a hollow CMU. In the finished wall, the joint-to-joint distance will be equal to one of the nominal block dimension. A hollow CMU may have two or three cells, referred to as two- or three-core units, respectively; a half unit has only one cell. The cells are used for the placement

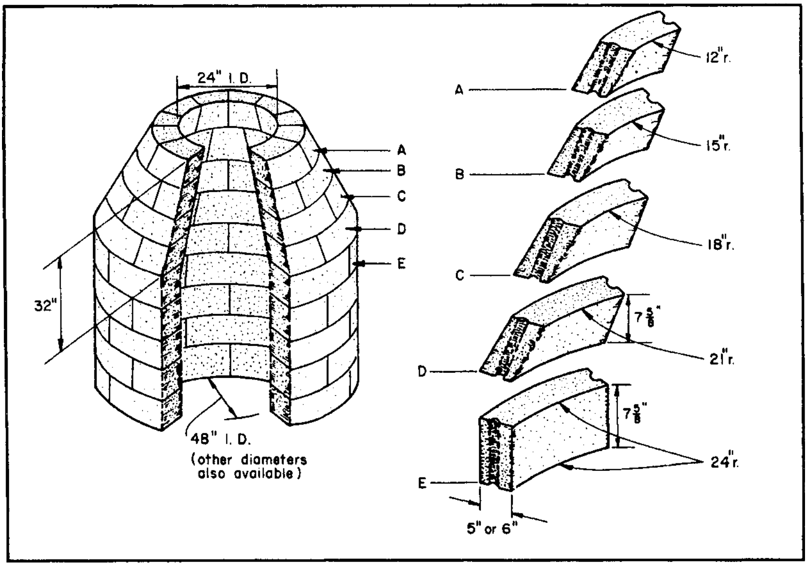


FIGURE 2.26 Radial units for manholes, catch basins, and valve vaults. (*Courtesy*: *PCA*.)

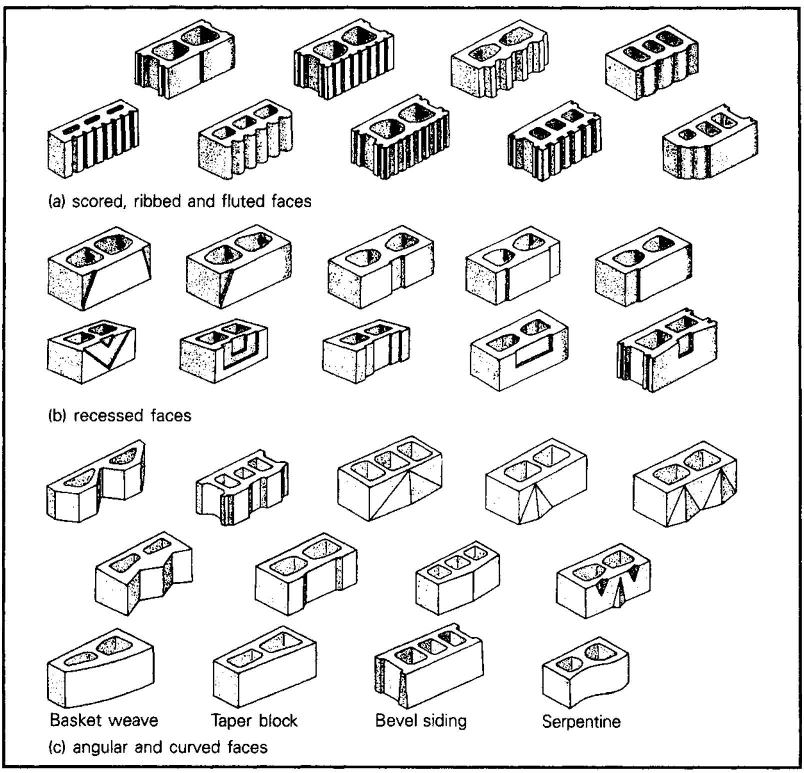


FIGURE 2.27 Examples of architectural units. (*Courtesy*: *PCA*.)

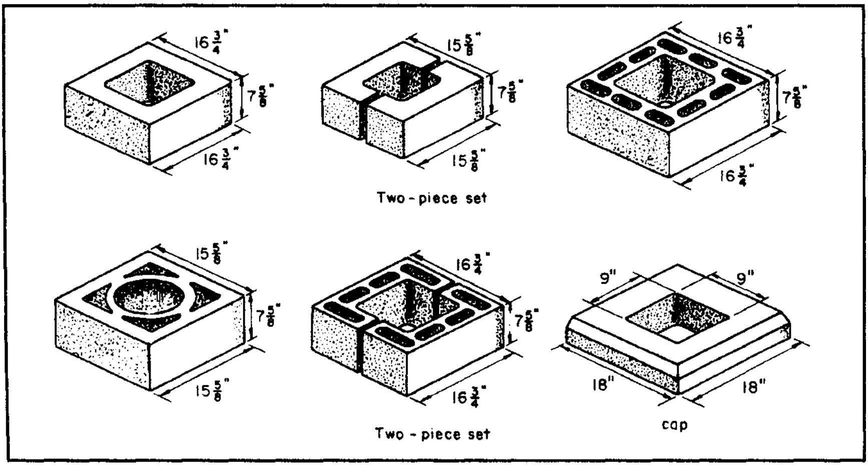


FIGURE 2.28 Customized column and chimney units, and chimney caps. (*Courtesy*: *PCA*.)

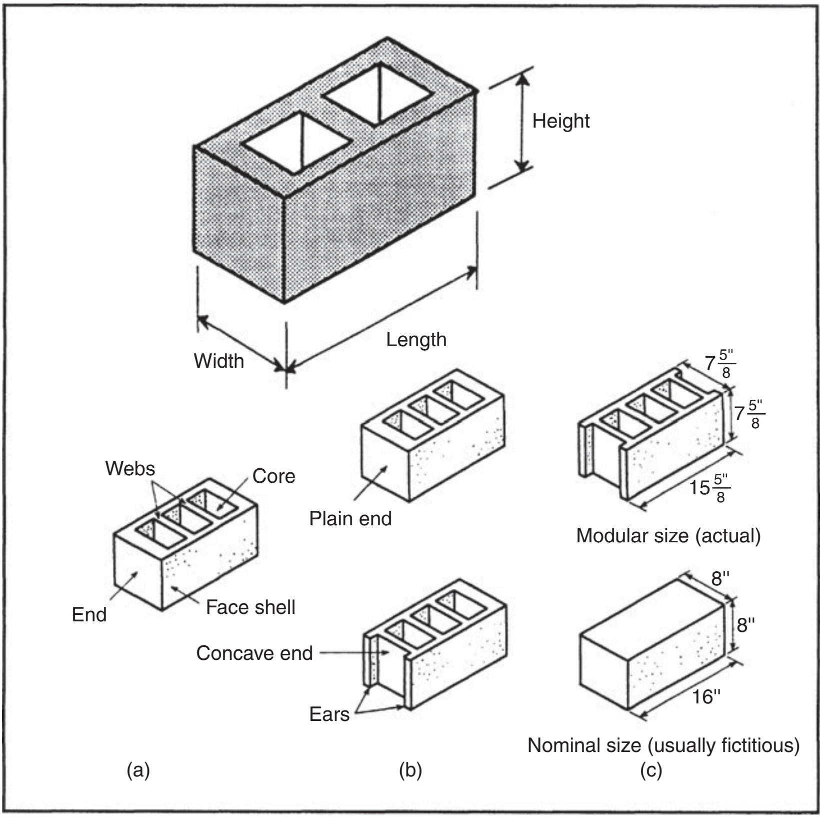


FIGURE 2.29 Nomenclature for typical concrete masonry units: (a) parts of a hollow block, each block has two face shells, two ends, and one or more webs, (b) common end types, (c) nominal and actual size of a typical 8 × 8 × 16 in. CMU. (*Courtesy*: *PCA*.)

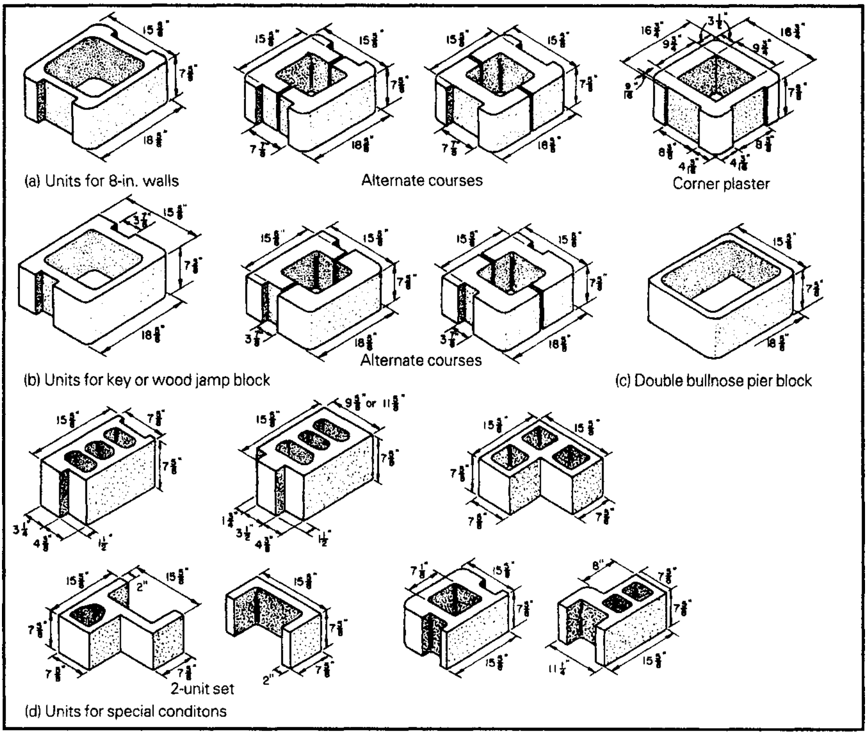


FIGURE 2.30 Special pilaster units in various shapes and sizes. (*Courtesy*: *PCA*.)

of reinforcement and/or grouting as necessary. The partition between the cells is called *web*. The two parts of the block perpendicular to the web are called the *face shells* (oriented along the length of a wall); those parallel to the web (or webs) are called the *ends* (oriented perpendicular to the wall).

Common concrete masonry unit nominal widths are 4, 6, 8, and 12 in.; common nominal heights are 4 and 8 in., and the common nominal length 16 in. Other sizes may be available on special order, depending on the producer. Also available are special blocks for columns and pilasters (Fig. 2.30). Figure 2.31 shows some sizes and shapes of concrete masonry units for sills, copings, bond beams, and lintels.

2.6.9 Surface Texture of Concrete Masonry Units

Concrete masonry units are produced with a variety of surface textures to satisfy the desired architectural requirements (Fig. 2.18). Textures are classified somewhat loosely, and with considerable overlapping, as open, tight, fine, medium, and coarse. Various degrees of smoothness (or roughness) can be achieved with any aggregate by changes in aggregate grading, mix proportions, wetness of mix, and the amount of compaction in molding. [2.1].

For example, slump blocks (or units) are produced by using a concrete mixture finer and wetter than usual. The units are squeezed to develop a bulging effect; the bulging faces

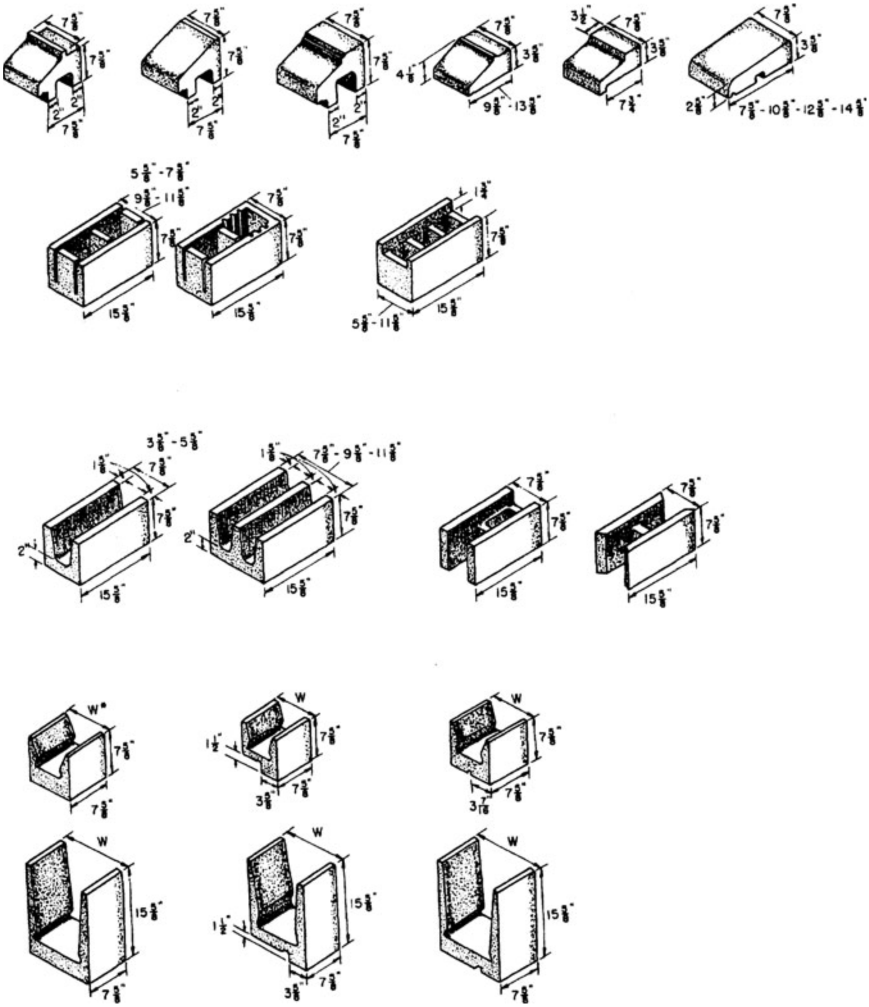


FIGURE 2.31 Concrete masonry units can be produced in various shapes and sizes. (*Courtesy*: *PCA*.)

resemble handmade adobe, producing a pleasing appearance (Fig. 2.32). Brick units can also be produced in a similar manner.

Split blocks are popular where aesthetic effects are important. Essentially, split blocks are solid or hollow units that are mechanically fractured lengthwise or crosswise to produce a stone-like texture. Split units may be obtained by simply striking a hammer blows to a heavy chisel placed on a concrete block along a scored line. The units are laid with their split surface exposed. When laid, the fractured faces of the units are irregular and expose the aggregate texture on the face of the wall (Fig. 2.33). A wide variety of colors, textures, and shapes are produced by varying cements, aggregates, color pigments, and unit size.

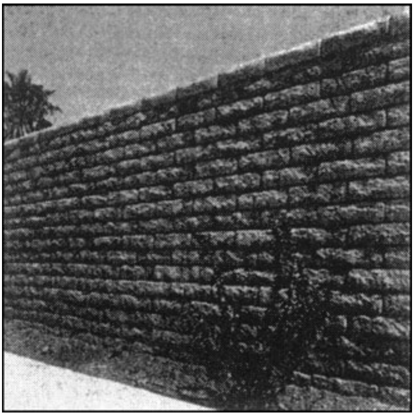


FIGURE 2.32 A slump block wall. (*Courtesy*: *PCA*.)

FIGURE 2.33 A wall with split-face concrete blocks. (*Courtesy*: *PCA*.)

2.6.10 Second-Hand Concrete Masonry Units

Discussion of second-hand clay masonry units presented in Section 2.5.8 applies to concrete masonry also.

2.6.11 Sound Absorbing Properties of Concrete Blocks

Concrete blocks with open exterior texture have excellent sound absorbing property. It is known that, upon striking a surface, sound waves are partly absorbed, partly reflected, and partly transmitted in varying amounts, depending upon the surface characteristics of concrete blocks. Surfaces which are smooth and dense, such as plaster and glass, reportedly, absorb only about 3 percent of the sound striking it. On the other hand, ordinary concrete blocks with rough surface characteristics can absorb between 18 and 68 percent of the sound that strikes it. In modern highway construction, concrete block sound barrier walls on the sides of the roadway are quite common. Design of sound barrier walls is discussed in Chap. 6.

2.7 BONDS AND PATTERNS IN MASONRY WORK

2.7.1 Bonds

The word "bond" is commonly used in the context of masonry design and construction. It is frequently references in building codes and specifications. Three kinds of *bond* are recognized [2.48]:

1. *Structural bond:* It refers to the method by which individual units are tied together so that the entire assembly acts integrally as a single structural unit.
2. *Pattern bond:* It refers to the pattern formed by the masonry units and the mortar joints on the face of a wall. The bond pattern may be the result of type of the structural bond used, or it may be purely decorative and unrelated to the structural bond.
3. *Mortar bond:* It refers to the adhesion of mortar to the masonry units or to reinforcement.

Techniques of obtaining bond in brick and masonry are essentially similar. However, differences exist because of the differences in the manner in which walls are constructed with clay masonry units (bricks) and concrete masonry units. Brick walls may be single wythe or multiwythe, whereas a wall built using concrete masonry units would usually be single wythe.

2.7.2 Bonds in Clay Masonry (Brick) Walls

All types of masonry units are placed together—one above the other, or side-by-side—with mortar joints between them, which is usually $\frac{3}{8}$ in. wide. Brick units can be laid in various positions: on the face or on the edge; the long dimension may be kept vertical or horizontal (parallel or perpendicular to wall face). These various positions are known as stretcher, header, soldier, shiner, rowlock or sailor (Fig. 2.4). Each layer of masonry units is called a *course*. The units can be arranged in courses in several ways. They may be laid in (a) regular courses of the same height, (b) in courses of two or more different heights, or (c) several sizes of units may be laid in a prearranged pattern. Within the same course in a two-wythe brick wall, the units can be laid (positioned) as headers or stretchers (Fig. 2.4).

A course consisting of units overlapping more than one wythe of masonry is called a *bond course*. Over the height of wall, the units can be placed so that the vertical joints are either continuous or discontinuous, resulting in many different kinds of appearances called *pattern bonds*. A comprehensive discussion on various types for brick masonry walls is presented in Ref. [2.48], which is summarized here.

Structural bonding of brick masonry walls is obtained in three ways:

1. By the overlapping (interlocking) of the masonry units,
2. By the use of metal ties embedded in connecting joints, and
3. By the adhesion of grout to adjacent wythes of masonry.

Traditionally, brick walls have been built with two types of overlapped bonds: *English bond* and *Flemish bond* (Fig. 2.34); other overlapped bonds are based on their variations. The English bond consists of alternating courses of headers and stretchers. The headers are centered on the stretchers and joints between stretchers in all courses are aligned vertically. By contrast, the Flemish bond consists of alternating courses of headers and stretchers in every course, so arranged that the headers and stretchers in every other course align vertically. In both cases, the stretchers, laid with the length of the wall develop longitudinal bonding, whereas the headers, laid across the width of the wall, bond the wall transversely. Building codes require that masonry bonded brick walls be bonded so that not less than 4 percent of the wall surface is composed

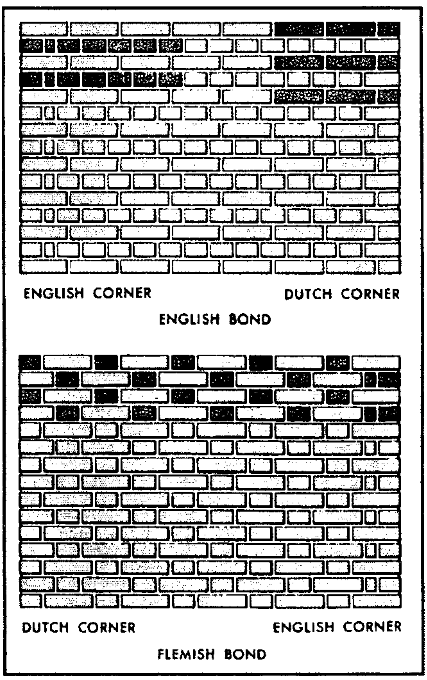


FIGURE 2.34 English and Flemish bonds. (*Courtesy*: *BIA*.)

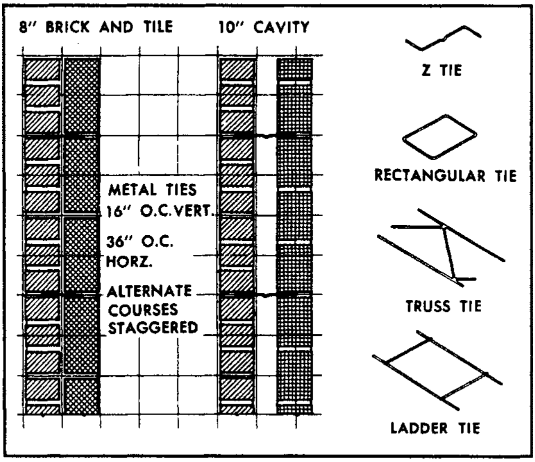


FIGURE 2.35 Metal-tied brick masonry walls. (*Courtesy*: *BIA*.)

of headers, with the distance between adjacent headers not exceeding 24 in., horizontally or vertically. Structural bonding of brick walls is accomplished with metal clips which can be used in both solid as well as cavity walls (Fig. 2.35).

Variations of English and Flemish bonds may be used as structural bonds, often referred to as pattern bonds. There are five basic types of structural bonds that are in common use and which create typical patterns (Fig. 2.36):

1. Running bond
2. Block or stack bond
3. Common or American bond
4. English bond
5. Flemish bond

In reinforced masonry construction, the masonry units are generally laid in one of two types of bond patterns: *running bond* and *stack bond*. MSJC-08 *Specification* ([2.4], Section 3.3A) specify masonry to be laid in running bond unless otherwise required. There are design implications (see next section) related to masonry layout in running bond and stack bond. To avoid ambiguity, the *Code* [2.3] defines running bond as "the placement of masonry units such that the head joints in successive courses are horizontally offset at least one-quarter the unit length." For reinforced masonry in running bond, the hollow masonry units are so placed that cells are aligned vertically to permit vertical reinforcing bars to continue through a wall without being interrupted. Stack bond is purely a pattern bond in which all units are laid as stretchers; vertically masonry units bear directly on the ones below them so that all joints align vertically; there is no overlapping of units.

Running bond is the simplest type of pattern bond. All units are laid as stretchers. Since there are no headers in this bond, metal ties are usually used. Running bond is primarily used in cavity wall construction, and veneered walls of brick. The vertical joints are staggered (discontinuous) by overlapping the units of adjacent courses by half or one-third length of the unit; however, joints themselves align in alternate courses. In the half running

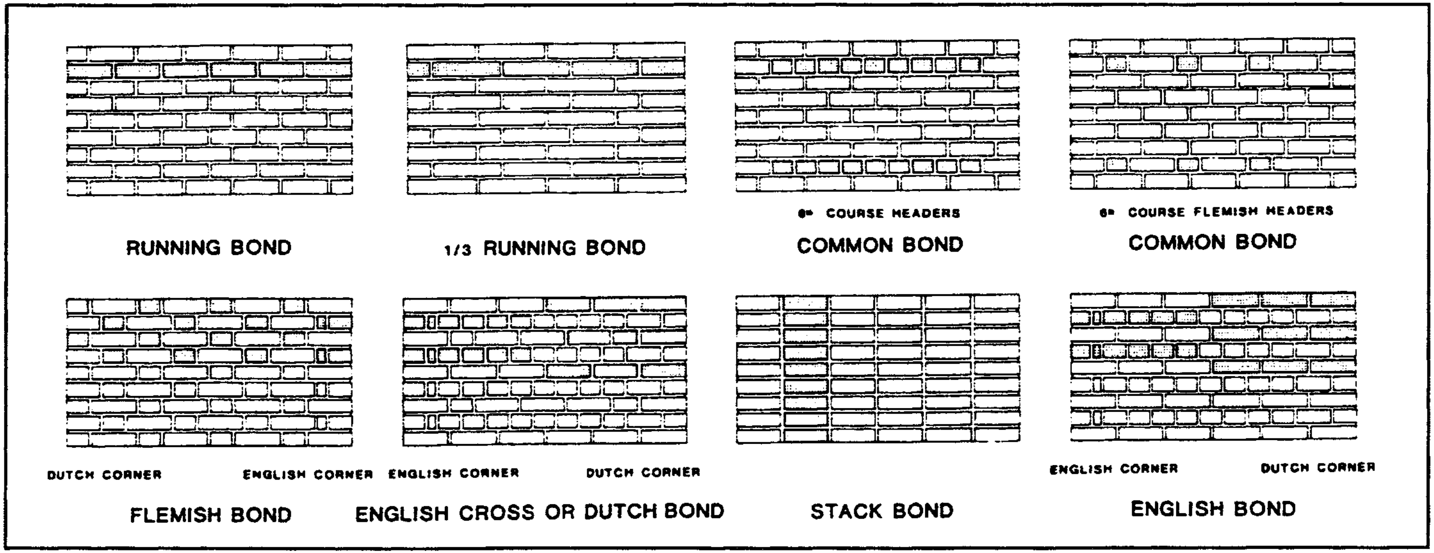


FIGURE 2.36 Traditional pattern bonds. (*Courtesy: BIA.*)

bond, joints are centered over the stretchers in alternate courses. In one-third running bond, the joints overlap the stretchers by one-third their lengths in alternate courses. The running bond pattern is the strongest and most economical.

English and Flemish bonds were described earlier. The Flemish bond may be varied by increasing the number of stretchers between headers in each course. *Garden wall bond* is formed when three stretchers alternate with a header. When there are two stretchers between headers, the bond pattern is called *double stretcher garden wall*. Garden wall bond may be formed with four or even five stretchers between headers. Figures 2.37 and 2.38 show some bond patterns which can be obtained by varying brick color. Figure 2.38 shows a garden wall bond with the pattern units in a dovetail fashion.

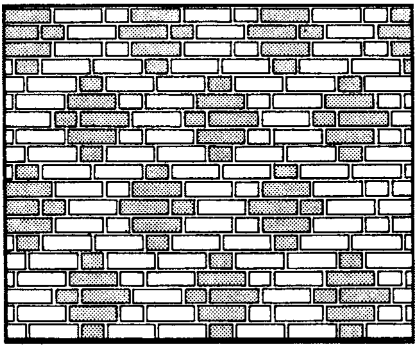


FIGURE 2.37 Double stretcher garden wall bond with units in diagonal lines. (*Courtesy*: *BIA*.)

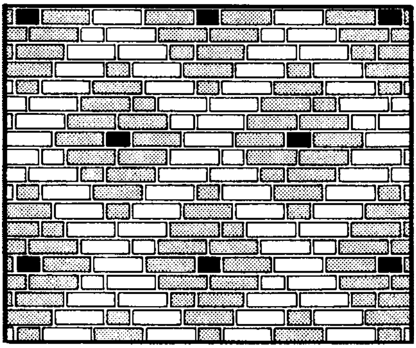


FIGURE 2.38 Garden wall bond with units in dovetail fashion. (*Courtesy*: *BIA*.)

English cross or *Dutch* bond is a variation of English bond in which joints between the stretchers in alternate courses do not align vertically; the joints themselves are centered on the stretchers in alternate courses.

Many variations and modifications of traditional bond patterns have been used to provide aesthetic and architectural effects. Variations of patterns may be created by projecting or depressing the faces of some units from the overall surface of the wall. Figure 2.39 shows uses of masonry in imaginative ways to form a pattern that is quite bold visually.

Units of different color and texture can be used in various pattern bonds to give desired architectural effect and aesthetic appeal. Before deciding upon a particular pattern, especially those of the stacked variety, its acceptability by the local building codes should be checked.

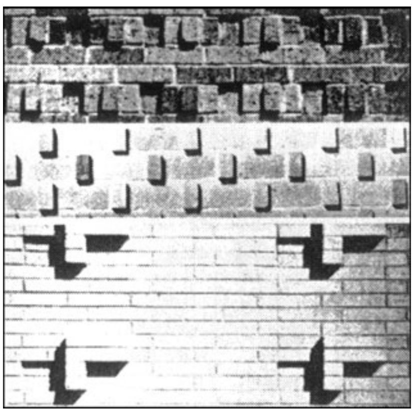


FIGURE 2.39 Contemporary bonds. (*Courtesy*: *BIA*.)

2.7.3 Bond Patterns in Concrete Masonry (Block) Walls

As in brick walls, the two most commonly used bonds are the running bond and the stack bond. Running bond may be half running bond or one-third running bond. Concrete masonry units are available in many thicknesses. In many cases, a variety of architectural effects can be obtained by a carefully planned mixing of these units. In addition to running and stack bond, concrete block can also be laid in diagonal and random patterns to produce almost any architectural and visual effects. Structural aspects of bonds in concrete masonry are discussed in Ref. [2.49], which are summarized in Chap. 6. Ref. [2.1] illustrates 42 patterns for concrete masonry walls; some examples are shown in Fig. 2.40.

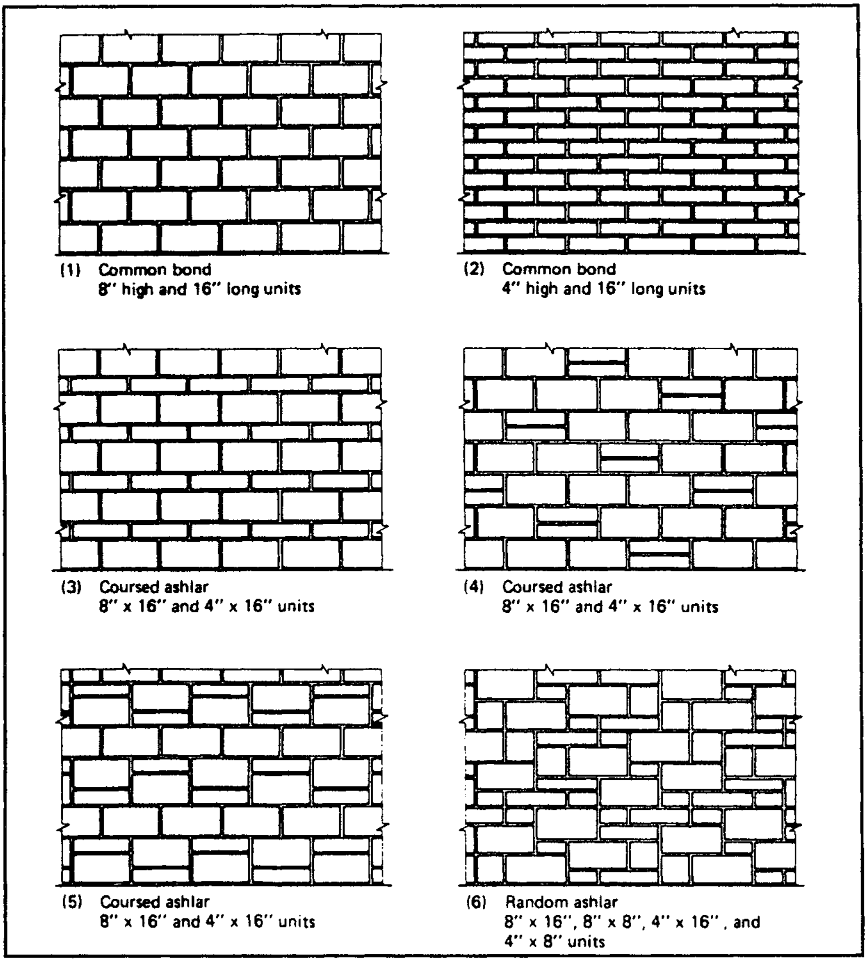


FIGURE 2.40 Bond patterns for concrete-block walls. (*Courtesy*: *NCMA*.)

2.8 STRUCTURAL REQUIREMENTS FOR MASONRY IN STACK BOND

From a design perspective, it is important to recognize these two types of bonds because of the many ramifications when masonry is in stack bond. Design requirements are more restrictive for masonry constructed in stack bond. The *Code* [2.3] specifies (1) special requirements that must be met when masonry is laid in stack bond and (2) limitations on strength when masonry is constructed in stack bond.

1. *Special requirements:* MSJC-08 Section 1.11 states that "for masonry in other than running bond*, the minimum area of horizontal reinforcement shall be 0.00028 times the gross-sectional area of the wall using specified dimensions. Horizontal reinforcement shall be placed in horizontal joints or in bond beams spaced not more than 48 in. on center." Furthermore, when stack bond is used for masonry walls in seismic regions, a much greater area of minimum horizontal reinforcement must be provided as compared to that for walls in running bond. MSJC-08 *Code* Sections 1.17.3 and 1.17.4 address specific requirements for reinforced masonry walls in structures in various Seismic Design Categories as follows:
   - **a.** Walls in *Seismic Design Category D:* Wythes of stack bond masonry to be constructed of fully grouted hollow open-end units, fully grouted hollow units laid with full head joints, or solid units. The minimum area of horizontal reinforcement to be 0.0007 times the gross cross-sectional area of wall; maximum spacing of reinforcement to be 24 in. (as compared to 48 in. for masonry in running bond).
   - **b.** Walls in *Seismic Design Category E and F:* Same as walls in Seismic Design Category D (above) with these additional requirements: (i) if the walls are *not* a part of lateral force resisting system, the minimum area of horizontal reinforcement to be at least 0.0015 times the gross cross-sectional area of masonry, and maximum spacing of horizontal reinforcement to be 24 in.; (ii) if the walls are a part of lateral force resisting system, the minimum area of horizontal reinforcement to be at least 0.0025 times the gross cross-sectional area of masonry, and maximum spacing of horizontal reinforcement to be 16 in.
2. *Strength limitations:* The *Code* [2.3, Section 3.1.8.2] places limits of strength of masonry in stack bond. The modulus of rupture for masonry in stack bond when the direction of flexural tensile stresses is parallel to bed joints is zero (Table 3.1.8.2.1, Ref. [2.2]). This is the case when beams and shear walls are subjected to loads (beams loaded transversely and shear walls carrying in-plane loads). This essentially means that transversely loaded members such as beams and shear walls cannot be constructed of masonry in stack bond.

The idea behind requiring some horizontal reinforcement as well as bond beams in stack bond masonry is to provide continuity across the head joints and ensure that all masonry units would act together as integral units rather than various stacks of masonry acting independently. Furthermore, masonry in intersecting walls is required to be in running bond (Ref. [2.2], Section 1.9.4.2).

In large wall areas and in load-bearing construction, it is advisable to reinforce the wall with steel placed in horizontal mortar joints. Dimensional accuracy of units is very important to maintain vertical alignment of the head joints.

---

*Masonry not overlapped a minimum of ¼ in. of the unit length is considered to be laid in "other than running bond."

2.9 MORTAR JOINTS

Mortar joints play an important role in the bond patterns and texture of a wall. Concrete masonry units are placed with a gap of ⅜ in. between the adjacent units, both horizontally and vertically, in order to provide for the mortar joints between them. Thus, the mortar joints appear as ⅜ in. wide stripes on the face of a masonry wall. However, these joints are not made solid through the wall thickness, that is, they are not continuous through the thickness (width) of a hollow concrete masonry wall. Rather, only the vertical head joints and horizontal bed joints are required to receive mortar; mortar is *not* placed on interior cross-webs or across the full width of head joints. Mortar joints are only as thick (measured perpendicular to the wall thickness) as the thickness of the face shell of a hollow concrete masonry unit. The face shell of a typical 8 in. (nominal) concrete masonry unit is 1¼ in. thick; therefore, mortar joints are 1¼ in. thick (measured perpendicular to wall) by ⅜ in. wide (measured on the face of the wall) on only the head and bed joints of the concrete masonry units. Thus, when the heads of two closed-end units are mortared together at a vertical joint, it results in a ⅜ in. wide air space between the adjacent masonry units that remains even if the wall is solid grouted. Of course, if necessary or preferred, a wall can also be built *without* any built-in air voids. This can be accomplished by using open-end units.

When the masonry surface is not to be plastered, all mortar joints of the masonry are finished as specified. In some cases, the joints are accentuated by deep *tooling*. Basically, three kinds of joint finishes are used in masonry construction, with some variations in each: *flush joints*, *tooled joints,* and *raked joints*. *Joint finish* is given with the help of appropriate finishing tools.

Various kinds of masonry joints are shown in Fig. 2.41. Flush joints are prepared by removing or cutting off the mortar flush with the face of the wall with a trowel. Tooled joints can be *concave* or *V-shaped*, and are prepared by striking the joints with a metal jointing tool. Racked joints are recessed joints prepared by racking out the mortar at a distance of ⅜ in. from the face and then tooled with a square jointer or the blade of a *tuck pointer* (Fig. 2.42). Another form of joint is the *squeezed joint*. This type of joint is obtained simply by using an excess of mortar while laying the units. During this process, some of the mortar is squeezed out as the units are set and pressed into place; this squeezed out mortar is not trimmed off but left in place.

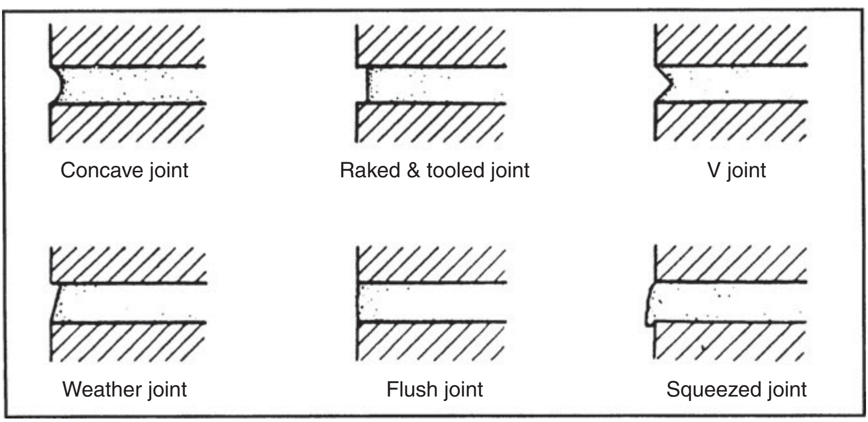


FIGURE 2.41 Various kinds of joint finishes for masonry walls. (*Courtesy*: *NCMA*.)

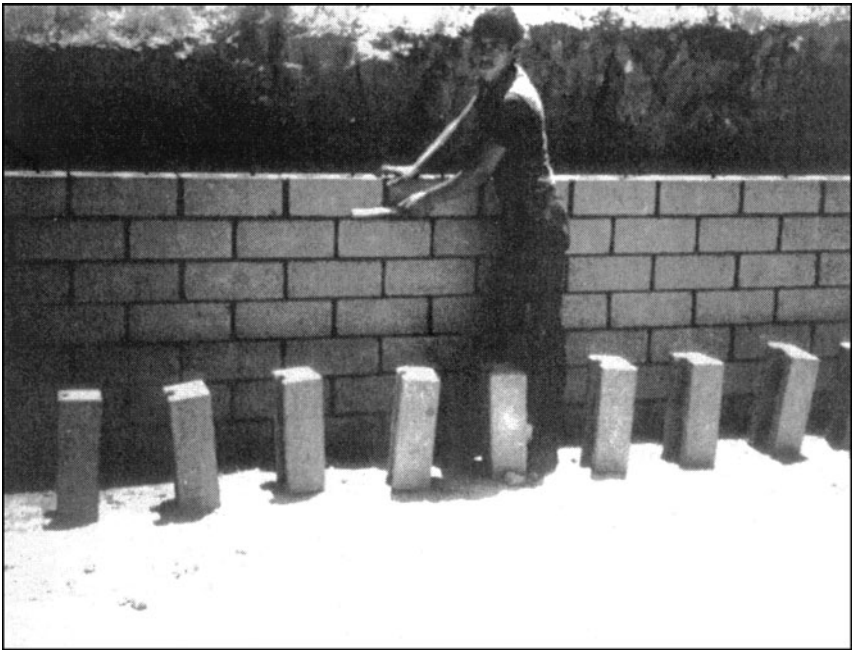


FIGURE 2.42 Tooling of masonry joints. (*Courtesy*: *Author.*)

2.10 TYPES OF WALL CONSTRUCTION

Columns and walls are examples of structural elements for which masonry construction employed. Design of these two elements is presented in Chaps. 5 and 6, respectively. A very brief summary of reinforced masonry construction is presented here.

2.10.1 Single and Cavity Walls

From the standpoint of construction, walls are commonly built in one of the two ways. The most frequently built is the *single wythe*, *single-block,* or *solid wall*. The term *solid* is rather a misnomer because the blocks may be cored or hollow. Figure 2.43 shows typical details of a reinforced masonry foundation wall built from concrete blocks.

The other type of wall is the *cavity wall* which consists of two separate wythes, separated by an air space and tied together with metal ties which are embedded in the horizontal mortar joints. Figure 2.44 shows details of a cavity wall. The two wythes can also be held together with bricks or blocks laid cross-wise between the two wythes at certain points. Regardless of how the two wythes are tied together, the purpose of these connectors is to transfer shear between the wythes. As a result of this inter-wythe shear transfer, if the two wythes are spaced close together and bonded with the closely spaced units, the behavior of a cavity wall approaches that of a single-wythe wall.

Cavity walls have several advantages. The continuous air barrier helps prevent moisture and, to some extent, heat from penetrating the wall. The cavity can also be filled with insulating material such as glass wool to give better insulating performance.

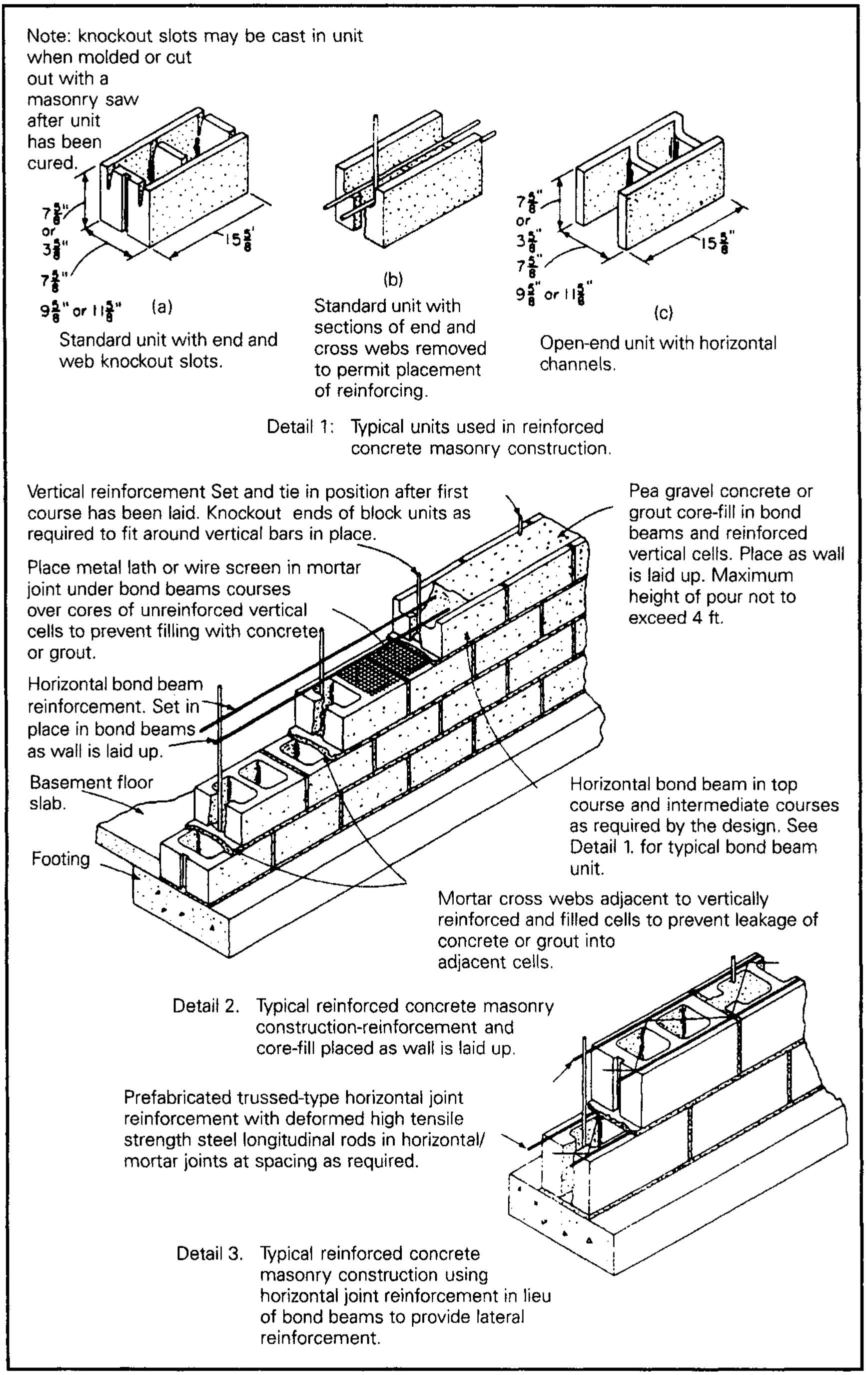


FIGURE 2.43 Details of a single-wythe wall for foundation. (*Courtesy*: *NCMA*.)

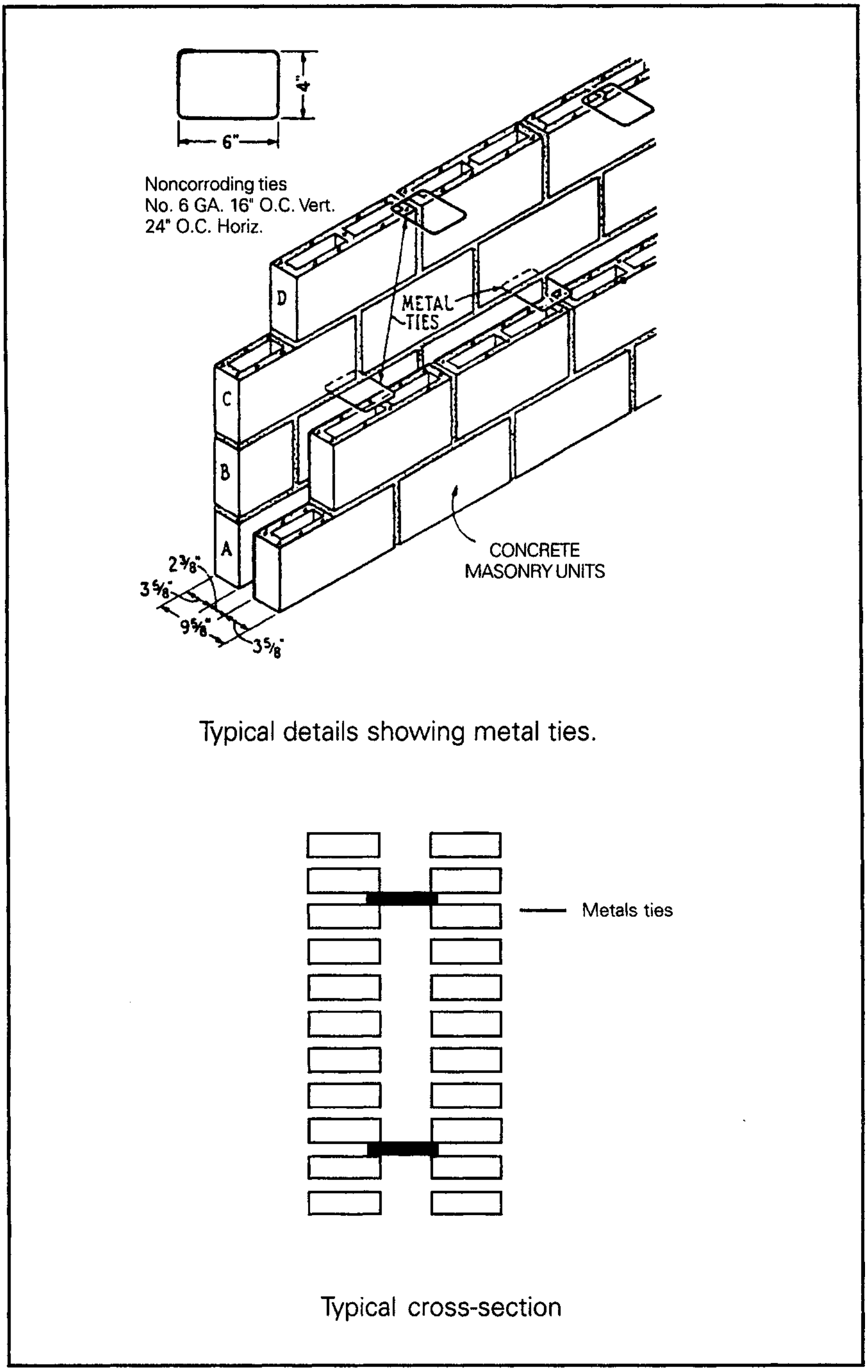


FIGURE 2.44 Details of a typical cavity wall. (*Courtesy*: *NCMA*.)

2.10.2 Modular Construction of Walls

Because blocks are available in specific sizes and depths, attention should be paid to the finished dimensions of walls and openings (length and height) shown on plans. It should be noted that certain number of blocks will result in a specific length of a wall. For example, a 20 ft long wall will contain exactly fifteen 15⅝-in. long (16-in. nominal) stretchers. Similarly, nine courses will be required to build a 6 ft high wall using units which are 7⅝ in. high (8 in. nominal). The overall dimensions of walls should be planned in such a way that an exact number of courses will produce the required heights without the necessity of cutting blocks. Figure 2.45 shows an elevation of a wall with openings for a door and a window showing blocks that have been cut to provide the required door opening. Figure 2.46 shows the elevation of the same wall where proper planning has resulted in the same soffit levels of lintels for both openings, requiring the same number of courses for both. *Coursing* for brick masonry construction is discussed in Ref. [2.11].

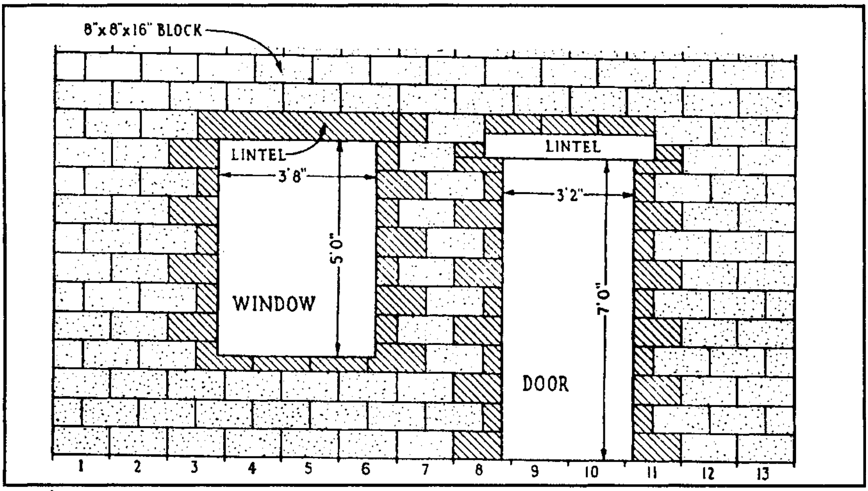


FIGURE 2.45 Example of wrong planning of a concrete block wall. The heights of the door (84 in.) and the window (60 in.) will not match the heights that can be formed by any whole number of 8-in.-high blocks (because the numbers 84 and 60 are not exactly divisible by 8). Similarly, their widths cannot be achieved by leaving out a combination of certain number of 8- and 16-in. long blocks. Note the clumsy arrangement of specially cut blocks around the openings. The dimensions of the openings should be proportioned so that they will match the multiples of the length (8 and 16 in.) and the height (8 in.) of the block. (*Courtesy*: *NCMA*.)

2.11 GLASS UNIT MASONRY

Glass block is considered masonry because it is laid up in cement mortar and uses the same type of joint reinforcement as other units. Glass unit masonry consists of glass units bonded together with mortar. They are manufactured by fusing two molded glass halves together to produce a partial vacuum in the center. Edges that will receive mortar are then treated with a polyvinyl butyral coating or latex-based paint to increase mortar bond and to provide an expansion and contraction to minimize cracking [2.50].

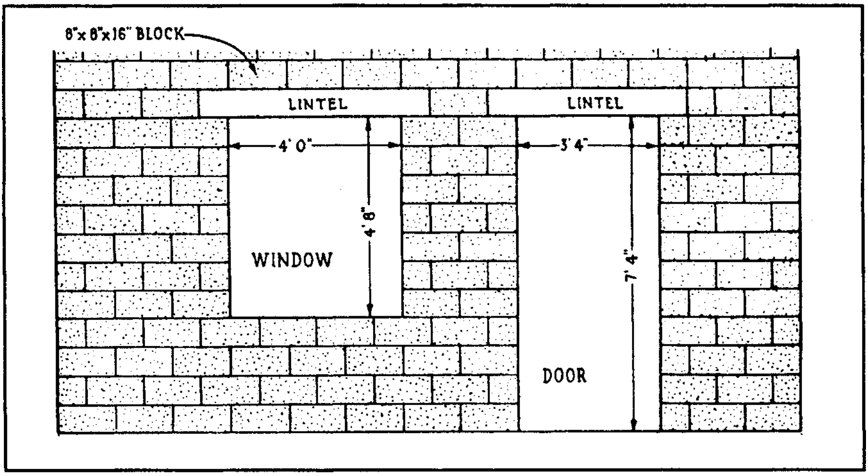


FIGURE 2.46 Example of correct planning of a concrete-block wall. Note the matching dimensions of the door and window openings. The 48-in. width of the window equals total length of three 16-in. blocks, and the height equals seven 8-in.-high courses. The door width is equal to a combination of two 16- and one 8-in. blocks, and the height equals eleven 8-in.-high courses. (*Courtesy*: *NCMA*.)

Glass units may be either solid or hollow. Typical minimum face thickness is $3/16$ in. Units with thicker faces, as well as 100 percent solid units, are also produced, which provide increased impact resistance, sound insulation, and fire ratings. These are produced in a variety of sizes, shapes, patterns, and textures. Shapes are also available to accommodate corners, radii, and exposed wall ends. Decorative blocks are available in clear, reflective, or color glass with smooth, molded, fluted, etched, or rippled or wavy texture, providing varying degree of light transmission and privacy. Most glass is made of clear, colorless variety that admits full spectrum of natural light. Light transmission characteristics of various types of glass units are summarized in Table 2.9 [2.7].

TABLE 2.9 Light Transmission Characteristics of Glass Units [2.7]

Type of glass block	Percent of light transmitted
Solid	80
Hollow	50–75
Diffusion	28–40
Reflective	5–20

Glass unit masonry is used for nonload-bearing elements in interior and exterior walls, partitions, and window openings. Window openings may be isolated, or in continuous bands. Glass unit masonry panels are required to be isolated so that in-plane loads would not be imparted to them.

Construction of glass masonry is covered in MSJC-08 Code Chap. 7: Glass Unit Masonry [2.3] and in 2009 IBC Section 2110 [2.4]. *Code* provisions are empirical, based on previous codes, successful performance, and manufacturers' recommendations. Since there is no consideration of stress in glass unit masonry, its compressive strength is not specified. The compressive strength, however, may vary between 400 and 800 psi [2.7].

Presently, no ASTM or any other type of material standard is available for glass units, and users depend on manufacturers' data, recommendations, and construction details.

Glass units are typically square, and produced in nominal 6, 8, and 12 in. sizes, although rectangular units are also available. Actual sizes are ¼ in. smaller, to allow for typical ¼-in. mortar joints. Unit thicknesses are either "standard" that are at least 3⅞ in. wide, or "thin" that are 3¼ and 3 in. wide, respectively, for hollow and solid units (MSJC Code-08 Ref. [2.3], Section 7.1.3). Glass units are required to be laid in Type S or N mortar. For exterior applications, integral water repellents can be incorporated into the mortar to help prevent water penetration. Glass unit masonry is typically constructed in stack bond. Edges of glass units are shaped to provide a mechanical key with the mortar, so full mortar beds should always be used to provide maximum bond between units and mortar. To preclude possibility of breakage or damage to units during installation, wood or rubber tools, rather than steel, should be used to tap units into the place.

Panel sizes for glass masonry units and other requirements are specified in building codes. They are, however, prescriptive in nature. They govern panel size, panel support, expansion joints, mortar, and joint requirements. The panel size limitations are based on structural and performance considerations. Height limits are more restrictive than length limits. The panel size requirements are based on history of successful performance, testing by independent laboratories, and manufacturers' recommendations, rather than on engineering principles or actual field experience.

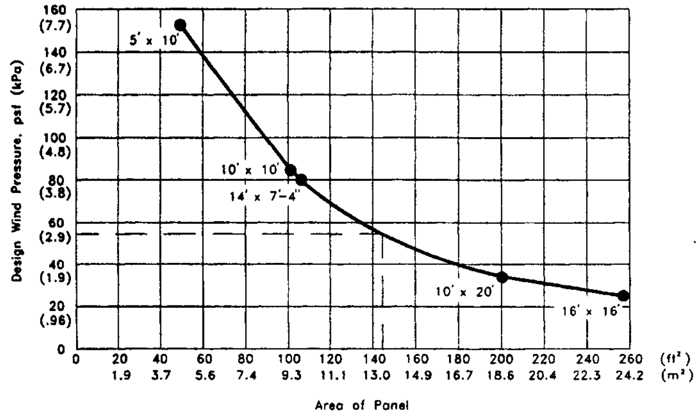
Maximum permitted panel sizes are shown in Table 2.10. For exterior applications (when panels are required to resist wind loads), maximum standard-unit glass masonry panel sizes can be determined from the wind load resistance curve for glass masonry panels shown in Fig. 2.47, which is a plot of glass masonry panel area and design wind pressure and is representative of the ultimate load limits for a variety of panel conditions. A factor of safety of 2.7 is suggested by the MSJC-08 *Code* (commentary to Section 7.2.1). For example, for a design wind pressure of 20 psf, the required panel area of 144 ft^2 (on the x-axis) is obtained by locating a point on the curve corresponding to $20 \times 2.7 = 54$ psf on the y-axis [2.33]. Interior walls are typically designed for lateral load of 5 psf. As a result, the interior glass unit masonry panels are permitted to be larger than similar exterior panels [2.3, 2.51].

TABLE 2.10 Maximum Panel Sizes for Glass Unit Masonry [2.3]

Panel description	Maximum panel size, ft^2	Maximum panel dimensions, ft	
		h	w
Exterior, standard unit	See Fig. 2.48	20	25
Exterior, thin unit	85	10	15
Interior, standard unit*	250	20	25
Interior, thin unit	150	20	25

*Thin unit panels may not be used in applications where the design wind pressure exceeds 20 psf.

Because glass masonry panels are not a structural part of the surrounding wall, code requires them to be laterally supported by panel anchors or channel-type restraints. Details for the panel anchor construction and channel-type restraint construction are shown in Figs. 2.48 and 2.49, respectively.



Example of how to use wind-load resistance curve: If using a design wind pressure of 20 psf (958 Pa), multiply by a safety factor of 2.7 and locate 54 psf (2586 Pa) wind pressure (on vertical axis), read across to curve and read corresponding 144 ft^2 (13.37 m^2) maximum area per panel (on horizontal axis).

FIGURE 2.47 Glass masonry ultimate wind load resistance [2.3, 2.51].

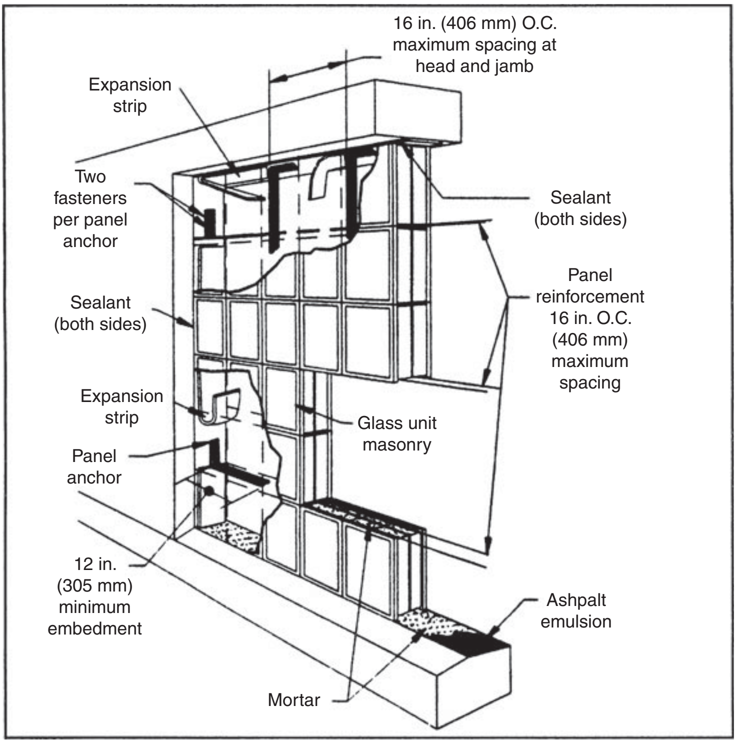


FIGURE 2.48 Panel anchor construction [2.3, 2.51].

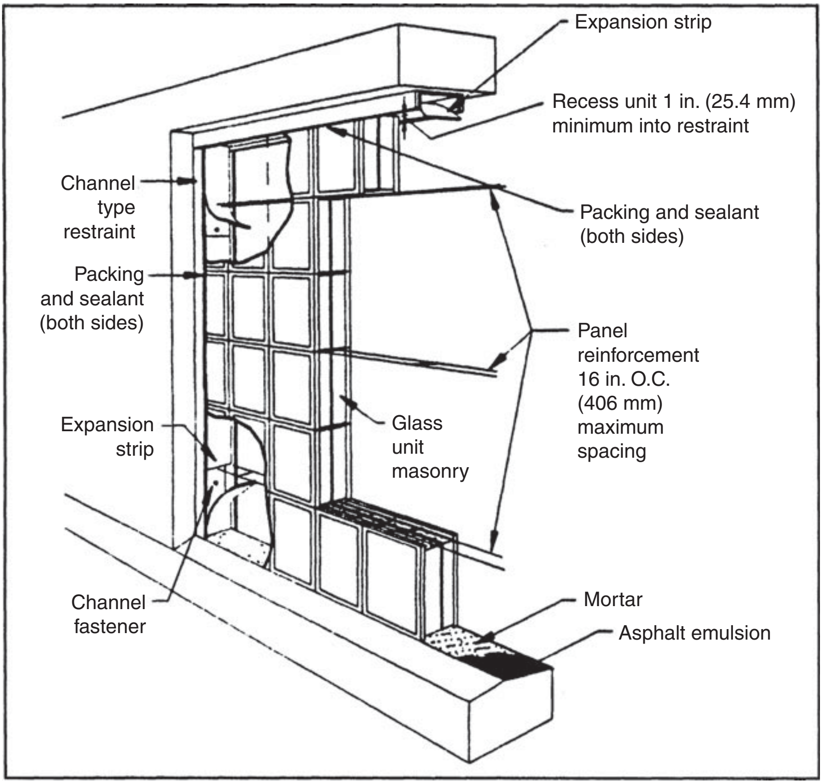


FIGURE 2.49 Channel-type restraint construction [2.3, 2.51].

Glass unit masonry panels are often rated for fire resistance, and for sound transmission. Solid and thick-faced units provide higher ratings in both areas. Technical data and other information on fire resistance ratings of glass unit masonry panels are available from manufacturers, the local building codes, or from the latest issue of the Underwriters Laboratory Building Materials Directory [2.52].

In order to limit the impact of wall movement on glass panels, it is required that they be structurally isolated to prevent the transfer of in-plane loads [2.2]. Vertical deflection of the supporting members is limited to *l*/600 to limit deflection cracking. Table 2.11 provides approximate dead weight of glass unit masonry panels, which may be used to design the supporting structures.

TABLE 2.11 Approximate Installed Weight of Glass Unit Masonry Panels [2.50]

Type of units	Dead weight, psf
Thin units	16
Standard units	20
Thick-faced units	30
Solid units	40

Use of glass block units is not permitted after being removed from an existing panel according to the MSJC-08 *Code*, although 2009 IBC, in general, permits reuse of second hand units if they conform to the requirement of new units.

2.12 MORTARLESS BLOCK SYSTEMS

Conventional or in-place masonry construction uses masonry units which are bonded together with mortar. Mortarless block systems are a new generation of concrete masonry units that symbolize innovation by the masonry construction industry. As the terms "mortarless block systems" indicates, these systems use concrete blocks that stack directly on one another without mortar. They are bonded together with materials trowelled onto or poured inside the wall. It is claimed that this system of installation requires less skill, and construction time is considerably reduced [2.53].

While new mortarless block systems continue to develop, several proprietary systems are already in use by the masonry industry. They have been used for building houses, high-rise apartment buildings, and industrial construction. A few mortarless systems reported in the literature are [2.53, 2.54]:

1. Azar dry-stack block system, developed by Azar Group of Companies, Windsor, ON, Canada.
2. Haener block system, developed by Haener Block Co., San Diego, CA, USA.
3. IMSI block system, developed by Insulated Masonry Systems Inc., Orem, Utah, USA.
4. Sparlock system, Sparlock Technologies, Inc., Montreal, Quebec, Canada.
5. Durisol block system, developed by Durisol Inc, Hamilton, ON, Canada.
6. Faswell block system, developed by the K-X Industries Inc., Windsor, SC, USA.

One or more of these systems can be modified to suit the desired applications. These innovative systems actually form a part of concrete home building systems that are finding wider acceptance, and more competitive in many more market segments than before. A detailed description of these systems can be found in the literature [2.53].

2.13 PREFABRICATED MASONRY

Prefabricated masonry evolved as a logical innovation from the experience gained in prefabricated building elements which have been used by the construction industry for a long time. The early development of prefabricated masonry took place in Europe shortly after World War II, in France, Switzerland, and Denmark. It evolved as an answer to the desperate need in Europe for fast and economical rebuilding with available manpower and materials. In the United States, development of prefabricated masonry panels started in the 1950s. The brick industry, through its research arm, the Structural Clay Products Research Foundation, (SCPRF), now the Engineering and Research Division of the Brick Institute of America (BIA), developed the SCR building panel. Although, this system, using special units, reinforcing, and grout backing, was successful on several projects in the Chicago area, it did not achieve wide acceptance by the masonry industry. However, several developments in the 1960s greatly affected the rapid growth of demand and the manufacturing capabilities for prefabricated unit masonry. A major trend which greatly increased the market potential for prefabricated masonry as well as for other materials and equipment was the "systems approach" to construction. The impetus for the systems was provided by federal agencies and others in an attempt to lower on-site construction costs [2.55, 2.56].

Prefabricated masonry panels are produced in many different ways. There are practically as many fabrication methods as there are firms producing these panels. Typically, these methods have been classified into five general categories: hand-laying, casting, special equipment, special units, and special mortars [2.55, 2.56]. A comprehensive discussion on various aspects of prefabricated masonry can be found in Ref. [2.55].

Panels have been prefabricated from both brick and concrete masonry units. They are designed in the same manner as hollow clay masonry walls. Each panel is self-contained structural system capable of handling the loads that are encountered during lifting, transportation, and installation. During lifting, the entire panel is hanging from the connectors at or near its top, which subjects the lower portion of the panel to tension. As a result, the panel behaves as a beam to support its own load. To preclude the possibility of masonry cracking in the tension zone, designers use standard mortar and special bricks for designing panels.

Prefabricated masonry panels offer great design flexibility since each one is custom made. However, on a project, there could be hundreds of panels, each one with a different configuration. Panels are produced to accommodate precast pieces (such as for window sills), and steel embedments for mounting windows. Also, complex shapes, such as sloping sills, arches, corbelled soffits, or articulated parapets that would be nearly impossible to build on the jobsite can be easily made in a factory [2.57]. Figure 2.50 shows prefabricated exterior load-bearing panels installed in building in Denver. The panels are 8-in. thick split-face concrete masonry laid up with a thin bed adhesive material. The bearing surfaces of the block units are ground to give better height control. The panels were preassembled at a factory site by Masonry Systems, Inc., Denver and hauled 25 mi to the jobsite for installation. Figure 2.51 shows a prefabricated curved brick panel being cured in a plant prior to shipping.

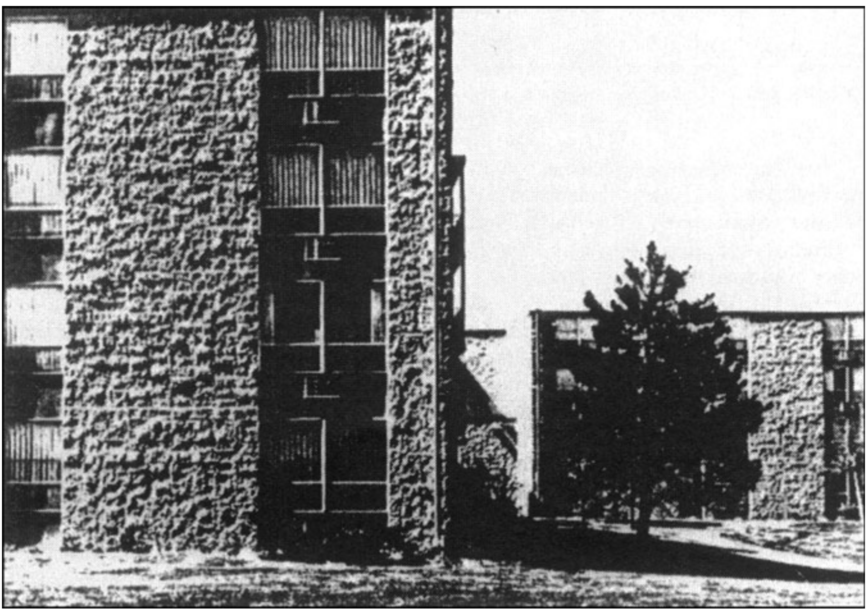


FIGURE 2.50 Preassembled exterior load-bearing panels. (*Courtesy*: *NCMA*.)

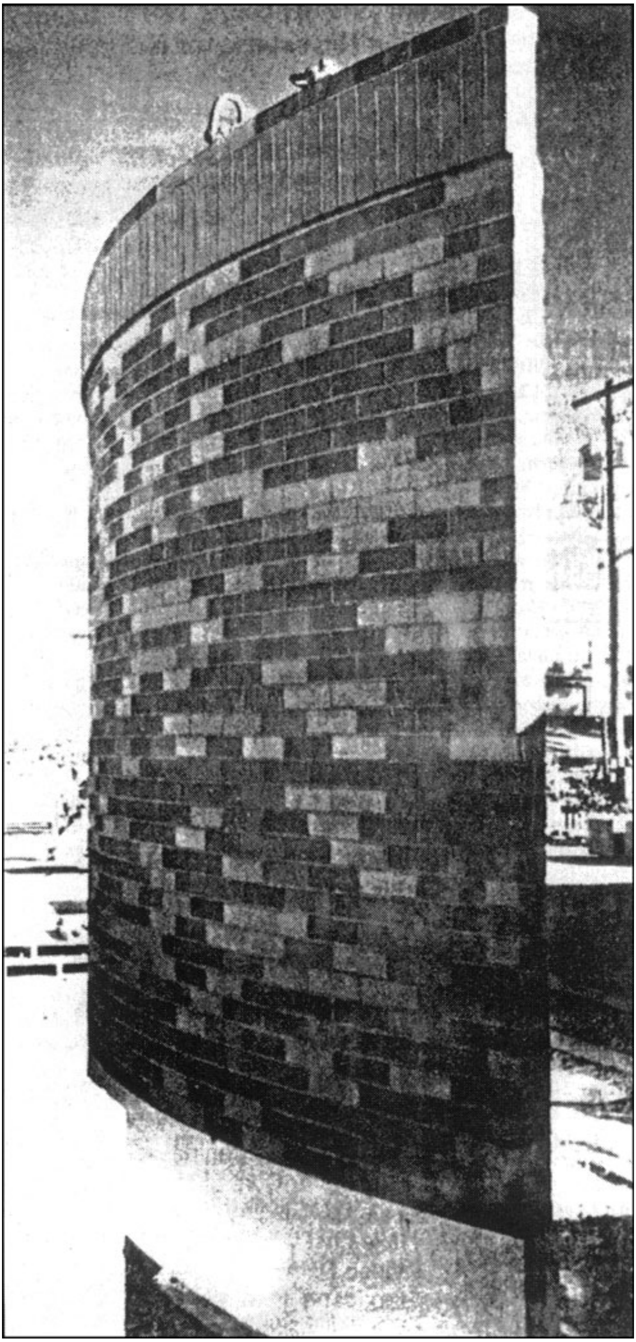


FIGURE 2.51 A prefabricated curved brick panel cures in a yard while waiting to be shipped. Panels can be moved the day after fabrication [2.57].

Prefabricated masonry panels are produced to conform to ASTM C901-01: Specification for Prefabricated Panels. Prefabrication construction is covered by MSJC-08 *Specification for Masonry Structures* [2.4].

Prefabricated masonry panels offer several advantages over conventional or in-place masonry construction. A major advantage of prefabrication is the quality control that can be exercised in a manufacturing plant to produce quality products. Since panels are made indoors, they can be fabricated 24 h a day if necessary and in any kind of weather. Another big advantage is that scaffolding is virtually eliminated since panels are installed by crane. Panelization on some projects may save construction time. For some projects, it is possible to build masonry panels as early as the groundbreaking for the project, thus keeping far ahead of the in-place construction work to permit panel erection when needed.

As with any other construction methodology, prefabrication has some disadvantages as well. The use of prefabricated panels is limited to certain types of construction. The use of prefabricated masonry is limited primarily by transportation and erection limitations. Architectural plan layout may, in some cases, preclude the use of prefabricated panels. Absence of adjustment capabilities during the construction process is another disadvantage of prefabricated masonry panels. In-place masonry construction allows the masons to build masonry to fit the other elements of the structure by adjusting the joint thicknesses over a large area so that it is not noticeable. This is not possible with prefabricated elements.

2.14 AUTOCLAVED AERATED CONCRETE

Autoclaved aerated concrete (AAC) is defined as low-density cementitious product calcium silicate hydrates, whose material specifications are defined in ASTM C1386 [2.3, 2.4]. Also called *autoclaved aerated concrete* or *autoclaved cellular concrete*, AAC is an innovative system of concrete blocks that are not molded. Unlike clay brick and concrete cellular blocks used for reinforced masonry construction, AAC units are solid blocks, but are much lighter than the conventional concrete masonry units. An AAC block weighs about half a normal-weight concrete masonry unit—20 lb per ft^2 versus 37 lb in an 8-in.-thick wall. However, the unit-by-unit comparison shows that one standard AAC block (8 in. wide × 10 in. high × 25 in. long) weighs 27 lb and creates 1.3 ft^2 of wall area while one standard concrete masonry unit block (8 × 8 × 16 in.) weighs 34 lb and creates 0.88 ft^2 of wall area. The AAC blocks are light because of their high air content—80 percent of their volume.

A description of manufacturing and using the AAC blocks for buildings can be found in Ref. [2.41]. AAC is made by mixing portland cement, silica-bearing sand, and water slurry with a finely powdered aluminum. Some of the plants under construction will substitute fly ash or mine tailings for sand. The aluminum powder reacts with other ingredients to release millions of small gas bubbles causing concrete to expand to 5 times its original volume. After the concrete sets initially, it is cut to size and then cooked in an autoclave to fully complete the curing process. Because the autoclave completes the hydration process, the AAC blocks do not experience the long term shrinkage problem associated with the conventional concrete masonry units.

Design of AAC masonry is covered in Appendix A of the MSJC-08 *Code*. In normal applications with AAC blocks, joint reinforcing is not required. AAC units are fully bedded when laid, so they distribute load over a full block face to create a very strong joint. This also eliminates problems of tension cracks and water penetration associated with joints when conventional concrete masonry unit are used. U-shaped units are available where bond beams are required. AAC can be easily cut with simple hand and power tools. The air cells in the AAC block make it a good thermal insulator. According to Ref. [2.42], a 4-in.-thick wall will provide up to 3 h of fire resistance, while an 8-in.-thick wall will provide up to 6 h of fire protection.

Developed by the world's largest AAC manufacturer, Hebel of Germany, the AAC has been a used as building material (known as *Hebel* wall system) in many parts of the world for over 70 years. However, its commercial applications in the United States are relatively recent [2.36]. The three types of AAC blocks that are commercially available in the United States are called *Hebel* AAC blocks, *Ytong's* AAC blocks, and *Contec's* AAC blocks [2.41].

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CHAPTER 3

MATERIALS OF MASONRY CONSTRUCTION

3.1 INTRODUCTION

A reinforced masonry member is built from four different components, viz., masonry units (concrete or clay), mortar, reinforcement, and grout. A typical construction sequence involves placing masonry units by hand in such a manner that both horizontal and vertical reinforcement can be positioned as required by design, place mortar (by hand) over and in between the bedded units, and pour grout in designated cells* as called for in the design (in only the cells having reinforcement or in all cells of the masonry units). Both clay and concrete masonry units were discussed in Chap. 2. Properties of mortar, grout, compressive strength and masonry, and steel reinforcement are discussed in this chapter.

Historically, mortars have been in use since 2690 B.C. when burned gypsum and sand mortars were used in Egypt. Later, in ancient Greece and Rome, mortars were produced from various materials such as burned lime, volcanic tuff, and sand. The development of modern mortar began with the advent of portland cement. The common use of portland cement began in the early part of the twentieth century and led to greatly strengthened mortar either when portland cement was used alone or in combination with lime [3.1].

Discussion of various materials in this chapter is based on the information provided in the MSJC-08 *Code* [3.2] and *Commentary* [3.3], MSJC-08 *Specification* [3.4] and *Commentary* [3.5], and 2009 IBC [3.6]. Constituents, testing, proportioning, and properties of various materials are governed by applicable ASTM Standards; Ref. 3.4 (Section 1.3) provides a list of ASTM Standards applicable to masonry construction.

3.2 MORTAR

3.2.1 Functions of Mortar

Masonry mortar is a versatile material capable of satisfying a variety of diverse requirements. It is one of the main constituents of a constructed masonry structure. Mortar is required to lay masonry units. As such, it must facilitate the placement of units, contribute to the serviceability of masonry structure, provide structural performance, and exhibit the desired appearance.

*All specified/required reinforcement must be placed and properly secured against displacement by wire positioners or other suitable devices prior to grouting; loss of bond and misalignment of reinforcement can occur if it is not placed and properly secured prior to grouting.

In masonry construction, mortar constitutes only a small proportion (approximately 7 percent) of the total wall area, but its influence on the performance of the wall is significant. At a first glance, mortar gives the appearance of simply being a jointing material for masonry units. Although the primary purpose of mortar in masonry is to bond masonry units into an assemblage, which acts as an integral element having desired functional characteristics, mortar also serves other functions [3.1]:

1. Bonds masonry units together into an integral structural assembly
2. Seals joints against penetration by air and moisture
3. Accommodates small movements within a wall
4. Bonds to joint reinforcement to assist in resisting shrinkage and tension
5. Bonds to ties and anchors so that all elements perform as an integral unit

3.2.2 Mortar Materials

Mortar consists of cementitious materials to which are added sufficient water and approved additives so as to achieve workable plastic consistency. The cementitious materials may be lime, masonry cement, mortar cement, and portland cement, and should not contain epoxy resins and derivatives, phenols, asbestos, fiber, or fireclays.

The American Society for Testing and Materials (ASTM) maintains national standards—ASTM C 270-05a [3.7]—for mortars, and materials commonly used as their constituents, under the jurisdiction of ASTM Committee C-12 on Mortars for Unit Masonry. It contains sections on scope, requirements, materials, test methods, construction practices, specification limitations, references, and an appendix. According to this standard, mortar is classified into three categories:

1. Cement-lime
2. Mortar cement
3. Masonry cement

All three mortars have been used successfully, with each offering unique benefits. A brief discussion of these types of mortar can be found in Ref. 3.9, which is summarized here.

Cement-lime mortars are mixed to meet minimum physical requirements. They are produced by blending lime-sand mortar with portland cement. A lime-sand mortar possesses excellent workability and high water retention, while portland cement increases setting time and provides additional strength. These mortars are further subclassified in various types. During the initial stages of mortar specification development, mortars were subclassified as Types A, B, C, and D. In 1954, the following designation for mortar types was adopted [3.9, 3.10]:

1. M (instead of A-1)
2. S (instead of A-2)
3. N (instead of A-B)
4. O (instead of A-C)
5. K (instead of A-D)

The new classification was apparently developed by dropping the alternate letters A, O, W, and R from the phrase MASON WORK. *Cement-lime* is subclassified into four types: M, S, N, and O. Both mortar cement and masonry cement are subclassified into six different types: M, M, S, S, N, and O. Proportioning requirements for mortar, by volume, are listed in ASTM C270-05a [3.7] (see Table A.1), and discussed in other references [3.8, 3.9].

Types M, S, and N mortars are typically specified in building codes. In Seismic Design Categories C, D, and E, as well as for empirical design of masonry, mortar types M or S are typically required. Glass units require Type S or N mortar [3.2, 3.6]. Neither mortar Type N nor masonry cement are permitted to be used as part of the lateral force resisting system in masonry structures in Seismic Design Category* D, E, and F [3.3 (Section 1.17.4.4.2.2)]. Note that the modulus of rupture of masonry using these mortars, regardless of the manner of laying masonry units (running bond or stack bond, discussed later), is significantly lower than for Type M or S mortars made from portland cement/lime or mortar cement (see Table A.7). It should be noted that the modulus of rupture values for Type N mortar made from portland cement/lime is typically 75 percent of those for Type M or S mortars; the corresponding values of the modulus of rupture for masonry cement are much lower.

ASTM A270 places requirements on constituents of mortar. A brief description of various constituents of mortars and governing requirements, summarized from Refs. 3.9 and 3.10 follows:

1. *Portland cement* (ASTM C150-52a [3.12]), one of the main constituents of masonry mortar, is hydraulic (sets and hardens with chemical interaction with water) cement. Types I, II, and III are permitted according to ASTM C270-02. Air-entrained portland cements (IA, IIA, and IIIA) may be used as alternates to each of these types.
2. *Masonry cement* (ASTM C91) is hydraulic cement consisting of a mixture of portland cement or blended hydraulic cement and plasticizing materials (such as limestone, hydrated or hydraulic lime) together with other materials introduced to influence such properties as setting time, workability, water retention, and durability.
3. *Mortar cement* is hydraulic cement similar to masonry cement, but mortar cement standard also includes a minimum bond strength requirement.
4. *Blended hydraulic cements* (ASTM C595) consists of standard portland cement or air-entrained portland cement combined through blending with such materials as blast furnace slag or fly ash.
5. *Quicklime* (ASTM C5) is calcinated (burned-decarbonated) limestone. Its major constituents are calcium oxide (CaO) and magnesium oxide (MgO). Quicklime must be slaked with water prior to use. The resultant lime putty must be stored and allowed to hydrate for at least for 24 h before. For this reason, quicklime is rarely used in mortar.
6. *Hydrated lime* (ASTM C270-05) is a dry powder obtained by treating quicklime with enough water to satisfy its affinity with water. ASTM C270-05 designates Type N (normal), Type S (special), and air-entraining (Type NA and Type SA) to hydrated limes. Slaking of hydrated lime is not required, thus it can be used immediately.
7. *Aggregates* (ASTM C144) [3.13] for mortar consist of either natural or manufactured sand. They represent the largest volume and weight constituent of the mortar. Sand acts as inert filler, providing economy, workability, and reduced shrinkage, while influencing compressive strength. An increase in sand content increases the setting time of a masonry mortar, but reduces potential cracking due to the shrinkage of the mortar joint. The special or standard sand required for certain laboratory tests may produce quite different test results from sand that is used in the construction mortar.
8. *Water* for masonry mortar (ASTM C270-05) must be clean and free of deleterious amounts of acid, alkalis, or organic materials. Potability of water is not in itself a consideration, but the water obtained from drinking supply sources is considered suitable for use.
9. *Admixtures* are functionally classified as agents promoting air-entrainment, water retentivity, workability, or accelerated set, among other things. Many substances, such as

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*Seismic Design Category is a classification assigned to a structure based on its occupancy category and the severity of design earthquake ground motion at the site [3.6]. See Chap. 7 for discussion on this topic.

tallow, salts of wood resins, and various chemicals have been employed as admixtures. Admixtures are commercially produced; accordingly, their composition is available from the manufacturers upon request. Admixtures containing chlorides accelerate the corrosion of steel reinforcing, ferrous ties and anchors, metal doors, and metal window frames. Therefore, admixtures containing chlorides are not permitted for use in mortar. *Specification for Masonry Structures* [3.4] does not permit use of admixtures containing more than 0.2 percent chloride ions.

Specification for Masonry Structures [3.4] requires that mortar be prepared at the jobsite by mixing cementitious materials and aggregates with a sufficient amount of water for 3 to 5 min in a mechanical batch mixer so as to produce workable consistency. Unless acceptable, mortar should not prepared by hand mixing. Workability of mortar should be maintained by remixing or retemping mortar that has begun to harden or stiffen due to hydration of cement. Mortar not used within 2½ h of initial mixing must be discarded. Temperature and wind velocity at the time of construction have a significant influence on useful life of mortar. If the ambient temperature exceeds 100 or 90°F with a wind velocity of 8 mph, then the mortar must be discarded if not used within 2 h of initial mixing.

3.2.3 Workability of Mortar

A mason's main requirement for mortar is workability. Without good workability, the chances for well-filled mortar joints in the masonry are very low [3.11]. Workable mortar can be spread easily with a trowel into the separations and crevices of the masonry unit. It also supports the weight of the masonry units when placed and facilitates alignment. It adheres to vertical masonry surfaces and readily extrudes from the mortar joints when mason applies pressure to bring the unit into alignment.

Workability is the result of the ball bearing effect of aggregate particles lubricated by the cementing paste. It is a combination of several properties such as plasticity, consistency, cohesion, and adhesion, all of which cannot be precisely measured in the laboratory. Although largely determined by aggregate grading, material proportions, and air content, the final adjustment to workability depends on water content [3.1]. However, the addition of water into a mortar mix is not governed by any specific volumetric requirements. The addition of water is left to the discretion of the mason contractor. The amount of water required to produce a workable mortar depends on the mortar type, the moisture content of the sand, the consistency desired, and the absorption rate of the masonry units. The mason best assesses the workability by observing the response of mortar to the trowel. Adding too little or too much water would produce mortar that is not workable. A workable mortar is obtained by adding just enough water to the mortar mix to produce a mortar that is "sticky" and adheres well to the specified masonry units [3.1].

3.2.4 Methods of Specifying Mortar

To satisfy minimum code requirements, mortar must conform to ASTM C270-05a which provides two methods of specifying mortar: (1) by proportions and (2) by properties. The *proportions specification* tells the contractor to mix the constituents in the volumetric proportions given in ASTM C270-05a Table C-1 (see Table A.1); no physical requirements are placed on the mortar itself. The *properties specification* instructs the contractor to develop a mortar mix that will yield the specified properties under laboratory testing conditions prior to construction. These are given in ASTM C270-05a Table C-2 (see Table A.2). The results are submitted to the owner's representative, and the proportions of ingredients as determined in the laboratory are maintained in the field. Water added in the field as determined by the mason for both methods of specifying mortar. Either proportions or properties specification, but not both, should be specified. If the project specifications do not indicate which of the two specifications is to be followed, the properties specifications govern, unless that data qualifying the mortar under the property specification are presented to and accepted by the specifier. Excessive amounts of pigments used to achieve mortar color may reduce both the compressive and bond strength of the masonry. As pointed out earlier, admixtures containing excessive amount of chloride ions are detrimental to items (reinforcing steel, joint reinforcement, wall ties, metal connectors and inserts, etc.) placed in mortar or grout.

3.2.5 Selection of the Right Mortar Type

There is no single mortar mix that is uniquely suitable for all applications. No one mortar type rates the highest in all areas of applications. No single mortar property defines mortar quality. Therefore, it is very important to understand the selection of the right type of mortar as it influences both the construction process and the quality of finished product. ASTM Standard specifications provide a means for specifications to identify acceptable materials and products without limiting those items to specific brands of manufacturers. Project specifications should reference ASTM C270-05a, the Standard Specification for Mortar of Unit Masonry.

The different mortar types are used for a variety of masonry applications. Type N mortar is a general-purpose mortar that provides good workability and serviceability. It is commonly used for interior walls, above-grade exterior walls under normal conditions, and for veneers. Type S mortar is used for structural load-bearing applications and for exterior applications for at or below grade. In addition, it also provides increased resistance to freeze-thaw deterioration. Type M is high-strength mortar, which may be considered for load-bearing or severe free-thaw applications. Type O is a low-strength mortar that is sometimes used for interior masonry or pointing. Special attention should be given when severe exposure conditions are expected. Type O mortar should not be used in saturated freezing conditions. Table 3.1 (adapted from ASTM C270-05a) provided guidelines for selecting mortar unreinforced (plain) masonry.

TABLE 3.1 Guide for Selection of Masonry Mortars

		Mortar type	
Location	Building segment	Recommended	Alternative
Exterior, above grade	Load-bearing walls,	N	S or M
	Non-load-bearing walls	O*	N or S
	Parapet walls	N	S
		S†	M or N†
Exterior, at or below grade	Foundation walls, retaining walls, manholes, sewers, pavements, walks, and patios	N	S or M
Interior	Non-load-bearing partitions	O	N

*Type O mortar is recommended for use where the masonry is unlikely to be frozen when saturated or unlikely to be subjected to high winds or other significant lateral loads. Types N or S mortars should be used in other cases.

†Masonry exposed to whether in nominally horizontal surface is extremely vulnerable to weathering. Mortar for such masonry should be selected with due caution.

According to the *Commentary to Specifications for Masonry Structures* [3.5], a good rule of thumb is to specify the weakest mortar that will perform adequately; stronger is not necessarily better when selecting/specifying mortar for masonry construction. For example, it is typically not necessary to use Type M mortar for high strength masonry; Type S mortar would provide comparable strength of masonry. The MSJC *Code* does not distinguish between the structural properties of masonry constructed from Type M mortar and Type S mortar. Of the two, masonry cement or cement-lime mortar, either can be used unless specified otherwise. Masonry cement mortar, generally offering improved convenience, workability, durability, and uniformity, is used in majority of masonry construction. Codes may also require the use of Type S cement-lime or mortar cement mortars for structural masonry in high seismic performance categories.

3.3 GROUT

3.3.1 Functions of Grout

Grout is a mixture of cementitious materials (lime and portland cement) and coarse or fine aggregate to which sufficient water has been added to achieve required fluidity so that it can be easily poured in the cells of the masonry blocks or between the wythes of masonry. When poured, grout should completely fill the voids in the masonry (cells) without segregation of the constituents, and completely encase the reinforcement. The two terms most associated with grout are *grout lift* and *grout pour*. The gout lift refers to an increment of grout within a total grout pour, whereas the latter refers to the total height of masonry wall to be grouted prior to erection of additional masonry. A grout pour may consist of one or more grout pours. Both are discussed further in Chap. 9.

Grout looks like very fluid mortar or concrete, but it is neither. Grouting forms a key phase in masonry construction; it is the structural backbone of masonry. Reinforced masonry derives its strength through grouting in the sense that reinforcement would not bond to the masonry without the grout and therefore would not contribute any strength to the structure. Grout serves many functions including the following:

1. Grout helps develop bonding between various masonry units to act together as one unit.
2. Grout is often used to structurally bond separate wall elements together. This is most commonly seen in reinforced construction, where grout is used to bond the steel reinforcement to the masonry so that the two elements to act integrally in resisting loads.
3. The grouted cells increase the bearing area for resisting higher compressive loads.
4. The grouted cells increase the stiffness of the walls, and thus increase their resistance to lateral loads. Masonry cantilever walls are often solidly grouted to increase the walls weight thereby increasing the resistance to overturning.
5. In two-wythe walls, the collar joint is grouted as one of the requirements for the two wythes to act integrally.
6. Grout increases volume for fire resistance.

3.3.2 Consistency of Grout

Grout contains high amounts of water to develop sufficient fluidity so that it can be pumped and poured easily and without segregation of its constituents. Fluidity allows grout to completely fill openings (or cells) in masonry units and to encapsulate steel reinforcement, the latter being necessary for reinforcement to participate in load sharing in the structure. The high initial water content of grout compensates for absorption of water by concrete masonry units after placement. Thus, grouts gain high strength despite the high initial water to cement ratios [3.14].

The fluid consistency of grout is described with slump requirements specified in building codes. *Specification for Masonry Structures* [3.4] requires a slump between 8 and 11 in. Lower slump-grout can be more difficult to place and ensure that the grout space has been completely filled. This high slump requirement for grout is in marked contrast with concrete for which 3 to 4 in. slump is commonly used in construction. The proper grout slump for a specific application would depend on how much water would be absorbed by the masonry. The more water the masonry units absorb, the more water the grout should contain to maintain fluidity. Grout slump requirement (between 8 and 11 in. slump) would depend on several factors as follows, and should be adjusted accordingly:

1. Initial rate of absorption (IRA) of masonry units. Masonry units with high IRA would absorb more water than units with low IRA.
2. Weather conditions: masonry constructed in hot and arid conditions would absorb water more rapidly than masonry constructed in cold or humid conditions.
3. Size of grout space: Amount of absorbed water depends on the masonry surface exposed to grout. Small, narrow grout spaces have a larger surface to volume ratio than wider grout spaces, and would absorb water at a greater rate.

Slump test of grout is performed in the same manner as for concrete (Fig. 3.1). First, the cone is dampened and placed on a flat, moist, nonabsorbent surface, and filled with grout in three layers. Each layer is rodded 25 times with a round steel rod (usually a ⅝-in. nominal diameter and 12 in. long, with rounded ends) to consolidate the grout. The second and third layers are rodded through the depth of the layer and penetrating the layer below. After the top layer is rodded, excess grout is struck off flush with the top of the cone. The mold (i.e., the cone) is immediately lifted, and the slump is measured by the distance between the top of the cone and the displaced original surface of the specimen.

3.3.3 Strength of Grout

When grout strength is specified, the minimum grout strength should equal or exceed the specified compressive strength of masonry (f'_m) but should not be less than 2000 psi, or greater than 5000 psi for concrete masonry and 6000 psi for clay masonry [3.2 (Section 3.1.8)]. Note that the upper limits on maximum compressive strength of masonry for design are 4000 and 6000 psi for concrete and clay masonry, respectively. When grout strength is specified, the grout must be sampled and tested in accordance with ASTM C1019-05: Standard Test Method for Sampling and Testing Grout [3.15].

3.3.4 Methods of Specifying Grout

ASTM C476-02**:** Specification for Grout for Masonry [3.16] contains standards for all materials used to make grout. This standard defines two types of grouts: *fine grout* and *coarse grout*. Fine grout contains only sand as its aggregate, whereas the coarse grout contains pea gravel or other acceptable aggregate (maximum aggregate size to pass through

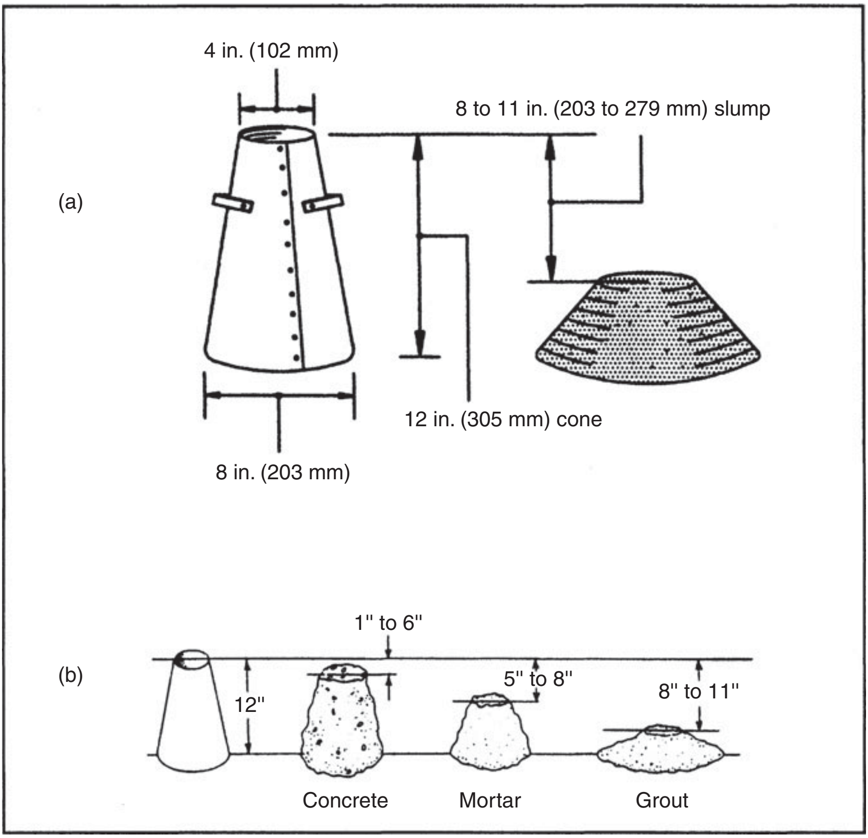


FIGURE 3.1 Slump test for grout. (*Courtesy: National Concrete Masonry Association.*)

a ⅜-in. opening) in addition to the sand, typically 1 to 2 times the sum of the volumes of the cementitious materials. All aggregates should conform to ASTM C476-02 Standard Specification for Aggregates for Masonry Grout [3.16]. Proportioning requirements by volume are given in ASTM C476-02, Table 1 (outlined as Table A.3).

The composition of the grout does not produce any appreciable difference between the strengths of the two types of grouts; however, the choice of grout depends mainly on the minimum clear cross-sectional dimensions of the grout space and the grout pour height. Criteria for the use of fine versus coarse grout based on these parameters are specified and listed in *Commentary on Specification for Masonry Structures*, Table 7 [3.5], summarized in Table 3.2. Coarse grout is typically preferred because it is more economical to produce [3.18]. However, the selection of the grout type depends on the size of the space to be grouted. Fine grout is selected for grout spaces with restricted openings.

ASTM C476-02 [3.16] requires that ingredients of grout be mixed thoroughly for a minimum of 5 min in a mechanical mixer with sufficient water to bring the mixture to desired consistency and workability. Hand mixing of grout may be permitted on small jobs, with the written approval of the purchaser outlining the hand-mixing procedure.

TABLE 3.2 Grout Space Requirements and Recommended Grout Pour Heights [3.4]

Specified grout type*	Maximum grout pour height, ft	Minimum width of grout space,†,‡ in.	Minimum grout space dimensions for grouting cells of hollow units,‡,§ in.^{2}
Fine	1	¾	1½ × 2
Fine	5	2	2 × 3
Fine	12	2½	2½ × 3
Fine	24	3	3 × 3
Coarse	1	1½	1½ × 3
Coarse	5	2	2½ × 3
Coarse	12	2½	3 × 3
Coarse	24	3	3 × 4

*Fine and coarse grouts are defined in ASTM C476. Grouts shall attain a minimum compressive strength of 2000 psi at 28 days.

†For grouting between masonry wythes.

‡Grout space dimensions between any masonry space protrusion and shall be increased by the diameters of the horizontal bars within the cross section of the grout space.

§Area of vertical reinforcement shall not exceed 6 percent of the area of the grout space.

Grout that has hardened or stiffened due to hydration of cement should not be used. In no case should grout be used 1½ h after water has been mixed to the dry ingredients at the jobsite.

To satisfy minimum code requirements, grout must conform to ASTM C476-02 which provides two methods of specifying grout: (1) by proportions and (2) by properties. The *proportions specification* tells the contractor to mix the constituents in the volumetric proportions given in ASTM C476-02 Table C-1 (see Table A.1); no physical requirements are placed on the grout itself. The *properties specification* instructs the contractor to develop a grout mix that will yield the specified properties under laboratory testing conditions prior to construction. When compressive strength is specified, the grout is required to have a *minimum* compressive strength of 2000 psi at 28 days and sampled and tested according to test method specified in ASTM C1019-05 [3.15]. In all cases the compressive strength of grout is required to equal or exceed the specified compressive strength of masonry, but not less than 2000 psi. Either proportions or properties specification, but not both, should be specified. If the project specifications do not indicate which of the two specifications is to be followed, the properties specifications govern, unless that data qualifying the grout under the property specification are presented to and accepted by the specifier. Excessive amounts of pigments used to achieve mortar color may reduce both the compressive and bond strength of the masonry. As pointed out earlier, admixtures containing excessive amount of chloride ions are detrimental to items (reinforcing steel, joint reinforcement, wall ties, metal connectors and inserts, etc.) placed in mortar or grout.

3.3.5 Compressive Strength Testing of Grout

Testing for the compressive strength of grout is performed according to ASTM C1019-05 [3.15], which is summarized as follows.

Grout specimens are required to have a square cross section, nominally 3 × 3 in. or larger, and twice as high as their width (so that the height-to-least lateral dimension ratio or the aspect ratio is 2). Dimensional tolerances are limited to within 5 percent of the nominal width selected. Three specimens constitute one sample at each stage of the test.

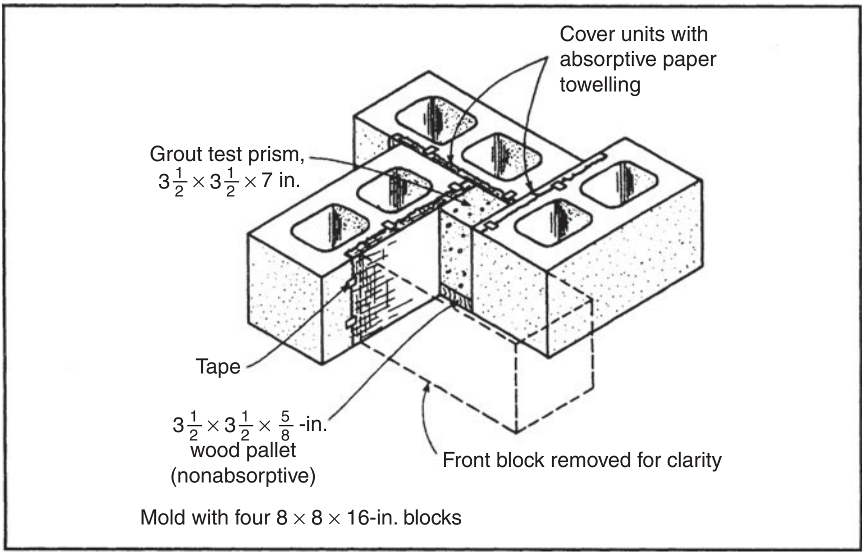


FIGURE 3.2 Grout mold for compressive strength testing per ASTM C1019-05. (*Courtesy: National Concrete Masonry Association.*)

The base of the test specimen consists of a square piece of wood of dimensions as specified in ASTM C1019-05 [3.15]. As a first step in the test, the piece of wood is placed on a level surface where the block mold will remain undisturbed and protected from freezing and extreme variations in temperatures for 48 h. Permeable paper, such as a paper towel, is taped to four masonry units to break the bond between the grout specimen and the masonry units (concrete or clay), but still allow the water to be absorbed by the masonry units. The units are then arranged around the piece of wood to form the mold (Fig. 3.2).

As a second step, the grout is poured in the mold in two lifts. Each layer is rodded 15 times, distributing the strokes uniformly over the cross section of the mold in order to eliminate any bubbles. While rodding the upper layer in the mold, the rod is required to penetrate at least ½ in. the bottom layer in mold. After the top layer is rodded, the top of the cube is leveled with a straight edge, and immediately covered with damp fabric or similar material to promote curing. After 48 h, the molds are dismantled and the specimens are shipped to the laboratory for testing where they are stored in a moist room.

In the laboratory, prior to testing, the specimens are capped as specified in ASTM C617: Standard Method of Capping Cylindrical Concrete Specimens [3.17]. This is followed by actual testing of the sample specimens according to the procedures of ASTM C39 [3.18].

Figure 3.3 shows some ASTM C1019-05 test results for grouts with various aggregate to cement ratios. The data show that, although a lower aggregate to cement ratio produces higher grout strengths, a direct relationship between the two does not exist.

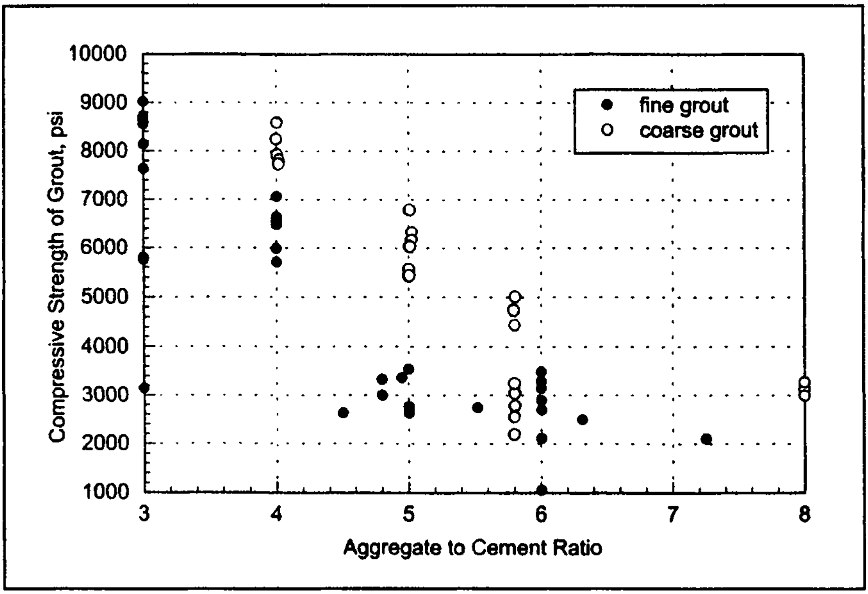


FIGURE 3.3 Relationship between grout mix and compressive strength. (*Courtesy: National Concrete Masonry Association.*)

3.4 DIFFERENCES BETWEEN MORTAR, GROUT, AND CONCRETE

To a casual observer, the only difference between mortar, grout, and concrete might be their fluidity. Although the three materials have the same principal constituents (cement mix, sand, gravel, water), there are many important differences between the three materials. They have different mix proportions, properties, and functions, and are governed by different ASTM Standards. Concrete is not used in masonry except for foundations. Concrete itself is a structural material just as masonry is; both are designed and used to support loads. The two can be used independently in the same structure at designer's discretion, for example, a reinforced concrete lintel or slab can be used in a building of masonry walls, columns, and pilasters. Mortar and grout are two different kinds of materials. Mortar is commonly used to bind masonry units into a single structural material (applied by hand by a mason using a trowel), whereas grout is poured into cells of masonry units to encapsulate reinforcement and provide a core. Mortar differs from concrete in working consistency, in method of placement, and in the curing environment.

Grout is neither concrete nor mortar. Both mortar and grout have their own specifications with regard to proportioning, testing, and strength. The strength of these two materials is correlated to specified compressive strength of masonry.

There are marked differences in water content and material composition of these materials. Concrete differs from grout in two distinct ways. Concrete contains much coarser aggregate, significantly lower water-cement ratio, and typically has a slump of 3 to 4 in. for typical construction; it can have even zero slump when used for specific purposes. It is poured with a minimum water concentration into nonporous forms, whereas grout is poured with a much higher water-cement ratio into what are essentially porous masonry forms. The initially high water-cement ratio of grout is rapidly reduced as the masonry absorbs water. Mortar differs from grout in that it often contains hydrated lime, finer aggregates, and only enough water to provide workability [3.7, 3.11].

3.5 COMPRESSIVE STRENGTH OF MASONRY

3.5.1 Compressive Strength of Masonry versus Compressive Strength of Masonry Units

Like reinforced concrete, the design of reinforced masonry structures is based on two key parameters: (1) the compressive strength of masonry (denoted by symbol f'_m) discussed in this section and (2) yield stress of reinforcing steel (denoted by symbol f_y) discussed in the next section. Similar to concrete, masonry is strong in compression and weak in tension. Accordingly, compressive strength of masonry is used as a basic design parameter in structural design of masonry, just as compressive strength of concrete is used as a basic design parameter in concrete design. Steel reinforcement is used in both reinforced masonry and concrete to resist tensile forces as well as compressive forces.

At the outset, it should be recognized that masonry consists of three separate materials: (1) masonry units, (2) mortar, and (3) grout. However, the parameter whose value is required/specified for design is the compressive strength of *masonry*, not that of masonry *units*. The specified compressive strength of masonry represents the compressive strength of structural unit made from these three different materials bonded together, unlike concrete for which the compressive strength of only one material—concrete—is required. As a rule, the strengths of these three materials measured separately are required to be at least equal to or greater than the specified compressive strength of masonry. Refer to Tables A.4 and A.5 for the strength requirements for clay and concrete masonry, respectively.

3.5.2 Methods of Evaluating Compressive Strength of Masonry

The *Code* [3.2] under its quality assurance program requires certification of compliance for the compressive strength of masonry prior to construction and for every 5000 ft^2 of masonry work during construction. There are two methods for arriving at the value of the compressive strengths of clay and concrete masonry as specified in MSJC-08 *Specification* Section 1.4A [3.4]: (1) the *unit* strength method and (2) *masonry prism* method. Typically, one of the two methods has to be specified by the engineer or the architect. When the method is not specified, the *Specification* [3.4] permits the contractor to select the method of determining compressive strength of masonry. The unit strength method is less expensive than the prism test method as it eliminates the costs associated with the making of test prisms and laboratory testing; however, it is more conservative than the prism test method. Both methods are discussed briefly in the following sections.

3.5.2.1 Unit Strength Method This method requires masonry units to be tested *prior to* and *during* construction to ensure their adequate strength. The value of f'_m (specified compressive strength of masonry) is based on the compressive strength of the masonry units and the type of mortar as listed in Tables A.4 and A.5. Clay and concrete masonry units should conform to their respective ASTM Specifications (discussed in Chap. 2). Clay masonry units should conform to the following ASTM specifications as applicable, and be sampled and tested as specified in ASTM C67-05c: Test Methods for Sampling and Testing Brick and Structural Clay Tile [3.19]:

1. ASTM C62-05: Specification for Building Brick (Solid Masonry Units Made from Clay or Shale) [3.20].
2. ASTM C216-05a: Specification for Facing Brick (Solid Masonry Units Made from Clay or Shale) [3.21].
3. ASTM C652-05a: Specification for Hollow Brick (Hollow Masonry Units Made from Clay or Shale) [3.22].

Likewise, concrete masonry units should conform to the following ASTM specifications as applicable, and be sampled and tested in accordance with ASTM C55-03a or ASTM C90-06.

ASTM C55-03**:** Specification for Concrete Brick [3.23].

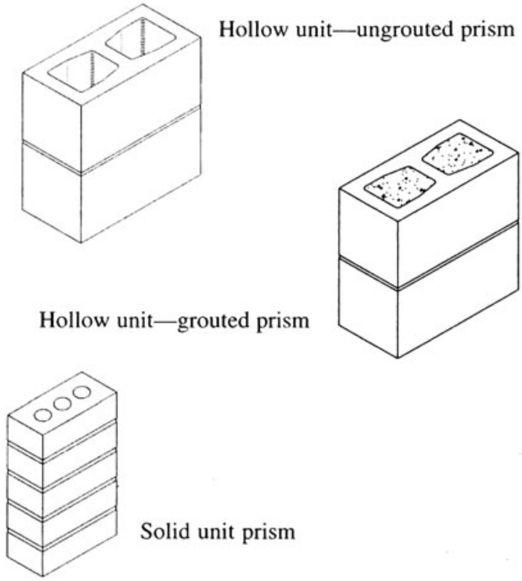
ASTM C90-06**:** Specification for Load Bearing Concrete Masonry Units [3.24].

For grouted masonry, the grout for both clay and concrete masonry should conform to ASTM C476 [3.16]; its compressive strength should equal or exceed the compressive strength of masonry but should not be less than 2000 psi. The thickness of bed joints for both types of masonry should not exceed ⅝ in.

3.5.2.2 Prism Test Method The prism test method provides a means of verifying that masonry materials used in construction result in masonry that meets the specified compressive strength. *Masonry prisms* are assemblages of masonry units, mortar, and grout (when applicable) that are prepared and tested for their compressive strength according to the ASTM C1314-03b**:** Standard Test Method for Constructing and Testing Masonry Prisms Used to Determine Compliance with Specified Compressive Strength of Masonry [3.25]. This method covers procedures for constructing masonry prisms, testing, and procedures for determining the compressive strength of masonry, f_{mt}, which is used to determine compliance with the *specified* compressive strength of masonry, f'_m.

The prisms are made from masonry units that are representative of those that are to be used in the construction. The test method requires constructing a *set* of prisms (i.e., at least three prisms) of the same material and testing them at the same age in accordance with specified procedure. A prism should consist of at least two masonry units. The aspect ratio (height-to-least lateral dimension ratio, h_p/t_p)* is required to be between 1.3 and 5.0. All prisms should be constructed by laying the units in stack bond in stretcher position (Fig. 3.4). The units should be oriented as in the corresponding construction and prepared with full mortar beds (i.e., mortar all the webs and face shells of hollow units). Both the mortar and the mortar joint thickness between the units, and the method of positioning and aligning the units during the preparation of the prisms should be representative of the construction. The prisms can be hollow or solid, being representative of the corresponding construction—hollow or solid grouted. If the corresponding construction is to be solidly grouted, the prisms are required to be grouted not less than 24 nor more than 48 h following the construction of the prisms. The grout, grout consolidation, and reconsolidation procedures should be representative of the corresponding construction. In case where the masonry is to be partially grouted, the test method requires constructing two sets of prisms: one set solidly grouted and the other ungrouted.

*h_p = prism height, t_p = least actual lateral dimension of the prism.



- Construct prisms in stack bond, one unit long and thick with a full mortar bed.
- Construct concrete masonry prisms with height to thickness ratios in the range from 1.33 to 5.0.
- Provide a minimum of one joint in hollow unit masonry prisms.

FIGURE 3.4 Construction of masonry prisms according to ASTM C1314-03b. (*Courtesy: National Concrete Masonry Association.*)

The prisms should be kept in airtight bags and should not be disturbed until 48 h prior to testing. After the initial 48 h, the bagged prisms should be maintained in an area with a temperature of 75 ± 15°F. The prisms are to be tested after 28 days or at the designated period. The moisture-tight bags containing the prisms are to be removed 2 days prior to testing and storing the prisms at 75 ± 15°F.

Calculation of the compressive strength of masonry from masonry prism tests is a three-step process. First, the masonry *prism strength* is calculated by dividing each prism's compressive load sustained by the net cross-sectional area of that prism (express the result to nearest 10 psi). Next, the compressive strength of masonry is determined based on the aspect ratio of the masonry test prisms. For masonry prisms having aspect ratio (h_p/t_p) of 2, the compressive strength of masonry is taken to be equal to the strength of masonry prism. For masonry prisms having aspect ratios between 1.3 and 5.0, the compressive strength of masonry is obtained by multiplying the masonry prism strength with correction factors listed in Table 3.3 [3.24]. Interpolation may be used for obtaining the correction factor for the aspect ratios not listed in the table. And finally, the compressive strength of masonry, f_{mt}, is obtained by averaging the values so obtained.

TABLE 3.3 Aspect Ratio Correction Factors for Masonry Prism Compressive Strength

t_p	1.3	1.5	2.0	2.5	3.0	4.0	5.0
Correction factor	0.75	0.86	1.0	1.04	1.07	1.15	1.22

3.6 STEEL REINFORCEMENT

Provisions for steel reinforcing bars and wire, and other metal accessories used in masonry construction are specified under the heading of "metal reinforcement and accessories" in 2009 IBC Section 2103.13 [3.6], which directs designers to MSJC-08 *Specification* Section 2.4 [3.2]. All three items (reinforcing bars, reinforcing wire, and metal accessories) are discussed in this section.

3.6.1 Reinforcement Bars

Steel reinforcing bars are used in masonry structures to enhance the strength of masonry members, both in tension and compression. Reinforcing bars are provided to carry tension in such members as lintels, bond beams, walls subjected to out-of-plane loads, retaining walls, etc. They are provided in columns to carry additional load in compression. In addition, the presence of reinforcement adds ductility to masonry structures.

Reinforcing bars used in masonry construction are of the same kind—*deformed bars* rather than *plain* bars—as those used for concrete construction. Deformed bars are hot-rolled; the ribbed projections are rolled onto their surfaces during the hot-rolling process. The reason for having a ribbed surface is for the bar to develop better bond with concrete and grout.

The bars used as reinforcement are circular in cross section, and are specified by their sizes. Plain round bars are specified by their diameters such as ¼, ½ in., etc. Deformed bars are specified by their average diameters expressed in whole numbers such as No. 3, No. 4, … No. 9…, etc.; the symbol # is often used to denote the bar size (e.g., #8 instead of No. 8, etc.). For bars from Nos. 3 to 8, the numbers denote *nominal* diameters in eighths of an inch. For example, No. 4 means ½-in. (= $^4/_8$ in.) nominal diameter, No. 6 means ¾-in. (= $^6/_8$ in.) nominal diameter, etc. Bar Nos. 9, 10, 11, 14, and 18 have diameters that provide cross-sectional areas of square bars as shown in Table 3.4.

It is common practice to use only bars Nos. 3 through 11 for design because they are readily available. In reinforced masonry construction, use of bars larger than No. 11 is not permitted when masonry is designed according to allowable stress design (ASD) method (MSJC-08 Section 1.15.2 [3.2]); maximum bar size permitted is No. 9 when masonry is designed according to the strength design provisions of the MSJC-08 *Code* (Section 3.3.3.1). These restrictions on bar sizes are based on bar sizes used in research, accepted engineering practice, and successful performance in construction. Bar sizes Nos. 14 and

TABLE 3.4 Cross-Sectional Area of Steel Reinforcing Bars Nos. 9 to 18

Bar No.	9	10	11	14	18
Square bar, in.^{2}	1×1	$1\frac{1}{8} \times 1\frac{1}{8}$	$1\frac{1}{4} \times 1\frac{1}{4}$	$1\frac{1}{2} \times 1\frac{1}{2}$	2×2
Area, in^2	1.0	1.27	1.56	2.25	4.0

18 are usually not readily available; they also are too large for use in practical masonry construction applications.

Properties of reinforcing bars and wire of different sizes used in masonry construction are listed in the appendix. Table A.8 lists the cross-sectional properties of various sizes of reinforcing bars. Table A.9 gives the areas of a group of reinforcing bars. Properties of steel reinforcing wire are listed in Table A.10. These tables are referenced in examples throughout this book.

Examples of deformed bars are shown in Fig. 3.5*a*. Reinforcing bars are classified according to the *minimum yield stress** of steel they are made from. They are designated by the word *Grade*, (often abbreviated as *Gr*.) followed by a two-digit number expressing *minimum yield stress* of steel in kips per square inch (usually expressed as *ksi*). The industry produces four *grades* of reinforcing bars: Grade 40, Grade 50, Grade 60, and Grade 75. A Grade 40 bar indicates a steel reinforcing bar with yield strength of 40 ksi, and so on. The ASTM specifications for billet-steel, rail-steel, axle-steel, and low-alloy reinforcing bars (A615, A616, A617, and A706, respectively) require identification marks to be rolled into the surface of the bar to denote the producer's mill designation, bar size, type of steel, and minimum yield designation, as shown in Fig. 3.5*b*. Grade 60 bars show these marks in the following order:

1. Producing m (usually the first initial of the mill's name)
2. Bar size number (Nos. 3 through 18)
3. Type of steel
4. Minimum yield strength designation

The minimum yield strength designation is used for Grade 60 bars only and may be either one single longitudinal line (grade line) or the number 60 (grade mark). A grade line is smaller and is located between the two main ribs, which are on the opposite sides of all bars produced in the United States. A grade line must be continued through at least five deformation spaces, and it may be placed on the side of the bar opposite the bar marks. A grade mark is the fourth mark on the bar. Grade mark numbers may be placed within separate consecutive deformation spaces to read vertically or horizontally. Bar identification marks may also be oriented to read horizontally—at 90° to those shown in Fig. 3.4*b*. Grade 40 and 50 bars are required to have only the first three identification marks (no minimum yield designation).

The type of reinforcing bars used in practice has changed over the past decade. Although both Grades 60 and 40 exhibit well-defined yield point and exhibit classic elastic-plastic strain behavior (Grade 75 steel does not†), present practice is to specify/use Grade 60 steel as primary reinforcement steel. Grade 40 is most ductile, followed by Grades 50, 60, and 75. Formerly, Grade 40 steel used to be more economical and also readily available. Presently (2009), Grade 60 bars are more economical and used by the design professionals for buildings and bridges. Grade 40-billet steel is available only in the smaller bars—Nos. 3 through 6. A major advantage of using Grade 60 reinforcement is that a smaller number of bars can be used to achieve the same strength as a larger number of Grade 40 bars. Having a smaller number of bars also helps alleviate the reinforcement congestion problem caused by a larger number of Grade 40 bars placed in the cells of masonry units, which can impede free flow of grout.

It should be recognized that codes and specifications may preclude the use of higher strength reinforcement specifically. Designers should be careful in selecting/specifying the

*The term "yield stress" refers to either yield point, the well-defined deviation from perfect elasticity (linear stress-strain relationship), or yield strength, the value obtained by a specified offset strain for a material that does not have a well-defined yield point, for example, Grade 75 steel.

†Grade 75 steel exhibits elastic-plastic behavior but does not exhibit a well-defined yield point.

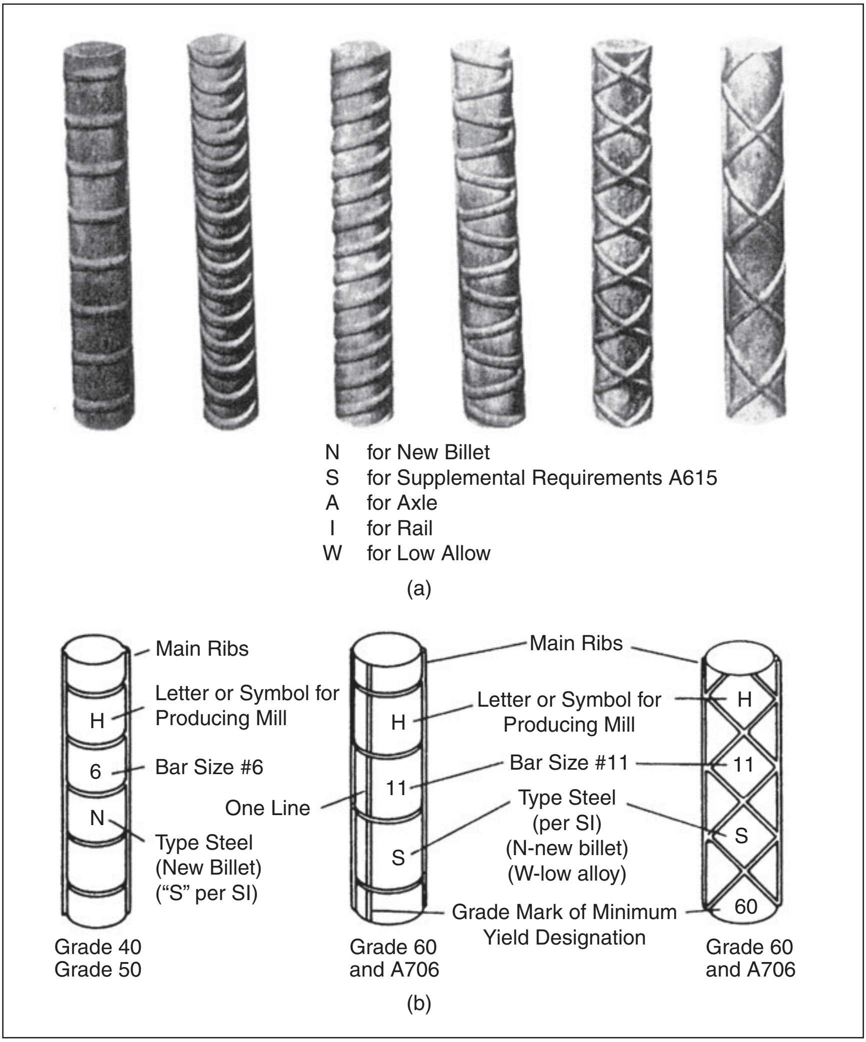


FIGURE 3.5 Deformed reinforcing bars. (*a*) Examples of reinforcing bars (CRSI). (*b*) Identification marks rolled on to the surface of bars. (*Courtesy: Concrete Reinforcing Steel Institute*.)

grade of reinforcing steel and the bar sizes as these may be subject to code-mandated restrictions. For example,

1. Both the MSJC-08 *Code* [3.2] and 2009 IBC [3.6] permit only Grades 40, 50, and 60 reinforcement (Grade 75 is not listed); however, the allowable stresses in tension are different for different grade steels as shown in Table 3.5 [3.2 (Section 2.3.2.1)].
2. The minimum diameter of bend measured on the inside of bars, other than for stirrups and ties, must not be less than that shown in Table 3.6 [3.3 (Section 1.15.6)]. The radius

TABLE 3.5 Allowable Stresses in Steel Reinforcement (*Source: Concrete Reinforcing Steel Institute*.)

Grade of steel	Minimum tensile strength f_u, lb/in.^{2}	Minimum yield stress f_y, lb/in.^{2}	Allowable stress in tension, lb/in.^{2}
40	70,000	40,000	$0.5f_y = 20{,}000$
50	80,000	50,000	$0.4f_y = 20{,}000$
60	90,000	60,000	$0.4f_y = 24{,}000$
75	100,000	75,000	Not listed
Wire joint reinforcement			30,000

TABLE 3.6 Minimum Inside Diameters of Bend for Reinforcing Bars

Grade of steel reinforcement	Bar No.	Minimum inside diameter
40	3 through 7	5 bar diameters
50 or 60	3 through 8	6 bar diameters
50 or 60	9, 10, and 11	8 bar diameters

of bend is referenced to the inside of the bars as it is easier to measure than the radius of the bend. The primary consideration for these limitations is the feasibility of bending without breakage.

Both 2009 IBC and MSJC-08 codes permit use of steel reinforcing bars conforming to the following ASTM designations, which specify certain chemical and mechanical properties, and dimensions:

1. ASTM A82/A82M-05a: Standard Specification for Steel Wire, Plain, for Concrete Reinforcement [3.26]. This specification covers most commonly used and commercially available *plain* bars in sizes up to ¾ in. The ¼-in. bar, the smallest diameter bar that is permitted by codes for use as ties in masonry members is a plain bar manufactured to this specification. The ultimate strength of the ¼-in. plain bar is 80 ksi and the yield strength is 70 ksi.
2. ASTM A615/A615M-5b: Standard Specification for Deformed and Plain Billet-Steel Bars for Concrete Reinforcement [3.27]. This specification covers most commonly used and commercially available bar sizes—sizes 3 through 18 in Grade 60, sizes 3 through 6 in Grade 40, and sizes 6 to 18 in Grade 75.
3. ASTM A706/A706M-05: Standard Specification for Low-Alloy Steel Deformed Bars for Concrete Reinforcement [3.28]. This specification covers bars intended for special applications where weldability, bendability, or ductility is important.
4. ASTM A767/A767M-05: Specification for Zinc-Coated (Galvanized) Steel Bars for Concrete Reinforcement [3.29]. The surface of these bars is coated with a layer of zinc as a means to provide protection from corrosion. The bars are galvanized after their fabrication. The specification covers requirements for the zinc-coating material, the galvanizing process, the class or weight of coating, finish, and adherence of coating. This specification has three supplementary requirements (S1, S2, and S3), which deal with condition assessment and special requirements and apply only when specified by the purchasers.

5. ASTM A775/A775M-01: Specification for Epoxy-Coated Reinforcing Steel Bars [3.30]. These steel bars are covered with a thin coat of epoxy of the order of 7 to 12 *mil* (1 mil = 1/1000) thick in order to provide protection from corrosion. The specification covers requirements for the epoxy-coating material, surface preparation of bars prior to application of the coating, the method of application of the coating material, limits on coating thickness, and acceptance tests to ensure that coating was properly applied.
6. ASTM A996/A996M-06: Standard Specification for Rail-Steel and Axle-Steel Deformed Bars for Concrete Reinforcement [3.31]. This specification covers reinforcing bars manufactured from discarded railroad rails or discarded train car axles; the bars are generally less ductile than billet steel.

Coated reinforcing bars are recognized by both concrete and masonry professionals as viable corrosion-protection systems. Both epoxy-coated and zinc-coated (galvanized) bars can be used for reinforced concrete and masonry construction when protection from corrosion is desired. Corrosion of steel reinforcement can occur when a structure is subjected to severe environmental conditions, such as marine environment or to deicing salts.

Performance of epoxy-coated bars has been controversial in the past. Many studies have shown that these bars can effectively reduce reinforcement corrosion and extend the service life of structures. Care should be taken during transportation, handling, storage, and placement of epoxy-coated bars to prevent damage (cracking, nicks, and cuts) to coating. Any damage to the coating must be repaired prior to placement of bars in the masonry. It is noted that epoxy coating of the bars leads to reduced resistance to slippage; consequently a longer development would be required for such bars. MSJC-08 Section 2.1.9.3 [3.2] requires that development length of epoxy-coated bars be 1½ times that required for uncoated bars.

Zinc-coated (galvanized) bars, like epoxy-coated bars, are covered with a thin layer of zinc as a means to provide protection from corrosion. Although zinc would protect the steel bar, zinc would corrode in concrete and eventually lead to corrosion of steel bars [3.32]. Unlike epoxy-coated bars, galvanized bars provide the same slip resistance as uncoated bars; consequently, modification of development length is not required for these bars.

An alternative to zinc-coated and epoxy-coated reinforcement used for corrosion protection is the fiber-reinforced polymer (FRP) reinforcement that has been successfully used for concrete structures. Typically available up to No. 8 size, FRP reinforcement is light, has high *specific strength* (ratio of tensile strength to weight), high *specific modulus* (ratio of modulus of elasticity in tension to weight), and is corrosion resistant [3.33]. The light weight of FRP reinforcing bars results in savings in transportation, handling, and erection costs. Also, because these bars are corrosion resistant, they can be used with smaller cover than required for steel reinforcing bars. The potential of this type of reinforcement for masonry structures has not been explored by research and field demonstration projects. Consequently, presently its use is not permitted by design codes for masonry structures.

3.6.2 Joint Reinforcement

Joint reinforcement is typically used in locations where available space would not permit placement of deformed bars with proper cover and clearance. Three types of joint reinforcement are generally used: (1) truss or ladder type joint reinforcement, (2) deformed reinforcing wire, and (3) welded wire fabric.

Truss- or ladder-type joint reinforcement conforms to ASTM A951-02: Standard Specification for Masonry Joint Reinforcement [3.34]. Typically, two types of joint reinforcement are permitted: the *truss* type (Fig. 3.6*a*) and the *ladder* type (Fig 3.6*b*). Specifications require maximum spacing of cross wires in ladder-type joint reinforcement and points of connections in the truss-type reinforcement to be 16 in. Both types of

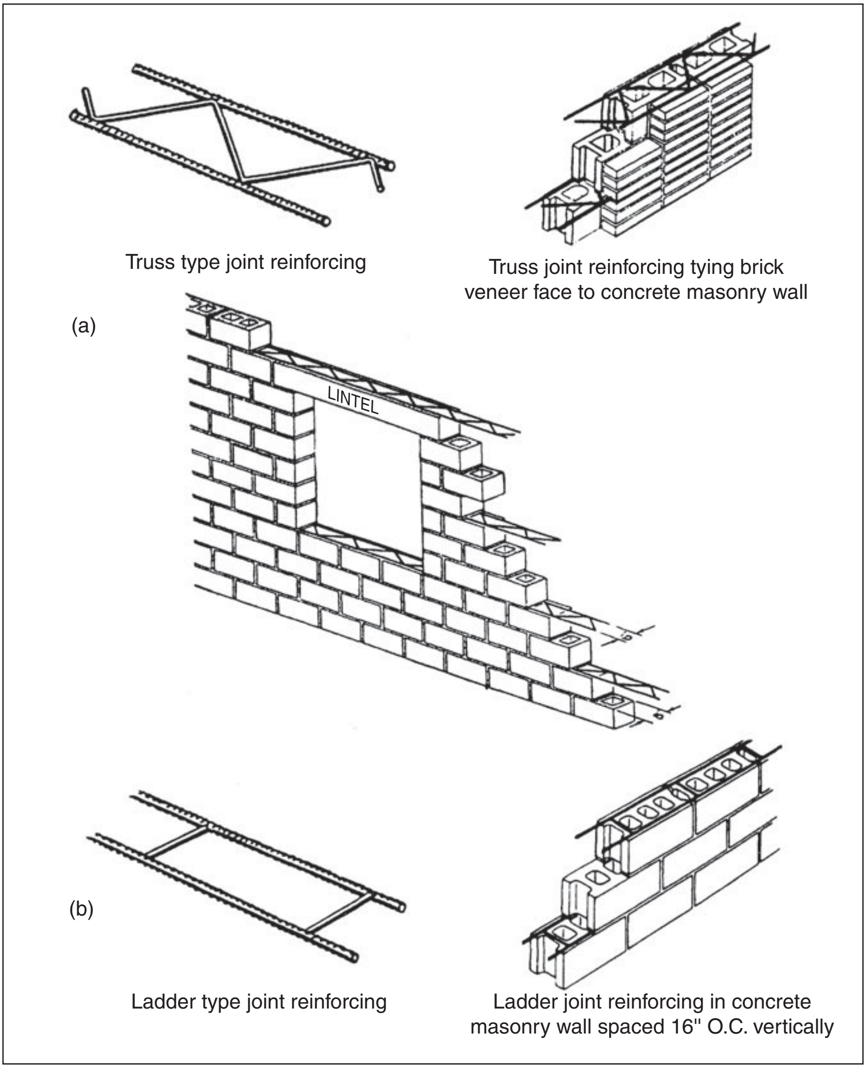


FIGURE 3.6 Prefabricated joint reinforcement. (*a*) Truss-type joint reinforcement. (*b*) Ladder-type joint reinforcement. (*Courtesy: National Concrete Masonry Association*.)

reinforcement can be conveniently placed in bed joints to reinforce walls in the horizontal direction.

1. Deformed reinforcing wire conforms to ASTM A496/A496M-05: Specification for Steel Wire, Deformed, for Concrete Reinforcement [3.35].
2. Welded wire fabric (WWF), which may consist of plain wire conforming to ASTM A185/A185M-06: Specification for Steel Welded Wire Reinforcement, Plain, for Concrete [3.36], or deformed wire conforming to ASTM A497/A497M-06: Standard

TABLE 3.7 Specifications for Anchors, Ties, and Accessories [3.6]

Items	ASTM Standard	Reference
1. Plate and bent bars anchors	ASTM A36/A36M	See Ref. 3.4
2. Sheet-metal anchors and ties	ASTM	
3. Wire mesh ties	A1008/A1008M	
4. Wire ties and anchors	ASTM A185	
5. Anchor bolts	ASTM A82	
6. Panel anchors for glass unit masonry	ASTM A307 Grade A	
Stainless steel items (AISI Type 304 or 316)		
1. Joint reinforcement	ASTM A580	
2. Plate and bent-bar anchors	ASTM A480 and A666	
3. Sheet-metal anchors	ASTM A480 and A240	
4. Wire ties and anchors	ASTM A580	

Specification for Steel Welded Wire Reinforcement, Deformed, for Concrete [3.37]. Unlike hot-rolled steel bars, the wire used in WWF *does not* have a well-defined yield point and is less ductile. Plain wire has a minimum yield stress of 56 and 65 ksi (respectively, for wire smaller than W1.2, and W1.2 and larger*) and a minimum tensile strength of 70 and 75 ksi (respectively, for wire smaller than W1.2, and W1.2 and larger). The wire is specified by symbol W (for smooth wires) or D† (for deformed wires), followed by a number that represents the cross-sectional area in hundredths of a square inch, varying from 1.5 to 31. On design drawings, welded wire fabric is indicated by symbol WWF followed by spacing of wires in two perpendicular directions. For example, designation WWF6 × 12-W16 × W8 indicates welded wire (smooth) fabric having longitudinal wires spaced at 6 in., transverse wires spaced at 12 in., longitudinal wire size W16 (i.e., cross-sectional area 0.16 in.^{2}), and the transverse size W8 (i.e., cross-sectional area 0.08 in.^{2}). A deformed wire fabric would be designated in the same style with the appropriate D-number. Detailed information on welded wire fabric is available from the Wire Reinforcement Institute [3.38].

3.6.3 Metal Accessories

Metal accessories include anchors, ties, and other accessories such as panel anchors for glass unit masonry. Requirements for metal accessories are covered in MSJC-08 *Specifications* [3.4]. Refer to Table 3.7 for various items and applicable ASTM Standards.

3.6.4 Protection for Reinforcement

Adequate *cover* should be provided to steel reinforcing bars to protect them from corrosion. Cover is measured from the exterior masonry surface to the nearest surface of the bar to which the cover requirement applies. It is measured to outer edges of stirrups or ties, when present (e.g., beams, columns, pilasters). Masonry cover includes the thickness of masonry

*The W-number represents the nominal cross-sectional area in square inches multiplied by 100 for smooth wires.

†The D-number represents the nominal cross-sectional area in square inches multiplied by 100 for deformed wires. A D-31 has a cross-sectional area of 31/100 = 0.31 in.^{2}.

TABLE 3.8 Protection of Steel reinforcement (MSJC-08 *Code* Section 1.15.4, [3.2])

Condition	Bar size	Required cover, in.
Masonry face exposed to earth or weather	Larger than No. 5	2
	No. 5 and smaller	1½
Masonry not exposed to earth or weather	All sizes	1½

shell, mortar, and grout surrounding the bar. At bed joints, cover includes total thickness of mortar and grout from the exterior of the mortar joint surface to the nearest surface of steel. Table 3.8 presents mandatory code requirements for the protection to steel reinforcing bars [3.2].

3.7 MODULUS OF ELASTICITY OF MASONRY MATERIALS

3.7.1 Modulus of Elasticity of Steel Reinforcement

The value of the modulus of elasticity of steel, E_s, is commonly taken as 29,000 ksi (MSJC-08 *Code* Section. 1.8.2.1, [3.2]).

3.7.2 Modulus of Elasticity of Masonry

The value of the modulus of elasticity of masonry, E_m, has been found to be dependent on the 28-day compressive strength of masonry (f'_m). This relationship is analogous to the modulus of elasticity of concrete which depends on the 28-day compressive strength of concrete. Accordingly, the values of modulus of elasticity of masonry are expressed as a function of the 28-day compressive strength of masonry prism as follows [MSJC-08 *Code* Section 1.8.2.2]:

$$\text{Clay masonry: } E_m = 700\, f'_m \tag{3.1}$$

$$\text{Concrete masonry: } E_m = 900\, f'_m \tag{3.2}$$

The above values of the moduli of elasticity are carryover from 1999 and 2002 MSJC *Codes*. In earlier masonry codes, the value of modulus of elasticity was specified as $1000\, f'_m$. Research has indicated a large variation in the relationship between the compressive strength of masonry and the modulus of elasticity [3.39, 3.40], and values of modulus of elasticity lower than $1000 f'_m$ may be more typical. Values of E_m given by Eqs. (3.1) and (3.2) are higher than indicated by the best fit of data relating the modulus of elasticity of masonry to the compressive strength of masonry. This is justified in view of the fact that actual compressive strength of masonry significantly exceeds the specified compressive strength, particularly for clay masonry. Readers should refer to *Commentary* ([3.4] Section 1.2) for a discussion on this topic.

Users of some of the earlier design codes such as 1997-UBC [3.41] would note that the aforestated values of the modulus of elasticity of masonry are different than those in 1997-UBC, which specified the same value for both clay and concrete masonry, given by Eq. (3.3) and its maximum value limited to 3×10^6 psi:

$$E_m = 750\, f'_m \tag{3.3}$$

The values of modulus of elasticity of both concrete and clay masonry as a function of compressive strength of masonry are listed in Table A.14. Also listed in Table A.14 are the values of the *modular ratio* (n) defined as the ratio of modulus of elasticity of steel to that of masonry as given by Eq. (3.4):

$$n = \frac{\text{modulus of elasticity of steel}}{\text{modulus of elasticity of masonry}} = \frac{E_s}{E_m} \tag{3.4}$$

The shear modulus of masonry (E_v) is given by Eq. (3.5):

$$E_v = 0.4E_m \tag{3.5}$$

3.7.3 Modulus of Elasticity of Grout

The modulus of elasticity of grout, E_g, is given by Eq. (3.6):

$$E_g = 500 f'_g \tag{3.6}$$

where f'_g = 28-day compressive strength of grout.

3.8 THERMAL EFFECTS ON MASONRY

All materials are inherently sensitive to fluctuations in temperature. Construction of structure usually takes place under normal temperature conditions. However, ambient temperatures may vary widely, cyclically, between summer and winter, by as much as 100°F between the two extremes. As a result of temperature changes, most materials expand (when the temperature rises above normal) and contract (when the temperature falls below normal). These thermal characteristics are described by a property called *coefficient of thermal expansion*. Table 3.9 lists coefficients of thermal expansion for some of the common building materials.

Masonry responds to temperature variations just like most other materials—expansion and contraction. The *Code* ([3.2] Sec 1.8.3) specifies the coefficient of thermal expansion, k_t, for clay and concrete masonry as follows:

$$\text{Clay masonry: } k_t = 4 \times 10^{-6} \text{ in./in./°F} \tag{3.7}$$

$$\text{Concrete masonry: } k_t = 4.5 \times 10^{-6} \text{ in./in./°F} \tag{3.8}$$

The coefficient of thermal expansion for concrete varies with the type of aggregate [3.46].

TABLE 3.9 Coefficients of Thermal Expansion of Some Common Building Materials*

Building material	Coefficient of thermal expansion ($\times 10^{-6}$ in./in./°F)
Clay, shale, or brick masonry	3.6
Normal-weight concrete masonry	5.2
Lightweight concrete masonry	4.3
Normal weight concrete	5.5–7.5
Structural steel	6.6

*Values of coefficients of thermal expansion in metric units (mm/mm/°C) can be obtained by multiplying the tabulated values by 1.8.

Expansion or contraction of masonry can be calculated based on the deviation of temperature from ambient temperature at the time of construction. For example, if the variation in the temperature is ΔT, the change in the length of a wall would be:

$$\Delta L = k_t L \Delta T \tag{3.9}$$

where ΔL = change in the length of wall (expansion or contraction)
L = original length of wall

It should be obvious that provision should be made in constructed walls to accommodate the thermal movement, failing which thermal stresses would be induced in the masonry, which could cause undesirable consequences. The thermally induced stress in the wall can be determined from Hooke's law as follows, provided the strain remains within the elastic limit:

$$\varepsilon_T = \frac{\Delta L}{L} \tag{3.10}$$

$$\sigma_T = \varepsilon_T E_m \tag{3.11}$$

where ε_m = thermal strain
σ_T = thermal stress (stress induced due to temperature variation)

Thermal movements are inevitable in a structure regardless of the type of building material used in construction. These movements can sometimes cause cracking in concrete or masonry. Large-scale effects of expansion and contraction are relieved by *volume change joints*. Various types of joints provided in masonry buildings are discussed in detail in Chap. 9 of this book.

Example 3.1 illustrates the thermal movements in a wall due to variation in the ambient temperature.

Example 3.1 Thermal movements in a wall.

A concrete masonry wall, constructed from nominal 8 × 8 × 16-in. medium-weight units is 24-ft long and 12-ft high. The wall is fully grouted; the grout weight being 140 lb/ft^3. The temperature in the local area varies from 70°F in the winter to 120°F in the summer. Calculate (a) the amount of thermal movement the wall would undergo due to temperature variation, (b) the stress that would be induced in the wall if thermal movement is prevented. The 28-day compressive strength of masonry is 2000 lb/in.^{2}.

Solution:

a. Thermal movement.

The wall would expand when the temperature rises beyond 70°F and shorten in length when the temperature falls below 70°F. We can calculate the elongation and shortening of the wall separately.

1. Elongation due to rise in temperature from 70 to 120°F:

$$\text{Length of wall, } L = 24 \text{ ft}$$

$$\text{Change in temperature, } \Delta T = 120 - 70 = 50$$

$$\text{For concrete masonry: } k_t = 4.5 \times 10^{-6} \text{ in./in./°F}$$

The change in the length of wall is given by Eq. (3.9):

$$\Delta L = k_t L \Delta T = (4.5 \times 10^{-6})(24 \times 12)(50°) = 0.0648 \text{ in.}$$

The thermal strain in the wall is calculated from Eq. (3.10):

$$\varepsilon_T = \frac{\Delta L}{L} = \frac{0.0648}{(24)(12)} = 2.25(10^{-4})\,in./in.$$

Modulus of elasticity of concrete masonry: $E_m = 900 f'_m = 900(2000) = 1.8(10^6)$ lb/in.^{2}
Thermal stress is given by Eq. (3.11):

$$\sigma_T = \varepsilon_T E_m = (2.25 \times 10^{-4})\ (1.8 \times 10^6) = 405 \text{ lb/in.}^2$$

2. Shortening of wall:

$$\text{Change in temperature, } \Delta T = 70 - 40 = 30$$

The change in the length of wall is given by Eq. (3.9):

$$\Delta L = k_t L \Delta T = (4.5 \times 10^{-6})(24 \times 12)(30°) = 0.039 \text{ in.}$$

3.9 INFLUENCE OF MOISTURE ON MASONRY: SHRINKAGE

Both clay and concrete masonry are affected by presence of moisture because both are porous materials. Although the response to presence of moisture is volume change in masonry units, clay and concrete respond differently. Clay masonry expands upon contact with moisture; the deformation is irreversible (i.e., the material does not return to its original size upon drying) [3.47]. Concrete masonry, on the other hand, shrinks because it is a cement-based material which shrinks due to moisture loss and *carbonation*. The main cause of shrinkage, however, is the loss of moisture as concrete dries and hardens.

Shrinkage is a time-dependent volume change phenomenon typical of concrete structures including those constructed from concrete masonry units. As the term implies, shrinkage reduces volume of concrete. Although there are many causes of shrinkage, the principal cause is loss of moisture in concrete. The primary type of shrinkage is called *drying shrinkage* or simply shrinkage, which results from evaporation of water in the capillaries (concrete is porous) and the *adsorbed water* (electrically bound water molecules) from the surface of gel particles. This water is about one molecule thick, or about 1 percent of the size of gel particles [3.43]. As concrete hardens, this water evaporates slowly and concrete continues to shrink.

The magnitude of shrinkage strain in concrete depends upon many variables, some of which are as follows:

1. *Relative humidity*: Humidity affects rate of evaporation. Shrinkage strain is largest for relative humidity of 40 percent or less. Concrete in highly humid climates shrinks less than in arid climates. This is the drying shrinkage, more than 90 percent of which occurs within the first few weeks after casting.
2. *Composition of concrete:* Composition of concrete. The hardened cement paste shrinks, whereas the aggregates do not. Thus, the larger the fraction of hydrated cement paste in a concrete mix, the greater the shrinkage. Shrinkage is also affected by the water/cement ratio; high water content reduces the volume of aggregates, which act to restrain shrinkage [3.30].
3. A secondary form of shrinkage is *carbonation shrinkage*, which occurs in carbon dioxide rich environments such as those found in parking garages. At 50 percent relative

humidity, the magnitude of carbonation shrinkage can be equal to the drying shrinkage, effectively doubling the total amount of shrinkage [3.44].

Shrinkage strain is partially recoverable upon rewetting (concrete swells when it gets soaked with water, and shrinks when it dries out again. Consequently, structures exposed to seasonal changes in humidity undergo slight expansion and contraction caused by changes in shrinkage strains. If these strains are prevented from occurring freely in a concrete masonry structure, the result would be cracking due to development and build up of tensile stresses (cracks act as a mechanism of relief from tensile stresses cause by prevention of free movement). Typically, this problem is solved by providing *expansion joints* and *volume change joints* (discussed in Chap. 9).

The following values of moisture coefficients are specified by the MSJC-08 *Code* [3.2 (Secs. 1.8.4 and 1.8.5)]:
Coefficient of moisture *expansion* for clay masonry:

$$k_e = 3 \times 10^{-4} \text{ in/in} \tag{3.12}$$

The shrinkage of clay masonry is negligible. Coefficient of *shrinkage* for concrete masonry:

$$k_m = 0.5s\ell \tag{3.13}$$

where s_l = total linear shrinkage of concrete masonry units determined in according to ASTM C426: Test Method Linear Drying Shrinkage of Concrete Masonry Units [3.45]. The maximum value of s_l is taken as 0.065 percent or 6.5×10^{-4} in./in., thus:

$$k_{m\,\max} = 0.5\ s\ell = 0.5\ (6.5 \times 10^{-4}) = 3.25 \times 10^{-4} \text{ in/in} \tag{3.14}$$

The maximum total shrinkage of a wall can be calculated by multiplying its length with the coefficient of shrinkage. If the shrinkage movement is prevented from occurring freely, the resulting stress can be calculated from Hooke's law. Example 3.2 illustrates the calculations for shrinkage deformation in a concrete masonry wall.

Example 3.2 Deformation due to shrinkage.

Calculate (a) total shrinkage in the concrete masonry wall described in Example 3.1, and (b) the maximum stress that would be created in the wall if the movement due to shrinkage is restrained.

Solution:

a. Total deformation due to shrinkage:
The coefficient of shrinkage for concrete masonry is

$$k_{m\,\max} = 0.5\ s\ell = 0.5\ (6.5 \times 10^{-4}) = 3.25 \times 10^{-4} \text{ in/in} \quad \text{(3.14 repeated)}$$

The length of masonry wall, $L = 24$ ft
Total maximum shrinkage, $\Delta_{m,\,\max} = k_m L = (3.25 \times 10^{-4} \text{ in/in})\ (24 \times 12) = 0.0936$ in.
b. Maximum stress caused by restraining the shrinkage movement:

$$\text{Stress} = (\text{strain})(\text{ modulus of elasticity, } E_m)$$

From Example 3.1, $E_m = 1.8 \times 10^6$ lb/in.^{2}

$$\sigma_{\max} = (\Delta_{m,\max})(E_m) = 3.25 \times 10^{-4})(1.8 \times 10^6) = 585 \text{ lb/in.}^2$$

Thus, a maximum tensile stress of 585 lb/in.^{2} might be caused if the wall movement due to shrinkage is prevented. This stress value is much higher than the allowable tensile stress in concrete masonry.

3.10 CREEP OF MASONRY

Deformation under loads is a common characteristic of structures. Deflection of beams and slabs (due to flexural loads), and shortening of columns (due to compressive loads) are examples of these deformations. Strain is simply a measurement of change per unit length of the material.

Materials such as masonry and concrete experience two types of deformations under loads: (1) *instantaneous deformation*, which is elastic and (2) *creep*. However, there is a difference in the manner in which the two occur. Instantaneous deformation occurs as soon as the load is applied, and is recoverable; deformation would disappear upon the removal of loads as along as the deformation was within the elastic limit. Creep, on the other hand, is a deformation that continues to occur slowly under *sustained* loading at stresses within the elastic range over a period of time. Furthermore, this deformation is inelastic (i.e., unrecoverable). Most deformation due to creep occurs during the first year and increases at a decreasing rate during the time of loading, and its magnitude can be 2 to 3 times the instantaneous (elastic) deformation. Thus, creep is a time- and load-dependent (i.e., depends on the magnitude of sustained loading) phenomenon [3.39, 3.40]; the greater the sustained load, the greater the creep. Creep is often associated with shrinkage because both continue to occur simultaneously over time, with the net effect of increases deformation with time.

Information is available for predicting creep [3.46, 3.47]. The internal mechanism of creep, or "plastic flow," as it is sometimes called, may be due to any one or a combination of the following factors:

1. Crystalline flow in the aggregate and hardened cement paste.
2. Plastic flow of the aggregate and hardened cement paste surrounding the aggregate.
3. Closing of internal voids.
4. Flow of water out of the cement gel due to external loading and drying.

A number of factors affect the magnitude of creep of concrete [3.44]. These include:

1. Constituents of concrete, such as size, grading and mineral content of aggregates, composition of cement, and the admixtures. Magnitude of creep is largest in concretes containing high cement-paste content; it is less in concretes containing a large percentage of aggregates because only the paste creeps, and aggregates act to restrain creep.
2. Mix proportion such as water content and water-cement ratio.
3. Curing temperature and humidity.
4. Relative humidity during period of use.
5. Age of loading.
6. Duration of loading.
7. Magnitude of sustained stress (i.e., stress due to sustained loads).
8. Surface-volume ratio of member.
9. Slump of concrete (which is affected by water-cement ratio).

Creep characteristics of a material are defined by a coefficient called *coefficient of creep* (k_c), which is the ratio of creep strain (after a very long time) to initial elastic strain. The coefficient of creep, k_c, for concrete masonry is much greater than that for clay masonry (which is negligible) [3.4]. This is attributed, in part, to the fact that concrete blocks are products of portland cements (the cement paste, not the aggregates, is the problem) [3.44].

The MSJC-08 *Code* [3.2 (Section 1.8.6)] specifies values of the coefficients of creep in terms of long-term deformation per unit compressive stress in masonry as follows:

$$\text{Clay masonry: } k_c = 0.7 \times 10^{-7} \text{ per psi} \tag{3.15}$$

$$\text{Concrete masonry: } k_c = 2.5 \times 10^{-7} \text{ per psi} \tag{3.16}$$

It is noted that this approach of specifying deformation due to creep is different than that of the Canadian code which specifies creep coefficients as a multiple of the corresponding elastic deformation—2 to 4 times the elastic strain for clay masonry and 3 to 5 times the elastic strain for lightweight concrete masonry. Research indicates that creep of concrete (1) is somewhat less than for ordinary concrete made from similar aggregates because of lesser portland cement and water contents and (2) lightweight aggregate blocks creep more than dense aggregate blocks [3.47, 3.48].

Example 3.3 illustrates calculations for creep deformation in a masonry wall.

Example 3.3 Creep deformation in a concrete masonry wall.

Determine deformation due to creep in the wall described in Example 3.1.

Solution:

For an 8-in. nominal wall, $b = 7.625$ in.

Height of wall = 12 ft

Cross-sectional area per foot length of wall = (7.625)(12) = 91.5 in.^{2}

For a fully grouted wall (grout weight = 140 lb/ft^3) constructed from medium-weight CMUs.

Self-weight of wall per foot of height = 78 lb (Table A.19)

Total weight of wall = (78)(12) = 936 lb

Calculate the average compressive stress in the wall. The entire wall weight does not contribute to creep. The bottommost layer supports the entire wall weight (so the compressive stress is maximum at this level), whereas the top layer supports none. Therefore, it is reasonable to average the weight of wall over its height.

Average self-weight of wall = ½ (936) = 468 lb

Live load on the wall = 1200 lb/ft length

Total compressive load on the wall = 468 + 1200 = 1668 lb

$$\text{Compressive stress in the wall due to total load} = \frac{1668}{91.5} = 18.2 \text{ lb/in}^2$$

Coefficient of creep for concrete masonry: kc = 2.5×10^{-7} per psi [Eq. (3.16)]

$$\text{Total creep deformation} = (2.5 \times 10^{-7})(18.2)(12)(12) = 6.55 \times 10^{-4} \text{ in.}$$

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CHAPTER 4
DESIGN OF REINFORCED MASONRY BEAMS

4.1 INTRODUCTION

The strength design method, initially called the *ultimate strength design* method, was originated by F. Stussi in 1931 [4.1] for design of reinforced concrete and prestressed concrete beams. Gradually, it became an accepted method for designing reinforced and prestressed concrete structures, and incorporated in the ACI *Code* [4.2]. Strength design method is also permitted by the design codes for reinforced masonry structures as incorporated in the MSJC-08 *Code* [4.3] and 2009 IBC [4.4]. According to this method, load effects, such as axial loads, and shears and moments in a beam, are determined by the methods of elastic analysis using the *factored loads*. Factored loads are obtained by multiplying service loads by code-specified load factors as specified in ASCE 7 Standard [4.5] and discussed in this chapter. The member cross section is then designed assuming that masonry and reinforcing steel have reached certain predefined strain values. The *nominal strength* of the member is then determined based on stresses corresponding to these specified strain limits. The *design strength* of a member is obtained from the nominal member strength by multiplying it by a *resistance factor* (which is less than 1.0) to reflect the uncertainty involved in the prediction of the material strength and in the analyses.

It should be recognized that masonry structures may be designed either by the *allowable stress design* (ASD) method (also called the *working stress method*, WSD, in the past) or the strength design method. A comprehensive discussion on design of reinforced masonry structures by the ASD method can be found in several references [4.6–4.8].

Minimum requirements for the strength design of masonry are presented in the MSJC-08 *Code* (hereinafter referred to as MSJC Code) Chap. 3 [4.3]. However, note that masonry design by the strength method is also required to comply with the general design requirements for masonry presented in Chap. 1 of the Code.

This chapter discusses analysis and design of reinforced masonry flexural elements (e.g., beams and lintels) for internal forces, that is, bending and shear, based on strength design philosophy. Discussion on the strength design and analysis of reinforced masonry walls subjected to flexure caused by out-of-plane loads, and in-plane loads is presented in Chaps. 6 and 7, respectively.

Philosophically speaking, the strength design approach for structural design, which is based on limit states design concept, involves two major considerations:

1. Design for strength
2. Design for serviceability

Design for strength includes considerations for the structure or structural member to carry maximum anticipated loads on the structure. For a beam, it means designing for moment and shear caused by factored loads. Design for serviceability deals with functional and performance aspects of a structure or structural member under service loads, such as deflections, fatigue, and vibration control. Deflection of reinforced masonry beams is discussed at the end of this chapter.

The readers should note that discussion on analysis and design of reinforced masonry presented in this and other chapters of this book are pertinent to both reinforced concrete masonry and reinforced clay masonry (brick masonry).

4.2 HISTORICAL DEVELOPMENT

The genesis of the provisions for the strength design of reinforced masonry as specified in the MSJC *Code* can be traced to *Technical Report* No. 4115 titled "Strength Design of One-to-Four Story Concrete Masonry Buildings" [4.9] published by the ICBO in February 1984. This report had gone through several independent reviews before and after its publication. Following these reviews, the report was renewed in February 1985 for two more years. Again, this report was reviewed by the newly formed ICBO Evaluation Services, Inc., Whittier, California, and renewed in September 1985 for two more years.

Design provisions based on strength design philosophy were first introduced in the 1985-UBC [4.10] in which Chap. 24 was completely revised from its earlier code versions. A strength design approach for slender wall was introduced in Section 2411 of 1985-UBC after it was approved by The Masonry Society (TMS) and the Structural Engineers Association of California (SEAOC). This was followed by the adoption of the strength design criteria for shear walls, which were introduced in Section 2412 of the 1988-UBC [4.11]. Codification of strength design approach also made it possible for engineers to design *slender walls* with *h/t* ratio in excess of 25 (briefly discussed in Chap. 6 of this book). In the 1997-UBC [4.12], provisions for strength design of reinforced masonry were covered in Chap. 21.

This chapter presents a discussion of the strength design philosophy as applicable to design of reinforced masonry beams. This is followed by several examples, which illustrate application of strength design philosophy and the provisions of MSJC *Code* [4.3] and 2009 IBC [4.4] to analysis and design of reinforced masonry structures.

4.3 STRENGTH DESIGN PHILOSOPHY

4.3.1 Basic Concepts

The "strength design" philosophy is based on the simple premise that a structure should be safe under the maximum loads that it may be required to carry during its service life. This principle can be restated by saying that the "strength" of a structure should be greater than or at least be equal to the *maximum anticipated loads* (also referred to as the "ultimate loads") that it may be called upon to resist. Mathematically, this principle can be stated as

$$\text{Design strength} \geq \text{required strength} \tag{4.1}$$

The term "strength" can be defined as the *resistance* that a structure (or a structural member) can offer. The strength of a structural member depends on the strength of the material (reinforced masonry in our case) from which it is constructed. The latter is determined from tests, which are always subject to some degree of uncertainty associated with them. We would therefore require that the design strength (or the maximum load-carrying capacity, or the maximum resistance it can offer) of the structure calculated from material properties be *greater* than that required to resist or support the maximum anticipated loads (also called the required strength or demand). For example, consider the design of a beam, for which we define the nominal strength, M_n, and the moment demand or the required strength, M_u, as follows:

M_n = nominal strength (or the moment resistance) calculated from material and cross-sectional properties

M_u = moment due to the anticipated loads that a beam is required to resist (or moment demand)

For design purposes, we require that M_n be slightly greater than, or at least equal to, M_u. Mathematically, this requirement is stated as

$$M_n \geq M_u \tag{4.2}$$

For practical purposes, we slightly modify Eq. (4.2) for computational purposes. To be able to use the *equal* sign, we multiply M_n by factor ϕ (phi) called the strength reduction factor (discussed later). Accordingly, Eq. (4.2) can be written as

$$\phi M_n \geq M_u \tag{4.3}$$

Because, by definition, $M_n > M_u$, one should intuitively recognize that $\phi < 1.0$ in Eq. (4.3). This ϕ-factor has a specific numerical value to be used for moment calculations as discussed later. In general, however, ϕ factors (*resistance* or *capacity reduction factors*) are applicable to strength calculations for all masonry components. We purposely use the term "factors" (plural term) rather than "factor" (singular term) because their values are different for different limit states.

The term "maximum anticipated loads" (or the *required strength*) needs to be clarified for use in design. Obviously, because of their probabilistic nature, these loads are not known a priori. Therefore, we must have a way to *predict* (or estimate) these loads. One way to do this is to simply *increase* the service loads in some proportion. This is accomplished by multiplying the service loads by certain factors called the "load factors" (specified as γ factors in some codes), and the loads so determined are called the "factored loads." Thus, Eq. (4.3) can be restated as

$$\phi \text{ (Nominal strength)} \geq \text{factored loads} \tag{4.4}$$

Equation (4.4) is quite general and is applicable regardless of the nature of the loads or forces involved. This is the fundamental equation that forms the basis of strength design, a design concept that parallels *limit states design* or *load and resistance factor design* (LRFD). The loads referred to here can be of any kind: bending moment, shear, axial loads, torsion, etc. We now see that the term "M_u" in Eq. (4.2), which is applicable to beams, is simply the design moment based on factored loads.

In this chapter, the terms "nominal strength" and "moment resistance" (both are denoted as M_n), which are determined from material strengths, would be used synonymously. Likewise, the terms "design moment" and "moment demand" (both are denoted as M_u), which are determined from the imposed loads, would be used synonymously.

4.3.2 Strength Reduction Factors and Load Factors

The load factors and the strength reduction factors (the ϕ-factors) alluded to in the preceding paragraph form the foundation of the strength design philosophy. Several considerations made in arriving at the values of these factors are discussed in this section.

4.3.2.1 Strength Reduction Factors (ϕ-Factors) The behavior of masonry, like that of reinforced concrete, is different under different loading conditions. For example, its behavior is different in bending than in shear, and it can be predicted with greater certainty in bending than in shear. Members under axial loads behave differently than in flexure. Plain masonry behaves differently than reinforced masonry. Thus, the values of ϕ-factors are different for members under different loading conditions.

There are several reasons for providing strength reduction factors:

1. To allow for the probability of understrength due to variations in material strengths and dimensions. The determination of the "nominal strength" is based on the *assumption* that the member would have exact dimensions and material properties used in the calculations. This assumption may not be close to reality.
2. To allow for the inaccuracies in the design equations.
3. To reflect the degree of ductility and required reliability of the member under the load effects being considered.
4. To reflect the importance of the member in the structure.

It is very important to recognize that the ϕ-factors to be used in design depend on both the type of the member, and the type of loading. Even for the same member, different values of ϕ may need to be used depending on the member strength to be determined. For example, two different values of ϕ-factors are used in design of beams—one value for determination of flexural strength and another for determination of *shear* strength. Table 4.1 lists strength reduction factors for various design considerations.

TABLE 4.1 Strength Reduction Factors, ϕ [MSJC-08; Ref. 4.3]

Design consideration	ϕ	MSJC-08 Section
Flexure	0.90	3.1.4.1
Axial compression	0.90	3.1.4.1
Axial compression and bending	0.90	3.1.4.1
Shear	0.80	3.1.4.3
Anchor bolts		3.1.4.4
Controlled by masonry breakout	0.50	
Controlled by anchor steel	0.90	
Controlled by anchor pullout	0.65	
Bearing	0.60	3.1.4.5

4.3.2.2 Load Factors Load factors are assigned in two ways. They are assigned to specific loads as well as to the combination of specific loads.

Different load factors are used to augment the values of the service loads in order to arrive at the maximum anticipated loads. The factor assigned to each load is influenced by the degree of certainty to which the load effect can be determined, and the expected variation in the load during the service life of a structure. For example, dead loads are determined based on known or assumed member sizes and therefore can be determined with reasonable degree of certainty. The live loads, on the other hand, are highly variable by their very nature, and cannot be determined with the same degree of certainty as the dead loads. Therefore, a higher value of load factor is assigned to the live load than to the dead load. Uncertainty in analysis and structural behavior of a structural system are other reasons for using load factors.

Load factor assigned to a particular kind of load can be different depending on whether the load is to be used singly or in combination with other loads for design purposes. For example, a load factor of 1.4 is assigned to dead load when considered acting alone, but a load factor of 1.2 is assigned to it when considered acting in conjunction with the live load. Furthermore, when dead and live loads are considered acting together, the live load is assigned a load factor of 1.6, much higher than the 1.2 factor assigned to the dead load. The reason for assigning a much higher value of load factor to the live load is the higher degree of uncertainty associated in estimating the live load than the dead load.

When load factors are assigned to various loads in a load combination, consideration is given to the probability of the simultaneous occurrence of those loads. Also, in deciding a load combination, it is very important to recognize whether the effects of one type of loading would offset the effects due to the other, the two effects being of the opposite types. For example, dead load, which is a gravity load, acts on a structure as a stabilizing force, which opposes the effects produced by the lateral loads (such as wind or seismic) which act as overturning forces and tend to destabilize the structure. Thus, the effects of these two types of forces are of the opposite types. In such cases, a conservative approach is taken, and the load factor assigned to the stabilizing force (dead load in this case) may be less than 1.0. Typically, when lateral loads are combined with the gravity loads, a load factor of 0.9 is assigned to the dead load. This is because the dead load reduces the effects of lateral loads, and that it might have been overestimated.

Due consideration must be given to various load combinations to determine the most critical design condition. Loads to be considered for a specific combination should be determined according to the applicable standards and codes such as ASCE 7 [4.5] or 2009 IBC [4.4]. A comprehensive discussion on the analysis of structural loads on buildings is presented in Ref. 4.12. See Chap. 7 for a brief discussion on this topic.

4.4 ASSUMPTIONS IN STRENGTH DESIGN PHILOSOPHY

Analytical model adopted for the strength design of reinforced masonry is similar to that used for design of reinforced concrete structures. Assumptions in strength design of reinforced masonry parallel those for design of reinforced concrete specified in ACI 318 [4.2]. Accordingly, readers familiar with the strength design of reinforced concrete would find it easier to understand the strength design of reinforced masonry.

At the very outset, it should be noted that the reinforced masonry beams are constructed, typically, from hollow concrete or clay units which, after placement of tension and shear reinforcing bars, are grouted solid. Solid grouting of beams is essential to ensure that masonry and reinforcement act in unison.

Provisions for strength design of reinforced masonry are stated in MSJC *Code* [4.3] and incorporated in 2009 IBC [4.4]. The assumptions for the strength design of reinforced masonry as specified in the MSJC *Code* can be summarized as follows (see Fig. 4.1):

1. Masonry, grout, and reinforcement act compositely in resisting applied loads by maintaining strain continuity between these three constituents of a masonry structure.
2. Nominal strength of reinforced masonry cross sections for combined flexure and axial loads shall be based on applicable conditions of equilibrium and compatibility of strains.
3. Strain in reinforcement and masonry shall be assumed to be directly proportional to the distance from the neutral axis, that is, strains vary linearly from zero at the neutral axis to maximum at the extreme fibers of the beam.

4. Maximum usable strain, ε_{mu}, in the extreme compression fibers of masonry shall be (Fig. 4.1*b*):

   a. Concrete masonry 0.0025

   b. Clay masonry 0.0035

5. Strain in steel reinforcement (ε_s) can be less than, equal to, or greater than the yield strain, ε_y. Balanced strain conditions are said to exist at a cross section when tension reinforcement reaches the strain corresponding to its specified yield strength, f_y (i.e., $\varepsilon_y = f_y E_s$) just as masonry in compression reaches its assumed ultimate strain of ε_{mu} as specified above (Fig. 4.1*c*).

6. Stress in reinforcement below specified yield strength f_y for grade of reinforcement used shall be taken (based on Hooke's law) as E_s times the steel strain (i.e., $f_s = \varepsilon_s E_s < f_y$). For strains greater that yield strain (i.e., $\varepsilon_s > \varepsilon_y$), stress in steel shall be considered to be independent of strain, and equal to f_y.

7. Masonry carries no tensile stress greater than the modulus of rupture. The tensile strength of masonry shall be neglected when determining flexural strength but shall be considered when determining deflection.

8. Relationship between masonry compressive stress (f'_m) and masonry strain shall be assumed as follows:

   Masonry stress of $0.80f'_m$ shall be assumed uniformly distributed over an equivalent compression zone bounded by edges of the cross section and a straight line located parallel to the neutral axis at a distance $a = 0.80c$ from the fiber of maximum compressive strain. Distance c form the fiber of maximum compressive strain to neutral axis shall be measured in a direction perpendicular to that axis (Fig. 4.1*d*).

The above stated design assumptions are based on principles that have been traditionally used successfully for design of reinforced masonry structures. Assumption 1 simply defines how reinforced masonry acts as a structural material. Assumption 2 provides a rational for analysis based on principles of structural mechanics. The values of maximum useable strains in concrete and clay masonry (assumption 4) are based on research. Assumption 6

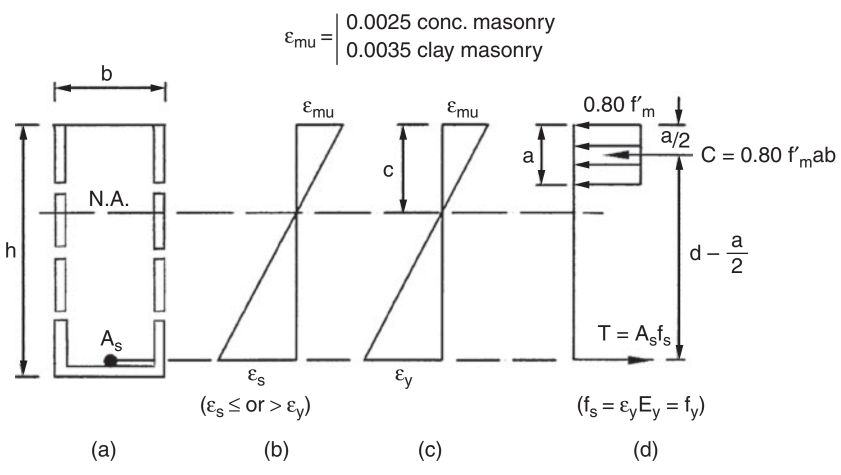


FIGURE 4.1 Assumptions in the strength design of masonry.

is a stipulation of Hooke's law. Tensile strength of masonry is very small and ineffective in resisting flexural loads; accordingly, it is ignored in calculation of flexural strength. However, it contributes to the overall stiffness of a masonry element and, therefore, is to be considered for deflection calculations (assumption 7).

The definition of distribution of compressive stress in a masonry element in flexure (assumption 8) is parallel to that used for design of a similar reinforced concrete element. For a reinforced concrete element in flexure, distance *a* is defined in ACI 318 [4.2] as $a = \beta_1 c$, where β_1 is a coefficient whose value depends on the compressive strength of concrete and varies between 0.85 and 0.65. For the commonly used range of masonry strengths (1500 psi $< f'_m <$ 6000 psi) that have been investigated, the value of β_1 is taken as a *constant* value of 0.80 for design of masonry structures (symbol β_1 is not used in the MSJC *Code*).

The analytical model based on the forgoing assumptions is illustrated in Fig. 4.1.

4.5 ANALYSIS OF RECTANGULAR SECTIONS IN FLEXURE

4.5.1 Principles of Flexural Analysis in Strength Design

The basis of the strength design approach is the assumed stress distribution in the cross section of a beam at failure (commonly referred to as the ultimate condition), which is completely different from the linear stress distribution (Fig. 4.2) assumed in the allowable stress design (ASD), which is discussed in the literature [4.6–4.8]. The assumption of rectangular stress distribution in the compression zone of masonry cross section is identical to that proposed for reinforced concrete in 1930s by Charles Whitney, which has been found to be simple and convenient for design calculations.

Figure 4.3 shows a comparison between the approximate parabolic stress distribution as well as Whitney's rectangular stress distribution in a rectangular reinforced masonry beam at ultimate load conditions. In the case of rectangular stress distribution, the average stress in the compression zone of the beam is assumed to have a constant value of $0.80 f'_m$ over its entire depth "a" measured from the extreme compression fibers. This stress block is

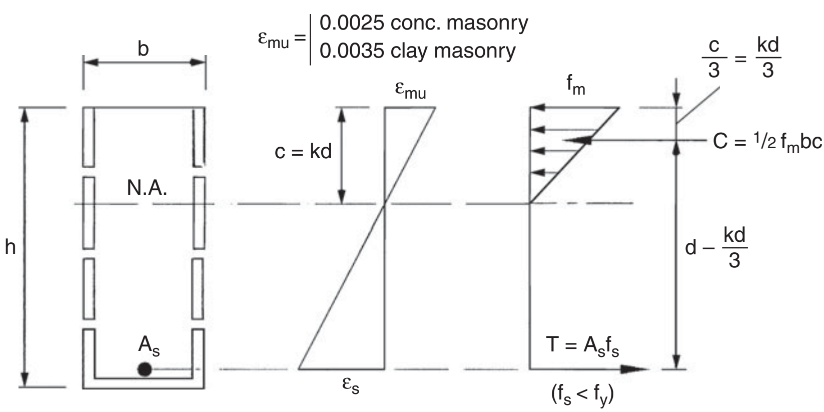


FIGURE 4.2 Linear stress distribution in reinforced masonry as the basis of working stress design.

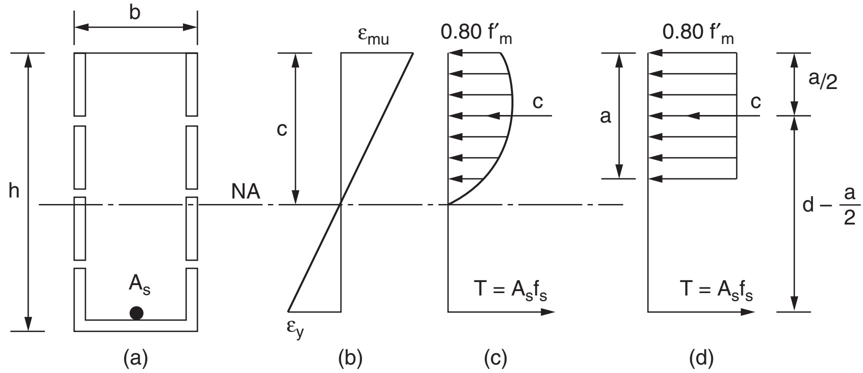


FIGURE 4.3 Analytical model for strength design of reinforced masonry.

referred to as the *equivalent* rectangular stress block, the qualifier "equivalent" being used to imply that the magnitude of the compression stress resultant C remains the same as that in the parabolic stress block, and that the neutral axis is located at the same distance "c" from the extreme compression fibers. The relationship between the depth of the compression block, a, and the depth of neutral axis, c, both measured from the extreme compression fibers of the beam, is defined by Eq. (4.5):

$$a = 0.80c \tag{4.5a}$$

Equation (4.5a) can also be expressed as

$$c = \frac{a}{0.80} \tag{4.5b}$$

Equation (4.5) is based on research and applies to both concrete and clay brick masonry.

Because the stress block is *rectangular*, its centroid is located at $a/2$ from extreme compression fibers of the beam, which is where the compression stress resultant C acts. The nominal strength (moment capacity) of the beam, M_n, can be obtained from the principles of static equilibrium. The value of the compression stress resultant C can be expressed as

$$C = \text{(ultimate stress in masonry)} \times \text{(area of masonry in the compression zone)}$$

$$C = 0.80 f'_m ab \tag{4.6}$$

Assuming that reinforcement yields prior to crushing of masonry so that stress in steel reinforcement equals the yield stress (i.e., $f_s = f_y$), the value of the tensile stress resultant, T, can be expressed as

$$T = A_s f_y \tag{4.7}$$

The line of action of the tensile stress resultant is assumed to be located at the centroid of the steel reinforcing bars. When all reinforcing bars in a beam are placed in one row, their centroid is obviously located at the same level as the centroid of all bars and in the vertical plane of symmetry of the beam cross section. However, when the bars are placed in more than one row, the centroid of the bar group must be located from principles of statics.

Equating $C = T$ from Eqs. (4.7) and (4.8) (for horizontal equilibrium), respectively, yields

$$0.80 f'_m ab = A_s f_y \tag{4.8}$$

The depth a of the compression zone of concrete is obtained from Eq. (4.8):

$$a = \frac{A_s f_y}{0.80 f'_m b} \tag{4.9}$$

The nominal strength of the beam, M_n, equals the moment provided by the C-T couple. Its magnitude can be determined by taking moment of the compressive stress resultant C about the line of action of T. Thus,

$$M_n = C\left(d - \frac{a}{2}\right) \tag{4.10}$$

Substitution for C from Eq. (4.6) in the above equation yields

$$M_n = 0.80 f'_m ab\left(d - \frac{a}{2}\right) \tag{4.11}$$

Alternatively, taking moment of T about the line of action of C and substituting $T = A_s f_y$ from Eq. (4.7), the nominal strength can be expressed as

$$\begin{aligned} M_n &= T\left(d - \frac{a}{2}\right) \\ &= A_s f_y\left(d - \frac{a}{2}\right) \end{aligned} \tag{4.12}$$

Multiplying both sides of Eq. (4.12) by the strength reduction factor, ϕ, we obtain

$$\phi M_n = \phi A_s f_y\left(d - \frac{a}{2}\right) \tag{4.13}$$

Equations (4.11) and (4.13) are based on principles of statics, and are used to determine the nominal strength of beams of rectangular cross sections. In practical beams, the amount of reinforcing steel is so limited (intentionally) that it yields before the onset of crushing of concrete. Therefore, Eq. (4.13) is used as the basic equation for determining the flexural strength of rectangular beams.

Equation (4.13) is based on the premise that the steel reinforcement has yielded, the validity of which must be checked. This is done by considering the compatibility of strains from the strain distribution diagram at the ultimate conditions (Fig. 4.4). For a member of effective depth d, and having neutral axis located at a distance c from the extreme compression fibers, we have (from similar triangles)

$$\frac{\varepsilon_s}{\varepsilon_{mu}} = \frac{d - c}{c} \tag{4.14}$$

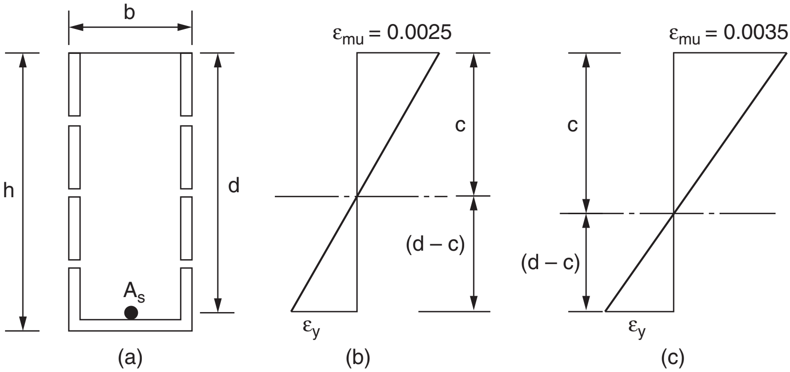


FIGURE 4.4 Linear strain relationship between ultimate strain in masonry and strain in tension reinforcement at balanced conditions: (*a*) beam cross section, (*b*) concrete masonry, (*c*) clay brick masonry.

The strain in reinforcement, ε_s, is obtained from Eq. (4.14):

$$\varepsilon_s = \left(\frac{d-c}{c}\right)\varepsilon_{mu} \tag{4.15}$$

where c = depth of neutral axis from the extreme compression fibers
ε_{mu} = maximum usable compressive strain in masonry
= 0.0025 for concrete masonry
= 0.0035 for clay masonry
ε_s = strain in steel reinforcement corresponding to ε_{mu}

Solving Eq. (4.15) gives the strain in steel, ε_s, which can be compared with the known value of yield strain obtained from Hooke's law [Eq. (4.16)] to verify if the steel reinforcement has yielded:

$$\varepsilon_y = \frac{f_y}{E_s} \tag{4.16}$$

For the commonly used Grade 60 reinforcement (f_y = 60 ksi), the yield strain is

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60}{29{,}000} = 0.00207$$

For calculation purposes, an approximate value of $\varepsilon_y = 0.002$ would be used (for Grade 60 reinforcement) throughout the book, a value which also is commonly used in professional practice. The same value has been adopted by the ACI *Code* [4.2] for design of reinforced concrete structures. The condition $\varepsilon_s \geq \varepsilon_y$ would indicate that reinforcement has yielded.

It should be noted from Eq. (4.15) that for a rectangular beam having a depth d, the strain in steel reinforcement can be calculated only if the location of neutral axis (i.e., distance c of the neutral axis from the extreme compression fibers) is known. For a given problem, distance c is determined from Eq. (4.5):

$$c = \frac{a}{0.80} \qquad \text{(4.5 repeated)}$$

where the depth of compression block (a) is calculated from Eq. (4.9):

$$a = \frac{A_s f_y}{0.80 f'_m b} \qquad \text{(4.9 repeated)}$$

Practical design of reinforced masonry beams is based on the premise that the tension reinforcement has attained certain level of yield strain ($\varepsilon_s \geq \varepsilon_y$) before the onset of crushing of masonry when the beam is subjected to ultimate loads. Therefore, when analyzing or designing a beam, it is required to check that reinforcement has yielded (i.e., $\varepsilon_s \geq \varepsilon_y$). Example 4.1 illustrates the procedure for calculating the values of parameters a, c, and the strain in the tension reinforcement, ε_s. Note that the strain in the tension reinforcement of the beam calculated in this example is based on the ultimate strain value for concrete masonry, $\varepsilon_{mu} = 0.0025$. If the beam were a clay masonry beam, we would use a strain value of $\varepsilon_{mu} = 0.0035$ (instead of 0.0025) to calculate strain in tension reinforcement.

Example 4.1 A nominal 10 × 40 in. concrete masonry beam is reinforced with two No. 6 Grade 60 bars (Fig. E4.1). The centroid of reinforcement is located 6 in. from the bottom of the beam. $f'_m = 1500$ psi. Calculate the depth of compression block a, distance c of the neutral axis from the extreme compression fiber, and the strain in tension steel reinforcement.

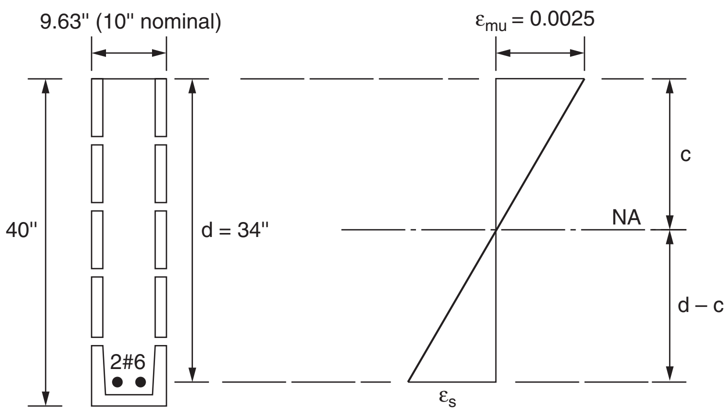


FIGURE E4.1 Beam cross section and the strain distribution diagram.

Solution

Given: $b = 9.63$ in. (10 in. nominal), $d = 40 - 6 = 34$ in., $f'_m = 1500$ psi, $f_y = 60$ ksi, $A_s = 0.88$ in.^{2} (two No. 6 bars).

From Eq. (4.9)

$$a = \frac{A_s f_y}{0.80 f'_m b} = \frac{(0.88)(60)}{0.80(1.5)(9.63)} = 4.57 \text{ in.}$$

From Eq. (4.5)

$$c = \frac{a}{0.80} = \frac{4.57}{0.80} = 5.71 \text{ in.}$$

From the strain distribution diagram, using Eq. (4.15):

$$\varepsilon_s = \frac{d-c}{c}(\varepsilon_{mu}) = \frac{34-5.71}{5.71}(0.0025) = 0.0124$$

It is noted that a strain of 0.0124 in the tension reinforcement is considerably larger than the yield strain value of 0.002 for Grade 60 reinforcement (therefore, the reinforcement is assumed to have yielded). The ratio of actual strain to yield strain is

$$\frac{\varepsilon_s}{\varepsilon_y} = \frac{0.0124}{0.002} = 6.2$$

The strain in tension reinforcement is 6.2 times its yield strain value.

4.5.2 Conditions for Yielding of Tension Reinforcement Based on Strain Compatibility

4.5.2.1 Conditions for Yielding of Tension Reinforcement in Beams at Balanced Conditions Fundamental relationships for yielding of steel reinforcement can be derived from compatibility of strains in a reinforced masonry beam. Figure 4.4 shows strain distribution across in a reinforced masonry beam at the balanced conditions. Under these conditions, the ultimate strain in masonry is assumed to be equal to ε_{mu} and the strain in the tension reinforcement is assumed to be equal to the yield strain ε_y.

From similar triangles of the strain distribution diagram, we observe that

$$\frac{c}{d} = \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \varepsilon_y} \tag{4.17}$$

The value of *c/d* ratio that would result in yielding of tension reinforcement can be obtained from Eq. (4.17) by substituting appropriate values of the ultimate strains in masonry and reinforcement.

a. *Concrete masonry:* Substitution of $\varepsilon_{mu} = 0.0025$ for concrete masonry and $\varepsilon_y = 0.002$ for Grade 60 reinforcement in Eq. (4.17) yields

$$\frac{c}{d} = \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \varepsilon_y} = \frac{0.0025}{0.0025 + 0.0020} = 0.555 \tag{4.18}$$

Equation (4.18) shows that in a concrete masonry beam, Grade 60 reinforcement would yield when the condition given by Eq. 4.19 is satisfied:

$$\frac{c}{d} \leq 0.555 \tag{4.19}$$

For all values of $c/d > 0.555$, the strain in steel would be less than the yield strain of 0.002.

An expression similar to Eq. (4.19) can be derived for Grade 40 reinforcement for which the yield strain is

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{40}{29{,}000} = 0.00138 \tag{4.20}$$

The ratio of yield strain in Grade 40 reinforcement and the ultimate strain in concrete masonry can be expressed as

$$\frac{c}{d} = \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \varepsilon_y} = \frac{0.0025}{0.0025 + 0.00138} = 0.644 \tag{4.21}$$

Equation (4.21) shows that in a concrete masonry beam, Grade 40 reinforcement would yield when the condition given by Eq. (4.22) is satisfied:

$$\frac{c}{d} \leq 0.644 \tag{4.22}$$

For all values of $c/d > 0.644$, the strain in Grade 40 reinforcement would be less than the yield strain of 0.00138.

b. *Clay masonry*: Proceeding as above, *c/d* ratios corresponding to yielding of tension reinforcement can be obtained for clay masonry for which $\varepsilon_{mu} = 0.0035$. The *c/d* ratio corresponding to yielding of Grade 60 tension reinforcement in clay masonry can be expressed by substituting $\varepsilon_{mu} = 0.0035$ (instead of 0.0025) and $\varepsilon_y = 0.002$ in Eq. (4.18). Thus,

$$\frac{c}{d} = \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \varepsilon_y} = \frac{0.0035}{0.0035 + 0.0020} = 0.636 \tag{4.23}$$

Equation (4.23) shows that in a clay masonry beam, Grade 60 reinforcement would yield when the condition given by Eq. (4.24) is satisfied:

$$\frac{c}{d} \leq 0.637 \tag{4.24}$$

For all values of $c/d > 0.636$, the strain in Grade 60 reinforcement would be less than the yield strain of 0.002.

Similarly, the *c/d* ratio corresponding to yielding of Grade 40 reinforcement in clay masonry beams can be expressed as

$$\frac{c}{d} = \frac{\varepsilon_{my}}{\varepsilon_{mu} + \varepsilon_y} = \frac{0.0035}{0.0035 + 0.00138} = 0.717 \tag{4.25}$$

Therefore, in a clay masonry beam, Grade 40 reinforcement would yield if the condition given by Eq. (4.26) is satisfied:

$$\frac{c}{d} \leq 0.717 \tag{4.26}$$

For all values of $c/d > 0.717$, the strain in Grade 40 reinforcement would be less than the yield strain of 0.00138.

Equations. (4.19), (4.22), (4.24), and (4.26) provide a check, based on strain compatibility, for verifying yielding of steel reinforcement in reinforced masonry beams. It is reiterated that the value of ε_{mu} for both concrete and clay masonry (0.0025 and 0.0035, respectively) is greater than ε_y for both Grades 60 and 40 steels (0.002 and 0.00138, respectively). One should intuitively recognize that because of linearity of strain distribution even at the ultimate conditions, if $c = d/2$, strain in tension reinforcement would be equal to the ultimate strain in masonry, and therefore, greater than ε_y for both Grades 60 and 40 reinforcing bars. For all cases when $c < d/2$, the strain in tension reinforcement (ε_s) would be greater than the ultimate strain in masonry, ε_{mu}. This is an important observation for it provides a quick check (without any additional calculations) whether the reinforcement has yielded.

TABLE 4.2 Limiting c/d Ratios for Yielding of Tension Reinforcement in Masonry Beams at Balanced Conditions

Type of masonry	Maximum compressive strain, ε_{mu}	Condition of tension reinforcement yielding	
		Grade 60 steel	Grade 40 steel
Concrete	0.0025	$\frac{c}{d} \le 0.555$ [Eq. (4.19)]	$\frac{c}{d} \le 0.644$ [Eq. (4.22)]
Clay	0.0035	$\frac{c}{d} \le 0.636$ [Eq. (4.24)]	$\frac{c}{d} \le 0.717$ [Eq. (4.26)]

A summary of conditions (*c/d* ratios) of yielding of Grades 60 and 40 tension reinforcement used in concrete and clay masonry beams, based on the assumed strain gradient at balanced conditions, is presented in Table 4.2.

4.5.2.2 General Conditions (c/d ratios) for Post-Yielding of Tension Reinforcement Based on Strain Compatibility In order to ensure sufficient yielding of tension reinforcement in elements of a masonry structure, MSJC *Code* (Section 3.3.3.5) imposes upper limits on maximum area of tension reinforcement that can be provided in various elements, such as beams, shear walls, etc. These limits, which are different for different structural elements, are specified corresponding to certain level of yield strain based on a strain gradient that is compatible with ultimate masonry strain, ε_{mu}. The *Code* specifies these upper limits corresponding to higher levels of yield strain, which may be stated as follows:

$$\varepsilon_s = \alpha \varepsilon_y \tag{4.27}$$

where ε_s = strain in tension steel reinforcement
ε_y = yield strain in steel reinforcement
α = tension reinforcement strain factor = $\varepsilon_s/\varepsilon_y$, $\alpha > 1.0$

We can now see from Eq. (4.27) that $\alpha = 6.2$ in Example 4.1.

Table 4.3 lists values of the tension reinforcement strain factor α which is required to be greater than 1.0. Obviously, if $\alpha < 1.0$, $\varepsilon_s < \varepsilon_y$, and the beam would be overreinforced beam (discussed hereinafter). Note that the value of α is different for different flexural elements depending on whether

1. Ratio $M_u/V_u d_v \ge 1.0$ or ≤ 1.0
2. The value of the seismic response modification factor R (whether $R \ge 1.5$ or ≤ 1.5 as defined in ASCE 7-05)

where M_u = maximum usable moment (= factored moment)
V_u = maximum usable shear (= factored shear force)
d_v = actual depth of masonry in the direction of shear considered (= h, the overall depth of a transversely loaded beam discussed in this chapter)

TABLE 4.3 Limiting Levels of Yield Strain in Steel Reinforcement (MSJC-08 Section 3.3.3.5)

Design element(s)	Tension reinforcement strain factor, α [Eq. (4.17)]	MSJC-05 Section
Masonry members where $M_u/V_ud_v \geq 1$	1.5	3.3.3.5.1
Intermediate reinforced masonry shear walls subject to in-plane loads where $M_u/V_ud_v \geq 1$	3	3.3.3.5.2
Special reinforced masonry shear walls subject to in-plane loads where $M_u/V_ud_v \geq 1$	4	3.3.3.5.3
Masonry members where $M_u/V_ud_v \leq 1$ and when designed using $R \leq 1.5$	No limit	3.3.3.5.4
when designed using $R \geq 1.5$	1.5	

Note that MSJC-08 Section 3.3.3.5.4 provides no upper limit for maximum reinforcement where the ratio $M_u/V_ud_v \leq 1.0$ and $R \leq 1.5$. This provision should not be construed to mean that a beam can be designed as an overreinforced beam. Section 3.3.3.5.4 actually is not applicable to transversely loaded beams discussed in this chapter. It is intended to apply to *squat* shear walls (which act as vertical beams and have large d_v, equal to the length of the shear wall, so that $M_u/V_ud_v \leq 1.0$) in regions of low seismicity ($R \leq 1.5$), where the shear failure rather than flexural failure (because moments are small) would be dominant.

Figure 4.5 shows a strain distribution diagram for a reinforced masonry beam in which the strain in masonry is ε_{mu} and the strain in steel reinforcement is $\alpha\varepsilon_y$, where α may be any

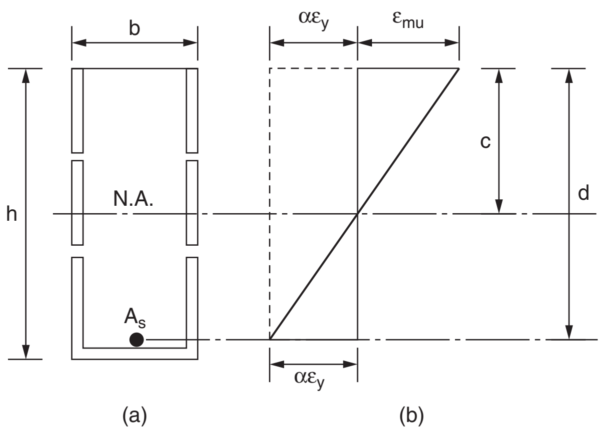


FIGURE 4.5 Strain distribution in a reinforced masonry beam at ultimate conditions based on compatibility of ultimate strain in masonry and post-yield strain in tension reinforcement.

desired number greater than 1.0 (so that $\alpha\varepsilon_y > \varepsilon_y$). Since the value of α (hence the strain in steel reinforcement) is based on strain gradient consistent with ultimate strain in masonry, it is possible to derive a mathematical relationship between *c/d* ratio and strains in masonry and reinforcement at ultimate conditions. From similar triangles in Fig. 4.5

$$\frac{c}{d} = \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \alpha\varepsilon_y} \tag{4.28}$$

Values of *c/d* ratio to be satisfied consistent with the desired value of tension reinforcement strain factor α for a given set of design condition can be determined from Eq. (4.28) for the specific value of ε_{mu}.

a. *Concrete Masonry:* For a concrete masonry beam, $\varepsilon_{mu} = 0.0025$, $\varepsilon_y = 0.002$ (Grade 60 reinforcement), and $\alpha = 1.5$ (assuming $M_u/V_u d_v \geq 1$, see Table 4.2). Substitution of these values in Eq. (4.28) yields

$$\frac{c}{d} = \frac{0.0025}{0.0025 + 1.5(0.002)} = 0.454 \tag{4.29}$$

Equation (4.29) states that for $c/d \leq 0.454$, the value of α in Eq. (4.28) would be greater than 1.5. Therefore, the condition that would indicate $\alpha \geq 1.5$ (the desired condition) can be stated as

$$\frac{c}{d} \leq 0.454 \tag{4.30}$$

Equation (4.30) states that for all values of $c/d > 0.454$, the value of α would be less than 1.5 in a concrete masonry beams having Grade 60 tension reinforcement, that is, the strain in the tension reinforcement would be less than 1.5 times its yield value.

Equations similar to Eq. (4.30) can be derived for Grade 40 reinforcement following the above procedure. The value of yield strain for Grade 40 reinforcement was calculated to be 0.00138. Thus from Eq. (4.28), we have for concrete masonry,

$$\frac{c}{d} = \frac{0.0025}{0.0025 + 1.5(0.00138)} = 0.547 \tag{4.31}$$

Equation (4.31) states that for all values of $c/d > 0.547$, the value of α would be less than 1.5 in clay masonry beams having Grade 40 tension reinforcement. Thus, for the tension reinforcement strain factor α to be equal to or greater than 1.5 in a concrete masonry beam having Grade 40 tension reinforcement, the condition given by Eq. (4.32) should be satisfied:

$$\frac{c}{d} \leq 0.547 \tag{4.32}$$

The significance of Eqs. (4.30) and (4.32) lies in the fact that they can be used (without calculating strain in the tension reinforcement) to determine if the strain in tension reinforcement is equal to greater than 1.5 times the yield strain, which is a critical condition for an acceptable beam design.

b. *Clay Masonry*: The values of *c/d* ratios for clay masonry beams can be derived by substituting ε_{mu}. = 0.0035 instead of 0.0025 in Eq. (4.29):

$$\frac{c}{d} = \frac{0.0035}{0.0035 + 1.5(0.002)} = 0.538 \tag{4.33}$$

Thus, for the tension reinforcement strain factor α to be equal to or greater than 1.5 in a clay masonry beam, the condition given by Eq. (4.34) should be satisfied:

$$\frac{c}{d} \le 0.538 \tag{4.34}$$

Equation (4.34) states that for all values of $c/d > 0.538$, the value of α would be less than 1.5 in clay masonry beams having Grade 60 tension reinforcement.

Similarly, for a clay masonry beam having Grade 40 reinforcement, the c/d ratio can be expressed as

$$\frac{c}{d} = \frac{0.0035}{0.0035 + 1.5(0.00138)} = 0.628 \tag{4.35}$$

Thus, for the tension reinforcement strain factor α to be equal to or greater than 1.5 in a clay masonry beam having Grade 40 reinforcement, the condition given by Eq. (4.36) should be satisfied:

$$\frac{c}{d} \le 0.628 \tag{4.36}$$

Equations for c/d ratios corresponding to the other values of the tension reinforcement strain factor α, for example, $\alpha = 3$ and 4 (as specified in Table 4.2), can be derived following the above procedure.

Equations (4.30), (4.32), (4.34), and (4.36) are summarized in Table 4.4 for use in analysis and design of reinforced masonry beams. These equations are used in the following examples to verify if the strain in the tension reinforcement has reached or exceeded 1.5 times the yield strain value. It would be observed from these examples that the condition $\varepsilon_s \ge 1.5\varepsilon_y$ is generally satisfied in practical masonry beams. Although the strain in tension reinforcement of a beam can always be calculated from strain distribution diagram (using similar triangles), it is much simpler to use c/d ratios (equations listed in Table 4.4) to verify if $\varepsilon_s \ge 1.5\varepsilon_y$. In the following examples, it is required that the condition $\varepsilon_s \ge 1.5\varepsilon_y$ be satisfied, for which these equations have been used.

TABLE 4.4 Limiting c/d ratios for Masonry Beams Based on Strain Gradient with ε_{mu} and $1.5\varepsilon_y$

Type of masonry	Maximum compressive strain, ε_{mu}	Condition of steel yielding	
		Grade 60 steel	Grade 40 steel
Concrete	0.0025	$\frac{c}{d} \le 0.454$ [Eq. (4.30)]	$\frac{c}{d} \le 0.547$ [Eq. (4.32)]
Clay	0.0035	$\frac{c}{d} \le 0.538$ [Eq. (4.34)]	$\frac{c}{d} \le 0.628$ [Eq. (4.36)]

Example 4.2 To determine the flexural strength of a reinforced masonry beam.

A nominal 10 × 40 in concrete masonry beam built from lightweight CMU is reinforced with two No. 6 Grade 60 bars for tension (Fig. E4.2). The centroid of reinforcement is located at 6 in. from the bottom of the beam. The beam carries a service live load of 1600 lb/ft over an effective span of 15 ft 8 in. in addition to its own weight. The grout weight is 140 lb/ft^3. $f'_m = 1500$ psi. Check if the beam is adequate to carry the imposed service loads.

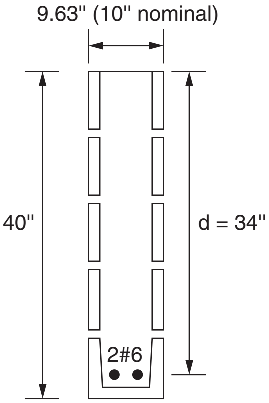


FIGURE E4.2 Beam cross section for Example 4.2.

Solution

For a 10-in. nominal lightweight CMU with a grout weight of 140 lb/ft^3, the dead weight = 93 lb/ft depth (h) of the beam (Table A.19).

Self-weight of the beam,

$$D = 93\left(\frac{40}{12}\right) = 310 \text{ lb/ft}$$
$$L = 1600 \text{ lb/ft}$$

Load combinations:

1. $U = 1.4\text{ D} = 1.4\ (310) = 434$ lb/ft
2. $U = 1.2\text{ D} + 1.6\text{ L} = 1.2\ (310) + 1.6\ (1600)$
   $= 2932$ lb/ft > 434 lb/ft

$$w_u = 2932 \text{ lb/ft (governs)}$$

Effective span = 15 ft 8 in. = 15.67 ft

$$M_u = \frac{w_u L^2}{8} = \frac{2932(15.67)^2}{8} = 89{,}994 \text{ lb-ft} \approx 90.0 \text{ k-ft}$$

Calculate the nominal strength of the beam. Assume that tension reinforcement has yielded (to be verified later).

$A_s = 0.88$ in.^{2} (two No. 6 bars, App. Table A.9), $b = 9.63$ in. (10 in. nominal)

Calculate the depth of compression block, a, from Eq. (4.9):

$$a = \frac{A_s f_y}{0.80 f'_m b} \tag{4.9}$$

$$a = \frac{(0.88)(60)}{0.80(1.5)(9.63)} = 4.57 \text{ in.}$$

Calculate the nominal strength from Eq. (4.12):

$$M_n = A_s f_y\left(d - \frac{a}{2}\right) \tag{4.12}$$

The centroid of reinforcing bars is at 6 in. from the bottom, $d = 40 - 6 = 34$ in.

$$M_n = (0.88)(60)\left(34 - \frac{4.57}{2}\right)$$
$$= 1674.5 \text{ kip-in} = 139.55 \text{ k-ft}$$

Check the $M_u/V_u d_v$ ratio, where $d_v = d = 34$ in. (see discussion in Section 4.10).

$$V_u = \frac{w_u L}{2} = \frac{(2.932)(15.67)}{2} = 22.97 \text{ kips}$$
$$d_v = d = 34 \text{ in.}$$
$$\frac{M_u}{V_u d_v} = \frac{(90)(12)}{(22.97)(34)} = 1.38 > 1.0$$

Verify from Eq. (4.30) that reinforcement has yielded and that $\varepsilon_s \geq 1.5\varepsilon_y$. From Eq. (4.5)

$$c = \frac{a}{0.80} = \frac{4.57}{0.80} = 5.71 \text{ in.}$$

$$\frac{c}{d} = \frac{5.71}{34.0} = 0.17 < 0.454 \tag{4.30}$$

Hence, the reinforcement has yielded, and $\varepsilon_s \geq 1.5\varepsilon_y$. Use $\phi = 0.9$.

$$\phi M_n = 0.9(139.55) = 125.6 \text{ k-ft} > M_u = 90.0 \text{ k-ft} \qquad \text{OK}$$

The beam is safe to carry the imposed loads.

(Alternatively, we could have calculated strain in the tension reinforcement from the strain distribution diagram to verify that $\varepsilon_s \geq 1.5\varepsilon_y$. From the similar triangles of the strain distribution diagram,

$$\frac{\varepsilon_s}{\varepsilon_{mu}} = \frac{d-c}{c} = \frac{34.0 - 5.71}{5.71} = 4.954$$

$$\varepsilon_s = 4.954\,\varepsilon_{mu} = 4.954(0.0025) = 0.0124$$

$$\frac{\varepsilon_s}{\varepsilon_y} = \frac{0.0124}{0.002} = 6.2 > 1.5$$

Therefore, the strain in the tension reinforcement is 6.2 times the yield strain value.

The simplicity of using *c/d* ratio in lieu of the above calculation to verify that $\varepsilon_s \geq 1.5\varepsilon_y$ should now be obvious.)

Example 4.3 A nominal 8- × 24-in. concrete masonry beam is reinforced with one No. 7 Grade 60 bar at an effective depth of 20 in. (Fig. E4.3). Determine the design moment strength, ϕM_n of the beam. Assume $f'_m = 2000$ psi, and that $M_u/V_u d_v \geq 1.0$.

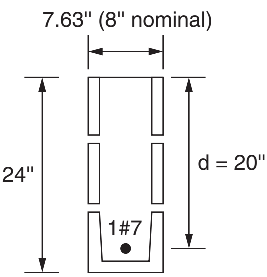


FIGURE E4.3 Beam cross section for Example 4.3.

Solution

Given: $A_s = 0.61$ in.^{2} (one No. 7 bar), $b = 7.63$ in. (8 in. nominal), $f'_m = 2000$ psi, $f_y = 60$ ksi.

Assume that reinforcement has yielded so that $f_s \geq f_y$ (to be verified later). Calculate the depth of compression block, a, from Eq. (4.9):

$$a = \frac{A_s f_y}{0.80 f'_m b} \tag{4.9 repeated}$$

$$a = \frac{(0.61)(60)}{0.80(2.0)(7.63)} = 3.0 \text{ in.}$$

From Eq. (4.13)

$$\phi M_n = \phi A_s f_y \left(d - \frac{a}{2} \right)$$

$$= 0.9(0.61)(60)\left(20 - \frac{3.0}{2} \right)$$

$$= 609.39 \text{ k-in.} = 50.78 \text{ k-ft}$$

Verify from Eq. (4.30) that reinforcement has yielded. From Eq. (4.5)

$$c = \frac{a}{0.8} = \frac{3.0}{0.8} = 3.75 \text{ in.}$$

$$\frac{c}{d} = \frac{3.75}{20.0} = 0.188 < 0.454 \qquad \text{(4.30 repeated)}$$

Hence, the reinforcement has yielded, and $\varepsilon_s \geq 1.5\varepsilon_y$. Use $\phi = 0.9$.

$$\phi M_n = 50.78 \text{ k-ft}$$

Example 4.4 Determine the design moment strength, ϕM_n, of the beam described in Example 4.3 if Grade 40 reinforcing bar were to be used instead of Grade 60 bar (Fig. E4.4). All other data are the same.

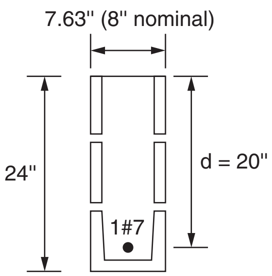


FIGURE E4.4 Beam cross section for Example 4.4.

Solution

Given: $A_s = 0.61$ in.^{2} (one No. 7 bar), $b = 7.63$ in. (8 in. nominal), $f'_m = 2000$ psi, $f_y = 40$ ksi.

Assume that the reinforcement has yielded. This would be verified later. Calculate the depth of compression block, a, from Eq. (4.9):

$$a = \frac{A_s f_y}{0.80 f'_m b} \qquad \text{(4.9 repeated)}$$

$$a = \frac{(0.61)(40)}{0.80(2.0)(7.63)} = 2.0 \text{ in.}$$

From Eq. (4.13)

$$\begin{aligned}\phi M_n &= \phi A_s f_y \left(d - \frac{a}{2} \right) \\ &= 0.9(0.61)(40)\left(20 - \frac{2.0}{2} \right) \\ &= 417.24 \text{ k-in.} = 34.77 \text{ k-ft}\end{aligned}$$

Verify from Eq. (4.32) that reinforcement has yielded. From Eq. (4.5)

$$c = \frac{a}{0.8} = \frac{2.0}{0.8} = 2.5 \text{ in.}$$

$$\frac{c}{d} = \frac{2.5}{20.0} = 0.125 < 0.547 \qquad \text{(4.32 repeated)}$$

Hence, the reinforcement has yielded, and $\varepsilon_s \geq 1.5\varepsilon_y$.

$$\phi M_n = 34.77 \text{ k-ft}$$

The next example presents a brick (or clay masonry) beam. The analysis and design procedures for structural clay masonry are same as for concrete masonry. The only difference is that the value of ultimate strain in clay masonry (0.0035) is much higher than ultimate strain in concrete masonry (0.0025). Therefore, strain in steel at ultimate load conditions, ε_s, would be determined based on $\varepsilon_{mu} = 0.0035$. Accordingly, Eqs. (4.34) (for Grade 60 steel) and (4.36) (for Grade 40 steel) can be used to verify if the reinforcement has yielded. Of course, if c is less than $d/2$ as before, $\varepsilon_s > \varepsilon_{mu} > \varepsilon_y$.

Example 4.5 A two-wythe 8 × 24 in. clay brick beam has an effective depth of 20 in. It is reinforced with one No. 8 Grade 60 bar for tension (Fig. E4.5). Determine the design moment strength, ϕM_n, of this beam. $f'_m = 2500$ psi. Portland cement Type *S* mortar would be used for construction. Assume that $M_u/V_u d_v \geq 1.0$.

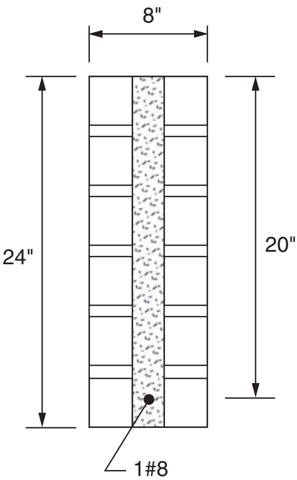


FIGURE E4.5 Beam cross section for Example 4.5.

Solution

Given: $b = 8$ in., $d = 20$ in., $A_s = 0.79$ in.^{2} (one No. 8 bar), $f'_m = 2500$ psi, $f_y = 60$ ksi.

Assume that $f_s \geq f_y$ (to be verified later). Calculate from Eq. (4.9) the depth a of the compression block.

$$a = \frac{A_s f_y}{0.80 f'_m b} \tag{4.9}$$

$$a = \frac{(0.79)(60)}{0.80(2.5)(8.00)} = 2.96 \text{ in.}$$

From Eq. (4.13)

$$\begin{aligned}\phi M_n &= \phi A_s f_y \left(d - \frac{a}{2}\right) \\ &= 0.9(0.79)(60)\left(20 - \frac{2.96}{2}\right) \\ &= 790 \text{ k-in.} = 65.83 \text{ k-ft}\end{aligned}$$

Verify that reinforcement has yielded. From Eq. (4.5),

$$c = \frac{a}{0.8} = \frac{2.96}{0.8} = 3.7 \text{ in.}$$

$$\frac{c}{d} = \frac{3.7}{20.0} = 0.185 < 0.538 \tag{4.34}$$

Hence, the reinforcement has yielded, and $\varepsilon_s \geq 1.5\varepsilon_y$.

$$\phi M_n = 65.83 \text{ k-ft}$$

4.5.3 Nominal Strength and Reinforcement Ratio

A relationship useful for both analysis and design of reinforced masonry beams is obtained by substituting the value of the depth of the compression block, *a,* from Eq. (4.9) into

Eq. (4.10). We define *reinforcement ratio* ρ (rho) as the ratio of the area of reinforcement to the cross-sectional area of the beam, bd:

$$\rho = \frac{A_s}{bd} \tag{4.37}$$

so that

$$A_s = \rho bd \tag{4.38}$$

where ρ = the reinforcement ratio.

Substitution of Eq. (4.38) into Eq. (4.9) yields

$$a = \frac{\rho f_y}{f'_m}\left(\frac{d}{0.80}\right) \tag{4.39}$$

Now, define

$$\omega = \frac{\rho f_y}{f'_m} \tag{4.40}$$

where the quantity ω (omega) is called the *mechanical reinforcement ratio* or the *reinforcing index.* With the substitution of Eq. (4.40) into Eq. (4.39), we can write

$$a = \frac{\omega d}{0.80} \tag{4.41}$$

Substitution for a from Eq. (4.41) into Eq. (4.11) yields the nominal strength of a reinforced masonry beam:

$$M_n = 0.80 f'_m ab\left(d - \frac{a}{2}\right) \tag{4.11 repeated}$$

$$M_n = f'_m bd^2\omega\left(1 - \frac{\omega}{2(0.80)}\right)$$

or

$$M_n = f'_m bd^2\omega(1 - 0.625\omega) \tag{4.42}$$

Multiplying both the sides of Eq. 4.42 with the strength reduction factor ϕ, we obtain

$$\phi M_n = \phi f'_m bd^2\omega(1 - 0.625\omega) \tag{4.43}$$

where ϕM_n = the design strength of a beam. For design purposes,

$$\phi M_n \geq M_u \tag{4.3 repeated}$$

Equation (4.43) can be used to determine the nominal strength M_n of a rectangular beam. Equation (4.43) shows that the nominal strength of a reinforced masonry beam, M_n, depends only on the cross section and material properties of a beam. Examples 4.6 and 4.7 illustrate the application of Eq. 4.43.

For design purposes, Eq. 4.43 can be expressed as

$$\frac{\phi M_n}{bd^2} = \phi[f'_m\omega(1 - 0.625\omega] \tag{4.44}$$

In Eq. (4.44), we substitute k_n for the bracketed quantity on the right-hand side:

$$k_n = f'_m \omega(1 - 0.625\omega) \tag{4.45}$$

Substitution of Eq. (4.45) in Eq. (4.44) yields

$$\frac{\phi M_n}{bd^2} = \phi k_n \tag{4.46}$$

A form of Eq. (4.46) that is useful for design can be obtained if we express the design moment M_u in kip-ft units, and b and d in inches. With these substitutions, Eq. (4.46) can be expressed as

$$\frac{\phi M_n}{\phi k_n} = \frac{bd^2}{12{,}000} \tag{4.47a}$$

$$\phi M_n = \phi k_n \left(\frac{bd^2}{12{,}000} \right) \tag{4.47b}$$

where M_n = nominal strength, k-ft. For economical designs, $\phi M_n = M_u$, where M_u is the design moment (or moment demand) calculated from the given loading conditions. Therefore, for design, Eq. (4.47) can be expressed as

$$\frac{M_u}{\phi k_n} = \frac{bd^2}{12{,}000} \tag{4.48a}$$

$$M_u = \phi k_n \left(\frac{bd^2}{12{,}000} \right) \tag{4.48b}$$

Equations (4.45) and (4.48) are very useful for flexural design of masonry elements as illustrated by Examples 4.8 and 4.9.

It is instructive to understand the significance of Eqs. (4.45) and (4.48) for design purposes. For a given or assumed value of the reinforcement ratio ω, the value of reinforcing index, ω, can be determined from Eq. (4.40) for the given values of f'_m and f_y. Knowing the value of ω, the value of k_n can be determined from Eq. (4.45). Then, for a calculated value of the design moment M_u, the value of the quantity bd^2 can be determined from either from Eq. (4.47) or (4.48). And finally, for a given or known value of the beam width b, the required value of depth d is easily determined.

Note that k_n is a function of reinforcement ratio (ρ), masonry compressive strength (f'_m), and the yield strength of reinforcing steel (f_y). Values of ϕk_n ($\phi = 0.9$) for $f_y = 60$ ksi and different combinations of ρ and f'_m are listed in Table A.13. Interpolation may be used for intermediate values of ρ to determine k_n without any appreciable error. See Examples 4.6 and 4.7.

Often in a design situation, the reinforcement ratio ρ is not known, in which case Eq. (4.45) cannot be used. In such cases, one can resort to some reasonable approximation in analysis, and use a trial-and-error procedure as discussed in Section 4.6.

Example 4.6 Solve Example 4.2 using Eq. (4.43).

Solution

$A_s = 0.88$ in.^{2} (two No. 6 bars), $f'_m = 1.5$ ksi, $f_y = 60$ ksi, $b = 9.625$ in. (10 in. nominal), $d = 34$ in.

From Eq. (4.37)

$$\rho = \frac{A_s}{bd} = \frac{(0.88)}{(9.63)(34)} = 0.00269$$

From Eq. (4.40)

$$\omega = \rho\frac{f_y}{f'_m} = (0.00269)\frac{(60)}{(1.5)} = 0.1076$$

From Eq. (4.43)

$$\begin{aligned}\phi M_n &= \phi[f'_m bd^2\omega(1-0.625\omega)]\\ &= 0.9[(1.5)(9.63)(34)^2(0.1076)(1-0.625\times 0.1076)]\\ &= 1508.3\ \text{lb-in.} = 125.69\ \text{k-ft}\end{aligned}$$

Alternatively, we could have determined ϕk_n from Table A.13. For $\rho = 0.00269$ and $f'_m = 1500$ psi, from interpolation,

$$\phi k_n = 103 + (150 - 103)(0.69) = 135.43\ \text{psi}$$

From Eq. (4.47),

$$\phi M_n = \phi k_n\left(\frac{bd^2}{12{,}000}\right) = (135.43)\left(\frac{(9.63)(34)^2}{12{,}000}\right) = 125.64\ \text{k-ft}$$

The above value ϕM_n is the same as obtained in Example 4.2. A check should be made to ensure that reinforcement has yielded, and $\varepsilon_s \geq 1.5\varepsilon_y$ as shown in Example 4.2 (calculations not repeated here).

$$\phi M_n = 125.7\ \text{k-ft}$$

Example 4.7 Using Eq. (4.43), (a) determine the design flexural strength, ϕM_n of the beam described in Example 4.3 (Fig. E4.7); (b) How would the flexural strength of the beam be affected if Grade 60 bar were replaced with a Grade 40 bar?

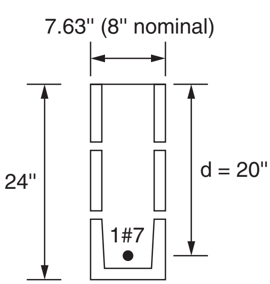


FIGURE E4.7 Beam cross section for Example 4.7.

Solution

a. Flexural strength:
From Example 4.3, $b = 7.63$ in. (8 in. nominal), $d = 20$ in, $f'_m = 2$ ksi, $f_y = 60$ ksi, $A_s = 0.61$ in.^{2}.
From Eq. (4.37)

$$\rho = \frac{A_s}{bd} = \frac{0.61}{(7.63)(20)} = 0.004$$

From Eq. (4.40)

$$\omega = \rho\frac{f_y}{f'_m} = 0.004\left(\frac{60}{2}\right) = 0.12$$

From Eq. (4.43)

$$\begin{aligned}\phi\ M_n &= \phi\left[f'_m bd^2\omega(1-0.625\omega)\right]\\ &= 0.9[2.0(7.63)(20)^2(0.12)(1-0.625\ \ x\ \ 0.12)]\\ &= 609.8\text{ k-in.} = 50.82\text{ k-ft}\end{aligned}$$

Alternatively, we could have determined ϕk_n from Table A13. For $\rho = 0.004$ and $f'_m = 2000$ psi, $\phi k_n = 200$ psi.
From Eq. (4.47),

$$\phi M_n = \phi k_n\left(\frac{bd^2}{12{,}000}\right) = (200)\left(\frac{(7.63)(20)^2}{12{,}000}\right) = 50.87\text{ k-ft}$$

The above result is the same as obtained in Example 4.3.
The above calculation assumes the steel has yielded, and $\varepsilon_s \geq 1.5\varepsilon_y$ as shown in Example 4.3 (calculations not repeated here).

b. Change in flexural strength when Grade 60 reinforcement is changed to Grade 40 reinforcement:
When the grade of reinforcing steel is changed from Grade 60 to 40, then the yield strength f_y changes from 60 to 40 ksi. However, the value of the reinforcement ratio, ρ (= 0.004) remains the same as calculated above. From Eq. (4.40),

$$\omega = \rho\frac{f_y}{f'_m} = 0.004\left(\frac{40}{2.0}\right) = 0.08$$

From Eq. (4.43)

$$\begin{aligned}\phi M_n &= 0.9[f'_m bd^2\omega(1-0.625\omega)]\\ &= 0.9[2.0(7.63)(20^2)(0.08)(1-0.625\times 0.08)]\\ &= 417.5\ \text{ k-in.} = 34.8\text{ k-ft}\end{aligned}$$

The above result is the same as obtained in Example 4.4.
The above calculation assumes the steel has yielded, and $\varepsilon_s \geq 1.5\varepsilon_y$ as shown previous examples (calculations not repeated here).
The reduction in the design flexural strength of the beam when the reinforcement grade is changed from Grade 60 to Grade 40:

$$(\phi M_n)_{\text{Gr }60} - (\phi M_n)_{\text{Gr }40} = 50.8 - 34.8 = 16.0\text{ k-ft}$$

$$\%\ \text{reduction} = \frac{16.0}{50.8}\times 100 = 31.5\%$$

Or, % increase when Grade 60 steel is used instead of Grade 40 steel:

$$\%\ \text{increase} = \frac{16.0}{34.8}\times 100 = 46\%$$

Example 4.8 For the beam described in Example 4.3, (a) determine the uniform service load the beam can carry over a span of 10 ft 8 in. if the dead-to-live load ratio is 0.7, (b) uniform live load the beam can carry safely if the only dead load on the beam is its own weight.

Solution

a. Span, L = 10 ft 8 in. = 10.67 ft From Example 4.3

$$\phi M_n = M_u = 50.78 \text{ k-ft}$$

For a uniformly loaded beam, $M_u = wL^2/8$, from which we obtain

$$\begin{aligned} w_u &= \frac{8M_u}{L^2} \\ &= \frac{8(50.8)}{(10.67)^2} = 3.57 \text{ k/ft} \end{aligned}$$

b. Uniform live load on the beam

$$D/L = 0.7, \text{ therefore, } D = 0.7L$$

Because $D/L < 8$, $U = 1.2\,D + 1.6L$ load combination governs.

$$w_u = 1.2D + 1.6L = 3.57 \text{ kips/ft}$$

$$1.2(0.7L) + 1.6(L) = 3.57 \text{ kips}$$

$$L = 1.463 \text{ kips/ft}$$

$$D = 0.7L = 0.7(1.463) = 1.024 \text{ kip/ft}$$

$$(\text{Check: } 1.2D + 1.6L = 1.2(1.024) + 1.6(1.463) \approx 3.57 \text{ kips/ft})$$

For an 8-in. nominal normal weight concrete masonry (grout weight 140 lb/ft^3), average weight = 84 lb/ft depth (h) of beam (Table A.19).

$$\text{Self-weight of beam} = 84\left(\frac{24}{12}\right) = 168 \text{ lb/ft}$$

Superimposed uniform load = 1024 – 168 = 856 lb/ft

4.6 MODULUS OF RUPTURE AND NOMINAL CRACKING MOMENT OF A MASONRY BEAM

4.6.1 Modulus of Rupture

Modulus of rupture refers to the tensile strength of an unreinforced beam, that is, maximum stress in an unreinforced beam when it cracks under bending. Beams are *required to be fully grouted* (code requirement). The modulus of rupture for masonry elements subjected to in-plane or out-of-plane bending, f_r, is specified in MSJC-08 *Code* Section 3.1.8.2. Modulus of rupture for masonry depends on several variables:

a. Layout of masonry units: stack or running bond.
b. Types of masonry units: solid or hollow.
c. Units are ungrouted, partially grouted, or fully grouted.
d. Type of mortar used in construction.

e. Direction of flexural stress (whether parallel or normal to the bed joints). For example, in a transversely loaded masonry beam, the flexural stresses are parallel to the bed joints, whereas, in flexural elements subjected to in-plane loads (such as shear walls), the flexural stresses are normal to the bed joints.

Values of modulus of rupture for hollow and solid grouted masonry are listed in MSJC-08 Table 3.1.8.1. Values are listed for two mortar types: (1) portland cement/lime or mortar cement and (2) masonry cement or air entrained portland cement/lime. For each category, values of modulus of rupture are listed for Types M or S (same values for both) and Type N mortars. Value of modulus of rupture for different masonry and mortar types are considerably different, so designers should carefully choose their values corresponding to the mortar type specified or to be used for a particular job.

Masonry beams typically are parts of masonry walls that are solid grouted. Therefore, in the context of beams, the value of modulus of rupture for solid grouted masonry should always be used. For partially grouted masonry, the modulus of rupture values can be obtained from linear interpolation between grouted hollow units and ungrouted hollow units, based on amount (percentage) of grouting.

4.6.2 Nominal Cracking Moment of Masonry Beams

The *nominal cracking moment* of a masonry beam is defined as its flexural strength without reinforcement (i.e., an unreinforced masonry beam). MSJC Section 3.3.4.2.2.2 requires that the nominal strength M_n of a reinforced beam be not less than 1.3 *times* the nominal cracking moment strength of the beam (M_{cr}) calculated based on modulus of rupture (i.e., the strength of an unreinforced or plain masonry beam). This relationship can be stated as

$$\text{(Strength of a reinforced masonry beam)} \not< [1.3(\text{strength of a plain masonry beam})] \tag{4.49}$$

For design purposes, Eq. (4.49) can be restated as Eq. (4.50):

$$M_n \geq 1.3M_{cr} \tag{4.50}$$

This requirement is intended to prevent brittle failures of flexural elements. Such a possibility exists where a beam may be so lightly reinforced that bending moment required to cause yielding of the reinforcement is less than the cracking moment (i.e., moment required to causing cracking). See Example 4.11.

The moment strength in this case is calculated based on the section properties of an uncracked section. Therefore, the value of nominal cracking moment can be calculated based on the convention flexure formula for a beam of homogeneous materials as follows:

$$M_{cr} = \frac{f_r y}{I} \tag{4.51}$$

where M_{cr} = nominal cracking moment of a masonry beam
f_r = modulus of rupture
y = distance of extreme fibers in tension measured from the neutral axis
I = moment of inertia of the gross cross section

The moment of inertia of a rectangular section having width b and total depth h is given by Eq. (4.52):

$$I = \frac{bh^3}{12} \tag{4.52}$$

For a rectangular section, $y = h/2$. Therefore, Eq. (4.51) can be expressed as

$$M_{cr} = \frac{f_r(bh^3/12)}{(h/2)} = f_r\left(\frac{bh^2}{6}\right) \tag{4.53}$$

Equation (4.53) can be expressed as

$$M_{cr} = f_r S \tag{4.54}$$

where S = section modulus of the gross or uncracked rectangular section = $bh^2/6$.

Examples 4.9 to 4.11 illustrate the calculation of nominal cracking moments of reinforced masonry beams and related code applications.

Example 4.9 A nominal 10 × 40-in. concrete masonry beam built from lightweight CMU is reinforced with two No. 6 Grade 60 bars for tension (Fig. E4.9). The centroid of reinforcement is located 6 in. from the bottom of the beam. The grout weight is 140 lb/ft^3. f'_m = 1500 psi. Check if the beam is adequate to carry the imposed loads. Calculate the nominal cracking moment of this beam. The beam consists of solid grouted masonry in running bond laid in Type S portland cement mortar. Does the beam satisfy the code requirement?

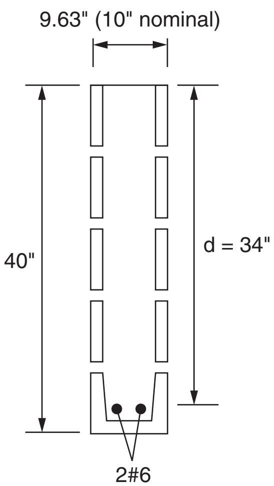


FIGURE E4.9

Solution

For a nominal 10 × 40 in. concrete masonry beam, b = 9.63 in., h = 40 in. From Eq. (4.53), the cracking moment of the beam is

$$M_{cr} = f_r\left(\frac{bh^2}{6}\right) \tag{4.53 repeated}$$

The modulus of rupture, f_r = 200 lb/in.^{2} for solid grouted masonry in running bond laid in Type S portland cement mortar (MSJC Table 3.1.8.2.1). Therefore,

$$M_{cr} = (0.20)\frac{(9.63)(40)^2}{6} = 531.6 \text{ k-in.} = 42.8 \text{ k-ft}$$

$$1.3M_{cr} = 1.3(42.8) = 55.64 \text{ k-ft}$$

The nominal strength of this beam, M_n, of this beam was calculated to be 139.55 kip-ft in Example 4.2 (calculations not repeated here), which is greater than $1.3M_{cr}$. Therefore, the beam satisfies the *Code* requirements for nominal cracking moment strength of the beam.

Example 4.10 A two-wythe 8 × 24 in. clay brick beam has an effective depth of 20 in. It is reinforced with one No. 8 Grade 60 bar for tension (Fig. E4.10). Determine the cracking moment of this beam. f'_m = 2500 psi. Portland cement Type *S* mortar would be used for construction. Does the beam satisfy the *Code* requirements?

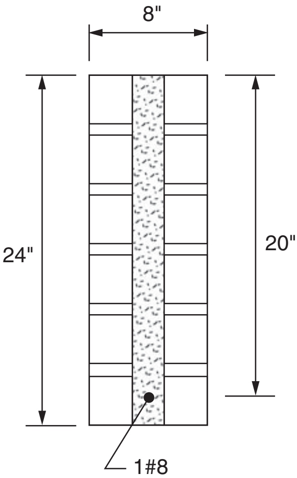


FIGURE E4.10 Beam cross section for Example 4.10.

Solution

For a nominal 8- × 24-in. clay brick beam, $b = 8.0$ in., $h = 24$ in. From Eq. (4.53) the nominal cracking moment of the beam is

$$M_{cr} = f_r\left(\frac{bh^2}{6}\right) \qquad (4.53\text{ repeated})$$

The modulus of rupture, f_r = 200 lb/in.^{2} for solid grouted masonry in running bond laid in Type S portland cement mortar (MSJC Table 3.1.8.2.1). Therefore,

$$M_{cr} = (0.20)\frac{(8.0)(24)^2}{6} = 153.6 \text{ k-in.} = 12.8 \text{ k-ft}$$

$$1.3M_{cr} = 1.3(12.8) = 16.64 \text{ k-ft}$$

The nominal strength of this beam, ϕM_n, of this beam was calculated to be 65.83 kip-ft in Example 4.5 (calculations not repeated here). Therefore,

$$M_n = \frac{\phi M_n}{\phi} = \frac{65.83}{0.9} = 73.14 \text{ k-ft} > 1.3M_{cr} = 16.64 \text{ k-ft}$$

The nominal strength of the beam (73.14 k-ft) is *greater* than 1.3 times the nominal cracking moment of the beam (16.64 k-ft). Therefore, this beam *satisfies* the code requirements for nominal cracking moment strength of the beam.

Example 4.11 Check if the beam described in Example 4.10 would satisfy the code requirements for cracking moment if it were reinforced with (a) one No. 3 Grade 60 bar instead of one No. 8 bar, (b) one No. 4 bar instead of one No. 8 Grade 60 bar. f'_m = 2500 psi, f_y = 60 ksi.

Solution

The cracking moment of the beam, M_{cr}, remains the same as calculated in Example 4.10 as its value is independent of the amount of tensile reinforcement, M_{cr} = 12.8 k-ft.

$$1.3M_{cr} = 1.3(12.8) = 16.64 \text{ k-ft}$$

Calculate the nominal strength of the beam, ϕM_n.

a. Beam reinforced with one No. 3 Grade 60 bar.

$b = 8$ in., $d = 20$ in., $A_s = 0.11$ in.^{2} (one No. 3 bar) $f'_m = 2500$ psi, $f_y = 60$ ksi.

Assume that the tension reinforcement has yielded (to be verified later). Calculate from Eq. (4.9) the depth a of the compression block.

$$a = \frac{A_s f_y}{0.80 f'_m b} \qquad (4.9\text{ repeated})$$

$$a = \frac{(0.11)(60)}{0.80(2.5)(8.00)} = 0.41 \text{ in.}$$

From Eq. (4.13)

$$\phi M_n = \phi A_s f_y \left(d - \frac{a}{2}\right)$$
$$= 0.9(0.11)(60)\left(20 - \frac{0.41}{2}\right)$$
$$= 117.58 \text{ k-in.} = 9.8 \text{ k-ft}$$

Verify that reinforcement has yielded and $\varepsilon_s \geq 1.5\varepsilon_y$ From Eq. 4.5,

$$c = \frac{a}{0.8} = \frac{0.41}{0.8} = 0.51 \text{ in.}$$

$$\frac{c}{d} = \frac{0.51}{20.0} = 0.026 < 0.538 \qquad \text{(Eq. 4.34)}$$

Hence, steel has yielded and the assumption $\varepsilon_s \geq 1.5\varepsilon_y$ is valid.

$$\phi M_n = 9.8 \text{ k-ft}$$

$$M_n = \frac{\phi M_n}{\phi} = \frac{9.8}{0.9} = 10.89 \text{ k-ft} < 1.3 M_{\text{cr}} = 16.64 \text{ k-ft}$$

The nominal strength of the beam (10.89 k-ft) is *less* than 1.3 times the cracking moment of the beam (16.64 k-ft). Therefore, this beam does *not* meet the code requirements for the nominal cracking moment strength of the beam.

b. Beam reinforced with one No. 4 Grade 60 bar.

$b = 8$ in., $d = 20$ in., $A_s = 0.20$ in.^{2} (one No. 4 bar) $f'_m = 2500$ psi, $f_y = 60$ ksi.

Assume that tension reinforcement has yielded and $\varepsilon_s \geq 1.5\varepsilon_y$ (to be verified later). Calculate from Eq. (4.9) the depth a of the compression block.

$$a = \frac{A_s f_y}{0.80 f'_m b} \qquad \text{(4.9 repeated)}$$

$$a = \frac{(0.20)(60)}{0.80(2.5)(8.00)} = 0.75 \text{ in.}$$

From Eq. (4.13)

$$\phi M_n = \phi A_s f_y \left(d - \frac{a}{2}\right)$$
$$= 0.9(0.20)(60)\left(20 - \frac{0.75}{2}\right)$$
$$= 211.95 \text{ k-in.} = 17.66 \text{ k-ft}$$

Verify that reinforcement has yielded and $\varepsilon_s \geq 1.5\varepsilon_y$. From Eq. (4.5),

$$c = \frac{a}{0.8} = \frac{0.75}{0.8} = 0.94 \text{ in.}$$

$$\frac{c}{d} = \frac{0.94}{20.0} = 0.047 < 0.538 \qquad \text{(4.34 repeated)}$$

Hence, steel has yielded and the assumption $\varepsilon_s \geq 1.5\varepsilon_y$ is valid.

$$\phi M_n = 17.66 \text{ k-ft}$$

$$M_n = \frac{\phi M_n}{\phi} = \frac{17.66}{0.9} = 19.62 \text{ k-ft} > 1.3M_{\text{cr}} = 16.64 \text{ k-ft}$$

The nominal strength of the beam (19.62 k-ft) is *greater* than 1.3 times the cracking moment of the beam (16.64 k-ft). Therefore, this beam *satisfies* the code requirements for the nominal cracking moment strength of the beam.

4.7 DESIGN OF MASONRY BEAMS

4.7.1 Ductility in Reinforced Concrete Beams: Balanced, Underreinforced, and Overreinforced Beams

Analysis of reinforced masonry beams presented in the preceding section is based on the premise that crushing of masonry and yielding of steel reinforcement in the beam occur simultaneously, the strain in masonry being ε_{mu} and the strain in the reinforcement as ε_y. Such a state of strain is defined as a *balanced condition*, and the corresponding reinforcement ratio is referred to as the *balanced steel ratio*, ρ_b. In a balanced beam, crushing of masonry would occur without a warning, resulting in a catastrophic failure. Therefore, it is highly desirable to provide an upper limit on the reinforcement ratio to ensure some degree of ductility in beams, that is, to cause the steel reinforcement to yield before the onset of crushing of masonry. Such beams are called *underreinforced* beams. Failures of such beams are characterized by excessive deflection accompanied by strains in steel reinforcement beyond the yield strain, ε_y, thus exhibiting ductile behavior. This type of behavior provides ample warning before the compression failure of masonry occurs. Accordingly, only underreinforced beams are permitted by the design codes. For practical purposes, the balanced beam should be considered only as a hypothetical beam.

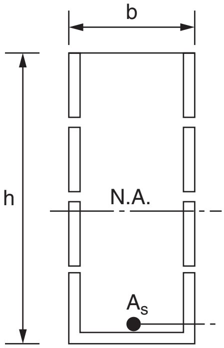
4.7.2 Determination of Balanced Steel Ratio, ρ_b

Balanced condition is defined by a set of two strain values: strain in masonry at the time of crushing, ε_{mu}, and the yield strain in steel, ε_y. Crushing strain in masonry is set at 0.0025 for concrete masonry and 0.0035 for clay masonry [MSJC *Code* Section 3.3.2.(c)]. The value of the balanced reinforcement ratio, ρ_b, can be derived in terms of the design strengths of concrete and reinforcement. It is assumed that the neutral axis can be located by the linear strain relationship as shown in Fig. 4.6. Based on this assumption, the relationship between strains in masonry and reinforcement was derived earlier as given by Eq. (4.14):

$$\frac{\varepsilon_s}{\varepsilon_{mu}} = \frac{d-c}{c} \qquad \text{(4.14 repeated)}$$

Equation (4.14) can be rearranged and expressed as

$$\frac{c}{d} = \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \varepsilon_s} \qquad (4.55)$$



(a)

FIGURE 4.6A Strain distribution diagram for balanced conditions in a reinforced concrete masonry beam.

The expressions for the balanced ratios for concrete and clay masonry can be derived form Eq. (4.55) as follows:

1. *Concrete masonry*:

$$\varepsilon_{mu} = 0.0025 \tag{4.56}$$

Substituting ε_{mu} = 0.0025 for concrete masonry, $\varepsilon_s = \varepsilon_y$ for strain in steel reinforcement, and writing $c = c_b$ in Eq. (4.55), we obtain the relationship between the depth of neutral axis and the effective beam depth at the balanced conditions as given by Eq. (4.57):

$$\frac{c_b}{d} = \frac{0.0025}{0.0025 + \varepsilon_y} \tag{4.57}$$

where c_b = depth of neutral axis under the balanced condition, measured from the extreme compression fibers.

Substitute in Eq. (4.57) for yield strain ε_y in terms of yield stress f_y from Hooke's law as expressed by Eq. (4.16):

$$\varepsilon_y = \frac{f_y}{E_s} \tag{4.16 repeated}$$

The resulting expression is

$$\frac{c_b}{d} = \frac{0.0025}{0.0025 + \dfrac{f_y}{29 \times 10^6}} \tag{4.58}$$

where f_y = yield strength of steel reinforcement, lb/in.^{2}
E_s = modulus of elasticity of steel = 29×10^6 lb/in.^{2}

Simplification of Eq. (4.58) yields

$$c_b = \left(\frac{72{,}500}{72{,}500 + f_y}\right)d \tag{4.59}$$

By definition,

$$c_b = \frac{a}{0.8} \tag{4.5 repeated}$$

From Eq. (4.9),

$$a = \frac{A_s f_y}{0.80 f'_m b} \tag{4.9 repeated}$$

Multiplying both the numerator and the denominator of Eq. (4.9) by d, we obtain

$$a = \left(\frac{A_s}{bd}\right)\left(\frac{f_y d}{0.80 f'_m}\right) \tag{4.60}$$

Substitution of $\rho = A_s/bd$ from Eq. (4.37) in Eq. (4.60) yields

$$a = \frac{\rho f_y d}{0.80 f'_m} \tag{4.61}$$

Combining Eqs. (4.5) and (4.61), we obtain

$$c_b = \frac{\rho f_y d}{0.80(0.80 f'_m)} \tag{4.62}$$

Equating the values of c_b from Eqs. (4.59) and (4.62), we obtain

$$\left(\frac{\rho f_y}{0.80(0.80) f'_m}\right) d = \left(\frac{72{,}500}{72{,}500 + f_y}\right) d \tag{4.63}$$

Substitution of $\rho = \rho_b$ in Eq. (4.63) yields

$$\rho_b = \left(\frac{72{,}500}{72{,}500 + f_y}\right)\left(\frac{0.64 f'_m}{f_y}\right) \tag{4.64}$$

Equation (4.64) gives the value of the balanced steel ratio, ρ_b, for concrete masonry. Values of ρ_b and $0.75\rho_b$ for concrete masonry for various combinations of f'_m and f_y are tabulated in Table 4.5.

TABLE 4.5 Values of ρ_b and ρ_{max} for Practical Combinations of f'_m and f_y for Concrete Masonry ($\varepsilon_{mu} = 0.0025$)

	$f_y = 60{,}000$ psi				$f_y = 40{,}000$ psi				
f'_m psi	ρ_b Eq. (4.64)	$0.75\rho_b$	ρ_{max} Eq. (4.75)	0.75 ρ_b/ρ_{max}	ρ_b Eq. (4.64)	$0.75\rho_b$	ρ_{max} Eq. (4.75)	0.75 ρ_b/ρ_{max}	f'_m psi
1500	0.0088	0.0066	0.0071	0.920	0.0155	0.0116	0.0131	0.883	1500
1800	0.0105	0.0079	0.0086	0.920	0.0186	0.0139	0.0158	0.883	1800
2000	0.0117	0.0088	0.0095	0.920	0.0206	0.0155	0.0175	0.883	2000
2500	0.0146	0.0109	0.0119	0.920	0.0258	0.0193	0.0219	0.883	2500
3000	0.0175	0.0131	0.0143	0.920	0.0309	0.0232	0.0263	0.883	3000
3500	0.0204	0.0153	0.0167	0.920	0.0361	0.0271	0.0306	0.883	3500
4000*	0.0233	0.0175	0.0190	0.920	0.0412	0.0309	0.0350	0.883	4000

*Values for $f'_m > 4000$ psi are not listed because of the *Code's* upper limit on the value of f'_m for concrete masonry.

2. *Clay masonry*: Following the above procedure, we can derive an expression for the balanced steel ratio for clay masonry as follows. For clay masonry,

$$\varepsilon_{mu} = 0.0035 \tag{4.65}$$

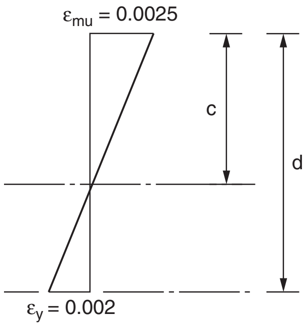


FIGURE 4.6B Strain distribution diagram for balanced conditions in a reinforced clay masonry beam.

From the strain distribution diagram shown in Fig. 4.6B., we obtain Eq. (4.66) for clay masonry:

$$\frac{c_b}{d} = \frac{0.0035}{0.0035 + \varepsilon_y} \tag{4.66}$$

Substitution for yield strain in steel in terms of the yield stress [from Eq. (4.16)] in Eq. (4.66) yields

$$\frac{c_b}{d} = \frac{0.0035}{0.0035 + \dfrac{f_y}{29 \times 10^6}} \tag{4.67}$$

where f_y = yield strength of steel reinforcement, lb/in.^{2}

E_s = modulus of elasticity of steel = 29 × 10^6 lb/in.^{2}

Simplification of Eq. (4.67) yields

$$c_b = \left(\frac{101{,}500}{101{,}500 + f_y}\right) d \tag{4.68}$$

With substitutions for c_b and a, as in the preceding derivation for concrete masonry, we obtain

$$\rho_b = \left(\frac{101{,}500}{101{,}500 + f_y}\right)\left(\frac{0.64 f'_m}{f_y}\right) \tag{4.69}$$

Equation (4.69) gives the balanced steel ratio, ρ_b, for clay masonry. Values of ρ_b and $0.75\rho_b$ for clay masonry for various combinations of f'_m and f_y are listed in Table 4.6.

In general, a beam having a reinforcement ratio $\rho = \rho_b$, the beam is called a *balanced* beam. When $\rho < \rho_b$, a beam is defined as an *underreinforced*, whereas when $\rho > \rho_b$, a beam is defined as an *overreinforced* beam.

It is important to recognize that because of the excessive amount of tensile reinforcement present ($\rho > \rho_b$), an overreinforced beam would fail in a brittle manner, characterized by sudden compression failure of masonry and without yielding of tensile reinforcement. An underreinforced beam, on the other hand, will fail in a ductile manner, characterized by yielding of tensile reinforcement prior to crushing of masonry in the compression zone. Appendix Tables A.11 and A.12 lists values of $0.375\rho_b$, $0.50\rho_b$, and ρ_{max} for several practical combinations of values of f'_m and f_y for concrete and clay masonry, respectively, which are useful for design purposes.

4.7.3 Minimum and Maximum Tensile Reinforcement in Beams

4.7.3.1 Minimum Reinforcement Requirements The *Code* provisions for limiting minimum reinforcement in beams are intended to ensure a minimum amount of ductility in beams and to prevent brittle failures. The *Code* requires that the nominal strength, M_n, of a beam be not less than 1.3 times the cracking moment of a beam calculated on the basis of moment of inertia of gross section (MSJC-08 Section 3.3.4.2.2.2). In some cases, a beam may be only lightly reinforced so that the reinforcement would yield under a nominal moment value less than the cracking moment of the beam. See Examples 4.9 and 4.10.

TABLE 4.6 Values of ρ_b, 0.75 ρ_b, and ρ_{max} for Practical Combinations of f'_m and f_y for Clay Masonry ($\varepsilon_{mu} = 0.0035$).

	f_y = 60,000 psi				f_y = 40,000 psi				
f'_m psi	ρ_b Eq. (4.69)	$0.75\rho_b$	ρ_{max} Eq. (4.76)	0.75 ρ_b/ρ_{max}	ρ_b Eq. (4.69)	$0.75\rho_b$	ρ_{max} Eq. (4.76)	0.75 ρ_b/ρ_{max}	f'_m psi
1500	0.0101	0.0075	0.0085	0.889	0.0172	0.0129	0.0151	0.856	1500
1800	0.0121	0.0091	0.0102	0.889	0.0207	0.0155	0.0181	0.856	1800
2000	0.0134	0.0101	0.0113	0.889	0.0230	0.0172	0.0201	0.856	2000
2500	0.0168	0.0126	0.0141	0.889	0.0287	0.0215	0.0251	0.856	2500
3000	0.0201	0.0151	0.0170	0.889	0.0344	0.0258	0.0302	0.856	3000
3500	0.0235	0.0176	0.0198	0.889	0.0402	0.0301	0.0352	0.856	3500
4000	0.0268	0.0201	0.0226	0.889	0.0459	0.0344	0.0402	0.856	4000
4500	0.0302	0.0226	0.0254	0.889	0.0516	0.0387	0.0453	0.856	4500
5000	0.0335	0.0251	0.0283	0.889	0.0574	0.0430	0.0503	0.856	5000
5500	0.0369	0.0277	0.0311	0.889	0.0631	0.0473	0.0553	0.856	5500
6000	0.0402	0.0302	0.0339	0.889	0.0689	0.0516	0.0603	0.856	6000

An approximate value of the minimum area of reinforcement required for masonry beams can be calculated by equating the expression for 1.3 times the nominal cracking moment ($1.3M_{cr}$) to the nominal strength of a beam as follows.

$$M_{cr} = f_r\left(\frac{bh^2}{6}\right) \qquad \text{(4.53 repeated)}$$

Assuming $h = 1.2d$, the nominal cracking moment can be expressed as

$$M_{cr} = f_r\left(\frac{b(1.2d)^2}{6}\right) = 0.24 f_r bd^2$$

$$1.3M_{cr} = (1.3)(0.24 f_r bd^2) = 0.312 f_r bd^2$$

For a reinforced masonry beam, the nominal strength can be expressed from Eq. (4.12):

$$M_n = A_s f_y\left(d - \frac{a}{2}\right) \qquad \text{(4.12 repeated)}$$

Assuming $(d - a/2) \approx 0.95d$, Eq. (4.12) can be expressed as

$$M_n = 0.95 A_s f_y d$$

Equating M_n to $1.3M_{cr}$ yields

$$0.95 A_s f_y = 0.312 f_r bd^2$$

whence

$$\frac{A_s}{bd} = 0.3284\left(\frac{f_r}{f_y}\right)$$

For $f_r = 200$ psi and $f_y = 60{,}000$ psi, the above expression yields

$$\rho_{\min} = \frac{A_s}{bd} = 0.3284\left(\frac{200}{60{,}000}\right) = 0.001097 \approx 0.0011$$

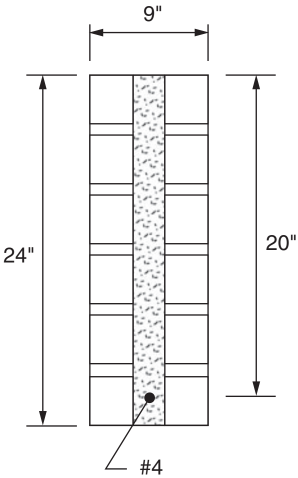


FIGURE E4.12 Beam cross section for Example 4.12.

The above result means that a reinforced beam, with $\rho = 0.0011$ would have its nominal strength equal to 1.3 times the nominal cracking moment. Accordingly, $\rho = 0.0011$ may be considered as the minimum reinforcement ratio for a reinforced masonry beam. See Example 4.12.

Example 4.12 A 9- × 24-in. brick beam is reinforced with a No. 4 Grade 60 bars placed at 20 in. from the compression face of the beam (Fig. E4.12). Calculate for this beam (a) the nominal cracking moment, (b) nominal moment strength, and (c) check if the beam complies with the code requirements. $f'_m = 1500$ psi.

Solution

Given: $b = 9$ in., $h = 24$ in., $d = 20$ in. $f_r = 200$ psi (MSJC-08 Table 3.1.8.2.1). $A_s = 0.20$ in.^{2}.

a. Calculate nominal cracking moment of the beam from Eq. (4.54):

$$M_{cr} = f_r\left(\frac{bh^2}{6}\right) = (200)\left(\frac{(9)(24)^2}{6}\right)\left(\frac{1}{12{,}000}\right) = 14.4 \text{ k-ft}$$

b. Calculate the nominal moment strength of the beam. From Eq. (4.9)

$$a = \frac{A_s f_y}{0.80 f'_m b} = \frac{(0.20)(60)}{0.80(1.5)(9)} = 1.11 \text{ in.}$$

From Eq. (4.12), the nominal strength of the beam is

$$M_n = A_s f_y\left(d - \frac{a}{2}\right) = (0.20(60)\left(20 - \frac{1.11}{2}\right)\left(\frac{1}{12}\right) = 19.45 \text{ k-ft}$$

c. $M_n = 19.45$ k-ft $> 1.3M_{cr} = 1.3(14.4) = 18.72$ k-ft. Therefore, the beam complies with the code requirement. Note that $\rho = A_s/bd = 0.20/(9)(20) = 0.0011$.

4.7.3.2 Maximum Reinforcement Requirements The code provisions for maximum reinforcement in beams are intended to ensure that the reinforcement would yield *sufficiently* so that the beam would exhibit a ductile behavior.

MSJC-08 Section 3.3.3.5.1 requires that for masonry members where the quantity $M_u/V_u d_v \geq 1$, the cross-sectional area of reinforcement shall not exceed the area required to maintaining a strain gradient corresponding to a strain in the reinforcing bar nearest the tension face of the member equal to 1.5 times the yield strain (i.e., $1.5\varepsilon_y$) and a maximum strain in masonry specified in MSJC Section 3.3.2(c) (i.e., $\varepsilon_{mu} = 0.0025$ for concrete masonry and 0.0035 for clay masonry). This code provision is intended to ensure that masonry compressive strains will not exceed their ultimate values, and that the compressive zone of the member would not crush before the tensile reinforcement develops inelastic strain consistent with the curvature ductility requirements.

Expressions for reinforcement ratio corresponding to 1.5 times the yield strain ($1.5\varepsilon_y$) in reinforcement can be obtained from compatibility of strains in masonry and reinforcement. We define the value of this reinforcement ratio as $\rho_{\max}$ for flexural elements. Figure 4.7 shows the strain compatibility diagram wherein strain in reinforcement has been shown to be $1.5\varepsilon_y$ and the strain in masonry as ε_{mu}, from which the following relationship is obtained:

$$\frac{c}{d} = \frac{\varepsilon_{mu}}{\varepsilon_{mu} + 1.5\varepsilon_y} \tag{4.70}$$

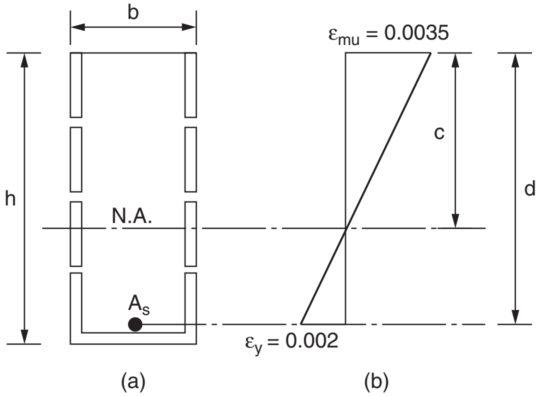


FIGURE 4.7 Strain compatibility diagram when $\varepsilon_m = \varepsilon_{mu}$ and $\varepsilon_s = 1.5\varepsilon_y$.

Equation (4.70) can be simplified and expressed as Eq. (4.71):

$$c = \left(\frac{\varepsilon_{mu}}{\varepsilon_{mu} + 1.5\varepsilon_y}\right) d \tag{4.71}$$

The relationship between the depths of compression block, a, and that of neutral axis, c, was obtained earlier in Eq. (4.9):

$$c = \frac{a}{0.8} \qquad \text{(4.9 repeated)}$$

The following relationship was derived earlier based on the force equilibrium

$$a = \frac{\rho f_y d}{0.80 f'_m} \qquad \text{(4.61 repeated)}$$

Equation (4.62) was obtained earlier for the depth of neutral axis corresponding to the balanced condition:

$$c_b = \frac{\rho f_y d}{0.80(0.80 f'_m)} \qquad \text{(4.62 repeated)}$$

Combining Eqs. (4.62) and (4.71), and substituting $d = d_t$ (where d_t = the distance from the extreme compression fibers to the tensile reinforcing bar closest to the tension face of the beam), we obtain

$$\frac{\rho f_y d_t}{0.80(0.80 f'_m)} = \left(\frac{\varepsilon_{mu}}{\varepsilon_{mu} + 1.5\varepsilon_y}\right) d_t \tag{4.72}$$

Substitution of $\rho = \rho_{\max}$ in Eq. (4.72) gives a general expression for the reinforcement ratio corresponding to 1.5 times the yield strain ε_y in steel:

$$\rho_{\max} = \left(\frac{\varepsilon_{mu}}{\varepsilon_{mu} + 1.5\varepsilon_y}\right)\left(\frac{0.64 f'_m}{f_y}\right) \tag{4.73}$$

Values of $\rho_{\max}$ depend on the values of strain in masonry, ε_{mu} (= 0.0025 for concrete masonry and 0.0035 for clay masonry). For concrete masonry, substitution of $\varepsilon_{mu} = 0.0025$ and $\varepsilon_y = f_y / E_s$, and $E_s = 29 \times 10^6$ psi in Eq. (4.73) yields

$$\rho_{\max} = \left(\frac{0.0025}{0.0025 + \dfrac{1.5 f_y}{29 \times 10^6}}\right)\left(\frac{0.64 f'_m}{f_y}\right) \tag{4.74}$$

Equation (4.74) can be simplified and expressed as Eq. (4.75):

$$\rho_{\max} = \left(\frac{72{,}500}{72{,}500 + 1.5 f_y}\right)\left(\frac{0.64 f'_m}{f_y}\right) \tag{4.75}$$

An expression similar to Eq. (4.75) can be obtained for clay masonry by substituting ε_{mu} = 0.0035 in Eq. (4.73):

$$\rho_{\max} = \left(\frac{101{,}500}{101{,}500 + 1.5 f_y}\right)\left(\frac{0.64 f'_m}{f_y}\right) \tag{4.76}$$

Tables 4.5 and 4.6 give the values of $\rho_{\max}$ determined form Eqs. (4.75) and (4.76) for concrete and clay masonry, respectively, for various combinations of f'_m and f_y.

It is important to understand the significance of Eqs. (4.75) and (4.76). They give the maximum reinforcement ratios for concrete and clay masonry, respectively. These limits represent threshold values that may not be exceeded if a beam is to remain ductile according the code. In cases where a beam is reinforced excessively (i.e., $\rho > \rho_{max}$), the design would not qualify as acceptable according the MSJC *Code* even though its nominal strength may be considerable. The reason: such a beam does not posses ductility; the masonry would fail by premature crushing without sufficient yielding of reinforcement. The value of the strength reduction factor, $\phi = 0.9$, associated with flexure, is intended to be used for ductile members, that is, only when $\rho < \rho_{\max}$, which ensures that $\varepsilon_s \geq 1.5\varepsilon_y$. Example 4.13 illustrates the intent of this code requirement.

Example 4.13 A nominal 8 × 24 in. concrete masonry beam is reinforced with one No. 9 Grade 60 reinforcing bar placed at d = 20 in. for tension (Fig. E4.13). The beam is required to carry a uniform service dead load of 1.0 k/ft (including its self-weight) and a service live load of 1.75 k/ft over an effective span of 12 ft. f'_m = 1500 psi.

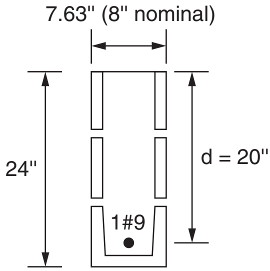


FIGURE E4.13 Beam cross section for Example 4.13.

a. Check if the beam is adequate to carry the load.
b. During the construction of this beam, it was found that No. 9 bar was not available and it was decided to use either one No. 10 or one No. 11 bar with the (erroneous) assumption that larger amount of steel would only make the beam stronger. Determine if the beam would be satisfactory using either No. 10 or No. 11 bar in lieu of one No. 9 bar provided for in the design.

Solution

a. Calculate the required moment strength moment, M_u.

$$D = 1.0 \text{ k/ft}, L = 1.75 \text{ k/ft}, L = 12 \text{ ft}$$

Load combinations:

1. $U = 1.4D = 1.4(1.0) = 1.4$ k/ft
2. $U = 1.2D + 1.6L = 1.2(1.0) + 1.6(1.75) = 4.0$ k/ft (governs)

$$M_u = \frac{w_u L^2}{8} = \frac{(4.0)(12)^2}{8} = 72 \text{ k-ft}$$

Calculate M_n assuming that reinforcement has yielded so that $f_s \geq f_y$.

$$b = 7.63 \text{ in. (8 in. nominal)}, d = 20 \text{ in.}, A_s = 1.0 \text{ in}^2 \text{ (one No. 9 bar).}$$

$$a = \frac{A_s f_y}{0.80 f'_m b} \qquad \text{(4.9 repeated)}$$

$$a = \frac{(1.0)(60)}{0.80(1.5)(7.63)} = 6.55 \text{ in.}$$

From Eq. (4.12)

$$M_n = A_s f_y \left(d - \frac{a}{2} \right)$$
$$= (1.0)(60)\left(20 - \frac{6.55}{2} \right)$$
$$= 1003.5 \text{ k-in.} = 83.63 \text{ k-ft}$$

Check the $M_u/V_u d_v$ ratio.

$$V_u = \frac{w_u L}{2} = \frac{(4.0)(12.0)}{2} = 24 \text{ kips}$$

$$d_v = d = 20 \text{ in.}$$

$$\frac{M_u}{V_u d_v} = \frac{(72)(12)}{(24)(20)} = 1.8 > 1.0$$

Verify from Eq. (4.30) that the tension reinforcement has yielded and that $\varepsilon_s \geq 1.5\varepsilon_y$. From Eq. (4.5),

$$c = \frac{a}{0.8} = \frac{6.55}{0.8} = 8.19 \text{ in.}$$

$$\frac{c}{d} = \frac{8.19}{20} = 0.41 < 0.454 \qquad \text{(4.30 repeated)}$$

Hence, steel has yielded and $\varepsilon_s \geq 1.5\varepsilon_y$. Check if the beam is ductile, that is, $\rho \leq 0.75\rho_b$.

$$\rho = \frac{A_s}{bd} = \frac{1.0}{(7.63)(20)} = 0.0066$$

From Table 4.5, for concrete masonry with f'_m = 1500 psi and Grade 60 reinforcement, $0.75\rho_b = 0.0066$, which is exactly equal to ρ_{provided}.
Therefore,

$$\phi = 0.9,$$

and

$$\phi M_n = (0.9)(83.63) = 75.27 \text{ k-ft} > M_u = 72 \text{ k-ft.}$$

Check that $M_n \nless 1.3M_{\text{cr}}$.

$$M_{\text{cr}} = f_r\left(\frac{bh^2}{6}\right) = 200\left(\frac{(7.63)(24)^2}{6}\right) = 146{,}496 \text{ in-lb} = 12.21 \text{ k-ft}$$

$$1.3M_{cr} = 1.3(12.21) = 15.87 \text{ k-ft}$$

$$M_n = 100.6 \text{ k-ft} > 1.3M_{cr} = 15.87 \text{ k-ft} \qquad \text{OK}$$

The beam is adequate to carry the imposed loads.

b. Check the nominal strength of the beam reinforced with one No. 10 bar. A_s = 1.27 in.^{2}. Assume that the reinforcement has yielded.

$$a = \frac{(1.27)(60)}{0.80(1.5)(7.63)} = 8.32 \text{ in.}$$

$$M_n = A_s f_y\left(d - \frac{a}{2}\right)$$

$$= (1.27)(60)\left(20 - \frac{8.32}{2}\right)$$

$$= 1207 \text{ k-in.} = 100.58 \text{ k-ft}$$

Verify from Eq. (4.30) that reinforcement has yielded and $\varepsilon_s \geq 1.5\varepsilon_y$. From Eq. (4.5),

$$c = \frac{a}{0.8} = \frac{8.32}{0.8} = 10.4 \text{ in.}$$

$$\frac{c}{d} = \frac{10.4}{20} = 0.52 > 0.454 \qquad \text{(4.30 repeated)}$$

Hence, $\varepsilon_s < 1.5\varepsilon_y$. Check if the beam is ductile, that is, $\rho \le 0.75\rho_b$.

$$\rho = \frac{A_s}{bd} = \frac{1.27}{(7.63)(20)} = 0.0083$$

From Table 4.5, for concrete masonry with f'_m = 1500 psi and Grade 60 reinforcement, $0.75\rho_b = 0.0066$, which is less than ρ_{provided}. (= 0.0083). Therefore, the beam is *not* ductile.

The beam design is not acceptable as it does not comply with the code requirements.

c. Check the nominal strength of the beam reinforced with one No. 11 bar. A_s = 1.56 in.^{2}. Assume that the reinforcement has yielded.

$$a = \frac{(1.56)(60)}{0.80(1.5)(7.63)} = 10.22 \text{ in.}$$

$$\begin{aligned} M_n &= A_s f_y\left(d - \frac{a}{2}\right) \\ &= (1.56)(60)\left(20 - \frac{10.22}{2}\right) \\ &= 1393.7 \text{ k-in.} = 116.14 \text{ k-ft} \end{aligned}$$

Verify from Eq. (4.30) that reinforcement has yielded and $\varepsilon_s \ge 1.5\varepsilon_y$. From Eq. (4.5),

$$c = \frac{a}{0.8} = \frac{10.22}{0.8} = 12.78 \text{ in.}$$

$$\frac{c}{d} = \frac{12.78}{20} = 0.64 > 0.454 \qquad \text{(4.34 repeated)}$$

Hence, $\varepsilon_s < 1.5\varepsilon_y$. Check the reinforcement ratio.

$$\rho = \frac{A_s}{bd} = \frac{1.56}{(7.63)(20)} = 0.0102 > \rho_{\max} = 0.0088$$

This beam is overreinforced and not acceptable because it does not comply with the code requirements. It is further noted that No. 11 bar is not permitted by the MSJC *Code* when using strength design philosophy for design of reinforced masonry.

4.8 PROCEDURE FOR FLEXURAL DESIGN OF BEAMS

4.8.1 General Procedure

Beam design is essentially a trial-and-error problem. Several examples that illustrate procedure for designing reinforced masonry beams, such as lintels, are presented in this section.

The design process begins with certain given information. In reinforced masonry, the beam width *b* is usually known because the beam is generally a part of a masonry wall (e.g., a wall above an opening). The beam width would typically be the actual width of the CMU (which is smaller than the nominal width) or the actual width of the brick. It is a common practice to use Grade 60 reinforcement because most research is based on tests on masonry using Grade 60 reinforcement. The strength of masonry, CMU or brick is specified according the specifications of a job. With this information given, a designer encounters two types of problems in design:

1. Determination of the amount of reinforcement when the depth of the beam is also known.
2. Determination of both the depth of the beam and the amount of reinforcement.

In the first case, when all the other information is given and the amount of reinforcement is to be determined, the problem is rather simple. Example 4.14 illustrates the procedure for determining the required amount of reinforcement when everything else is known. In the second case, when both the beam depth and the amount of reinforcement are to be determined, the problem is a bit more laborious. Examples 4.15 and 4.16 illustrate the procedure for determining both the required depth of the beam and amount of reinforcement when everything else is known.

In cases where the depth of the beam is unknown (as is usually the case in design), it is necessary to guess the nominal depth (h) of the beam for the initial trial, estimate its dead weight, and proceed with deign. Although no firm guidelines can be established to estimate a reasonable depth of a beam, one can begin with an assumed nominal beam depth greater than or equal to 8 in., which is the minimum nominal depth requirement for a beam [MSJC-08 Section 3.3.4.2.5(b)]. Therefore, a nominal beam depth (h) of 16 in. (i.e., two 8-in. high CMU blocks) or 24 in. (three 8-in. high CMU blocks) can be assumed for initial design, and the final design depth can be determined from subsequent trial-and-error procedure. Once the initial trial nominal depth is chosen, the next step is to select the design depth d of the beam (the distance from the extreme compression fibers to the centroid of tension reinforcement). This depth can be determined by considering the thickness of the bottom face of a bond beam or a lintel unit, and the grout cover requirements. The thickness of the bottom face of the masonry unit can be assumed to be approximately 2 in. All beams and lintels are required to be grouted solid. MSJC-08 Section 1.15.3.5 requires reinforcement embedded in grout to have a thickness of grout between the reinforcement masonry units not less than ¼ in. for fine grout and ½ in. for coarse grout. With a ½ in. grout cover, and assuming a No. 8 bar for flexural reinforcement, the minimum distance from the bottom of the masonry unit to the centroid of the bar works out to be 2¾ in. This can be rounded off to 3 in. Therefore, as a maximum, the depth d (also sometimes referred to as the effective depth) can be reasonably assumed as the nominal depth of the beam *minus* 3 in. See Fig. 4.8. The assumption here is that the tensile reinforcement consists of one bar (as in most cases). In case the flexural reinforcement consists of two bars (e.g., one bar above the other; *bundling of bars is not permitted in masonry* (MSJC-08 Section 3.3.3.6) the depth d is to be measured from the centroid of the bar group, and it should be verified that the strain in the lowermost reinforcing bar (i.e., nearest the tension face of the beam) satisfies the condition $\varepsilon_s \geq 1.5\varepsilon_y$. Furthermore, the variation in the size of reinforcing bras in a beam should not be greater than one bar size, and no more than two bar sizes should be used in a beam (MSJC-05 Section 3.3.4.2.2.1). The purpose of this restriction is to increase the depth of the compression zone of the beam and to increase ductility. When two bars of significantly different sizes are placed in a beam, the larger bar requires a much higher load to reach yield strain. Note that MSJC-08 Section 3.3.3.1 does not permit reinforcing bars larger than No. 9 for strength design of reinforced masonry.

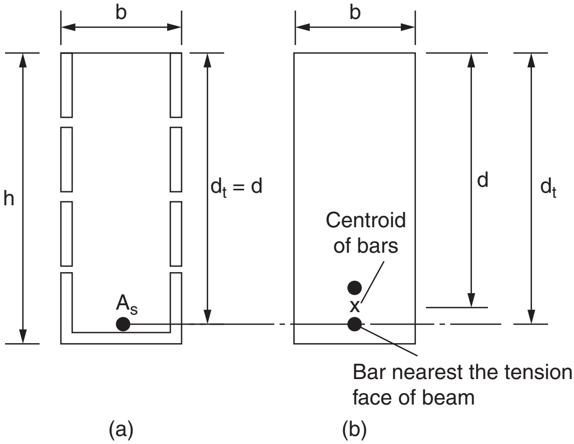


FIGURE 4.8 Estimation of the design depth *d* of a beam: (*a*) one layer of reinforcing bars, (*b*) two layers of reinforcing bars.

Another assumption that needs to be made in designing a beam involves calculating the area of tension reinforcement required for flexure, which can be calculated from Eq. (4.77) which is obtained by rearranging Eq. (4.12):

$$M_n = A_s f_y \left(d - \frac{a}{2} \right) \tag{4.12 repeated}$$

$$A_s = \frac{M_u}{f_y \left(d - \frac{a}{2} \right)} \tag{4.77}$$

Use of Eq. (4.77) requires that the value of the term ($d - a/2$) be known a priori, but the value of a is not known a priori. Based on the fact that masonry beams are lightly reinforced, it would be reasonable to assume a trial value of ($d - a/2$) as given by Eq. (4.78):

$$d - \frac{a}{2} \approx 0.95d \tag{4.78}$$

This is equivalent to assuming the depth of compression block, $a = 0.10d$. Later, based on the actual area of tension reinforcement provided, the exact value of a can be calculated as illustrated in Examples 4.14 and 4.15.

And finally, the design span, L, should be known in order to calculate the design moment (or moment demand). The design span (L) depends on the condition of beam supports. MSJC-08 Section 1.13.1 defines the span lengths as shown in Table 4.7, with the stipulation that the length of the bearing of beams on their supports be at least 4 in. (MSJC-08 Section 2.3.3.3).

The preceding discussion presents mechanics of calculations involved in flexural design of a reinforced masonry beam. In all cases of flexural analysis and design of beams, the following code requirements, discussed hereinbefore, should be satisfied:

TABLE 4.7 Design Span Lengths of Beams

Support conditions	Span length
Members not integrally built with supports	Clear span plus depth of member, but need not exceed the distance between centers of supports
Members continuous over the supports	Distance between centers of supports

a. The nominal flexural strength at any section of a member shall not be less than one-fourth of the maximum nominal flexural strength at the critical section (MSJC-08 Section 3.3.4.1.1). The intent of this requirement seems to be to ensure that a flexural member must have tensile reinforcement over its entire length even though reinforcement may not be required in the regions of very low moment demand (e.g., near simple supports). This requirement can be stated as follows:

$$(M_n)_{\text{any section}} \nless 0.25(M_n)_{\text{critical section}}$$

b. The nominal flexural strength of beam shall not be less than 1.3 times the nominal cracking moment strength of the beam, that is, $M_n \nless 1.3M_{\text{cr}}$. As discussed earlier, this code provision is intended to ensure that a flexural member possesses a minimum amount of ductility. (MSJC-08 Section 3.3.4.2.2.2).

c. For masonry members where $M_u/V_ud_v \geq 1$, the cross-sectional area of the flexural tensile reinforcement area shall not exceed the area required to maintain axial equilibrium (i.e., $C = T$) consistent with an assumed strain gradient corresponding to a strain in the extreme tensile reinforcement equal to 1.5 times the yield strain (i.e., $\varepsilon_s \geq 1.5\varepsilon_y$) and a maximum strain masonry specified in MSJC-08 Section 3.3.2(c), $\varepsilon_{mu} = 0.0025$ and 0.0035, respectively, in concrete and clay masonry (Section 3.3.3.5.1 see Table 4.4).

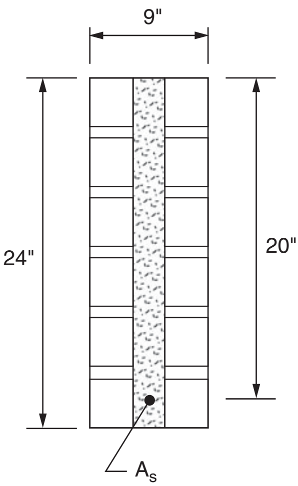


FIGURE E4.14 Beam cross section for Example 4.14.

Example 4.14 Determination of the amount of steel reinforcement when both *b* and *d* are known.

A simply supported clay brick beam is 9 in. wide and 24 in. deep with the tension reinforcement located at 20 in. from the top of the beam (Fig. E4.14). The beam has an effective span of 16 ft and carries a service live load of 1200 lb/ft in addition to its own weight. Use $f'_m = 2500$ psi and Grade 60 steel. Determine the flexural reinforcement required for this beam.

Solution

Calculate the self-weight of the beam based on 10 lb/in. width per foot height of the beam (Table A.21).

$$D = (10)(9)\left(\frac{24}{12}\right) = 180\,\text{lb/ft}, \; L = 1200 \text{ lb/ft}$$

Calculate the factored loads.
Load combinations:

1. $U = 1.4D = 1.4(180) = 252$ lb/ft
2. $U = 1.2D + 1.6L = 1.2(180) + 1.6(1200) = 2136$ lb/ft (governs)

$$w_u = 2.136 \text{ k/ft (governs)}$$

$$M_u = \frac{w_u L^2}{8} = \frac{2.136(16)^2}{8} = 68.35 \text{ k-ft}$$

From Eq. (4.43)

$$\phi M_n = \phi[f'_m b d^2 \omega(1 - 0.625)]$$

Substitute the following into the above equation:

$$\phi M_n = M_u = 68.35 \text{ k-ft},\ b = 9.0 \text{ in.},\ d = 20 \text{ in. (given)}$$

$$(68.35)(12) = 0.9[(2.5)(9)(20)^2\ \omega(1 - 0.625\omega)]$$

$$820.2 = 8100\omega(1 - 0.625\omega)$$

$$0.625\omega^2 - \omega + 0.10126 = 0$$

$$\omega = \frac{1 \pm \sqrt{1 - 4(0.625)(0.10126)}}{2(0.625)} = 0.10864$$

$$\omega = \rho\frac{f_y}{f'_m} \qquad \text{(4.40 repeated)}$$

Therefore,

$$0.10864 = \rho\left(\frac{60}{2.5}\right)$$

which yields $\rho = 0.00453$.
From Eq. (4.36)

$$A_s = \rho b d = 0.00453(9)(20) = 0.82 \text{ in.}^2$$

$$\text{Try one No. 9 bar, } A_s = 1.0 \text{ in.}^2$$

Calculate ϕM_n for the beam with $A_s = 1.0$ in.^{2}. From Eq 4.9, assuming that steel has yielded

$$a = \frac{A_s f_y}{0.8 f'_m b} \qquad \text{(4.9 repeated)}$$

$$a = \frac{(1.0)(60)}{0.80(2.5)(9.00)} = 3.33 \text{ in.}$$

From Eq. (4.13)

$$\phi M_n = \phi A_s f_y \left(d - \frac{a}{2} \right)$$
$$= 0.9(1.0)(60)\left(20 - \frac{3.33}{2} \right)$$
$$= 990.1 \text{ lb-in.} = 82.51 \text{ k-ft}$$
$$\phi M_n = 82.51 \text{ k-ft} > M_u = 68.35 \text{ k-ft} \qquad \text{OK}$$

Check the $M_u/V_u d_v$ ratio.

$$V_u = \frac{w_u L}{2} = \frac{(2.136)(16)}{2} = 17.09 \text{ kips}$$
$$\frac{M_u}{V_u d_v} = \frac{(68.35)(12)}{(17.09)(20)} = 2.4 > 1.0$$

Verify that reinforcement has yielded and $\varepsilon_s \geq 1.5\varepsilon_y$. From Eq. (4.5):

$$c = \frac{a}{0.8} = \frac{3.33}{0.8} = 4.16 \text{ in.}$$
$$\frac{c}{d} = \frac{4.16}{20.0} = 0.208 < 0.538 \qquad \text{(4.34 repeated)}$$

Hence, steel has yielded and $\varepsilon_s \geq 1.5\varepsilon_y$. Check that M_n is not less than $1.3M_{cr}$ (MSJC-08 Section. 3.3.4.2.2.2). From Eq. (4.53)

$$M_{cr} = f_r \left(\frac{bh^2}{6} \right)$$

where h = the total depth of the beam. The modulus of rupture, $f_r = 200$ lb/in.^{2} for solid grouted masonry in running bond laid in mortar S portland cement/lime or mortar cement (MSJC-08 Table 3.1.8.2.1)

$$M_{ct} = 200(9)\frac{(24)^2}{6} = 172{,}800 \text{ lb-in.} = 14.4 \text{ k-ft}$$
$$1.3M_{cr} = 1.3(14.4) = 18.72 \text{ k-ft}$$
$$M_n = \frac{\phi M_n}{\phi} = \frac{82.51}{0.9} = 91.68 \text{ k-ft} > 18.72 \text{ k-ft} \qquad \text{OK}$$

Check that $\rho \leq 0.75\rho_b$.

$$\rho = \frac{A_s}{bd} = \frac{1.0}{(9)(20)} = 0.0056 < 0.75\rho_b = 0.0126 \text{ (Table 4.6)} \qquad \text{OK}$$

Final design: Provide one No. 9 Grade 60 bar, with d = 20 in.

Commentary: We could have also used two No. 6 bars ($A_s = 0.88$ in.^{2}) instead of one No. 9 bar ($A_s = 1.0$ in.^{2}) if necessary.

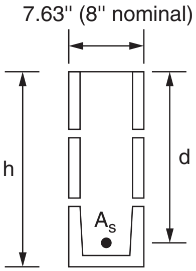


FIGURE E4.15 Beam cross section for Example 4.15.

Examples 4.15 Determination of beam depth and reinforcement when only the beam width *b* is known. An 8-in. wide CMU lintel with a clear span of 12 ft has to carry a superimposed dead load of 850 lb/ft and a live load of 1200 lb/ft. Use f'_m = 2000 psi and Grade 60 reinforcement (Fig. E4.15). Determine the depth of the lintel and the area of reinforcement. The masonry would be built from normal weight CMU with a grout weight of 140 lb/ft^3. Assume that length of bearing on each support is 8 in.

Solution

$$\text{Clear span} = 12 \text{ ft}$$

Assuming the bearing width of 8 in. on each side of the beam,

$$\text{Effective span} = 12 \text{ ft} + 8 \text{ in.} = 12.67 \text{ ft}$$

Assume $h = 24$ in. and the centroid of reinforcement as 4 in. from the bottom of the CMU.

$$d = 24 - 4 = 20 \text{ in.}$$

For normal weight 8-in. wide CMU with a grout weight of 140 lb/ft^3
Dead weight of lintel masonry = 84 lb/ft depth of the lintel (Table A.19)

$$\text{Self-weight of the lintel} = 84\left(\frac{24}{12}\right) = 168 \text{ lb/ft}$$

$$\text{Total dead weight, } D = 168 + 850 = 1018 \text{ lb/ft}$$

Load combinations:

1. $U = 1.4D = 1.4(1018) = 1425$ lb/ft
2. $U = 1.2D + 1.6L = 1.2(1018) + 1.6(1200) = 3142$ lb/ft >1425 lb/ft

$$w_u = 3142 \text{ lb/ft} = 3.142 \text{ k/ft (governs)}$$

$$M_u = \frac{w_u L^2}{8} = \frac{(3.142)(12.67)^2}{8} = 63.05 \text{ k-ft}$$

For an 8-in. wide CMU, $b = 7.63$ in. From Eq. (4.48)

$$bd^2 = \frac{12{,}000 M_u}{\phi k_n} \quad (M_u\text{, k-ft, } b \text{ and } d \text{ in inches})$$

From Eq. (4.45)

$$\phi k_n = \phi[f'_m \omega(1 - 0.625\omega)]$$

Assume $\rho = 0.5\rho_b$
For f'_m = 2000 psi and Grade 60 reinforcement, $0.50\rho_b = 0.0058$ (Table A.11).
From Eq. (4.40)

$$\omega = \rho\frac{f_y}{f'_m} = 0.0058\left(\frac{60{,}000}{2000}\right) = 0.174$$

$$\begin{aligned}\phi k_n &= 0.9[(2000)(0.174)(1 - 0.625\,x\,0.174)] \\ &= 279.14 \text{ psi}\end{aligned}$$

$$bd^2 = \frac{12{,}000M_u}{\phi k_n}$$
$$= \frac{12{,}000(63.05)}{279.14} = 2710.5 \text{ in.}^3$$
$$d = \sqrt{\frac{2710.5}{b}} = \sqrt{\frac{2710.5}{7.63}} = 18.85 \text{ in.}$$

With the assumed value of total beam depth, h, equal 24 in., the design depth d would be 24 − 4 in. (cover) = 20 > 18.85 in. So, d = 20 in. as assumed is adequate. Calculate the required amount of reinforcement based on d = 20 in. From Eq. (4.13)

$$\phi M_n = \phi A_s f_y \left(d - \frac{a}{2}\right)$$

Assume $\left(d - \frac{a}{2}\right) \approx 0.95d$ [Eq. (4.78)] and estimate the area of tension reinforcement, A_s.

$$\phi M_n = M_u = 63.05 \text{ k-ft}$$
$$(63.05)(12) = 0.9A_s(60)[(0.95)(20)]$$
$$A_s = 0.74 \text{ in.}^2$$

Try one No. 8 bar, $A_s = 0.79$ in.^{2}. Calculate M_u with d = 20 in. and $A_s = 0.79$ in.^{2}. From Eq. (4.9)

$$a = \frac{A_s f_y}{0.80 f'_m b} = \frac{(0.79)(60)}{0.80(2.0)(7.63)} = 3.88 \text{ in.}$$

From Eq. (4.13),

$$\phi M_n = \phi A_s f_y \left(d - \frac{a}{2}\right)$$
$$= 0.9(0.79)(60)\left(20 - \frac{3.88}{2}\right)$$
$$= 770.4 \text{ k-in.} = 64.2 \text{ k-ft}$$

$$\phi M_n = 64.2 \text{ k-ft} > M_u = 63.05 \text{ k-ft} \qquad \text{OK}$$

Check the $M_u/V_u d_v$ ratio.

$$V_u = \frac{w_u L}{2} = \frac{(3.142)(12.67)}{2} = 19.9 \text{ kips}$$
$$\frac{M_u}{V_u d_v} = \frac{(63.05)(12)}{(19.9)(20)} = 1.9 > 1.0$$

Verify that reinforcement has yielded and that $\varepsilon_s \geq 1.5\varepsilon_y$ from Eq. (4.30). From Eq. (4.5)

$$c = \frac{a}{0.80} = \frac{3.88}{0.80} = 4.85 \text{ in.}$$

$$\frac{c}{d} = \frac{4.85}{20} = 0.243 < 0.454 \qquad \text{(Eq. 4.30)}$$

Therefore, steel has yielded and $\varepsilon_s \geq 1.5\varepsilon_y$.
Check that M_n is not less than 1.3 M_{cr} (MSJC-08 Section 3.3.4.2.2.2). Calculate M_{cr} from Eq. (4.53).

$$M_{cr} = f_r\left(\frac{bh^2}{6}\right)$$

where h = the total depth of the beam. The modulus of rupture, f_r = 200 lb/in.^{2} for solid grouted masonry in running bond laid in mortar S portland cement/lime or mortar cement (MSJC-08 Table 3.1.8.2.1)

$$M_{\text{cr}} = (200)\frac{(7.63)(24)^2}{6} = 146{,}496 \text{ lb-in.} = 12.21 \text{ k-ft}$$

$$1.3M_{cr} = 1.3(12.21) = 15.88 \text{ k-ft}$$

$$M_n = \frac{\phi M_n}{\phi} = \frac{64.2}{0.9} = 71.33 \text{ k-ft} > 1.3M_{\text{cr}} = 15.88 \text{ k-ft} \qquad \text{OK}$$

Check that $\rho \leq 0.75\rho_b$.

$$\rho = \frac{A_s}{bd} = \frac{0.79}{(7.63)(20)} = 0.0052 < 0.75\rho_b = 0.0088 \text{ (Table 4.5)} \qquad \text{OK}$$

$$\phi M_n = 64.2 \text{ k-ft}$$

Final design: Provide a nominal 8 × 24 in. CMU lintel with one No. 8 Grade 60 bar, and d = 20 in.

Example 4.16 Solve Prob. 4.15 assuming that high strength masonry with $f'_m = 4000$ psi would be used. All other data remain the same as in Example 4.15.

Solution

From Example 4.15, M_u = 63.05 k-ft. From Eq. (4.48)

$$\frac{M_u}{\phi k_n} = \frac{bd^2}{12{,}000} \quad (M_u, \text{ k-ft, } b \text{ and } d \text{ in inches})$$

From Eq. (4.45)

$$\phi k_n = \phi[f'_m\omega(1 - 0.625\omega)]$$

Assume $\rho = 0.50\rho_b$. For $f'_m = 4000$ psi and Grade 60 reinforcement, $0.50\rho_b = 0.0117$ (Table A.11). From Eq. (4.40)

$$\omega = \rho\frac{f'_y}{f'_m} = 0.0117\left(\frac{60}{4}\right) = 0.1755$$
$$\phi k_n = 0.9(4000)(0.1755)[1-(0.625)(0.1755)]$$
$$= 562.5 \text{ psi}$$

Alternatively, we could have determined ϕk_n from Table A.13. For $\rho = 0.0117$ and $f'_m = 4000$ psi, from interpolation,

$$\phi k_n = 533 + (575 - 533)(0.7) = 562.4 \text{ psi}$$

From Eq. (4.47),

$$\phi M_n = \phi k_n\left(\frac{bd^2}{12{,}000}\right) = (562.4)\left(\frac{(7.63)(20)^2}{12{,}000}\right) = 143.04 \text{ k-ft}$$

From Eq. (4.48)

$$bd^2 = \frac{12{,}000 M_u}{\phi k_n} = \frac{12{,}000(63.05)}{562.4} = 1345 \text{ in.}^3$$

with $b = 7.63$ in.,

$$d = \sqrt{\frac{1345}{7.63}} = 13.3 \text{ in.}$$

With 4-in. cover, $h_{\text{reqd}} = 13.3 + 4 = 17.3$ in. Try a nominal 8×24 in. beam. With the centroid of reinforcement at 4 in. from the tension face of the beam, $d = 24 - 4 = 20$ in. Calculate the required amount of reinforcement assuming $d = 20$ in. From Eq. (4.13)

$$M_u = \phi M_n = \phi A_s f_y\left(d - \frac{a}{2}\right)$$

Assume $\left(d - \frac{a}{2}\right) \approx 0.95d$ [Eq. (4.78)] and estimate A_s.

$$63.05\,(12) = 0.9A_s\,(60)[(0.95)(20)]$$
$$A_s = 0.74 \text{ in.}^2$$

Try one No. 8 bar, $A_s = 0.79$ in.^{2}, or two No. 5 bars, $A_s = 0.88$ in.^{2}. Of these two choices, try one No. 8 bar (a better choice). Calculate ϕM_n with $d = 20$ in. and $A_s = 0.79$ in.^{2}. From Eq. (4.9)

$$a = \frac{A_s f_y}{0.80 f'_m b} = \frac{(0.79)(60)}{0.80(4.0)(7.63)} = 1.94 \text{ in.}$$

From Eq. (4.13)

$$\begin{aligned}\phi M_n &= \phi A_s f_y\left(d-\frac{a}{2}\right)\\ &= 0.9(0.79)(60)\left(20-\frac{1.94}{2}\right)\\ &= 811.8 \text{ k-in.} = 67.65 \text{ k-ft}\\ \phi M_n &= 67.7 \text{ k-ft} > M_u = 63.05 \text{ k-ft} \qquad \text{OK}\end{aligned}$$

Check that reinforcement has yielded and $\varepsilon_s \geq 1.5\varepsilon_y$, which should be verified from Eq. (4.30). From Eq. (4.5)

$$c = \frac{a}{0.80} = \frac{1.94}{0.80} = 2.43 \text{ in.}$$

$$\frac{c}{d} = \frac{2.43}{20} = 0.12 < 0.454 \qquad \text{(4.30 repeated)}$$

Therefore, steel has yielded and the assumption $\varepsilon_s \geq 1.5\varepsilon_y$ is satisfied.

$$\phi M_n = 67.7 \text{ k-ft}$$

Check that M_n is not less than $1.3M_{\text{cr}}$ (MSJC-08 Section. 3.3.4.2.2.2). Calculate M_{cr} from Eq. (4.53).

$$M_{cr} = f_r\left(\frac{bh^2}{6}\right)$$

where h = the total depth of the beam. The modulus of rupture, f_r = 200 lb/in.^{2} for solid grouted masonry in running bond laid in mortar S portland cement/lime or mortar cement (MSJC-08 Table 3.1.8.2.1). The value of M_{cr} remains the same as in Example 4.15, but calculations are repeated here for completeness.

$$M_{\text{cr}} = (200)\frac{(7.63)(24)^2}{6} = 146{,}496 \text{ lb-in.} = 12.21 \text{ k-ft}$$

$$1.3M_{cr} = 1.3(12.21) = 15.88 \text{ k-ft}$$

$$M_n = \frac{\phi M_n}{\phi} = \frac{67.7}{0.9} = 75.22 \text{ k-ft} > 1.3M_{\text{cr}} = 15.88 \text{ k-ft} \qquad \text{OK}$$

Check that $\rho \leq 0.75\rho_b$.

$$\rho = \frac{A_s}{bd} = \frac{0.79}{(7.63)(20)} = 0.0052 < 0.75\rho_b = 0.0175 \ \text{(Table 4.5)} \qquad \text{OK}$$

Final design: Provide a nominal 8 × 24 in. CMU lintel with one No. 8 Grade 60 bar, and d = 20 in.

4.8.2 Determination of A_S When the Beam Width b and Depth d are Known: Direct Solution

Instead of using the iterative procedure described in the preceding section, it is possible to develop a procedure for direct determination of the required area of reinforcement if the beam width b and the depth d are known. This can be done by combining Eqs. (4.9) and (4.13) (repeated as follows):

$$a = \frac{A_s f_y}{0.80 f'_m b} \qquad \text{(4.9 repeated)}$$

$$M_n = A_s f_y \left(d - \frac{a}{2} \right) \qquad \text{(4.13 repeated)}$$

Substitution of Eq. (4.9) into (4.13) and multiplying both sides with ϕ yields

$$M_u = \phi \left[A_s f_y \left(d - \frac{A_s f_y}{1.6 f'_m b} \right) \right] \qquad (4.79)$$

Equation (4.79) can be rearranged as a quadratic in A_s (of the form $Ax^2 + Bx + C = 0$):

$$\left(\frac{\phi f_y^2}{1.6 f'_m b} \right) A_s^2 - (\phi f_y d) A_s + M_u = 0 \qquad (4.80)$$

Equation (4.80) can be simplified and expressed as Eq. (4.81):

$$A_s^2 - 1.6bd \left(\frac{f'_m}{f_y} \right) A_s + \left(\frac{1.6bf'_m}{f_y^2} \right) \left(\frac{M_u}{\phi} \right) = 0 \qquad (4.81)$$

With the known values of f'_m, f_y, b, d, and M_u, either Eqs. (4.80) or (4.81) can be solved as quadratic equations for the required value of A_s. This procedure is illustrated in Example 4.17. Note that both these equations contain the strength reduction factor ϕ. Its value can be taken as 0.9 for solving these equations, but must be verified later based on the value of inelastic strain in the tension reinforcement ($\varepsilon_s \geq 1.5\varepsilon_y$).

Example 4.17 Solve Example 4.16 by Eq. (4.81).

Solution

Equation (4.80) is

$$A_s^2 - 1.6(bd) \left(\frac{f'_m}{f_y} \right) A_s + \left(\frac{1.6bf'_m}{f_y^2} \right) \left(\frac{M_u}{\phi} \right) = 0 \qquad \text{(4.81 repeated)}$$

Given: $b = 7.63$ in., $d = 20$ in., $f'_m = 4000$ psi, $f_y = 60$ ksi, $M_u = 63.05$ k-ft $= 756.6$ k-in. (calculated in Example 4.16), $\phi = 0.9$ (assumed, verified in Example 4.16)

Calculate the coefficients in Eq. (4.80).

$$1.6(bd)\left(\frac{f'_m}{f_y} \right) = 1.6(7.63)(20)\left(\frac{4.0}{60} \right) = 16.28$$

$$\left(\frac{1.6bf'_m}{f_y^2} \right) \left(\frac{M_u}{\phi} \right) = \frac{1.6(7.63)(4.0)}{(60)^2} \frac{(756.6)}{0.9} = 11.4$$

Substituting in Eq. (4.80), we obtain

$$A_s^2 - 16.28A_s + 11.4 = 0$$
$$A_s = 0.73 \text{ in}^2$$

Provide one No. 8 bar, $A_s = 0.79$ in^2

This is the same result that was obtained in Example 4.16. Other calculations should follow as shown in Example 4.16, that is, check M_u with $d = 20$ in. and $A_s = 0.79$ in.^{2}, and that assumption $\varepsilon_s \geq 1.5\varepsilon_y$ is satisfied. These calculations would be exactly the same as in Example 4.16. A check should also be made to ensure that ρ is not greater than $0.75\rho_b$, as illustrated in Example 4.16.

4.9 OVERREINFORCED BEAMS

In some cases, a masonry beam may contain excessive tension reinforcement, which would make it an overreinforced beam. The implication of a beam being overreinforced is that the tension reinforcement would not yield, that is, the stress in the reinforcement would remain in the elastic range. Under ultimate loads, such a beam may experience sudden (brittle) failure initiated by crushing of the masonry in the compression zone of the beam. In such cases, the analyses presented in preceding sections cannot be used because they are based on the premise of tension reinforcement having yielded.

Overreinforced beams can be analyzed based on the same general equilibrium condition ($C = T$) that was used for analyzing underreinforced beams except for the fact that the stress in reinforcement, f_s, is not equal to the yield stress f_y (it is less than f_y). Because the stress in reinforcement is in the elastic range, it can be expressed as a function of elastic strain ε_s and the modulus of elasticity of steel, E_s. The analysis follows.

Equilibrium of compressive and tensile force resultants in the beam gives

$$C = T \tag{4.82}$$

where the value of C is given by Eq. (4.6), and T can be expressed as a product of the reinforcement area and stress in it. Thus,

$$C = 0.80 f'_m ab \qquad \text{(4.6 repeated)}$$

$$T = A_s f_s \qquad \text{(4.7 repeated)}$$

The stress in reinforcement can be expressed in terms of strain based on Hooke's law:

$$f_s = \varepsilon_s E_s \tag{4.83}$$

Substitution for f_s from Eq. (4.83) in Eq. (4.82) yields

$$T = A_s \varepsilon_s E_s \tag{4.84}$$

Substitution of Eqs. (4.6) and (4.84), and $A_s = \rho bd$ [Eq. (4.38)] in Eq. (4.81) yields

$$0.80 f'_m ab = \rho E_s \varepsilon_s bd \tag{4.85}$$

From strain distribution diagram (Fig. 4.9), we obtain,

$$\varepsilon_s = \varepsilon_{mu}\left(\frac{d-c}{c}\right) \tag{4.86}$$

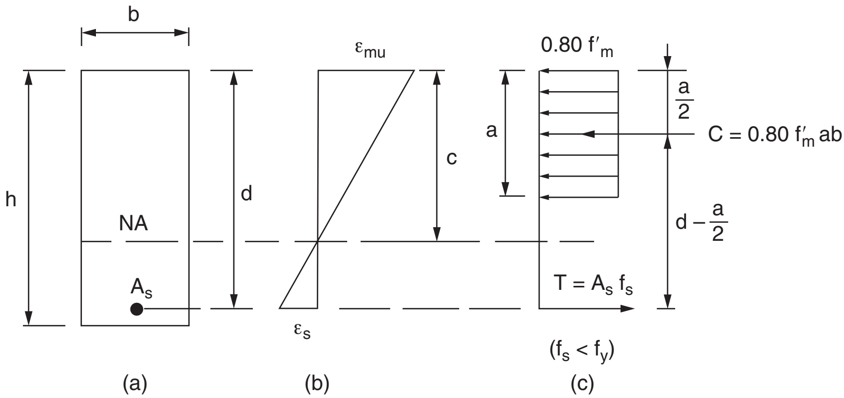


FIGURE 4.9 Stress and strain distribution diagram for an overeinforced masonry beam.

Substituting for ε_s from Eq. (4.86) in Eq. (4.85), and noting that $c = a/0.8$ [Eq. (4.5)], we obtain

$$0.80 f'_m ab = \rho bdE_s \left(\frac{d - a/0.8}{a/0.8} \right) \varepsilon_{mu} \tag{4.87}$$

Equation (4.87) can be simplified and expressed as

$$0.80 f'_m ba^2 = \rho bdE_s (0.8d - a)\varepsilon_{mu} \tag{4.88}$$

Equation (4.88) can be expressed as a quadric in a (of the form $Ax^2 + Bx + C = 0$):

$$\left(\frac{f'_m}{\rho E_s \varepsilon_{mu}} \right) a^2 + (1.25d)a - d^2 = 0 \tag{4.89}$$

Equation (4.89) can be solved for a, the depth of compression block. Once the value of a is known, the nominal strength of the beam can be determined by taking the moment of the compression force resultant about the centroid of tension reinforcement (Fig. 4.3) as given by Eq. (4.90):

$$\begin{aligned} M_n &= C\left(d - \frac{a}{2} \right) \\ &= 0.80 f'_m ab \left(d - \frac{a}{2} \right) \end{aligned} \tag{4.90}$$

Note that the value of M_n cannot be determined by taking the moment of T about C because the value of f_s (and, hence, the value of T) is not known. Example 4.18 presents a procedure for calculating the nominal strength of an overreinforced beam. Of course, such a beam design is not permitted by the code.

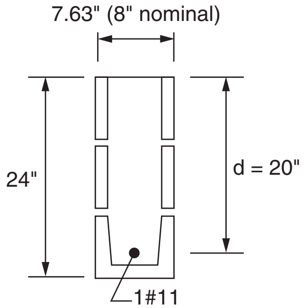


FIGURE E4.18 Beam cross section for Example 4.18.

Example 4.18 A nominal 8 × 24 in. concrete masonry beam is reinforced with one No. 11 Grade 60 reinforcing bar placed at $d = 20$ in. for tension (Fig. E4.18). The beam is required to carry a uniform service dead load of 1.0 k/ft (including its self-weight) and a service live load of 1.75 k/ft over a span of 12 ft. $f'_m = 1500$ psi. Check if the beam is adequate to support the loads.

Solution

The above problem was solved earlier in Example 4.13(c). The design moment, M_u, was calculated to be 72 k-ft (calculations not repeated here). However, it was determined that the beam was overreinforced ($\rho = 0.0102 > \rho_{\max} = 0.0088$), and therefore, not acceptable according to the code. Theoretically, however, the nominal strength, M_n, of the beam can be calculated from Eq. (4.90) where the distance a, the depth of compression block of the beam, is determined from Eq. (4.89).

$$\left(\frac{f'_m}{\rho E_s \varepsilon_{mu}}\right) a^2 + (1.25d)a - d^2 = 0 \qquad \text{(4.89 repeated)}$$

Given: $f'_m = 1500$ psi, $E_s = 29(10^6)$ psi, $\varepsilon_{mu} = 0.0025$ (concrete masonry), $d = 20$ in.

$$\rho = \frac{A_s}{bd} = \frac{1.56}{(7.63)(20)} = 0.0102 > 0.75\rho_b = 0.0066$$

$$\left(\frac{f'_m}{\rho E_s \varepsilon_{mu}}\right) = \left(\frac{1500}{(0.0102)(29x10^6)(0.0025)}\right) = 2.0284$$

Substitution of the above value in Eq. (4.87) yields the quadratic in a:

$$(2.0284)a^2 + (1.25 \times 20)a - (20)^2 = 0$$

The solution of the above quadratic is: $a = 9.17$ in.
From Eq. (4.90), we obtain

$$\begin{aligned} M_n &= C\left(d - \frac{a}{2}\right) \\ &= 0.80 f'_m ab\left(d - \frac{a}{2}\right) \\ &= 0.80(1.5)(9.17)(7.63)\left(20 - \frac{9.17}{2}\right) \\ &= 1294.25 \text{ k-in.} = 107.85 \text{ k-ft} \end{aligned}$$

As noted earlier, this beam is overreinforced and not acceptable as it does not comply with the code requirements for reinforced masonry beams. Note also that No. 11 bar is not permitted by MSJC-8 *Code* for strength design of reinforced masonry.

4.10 DESIGN FOR SHEAR IN REINFORCED MASONRY BEAMS

4.10.1 MSJC Provisions for Estimating Shear Strength of a Masonry Beam

The provisions for shear strength of reinforced masonry beams are covered by MSJC-08 Section 3.3.4.1.2, which are discussed in this section.

Masonry beams without any shear reinforcement are capable of providing a certain amount of resistance to shear in beams caused by transverse loads. Tests indicate that the shear resistance of masonry depends on the following parameters:

1. Compressive strength of masonry (varies with $\sqrt{f'_m}$)
2. Net cross-sectional area of masonry
3. Shear span-to-depth ratio (a dimensionless parameter), M_u/V_ud_v
4. Axial force, if any, in the member

The beneficial effect due to the presence of axial load occurs because of the resulting improved aggregate interlock. This benefit does not apply to transversely loaded members such as beams and lintels, however, because those members are subjected to negligible axial loads.

Accordingly, the nominal shear resistance V_{nm} of masonry alone is given by Eq. (4.91) [MSJC-08 Eq. (3.22)]:

$$V_{nm} = \left[4.0 - 1.75\left(\frac{M_u}{V_ud_v}\right)\right]A_n\sqrt{f'_m} + 0.25P_u \tag{4.91}$$

where V_{nm} = nominal shear strength of the masonry (alone)
M_u = factored moment in the beam
V_u = shear due to factored loads
d_v = depth of the beam in the direction of the shear considered
A_n = net cross-sectional area of masonry member
P_u = factored axial load (if present)

Equation (4.91) is based the on research. The value of shear-span to depth ratio, M_u/V_ud_v, is to be taken as a positive number and need not be taken greater than 1.0 (MSJC-08 Section 3.3.4.1.2.1).

Equation (4.91) shows that increasing the ratio M_u/V_ud_v has an adverse influence on the nominal shear strength of a masonry beam. Regardless of the amount of transverse reinforcement provided in the beam, the maximum value of its nominal shear strength V_n is limited by MSJC-08 Eqs. (3-19) and (3-20) as follows:

1. Where $M_u/V_ud_v \le 0.25$,

$$V_n \le 6A_n\sqrt{f'_m} \tag{4.92}$$

2. Where $M_u/V_ud_v \ge 1.00$,

$$V_n \le 4A_n\sqrt{f'_m} \tag{4.93}$$

3. Values of V_n given by Eqs. (4.92) and (4.93) are the maximum values of V_n for the specified limits of M_u/V_ud_v ratios. For M_u/V_ud_v ratios between 0.25 and 1.0, the value of V_n is

to be obtained from linear interpolation of the values given by Eqs. (4.92) and (4.93). Note that as the value of ratio M_u/V_ud_v increases from 0.25 to 1.00, the maximum value of V_n decreases by one-third.

It is noteworthy that limitations for V_n given by Eqs. (4.92) and (4.93) were derived for shear walls, which act as vertical beams. For transversely loaded beams, the ratio M_u/V_ud_v would almost always be greater than 1.0, and Eq. (4.93) would apply.

Shear cracking in masonry is caused primarily by diagonal tension. The last term in Eq. (4.91) (expressed as $0.25P_u$) provides for the influence of axial compressive force that may be present in the member (e.g., a prestressed masonry beam or a shear wall subjected to gravity loads), which tends to counteract the horizontal component of diagonal tension in the beam. MSJC-08 Section 3.3.4.2.1 specifies an upper limit of the factored axial load P_u in a beam [to be included in Eq. (4.91)] as given by the following expression:

$$\mathrm{P_u} \ngtr 0.05\mathrm{A_n}\, f'_m \tag{4.94}$$

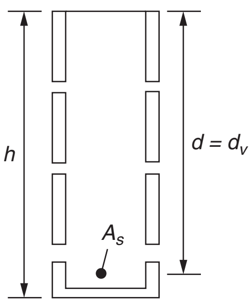


FIGURE 4.10 Definitions of d and d_v for beams.

The term d_v included in Eq. (4.91) whether it should be equal to h or d, remains debatable. This term is intended to be used for shear walls, which act as vertical beams, and which also carry a significant amount of factored load P_u (due to gravity loads). In the case of a shear wall, d_v represents the length of the wall measured parallel to the horizontal shear force it is designed to resist. In a shear wall, the difference between distances d_v and d (the distance between the extreme compression fibers and the centroid of tension reinforcement) is relatively negligible (i.e., $d_v \approx d$). However, such is not the case in a transversely loaded beam. The difference between the two terms (dimensions h and d) is illustrated in Fig. 4.10 for clarity. Therefore, when applying Eq. (4.91) to transversely loaded beams, d_v should be replaced with d, the distance from the extreme compression fibers to the centroid of tension reinforcement (the same as used for flexural calculations, a conservative approach used in this book (see Examples 4.14 and 4.15). Thus, for transversely loaded members such as beams and lintels, Eq. (4.91) can be rewritten as follows:

$$V_{nm} = \left[4.0 - 1.75\left(\frac{M_u}{V_u d_v}\right)\right] A_n\sqrt{f'_m} + 0.25P_u \tag{4.95}$$

In transversely loaded beams, the value of ratio M_u/V_ud_v would more likely be greater than 1.0, in which case $M_u/V_ud = 1.0$ in Eq. (4.95). Therefore, if $P_u = 0$, Eq. (4.95) can be expressed as

$$\begin{aligned} V_{nm} &= \left[4.0 - 1.75\left(\frac{M_u}{V_u d_v}\right)\right] A_n\sqrt{f'_m} \\ &= [4 - 1.75(1.0)]A_n\sqrt{f'_m} \\ &= 2.25A_n\sqrt{f'_m} \end{aligned} \tag{4.96}$$

Equation (4.96) gives the value of shear contribution of masonry alone in a transversely loaded beam. Note that the nominal shear strength of masonry is proportional to the square root of the specified compressive strength of masonry f'_m. As a result, the nominal shear strength of masonry increases only moderately when f'_m is increased even by as much as 30 percent. See Example 4.19.

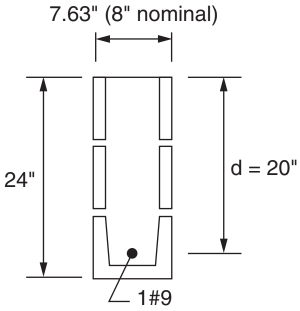


FIGURE E4.19

Example 4.19 A nominal 8- × 24-in. CMU beam is reinforced with one No. 9 Grade 60 reinforcing bar with d = 20 in. (Fig. E4.19). The beam is required to carry a service dead load of 1.0 k/ft (including its self-weight) and a service live load of 1.0 k/ft over an effective span of 12 ft. $f'_m = 1500$ psi. (a) Calculate the shear resistance V_m provided by masonry, (b) what would be the increase in the shear resistance provided by the masonry if the specified strength of masonry f'_m is increased to 2000 psi from 1500 psi.

Solution

a. Since no axial force is present in the beam, the shear strength provided by the masonry can be calculated from Eq. (4.96):

$$b = 7.63 \text{ in. (8-in. nominal)}, \; d = 20 \text{ in.}, \; f'_m = 1500 \text{ psi}$$

$$V_{nm} = 2.25A_n\sqrt{f'_m} = 2.25(7.63)(20)(\sqrt{1500}) = 13{,}298 \text{ lb} \approx 13.3 \text{ kips}$$

b. When $f'_m = 2000$ psi, the value of V_{nm} is

$$V_{nm} = 2.25A_n\sqrt{f'_m} = 2.25(7.63)(20)(\sqrt{2000}) = 15{,}355 \text{ lb} \approx 15.36 \text{ kips}$$

The increase in shear strength of masonry is

$$\Delta V = (V_m)_{2000 \text{ psi}} - (V_m)_{1500 \text{ psi}} = 15.36 - 13.3 = 2.06 \text{ kips}$$

$$\% \text{ increase} = \frac{15.36 - 13.3}{13.3} \approx 15.5\%$$

Conclusion: Increasing the compressive strength of masonry (f'_m) from 1500 psi to 2000 psi, a one-third increase, results in only a 15.5 percent increase in the shear strength of masonry.

4.10.2 Nominal Shear Strength of Reinforced Masonry

Shear reinforcement is not required in masonry beams when the reduced shear strength provided by the masonry, ϕV_{nm}, exceeds the factored shear V_u (i.e., when $\phi V_{nm} \geq V_u$), where ϕ = strength reduction factor for shear (Table 4.1). See Example 4.20.

Example 4.20 A nominal 10- × 40-in. CMU beam is reinforced with two No. 6 Grade 60 reinforcing bars for tension. The centroid of reinforcement is located at 6 in. from the bottom of the beam (Fig. E4.20). The beam carries a service load of 1600 lb/ft over an effective span of 15 ft 8 in. in addition to its own weight. The grout unit weight is 140 lb/ft^3. $f'_m = 1500$ psi. Check if shear reinforcement is required for this beam. A check for flexural calculations is not required for this problem*.

Solution

Given: $b = 9.63$ in., $d = 34$ in., effective span = 15.67 ft, $f'_m = 1500$ psi, $w_u = 2.932$ k/ft (see Example 4.2 for details).

(*See Example 4.2 for flexural calculations for this problem.)

Calculate the factored shear (or shear demand):

$$V_u = \frac{w_u l}{2} = \frac{(2.932)(15.67)}{2} = 22.97 \text{ kips}$$

Calculate the shear strength of masonry (V_m) from Eq. (4.96):

$$V_{nm} = 2.25 A_n \sqrt{f'_m} = 2.25(9.63)(34)(\sqrt{1500})$$
$$= 28{,}532 \text{ lb} = 28.53 \text{ kips}$$
$$\phi V_{nm} = (0.8)(28.53) = 22.82 \text{ kips} \approx V_u = 22.97 \text{ kips}$$

Since $\phi V_{nm} \approx V_u$, shear reinforcement is theoretically not required for this beam. However, provide minimum shear reinforcement for the beam, say No. 3 Grade 60 bar single-legged stirrup at 48 in. on center as discussed in the next section to provide ductility in the beam.

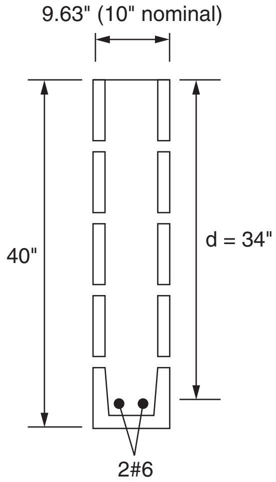


FIGURE E4.20 Beam cross section for Example 4.20.

4.10.3 Shear Strength of Reinforced Masonry Beams with Shear Reinforcement

Often, shear in flexural elements is greater than that can be resisted by masonry alone (i.e., $V_u \geq \phi V_{nm}$). In such cases, shear reinforcement must be provided to resist shear that may be in excess of the shear resistance of the masonry alone. Based on this premise, the nominal shear strength of a reinforced masonry beam (V_n) may be considered as a sum of two strength components: (1) shear strength provided by the masonry (V_{nm}), and (2) shear strength (resistance) provided by the shear reinforcement (V_{ns}). This statement can be expressed as stated by MSJC-08 Eq. (3.19):

$$V_n = V_{nm} + V_{ns} \tag{4.97}$$

Note that the value of V_n in Eq. (4.97) is limited to that given by Eq. (4.93).

The shear resistance to be provided by transverse reinforcement in a masonry beam can be expressed by multiplying both sides of Eq. (4.97) with the strength reduction factor for shear ϕ (Table 4.1) and rearranging the terms as follows:

$$\begin{aligned} \phi V_n &= \phi(V_{nm} + V_{ns}) \\ \phi V_{ns} &= \phi V_n - \phi V_{nm} \end{aligned} \tag{4.98}$$

In Eq. (4.98), the term ϕV_{ns} accounts for shear contribution of the shear reinforcement provided in the beam. Note that if V_u exceeds ϕV_n, a need for larger beam cross section or a higher compressive strength of masonry is indicated. This is an important concept in the sense that the nominal shear strength of a beam with transverse reinforcement (ϕV_n) is controlled by the shear strength provided by the masonry (ϕV_{nm}), for when V_u exceeds ϕV_n [$\leq \phi 4 A_n \sqrt{f'_m}$, Eq. (4.93)] no amount of shear reinforcement would make the beam adequate for shear, and the shear strength provided by the masonry (ϕV_{nm}) must be increased either by providing a larger effective depth (d) of beam or much higher compressive strength of masonry (shear strength of masonry increases in proportion to $\phi\sqrt{f'_m}$). This may very well be the case in a practical situation where a beam having short spans may be adequate in flexure but not in shear. See Example 4.21.

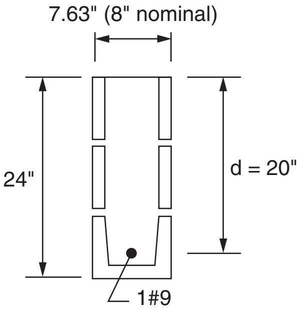


FIGURE E4.21 Beam cross section for Example 4.21.

Example 4.21 A nominal 8- × 24-in. CMU beam is reinforced with one No. 9 Grade 60 reinforcing bar for tension with d = 20 in. (Fig. E4.21). The beam is required to carry a service dead load of 1.0 k/ft (including its self-weight) and a service live load of 1.75 k/ft over an effective span of 12 ft. f'_m = 1500 psi. Check if the beam section is adequate for shear. If not, suggest alternatives for the beam to be adequate in shear.

(*Note:* This example was presented earlier as Example 4.13 for flexural calculations.)

Solution

Given: b = 7.63 in., d = 20 in., effective span = 12.0 ft, f'_m = 1500 psi, w_u = 4.0 k/ft (see Example 4.13 for details).

Calculate the factored shear (or shear demand):

$$V_u = \frac{w_u l}{2} = \frac{(4.0)(12)}{2} = 24.0 \text{ kips}$$

Calculate the shear strength of masonry (V_m) from Eq. (4.96):

$$V_{nm} = 2.25A_n\sqrt{f'_m} = 2.25(7.63)(20)(\sqrt{1500}) = 13{,}298 \text{ lb} = 13.3 \text{ kips}$$

$$\phi V_{nm} = (0.8)(13.3) = 10.64 \text{ kips} < V_u = 24.0 \text{ kips}$$

Since $\phi V_{nm} < V_u$, shear reinforcement is required for this beam. Check the maximum permissible value of V_n from Eq. (4.93).

$$(V_n)_{\max} \le 4A_n\sqrt{f'_m} = 4(7.63)(20)\sqrt{1500} = 23{,}641 \text{ lb} = 23.64 \text{ kips}$$

$$(\phi V_n)_{\max} = (0.8)(23.64) = 18.91 \text{ kips} < V_u = 24 \text{ kips}$$

Because ϕV_n (= 18.91 kips) < V_u (= 24 kips), the beam cross section is not acceptable. Therefore, either (a) a larger beam size or (b) higher compressive strength of masonry should be provided. Both options are explored as follows:

a. Larger beam size: The given beam is 24 in. deep (three 8-in. blocks). Try four 8-in. blocks so that h = 32 in. and d = 32 − 4 = 28 in. Calculate the revised maximum permissible value of V_n from Eq. (4.93).

$$(V_n)_{\max} = 4A_n\sqrt{f'_m} = 4(7.63)(28)\sqrt{1500} = 33{,}097 \text{ lb} = 33.1 \text{ kips}$$

$$(\phi V_n)_{\max} = (0.8)(33.1) = 26.48 \text{ kips} > V_u = 24 \text{ kips} \qquad \text{OK}$$

Increase the beam cross section to nominal 8 × 32 in. to satisfy shear requirements.

b. *Higher compressive strength of masonry*: Calculate the required value of f'_m from Eq. (4.93) by substituting $\phi V_n = V_u$.

$$\phi V_n = (0.8)(4)A_n\sqrt{f'_m} = 24{,}000 \text{ lb}$$
$$\sqrt{f'_m} = \frac{24{,}000}{(0.8)(4)(7.63)(20)} = 49.15$$

whence $f'_m = 2416$ psi. Therefore, increase the compressive strength of masonry to 2500 psi (rounded-off figure).

$$\phi(V_n)_{\max} = \phi 4A_n\sqrt{f'_m} = (0.8)(4)(7.63)(20)\sqrt{2500}$$
$$= 24{,}416 \text{ lb} = 24.42 \text{ kips} > V_u = 24 \text{ kips} \qquad \text{OK}$$

Therefore, the beam section is acceptable. Calculate ϕV_{nm}, shear strength provided by the masonry.

$$\phi V_{nm} = \phi 2.25A_n\sqrt{f'_m} = (0.8)(2.25)(7.63)(20)(\sqrt{2500}) = 13{,}734 \text{ lb} = 13.73 \text{ kips}$$

$\phi V_{nm} = 13.73$ kips $< V_u = 24$ kips; shear reinforcement required.

Commentary: The two options presented above would result in a satisfactory beam design. Alternative (a) does not require shear reinforcement as the masonry alone in the larger beam size would be able to resist the entire shear in the beam. In alternative (b), if the masonry strength is increased to 2500 psi, the nominal 8- × 24-in. beam can be used if shear reinforcement is provided. See Example 4.22.

4.10.4 Determination of Shear Reinforcement for Masonry Beams

For design purposes, we can substitute V_u (the factored shear or shear demand) for ϕV_n in Eq. (4.98) resulting in the following expression:

$$V_u = \phi(V_n + V_{ns}) \tag{4.99}$$

The shear strength required to be provided by shear reinforcement (V_{ns}) can be expressed by rearranging Eq. (4.99) as follows:

$$V_s = \frac{V_u - \phi V_{nm}}{\phi} \tag{4.100}$$

The shear reinforcement in masonry beams is provided typically in the form of single transverse bars placed in the grouted cells of masonry beams and hooked around horizontal reinforcement. Inclined stirrups, which sometimes are provided in reinforced concrete beams, are not practical for masonry which is built from precast units. The relationship between the nominal shear strength provided by shear reinforcement and the area of shear reinforcement is given by Eq. (4.101) [MSJC-08 Eq. (3-23)]:

$$V_{ns} = 0.5\left(\frac{A_v}{s}\right)f_y d \tag{4.101}$$

where A_v = cross-sectional area of shear reinforcement (single transverse bars)
s = spacing of shear reinforcement
f_y = yield strength of shear reinforcement
d = depth of member measured for the extreme compression fibers to the centroid of tension reinforcement

In a design situation, the quantity V_{ns} is obtained from Eq. (4.100), and we need to determine either the area of shear reinforcement (A_v), or the spacing of shear reinforcement (s). In a typical concrete masonry construction, it is required to provide shear reinforcement in the form of single-legged vertical stirrups placed in individual cells which are later grouted solid (this is mandatory requirement per MSJC-08 Section. 3.3.4.2.4). Thus, the spacing of stirrups would be a multiple of cell spacing. For example, in typical 8 × 8 × 16-in. block masonry, the cells are spaced at 8 in. on center; therefore, the stirrup spacing would be a multiple of 8 in. (i.e., 8, 16, 24 in., etc.). In a two-wythe clay (brick) masonry construction, the shear reinforcement can be placed at the desired spacing in the cavity between the two wythes. In both cases, Eq. (4.101) can be used in one of the two ways:

1. To determine the spacing (s) for a chosen A_v (cross-sectional area of shear reinforcing bar):

$$s = \frac{A_v f_y d}{2V_{ns}} \tag{4.102}$$

2. To determine A_v for a chosen shear reinforcement spacing (s):

$$A_v = \frac{2V_{ns}(s)}{f_y d} \tag{4.103}$$

It is a common practice to choose a No. 3 or No. 4 reinforcing bar for stirrups in masonry beams, which would found to be adequate in most cases. Because of the space limitation in a grouted cells, shear reinforcement is provided in the form of *single* bars (as opposed to U-stirrups) placed transverse to and hooked around horizontal reinforcement in the beams. Depending on the available space for placement of shear reinforcement, the stirrup may be one-legged or two-legged (Fig. 4.11). For example, in a typical CMU beam built from 8 × 8 × 16 in. blocks, limitation of space within the grouted cells permits only single-legged stirrups. In such cases, A_v = cross-sectional area of *one* bar in Eqs. (4.102) and (4.103). On the other hand, in a two-wythe masonry construction, stirrups are placed in the cavity between the wythes, which may accommodate one- or two-legged stirrups. If a two-legged stirrup were to be used, A_v would be equal to *twice* the cross-sectional area of the bar in Eqs. (4.102) and (4.103).

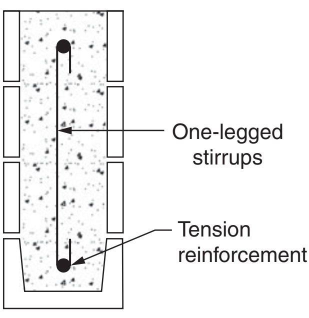


FIGURE 4.11 Shear reinforcement in masonry beams.

The reinforcement for shear is to be provided in a beam only in that portion where V_u exceeds ϕV_{nm}. Shear reinforcement is not required in the beam where $V_u < \phi V_{nm}$, typically a few feet on either side of the center of a uniformly loaded simply supported beam. See Example 4.21.

4.10.5 MSJC Provisions for Transverse (Shear) Reinforcement

In reinforced masonry beams, transverse reinforcement is required when the shear demand (V_u) exceeds ϕV_n. In such cases, MSJC-08 Section 3.3.4.2.3 specifies the following requirements:

1. Transverse reinforcement shall be a single bar with a 180° hook at each end.
2. Transverse reinforcement shall be hooked around the longitudinal reinforcement.
3. Shear reinforcement shall extend the depth of the member less cover distances.
4. The minimum area of transverse reinforcement shall be $0.0007bd$ ($0.0007bd_v$ for shear walls).
5. The first transverse bar shall not be located more than one-fourth of the beam depth d (d_v for shear walls) from the end of the beam.
6. The maximum spacing of transverse reinforcement shall not exceed one-half of the beam depth or 48 in.

There are many practical reasons that form the basis of above requirements. Hooking the transverse reinforcement around the longitudinal bars facilitates construction, confines the longitudinal reinforcement, and helps ensure the development of the shear reinforcement. The minimum area of shear reinforcement is intended to prevent brittle shear failures. The limitations on the maximum spacing of shear reinforcement are intended to increase member ductility.

The spacing limitation specified in item 5 above is intended to ensure that a 45° crack would be intercepted by at least one transverse bar (Fig. 4.12).

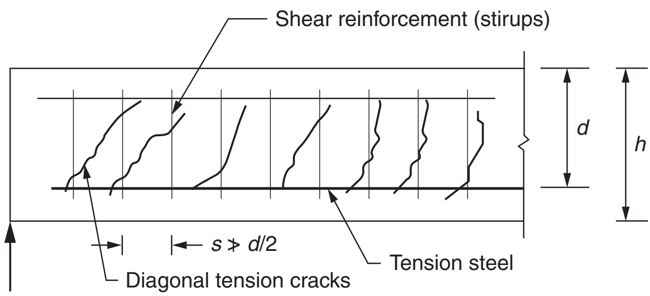


FIGURE 4.12 Shear reinforcement in masonry beams; spacing not to exceed $d/2$.

4.10.6 Examples for Design for Shear Reinforcement

Examples 4.22 to 4.25 illustrate design procedure for determining shear reinforcement for reinforced masonry beams. These examples present only the shear calculations; calculations for flexural strength of beams are presented in previous examples in this chapter.

Example 4.22 Shear reinforcement for a CMU beam.
A nominal 8×24 in. CMU beam is reinforced with one No. 9 Grade 60 reinforcing bar for tension with $d = 20$ in. (Fig. E4.22a). The beam is required to carry a service dead load of 1.0 k/ft (including its self-weight) and a service live load of 1.75 k/ft

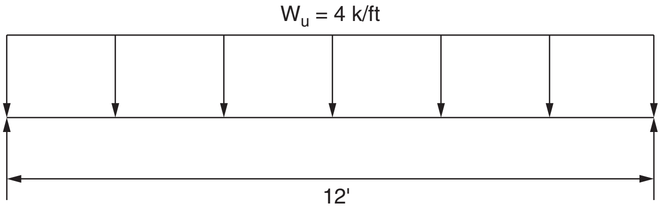


FIGURE E4.22A Beam cross section for Example 4.22.

over an effective span of 12 ft. $f'_m = 2500$ psi. Design the shear reinforcement for this beam.

(Note: This example was presented earlier as Example 4.13 for flexural calculations.)

Solution

Given: $b = 7.63$ in., $d = 20$ in., effective span = 12.0 ft, $f'_m = 2500$ psi, $w_u = 4.0$ k/ft (see Example 4.13 for details).

Calculate the factored shear or shear demand:

$$V_u = \frac{w_u l}{2} = \frac{(4.0)(12)}{2} = 24.0 \text{ kips}$$

Calculate the shear strength of masonry (V_{nm}) from Eq. (4.96):

$$V_{nm} = 2.25 A_n \sqrt{f'_m} = 2.25(7.63)(20)(\sqrt{2500}) = 17{,}168 \text{ lb} = 17.17 \text{ kips}$$

$$\phi V_{nm} = (0.8)(17.17) = 13.74 \text{ kips} < V_u = 24.0 \text{ kips}$$

Since $\phi V_{nm} < V_u$, shear reinforcement is required for this beam. Check the maximum permissible value of V_n from Eq. (4.93).

$$(V_n)_{\max} \le 4 A_n \sqrt{f'_m} = 4(7.63)(20)\sqrt{2500} = 30{,}520 \text{ lb} = 30.52 \text{ kips}$$

$$(\phi V_n)_{\max} = (0.8)(30.52) = 24.42 \text{ kips} > V_u = 24 \text{ kips} \qquad \text{OK}$$

Beam size is adequate. Calculate shear from Eq. (4.100) that must be carried by shear reinforcement:

$$V_{ns} = \frac{V_u - \phi V_{nm}}{\phi} = \frac{24 - 13.74}{0.8} = 12.83 \text{ kips}$$

Try 8 in. spacing for vertical bars as shear reinforcement. Calculate A_v from Eq. (4.103):

$$A_v = \frac{2V_{ns}(s)}{f_y d} = \frac{2(12.83)(8)}{(60)(20)} = 0.17 \text{ in}^2$$

Provide No. 4 Grade 60 bar as vertical bars (shear reinforcement), $A_v = 0.2$ in.^{2} OK

Check maximum permissible spacing of stirrups.

$$s_{\max} = d/2 = 20/2 = 10 \text{ in.} > s = 8 \text{ in. (provided)} \qquad \text{OK}$$

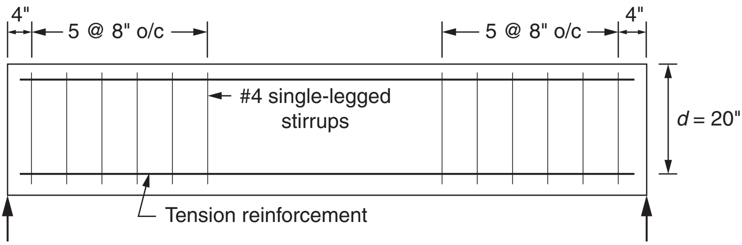


FIGURE E4.22B Details of shear reinforcement.

Check minimum reinforcement:

$$\text{Minimum } A_v \text{ per linear foot of beam} = 0.0007bd = 0.0007(7.63)(20) = 0.11 \text{ in.}^2$$

$$\text{Actual } A_v \text{ provided per foot length of beam} = \frac{(0.2)(12)}{8} = 0.3 \text{ in}^2 > 0.11 \text{ in.}^2 \quad \text{OK}$$

Calculate distance from the center of the span in which shear reinforcement is not required (i.e., where $V_u < \phi V_{nm}$). For a value of $w_u = 4.0$ k/ft and $\phi V_{nm} = 13.74$ kips, this distance is calculated as x on either side of the midspan:

$$x = \frac{(13.74)(12)}{4.0} = 41.22 \text{ in., say 42 in. or 3 ft 6 in.}$$

With cells in the CMU spaced at 8 in. on center, theoretically shear reinforcement is not required for a distance of 3 ft 6 in. on either side of the midspan. This may not result in much of a saving, and often designers choose to provide shear reinforcement in every cell for ductility.

Provide No. 4 Grade 60 vertical bars @ 8 in. on center for shear reinforcement as shown in Fig. E4.22B.

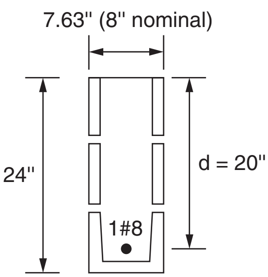


FIGURE E4.23A Beam cross section for Example 4.23.

Example 4.23 Shear reinforcement for a CMU beam.

A nominal 8 × 24 in. simply supported CMU beam is reinforced with one No. 8 Grade 60 reinforcing bar for tension with $d = 20$ in. (Fig. E4.23A). The beam is required to carry a service dead load of 1.0 k/ft (including its self-weight) and a service live load of 1.125 k/ft over an effective span of 12 ft. $f'_m = 1500$ psi. Design the shear reinforcement for this beam. Assume that beam is safe in flexure.

Solution

Given: $b = 7.63$ in., $d = 20$ in., effective span = 12.0 ft, $f'_m = 1500$ psi, $D = 1.0$ k/ft, $L = 1.125$ k/ft

Calculate factored loads on the beam.

Load combinations:

1. $U = 1.4D = 1.4(1.0) = 1.4$ k/ft

2. $U = 1.2D + 1.6L = 1.2(1.0) + 1.6(1.125) = 3.0 \text{ k/ft} > 1.4 \text{ k/ft}$

$$w_u = 3.0 \text{ k/ft (governs)}$$

Calculate the factored shear (or shear demand) V_u.

$$V_u = \frac{w_u l}{2} = \frac{(3.0)(12)}{2} = 18.0 \text{ kips}$$

Calculate the shear strength provided by the masonry (V_{nm}) from Eq. (4.96):

$$V_{nm} = 2.25 A_n \sqrt{f'_m} = 2.25(7.63)(20)(\sqrt{1500}) = 13{,}298 \text{ lb} = 13.3 \text{ kips}$$

$$\phi V_{nm} = (0.80)(13.3) = 10.64 \text{ kips} < V_u = 24.0 \text{ kips}$$

Since $\phi V_m < V_u$, shear reinforcement is required for this beam. Check the maximum permissible value of V_n from Eq. (4.93).

$$(V_n)_{\max} \le 4 A_n \sqrt{f'_m} = 4(7.63)(20)\sqrt{1500} = 23{,}641 \text{ lb} = 23.64 \text{ kips}$$

$$(\phi V_n)_{\max} = (0.8)(23.64) = 18.91 \text{ kips} > V_u = 18 \text{ kips} \qquad \text{OK}$$

Beam size is adequate. Calculate shear from Eq. (4.100) that must be carried by shear reinforcement:

$$V_{ns} = \frac{V_u - \phi V_{nm}}{\phi} = \frac{18 - 10.64}{0.8} = 9.2 \text{ kips}$$

Try 8 in. spacing for single vertical bars as shear reinforcement. Calculate A_v from Eq. (4.103):

$$A_v = \frac{2V_{ns}(s)}{f_y d} = \frac{2(9.2)(8)}{(60)(20)} = 0.12 \text{ in.}^2$$

Provide No. 3 Grade 60 vertical bars as shear reinforcement, $A_v = 0.11$ in.^{2} OK
Check maximum permissible spacing of shear reinforcement:

$$s_{\max} = d/2 = 20/2 = 10 \text{ in.} > s = 8 \text{ in. (provided)} \qquad \text{OK}$$

Check minimum reinforcement:

$$\text{Minimum } A_v \text{ per foot of beam} = 0.0007bd = 0.0007(7.63)(20) = 0.11 \text{ in.}^2$$

$$\text{Actual } A_v \text{ provided per foot length of beam} = \frac{(0.11)(12)}{8} = 0.17 \text{ in}^2 > 0.11 \text{ in.}^2 \quad \text{OK}$$

Calculate distance from the center of the span in which shear reinforcement is not required (i.e., where $V_u < \phi V_{nm}$). For a value of $w_u = 3.0$ k/ft and $\phi V_{nm} = 10.64$ kips, this distance is calculated as x on either side of the midspan:

$$x = \frac{(10.64)(12)}{3.0} = 42.56 \text{ in.}$$

With cells in the CMU spaced at 8 in. on center, theoretically shear reinforcement is not required for a distance 3 ft 6.56 in. on either side of the midspan. This may not result in much of a saving, and some designers may choose to provide shear reinforcement in every cell for ductility.

Provide No. 3 Grade 60 vertical bars @ 8 in. on center for shear reinforcement as shown in Fig. E4.23B.

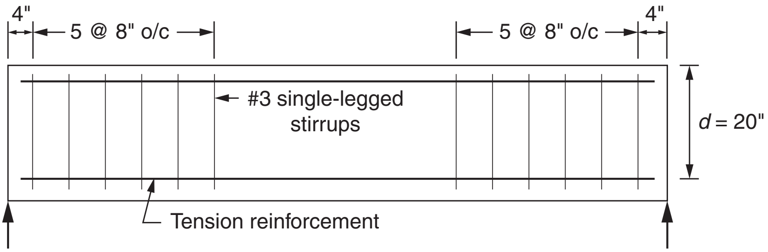


FIGURE E4.23B Details of shear reinforcement.

*Flexural adequacy of the beam in this example can be quickly checked as follows:

$$a = \frac{A_s f_y}{0.8 f'_m b} = \frac{(0.79)(60)}{(0.8)(1.5)(7.63)} = 5.18 \text{ in.}$$

$$\begin{aligned}\phi M_n &= \phi A_s f_y \left(d - \frac{a}{2} \right) \\ &= (0.9)(0.79)(60)\left(20 - \frac{5.18}{2} \right) \\ &= 742.7 \text{ k-in.} = 61.9 \text{ k-ft}\end{aligned}$$

This value is much greater than the factored moment M_u:

$$M_u = \frac{wL^2}{8} = \frac{(3.0)(12)^2}{8} = 54 \text{ k-ft}$$

Other checks for the beam's code compliance would need to be made as illustrated in several previous examples, and are left as readers' exercise (not performed here).

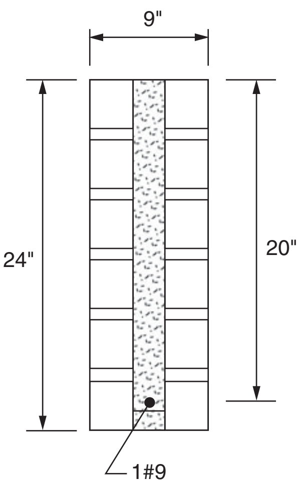


FIGURE E4.24A Beam for Example 4.24.

Example 4.24 Shear reinforcement for a brick beam.

A 9 × 24 in. brick lintel carries a factored gravity load of 2.136 k/ft over an effective span of 16 ft. The tension reinforcement consists of one No. 9 Grade 60 bar placed at $d = 20$ in. from the top (Fig. E4.24*a*). Design shear reinforcement for this beam. Assume $f'_m = 2500$ psi.

Solution

(Note: See Example 4.14 for flexural calculations for this beam.)

Given: $b = 9$ in. (brick masonry), $d = 20$ in., $w_u =$ 2.136 k/ft, effective span = 16 ft, $f'_m = 2500$ psi.

Calculate the factored shear V_u.

$$V_u = \frac{w_u l}{2} = \frac{(2.136)(16)}{2} = 17.09 \text{ kips}$$

Calculate the shear strength provided by the masonry, V_{nm}, from Eq. (4.96):

$$V_m = 2.25A_n\sqrt{f'_m} = 2.25(9)(20)(\sqrt{2500}) = 20{,}250 \text{ lb} = 20.25 \text{ kips}$$

$$\phi V_{nm} = (0.80)(20.25) = 16.2 \text{ kips} < V_u = 17.09 \text{ kips}$$

Since $\phi V_{nm} < V_u$, shear reinforcement is required for this beam. Check the maximum permissible value of nominal shear strength, V_n, from Eq. (4.93).

$$(V_n)_{\max} \le 4A_n\sqrt{f'_m} = 4(9)(20)\sqrt{2500} = 36{,}000 \text{ lb} = 36.0 \text{ kips}$$

$$(\phi V_n)_{\max} = (0.8)(36.0) = 28.8 \text{ kips} > V_u = 17.09 \text{ kips} \qquad \text{OK}$$

Beam size is adequate. Calculate shear from Eq. (4.100) that must be carried by shear reinforcement:

$$V_{ns} = \frac{V_u - \phi V_{nm}}{\phi} = \frac{17.09 - 16.2}{0.8} = 1.11 \text{ kips}$$

Try No. 3 Grade 60 vertical bars as shear reinforcement. Calculate the required spacing of bars from Eq. (4.102):

$$s = \frac{A_v f_y d}{2V_{ns}} = \frac{(0.11)(60)(20)}{2(1.11)} = 59.46 \text{ in.}$$

Check maximum permissible spacing of shear reinforcement.

$$s_{\max} = d/2 = 20/2 = 10 \text{ in.} < 48 \text{ in.}$$

Therefore, use a spacing of 10 in. Check minimum reinforcement:

$$\text{Minimum } A_v \text{ per foot of beam} = 0.0007bd = 0.0007(9)(20) = 0.13 \text{ in.}^2$$

$$\text{Actual } A_v \text{ provided per foot length of beam} = \frac{(0.11)(12)}{10} = 0.13 \text{ in.}^2 \qquad \text{OK}$$

Calculate distance from the center of the span in which shear reinforcement is not required (i.e., where $V_u < \phi V_{nm}$. For a value of $w_u = 2.136$ k/ft and $\phi V_{nm} = 16.2$ kips, this distance is calculated as x on either side of the midspan:

$$x = \frac{(16.2)(12)}{2.136} = 91 \text{ in.}$$

Therefore, shear reinforcement is required for only a distance of 8(12) – 91 = 5 in. from each support. This would need only one stirrup. However, provide No. 3 stirrups @ 10 in. on center for construction purposes.

Provide No. 3 Grade 60 one-legged stirrups @ 10 in. on center for shear reinforcement for the entire beam as shown in Fig. E4.24*b*. The first stirrup is to be placed at $d/4 = 20/4 = 5$ in. from the support.

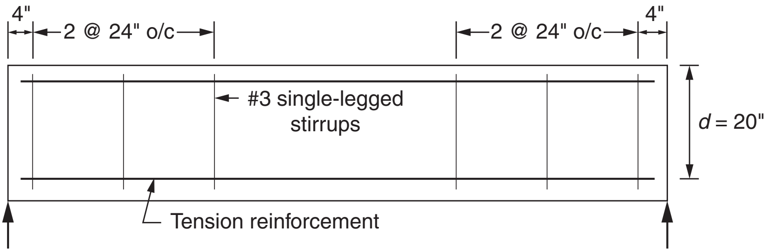


FIGURE E4.24B Details of shear reinforcement.

4.11 LATERAL SUPPORT OF MASONRY BEAMS

Because of the presence of compression, masonry beams, depending on their span lengths, may be subjected to lateral buckling. To ensure that they do not buckle laterally, either their spans should be short or they must be braced laterally at certain intervals. To prevent the lateral buckling of the compression flange of a reinforced masonry beam, MSJC-08 Section 1.13.2 requires that the clear distance between lateral supports of a beam be not more than 32 times the beam thickness. This requirement is analogous to that for flexural members of other materials (steel, reinforced concrete, and wood).

The most common use of masonry beams in buildings occurs for lintels. For example, consider a simply supported 8-in. CMU beam (so the nominal beam width is 8 in.). This beam could have an unsupported span length of $32(7.63) \approx 244$ in. (or 20 ft 4 in.) without any lateral bracings between the supports. In reinforced masonry structures, such span lengths are hardly exceeded. Accordingly, lateral stability is generally not a problem with reinforced masonry beams.

4.12 ANALYSIS OF DOUBLY REINFORCED MASONRY BEAMS

4.12.1 Introduction

Heretofore we have discussed beams that were reinforced only on the tension side, the reinforcement being referred to as tension reinforcement. Such beams are called *singly reinforced* beams. However, the reinforcement can also be provided near the compression side of a beam (i.e., in the compression zone of the beam), in which case the beam is called a *doubly reinforced* beam and the reinforcement is referred to as compression reinforcement.

It is not common to design doubly reinforced masonry beams as singly reinforced beams are generally adequate to carry design loads. The most common use of reinforced masonry beams occurs in the form of lintels which support loads over door and window openings; singly reinforced masonry beams would generally be adequate to carry design loads in such cases. However, in exceptional cases, a doubly reinforced beam may be required to carry loads beyond the capacity of a practical singly reinforced beam.

Reinforced beams, masonry or concrete, are, typically provided with hanger bars which are placed close to the compression face of the beam. These bars are used to secure shear stirrups in the compression zone of concrete or masonry. It is a common practice to ignore their presence when analyzing a beam for flexure as ignoring them simplifies the analysis

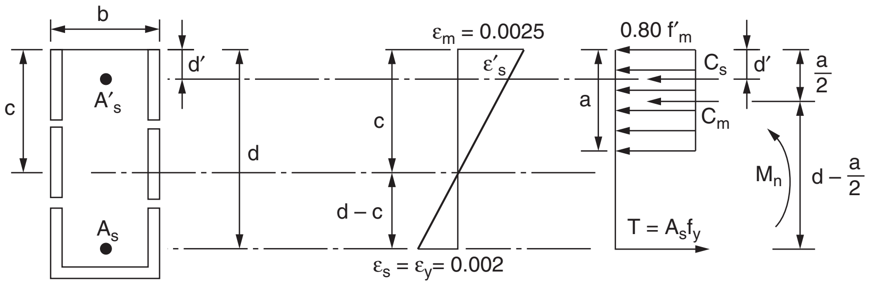


FIGURE 4.13 Strain distribution and forces in a doubly reinforced beam.

considerably. However, their contribution to flexural strength can be accounted for by including them in the analysis, in which case these bars would play the role of compression reinforcement. The compression reinforcement is typically denoted as A_s'; the prime notation distinguishes it from the tensile reinforcement which is denoted as A_s.

A major advantage of a doubly reinforced beam is that a cross section smaller than that of singly reinforced beam can be used to support design loads. For example, in some cases, available beam depth may be limited by architectural or aesthetic constraints. In such cases, a doubly reinforced beam may be required to support loads because the limited depth of a singly reinforced beam may be inadequate to provide the required flexural capacity. Compression reinforcement increases the stiffness of the flexural member resulting in smaller long-term deflections. Additionally, compression reinforcement adds significantly to the ductility of beams which enables them to withstand large levels of movements and deformations that might be caused by lateral loads such as earthquakes.

4.12.2 Analysis of Doubly Reinforced Beams

Determination of the nominal moment capacity of a doubly reinforced beam is similar to that for the singly reinforced beams except that the force in the compression reinforcement must be accounted for in the analysis.

Figure 4.13 shows strain distribution and forces in a doubly reinforced beam. The areas of reinforcement in compression and tension are shown as A_s' and A_s, respectively. As a consequence of being a doubly reinforced beam, the figure shows three forces: compression resultant force in masonry C_m, compression force, C_s, in compression reinforcement (which would not be included if the beam were to be analyzed as a singly reinforced beam), and the tensile force T in tension reinforcement. The stress in tension steel is assumed as being equal to its yield strength f_y as in the case of singly reinforced beams. The stress in compression reinforcement is determined from the strain distribution diagram assuming compressive strain in extreme fibers of the beam equal to ε_m (0.0025 for concrete masonry and 0.0035 for clay masonry). Referring to Fig. 4.13, the values of these forces are obtained as follows:

Compression force in masonry C_m:

$$C_m = 0.80 f_m' ab \qquad \text{(4.6 repeated*)}$$

where a = depth of the compressions block in the beam cross section.

* "C" in Eq. (4.6) has been replaced by C_m in this discussion for clarity, and to distinguish it from the constant C of the quadratic equation that appears later in Eq. (4.116).

Compression force in steel C_s:

$$C_s = A'_s f'_s \tag{4.104}$$

where A'_s = area of compression reinforcement
f'_s = stress in compression reinforcement and calculated from Hooke's law:

$$f'_s = \varepsilon'_s E_s \le f_y \tag{4.105}$$

where ε'_s = strain in compression reinforcement
E_s = modulus of elasticity of reinforcing steel = 29,000 ksi

Note that stress in compression reinforcement, f'_s, is limited to a maximum of yield strength of reinforcing steel, f_y. The strain in compression reinforcement is obtained from similar triangles of strain distribution diagram (Fig. 4.13) in which the neutral axis is located at a distance c from the compression face of the beam.

$$\frac{\varepsilon'_s}{\varepsilon_m} = \frac{c - d'}{c} = 1 - \frac{d'}{c} \tag{4.106}$$

whence

$$\varepsilon'_s = \left(1 - \frac{d'}{c}\right)\varepsilon_m \tag{4.107}$$

Substituting Eq. (4.107) in Eq. (4.105), we obtain

$$f'_s = \varepsilon'_s E_s = \left(1 - \frac{d'}{c}\right)\varepsilon_m E_s \tag{4.108}$$

Substituting Eq. (4.108) in Eq. (4.104), we obtain the force in compression steel:

$$C_s = A'_s\left(1 - \frac{d'}{c}\right)\varepsilon_m E_s \tag{4.109}$$

Equation (4.6) gives an overestimated value of the force in compression area of masonry because the area of compression reinforcement, A'_s, was not deducted from the compression area of masonry (actual area of masonry in compression = $ab - A'_s$). To compensate for this overestimation, the force in compression reinforcement can be expressed as

$$C_s = A'_s\left[\left(1 - \frac{d'}{c}\right)\varepsilon_m E_s - 0.80 f'_m\right] \tag{4.110}$$

Tension force in tension reinforcement T:

$$T = A_s f_y \tag{4.7 repeated}$$

Equating sum of all horizontal forces to zero for equilibrium in the horizontal direction, we have,

$$C_m + C_s - T = 0 \tag{4.111}$$

Substitution of values of various parameters in Eq. (4.111) yields

$$0.80 f'_m ab + A'_s\left[\left(1 - \frac{d'}{c}\right)\varepsilon_m E_s - 0.80 f'_m\right] - A_s f_y = 0 \tag{4.112}$$

In Eq. (4.112), the value of a is unknown. By definition,

$$a = 0.80c \qquad \text{(4.5a repeated)}$$

With the above substitution, Eq. (4.112) can be written as Eq. (4.113):

$$0.8(0.80f_m'bc) + A_s'\left[\left(1-\frac{d'}{c}\right)\varepsilon_m E_s - 0.80f_m'\right] - A_s f_y = 0 \qquad (4.113)$$

Equation (4.113) can be simplified and written as Eq. (4.114):

$$0.64f_m'bc^2 + A_s'\varepsilon_m E_s c - A_s'\varepsilon_m E_s d' - 0.80f_m'A_s'c - A_s f_y c = 0 \qquad (4.114)$$

Equation (4.114) can be expressed as a quadric in c:

$$(0.64f_m'b)c^2 + (A_s'\varepsilon_m E_s - A_s f_y - 0.80f_m'A_s')c - A_s'\varepsilon_m E_s d' = 0 \qquad (4.115)$$

Equation (4.115) is a quadratic of the form: $Ax^2 + Bx + C = 0$, which can be solved for x [= c in Eq. (4.115)]:

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \qquad (4.116)$$

Note that Eq. (4.115) has two roots of x which are given by Eq. (4.116); the negative root should be ignored as it has no significance in this problem. Once c is known, $a = 0.8c$, and quantities C_m and C_s are easily determined. Finally, the magnitude of M_n can be determined by summing up moments due to C_m and C_s about T:

$$M_n = C_m\left(d - \frac{a}{2}\right) + C_s(d - d') \qquad (4.117)$$

Example 4.25 presents calculations for determining the nominal strength of doubly reinforced beam based on the aforederived relationships.

It is noted that reducing the stress in compression reinforcement by $0.80f_m'$ in Eq. (4.110) is a conservative approach, and is not necessary for practical purposes as ignoring this quantity would not affect the results significantly. However, to maintain accuracy of results, Eq. (4.110) would be used in this book to calculate C_s.

If one wishes to ignore the $0.80f_m'$ term from Eq. (4.110), the force in compression reinforcement can be expressed as

$$C_s = A_s'\left[\left(1-\frac{d'}{c}\right)\varepsilon_m E_s\right] \qquad (4.118)$$

Equation (4.118) should also be used to determine C_s if the neutral axis is so located that $c = d'$ in which case the strain in compression reinforcement would be zero (so that $C_s = 0$), or if $c < d'$ in which case the strain in the compression reinforcement would be tensile and the force in the compression force would be tensile. In such cases, Eq. (4.113) would then be simplified to Eq. (4.119):

$$0.8(0.80f_m'bc) + A_s'\left(1-\frac{d'}{c}\right)\varepsilon_m E_s - A_s f_y = 0 \qquad (4.119)$$

Equation (4.119) can be simplified and written as Eq. (4.120):

$$0.64 f'_m bc^2 + A'_s \varepsilon_m E_s c - A'_s \varepsilon_m E_s d' - A_s f_y c = 0 \tag{4.120}$$

Equation (4.120) can be expressed as a quadric in c:

$$(0.64 f'_m b)c^2 + (A'_s \varepsilon_m E_s - A_s f_y)c - A'_s \varepsilon_m E_s d' = 0 \tag{4.121}$$

Equation (4.121) can be solved for c, and the magnitude of M_n can be determined by summing up moments due to C_m and C_s about T as given by Eq. (4.117).

Example 4.25 Nominal strength of a doubly reinforced beam.

A nominal 8 × 24 in. simply supported concrete masonry beam is reinforced with one No. 8 Grade 60 bar for tension reinforcement and one No. 8 Grade 60 bar for compression reinforcement as shown in Fig. E4.25. Assuming f'_m = 2000 psi, calculate the nominal strength of the beam as (a) singly reinforced beam, (b) a doubly reinforced beam. What is the percentage increase in the nominal strength of the beam as a doubly reinforced beam?

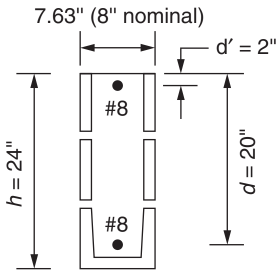


FIGURE E4.25 Cross section of beam in Example 4.25.

Solution

a. Nominal strength as a singly reinforced beam:

The nominal strength of this beam was determined earlier in Example 4.15 (calculations not repeated here).

$$\phi M_n = 64.2 \text{ k-ft}$$

b. Doubly reinforced beam

Given: $b = 7.625$ in., $d = 20$ in., $d' = 2$ in., $A_s = 0.79$ in.^{2} (one No. 8 bar), $A'_s = 0.79$ in.^{2} (one No. 8 bar), f'_m = 2000 psi, f_y = 60 ksi.

The location of neutral axis for a doubly reinforced beam is determined from Eq. (4.115):

$$(0.64 f'_m b)c^2 + (A'_s \varepsilon_m E_s - A_s f_y - 0.80 f'_m A'_s)c - A'_s \varepsilon_m E_s d' = 0 \qquad \text{(4.115 repeated)}$$

Equation (4.115) is a quadratic of the form: $Ax^2 + Bx + C = 0$, which can be solved for x [= c in Eq. (4.115)]:

where

$$A = 0.64 f'_m b = 0.64(2.0)(7.625) = 9.76 \text{ k/in.}$$
$$B = A'_s \varepsilon_m E_s - A_s f_y - 0.80 f'_m A_{sy}$$
$$= (0.79)(0.0025)(29{,}000) - (0.79)(60) - (0.80)(2.0)(0.79)$$
$$= 8.61 \text{ kips}$$
$$C = A'_s E_s \varepsilon_m d' = (0.79)(29{,}000)(0.0025)(2.0) = 114.55 \text{ k-in.}$$

Substituting in Eq. (4.115), we obtain

$$9.76c^2 + 8.61c - 114.55 = 0$$

whence

$$c = +3.01 \text{ in.}, -3.9 \text{ in.}$$

Use $c = 3.01$ in.

$$a = 0.80c = 0.80(3.01) = 2.41 \text{ in.}$$

$$C_m = 0.80 f_m' ab = 0.80(2.0)(2.41)(7.625) = 29.4 \text{ kips}$$

$$\varepsilon_s' = \left(1 - \frac{d'}{c}\right)\varepsilon_m = \left(1 - \frac{2.0}{3.01}\right)(0.0025) = 0.00084 < \varepsilon_y = 0.002$$

Hence, the compression reinforcement has not yielded, and stress in it is calculated from Hooke's law:

$$f_s' = \varepsilon_s' E_s = (0.00084)(29{,}000) = 24.36 \text{ ksi} < f_y = 60 \text{ ksi}$$

$$C_s = A_s'\left[\left(1 - \frac{d'}{c}\right)\varepsilon_m E_s - 0.80 f_m'\right] = (0.79)[24.36 - 0.80(2.0)] = 17.98 \text{ kips}$$

$$T = A_s f_y = (0.79)(60) = 47.4 \text{ kips}$$

Check equilibrium using Eq. (4.111):

$$C_s + C_m - T = 0$$

$$29.4 + 17.98 - 47.4 = -0.02 \text{ kip} \approx 0 \qquad \text{OK}$$

Calculate M_n from Eq. (4.116):

$$\begin{aligned} M_n &= C_m\left(d - \frac{a}{2}\right) + C_s(d - d') \\ &= 29.4\left(20 - \frac{2.41}{2}\right) + 17.98(20 - 2.0) \\ &= 876.21 \text{ k-in.} = 73.02 \text{ k-ft} \\ \phi M_n &= (0.9)(73.02) = 75.72 \text{ k-ft} \end{aligned}$$

$$\text{Percentage increase in moment strength} = \frac{75.72 - 64.2}{64.2}(100) \approx 18\%$$

4.13 LINTELS

4.13.1 General Description

A *lintel* is simply a transversely loaded beam placed over an opening in a wall to support the loads above (Fig. 4.14). These loads might consist of distributed gravity loads (dead load from the wall above plus the service loads from the floors and the roof). The load from the wall might be due to the entire height of wall above (Fig. 4.15*c*) or that from the wall portions contained between two or more openings at different levels in a wall (Fig. 4.15*a*

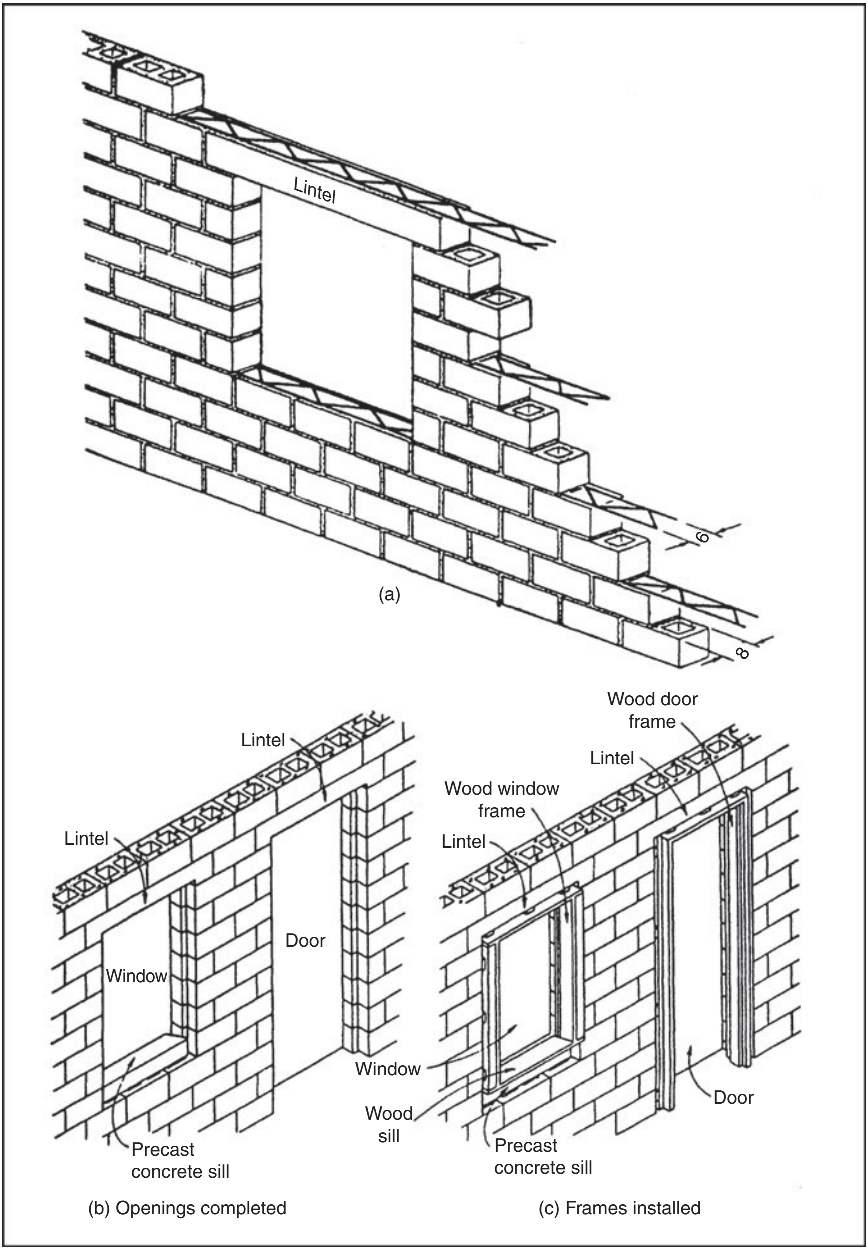


FIGURE 4.14 Lintels over openings in a wall. (*Courtesy: NCMA.*)

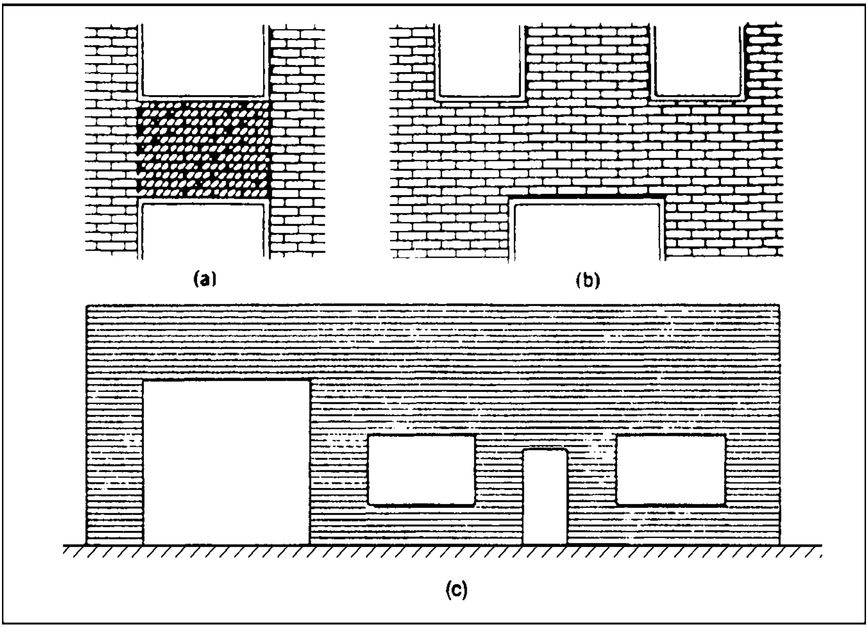


FIGURE 4.15 Wall loads on lintels.

and *b*). In addition, a lintel might have to support concentrated loads from roof joists/floor beams, or other beams framing into the wall.

A variety of schemes are used to provide lintels in masonry buildings. Lintels may be constructed from materials such as steel, reinforced concrete (precast or cast-in-place), wood, and reinforced masonry. Figure 4.16 shows several schematics of steel lintels used in masonry structures. Those shown in Fig. 4.16*a*, *b,* and *c* are made up of steel *angles*, and are used for relatively small openings. For longer spans and heavier loads, steel W-shapes with suspended soffit plates (Fig. 4.16*d*) may be used. Figure 4.17 shows steel W-shapes installed as lintels in a masonry building under construction.

Figure 4.18 shows reinforced concrete masonry lintels under construction. Figures 4.19 to 4.21 show several commonly used schemes of concrete and brick masonry lintels. In general, masonry lintels make it easy to maintain the bond pattern, color, and surface texture of the surrounding masonry, an important consideration for a building's aesthetics and architecture. Masonry lintels provide an advantage over steel or precast concrete lintels in that no special lifting equipment is required for their construction.

Masonry lintels must be reinforced because of the masonry's inherent weakness in tension. Typical reinforced brick and concrete masonry beams and lintels are shown in Figs. 4.18 to 4.20. Figure 4.20*a* and *b* show lintels that can be constructed from single or multiwythe bricks to attain the desired widths. Note the special U-shaped lintel units that can be used to construct reinforced masonry lintels (Fig. 4.21*c*). To carry tension in the bottom fibers of a masonry lintel, it is a common practice to provide typically one or two reinforcing bars which are grouted, which enables them to act in unison with masonry lintel blocks after the grout sets. In general, masonry lintels are integral part of the masonry walls, and therefore, their widths are same as that of the wall of which they are a part.

With the exception of steel and precast concrete lintels, some sort of temporary support is required as the masonry is laid for a lintel or a beam as shown in Fig. 4.18. The function of

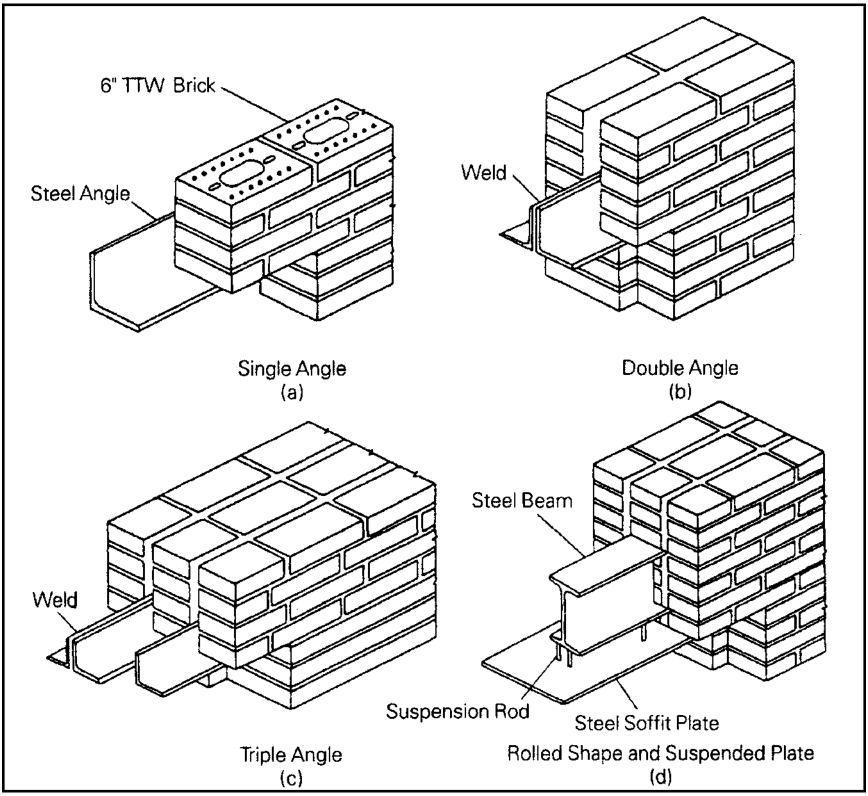


FIGURE 4.16 Steel lintels. (*Courtesy: NCMA.*)

such a support is simply to act as a temporary support until the grout in the masonry sets. Some types of supports, such as a steel plate or angles, may even be left in place permanently.

This section presents a simplified approach to analysis and designs of reinforced masonry lintels. Regardless of the type of masonry (concrete or clay) used for lintels, once the masonry walls are completed, the lintels become integral part of these walls and are usually indistinguishable from the wall (Fig. 4.15*a*). The segment of the wall supported directly above the opening, which is an integral part of the surrounding wall, is often referred to as a "wall beam" or "deep wall beam" and may be analyzed as a deep beam (which is a rigorous analysis similar to that for reinforced concrete deep beams) as discussed in the next section.

4.13.2 Design Considerations

4.13.2.1 Effective Span Lintels must bear on supporting masonry on each side of the opening. The effective span of a lintel is assumed as the distance between the centers of bearings on either side of the lintel. MSJC-08 Section 2.3.3.3 [4.2] *requires* that lintels have a *minimum* bearing of 4 in. This minimum bearing length of 4 in. in the direction of span is considered a reasonable minimum for masonry beams over door and window openings in

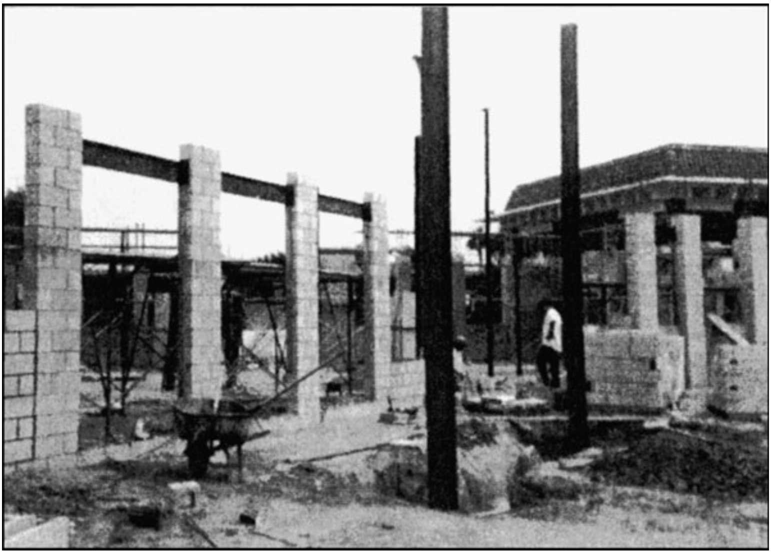


FIGURE 4.17 Rolled steel W-shapes installed as lintels in a concrete masonry building under construction. (*Courtesy: Author.*)

FIGURE 4.18 Concrete masonry lintels under construction. (*Courtesy: Author.*)

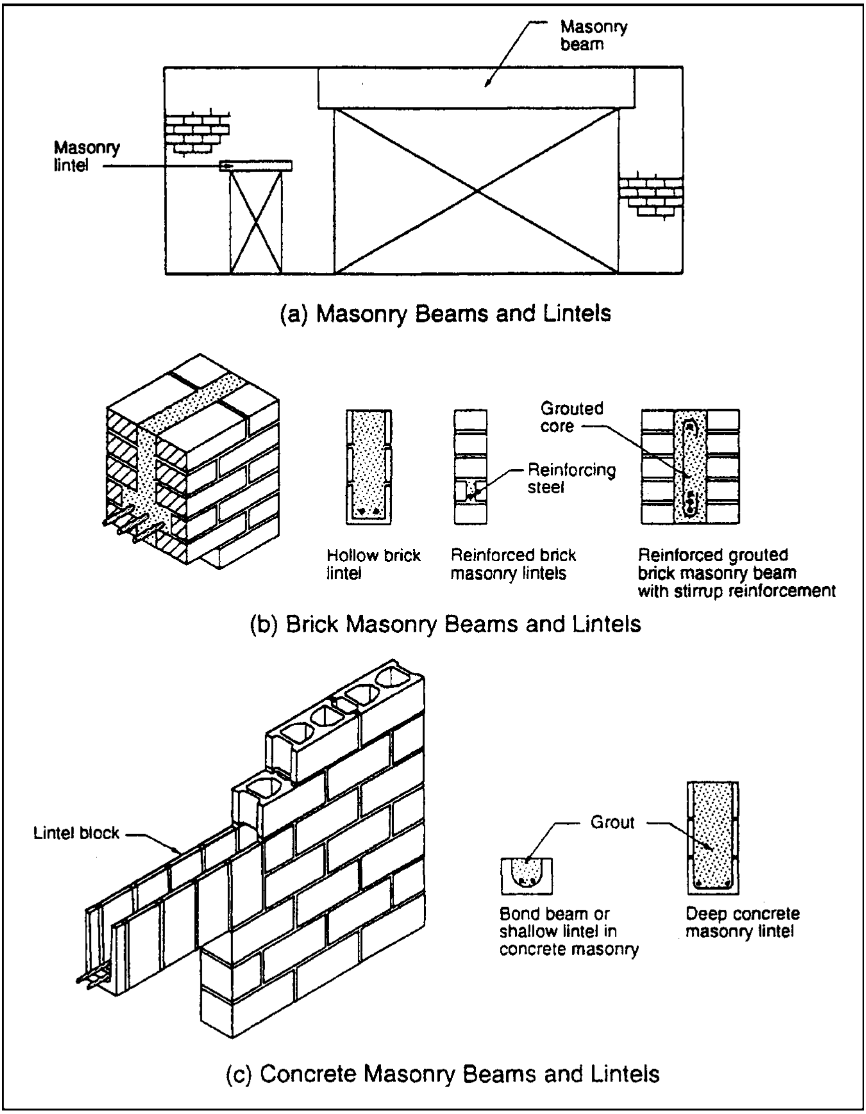


FIGURE 4.19 Typical masonry beams and lintels. (*Courtesy: NCMA.*)

order to prevent concentrated compressive stresses at the edges of the opening. When nominal $8 \times 8 \times 16$ in. concrete masonry units are used to build a lintel, it is customary to assume the length of bearing on each support to be about 8 in. (half of 16 in.). Consequently, it is a common practice to assume the *effective span* of a lintel as distance between the center of these supports, that is, clear span (width of opening) plus 8 in. Practices in this regard might vary, however. In the examples to follow, the effective span is assumed as the clear span *plus* 8 in.

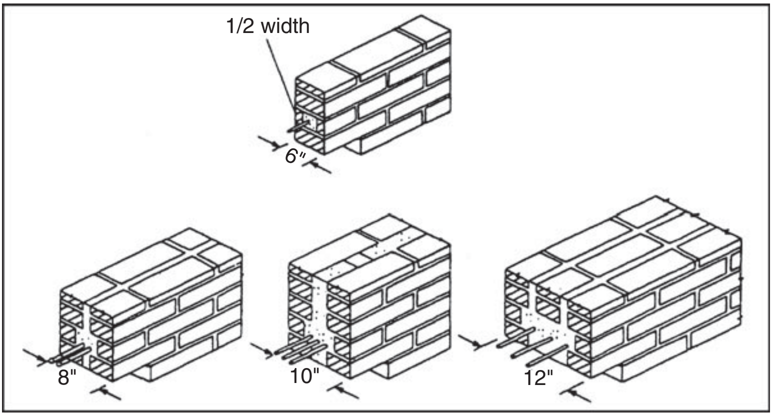


FIGURE 4.20 Reinforced brick lintels. (*Courtesy: NCMA.*)

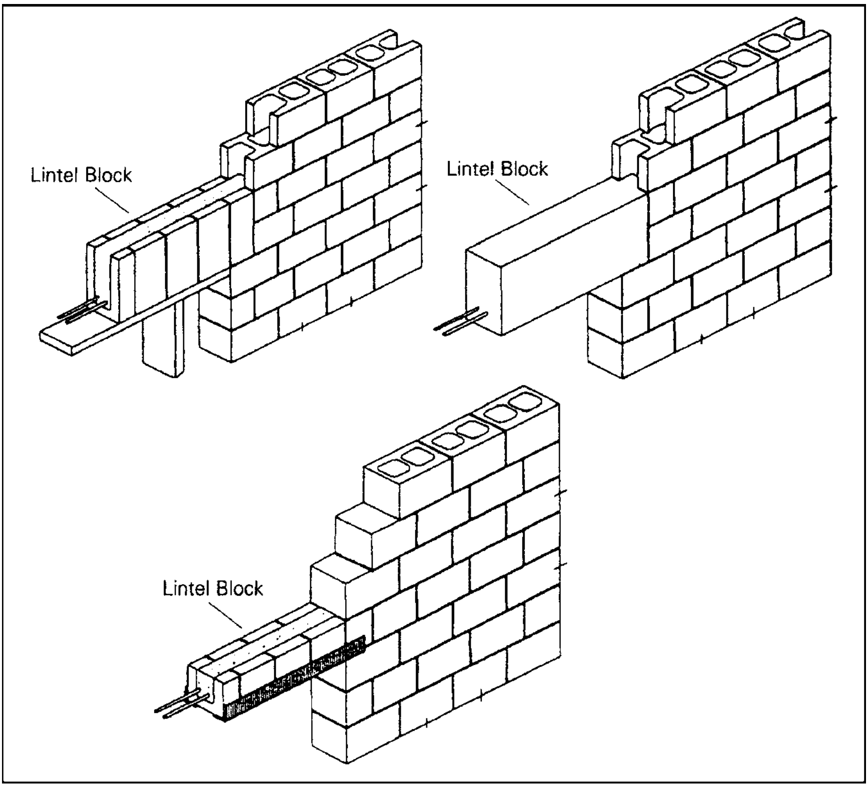


FIGURE 4.21 Concrete lintel blocks in place in a CMU wall. (*Courtesy: NCMA.*)

4.13.2.2 Arching Action in Masonry Walls Because the exact load distribution in a wall above an opening is uncertain, the load carried by a lintel is really indeterminate. The span of a lintel varies depending on the width of the wall opening (for doors and windows). The height of the wall above a lintel may be small or large relative to its span, which depends on the building configuration (heights of doors, and floor-to-floor, or floor-to-roof heights, see Fig. 4.13*a*).

Design loads for lintels can be estimated based on certain assumptions. For a lintel supporting a wall at least as high as its span, and when there are also substantial piers (wall-segments) on both sides of the openings, it may be assumed that *arching action* develops in the wall, which transfers a good portion of the wall load over the lintel to the masonry on either sides of the openings as shown in Fig. 4.22*a*, and only a small triangular

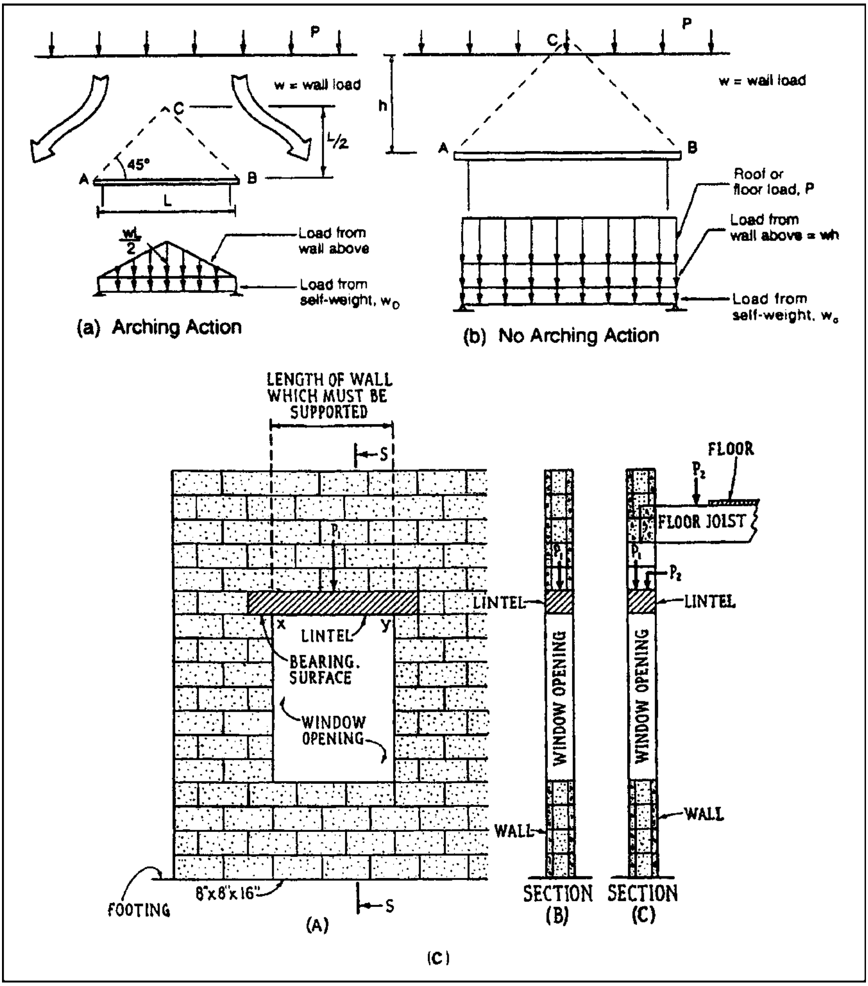


FIGURE 4.22 Load distribution on lintels.

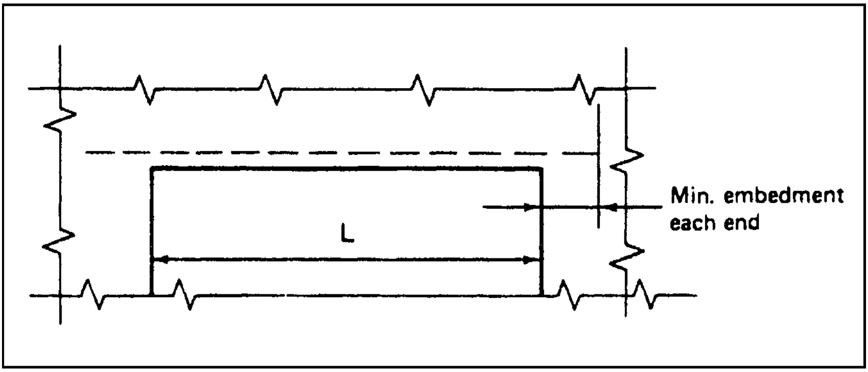


FIGURE 4.23 Minimum embedment for lintel reinforcement.

portion of the wall load to lintel. This assumed arching action amounts to a tacit assumption that if a lintel fails or is removed, only a triangular portion of the wall immediately above the opening would collapse because the masonry will form an arch over the opening, which will support the load above [4.14, 4.15]. Because a lintel is a horizontal member, it is sometimes referred to as *flat arch*. In the absence of the arching action, the load distribution as shown in Fig. 4.22 (similar to load distribution to a conventional beam) is assumed, that is, load from the entire wall height above and the contributory service loads (superimposed loads) are to be supported by the lintel. These loads are shown in Fig. 4.22*b*; all loads applied below the level of point *C* must be carried by the lintel.

It is important to clearly understand the conditions under which arching action may be assumed to exist over an opening in a wall. The development of arching action in a wall over an opening is contingent upon the mass of masonry on either side of the opening providing sufficient restraint to resist the horizontal thrust developed by arching. In areas where limited mass of masonry is available, for example, near corners or between the adjacent openings, it may be necessary to check the resistance of the wall to the horizontal thrusts. If the adjacent masonry is found to be incapable of resisting this horizontal thrust, tension ties should be provided to resist this force, for the lintel or the beam reinforcement cannot be counted upon to provide the required horizontal resistance [4.14]. The required tension ties can be provided by sizing the lintel reinforcement to resist both the beam stress and to provide the tie for the arching action over the opening. For the tensile reinforcement to also serve as tension tie to resist the arch thrust, it must be adequately anchored within the masonry piers on both sides of the openings by providing minimum embedment lengths equal to the required development lengths (discussed in Section 4.18) as shown in Fig. 4.23. Arching action also requires that adequate depth of masonry be present above point *C* in Fig. 4.22 to carry the horizontal compressive forces from the arching thrusts.

Uncertainty in the exact nature of load distribution over openings in a wall makes the load to be carried by a lintel an indeterminate quantity. There is also a great degree of uncertainty as to the ratio of the depth of masonry above the lintel to its span for the arch action to occur. It is common practice to assume existence of arching action if the depth of masonry above the top of the lintel is at least half its effective span lintel plus 8 in., where effective span is equal to clear span plus 8 in. (assuming 8 in. end bearings). This recommended practice assumes arching action to be contingent on meeting the following criteria [4.14–4.15]:

1. Masonry is laid in running bond
2. Sufficient wall height above the lintel exists to form a 45° triangle

3. Wall height above the arch height (i.e., above the apex of 45° triangle) is at least 8 in.
4. Minimum end bearing, typically 4 in., is maintained
5. Control joints are not located adjacent to the lintel
6. Sufficient masonry exists on each side of the opening to resist lateral thrust from arching action

Some of the aforestated criteria, which are based on sound engineering judgment, need clarification. It is assumed that load disperses in the wall at an angle of 45° from the vertical. For proper dispersion of load through the wall, the masonry units in one course should overlap the units in the course below, hence the requirement for wall to be in *running* bond. In a wall laid in *stack* bond, masonry units are stacked on top of each other, without any overlapping of units. Such an arrangement of units makes it impossible for the load to disperse in masonry units of the bottom courses*. For item 3, the arch height is measured from the top of the lintel. This means that masonry should extend half the effective span plus 8 in. above the top of lintel for arching action to exist. Minimum end bearing of 4 in. is a code requirement (MSJC-08 Section 2.3.3.3, Ref. 4.3). Control joints by their very nature create discontinuity in a masonry structure. If control joints are provided adjacent to the lintel, load dispersion cannot occur across them, and arching action cannot be assumed to exist. And finally, sufficient masonry should be present on each side of the opening and above the lintel to absorb the horizontal thrust created by the arching action in the masonry.

It is very important for a designer to clearly understand the ramifications of assuming the presence of arching action over an opening in a wall because this assumption *reduces* considerably the design load for the lintel. Where uncertainty exists, it would be wise to ignore the arching action and design the lintel for the full load supported over it.

4.13.2.3 Design Loads for Lintels Design loads for lintels comprise the following:

1. Self-weight (dead weight) of lintel
2. Dead load of wall above the opening
3. Dead load and live load transferred from the floor or the roof supported by the wall over the opening

Items 1 and 2 are shown schematically by load P_1 in Fig. 4.22*c*, Item 3 by load P_2. These loads are determined as follows:

1. Self-weight of lintel

   All masonry beams and lintels must be solid grouted. The self-weight of a lintel can be determined if its cross-sectional dimensions are known.

   In a design situation, the cross-sectional dimensions are assumed for preliminary design. Because a masonry lintel forms an integral part of a wall, its width is the same as the wall width; therefore, only the depth of a lintel is required to be assumed. For preliminary design, the overall depth (h) of a lintel may be assumed as ¾ in. per liner foot of span, rounded off to a multiple of 8 in. (for typical nominal 8-in. concrete masonry units), subject to a minimum of $h = 8$ in. for beams as required by the code. The self-weight of a masonry lintel can be determined based type of masonry units (lightweight, medium weight, or normal weight) and the unit weight of grout (105 lb/ft^3 or 140 lb/ft^3) used for the wall. Refer to Tables A.19 and A.20 for pertinent information. Alternatively, estimated dead weights of lintels having specific sizes (width and depths) as suggested by NCMA [4.14] and shown in Table 4.8 may be used without any appreciable error.

*Refer to Chap. 6 for more discussion on this topic.

TABLE 4.8 Lintel Weights in Pounds per Linear Foot* [4.13].

Lintel height, in. (nominal)	Wall thickness, in. 4	6	8	10	12
	Lightweight concrete masonry units				
8	24	39	54	69	87
16	49	78	110	141	180
24	72	117	162	207	262
	Normal weight concrete masonry units				
8	29	45	61	77	94
16	58	90	122	155	197
24	87	135	183	231	282

*Face shell mortar bedding. Listed weights are based on grout weight of 145 lb/ft^3 and unit weight of masonry units as 100 lb/ft^3 for lightweight units and 145 lb/^{3} for normal weight.

2. Dead load of wall above the lintel

If arching action is assumed to be present, the lintel should be designed to support the sum of the dead weight of the masonry contained in a 45°-triangular area above the lintel and the self-weight of the lintel (Fig. 4.22*a*). One may consider triangles formed by 45 to 60° over the effective lintel span to determine the wall tributary area whose weight is to be carried by the lintel (45° shown in Fig. 4.22*a*). Obviously, the area contained within a 60° triangle (i.e., an equilateral triangle, area $A = 0.433L^2$, L = base length) is much larger than that contained within the 45° triangle (area $A = 0.25L^2$). However, the recommended practice is to use 45° triangles for computing the dead load from the wall to be carried by the lintel [4.14]. In case of uncertainty regarding the presence of arching action in the masonry above the lintel, or on both sides of the opening to form the arch, the arching action should be disregarded, and the lintel should be designed for the full load based on the tributary area basis.

Estimation of wall height above the top of lintel requires prior knowledge of the depth of lintel which is unknown. For preliminary design, a nominal lintel depth of 8 in. (minimum permissible depth for a concrete masonry beam) for short spans and larger depths (16 or 24 in.) for longer spans may be assumed. These assumed depths may be revised later as necessary for the final design.

For wall heights above the lintel equal to or less than half the effective span of lintel plus 8 in., arching action, being ineffective, is ignored. In such cases, the lintel should be designed to carry the dead weight of the entire rectangular portion of the wall over the effective span (Fig. 4.22*b*).

3. Dead and live load from floor or roof

Dead and live load that may be transferred to a lintel might be (a) a uniform load or (b) a concentrated load. Both types of loads may be determined as follows:

a. Uniform load

Uniform load occurs on a lintel, for example, when floor or roof sheathing is nailed to a ledger beam, which is bolted to the wall directly supported over the opening as shown in Fig. 4.24. The ledger beam, in turn, transfers the floor or the roof load to the wall, which may have an opening. Two load cases may be considered. If the floor or the roof line lies above a distance *from the top of lintel* equal to half the effective

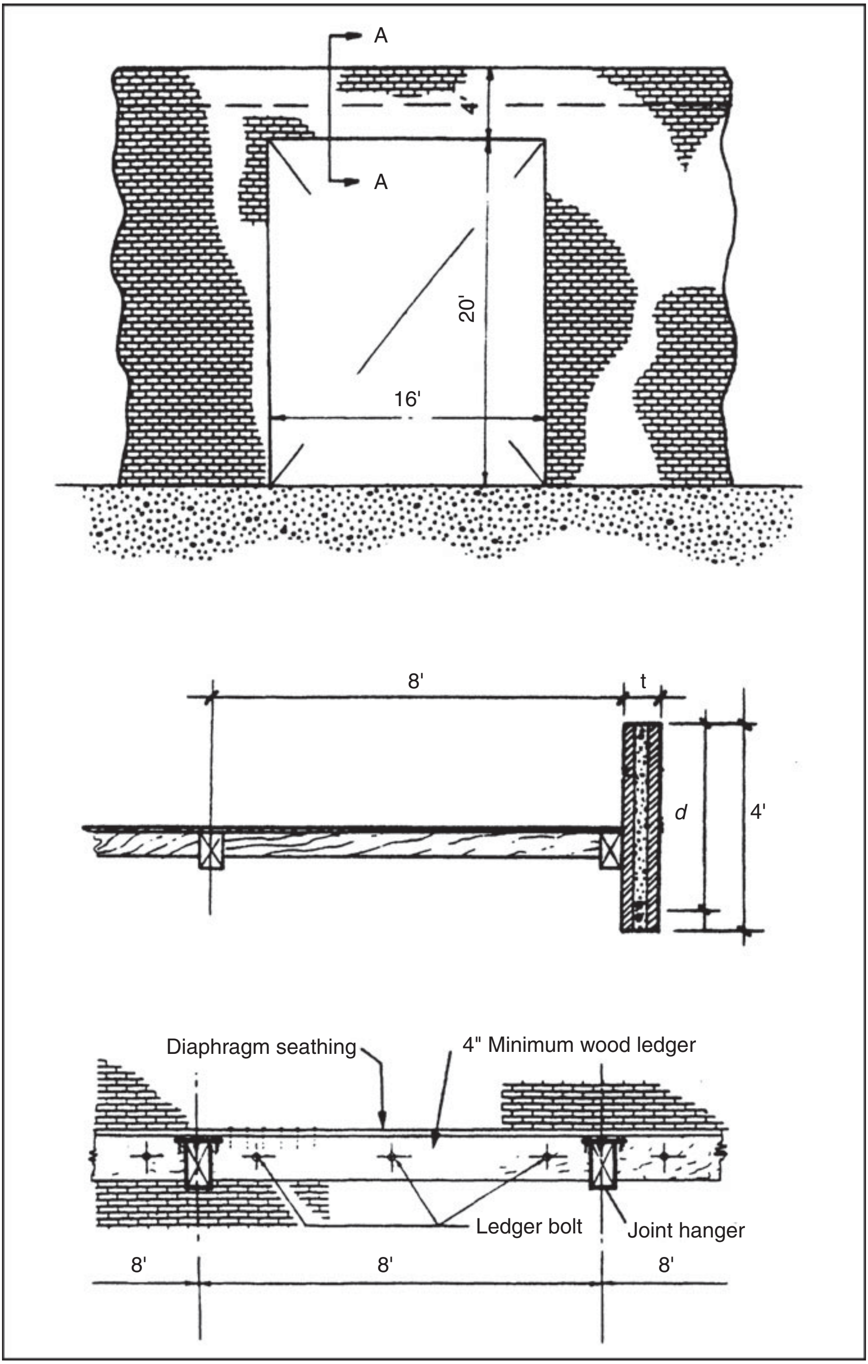


FIGURE 4.24 Uniform load from the roof is transferred to the wall through a ledger beam bolted to the wall.

span plus 8 in., arching action is assumed to be present and the dead and the live loads are transferred to the adjacent masonry through the arching action; no part of this load is carried by the lintel. On the other hand, if the floor or the roof line lies below this height, the load from the wall must be carried as a uniform load by the lintel distributed over the entire span (Fig. 4.22*b*). In addition, there may be loads distributed uniformly only on a portion of the span.

b. Concentrated load

A method for determination of dispersion/distribution of concentrated loads on walls is recommended by NCMA [4.14] and BIA [4.15], which is based on test results reported in the literature [4.16, 4.17].

Reference 4.16 reports results of tests on a wide variety of specimens subjected to concentrated loads, including both concrete block and clay brick masonry, and AAC masonry. It suggests that a concentrated load can be dispersed at a 2:1 slope, terminating at half the wall height (measured from the point of application of the load to the wall footing (Fig. 4.25). In another study [4.16], tests on load dispersion through a bond beam on top of hollow masonry resulted in an angle from horizontal of 59° for a one-course CMU bond beam, 65° for a two-course CMU bond beam, and 58° for a two-course clay brick bond beam, or approximately a 2:1 slope [4.16]. Accordingly, for simplicity in design, a 2:1 slope is used for all cases of load dispersion of a concentrated load. Figure 4.25*a* shows dispersion of a concentrated load (to be always supported on a bearing plate) through a bond beam for both running bond and stack bond. Figure 4.25*b* illustrates the effective length of a wall over which a concentrated load is assumed to be dispersed.

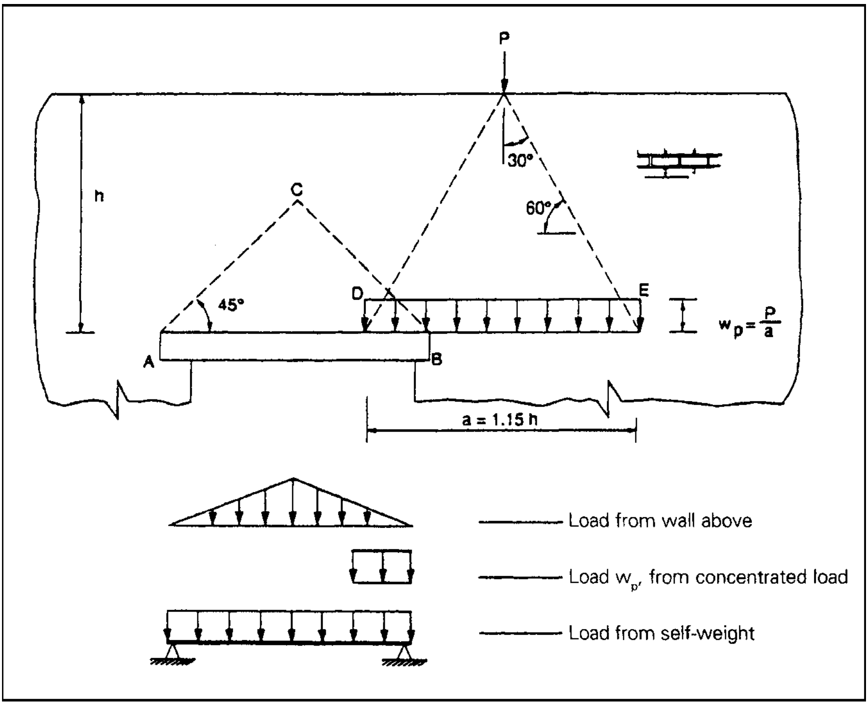


FIGURE 4.25 Load distribution on lintel due to concentrated loads.

For design of lintels, effects of concentrated loads on a wall may have to be checked at various locations, for example, under a bond beam and at midheight. As shown in Fig. 4.25*b*, the effective length of wall subjected to concentrated load dispersion can be expressed by Eq. (4.122):

$$\text{Effective length} = h/2 + b_{pl} \tag{4.122}$$

where h = height of wall measured from the footing to the load point
b_{pl} = width of bearing plate

When two concentrated loads are placed adjacently so that the distance between them is smaller than half the wall height, then the effective length of the wall under each load is given by Eq. (4.123):

$$\text{Effective length} = 2(a + b_{pl}) \tag{4.123}$$

where a = distance between the inside edges of bearing plates

The contribution of the concentrated load on the lintel is equal to the uniform load, w_P, which is calculated based on the effective lengths. For the arbitrary position of the load in Fig. 4.25, only partial length of the lintel (segment DB) carries this uniform load.

According to another approach, the concentrated load is first distributed over a certain limited length of the wall as specified in codes. When a wall is laid up in running bond, this specified length is taken as the smaller of (1) the width of the bearing area plus 4 times the wall thickness or (2) the center-to-center distance between the concentrated loads (MSJC-05 Section 2.1.9.1, Ref. 4.3). A concentrated load (e.g., reaction from a glued-laminated or a steel beam) is typically supported on a bearing plate placed on the top of a wall (Fig. 4.26), and is assumed to be dispersed in the wall at 30° angles (to vertical) from the edges of the bearing plate. The resulting uniform load, w (Fig. 4.26), can be obtained from Eqs. (4.124) and (4.125) (choose the larger value of w):

$$w = \frac{\text{Concentrated load}}{\text{Width of bearing} + 4t} \tag{4.124}$$

$$w = \frac{\text{Concentrated load}}{\text{Distance between concentrated loads}} \tag{4.125}$$

where t = actual thickness of wall. The larger of the above two values should be used for design purposes. This load is then transferred to the lintel as uniform load w. Concepts of load distribution in lintels are illustrated in Examples 4.25 and 4.26.

4.13.2.4 Lintel Depth Considerations Where there is a considerable height of wall above the opening, it is difficult to define the portion of the wall height that exactly constitutes the depth of lintel. Because the lintel is a part of the wall, the width of the lintel is the same as the width of the wall. It has been suggested that for wall heights up to 3 ft above the soffit of the beam, the full height could be considered as the height of a lintel. For greater wall heights above the soffit of the beam, the lintel depth d could be arbitrarily assumed.

A common engineering practice is to establish a depth of lintel which can resist the entire shear in the lintel without shear reinforcement, and which can be determined as discussed in Section 4.10. The nominal shear strength of a transversely loaded beam without any shear reinforcement, V_m, can be determined from Eq. (4.96):

$$V_m = 2.25 A_n \sqrt{f'_m} \tag{4.96 repeated}$$

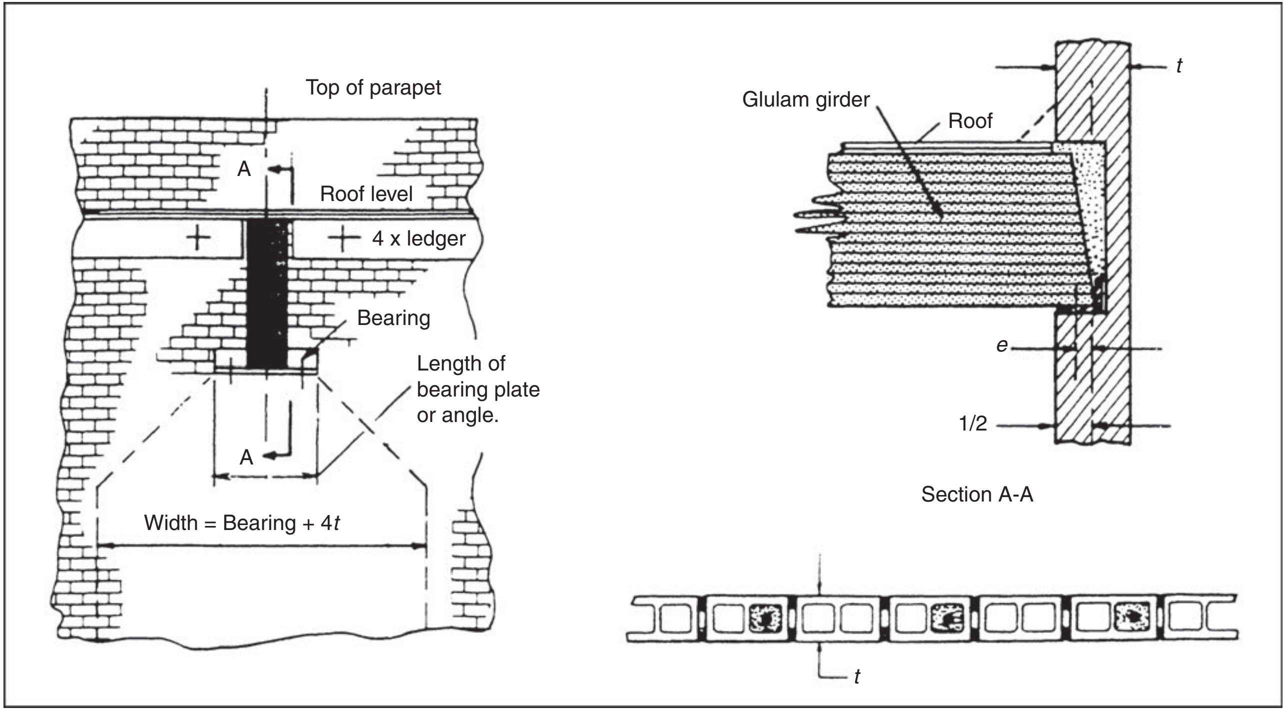


FIGURE 4.26 Distribution of a concentrated load supported on a bearing plate placed on a wall. [4.6].

where A_n = net cross-sectional area of lintel = bd. Thus,

$$V_m = 2.25bd\sqrt{f'_m} \tag{4.126}$$

$$d_{\text{reqd}} = \frac{V_u}{2.25b\sqrt{f'_m}} \tag{4.127}$$

where V_u = the maximum factored shear in the lintel.

When transverse reinforcement is provided, the nominal shear strength, V_n, of the lintel (or a beam) is limited by Eq. (4.93):

$$V_n = 4A_n\sqrt{f'_m} \qquad \text{(4.93 repeated)}$$

The depth of the beam so computed is assumed as the depth for flexural design, and the required tensile reinforcement is computed accordingly. This is not an exact method, but considered reliable and safe. Note that the overall depth of the beam so determined will need to be fully grouted. Design of flexural members involving this concept is illustrated in Example 4.26.

In some cases, the wall height present above the opening, which constitutes the depth of lintel, may be rather short and insufficient to resist shear present in lintel. In that case, the lintel should be designed with the available wall height as its depth, and the necessary reinforcement provided to carry the *entire* shear, since the masonry cannot be permitted to carry any of it. Another situation occurs when the available wall height (i.e., the depth of lintel) may not be sufficient to carry the flexural loads as a singly reinforced beam. In that case, the lintel may be designed as a doubly reinforced beam.

4.13.2.5 Design Moments A lintel is usually designed as a *simple* beam unless other design conditions are warranted. The design load for the lintel is obtained by considering appropriate combination of dead load and applicable live load (roof or the floor load, as the case may be). The magnitude of load to be supported by the lintel depends on whether it qualifies for the arching action or not, and can be obtained as follows:

When a lintel does not qualify for the arching action, the deign moment is determined as a simple beam moment: $M_u = w_u L^2/8$, where w_u = design uniform load resulting from appropriate load combinations.

When a lintel qualifies for arching action, the design moment is obtained by considering a triangular load as discussed earlier. The total load in this case might be due to the dead load of the triangular portion of the wall supported by the lintel plus load dispersed from any concentrated loads from the floor or the roof. When a mass of masonry included in a 45° triangle is assumed as the load on lintel, its height will be equal to half its effective span (Fig. 4.25*b*). For a wall built with material weighing w lb/ft^2 of the wall surface, the maximum (simple beam) moment at the center of the lintel can be obtained from Eqs. (4.128) or (4.129):

$$M_u = \frac{w_u L^3}{24} \tag{4.128}$$

$$M_u = \frac{W_u L}{6} \tag{4.129}$$

in which W_u (= $w_u L^2/4$) is the weight of the masonry enclosed within the 45° triangle. In addition, the lintel must carry its own dead weight.

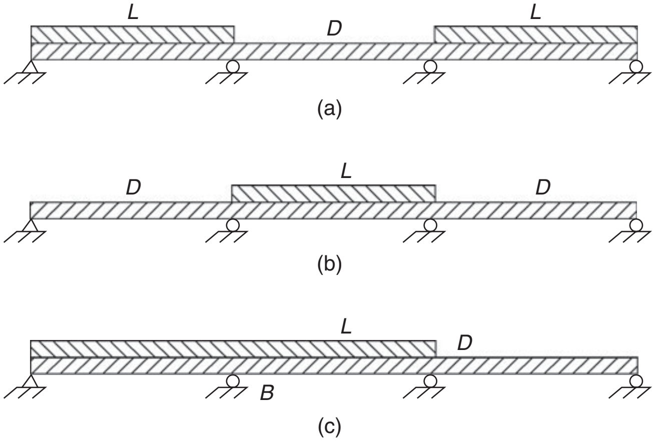


FIGURE 4.27 (*a*) Position of live load for maximum positive moment and deflection in the exterior span, (*b*) live load position for maximum positive moment and maximum deflection in the interior span, (*c*) live load position for maximum negative moment at support *B*.

In regard to designing a lintel for shear, it is instructive to note that the shear in a uniformly loaded simple beam is zero at the midspan, but such is not the case when the dead load is placed on the entire span and the live load is placed on only half the span. In such cases, values of design shears can be determined by drawing shear envelopes.

When a sufficient mass of masonry is present on both sides of a wall opening that might provide full restraint to lintel, then a single-span lintel may be assumed to behave as a *fixed* beam, and the maximum moment in the lintel may be calculated as $-w_u L^2/12$ at the support, and $+w_u L^2/24$ at the midspan. Designers would be well advised to use their discretion carefully in making such an assumption. Note that these moments are considerably smaller than the midspan moment of $wL^2/8$ in a simple span.

In case of a continuous lintel (or a beam), the maximum positive and negative moments would depend on the number of spans. Note that reinforcement would need to be provided for both positive moments (in spans) and negative moments (over the supports). Careful engineering judgment should be exercised in selecting appropriate load combination for determining the magnitude of the uniform load w_u required for calculating maximum positive and negative moments. As discussed earlier, these would be either Load Combination 1 (factored dead load only) or Load Combination 2 (factored dead load plus factored live load plus factored roof live load or snow load or rain load, whichever is critical). It is important to recognize that maximum shears and moments in a continuous span do not occur when both dead and live loads are placed on all contiguous spans. For example, for a continuous span having three spans, the following loading pattern (Fig. 4.27) should be considered for maximum force effects:

1. The maximum positive moment and deflection in an interior span occurs when the dead load is placed on all spans (dead load is always present on all spans) plus live load only on the interior span.
2. For maximum negative moment and maximum shear at an interior support, the dead load should be placed on all spans and live load on only the adjacent spans (i.e., spans on either side of the support under consideration).
3. For maximum moment and deflection in an exterior span, place dead load on all spans and live load only on the exterior spans.

For more than three continuous spans, load positions for maximum effects would be different. All possible loading combinations and load positions should be thoroughly analyzed in order to properly reinforce continuous masonry beams. A comprehensive discussion on analysis of structural loads on buildings can be found in Ref. 4.13. A discussion on design of lintels can be found in Refs. [4.14, 4.15].

4.13.2.6 Deflection Considerations for Lintels Deflection of a lintel may become a limiting design criterion in some cases from the serviceability standpoint. For example, if the lintel supports some sort of brittle nonstructural element such as a plastered ceiling, deflection of beam may be detrimental to that element. Similarly, appreciable deflection of a lintel may cause malfunction while operating the doors of the openings. In such cases, code-imposed deflection limitations should be observed.

Beam deflections constitute serviceability criteria under strength design philosophy, and are discussed later in this chapter.

4.13.2.7 Examples on Lintels Example 4.26 presents analysis of loads on lintels based on the above described simplified approach. The example involves two types of openings: one that qualifies for the arching action and the other that does not. Once design forces (shears and moments) on lintels are determined, they can be designed as beams illustrated in previous examples.

Example 4.26 Analysis of loads on lintels.

Establish load distribution on and design shears and moments for the lintels over various openings shown in Fig. E4.26A. The wall is constructed from normal weight nominal $8 \times 8 \times 16$ in. hollow concrete masonry units, which are grouted solid (grout weight 140 lb/ft^3). Total dead and live load from the roof is 32 lb/ft^2 ($D = 12$ lb/ft^2 and $L = 20$ lb/ft^2). The tributary width for the lintel for roof loads is 10 ft.

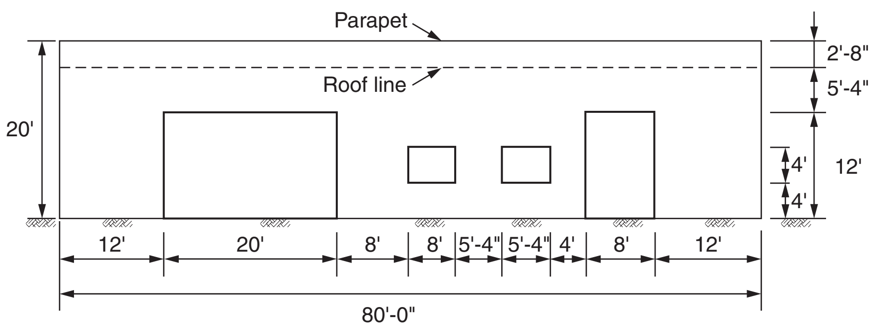


FIGURE E4.26A Wall elevation of a reinforced masonry building.

Solution

The subject wall is the *east wall* of the building. Its roof is supported by transverse girders spaced at 20 ft on centers. Thus, the tributary width for each girder is 20 ft, and that for the subject wall is 10 ft.

The wall contains four openings to be spanned by lintels, which are located at different levels measured from the floor. For the two side openings, the roof dead and live loads are transferred to lintels at a height of 12 ft from the floor; the total height of the wall above these two openings is 8 ft. Similarly, the total wall depth over the two middle openings is 12 ft. Separate checks will be made in each of these two cases to determine if the lintels over these opening qualify for the arching action, and the loads that the lintels must carry will be calculated accordingly.

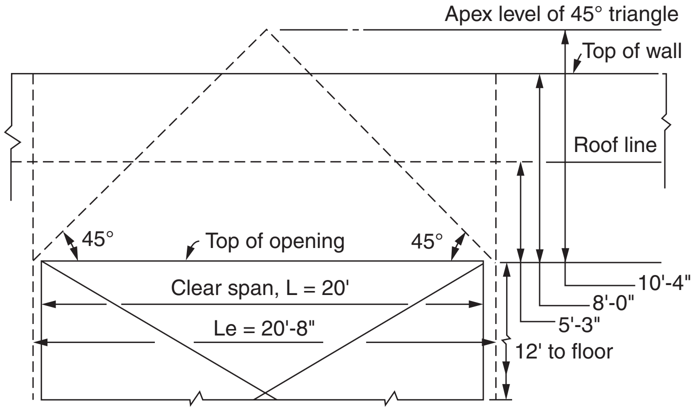


FIGURE E4.26B

a. Left opening (Fig. E4.26B)
Clear span (width of opening),
$L = 20$ ft
Length of bearings on each side of the opening = 8 in. (assumed).
Effective span,

$$L_e = \text{center-to-center of bearings} = 20 + 8/12 = 20.67 \text{ ft.}$$

Assume overall depth of lintel,

$$h = 16 \text{ in.} = 1.33 \text{ ft}$$

Height of masonry above the lintel to the top of wall = 20 − 12 − 1.33 = 6.67 ft.

Minimum height of wall required above the lintel for arching action to exist = ½ (20.67) + 8/12 = 11 ft. This is less than the wall height above the lintel (= 6.67 ft). Hence, the arching action cannot be assumed to exist, and the lintel must be designed to carry all loads based on the tributary width basis.

The dead weight of the fully grouted masonry wall (nominal 8 in. wide, medium weight units, grout weight = 140 lb/ft^3) is estimated as 78 lb/ft^2 of the wall area (Table A.19). Therefore, the dead load is calculated as follows:

Dead load due to the 8 ft high wall (including lintel) = 8(78) = 624 lb/ft length of lintel

Roof dead load = (12) (10) = 120 lb/ft
Total dead load on lintel, $D = 624 + 120 = 744$ lb/ft
Live load from the roof, $L_r = (20)(10) = 200$ lb/ft length of lintel

Load combinations:

1. $U = 1.4D = 1.4(744) = 1042$ lb/ft
2. $U = 1.2D + 1.6L_r = 1.2(744) + 1.6(200) = 1213$ lb/ft > 1042 lb/ft (governs)

$$\text{Design moment, } M_u = \frac{w_u L_e^2}{8} = \frac{(1.213)(20.67)^2}{8} = 64.78 \text{ k-ft}$$

$$\text{Design shear, } V_u = \frac{w_u L_e}{2} = \frac{(1.213)(20.67)}{2} = 12.54 \text{ kips}$$

b. 5 ft 4 in. wide window openings (Fig. E4.26C)

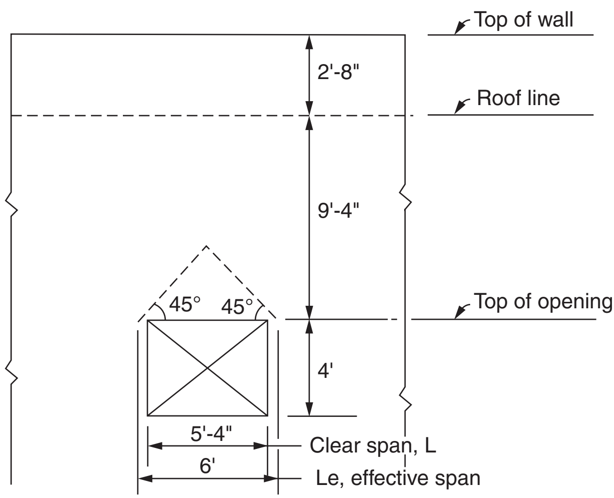


FIGURE E4.26C

Clear span (width of opening),

$$L = 5 \text{ ft } 4 \text{ in.} = 5.33 \text{ ft}$$

Length of bearings on each side of the opening = 8 in. (assumed)
Effective span,

$$L_e = 5.33 + 8/12 = 6 \text{ ft}$$

The minimum depth of masonry above the lintel required for arching action to exist is

$$\frac{L_e}{2} + \frac{8}{12} = \frac{6}{2} + 0.67 = 3.67 \text{ ft}$$

Height of opening above the floor = 4 ft

Assume overall depth of lintel = 8 in. = 0.67 ft (least beam depth permitted)

Height of masonry above the lintel to the top of wall = 20 – 4 – 4 – 0.67 = 11.33 ft > 3.67 ft.

Hence, the arching action can be assumed to be present, and the lintel may be designed to carry the dead load due to the mass of masonry contained in the 45° triangle (with base equal to the effective span of lintel) and its own self-weight. Roof loads are assumed dispersed through arching action in the wall, and not transmitted to the lintel.

$$\text{Unit weight of masonry wall} = 78 \text{ lb/ft}^2$$

Load due to the triangular mass of masonry wall,

$$W = \tfrac{1}{2}(6)\ (6/2)\ (78) = 702 \text{ lb}$$
$$\text{Self-weight of 8-in. deep lintel} = 78 \text{ lb/ft}$$

Load combinations:

The lintel does not carry any live load from the roof. Thus, the factored load due to the triangular mass of the masonry is

$$U = 1.4D = 1.4(702) = 983 \text{ lb}$$

Design moment due to the triangular mass of masonry,

$$M_{u1} = \frac{W_u L_e}{6} = \frac{(983)(6)}{6} = 983 \text{ lb-ft}$$

Design shear due to the triangular mass of masonry, $V_{u1} = \dfrac{W_u}{2} = \dfrac{983}{2} = 492 \text{ lb}$

In addition to the above loads, the lintel must carry its own weight.

Dead load due to nominal 8-in. deep lintel,

$$w = 78 \text{ lb/ft length}$$

Factored dead load, $w_u = 1.4w = 1.4(78) = 109.2$ lb/ft

Factored moments and shear due to the dead load of lintel are

$$M_{u2} = \frac{w_u L_e^2}{8} = \frac{(109.2)(6)^2}{8} = 491.4 \text{ lb-ft}$$
$$V_{u2} = \frac{w_u L_3}{2} = \frac{(109.2)(6)}{2} = 327.6 \text{ lb}$$

Design forces:

$$M_u = M_{u1} + M_{u2} = 983 + 491 = 1474 \text{ lb-ft}$$
$$V_u = V_{u1} + V_{u2} = 492 + 328 = 820 \text{ lb}$$

c. Right side opening (Fig. E4.26D)

Clear span (width of opening),

$$L = 8 \text{ ft}$$

Length of bearings on each side of opening = 8 in.

Effective span = Center-to-center of bearings,

$$L_e = 8 + 8/12 = 8.67 \text{ ft}$$
$$\text{Height of masonry wall} = 20 \text{ ft}$$
$$\text{Height of opening} = 12 \text{ ft}$$

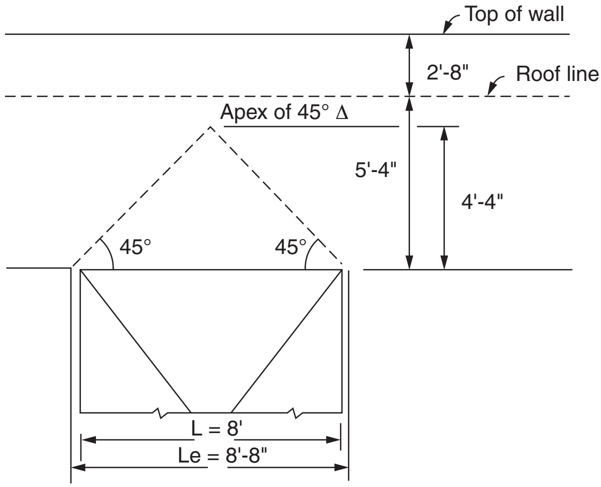


FIGURE E4.26D

Assume the overall depth of lintel = 8 in. = 0.67 ft (least permitted depth)

The minimum depth of masonry above the lintel required for arching action to exist is

$$\frac{L_e}{2}+\frac{8}{12}=\frac{8.67}{2}+0.67=5 \text{ ft}$$

Height of masonry above the opening to the top of wall = 20 – 12 – 0.67 = 7.33 ft. This is greater than 5 ft height of wall required to develop arching action. Hence, the arching action is assumed to exist, and lintel should be designed to carry the dead load due to the mass of masonry contained in the 45° triangle (with a base equal to the effective span of the lintel) and its own self-weight.

Unit weight of masonry wall = 78 lb/ft^2

Load due to the triangular mass of masonry wall,

$$W = ½(8.67)\,(8.67/2)(78) = 1466 \text{ lb}$$

Self-weight of 8-in. deep lintel = 78 lb/ft

Load combinations:

The lintel does not carry any live load from the roof. Thus, the factored load due to the triangular mass of the masonry is

$$U = 1.4D = 1.4(1466) = 2052 \text{ lb}$$

Design moment due to the triangular mass of masonry,

$$M_{u1}=\frac{W_u L_e}{6}=\frac{(2052)(8.66)}{6}=2962 \text{ lb-ft}$$

Design shear due to the triangular mass of masonry,

$$V_{u1} = \frac{W_u}{2} = \frac{2052}{2} = 1026 \text{ lb}$$

In addition to the above loads, the lintel must carry its own weight.

Dead load due to nominal 8-in. deep lintel,

$w = 78$ lb/ft length

Factored dead load,

$w_u = 1.4w = 1.4(78) = 109.2$ lb/ft

Factored moments and shear due to the dead load of lintel are

$$M_{u2} = \frac{w_u L_e^2}{8} = \frac{(109.2)(8.67)^2}{8} = 1026 \text{ lb-ft}$$

$$V_{u2} = \frac{w_u L_3}{2} = \frac{(109.2)(8.67)}{2} = 473 \text{ lb}$$

Design forces:

$$M_u = M_{u1} + M_{u2} = 2962 + 1026 = 3988 \text{ lb-ft} \approx 4 \text{ k/ft}$$
$$V_u = V_{u1} + V_{u2} = 1026 + 473 = 1499 \text{ lb} \approx 1.5 \text{ k/ft}$$

Design loads are summarized in the following table:

	Left opening	Middle opening	Right opening
Design forces	$L_e = 20.67$ ft	$L_e = 6$ ft	$L_e = 8.67$ ft
Moment, M_u	64.78 k-ft	1.47 k-ft	4 k-ft
Shear, V_u	12.54 kips	0.82 kip	1.5 kips

Example 4.27 Design of a lintel: simplified approach.

Figure E4.27a shows details of a concrete masonry wall having a 5 ft 4 in. wide opening. The wall is 8 in. wide (nominal) and grouted solid (medium weight units, grout weight 140 lb/ft^3). The reinforced masonry over the opening is to be utilized as lintel. The wall supports two superimposed concentrated loads 12 kips each (30 percent dead load and 70 percent live load), transferred to it as reactions from roof trusses spaced at 8 ft o.c. The parapet extends 2 ft 8 in. above the roof level as shown in the figure. Determine the tensile reinforcement (Grade 60) required for the lintel if $f'_m = 2000$ lb/in.^{2}.

Solution

Commentary: In this problem, the masonry above the opening is to be used as lintel. The wall is solid grouted. Because there is ample depth (7 ft 6 in.) of masonry above the opening, we will assume that sufficient depth of masonry lintel is available so that the lintel can carry shear without any transverse reinforcement. Alternatively, we can determine the smallest depth of masonry required for flexure, and provide the necessary reinforcement to carry shear. This example uses the first of the two suggested alternatives because of its simplicity. The problem is solved in two parts: (a) analysis of loads and (b) strength design of lintel.

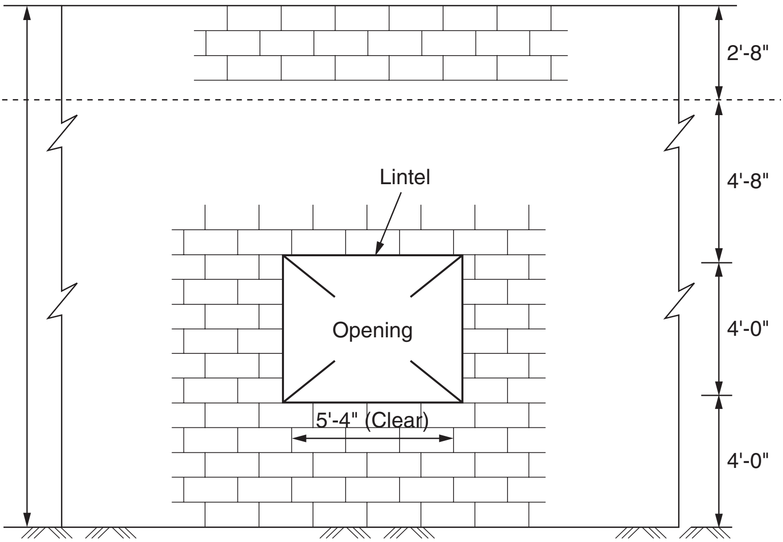


FIGURE E4.27A Design of a lintel.

a. Analysis of loads on lintel
 Calculate the depth of lintel adequate to carry shear without shear reinforcement.

$$\text{Clear span (width of opening),}\quad L = 5\text{ ft }4\text{ in.} = 5.33\text{ ft}$$

Length of bearings on each side of lintel = 8 in. = 0.67 ft

$$\text{Effective span,}\quad L_e = 5.33 + 0.67 = 6\text{ ft}$$

Assume depth of lintel, $d = 8$ in. (to be revised later if necessary)

$$\text{Height of parapet} = 2\text{ ft }8\text{ in.} = 2.67\text{ ft}$$

$$\text{Height of masonry above the top of lintel} = 4 + 2.67 = 6.67\text{ ft}$$

Height of wall required above the top of lintel to develop arching action =

$$\frac{L_e}{2} + 8\text{ in.} = \frac{6}{2} + 0.67 = 3.67\text{ ft} < 6.67\text{ ft}$$

Hence there is sufficient masonry wall depth above the lintel and arching action can be assumed to exist. The superimposed concentrated loads are applied at a height of 4 ft above the top of lintel, which is 4 in. (4 – 3.67 = 0.33 ft = 4 in.) above the apex of the 45° traingle; therefore, these loads would disperse through arching action at an angle of 60°.

Calculate the weight of masonry, W, contained in the 45° triangle with base equal to 6 ft (Fig. E4.27B). The unit weight of masonry is 78 lb/ft^2 of wall (Table A.19). Thus,

$$W = \frac{1}{4} w L_e^2 = \frac{1}{4}(78)(6)^2 = 702\text{ lb}$$

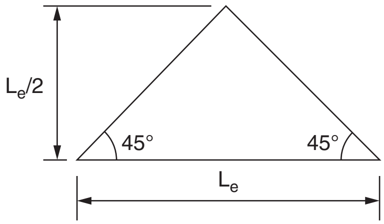


FIGURE E4.27B Load on lintel (unfactored).

Calculate the distribution of concentrated loads (superimposed loads) over the lintel. The 12-kip concentrated loads are placed at a height of $h = 4$ ft above the lintel. The base length of the 60° triangle is given by Eq. 5.2:

$$a = 1.155\ (h) = 1.155(4) = 4.62 \text{ ft}$$

Distributed load due to each concentrated load

$$w = \frac{P}{a} = \frac{12{,}000}{4.62} = 2597 \text{ lb/ft} \approx 2600 \text{ lb/ft}$$

The end distance on which the distributed load acts (Fig. E4.27c),

$$x = 3 - (4 - a/2) = 3 - (4 - 4.62/2) = 1.31 \text{ ft}$$

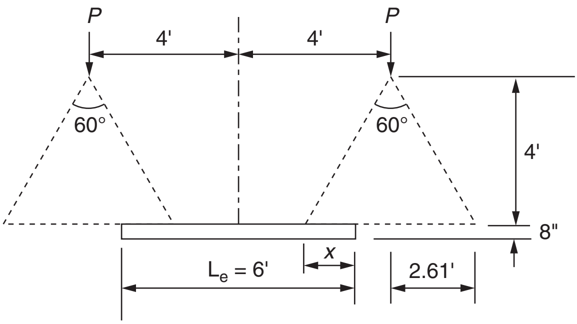


FIGURE E4.27C

Therefore, the design loads (factored loads) on the lintel are as shown on Fig. E4.27D. Factored loads:

Load Combination 1: $U = 1.4D$

Due to triangular load, $U = 1.4\ (702) = 982.8 \text{ lb} \approx 983 \text{ lb}$

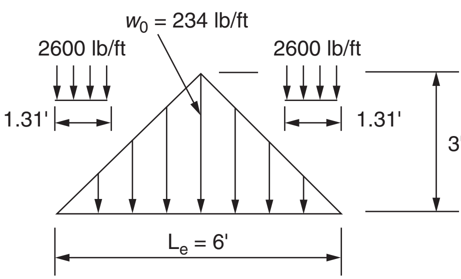


FIGURE E4.27D Load on lintel (factored).

The concentrated loads consist of 30 percent dead load and 70 percent live load. Therefore, the unfactored loads on the end segments of the lintel are

$$D = 0.3\ (2600)\ 780\ \text{lb/ft}$$
$$L = 0.7\ (2600) = 1820\ \text{lb/ft}$$

The factored dead load (live load not considered in Load Combination 1) on the end segment of the lintel is (Fig. E4.27E):

$$U = 1.4D = 1.4\ (780) = 1092\ \text{lb/ft}$$

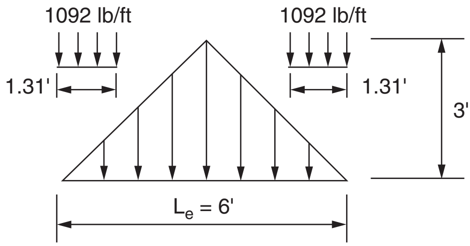


FIGURE E4.27E

Calculate shears and moments due to combination 1 loads.
Due to triangular load on the lintel:

$$V_u = ½\ W_u = ½\ (983) = 492\ \text{lb}$$

Due to distributed load on the end segments (Fig. E4.27F):

$$V_u = 1092\ (1.31) = 1431\ \text{lb}$$

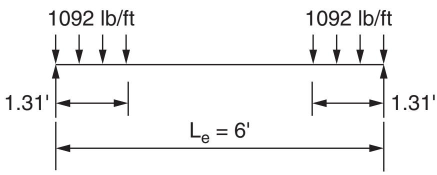


FIGURE E4.27F

Total factored shear = 492 + 1431 = 1923 lb
Moment due to the triangular load (W_u = 708 lb) on the lintel,

$$M_u = \frac{W_u L_e}{6} = \frac{(983)(6)}{6} = 983\ \text{lb-ft}$$

Reaction due to distributed load on the end segments,

$$R = 1431\ \text{lb}$$

Moment due to distributed load on the end segments of the lintel is,

$$M_u = 1431(1.3) - 1092(1.31)\left(3 - \frac{1.31}{2}\right) = 938 \text{ lb-ft}$$

Total factored moment due to Load Combination 1 is, $M_{u1} = 983 + 938 = 1921$ lb-ft
Load Combination 2: $U = 1.2D + 1.6L_r$
The triangular load is all dead load due to the wall.

$$U = 1.2\,(702) = 842 \text{ lb} = W_u$$

$$V_u = ½W_u = 842/2 = 421 \text{ lb}$$

$$M_u = \frac{W_u L_e}{6} = \frac{(842)(6)}{6} = 842 \text{ lb-ft}$$

Due to the distributed load on the end segments of the beam,

$$U = 1.2D + 1.6L = 1.2\,(780) + 1.6\,(1820) = 3848 \text{ lb/ft}$$

Support reaction,

$$V_u = 3848\,(1.31) = 5041 \text{ lb}$$

$$M_u = 5041(3) - 3848(1.31)\left(3 - \frac{1.31}{2}\right) = 3302 \text{ lb-ft}$$

Total factored moment due to Load Combination 2 is

$$M_{u2} = 842 + 3302 = 4144 \text{ lb-ft} > M_{u1} = 1921 \text{ lb-ft}$$

Total factored shear is

$$V_{u2} = 421 + 5041 = 5462 \text{ lb}$$

Thus, the design forces for the lintel are:

$$V_u = 5462 \text{ lb}$$
$$M_u = 4144 \text{ lb-ft}$$

b. Strength design of lintel:
Calculate the depth of lintel to resist the entire shear without any transverse reinforcement. From Eq. (4.127), the depth of lintel required is

$$d_{\text{reqd}} = \frac{V_u}{2.25b\sqrt{f'_m}} = \frac{5462}{2.25(7.625)\sqrt{2000}} = 7.12 \text{ in.}$$

The overall depth of lintel required is, $h = 7.12 + 4 = 11.12$ in., say 12 in.

Therefore, of the total height of wall above the opening, only 12 in. depth of masonry is required to resist the entire shear. This is much smaller than 7.25 ft height of masonry available above the opening. Therefore, no transverse reinforcement is required in the lintel to resist shear.

With $h = 12$ in., it is observed that the apex of the 45° traingle is at the same level as the superimposed loads; arching action is justified. Calculate the amount of tensile reinforcement in the lintel assuming $d = 12 - 4 = 8$ in. Alternatively, try one No. 5 Grade 60 bar (typical), $A_s = 0.31$ in.^{2}, and calculate the moment strength of the lintel. From Eq. (4.9),

$$a = \frac{A_s f_y}{0.8 f'_m b} = \frac{(0.31)(60)}{0.8(2.0)(7.625)} = 1.52 \text{ in.}$$

$$\phi M_n = \phi A_s f_y \left(d - \frac{a}{2} \right) = 0.9(0.31)(60)\left(8 - \frac{1.52}{2} \right)$$
$$= 121.2 \text{ k-in.} = 10{,}100 \text{ lb-ft} > 4144 \text{ lb-ft}$$

The moment strength of the lintel is 15,683 lb-ft > 4144 lb-ft (design moment).

Commentary:

a. The design moment in the above example is relatively small compared to the moment strength of the lintel. This is because the width of the opening is rather small, which qualifies for the arching action in the supported masonry. The design shears and moments would be significantly large in the case of wider openings as illustrated in Example 4.26.
b. In the above example, one No. 5 bar has been used for tensile reinforcement. It would have been just as satisfactory to use two No. 4 Grade 60 bars (A_s = 0.4 in.^{2}).

4.14 MASONRY WALL BEAMS (DEEP WALL BEAMS)

4.14.1 Wall Beams Defined

Section 4.13 presented a *simplified* approach to analysis and design of masonry lintels that are parts of masonry wall panels spanning openings. The qualifier "simplified" is used here to emphasize the fact that the masonry lintel is not an isolated flexural member; rather it is a part of the entire wall panel spanning the opening. The simplified approach assumed the existence of a lintel of limited depth within the wall panel, which carried loads from the supported wall panel (with or without arching effect). Wall panels that span openings instead of acting as bearing walls with continuous line support along the bottom (Fig. 4.28) are often referred to as "wall beams"[4.16]. "Deep beams" and "deep wall beams" are other terms used in the literature to describe wall beams. The term "wall beams" would be used throughout the following discussion.

There are no design provisions or guidance for analysis and design of wall beams in the MSJC *Code* [4.3], presumably because of the lack of test data on the behavior of wall beams (or deep masonry beams), unlike reinforced concrete, for which guidance for analysis and design (based on *strut-and-tie* model) is provided in App. A of the ACI *Code* (ACI 318-05, Ref. 4.2).

4.14.2 Concept of Deep Beams

ACI 318-05 (Section 10.7, Ref. 4.2) defines *deep beams* as "members loaded on one face and supported on the opposite face so that compression struts can develop between the load and the supports, and have either:

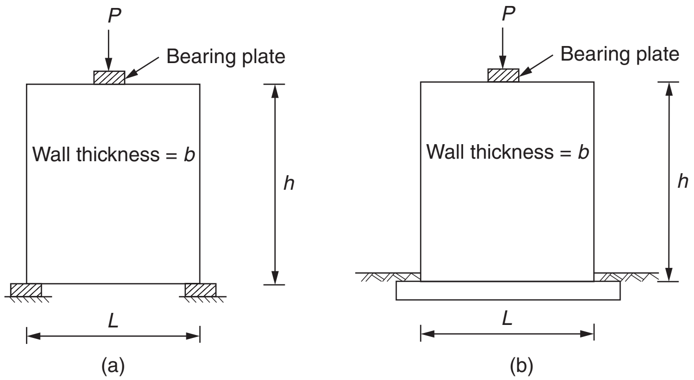


FIGURE 4.28 (*a*) A wall beam (or deep wall beam or deep beam, supported on discrete supports at the ends), (*b*) conventional wall (continuously supported at the bottom along its entire length).

1. Clear span ℓ_n equal to or less than four times the overall member depth, or

2. Regions with concentrated loads within twice the member depth from the face of the support."

ACI 318-05 (Section 10.7, Ref. 4.2) also requires deep beams to be designed either taking into account nonlinear distribution of strain or as suggested in App. A (of ACI 318-05), which describes a strut-and-tie model. A discussion on the behavior of deep concrete flexural members can be found in the literature [4.19–4.22].

4.14.3 Examples on Deep Wall Beams

The following examples illustrate the design approach discussed in the preceding section. These examples*, originally based on the allowable stress design (ASD) philosophy as presented in Ref. 4.18 (published in 1976), have been modified to reflect both the current allowable stress design and strength design philosophy. The MSJC-08 *Code* [4.3] does not provide any guidance for any analysis or design approach. These examples present the ASD approach for design of wall beams.

Example 4.28 Design of a single-span deep beam: allowable stress design.
A nominal 8 in wide and 24 ft deep concrete masonry wall panel is supported over a span (center-to-center of supports) 24 ft. The length of supports is 2 ft each as shown in Fig. E4.28. The wall panel is fully grouted and weighs 100 lb/ft^2. The wall panel carries a uniform load of 2500 lb/ft including its own weight. Calculate (a) the load-carrying capacity of the wall panel and check if the panel can safely support its weight, (b) flexural reinforcement requirements, and (c) shear reinforcement requirements. Use f'_m = 1500 lb/in.^{2} and Grade 60 reinforcement.

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*These examples (based on ASD) are taken from Ref. 4.18 with permission from National Concrete Masonry Association, Herndon, VA., and have been expanded by the author to reflect the provisions of both the allowable stress design and strength design philosophy of MSJC-08 *Code* [4.3].

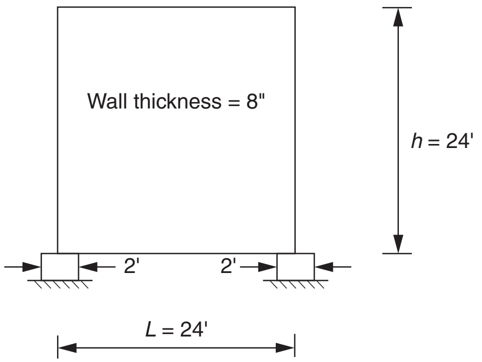


FIGURE E4.28 Wall panel supported over bearing pads.

Solution

Given: f'_m = 1500 lb/ft², wall thickness, $b = t = 7.625$ in. (8 in. nominal), wall height, $h = 24$ ft, effective span $L = 24$ ft, length of support = 2.0 ft., dead weight of wall = 100 lb/ft²

$$\text{Gravity load on wall} = 2500 \text{ lb/ft}$$

Maximum permitted bearing length of supports

$$= 0.1 \text{ (distance between vertical supporting edge members)}$$
$$= 0.1(24)$$
$$= 2.4 \text{ ft} > 2.0 \text{ ft} \qquad \text{OK}$$

Allowable stress design approach (Ref. 4.18):

a. Calculate allowable compressive stress:

$$F_a = 0.20 f'_m\left[1-\left(\frac{h}{48t}\right)^3\right] = 0.20(1500)\left[1-\left(\frac{(24)(12)}{(48)(7.625)}\right)^3\right] = 154 \text{ lb/ft}^2$$

Actual compressive stress is calculated on area equal to the wall thickness times the length of bearing plus twice the panel thickness.

$$\text{Reaction at supports} = \frac{wL}{2} = \frac{2500(24)}{2} = 30{,}000 \text{ lb}$$

$$\text{Actual compressive stress} = \frac{30{,}000}{(7.625)[12+2(7.625)]} = 144 \text{ lb/ft}^2 < F_a = 154 \text{ lb/ft}^2 \qquad \text{OK}$$

Commentary: If we calculate the allowable compressive stress based on MSJC-08, its value would be as follows:

The radius of gyration for an 8 in. nominal wall,

$$r = 0.289t = 0.289\ (7.625) = 2.2 \text{ in.}$$

$$\frac{h}{r} = \frac{(24)(12)}{2.2} = 130.91 > 99$$

$$F_a = 0.25 f'_m \left[\left(\frac{70r}{h} \right)^2 \right] = 0.25(1500) \left[\left(\frac{70}{130.91} \right)^2 \right] = 107.22 \;\text{lb/ft}^2 < 144 \;\text{lb/ft}^3 \qquad \text{NG}$$

b. Calculate flexural stress and required reinforcement:
For single span beams, $\varepsilon = ½$

$$\beta = \frac{h}{2l} = \frac{24}{2(24)} = \frac{1}{2}$$

$$\text{Flexural stress} = (\text{coefficient from Table 4.9}) \left(\frac{w}{b} \right)$$

where

$$w = \text{load per inch of wall} = \frac{2500}{12} = 208.3 \;\text{lb/in.}$$

From Table 4.9, for $\beta = ½$, Coefficient = +0.75 (top of beam)
Coefficient = −1.2 (bottom of beam)

$$\text{Flexural stress (top)} = 0.75 \left(\frac{w}{b} \right) = 0.75 \left(\frac{208.3}{7.625} \right) = 20.5 \text{ lb/in.}^2 \text{ (compression)}$$

$$\text{Allowable compressive stress} = 0.33 \; f'_m = 0.33(1500)$$
$$= 495 \text{ lb/in.}^2 > 20 \text{ lb/in.}^2 \qquad \text{OK}$$

$$\text{Flexural stress (bottom)} = -1.2 \left(\frac{w}{b} \right) = -1.2 \left(\frac{208.3}{7.625} \right) = -32.8 \text{ lb/in.}^2 \text{ (tension)}$$

TABLE 4.9 Moment Stress Coefficients for Simply Supported Single-Span Uniformly Loaded Wall Beams: $\varepsilon = ½$*

$\beta = h/2l$	Top of beam	Bottom of beam
½	+0.75	−1.2
2/3	+0.33	−1.05
1	+0.06	−0.97

*Based on data contained in "Design of Deep Girders," Portland Cement Association, 1951 [4.20]. Positive values indicate compression; negative values indicate tension.

TABLE 4.10 Moment Stress Coefficients for Continuous Uniformly Loaded Wall Beams* [4.18]

	Midspan		Centerline of support					
$\beta = h/2l$	ε = 1/20 to 1/5		ε = 1/20		ε = 1/10		ε = 1/5	
	Top	Bottom	Top	Bottom	Top	Bottom	Bottom	Top
½	+1.05	−1.31	−1.25	+19.32	−1.25	+9.32	−1.15	+4.30
2/3	+0.48	−1.05	−0.50	+19.07	−0.50	+9.07	−0.48	+4.06
1	+0.08	−1.00	−0.08	+19.00	−0.08	+9.00	−0.07	+4.00

*Based on data contained in "Design of Deep Girders," Portland Cement Association, 1951 [4.22]. Positive values indicate compression; negative values indicate tension.

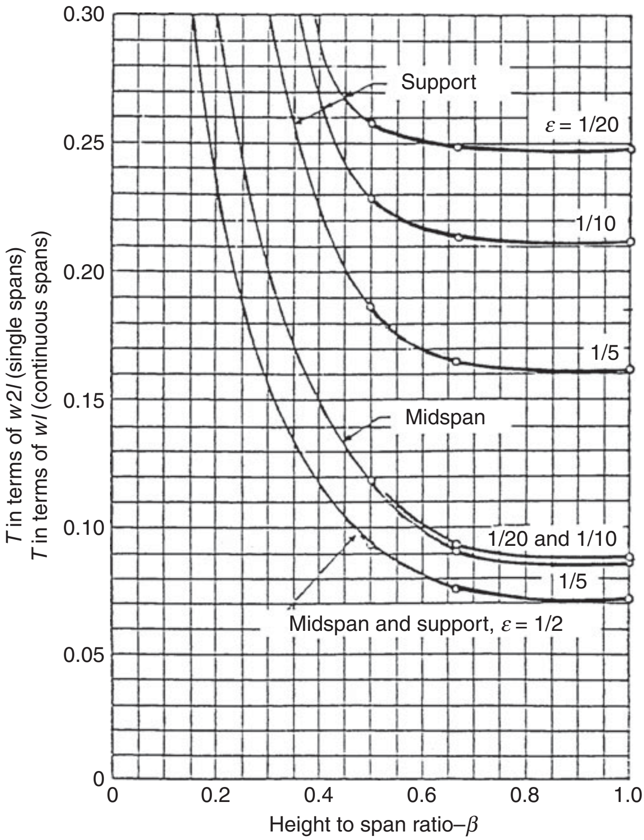


FIGURE 4.29 Determination of resultant T for Example 4.28 [4.18].

From Fig. 4.29, find coefficient for T, resultant of tensile stresses, and multiply by $w2l$ (for single-span beams), where w = load on wall per linear foot.

$$\begin{aligned} T &= (0.095)\,(w2l) \\ &= (0.095)\,(2500)\,(2)\,(24) \\ &= 11{,}400 \text{ lb} \end{aligned}$$

Calculate the area of tension reinforcement:

$$A_s = \frac{T}{F_s} = \frac{11{,}400}{24{,}000} = 0.48 \text{ in.}^2$$

Select one No. 7 bar,

$$A_s = 0.6 \text{ in.}^2 > 0.48 \text{ in.}^2 \qquad \text{OK}$$

Assume that reinforcing bar is located near the bottom of beam, $d = (24)\,(12) - 4 = 284$ in.

c. Calculate the shear stress and the required shear reinforcement. Assume that critical section is located at 0.15 l_n from the face of support, where l_n clear span between supports.

$$0.15 l_n = 0.15\ (24 - 2) = 3.3 \text{ ft}$$

At the critical section, the maximum shear is

$$V = w\left[\frac{L}{2} - (3.3 + 1)\right] = (2500)\left[\frac{24}{2} - (3.3 + 1)\right] = 19{,}250 \text{ lb}$$

The shear stress at the critical section is

$$v = \frac{V}{bd} = \frac{19{,}250}{(7.625)(284)} = 8.9 \ \text{lb/in.}^2$$

$$\text{Allowable shear stress} = 1.1\sqrt{f'_m} = 1.1(\sqrt{1500}) = 42.6 \text{ lb/in.}^2$$
(without shear reinforcement)

Therefore, no special shear reinforcement is required.

Minimum wall reinforcement = 0.002 (7.625)(12) = 0.183 in.^{2}/ft*
(*Total in both direction; no more than $^2/_3$ in one direction)

Provide No. 5 bars at 32 in. o.c. in horizontal direction, $A_s = 0.115$ in.^{2}
Provide No. 5 bars at 48 in. o.c. in vertical direction, $A_s = 0.077$ in.^{2}
Total reinforcement,

$$A_s = 0.115 + 0.077 = 0.192 \text{ in.}^2 > 0.183 \text{ in.}^2 \qquad \text{OK}$$

Example 4.29 Design of an interior span of a continuous deep beam: allowable stress design.
The interior span of a nominal 12 in. wide and 15 ft high continuous wall panel measures 30 ft center-to-center of supports which are 3 ft wide (Fig. E4.29). The wall panel carries a uniform superimposed load of 3500 lb/ft in addition to its own dead weight of 1500 lb/ft (100 lb/ft^2 of its surface area). Determine (a) compressive stress at supports, (b) flexural stresses and required tensile reinforcement, and (c) shear stress and required transverse reinforcement. Assume $f'_m = 2000$ lb/in.^{2}, and use Grade 60 reinforcement.

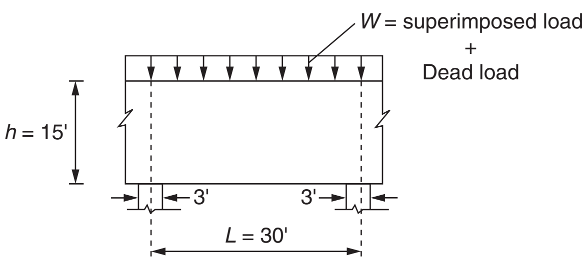


FIGURE E4.29

Solution

Given: $f'_m = 2000$ lb/in.^{2}, length of support, $c = 3$ ft, length of span = 30 ft, height of beam = 15 ft, width of beam = 11.625 in. (12 in. nominal), uniform load = 1500 + 3500 = 5000 lb/ft = 416.67 lb/in.

a. Check compressive stress at support.

$$F_a = 0.20 f'_m\left[1-\left(\frac{h}{48t}\right)^3\right] = 0.20(2000)\left[1-\left(\frac{(15)(12)}{(48)(11.625)}\right)^3\right] = 387 \text{ lb/in.}^2$$

Actually compressive is calculated on area equal to the wall thickness times the length of bearing plus twice the panel thickness.

$$\text{Actual compressive stress } = \frac{(5000)(33)}{(11.625)[3(12)+2(11.625)]} = 239 \text{ lb/in.}^2 < F_a$$
$$= 387 \text{ lb/in.}^2 \qquad \text{OK}$$

Commentary: If we calculate the allowable compressive stress based on MSJC-08, its value would be as follows:

The radius of gyration for a 12 in. nominal wall, $r = 0.289t = 0.289\ (11.625) = 3.36$ in.

$$\frac{h}{r} = \frac{(15)(12)}{3.36} = 53.57 < 99$$

$$F_a = 0.25 f'_m\left[1-\left(\frac{h}{140r}\right)^2\right] = 0.25(2000)\left[1-\left(\frac{53.57}{140}\right)^2\right] = 427 \text{ lb/in.}^2 > 239 \text{ lb/in.}^2 \quad \text{OK}$$

b. Calculate flexural stress and the reinforcement requirement:

$$\text{For continuous span beams, } \varepsilon = \frac{c}{l} = \frac{3}{30} = \frac{1}{10}$$

$$\beta = \frac{h}{l} = \frac{15}{30} = \frac{1}{2}$$

$$\text{Flexural stress} = (\text{coefficient from Table 4.9})\left(\frac{w}{b}\right)$$

where

$$w = \text{load per inch of wall} = 416.67 \text{ lb/in.}$$

From Table 4.9, for $\beta = ½$, Coefficient = +1.05 (top of beam)
Coefficient = −1.31 (bottom of beam)

$$\text{Flexural stress (top)} = 1.05\left(\frac{w}{b}\right) = 1.05\left(\frac{416.67}{11.625}\right) = 38 \text{ lb/in.}^2 \text{ (compression)}$$

$$\text{Allowable compressive stress} = 0.33\ f'_m = 0.33\ (2000)$$
$$= 660 \text{ lb/in.}^2 > 38 \text{ lb/ft}^2 \qquad \text{OK}$$

$$\text{Flexural stress (bottom)} = -1.31\left(\frac{w}{b}\right) = -1.31\left(\frac{416.67}{11.625}\right) = -47 \text{ lb/in.}^2 \text{ (tension)}$$

From Fig. 4.29, find coefficient for T, resultant of tensile stresses, and multiply by wl (for continuous span beams), where w = load on wall per linear foot.
Midspan:

$$T = 0.12(wl) = (0.12)\ (5000)\ (30) = 18{,}000 \text{ lb}$$

Support:

$$T = (0.23)\ (5000)\ (30) = 34{,}500 \text{ lb}$$

Calculate the area of tension reinforcement:

$$\text{Midspan: } A_s = \frac{T}{F_s} = \frac{18{,}000}{24{,}000} = 0.75 \text{ in.}^2$$

Provide two No. 6 bars, $A_s = 0.88$ in.$^2 > 0.75$ in.^{2} OK
Assume that reinforcing bar is located near the bottom of beam,

$$d = (15)\ (12) - 4 = 176 \text{ in.}$$

Support:

$$A_s = \frac{T}{F_s} = \frac{34{,}500}{24{,}000} = 1.44 \text{ in.}^2$$

Provide two No. 8 bars, $A_s = 1.57$ in.$^2 > 1.44$ in.^{2} OK
Locate the bars near the top of the beam.

c. Calculate the shear stress and required shear reinforcement. Assume that critical section is located at 0.15 l_n from the face of support, where l_n clear span between supports.

$$0.15 l_n = 0.15\ (30 - 3) = 4.05 \text{ ft}$$

At the critical section, the maximum shear (assuming three continuous spans) is

$$V = w\left[\frac{L}{2} - (4.05 + 1.5)\right] = (5000)\left[\frac{33}{2} - (4.05 + 1.5)\right] = 54{,}750 \text{ lb}$$

The shear stress at the critical section is

$$v = \frac{V}{bd} = \frac{54{,}750}{(11.625)(176)} = 26.8 \text{ lb/in.}^2$$

Allowable shear stress = $1.1\sqrt{f'_m} = 1.1(\sqrt{2000}) = 49.2$ lb/in.^{2} (without shear reinforcement)
Therefore, no special shear reinforcement is required.

Minimum wall reinforcement = 0.002(11.625)(12) = 0.28 in.^{2}/ft*
(*Total in both direction; no more than $^2/_3$ in one direction)

Provide No. 5 bars @ 24 in. o.c. in horizontal direction, $A_s = 0.15$ in.^{2}
Provide No. 5 bars at 24 in. o.c. in vertical direction, $A_s = 0.15$ in.^{2}
Total reinforcement,

$$A_s = 0.15 + 0.15 = 0.30 \text{ in.}^2 > 0.28 \text{ in.}^2 \qquad \text{OK}$$

4.15 BOND BEAMS

4.15.1 Definition and Functions of a Bond Beam

"Bond beam" is a special term that is encountered frequently in the context of masonry construction. A *bond beam* is not a *beam* in the conventional sense of a structural member that typically supports transverse loads; for example, a beam supporting a part of a roof or a floor. Rather, it is an integral part of a wall that performs the following important functions:

1. It ties the reinforced masonry structure together around its perimeter, resulting in stronger unit.
2. It acts as a *chord* of a diaphragm in resisting lateral loads.
3. It acts as an effective restraint against wall movement and helps reduce the formation of cracks.

To perform the aforestated functions, bond beams may be located at all floor and roof levels, above and below the openings, and around the tops of walls and foundations as means of tying all blocks together and distributing the loads to better advantage. Bond beams located at the top of walls stiffen them.

In load-bearing masonry walls, bond beams are commonly placed in the top course of the wall beneath the roof framing system to act as *diaphragm chords* to resist lateral loads due to wind or earthquakes. In this case, the bond beam has to resist axial forces—compressive and tensile—on the opposite edges of the diaphragm system. Because of the inherent weakness of masonry in tension, adequate amount of reinforcing must be provided in the bond beam to resist the tensile chord force. Diaphragm action is discussed in the next section.

Bond beams in concrete masonry walls can be accommodated either by using special channel-shaped units called bond beam units (Fig. 2.31, Chap. 2) or by saw-cutting out of a standard unit. Bond beam units are fabricated either with reduced webs or with "knock-out" webs that are removed prior to their placement in the wall. The width and the depth of bond beam units is the same as those of the other masonry units used in the wall in which they are placed.

4.15.2 Typical Reinforcement in Bond Beams

The size of reinforcing bars used in a bond beam depends on the thickness of the wall. It is a common practice to provide horizontal reinforcement in bond beams as two No. 4 bars in 8-in.-wide bond beams and two No. 5 bars in 10- and 12-in.-wide bond beams. After the placement of horizontal reinforcement, the units are filled with grout or concrete. Upon hardening of the grout or the concrete, the beam behaves as a solid reinforced masonry member. Building codes in some earthquake-prone areas require a 16-in.-deep bond beam with two additional No. 4 bars located in the top of the beam.

It is important to note that the horizontal bars in bond beams may act as *structural* reinforcement, such as bond beam reinforcement in roof and floor diaphragms that resist diaphragm chord tension. Consequently, they must be continuous or lapped, and bent around corners as required by the applicable code. Continuity in bond beam reinforcement is required even through the control joints, which provide complete separation between two segments of a wall. Control joints are provided to permit free longitudinal movement, but may need to transfer shear or lateral loads. Nonstructural reinforcement, such as horizontal joint reinforcement, should not be continuous across the control joint.

In nonload-bearing masonry walls, bond beams may be placed in any of the top three courses below the roof slab. In this case, it functions as a structural tie, integrating the nonload-bearing walls with the adjacent walls, and also a crack control device. Also, in this

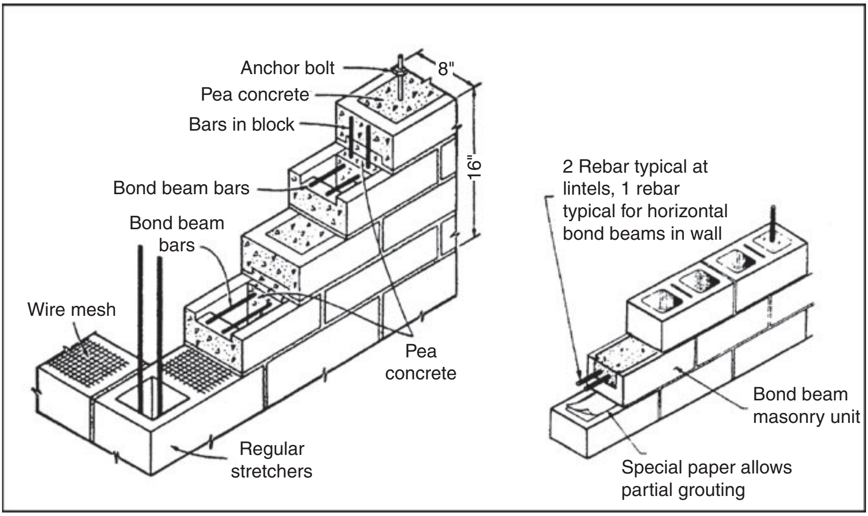


FIGURE 4.30 Section of a typical bond beam.

case, the bond beam can be designed to carry gravity loads from the roof or the floor, which cannot be transferred to these walls since they are designed as nonload-bearing walls.

In retaining walls, bond beams can be used as an alternative to providing horizontal reinforcement to distribute stresses that occur due to expansion and contraction of the wall. Typically, bond beam is provided at the top of the wall, and at 16 in. on center below. For 8-in. thick retaining walls, it is a common practice to provide two No. 4 bars in all bond beam courses. For 12-in. thick retaining walls, two No.5 bars are provided in the bond beam course at the top, and two No. 4 bars in the courses below.

Figure 4.30 shows details of a typical bond beam. Figure 4.31 shows a bond beam located at the roof level of a reinforced masonry building.

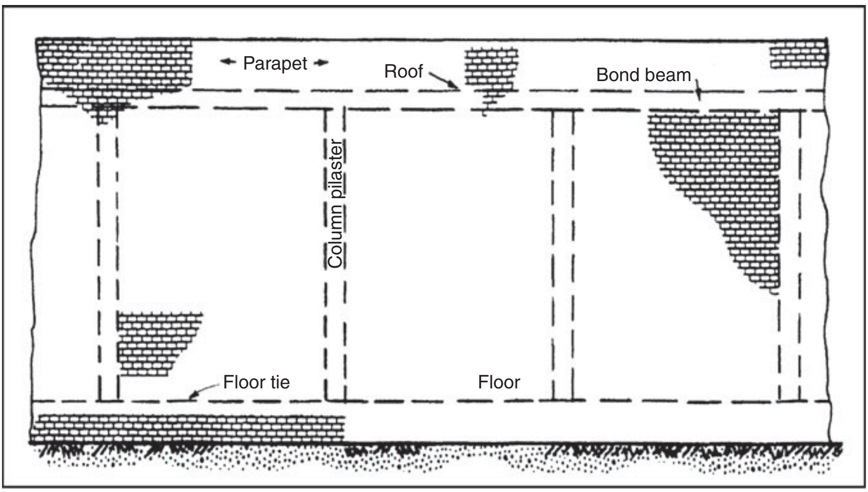


FIGURE 4.31 Bond beam located at roof level. (*Courtesy: NCMA.*)

4.16 DIAPHRAGM ACTION

4.16.1 In-Plane Loads in a Diaphragm

To understand the mechanism by which axial force is developed in a bond beam, it is essential to understand the diaphragm action in a building. Essentially, walls are subjected to lateral loads due to wind or earthquakes as shown in Fig. 4.32. These loads are transmitted to the lateral load resisting elements of a building on the tributary area basis. Typically, in a single-story building, lateral loads on the lower half of the wall are transmitted to the floor, whereas those from the upper half of the wall, including those from the parapet, are transferred to the roof. The roof then acts as a large horizontal member resisting lateral loads in its own plane (i.e., acts as a diaphragm)—this mechanism is known as *diaphragm* action, and causes *in-plane* loads in the diaphragm. The roof, in turn, transfers these in-plane loads to shear walls through the connection hardware. And finally, the shear walls transfer all loads to the foundation.

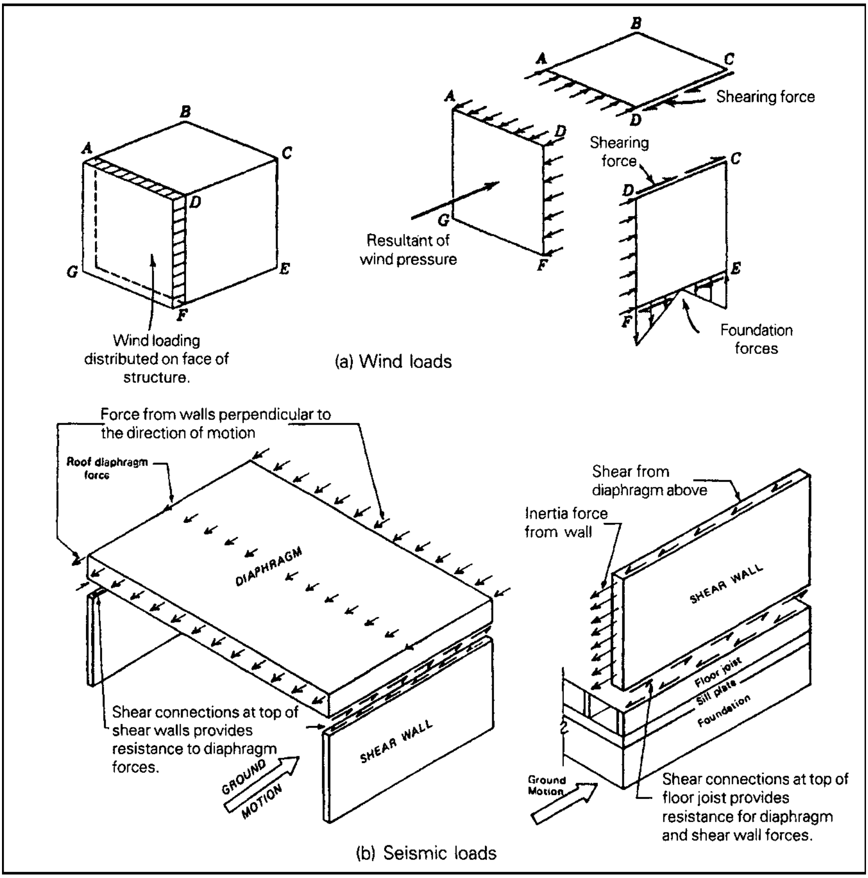


FIGURE 4.32 Diaphragm action in a roof due to lateral loads.

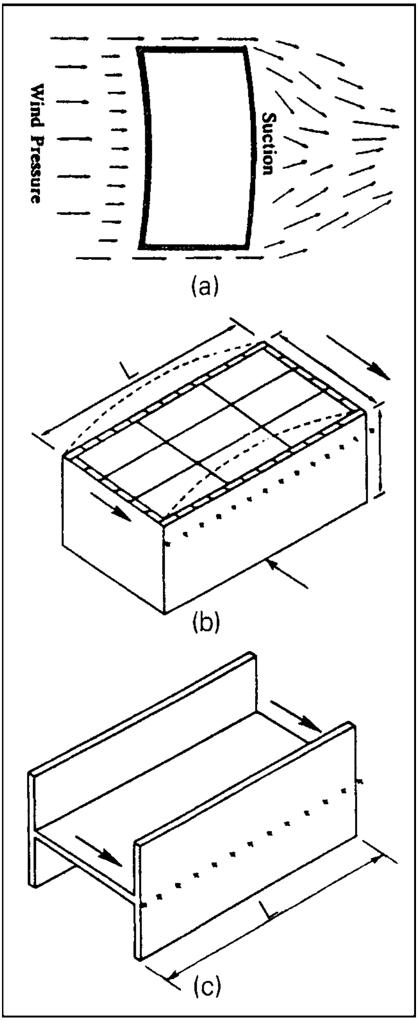


FIGURE 4.33 Lateral loads on a diaphragm: (*a*) wind, (*b*) seismic, and (*c*) comparison with a W-shape beam.

From a structural standpoint, a *diaphragm* is assumed to act like a simple beam with a very deep web, bending in the horizontal plane (unlike conventional beams that support gravity loads and bend in vertical plane), with span equal to the distance between shear walls to which it transfers the lateral load (Fig. 4.33). This distance is perpendicular to the direction of the in-plane loads (L in Fig. 4.33*b*). For analytical purposes, the behavior of a diaphragm can be idealized as bending of a transversely loaded *wide-flange* (W-shape) beam (Fig. 4.33*c*) in a horizontal plane. The *chords* of a diaphragm are akin to compression and tension flanges of this beam. There is, however, a minor difference between this assumed behavior of the diaphragm and a conventional beam—the chords of a diaphragm are assumed to carry *entire* tension and compression forces; none of it is assumed to be carried by the web, whereas the web of a conventional beam carries some tension and compression (as defined by stress distribution diagram). Because lateral loads can reverse in direction, either chord can be a tension or a compression chord. The *web* of the diaphragm (decking or sheathing) is assumed to carry shear, analogous to the web of a W-shape beam (but no bending), in addition to the (out-of-plane) gravity loads (or sheathing loads) that it has to carry. Under the action of lateral loads, it is further assumed that the diaphragm action occurs, independently, in both directions—transverse and longitudinal. Thus, at any given time, any two opposite sides of a diaphragm act as chords. Furthermore, each of the two opposite chords may be in compression (loaded side) *or* tension (far side) because of the reversible nature of lateral loads.

4.16.2 Analysis for the Diaphragm Action and the Bond Beam

Force in a bond beam can be estimated with reference to schematics shown in Fig. 4.34. Let

w = diaphragm load (uniform)

L = length of diaphragm *perpendicular* to loads

d = depth of diaphragm (dimension *parallel* to loads)

T = Tensile force in the chord

C = compression force in the chord

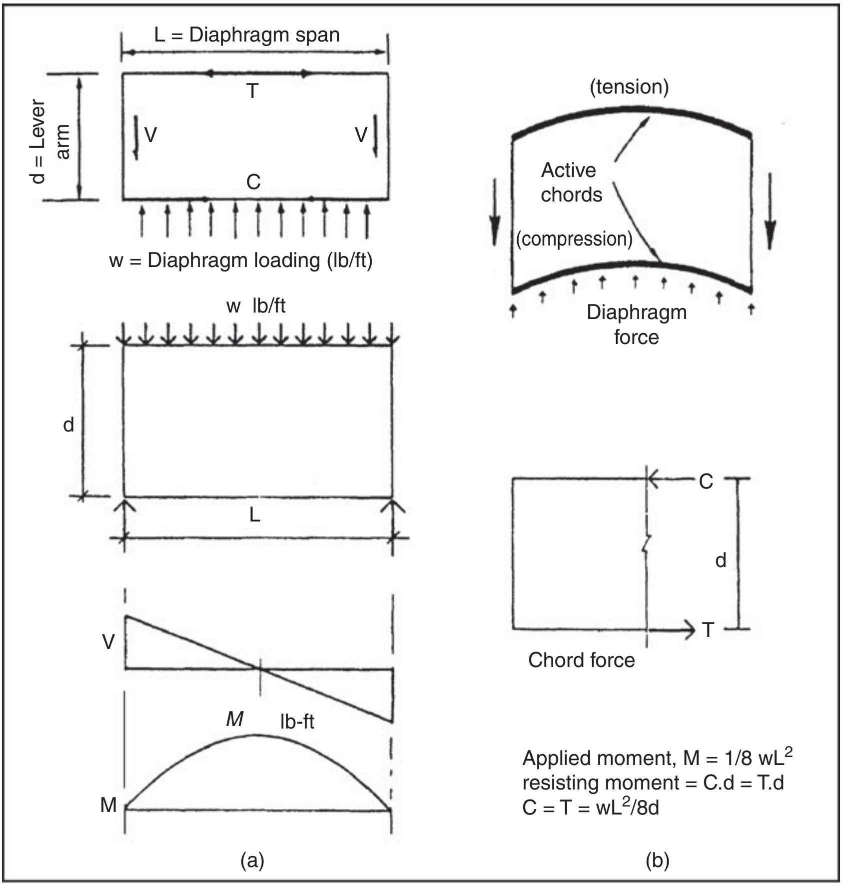


FIGURE 4.34 Idealization of a diaphragm as transversely loaded beam: (*a*) diaphragm loads, (*b*) calculation of chord forces.

Moment due to the distributed load, w, is given by

$$M = \frac{wL^2}{8} \tag{4.130}$$

For horizontal equilibrium, $C = T$

Moment due to couple $= C.d = T.d$

$$\text{Thus, } M = \frac{wL^2}{8} = C.d = T.d$$

which yields

$$C = T = \frac{M}{d} = \frac{wL^2}{8d} \tag{4.131}$$

Maximum diaphragm shear V is computed by treating diaphragm as a simple beam, that is, computed as beam reactions (which are transmitted at the top of shear walls):

$$V = wL/2 \tag{4.132}$$

The maximum *unit* diaphragm shear, v, is computed by dividing the maximum diaphragm shear, V (= $wL/2$) by the diaphragm *depth* (measured parallel to the diaphragm loads):

$$v = wL/2d \tag{4.133}$$

For design purposes, the diaphragm load w is to be computed for wind as well as seismic loads as shown in Fig. 4.32, and the chord forces should be computed for the governing diaphragm loads. According to equivalent static force procedures, diaphragm loads are computed on the basis of tributary heights of walls or the building. In a typical reinforced masonry building (symmetrical in plan and elevation), the chord forces so computed are also the axial forces in the bond beams, which must resist them as axial loads. As alluded to in the previous section, because of the reversible nature of lateral loads, any of the two opposite sides of a diaphragm may be in tension. Because masonry is not permitted to resist tensile forces, the entire tensile chord force, T, must be resisted by the reinforcement provided in each bond beam. The required amount of reinforcement can be calculated simply by dividing the chord force by the yield stress in reinforcement:

$$A_s = \frac{T}{\phi f_y} \tag{4.134}$$

where $\phi = 0.9$ for a reinforced masonry member subjected to axial loads.

Application of these concepts discussion in this section is presented in Example 4.30.

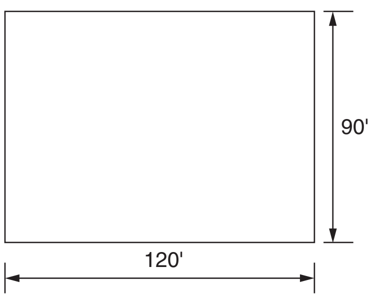


FIGURE E4.30A

Example 4.30 Design of reinforcement for a bond beam.

A roof diaphragm 120 × 90 ft (Fig. 4.30A) is subjected to a strength level lateral load of 1200 lb/ft perpendicular to its long side and supported over reinforced masonry walls. Determine the reinforcement for the bond beam.

Solution

Determine bending moment in the diaphragm.

Span, $L = 120$ ft, $w_u = 1200$ lb/ft = 1.2 k/ft

$$M = \frac{w_u L^2}{8} = \frac{1.2(120)^2}{8} = 2160 \text{ kip-ft}$$

The tensile chord force in the bond beam is [Eq. (4.131)]:

$$T = \frac{M}{d} = \frac{2160}{90} = 24 \text{ kips}$$

Alternatively, the tensile chord force in the bond beam could be determined directly from Eq. (4.131):

$$T = \frac{w_u L^2}{8d} = \frac{1.2(120)^2}{8(90)} = 24 \text{ kips}$$

The bond beam is thus subjected to an axial force of 24 kips, which must be resisted by the reinforcement only. Calculate the tensile steel required from Eq. (4.134):

$$A_s = \frac{T}{\phi\ f_y} = \frac{24}{0.9(60)} = 0.44 \text{ in.}^2$$

Provide two No. 5 Grade 60 bars,
$A_s = 0.61$ in.^{2} OK (Table A.9)
Reinforcement details are shown in Fig. E4.30B.

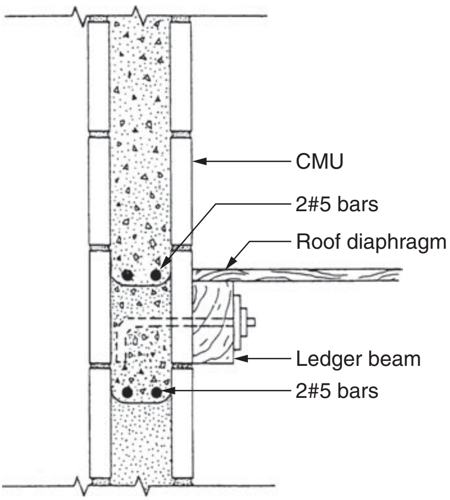


FIGURE E4.30B

4.17 FLEXURAL STRENGTH OF A WALL DUE TO IN-PLANE LOADS

In many cases, walls are subjected to flexure induced by in-plane loads; masonry and concrete shear walls fall into this category. This section presents procedure for calculating the nominal strength of a reinforced masonry wall subjected to in-plane loads.

The theory of flexural strength of reinforced masonry elements is applicable for calculating the flexural strength of a wall subjected to in-plane loads. A beam is a transversely loaded horizontal member (subjected to gravity loads), whereas, a wall is a vertical element subjected to flexure by in-plane lateral loads (such as wind and earthquake). Such walls are called shear walls, and their flexural behavior can be analyzed by treating them like vertical cantilever beams—one end of the wall being in compression and the other end in tension. Also, like a reinforced masonry beam, the wall is grouted solid. However, some differences between the configurations of a beam and a wall should be noted as illustrated in Fig. 4.35. There is also some difference in the analyses procedures.

1. *Difference in member configurations*: Unlike beams, in which tension-resisting reinforcing bars are placed horizontally near the tension face of the beam, a wall contains several vertical reinforcing bars equally spaced along its entire length.

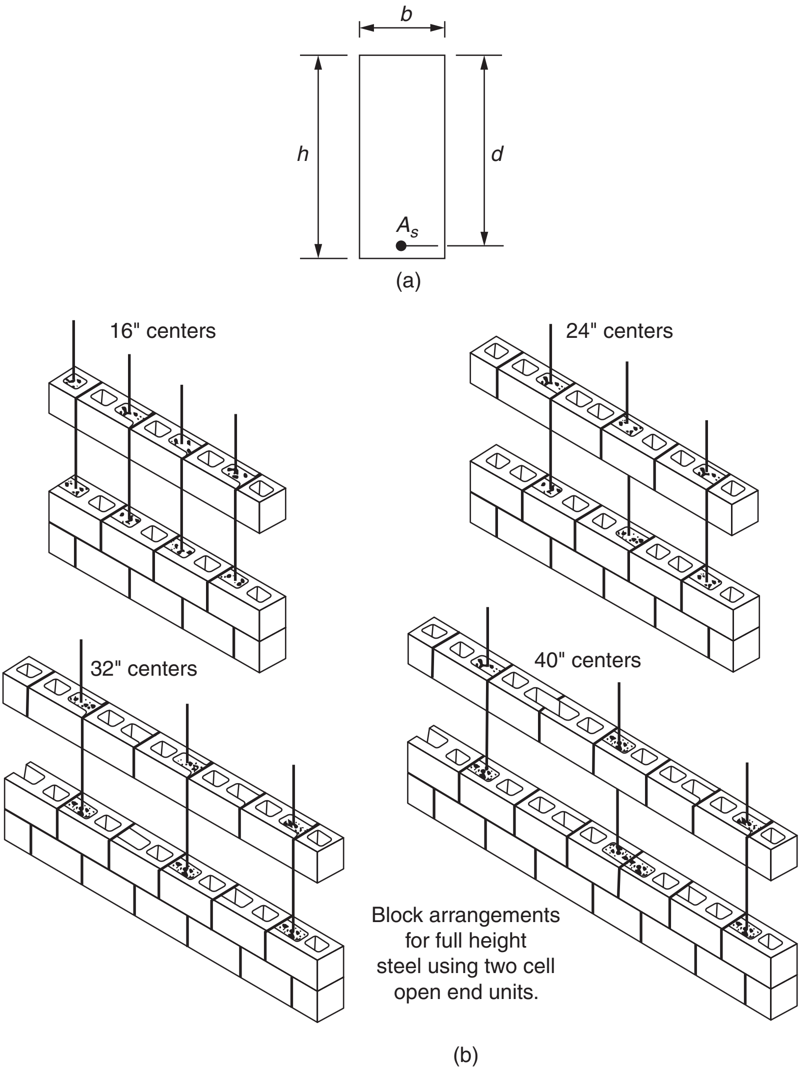


FIGURE 4.35 (*a*) cross section of reinforced beam, (*b*) cross section of reinforced masonry wall. (*Courtesy: NCMA.*)

2. *Difference in analyses*: There is a slight difference in the flexural analysis procedure for beams and walls. The neutral axis of a reinforced masonry beam is easily determined by equating compression and tension forces acting on its cross section [see Eqs. (4.8), (4.9), and (4.5b)]. This procedure works well for a beam because the tension reinforcement, which is assumed to have yielded under the ultimate loading condition, is located at one level (close to the tension face of the beam). Because the reinforcement in a wall consists of several equally spaced vertical bars, the neutral axis of the wall is located by a trial-and-error procedure because of uncertainty of stresses in various bars—all bars cannot be assumed to have yielded. Therefore, several trials may be necessary to determine the position of neutral axis in a wall. The procedure begins by selecting an arbitrary (but reasonable) value of the distance of neutral axis from the compression face of the wall (c). Also, in the wall analysis, the compression forces in bar(s) located in the compression zone of the wall (i.e., bars located near the compression face of wall) is taken into account when considering equilibrium (no such consideration for analysis of typical reinforced masonry beams). Once the position of neutral axis is located, stresses (and hence the forces) in reinforcing bars, which may be in compression or tension, are determined based on strain in them, which, in turn, are calculated from Hooke's law (maximum bar stress limited to 60 ksi for Grade 60 bars, which corresponds to a yield strain of 0.00207). The analysis procedure for deter mining the flexural strength walls subjected to in-plane loads is presented in Chap. 7.

4.18 DEVELOPMENT LENGTHS FOR REINFORCING BARS

Reinforced masonry members are designed on the basic premise that grout surrounding reinforcing bars would transfer forces from reinforcing bars to the masonry. (The same is true for reinforced concrete members which are designed on the premise that the surrounding concrete would transfer forces from reinforcing bars to the concrete.) The length of the reinforcing bars over which forces (or stresses) are transferred from the grout to the masonry is called the *development length.*

The requirements for development lengths of reinforcing bars are the same for both allowable stress design (ASD) and strength design, and are covered in MSJC-08 Section 3.3.3. All reinforcing bars (in compression or tension) must extend from the point of maximum moment in either direction a distance equal to the development length as determined from Eq. (4.135), but not less than 12 in.:

$$l_d = \frac{0.13 d_b^2 f_y \gamma}{K\sqrt{f'_m}} \tag{4.135}$$

where l_d = development length of bar
d_b = diameter of bar
f_y = yield stress of reinforcement, lb/in.^{2}
K = the least of (1) masonry cover, (2) spacing between the adjacent reinforcement, (3) 5 times d_b
γ = reinforcement size factor as listed in Table 4.11

TABLE 4.11 Value of Reinforcement Size Factor, γ, for Use in Eq. (4.135)

Bar no.	γ
3, 4, and 5	1.0
6 and 7	1.3
8 and 9	1.5

2006 IBC (Section 2108.2) specifies that the development length given by Eq. (4.135) need not exceed $72d_b$ (no such upper limit in MSJC-08).

Development lengths for epoxy-coated bars are required to be 50 percent longer than required by Eq. (4.135). Bars spliced by noncontact lap splices can be farther apart no more than one-fifth the required length of lap or 8 in.

Perusal of Eq. (4.135) shows that the development length of a reinforcing bar is proportional to the square of its diameter. Therefore, the larger the bar diameter, the longer would be the required development length. Also, the reinforcement size factor, γ, increases as the bar diameters increase (Table 4.11). For these reasons, it is advisable to use smaller diameter bars; the use of larger diameter bars would require relatively longer development lengths that may not be practical. Factor K has a significant influence on the required development length for a bar depending on whether there is only one bar or two closely spaced bars in a grouted cell. See Example 4.31.

According to the MSJC-08 *Commentary*, the moment diagrams customarily used in design are approximate as some shifting of the point of maximum moment may occur due to changes in loading, settlement of supports, lateral loads, or other causes. A diagonal tension crack in a flexural member without stirrups may shift the location of calculated tensile stress approximately a distance d toward a point of zero moment. When stirrups are provided, this effect is less severe, but still present. To provide for this uncertainty, the MSJC *Code* requires the extension of reinforcement a distance equal to a distance d or $12d_b$ beyond the point at which it is theoretically no longer required to resist flexure.

The longitudinal reinforcing bars may be spliced as required. The above requirement applies to contact splices of standard deformed bars as well. Normally, these bars would extend from the column footing, which may require splicing as construction progresses. Splice lengths should be sufficient to transfer the loads in reinforcing bars by proper *development length* (MSJC-08 Section 3.3.3.3).

Example 4.31 Development lengths.

Calculate the developments for No. 5 Grade 60 bar placed in a standard 8 × 8 × 16 CMU when (a) only one bar is placed in each cell, (b) when two bars are placed in each cell. Use $f'_m = 1500$ psi.

Solution

Figure E4.31 shows cross sections of standard 8 × 8 × 16 CMUs with No. 5 bars. The development length for a reinforcing bar is given by Eq. (4.135):

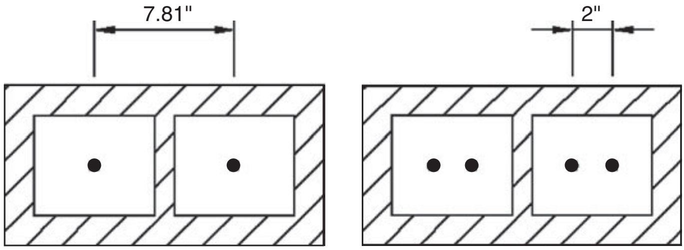
$$l_d = \frac{0.13 d_b^2 f_y \gamma}{K\sqrt{f'_m}}$$

a. One No. 5 Grade 60 bar in each cell.

$d_b = 0.625$ in., $f_y = 60{,}000$ psi, $f'_m = 1500$ psi, $\gamma = 1.0$ (Table 5.1). K is the smallest of

1. Masonry cover = ½(7.625) – ½(0.625) = 3.5 in.
2. Spacing between the adjacent reinforcement = ½(15.625) – 0.625 = 7.19 in.
3. 5 times $d_b = 5(0.625) = 3.125$ in. ← governs

$$l_d = \frac{0.13 d_b^2 f_y \gamma}{K\sqrt{f'_m}} = \frac{(0.13)(0.625)^2(60{,}000)(1.0)}{(3.125)(\sqrt{1500})} = 25.17 \text{ in.}$$



(a) One bar in each cell (b) Two bars in each cell

FIGURE E4.31

b. Two No. 5 Grade 60 bars in each cell.
d_b = 0.625 in., f_y = 60,000 psi, f'_m = 1500 psi, γ = 1.0 (Table 5.1). K is the smallest of

1. Masonry cover = ½(7.625) – ½(0.625) = 3.5 in.
2. Spacing between the adjacent reinforcement = 2 in. ← governs
3. 5 times d_b = 5(0.625) = 3.125 in.

$$l_d = \frac{0.13 d_b^2 f_y \gamma}{K\sqrt{f'_m}} = \frac{(0.13)(0.625)^2(60{,}000)(1.0)}{(2.0)(\sqrt{1500})} = 39.33 \text{ in.}$$

It is noted that the required development length is 1½ times as long when two bars are placed in a cell as compared to when only one bar is placed in the same cell.

4.19 SERVICEABILITY CRITERIA FOR BEAMS

Serviceability criteria for a structure include such functional (as opposed to structural) considerations as deflection and vibration control. Deflection criteria for masonry beams are discussed in this section.

Deflections, being a serviceability criterion, are calculated for service load conditions (i.e., for service or unfactored loads on the beam), as opposed to factored loads, which are used for design (strength criterion). The reason for this engineering practice is that when factored loads are used, we are concerned with the strength of a beam rather than deflection, which is likely to be large but not of concern because at the ultimate load condition, a beam is considered unusable anyway. It is the service loads under which a beam has to perform satisfactorily during its service life. The allowable beam deflection is to be compared with deflection due to service loads (not due to factored or ultimate loads).

In general, excessive deflection may interfere with the use of a structure. For example, deflection would be a problem if a member were to support machinery. In such a case, deflection limitation would be a criterion which must be satisfied by proper structural design. Uncontrolled or excessive deflections may cause problems with the drainage of floors and roofs. Weight of undrained or accumulated water on the roof causes additional deflections, which would permit additional water to accumulate, causing still more deflection. This phenomenon of progressively increasing deflections and more accumulated water may lead to a *ponding failure* and consequent considerable structural damage [4.13].

In some cases, deflection limitations of flexural elements such as lintels may be necessary for proper functioning of windows and doors below the opening. However, this constitutes a serviceability problem, not structural. In general, deflections that may lead to structural damage are several times larger than those causing serviceability problems. If a member deflects so much that it comes in contact with another member, the load paths may change, causing cracking.

The *Code* limitations for deflections of masonry beams are empirical. MSJC-08 Section 1.13 provides requirements governing beam deflections. It requires beams and lintel to be designed to have adequate stiffness to limit deflections that adversely affect strength or serviceability. To this effect, it is required that deflections of beams and lintels due to unfactored dead plus live loads (i.e., service loads) not exceed 1/600 of clear span when providing vertical support to unreinforced or empirically designed masonry. These empirical requirements are intended to limit excessive deflections which may cause damage to the supported unreinforced (plain) masonry, or reinforced masonry with vertical reinforcement only. According to MSJC-08 *Commentary* [4.3], reinforced masonry beams with span lengths of 8 times d have immediate deflections of approximately 1/600 of span lengths. Masonry beams and lintels with shorter spans should have sufficient stiffness to prevent serviceability problems, and therefore deflections need not be checked. Additionally, it is noted that most masonry beams have some end restraint due to being built integrally with a wall. Tests have shown that those end support conditions reduce the deflections from about 20 to 45 percent from those with simply supported specimens [4.29].

4.20 SERVICE LOAD ANALYSIS OF REINFORCED MASONRY BEAMS

4.20.1 Concept of Transformed Section

Reinforced masonry is constructed from four dissimilar materials: masonry units (concrete or clay), mortar, grout, and steel reinforcement. The elastic properties of all these materials are different from each other. Consequently, like reinforced concrete, masonry is a non-homogeneous material. Beams made from dissimilar are referred to as *composite* materials.

The conventional theory of flexure ($f = My/I$) is based on the fundamental principles that (1) the material is homogeneous and that (2) the plane sections remained plane so the strains varied directly with their distance from the neutral axis. This theory does not apply directly to beams of nonhomogeneous materials. However, by suitable modifications, an equivalent section can be obtained in terms of one material to which the theory can be applied. This equivalent section is called *transformed* section. When applying the bending theory to composite beams, only one assumption is retained, that is, plane sections remain plane so that strains vary directly with their distances from the neutral axis. In general, the concept of transformed section is applicable to any beam consisting of dissimilar materials, and the conventional theory of flexure can be applied.

In sections of homogeneous materials such as steel or aluminum, the neutral axis is located at the *geometric centroid* of the section. But because of nonhomogeneity, the neutral axis in a reinforced masonry beam is *not* located at the geometric centroid of the section.

Consequently, its location is determined using the concept of *transformed section*. A transformed section is a fictitious section in which the cross-sectional area of one material is converted (transformed) into an *equivalent* area of the other. In general, this concept can be used for sections consisting of any dissimilar materials such as concrete and steel, concrete and wood, wood and steel, composites, etc.

In the analysis of transformed sections, one material is selected as a reference material, and other materials are transformed into an equivalent area of the reference material. This transformation is achieved by multiplying the areas of the other materials by the respective ratios of the moduli of elasticity of the other materials to that of the reference material. The centroidal distance of the transformed area measured perpendicular to a given axis, as well as the effective height or length of the element, remain unchained.

The ratio of moduli of elasticity of two materials is called *modular ratio* and denoted by n. For example, the modular ratio n of two materials having different moduli of elasticity, E_1 and E_2 ($E_2 > E_1$) respectively, can be expressed as Eq. (4.136):

$$n = \frac{E_2}{E_1} \tag{4.136}$$

where E_1 = modulus of elasticity of the reference material
E_2 = modulus of elasticity of the material whose area is to be transformed to the equivalent area of the reference material

The transformation to an equivalent area is accomplished in one of the two ways depending on the reference material:

1. The cross-sectional area of the *stiffer* material (i.e., one with larger modulus of elasticity) is *multiplied* by the modular ratio, the product being the transformed area equivalent to the less stiff material (i.e., one with smaller value of modulus of elasticity). This method is followed for allowable stress design (ASD, also called working stress design, WSD) of reinforced concrete and masonry. In the latter case, the modular ratio is defined as the ratio of moduli of elasticity of reinforcing material to that of masonry. The cross-sectional area of reinforcement is multiplied by the modular ratio, the result being the transformed area of reinforcing material (steel in our case). For example, in the design of reinforced masonry structures, the modular ratio for reinforcing steel is expressed as the quotient of moduli of elasticity of steel and masonry, E_s and E_m, respectively. Thus,

$$n = \frac{E_s}{E_m} \tag{4.137}$$

The transformed area of reinforcing steel is then expressed as nA_s, with its centroid located at the location of the centroid of reinforcement.

2. Conversely, the area of the less stiff material can be *divided* by the modular ratio, the quotient then being the transformed area equivalent to that of the stiffer material. This method is followed in composite steel design. The width of concrete cross section, b, is divided by the modular ratio, resulting in a new width called the *effective width* b_e (= b/n), the resulting area (effective width times the depth of concrete cross section) being the transformed area of concrete.

In reinforced masonry, the transformed section concept is applied to sections consisting of masonry and steel, in which it is assumed that masonry in the tension zone of the beam is cracked and structurally absent. The area of steel reinforcement is transformed into an equivalent area of the masonry, with its centroid located at the centroid of tension reinforcement so that the resultant tensile force is assumed to act at this level. This principle is equally applicable to composite sections that consist of brick *and* concrete masonry units, in which case the area of one type of masonry unit (e.g., brick) can be transformed into an equivalent area of the masonry unit of the other type (e.g., concrete block). The general principle of transformed section is applied in the following derivations for reinforced masonry sections.

4.20.2 Location of Neutral Axis in a Reinforced Masonry Beam under Service Loads

Figure 4.36 shows a rectangular singly reinforced masonry section in flexure having a total cross-sectional area of tensile reinforcement equal to A_s. It is assumed that there is no compression reinforcement present in the beam. Although reinforced masonry is neither homogeneous nor isotropic, it will be assumed that the assumptions of theory of bending apply as discussed above. Accordingly, both strains and stresses will be assumed to have linear distribution as shown in Fig. 4.36. It is assumed that there is no slip between the reinforcing steel and the surrounding grout so the strain in the reinforcement (ε_s) would be the same as the strain in the surrounding grout at the level of reinforcement (ε_m), that is,

$$\varepsilon_m = \varepsilon_s \tag{4.138}$$

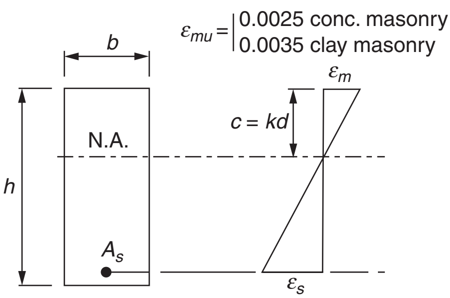


FIGURE 4.36 A singly reinforced masonry section in flexure.

Within elastic limits, strains in masonry and steel reinforcement can be expressed in terms of stresses and the modulus of elasticity based on Hooke's law, so that $\varepsilon_m = f_m/E_m$. and $\varepsilon_s = f_s/E_s$. Substitution of these values in Eq. (4.138) yields

$$\frac{f_m}{E_m} = \frac{f_s}{E_s}$$

or

$$\frac{f_s}{f_m} = \frac{E_s}{E_m} = n \tag{4.139}$$

where $E_s/E_m = n$ (modular ratio)

Let A_m = equivalent area of masonry required to carry the same force as the area of tension reinforcement. Then, for equilibrium in the horizontal direction, we must have

$$\text{Force in equivalent area of masonry} = \text{Force in steel reinforcement}$$
$$P_m = P_s \tag{4.140}$$

where P_m = force in masonry in the equivalent area of masonry
P_s = force in steel reinforcement

Since $P_m = A_m f_m$, and $P_s = A_s f_s$, we have, from Eq. (4.140)

$$A_m f_m = A_s f_s$$

which yields

$$A_m = \left(\frac{f_s}{f_m}\right) A_s = nA_s \tag{4.141}$$

in which $(f_s / f_m) = n$ has been substituted from Eq. (4.139).

Equation (4.141) shows that in a reinforced masonry section with tension reinforcement area equal to A_s, the equivalent masonry area can be considered as being equal to nA_s, which is assumed concentrated at the centroid of reinforcing steel, A_s.

As explained heretofore, the fundamental premise in analyzing reinforced masonry beams is that masonry below the neutral axis (i.e., in the tensile zone of the cross section) is cracked (i.e., it is structurally absent) and the tensile force in the beam is carried by only the reinforcement. Thus, for the purpose of locating the neutral axis, the area of masonry below the neutral axis is omitted, and only the transformed area equal to nA_s is assumed present with its centroid at the level of centroid of reinforcing steel.

Referring to Fig. 4.36, the neutral axis of the transformed section is located at some distance kd ($< d$) from the extreme compression fibers so that the cross-sectional area in compression is bkd (i.e., width times depth of masonry in compression). The distance kd can be determined from statics (by equating the moments of compression area bkd and the equivalent tension area nA_s, taken about the neutral axis):

$$(b)(kd)\left(\frac{kd}{2}\right) = nA_s(d - kd) \tag{4.142}$$

where d = depth of section from extreme compression fibers to the centroid of tension reinforcement
kd = depth of neutral axis measured from extreme compression fibers
k = neutral axis factor (< 1.0)
b = width of beam cross section

For design purposes, we modify Eq. (4.142) by introducing ρ, defined as the ratio of tension reinforcement A_s to the cross-sectional area of masonry bd [i.e., $\rho = A_s/bd$, Eq. (4.38)] so that $A_s = \rho bd$. Substituting for A_s in Eq. (4.142) and simplifying yields

$$b(kd)^2 + 2n\rho kd^2 - 2n\rho bkd^2 = 0$$

or

$$k^2 + 2n\rho k - 2n\rho = 0 \tag{4.143}$$

Equation (4.143) is a quadratic in k of the form: $Ax^2 + Bx + C = 0$, and can be solved for k:

$$k = \sqrt{(n\rho)^2 + 2n\rho} - n\rho \tag{4.144}$$

Equation (4.144) locates the neutral axis of a section whose properties are completely known (i.e., size of the beam, and the location and area of tension reinforcement, and no compression steel present). See Example 4.32. For computational expediency, values of k given by Eq. (4.144) are tabulated for various values of $n\rho$ (Table A.15).

4.20.3 Modular Ratio, *n*

The modular ratio, n, was defined earlier as the ratio of modulus of elasticity of steel, E_s, to that of masonry, E_m (both were discussed in Chap. 3). Whereas the value of E_s is commonly taken as a standard 29,000 ksi (MSJC-08 Section 1.8), the value of the modulus of elasticity of masonry, E_m, is based on the 28-day compressive strength of *masonry prism*. This relationship is analogous to the modulus of elasticity of concrete which also is based on the 28-day compressive strength of a concrete cylinder.

MSJC-08 Section 1.8 specifies the following values of E_m for clay and concrete masonry:

$$\text{For clay masonry: } E_m = 700\ f'_m$$

$$\text{For concrete masonry: } E_m = 900\ f'_m$$

The above values of E_m are based on the chord modulus values determined from stress values of 5 to 33 percent of the compression strength of masonry. The chord modulus is defined as the slope of a line intersecting the stress-strain curve at two points, neither of which is the origin of the curve. Readers are referred to the commentary to MSJC *Code* [4.3] for a discussion on research that forms the basis of the specified E_m values.

Values of n corresponding to a few typical values of f'_m are given in Table A.14. Referring to Fig. 4.36, the moments of inertia of the gross section (uncracked) taken about its centroidal axis is given by Eq. (4.144):

$$I_g = \frac{bh^3}{12} \tag{4.145}$$

Note that the moment of inertia of the gross section, I_g [Eq. (4.145)], does not take into account the presence of tension or compression reinforcement present in the uncracked section. This is a conservative approach, but this is how it has been traditionally specified in design codes. If the transformed area of reinforcement present in the beam were to be considered for calculating the moment of inertia of the gross section, defined as *gross transformed moment of inertia* I_{gt}, its value would be greater than I_g (i.e., $I_{gt} > I_g$). However, the increase in the value of I_g is small.

The moment of inertia of the transformed cracked section taken about the centroidal axis of the cracked section can be expressed as Eq. (4.146):

$$I_{\text{cr}} = \frac{b(kd)^3}{3} + nA_s\left(d - kd\right)^2 \tag{4.146}$$

Example 4.32 illustrates the procedure for calculating I_{cr} of a rectangular reinforced masonry section. Example 4.33 shows calculations for I_{gt} for the same section.

Example 4.32 Moment of inertia of a cracked rectangular reinforced masonry section.

A simply supported nominal 8 × 24 in. CMU beam is reinforced with one No. 9 Grade 60 bar for tension with its centroid located at 20 in. from the compression face of

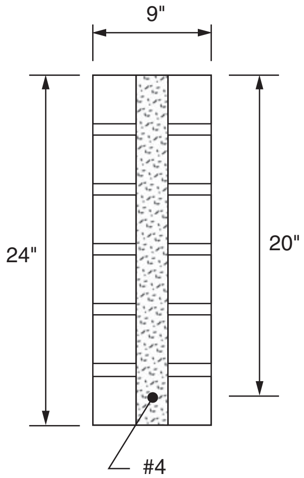


FIGURE E4.32

the beam (Fig. E4.32). Calculate the moments of inertia of (a) gross section, (b) cracked section. Assume f'_m = 1500 psi. (Note: This data is taken from Example 4.12 which illustrated flexural calculations.)

Solution

a. For a nominal 8-in. wide CMU, actual width $b = 7.63$ in. The moment of inertia of the gross section is [Eq. (4.145)]

$$I_g = \frac{bh^3}{12} = \frac{(7.63)(24)^3}{12} = 8790\,\text{in.}^4$$

b. Since the centroid of tension reinforcement is located at 20 in. from the compression face of the beam, the effective depth $d = 20$ in. $A_s = 1.0$ in.^{2} (one No. 9 bar).

$$\rho = \frac{A_s}{bd} = \frac{1.0}{(7.63)(20)} = 0.0066$$

$$n = \frac{E_s}{E_m} = \frac{29(10^6)}{700(1500)} = 27.62$$

$$n\rho = 27.62(0.0066) = 0.1823$$

$$k = \sqrt{(n\rho)^2 + 2n\rho} - n\rho$$
$$= \sqrt{(0.1823)^2 + 2(0.1823)} - (0.1823)$$
$$= 0.4484$$

(Alternatively, from Table A.15, by interpolation, $k = 0.4484$)
Calculate I_{cr} from Eq. (4.146):

$$I_{cr} = \frac{1}{3} b\,(kd)^3 + nA_s\,(d - kd)^2$$

$$kd = 0.4484(20) = 8.97\text{ in.}$$

$$d - kd = 20 - 8.97 = 11.03\text{ in.}$$

$$I_{cr} = \frac{(7.63)(8.97)^3}{3} + (27.62)(1.0)(11.03)^2$$
$$= 5196\text{ in.}^4$$

Example 4.33 Gross moment of inertia of a transformed section.

Calculate the gross moment of inertia of the transformed section for the rectangular beam described in Example 4.32.

Solution

Given: CMU beam, $b = 7.65$ in., $h = 24$ in., $d = 20$ in., $A_s = 1.0$ in.^{2}, $f'_m = 1500$ psi.

Assuming that there is no compression steel, the transformed area of steel reinforcement in the uncracked beam cross section is calculated as follows:

$$n = \frac{E_s}{E_m} = \frac{29(10^6)}{700(1500)} = 27.62$$

Transformed area of steel = $(n - 1)A_s = (27.62 - 1)(1.0) = 26.62$ in.^{2}
Total area of masonry (including the transformed area of reinforcement)

$$= 7.63\,(24) + 26.62 = 209.74 \text{ in.}^2$$

The distance of the centroidal axis of the beam from its top face, y_t, is calculated as

$$y_t = \frac{(7.63)(24)(12) + (26.62)(20)}{209.74} = 13.015 \text{ in.}$$

The moment of inertia of the transformed section, I_{gt}, is calculated as follows (using parallel-axis theorem):

$$I_{gt} = \frac{(7.63)(24)^3}{12} + (7.63)(24)(13.015 - 12)^2 + 26.62(20 - 13.015)^2 = 10{,}277 \text{ in.}^4$$

The gross moment of inertia of the section (ignoring reinforcement) was calculated in the previous example to be 8790 in.^{4}. Thus, I_{gt} is greater than I_g by 10,277 − 8790 = 1487 in.^{4}. The difference amounts to

$$\left(\frac{1487}{8790}\right)(100) = 16.92\%$$

4.21 DEFLECTIONS OF REINFORCED MASONRY BEAMS

4.21.1 Concept of Effective Moment of Inertia of a Reinforced Masonry Beam

The MSJC-08 *Code* [4.3] does not specify a methodology for computing deflections in masonry beams. However, *Code* Section 3.1.5.2 requires that "deflection calculations of reinforced masonry members shall consider the effects of cracking and reinforcement on member stiffness." It states further that "flexural and shear stiffness properties assumed for deflection calculations shall not exceed one-half of the gross section properties, unless a cracked-section analysis is performed."

Very little data is available on the deflections of masonry beams and related computational methods. Deflections of beams are inversely proportional to their flexural stiffness, EI (E_mI for masonry beams). There is always some uncertainty about the value of modulus of elasticity, E_m, since testing for its value is uncommon even when deflection is important to the design. Deflection calculations in both concrete and masonry structures are therefore generally approximate. Fortunately, it is uncommon for a masonry element to be limited or sized based on deflection limitations.

Under full service loads, masonry beams, like reinforced concrete beams, develop cracks which create uncertainty in the values of their moments of inertia. This is because the extent of cracking varies along the span due to the variation in the bending moment along the span. This causes a change in the position of beam's neutral axis along the span and a consequent change in the value of the moment of inertia of the beam along the span.

When the maximum bending moment present in the beam is equal to or less than the cracking moment (M_{cr}), the flexural stresses in masonry are below or equal to the modulus of rupture (f_r), and the beam section is assumed to remain uncracked. In that case, the full cross section of the beam can be counted upon resist deflection, and the moment of inertia of gross section (I_g) can be used to calculate deflections. Under full service loads, however, cracking develops in the tension zone of the beam. The extent and pattern of this cracking varies along the span (closely spaced and wider cracks near the midspan where moments are large, and fewer and narrower near the supports where moments are small), which complicates the problem of determining the value of true moment of inertia of a cracked beam. At the cracked sections, the moment of inertia is close to I_{cr}, but in between the cracks (where the section is uncracked), the moment of inertia is perhaps closer to I_g. In essence, the flexural stiffness of the beam, *EI*, varies along the span. To account for these uncertainties, it becomes necessary to use the effective moment of inertia, I_{eff}, to calculate deflections of beams under service loads.

To the extent that masonry beams behave similar to reinforced concrete beams under service loads, the methods that have been developed for the latter may be also used for masonry beams. For reinforced concrete members, Branson [4.23, 4.24] suggested the following expression for the effective moment of inertia of reinforced concrete section:

$$I_{\text{eff}} = \left(\frac{M_{\text{cr}}}{M_a}\right)^a I_{gt} + \left[1 - \left(\frac{M_{\text{cr}}}{M_a}\right)^a\right] I_{\text{cr}} \tag{4.147}$$

where I_{eff} = effective moment of inertia
I_{cr} = moment of inertia of the cracked section transformed to masonry
I_{gt} = gross-transformed moment of inertia
M_a = maximum moment in the member at the loading stage for which the moment of inertia is being considered or at any other previous loading stage [4.25]
M_{cr} = nominal cracking moment strength

In reinforced concrete members under very high loads, the tensile force in the concrete is insignificant compared to with that in tension reinforcement and the member approximates a completely cracked section. The effect of the tensile forces in the concrete on the *EI* is referred to as *tension stiffening* [4.25]. A somewhat similar condition may be assumed in masonry beams at ultimate load conditions. It is also clear from Eq. (4.146) that the effective moment of inertia I_{eff} would decrease as the loads (and hence M_a) would increase. This means that I_{eff} is larger when only the dead load is acting than when both dead and live loads are acting together.

For regions of constant moment, Branson [4.22] found the value of exponent *a* in Eq. (4.147) to be 4. For simply supported beams, Branson suggested that both the tension stiffening and the variation in flexural stiffness along the span could be accounted for by assuming $a = 3$. Thus, Eq. (4.147) can be modified and expressed as Eq. (4.148) as specified in the ACI *Code* [4.2], wherein, for simplicity, I_g has been substituted for I_{gt} ignoring the small contribution of the tension reinforcement to the moment of inertia:

$$I_{\text{eff}} = \left(\frac{M_{\text{cr}}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{\text{cr}}}{M_a}\right)^3\right] I_{\text{cr}} < I_g \tag{4.148}$$

where I_g = gross moment of inertia of masonry section (neglecting reinforcement)

An alternatively form of Eq. (4.148) is Eq. (4.149):

$$I_{\text{eff}} = I_{\text{cr}} + (I_g - I_{\text{cr}})\left(\frac{M_{\text{cr}}}{M_a}\right)^3 < I_g \tag{4.149}$$

It is seen from Eq. (4.148) or Eq. (4.149) that the effective moment of inertia I_{eff} is a function of the dimensionless ratio M_{cr}/M_a which provides a transition between the upper bounds of I_g and I_{cr}. The current practice is to use the *effective moment of inertia*, I_{eff}, to calculate the deflection in concrete beams [4.2]. In using either Eq. (4.148) or Eq. (4.149), M_a needs to be calculated at the stage of loading of the beam. Consider, for example, a reinforced masonry beam. When the beam is supporting its own weight, the moment would likely be smaller than its cracking moment (M_{cr}) so the beam would remain uncracked. The deflection under this loading condition (call it stage 1) can be calculated using the gross moment of inertia, I_g. When superimposed dead load is applied, the moment increases. If the total moment at this stage (call it stage 2) is greater than M_{cr}, the beam would have cracked; therefore, the deflection due to the superimposed load should be calculated using the effective moment of inertia of the beam (I_{eff}). The applied moment in stage 2 loading, M_a, to be used in Eq. (6.148) or Eq. (4.149), equals the sum of moments due to self-weight of the beam and the superimposed dead load. Next, when the live load is applied, the moment increases again (call it stage 3), which causes increased cracking of the beam. The applied moment, M_a, now equals the sum of moments due to (1) self-weight of the beam, (2) superimposed dead load, and (3) service load. The deflection due to service live load should be calculated using the effective moment of inertia, I_{eff}, calculated based on the value of M_a at stage 3 (which would be different than M_a at stage 2). The total deflection in the beam would be the sum of deflections at the three loading stages just described. See Example 4.34.

Equation (4.148) or Eq. (4.149) is applicable to simply supported beams. For continuous beams, the value of I_e would be different. Reference 4.26 suggests the following average values of I_{eff}:

Beams with two ends continuous:

$$I_{\text{eff, avg}} = 0.70 I_{em} + 0.15(I_{e1} + I_{e2}) \tag{4.150}$$

With one end continuous:

$$I_{\text{eff,avg}} = 0.85 I_{em} + 0.15(I_{e,\text{ continuous end}}) \tag{4.151}$$

where I_{em} = value of I_{eff} at the midspan
I_{e1}, I_{e2} = values of I_{eff} at the two ends

For deflection of masonry members, MSJC-08 Section 1.13.3.2 requires deflection of reinforced masonry beams to be computed from Eq. (4.152) [a modified version of Eq. (4.148)] unless effective stiffness properties are determined from a more comprehensive analysis:

$$I_{\text{eff}} = \left(\frac{M_{\text{cr}}}{M_a}\right)^3 I_n + \left[1 - \left(\frac{M_{\text{cr}}}{M_a}\right)^3\right] I_{\text{cr}} \le I_n \le 0.5 I_g \tag{4.152}$$

where I_n = moment of inertia of the net masonry section.

The upper limit ($0.5I_g$) in Eq. (4.152) applies to reinforced masonry designed under the strength design provisions of MSJC-08 code (Section 3.5.1.2).

For continuous members, I_{eff} may be computed as the average of values obtained from Eq. (4.152) for the critical positive and negative moment regions.

More accurate deflections can be calculated by performing moment-curvature analysis. This is particularly true for lightly reinforced sections with service load moments close to the cracking moment. If the curvatures ϕ are known at the ends and midspan of a concrete beam, and if they are assumed to be parabolically distributed along the span, the midspan deflection, Δ, of a simple or a continuous beam can be calculated from Eq. (4.153) [4.25]:

$$\Delta = \frac{L^2}{96}(\phi_1 + 10\phi_2 + \phi_3) \tag{4.153}$$

where L = length of span, and ϕ_1, ϕ_2, and ϕ_3 are curvatures at the one end, midspan, and at the other end, respectively. A discussion on the use of Eq. (4.153) can be found in Refs. 4.27 and 4.28.

For a uniformly loaded simple beam made from homogeneous materials, the maximum deflection occurs at the midspan as given by Eq. (4.154):

$$\Delta_{\max} = \frac{5wL^4}{384EI} \tag{4.154}$$

For reinforced masonry beams, Eq. (4.154) can be expressed as Eq. (4.155) wherein E_mI_e has been substituted for EI:

$$\Delta_{\max} = \frac{5wL^4}{384E_mI_e} \tag{4.155}$$

Alternatively, noting that in a simple beam, $M = wL^2/8$, Eq. (4.155) can be expressed as Eq. (4.156):

$$\Delta_{\max} = \frac{5ML^2}{48E_mI_e} \tag{4.156}$$

Application of Eqs. (4.152) and (4.155) is illustrated in Example 4.34.

Example 4.34 Deflection in a uniformly loaded CMU beam.
A simply supported nominal 8 × 24 in. CMU beam is reinforced with one No. 9 Grade 60 bar for tension with its centroid located at 20 in. from the compression face of the beam (see Fig. E4.32). It carries a uniform service dead load of 1.0 k/ft in addition to its own weight, and a uniform service live load of 1.75 k/ft over an effective span of 12 ft. Calculate the maximum deflection in the beam. Assume $f'_m = 1500$ psi. (Note: This data is taken from Example 4.13 which illustrated flexural calculations.)

Solution
Given: $D = 1.0$ k/ft (plus self-weight), $L = 1.75$ k/ft, effective span, $L_e = 12$ ft, $f'_m =$ 1500 psi, $h = 24$ in., $d = 20$ in.

Calculate moments at various loading stages.

Loading Stage 1: Moment due to self-weight of beam (M_{SW}).

Assuming normal weight masonry (with 140 lb/ft^3 grout weight), the self weight of a nominal 8-in. wide beam is 84 lb/ft^2 of beam height. Therefore, self-weight of beam = (84)(24/12) = 168 lb/ft

$$M_{SW} = \frac{wL^2}{8} = \frac{(0.168)(12)^2}{8} = 3.02\text{ k-ft}$$

Calculate the cracking moment M_{cr} from Eq. (4.54), and I_g from Eq. (4.113):

$$f_r = 200 \text{ psi (MSJC-08 Table 3.1.8.2.1)}$$

$$M_{cr} = f_r\left(\frac{bh^2}{6}\right) = (200)\frac{(7.63)(24)^2}{6} = 146{,}496 \text{ lb-in.} \approx 12.21 \text{ k-ft}$$

$$M_{cr} = 12.21 \text{ k-ft} > M_a = 3.02 \text{ k-ft}$$

Because, the applied moment is less than the cracking moment for the beam, the beam is not cracked. Use the moment of inertia of gross section for calculating deflection at this stage of loading.

$$I_g = 8790 \text{ in.}^4 \text{ (calculated in Example 4.32)}$$

$$E_m = 700 f'_m = (700)(1500) = 1.05(10^6)\ psi = 1050 \text{ ksi}$$

$$\Delta_{SW} = \frac{5ML^2}{48E_m I_e} = \frac{5(3.02)(12)^2(12)^3}{48(1050)(8790)} = 0.008 \text{ in.}$$

Loading Stage 2: Moment due to superimposed dead load on the beam (M_{SD}).

$$M_{SD} = \frac{wL^2}{8} = \frac{(1.0)(12)^2}{8} = 18 \text{ k-ft} > M_{cr} = 12.21 \text{ k-ft}$$

Because the moment due to superimposed dead load is greater than the cracking moment, the beam is cracked, so use the effective moment of inertia to calculate deflection.

$$M_a = M_{SW} + M_{SD} = 3.02 + 18.0 = 21.02 \text{ k-ft}$$

Calculate the effective moment of inertia from Eq. (4.148):
$I_g = 8790$ in^4, $I_{cr} = 6773$ in^4 (calculated in Example 4.32)

$$\begin{aligned} I_{eff} &= \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \\ &= \left(\frac{12.21}{21.02}\right)^3 (8790) + \left[1 - \left(\frac{12.21}{21.02}\right)^3\right](6773) \\ &= 1722.8 + 5445.5 = 7168 \text{ in}^4 \end{aligned}$$

Deflection due to superimposed dead load is Eq. (4.156)

$$\Delta_{SD} = \frac{5ML^2}{48E_m I_e} = \frac{5(18.0)(12)^2(12)^3}{48(1050)(7168)} = 0.062 \text{ in.}$$

Loading Stage 3: Moment due to service live load on the beam (M_L).

$$M_L = \frac{wL^2}{8} = \frac{(1.75)(12)^2}{8} = 31.5 \text{ k-ft}$$

$$M_a = M_{SW} + M_{SD} + M_L = 3.02 + 18.0 + 31.5 = 52.52 \text{ k-ft}$$

Calculate the effective moment of inertia using $M_a = 52.52$ k-ft.

$$\begin{aligned} I_{eff} &= \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \\ &= \left(\frac{12.21}{52.52}\right)^3 (8790) + \left[1 - \left(\frac{12.21}{52.52}\right)^3\right](6773) \\ &= 110 + 6688 = 6798 \text{ in}^4 \end{aligned}$$

The deflection due to live load is

$$\Delta_L = \frac{5ML^2}{48E_m I_e} = \frac{5(31.5)(12)^2(12)^3}{48(1050)(7168)} = 0.108 \text{ in.}$$

The total beam deflection is sum of deflections in the three stages of loading. Thus,

$$\Delta_{total} = \Delta_{SW} + \Delta_{SD} + \Delta_L = 0.008 + 0.062 + 0.108 = 0.178 \text{ in.} \approx 0.18 \text{ in.}$$

In case this beam were to support unreinforced masonry, the allowable deflection would be

$$\frac{L_e}{600} = \frac{(12)(12)}{600} = 0.24 \text{ in.} > 0.18 \text{ in.} \qquad \text{OK}$$

However, no such restriction is specified in this problem.

Commentary: Note that as the beam cracks, the moment of inertia decreases progressively as the applied moment increases. However, if the deflection is calculated using I_e from Eq. (4.151) we would have

$I_e = 0.5I_g = 0.5(8790) = 4395$ in.^{4} which is less than I_e calculated above. Therefore, use $I_e = 4395$ in.^{4}.

$$\Delta_{total} = \frac{5ML^2}{48E_m I_e} = \frac{5(52.52)(12)^2(12)^3}{48(1050)(4395)} = 0.295 \text{ in.}$$

This deflection value is larger than $L/600$ limit (= 0.24 in.), but is less that $L/240$ limit commonly prescribed for beams under service loads:

$$\frac{L}{240} = \frac{12(12)}{240} = 0.6 \text{ in.}$$

However, no such deflections limits are prescribed in this problem.

Problems

4.1 An 8 × 24 in. (nominal) concrete masonry beam is reinforced with one No. 6 Grade 60 reinforcing bar positioned at 20 in. from the compression face (Fig. P4.1). Calculate (a) moment of inertia of gross section, (b) moment of inertia of cracked section, (c) cracking moment, (d) ϕM_n of the beam, and (e) check if the beam satisfies the code requirements. Assume f'_m = 1500 psi.

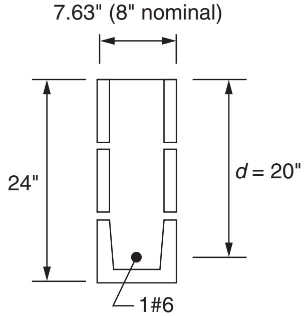


FIGURE P4.1

4.2 A 10 × 40 in. (nominal) concrete masonry beam is reinforced with two No. 7 Grade 60 reinforcing bars positioned at 34 in. from the compression face (Fig. P4.2). Calculate (a) moment of inertia of gross section, (b) moment of inertia of cracked section (c) cracking moment, (d) ϕM_n of the beam, and (e) check if the beam satisfies the code requirements. Assume $f'_m = 2000$ psi.

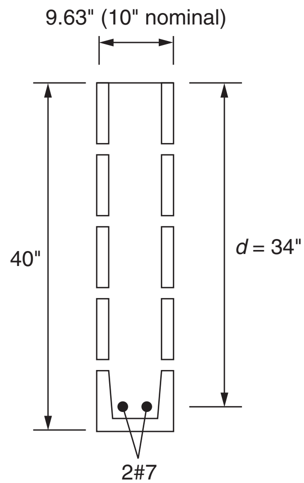


FIGURE P4.2

4.3 An 8 × 24 in. (nominal) concrete masonry beam is reinforced with one No. 5 Grade 60 reinforcing bar positioned at 20 in. from the compression face (Fig. P4.3). Calculate (a) moment of inertia of cracked section (b) cracking moment, (c) ϕM_n of the beam, and (d) check if the beam satisfies the code requirements. Assume $f'_m = 1800$ psi.

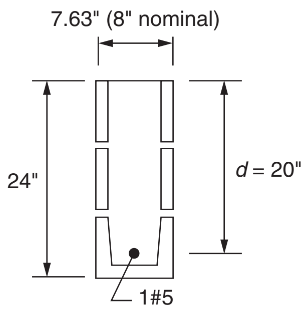


FIGURE P4.3

4.4 An 8 × 24 in. (nominal) concrete masonry beam is reinforced with one No. 8 Grade 60 reinforcing bar positioned at 20 in. from the compression face (Fig. P4.4). Calculate (a) cracking moment, and (b) ϕM_n of the beam, and (c) check if the beam satisfies the code requirements. Assume $f'_m = 2000$ psi.

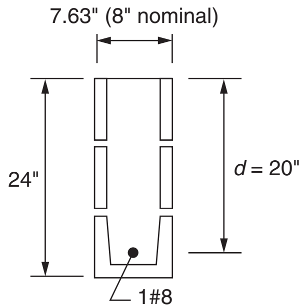


FIGURE P4.4

4.5 An 8 × 24 in. (nominal) concrete masonry beam is reinforced with one No. 8 Grade 60 reinforcing bar positioned at 20 in. from the compression face (Fig. P4.5). Calculate (a) cracking moment, (b) ϕM_n of the beam, and (c) check if the beam satisfies the code requirements. Assume $f'_m = 1500$ psi.

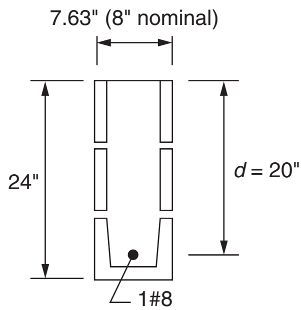


FIGURE P4.5

4.6 A nominal 10 × 40 in. concrete masonry beam is reinforced with two No. 5 Grade 60 bars. The centroid of reinforcement is located 6 in. from the bottom of the beam (Fig. P4.6). $f'_m = 2000$ psi. Calculate a, c, and the strain in tension steel reinforcement.

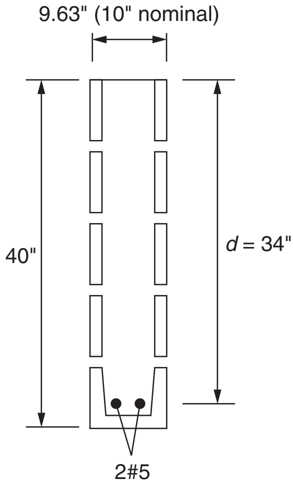


FIGURE P4.6

4.7 A nominal 10 × 40 in concrete masonry beam built from lightweight CMU is reinforced with two No. 5 Grade 60 bars for tension. The centroid of reinforcement is located at 6 in. from the bottom of the beam (Fig. P4.7). The beam carries a service live load of 1000 lb/ft in addition to its own weight over an effective span of 15 ft 8 in. The grout weight is 140 lb/ft^3. $f'_m = 1500$ psi. Check if the beam is adequate to carry the imposed loads.

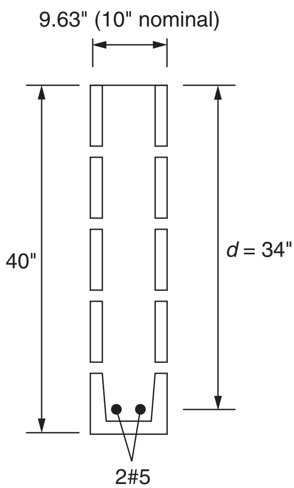


FIGURE P4.7

4.8 A nominal 8 × 24 in. concrete masonry beam is reinforced with one No. 5 Grade 60 bar at an effective depth of 20 in. (Figure P4.3). Determine the moment capacity ϕM_n of the beam. Assume $f'_m = 1500$ psi.

4.9 Determine the moment capacity, ϕM_n of the beam described in Prob. 4.3 if Grade 40 reinforcing bar were to be used instead of Grade 60 bar. All other data are the same.

4.10 A two-wythe 8 × 24 in. clay brick beam has an effective depth of 20 in. It is reinforced with one No. 7 Grade 60 bar for tension. (Fig. P4.10). Determine the moment capacity, ϕM_n, of this beam. $f'_m = 2000$ psi. Portland cement Type *S* mortar would be used for construction.

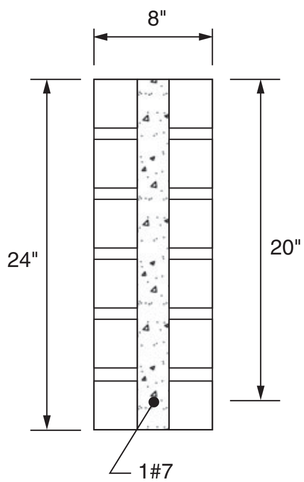


FIGURE P4.10

4.11 Using Eq. (4.43), (a) determine the flexural strength, ϕM_n, of the beam described in Prob. 4.3; (b) How would the flexural strength of the beam be affected if Grade 60 bar were replaced with a Grade 40 bar?

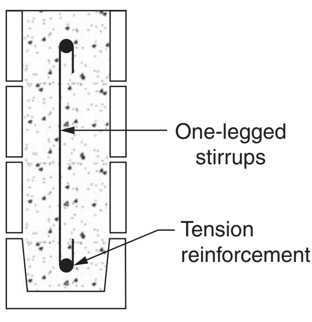


FIGURE P4.11

4.12 For the beam described in Prob. 4.3, (a) determine the uniform load the beam can carry over a span of 10 ft 8 in. if the dead-to-live load ratio is 0.7, (b) uniform live load the beam can carry safely if the only dead load on the beam is its own weight.

4.13 A two-wythe 8 × 24 in. clay brick beam has an effective depth of 20 in. It is reinforced with one No. 7 Grade 60 bar for tension. Determine the cracking moment of this beam (as shown in Prob. 4.10). $f'_m = 2000$ psi. Portland cement Type *S* mortar would be used for construction. Does the beam satisfy the code requirements?

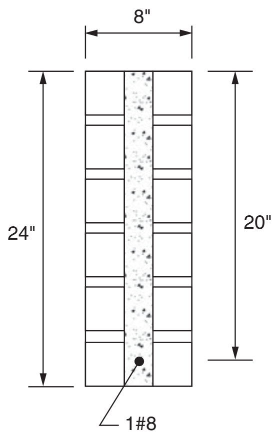


FIGURE P4.13

4.14 Check if the beam described in Prob. 4.8 would satisfy the code requirements for cracking moment if it were reinforced with (a) one No. 4 Grade 60 bar instead of one No. 5 bar, (b) one No. 7 bar instead of one No. 5 Grade 60 bar. $f'_m = 2500$ psi, $f_y = 60$ ksi.

4.15 A nominal 8 × 24 in. concrete masonry beam is reinforced with one No. 9 Grade 60 reinforcing bar placed at $d = 20$ in. for tension (Fig. P4.15). The beam is required to carry a uniform service dead load of 1.2 k/ft (including its self-weight) and a service live load of 1.5 k/ft over a span of 12 ft. $f'_m = 1500$ psi. Check if the beam is adequate to carry the load.

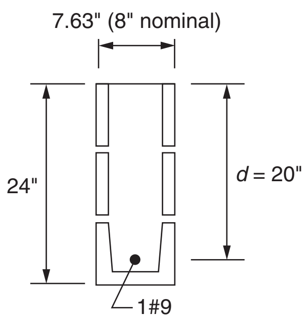


FIGURE P4.15

4.16 A simply supported clay brick beam is 9 in. wide and 24 in. deep with the tension reinforcement located at 20 in. from the top of the beam. The beam has an effective span of 12 ft and carries a service live load of 1200 lb/ft in addition to its own weight. Use f'_m = 2000 psi and Grade 60 steel. Determine the flexural reinforcement required for this beam.

4.17 An 8-in. wide CMU lintel with a clear span of 12 ft has to carry a superimposed dead load of 750 lb/ft and a live load of 800 lb/ft. Use f'_m = 1500 psi and Grade 60 reinforcement. Determine the depth of the lintel and the area of reinforcement. The masonry would be built from normal weight CMU with a grout weight of 140 lb/ft^3. Assume that length of bearing on each support is 8 in.

4.18 A nominal 8 × 24 in. CMU beam is reinforced with one No. 8 Grade 60 reinforcing bar with d = 20 in. The beam is required to carry a service dead load of 1.0 k/ft (including its self-weight) and a service live load of 1.0 k/ft over a span of 12 ft. f'_m = 1500 psi (Fig. P4.18) (a) Calculate the shear resistance V_m provided by masonry, (b) what would be the increase in the shear resistance provided by the masonry if the specified strength of masonry f'_m is increased from 1500 psi to 2000 psi.

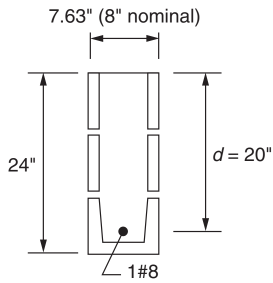


FIGURE P4.18

4.19 A nominal 10 × 40 in. CMU beam is reinforced with two No. 5 Grade 60 reinforcing bars for tension. The centroid of reinforcement is located at 6 in. from the bottom of the beam (Fig. P4.19). The beam carries a service load of 1200 lb/ft over an effective span of 15 ft 8 in. in addition to its own weight. The grout unit weight is 140 lb/ft^3. f'_m = 1500 psi. Check if shear reinforcement is required for this beam.

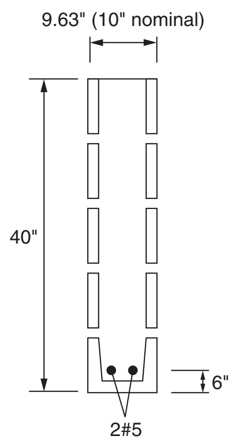


FIGURE P4.19

4.20 A nominal 8 × 24 in. CMU beam is reinforced with one No. 8 Grade 60 reinforcing bar for tension with $d = 20$ in (as shown in Fig P4.18). The beam is required to carry a service dead load of 1.0 k/ft (including its self-weight) and a service live load of 1.5 k/ft over a span of 12 ft. $f'_m = 1500$ psi. Design the shear reinforcement for this beam.

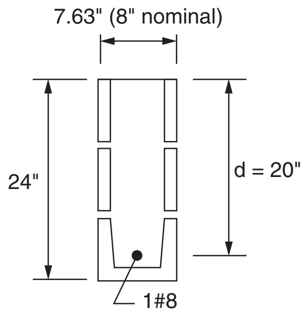


FIGURE P4.20

4.21 A nominal 8 × 24 in. simply supported CMU beam is reinforced with one No. 8 Grade 60 reinforcing bar for tension with $d = 20$ in. (as shown in Fig P4.18.) The beam is required to carry a service dead load of 1.2 k/ft (including its self-weight) and a service live load of 1.0 k/ft over a span of 12 ft. $f'_m = 1500$ psi. Design the shear reinforcement for this beam.

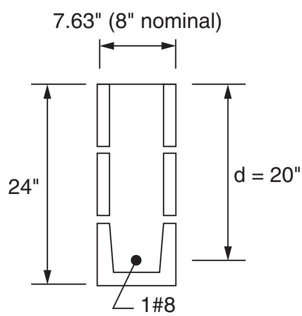


FIGURE P4.21

4.22 Figure P4.22 shows details of a concrete masonry wall having an 8 ft wide opening. The wall is 8-in. thick (nominal) and grouted solid (medium weight units, grout weight 140 lb/ft^3). The reinforced masonry over the opening is to be utilized as lintel. The wall supports two superimposed concentrated loads 8 kips each (40 percent dead load and 60 percent live load), transferred to it as reactions from roof trusses spaced at 8 ft o.c. The parapet extends 2 ft 8 in. above the roof level as shown in the figure. Determine the tensile reinforcement (Grade 60) required for the lintel if $f'_m = 1500\,\text{lb/in.}^2$.

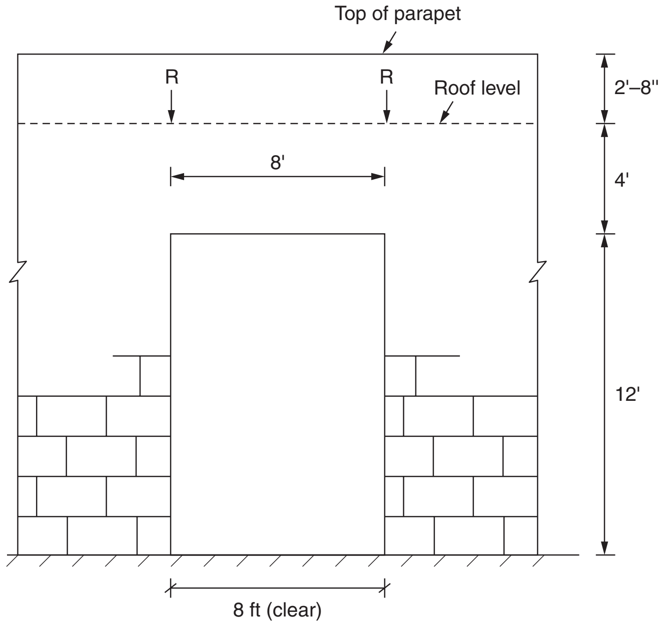


FIGURE P4.22

4.23 A roof diaphragm 96 × 60 ft is subjected to a strength level lateral load of 1000 lb/ft perpendicular to its long side and supported over reinforced masonry walls. Determine the reinforcement for the bond beam.

4.24 Calculate the development length for No. 6 Grade 60 bar placed in a standard 8 × 8 × 16 CMU when (a) only one bar is placed in each cell, and (2) when two bars are placed in each cell. Use $f'_m = 1500$ psi.

4.25 A simply supported nominal 8 × 24 in. CMU beam is reinforced with two No. 6 Grade 60 bars for tension with centroid located at 20 in. from the compression face of the beam. Calculate the moments of inertia of (a) gross section, (b) cracked section. Assume $f'_m = 1500$ psi.

4.26 A simply supported nominal 8 × 24 in. CMU beam is reinforced with two No. 6 Grade 60 bars for tension with centroid located at 20 in. from the compression face of the beam. It carries a uniform service dead load of 1.2 k/ft in addition to its own weight, and a uniform service live load of 1.5 k/ft over an effective span of 12 ft. Calculate the maximum deflection in the beam. Assume $f'_m = 1500$ psi.

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CHAPTER 5
COLUMNS

5.1 INTRODUCTION

Vertical members designed to carry axial compression loads, alone or in combinations with flexure, are called columns. The MSJC *Code* [5.1] defines *column* as an isolated vertical compression member having its horizontal dimension measured at right angles to its thickness not exceeding 3 times its thickness and having its height greater than 4 times its thickness (Fig. 5.1). The descriptive “isolated” is not intended to imply that the column is isolated from other parts of a masonry structure or lateral force resisting system; it is intended to be used for the application of the MSJC *Code* provisions for columns as specified in *Code* Sections 1.14 and 3.3.4:

1. Minimum side dimension of a column shall be 8 in.
2. The nominal depth of a column shall be not less than 8 in. and not be greater than 3 times its width.
3. The distance between lateral supports of a column shall not exceed 30 multiplied by its nominal width.

The above descriptors of a column are usually satisfied in masonry structures. For example, an 8-in.-deep (side dimension) reinforced masonry column is not likely to have a width greater than 24 in. (= 3 × 8), and a height of greater than 60 ft (= 30 × 24 in.) between its lateral supports (usually top and bottom), and so on. The later requirement is expressed as h/t limit in examples in this chapter (i.e., $h/t \leq 30$).

Vertical loads, in combination with or without flexure, may be also carried by *wall columns* that really are parts of a wall which act as columns. When they are fully contained in the wall, they are called *flush wall columns* or *in-wall columns*. Often the wall columns may project from one or both faces of the wall in which case they are called *pilasters* (Fig. 5.2). In addition to carrying vertical loads, pilasters aid in lateral force resistance of masonry walls of which they are a part. Figure 5.2 shows a typical pilaster projecting only from one side of the wall.

Reinforced masonry columns can be constructed from either concrete masonry units (CMU) or bricks (clay masonry) (Fig. 5.3). Both columns and pilasters can be constructed from standard CMU and bricks, or from special pilaster or chimney units. In both cases, masonry units should be arranged in running bond so as to avoid continuous head joints.

Masonry columns, like other masonry members, can be designed by either allowable stress design (ASD) method or the strength method (Fig. 5.4). This chapter presents a discussion on analysis and design of columns based on strength design philosophy in conformance to MSJC-08 *Building Code Requirements for Masonry Structures* [5.1], referred to hereinafter as *Code* and MSJC-08 *Specification for Masonry Structures* [5.2], and referred

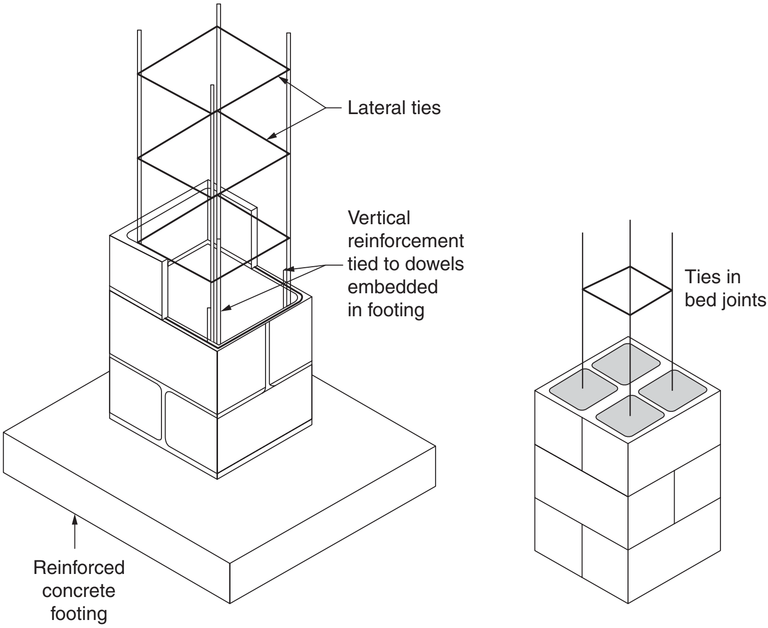


FIGURE 5.1 A reinforced masonry column.

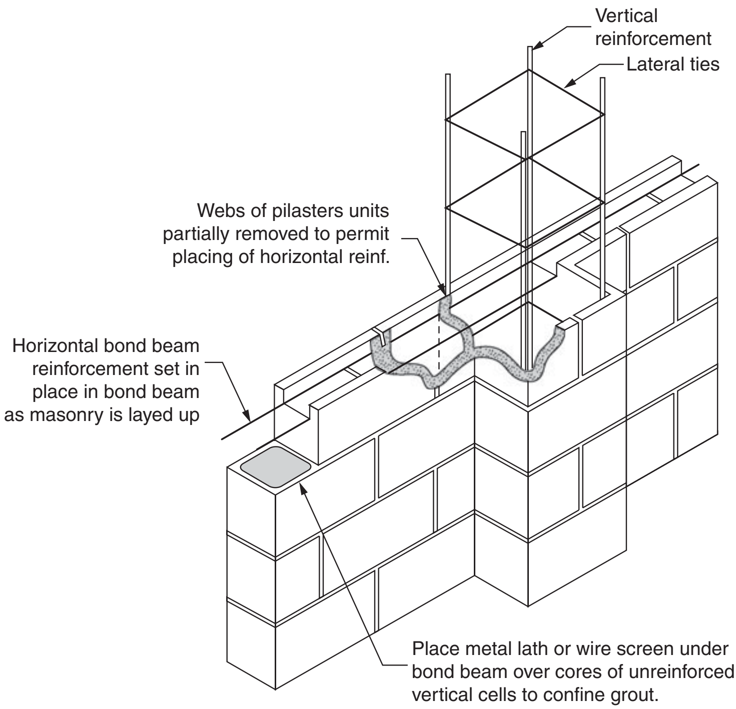


FIGURE 5.2 A pilaster.

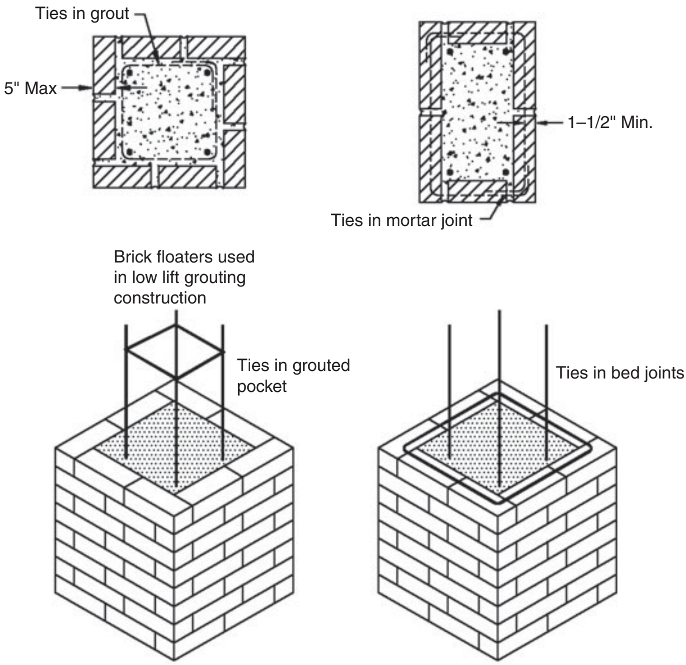


FIGURE 5.3 Column cross sections for brick masonry columns.

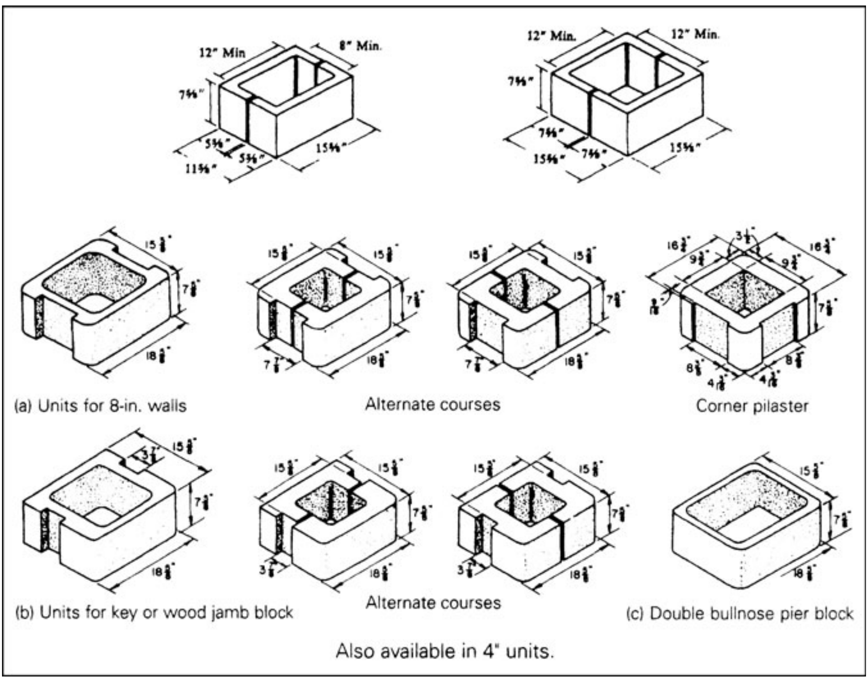


FIGURE 5.4 Concrete masonry units for columns and pilasters.

to hereinafter as *Specification*. Frequent references in this discussion are made to two documents supporting the *Code* and the *Specification*, viz., the *Commentary on Building Code Requirements* [5.3], and *Commentary on Specification* [5.4]. It is noted that Ref. 5.1 is the reference document for design requirements (with few exceptions) for strength design of masonry columns specified in 2009 *International Building Code* (2009 IBC) [5.5]. Analysis and design of columns according to the ASD are presented in several references and are not discussed in this book [5.6–5.8].

5.2 BEHAVIOR OF AXIALLY LOADED COLUMNS

5.2.1 Buckling Strength of Columns

Behavior of long columns of elastic and homogeneous materials that follow Hooke's law is well documented in texts on strength of materials. Long columns are those that fail due to instability, commonly described as buckling failure. Because this type of failure occurs *before* the material reaches its yield strength, it is referred to as the *elastic buckling failure*. The *critical* or *buckling load*, also known as *Euler's load* for columns is given by Eq. (5.1):

$$P_{cr} = \frac{\pi^2 EI}{h^2} \tag{5.1}$$

where P_{cr} = critical or buckling load
E = modulus of elasticity
I = moment of inertia of column cross-section
h = effective height of the column

Noting that $I = Ar^2$, where A and r, respectively, are the cross-sectional area and the radius of gyration of the column cross section, Eq. (5.1) can be rewritten as Eq. (5.2):

$$P_{cr} = \frac{\pi^2 EA}{(h/r)^2} \tag{5.2}$$

From the standpoint of failure, columns made from elastic, homogenous materials that follow Hooke's law are classified as *short*, *intermediate,* and *long*. Short columns are characterized by failure by crushing or yielding of material. The failure of both the intermediate and the long columns is controlled by the stability of column cross section. The failure of intermediate columns is initiated by crushing or yielding of material in some portion of the cross section, followed by buckling. Long columns are characterized by pure buckling failure.

Failure modes of columns made from an ideal material such as steel can be well defined or predicted easily as described above. However, such is not the case for columns made from nonhomogeneous material such as reinforced masonry, since no well-defined limiting stress, such as yield stress, occurs in these columns. To date, not enough research on full-size reinforced masonry columns has been conducted to define their behavior under failure loads, and no rational expressions have been developed to form a basis for design.

For reinforced masonry, Eq. (5.2) can be expressed as Eq. (5.3) wherein E_m (modulus of elasticity of masonry) has been substituted for E:

$$P_{cr} = \frac{\pi^2 E_m A}{(h/r)^2} \tag{5.3}$$

5.2.2 Effective Column Height: Effects of End Restraints

The effective height, h, which appears in Euler's formula for the buckling load of a column, is valid only for a column with pin-ended supports, h being the distance between the pinned ends. This formula must be modified if the end-restraint conditions are different from the pin-ended conditions. This modification is easily done by considering the effective column height Kh, defined as the distance between the inflection points on the buckled configuration of the column, which depends on the end-restraint conditions. In order to account for the effective column height, Eq. (5.3) can be expressed as Eq. (5.4) wherein Kh has been substituted for h.

$$P_{cr} = \frac{\pi^2 EA}{(Kh/r)^2} \tag{5.4}$$

where K = the *effective length factor* whose value depends on the end restraint conditions.

It is important to recognize that the load-carrying capacity of a column is inversely proportional to the square of the effective height as indicated by Eq. (5.4). Thus, determination of effective height of a column is an important consideration in design. For a column with pinned ends, value of the effective length factor K may be taken as unity ($K = 1.0$). Values of K for other end-restraint conditions for columns, walls, and pilasters would be different. Figure 5.5 shows examples of determining effective h for some practical cases. Columns, walls, and pilasters, all of which might span several floors may be assumed braced at the floor locations. For these elements, the effective height may be taken as the *clear height* between the floors. For a cantilevered column, wall, or a pilaster, the effective height may be measured from the top of the floor or the roof as applicable. Because of the nature of masonry construction, the MSJC *Code* does not provide the values of effective length factor, K, for various end-restraint conditions.

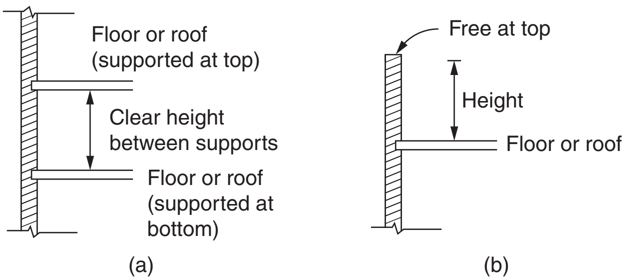
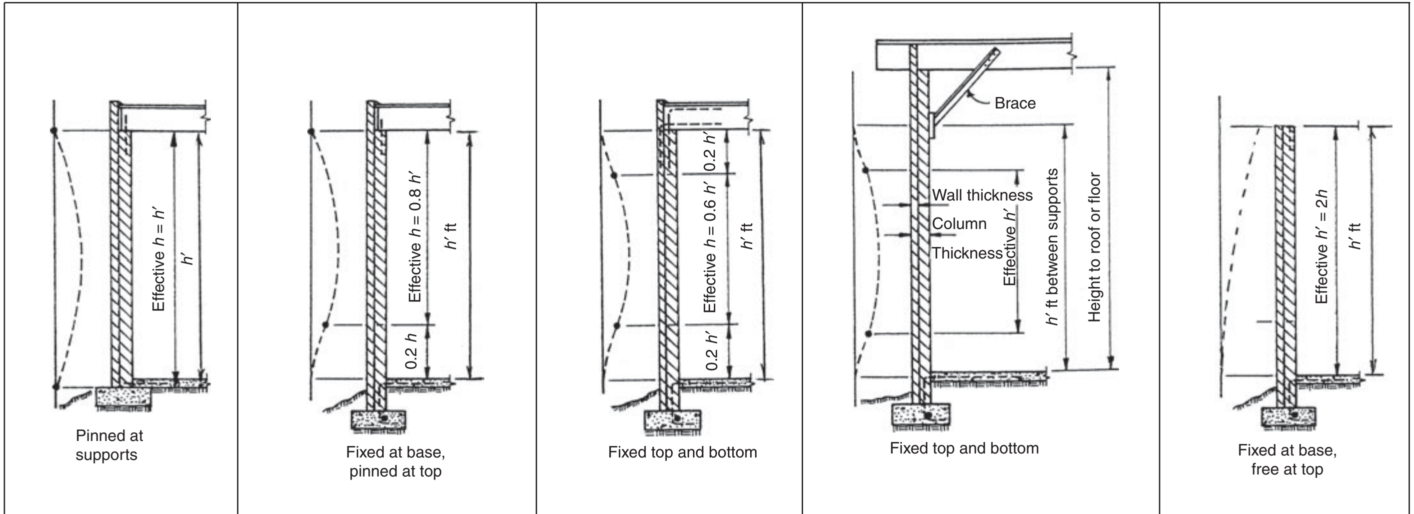


FIGURE 5.5 Effective height of a column, wall, or a pilaster [5.3]. (*a*) Column, wall, or pilaster braced at support: h = clear height. (*b*) Column, wall, or pilaster fixed at base: $h = 2 \times$ height.

If there is reliable restraint against translation and rotation at the supports, the "effective height" may be taken as low as the distance between points of inflection for the loading case under consideration.

Figure 5.6 presents end restraints for several practical cases. In all the examples in this chapter, the specified column height is considered as the effective height unless stated otherwise.

Assumed buckled shape of columns, walls, and pilasters is shown by dashed lines.



$K = 1.0$	$K = 0.8$	$K = 0.6$	$K = 0.6$	$K = 2.0$
$K^* = 1.0$	$K = 0.8$	$K = 0.65$		$K = 2.10$

*AISC recommended values of K [5.9]

FIGURE 5.6 Effective height of a column, wall, or a pilaster for practical end conditions. (Adapted from Ref. [5.6].)

5.3 AXIAL STRENGTH OF REINFORCED MASONRY COLUMNS

Load-carrying capacity of unreinforced masonry columns is limited by the compressive strength of masonry units. This limited capacity can be augmented by providing longitudinal reinforcement to carry additional compressive loads. The presence of reinforcement also adds ductility to column behavior, an important requirement for their performance.

Philosophically, the nominal axial strength P_n of a short reinforced masonry column is taken as the sum of the separate nominal axial strengths of masonry, $(P_n)_m$, and the reinforcement, $(P_n)_s$. Theoretically, these two strength components can be expressed as

$$(P_n)_m = 0.80 f'_m (A_n - A_{st}) \tag{5.5}$$

$$(P_n)_s = f_y A_{st} \tag{5.6}$$

where f'_m = specified compressive strength of masonry
A_n = net cross-sectional area of masonry
A_{st} = area of longitudinal reinforcement
f_y = yield stress of steel in compression

Again, theoretically, the nominal strength of an axially loaded reinforced masonry column can be expressed as the sum of two strength components given by Eqs. (5.5) and (5.5):

$$P_n = (P_n)_m + (P_n)_s = 0.80 f'_m (A_n - A_{st}) + f_y A_s \tag{5.7}$$

Equation (5.7) gives the nominal strength of a perfectly straight, short, axially loaded reinforced masonry column. For practical columns, Eq. (5.7) is modified for two important reasons:

1. There is always some unintentional (or accidental) eccentricity present in columns, which would cause some bending with consequent loss in axial load capacity. To account for this eventuality, the nominal strength given by Eq. (5.7) is multiplied by a factor of 0.8, which is different from the 0.8 multiplier applied to compressive strength of masonry (as in 0.8 f'_m) in Eq. (5.7). It is noted that columns designed under the allowable stress design (ASD) provisions of the MSJC *Code* [5.1] are required to be designed, as a minimum, for an eccentricity of 0.1 times each side dimension, independently about each axis (MSJC *Code* Section 2.1.6.2). This minimum eccentricity of axial loads results from construction imperfections not otherwise anticipated in analysis [5.2]. There is no such requirement under the strength design provisions of the code; the 0.8-factor is used in lieu of that requirement.
2. To account for slenderness of the columns, the nominal axial strength given by Eq. (5.7) is multiplied by yet another reduction factor, which is a function of the slenderness ratio h/r of the column.

Masonry columns, like concrete columns, behave differently when their slenderness increases beyond certain limits. Accordingly, Eq. (5.7) is expressed as two separate equations applicable to two different ranges of slenderness ratios:

For $h/r \le 99$

$$\begin{aligned} P_n &= 0.80[P'_m + P'_s]\left[1 - \left(\frac{h}{140r}\right)^2\right] \\ &= 0.80[0.80 f'_m (A_n - A_{st}) + f_y A_{st}]\left[1 - \left(\frac{h}{140r}\right)^2\right] \end{aligned} \tag{5.8}$$

For $h/r > 99$

$$P_n = 0.80[P'_m + P'_s]\left(\frac{70r}{h}\right)^2$$

$$= 0.80[0.80f'_m(A_n - A_{st}) + f_y A_{st}]\left(\frac{70r}{h}\right)^2 \tag{5.9}$$

It is noted that for the slenderness ratio $h'/r = 99$, Eqs. (5.8) and (5.9) give the same values of P_n, for when $h/r = 99$,

$$\left[1-\left(\frac{h}{140r}\right)^2\right] = \left(\frac{70r}{h}\right)^2 = 0.5$$

Figure 5.7 shows the slenderness effects on axial strengths of columns [5.3].

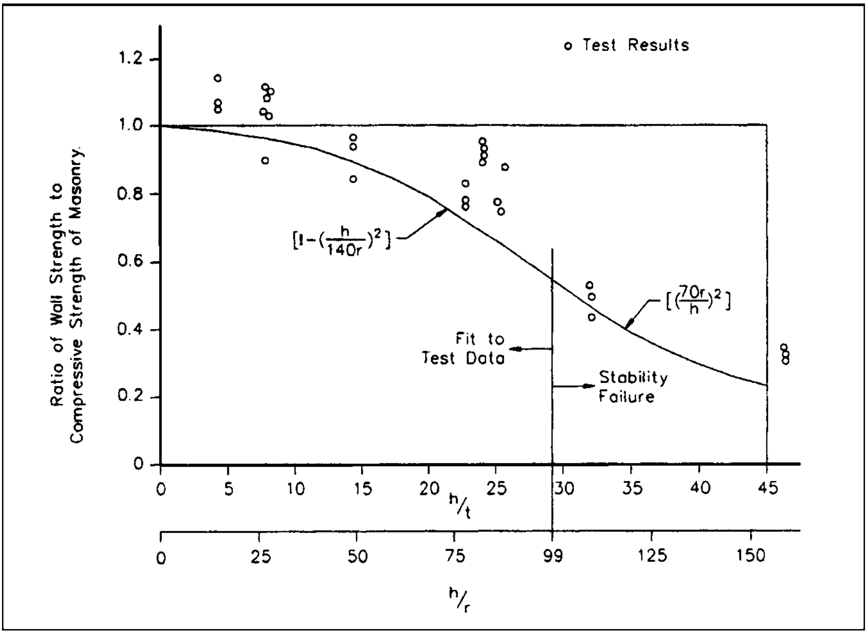


FIGURE 5.7 Slenderness effects on axial compressive strengths of masonry columns [5.3].

Equations (5.8) and (5.9) are applicable to columns having pinned-end conditions at both ends, which would result in symmetric deflection (curvature of buckled configuration) about the mid-height of the column. The term A_n in Eqs. (5.8) and (5.9) represents the *net* area of the masonry cross section, whereas the area $(A_n - A_{st})$ represents the *effective* net area of the column cross section.

For design purposes, the factored load P_u should not exceed ϕP_n where $\phi = 0.9$, strength reduction factor for axially loaded members (MSJC Section 3.1.4.1). Therefore, Eqs. (5.8) and (5.9) can be written as follows:

For $h/r \le 99$

$$\phi P_n = 0.80\phi[P'_m + P'_s]\left[1-\left(\frac{h}{140r}\right)^2\right]$$

$$= 0.80\phi[0.80f'_m(A_n - A_{st}) + f_y A_{st}]\left[1-\left(\frac{h}{140r}\right)^2\right] \tag{5.10}$$

For $h/r > 99$

$$\phi P_n = 0.80\phi[P'_m + P'_s]\left(\frac{70r}{h}\right)^2$$

$$= 0.80\phi[0.80f'_m(A_n - A_{st}) + f_y A_{st}]\left(\frac{70r}{h}\right)^2 \tag{5.11}$$

Alternatively, Eqs. (5.10) and (5.11) can be written as a single equation given by Eq. (5.12):

$$\phi P_n = 0.80\phi[0.80f'_m(A_n - A_{st}) + f_y A_{st}]C_P \tag{5.12}$$

In Eq. (5.12), C_P is a modifier (less than 1.0), which accounts for lateral stability of the column. It is a function of limiting column slenderness ratio h/r:

For $$\frac{h}{r} \le 99, C_P = \left[1-\left(\frac{h}{140r}\right)^2\right] \tag{5.13}$$

For $$\frac{h}{r} > 99, C_P = \left(\frac{70r}{h}\right)^2 \tag{5.14}$$

Use of Eqs. (5.13) and (5.14) require calculating radius of gyration r of the column cross section. Masonry columns are generally rectangular, so the radius of gyration for such columns can be expressed as

$$r = \sqrt{I_{\min}/A_n} \tag{5.15}$$

where $A_n = bt$
b = actual width of column
t = actual depth of column ($b > t$), the least cross-sectional dimension
$I_{\min} = bt^3/12$.

With the above substitutions, Eq. (5.15) can be expressed as

$$\mathrm{r} = \sqrt{\frac{(bt^3/12)}{bt}} = \frac{t}{\sqrt{12}} \tag{5.16a}$$

Alternatively, Eq. (5.16a) may be expressed in a simpler form as

$$r \approx 0.289t \tag{5.16b}$$

Most practical masonry columns would fall into the column height category for which Eq. (5.13) would apply; Eq. (5.14) would apply for unusually long columns. For example, for a 24 × 24 in. (actual dimensions) column,

$$r = 0.289t = 0.289(24) = 6.936 \text{ in.}$$

$$\text{For } \frac{h}{r} = 99 \text{ and } r = 6.936 \text{ in., } h = (99)(6.936)/12 \approx 57 \text{ ft}$$

Similarly, for a 16 × 16 in. (actual dimensions) column,

$$r = 0.289t = 0.289(16) = 4.625 \text{ in.}$$

$$\text{For } \frac{h}{r} = 99 \text{ and } r = 4.625 \text{ in.}, h = (99)(4.625)/12 \approx 38 \text{ ft}$$

Both effective column heights are unusually tall for a practical structure if the columns are not laterally supported at some points between their ends.

It is noted that determination of nominal strength of a masonry column requires determining the value of the axial stress reduction coefficient C_P from Eqs. (5.12) and (5.13). It is possible to simplify these computations if C_P can be expressed as a function of *h*/*t* ratio. The derivation follows:

Case 1: $h/r \leq 99$

The radius of gyration, *r*, for a rectangular section is given by $r = t/\sqrt{12}$ [Eq. (5.16)], which when substituted in Eq. (5.13) yields

$$C_P = 1 - \left(\frac{h}{140(t/\sqrt{12})}\right)^2$$
$$= 1 - 0.000612244\left(\frac{h}{t}\right)^2 \tag{5.17}$$

Note that for $h/t = 28.577$, Eq. (5.17) gives $C_P = 0.5$. Consequently, Eq. (5.17) applies for $h/t \leq 28.577$, which corresponds to $h/r \leq 99$.

Case 2: $h/r > 99$

A similar expression for C_P can be derived in terms for *h*/*t* ratio corresponding to $h/r \geq 99$ by substituting $r = t/\sqrt{12}$ in Eq. (5.13). Thus,

$$C_P = \left(\frac{70r}{h}\right)^2 = \left(\frac{(70(t/\sqrt{12})}{h}\right)^2 = \frac{408.33}{(h/t)^2} \tag{5.18}$$

Note that for $h/t = 28.577$, Eq. (5.18) gives $C_P = 0.5$. Consequently, Eq. (5.18) applies for $h/t > 28.577$, which corresponds to $h/r > 99$. For all values of *h*/*t* ratios greater than 28.577, $C_P < 0.5$.

The values of C_P for a practical range of *h*/*t* ratios are given in Table A.16. For intermediate values of *h*/*t* ratios, C_P can be calculated by interpolation without any appreciable error as illustrated in several examples that follow.

5.4 *MSJC* CODE *PROVISIONS FOR REINFORCED MASONRY COLUMNS*

All masonry columns and pilasters are required to be *fully grouted*. Columns should comply with the code requirements governing their sizes (cross-sectional dimensions), and height–to–least cross-sectional dimension ratio (*h*/*t* ratio), and configuration of reinforcement presented in this section. In general, these requirements, some being prescriptive, are based on successful past construction practices, experience from historical performance of as-built structures in earthquakes and windstorms (hurricanes and tornadoes), and to some extent, on research findings. In addition, there are practical and constructibility considerations, such as ensuring that required cover is provided to reinforcement, that there is no congestion (crowding) in the cells due to too many closely placed reinforcing bars that would impede flow of grout, and spacing of ties (to match coursing).

It is noted that per MSJC-08 *Code* Section 1.17.4.4.2.2, neither Type N mortar nor masonry cement is permitted to be used to construct elements (such as columns) that participate in lateral force resisting system (LFRS) in Seismic Design Category D, E, and F (discussed in Chap. 7). There is no such restriction for LFRS in other Seismic Design Categories.

The compressive strength of masonry is limited to 4000 psi for concrete masonry and 6000 psi for clay masonry. The nominal yield strength of steel reinforcement is specified to be 60,000 psi. The MSJC *Code* [5.6] is specific to these limitations for design purposes because these are the material strengths of masonry structural components on which research has been conducted.

5.4.1 Dimensional Limits

General provisions for reinforced masonry columns are specified in MSJC *Code* Sections 1.14.1 and 3.3.4.4 [5.1] as follows, which are judgment-based and intended to prevent local instability or buckling failures:

1. Minimum side dimension shall be not less than 8 in. nominal.
2. The nominal depth of a column shall not be less than 8 in. and not greater than 3 times its nominal width.
3. The distance between the lateral supports of a column should not exceed 30 times its nominal width, that is, the effective height–to–least nominal lateral dimension ratio, that is, h/t ratio, should not exceed 30.

The minimum column width limitation to 8 in. is based on experience with masonry columns. The h/t ratio limitation of a maximum of 30 is based also on experience; sufficient data are not available to justify a higher h/t ratio [5.3].

5.4.2 Cross-Sectional Areas of Masonry Columns

Masonry column cross sections are generally square or rectangular. It is a common practice to express their cross sections (sizes) as thickness *times* depth, for example, 24 × 24 in, 16 × 24 in., etc. The *thickness* (t) of a column cross section is the smaller dimension, which is typically used in determining design parameters such as h/r ratios. As noted in earlier chapters, concrete masonry units (CMUs) are manufactured, typically, in 6-, 8-, 10-, 12-, or 16-in. nominal widths, the actual width being 3/8 in. smaller. Columns constructed from masonry units are specified by their *nominal* size, the actual size being 3/8 in. smaller on each side. For example, a nominal 24 × 24 in. column constructed from CMUs would actually be 23 5/8 × 23 5/8 in. (so the cross-sectional area = 23.625 × 23.625 = 551.14 in.^{2}, not, 24 × 24 = 576 in.^{2}), a nominal 16 × 24 in. CMU column would actually be 15 5/8 × 23 5/8 in. (so the cross-sectional area would be 15.625 × 23.625 = 369.14 in.^{2}, not 16 × 24 = 384 in.^{2}), and so on. On the other hand, when clay masonry (bricks) is used to build columns, the nominal size, expressed in whole numbers (e.g., 16 × 16, 18 × 18, 24 × 24 in., etc.) and the actual size are generally the same. In all cases, only the net cross-sectional areas (A_n) should be used in design.

5.4.3 Reinforcement for Columns

Reinforcement in masonry columns and pilasters consists of two different forms: longitudinal (or vertical) and lateral which is provided in the form of ties (as in reinforced concrete columns). The longitudinal reinforcing bars participate in carrying their share of load (i.e., the difference between the total load and that carried by masonry alone), but they must be laterally supported by ties, for the contribution of the longitudinal reinforcement as load-carrying elements cannot be relied upon in the absence of lateral ties. The code specifies limitations on clear spacing between bars, and the percentage of cell area for reinforcement, which are indirect methods of controlling problems associated with overreinforcing and grout consolidation.

The following code requirements pertain to longitudinal reinforcement and lateral ties:

5.4.3.1 Longitudinal Reinforcement The following requirements are specified for longitudinal (vertical) reinforcement for rectangular or square columns (MSJC-08 Section 1.14):

1. At least four longitudinal bars, one placed in each corner, must be provided.
2. Maximum reinforcement area should not exceed $0.04A_n$.
3. The minimum vertical reinforcement should not be less than $0.0025A_n$.
4. Longitudinal reinforcement should be uniformly distributed throughout the depth of the member.
5. Bundling longitudinal of bars is not permitted when using the strength design provisions of the MSJC *Code* (Section 3.3.3.6). A maximum of two bars may be bundled if masonry is designed under the provisions of allowable stress design (*Code* Section 1.15.3.4). Restriction on bundling of bars is arbitrary and imposed because of lack of research on masonry constructed with bundled bars.
6. The nominal bar diameter should not exceed the following limitations: (MSJC-08 Section 3.3.3.1):
   (a) No. 9
   (b) One-eighth of the nominal member thickness
   (c) One-quarter of the least clear dimension of the cell, course or collar joint in which the bar is placed.

   Practically speaking, restriction on using large diameter bars for column longitudinal reinforcement has been imposed because of grout space limitations; use of smaller bars would be usually necessary where splicing would be required. The restriction to using No. 9 bar applies when using the strength provisions of the MSJC *Code*; it is based on tests conducted to investigate strength of reinforced masonry for which bars larger than No. 9 were not used. Bars up to size No. 11 may be used if allowable stress design provisions of the code are used. Although the smallest bar size to be used as longitudinal steel has not been specified in the *Code*, No. 3 deformed bars, which are the smallest size commercially available deformed bars, should be used as smallest size longitudinal reinforcement as a matter of good engineering practice.
7. Minimum clearance requirements between longitudinal (vertical) bars for both columns and pilasters are specified by MSJC Section 1.15.3. The minimum clear distance between parallel bars is to be not less than 1½ times the bar diameter ($1.5d_b$) nor 1½ in. Number of vertical bars arranged along a column face should be checked to satisfy this requirement. Table A.17 gives the minimum core width required for placement of various numbers of bars along a column face. The minimum clear distance between the surface of the bar and any surface of a masonry unit is required to be ¼ in. for fine grout and ½ in. for the coarse grout.

The code's minimum reinforcement requirement is intended to provide ductility and to prevent brittle collapse of a column. The maximum percentage limit is based on experience and to avoid possibility of congestion in the core, which may be problematic for grout consolidation. Four bars are required so that ties can be used conveniently around them to provide a confined core of grout. In all cases, longitudinal bars should be placed symmetrically about both column axes as the design assumes implicitly that all bars are loaded equally.

Example 5.1 illustrates the provision for limitation on the maximum bar size. Although the limitation of No. 9 size bar is arbitrary, there is little masonry work large enough to accommodate bars larger than No. 9 bars. There is also a disadvantage with using larger diameter bars. Because the larger diameter bars carry larger loads, they require longer development lengths which are proportional to the square of the bar diameters. For example, a No. 8 bar would require almost twice as long a development length as a No. 6 bar. In addition, sufficient information is not available to justify the stress levels, which might occur around fully loaded larger bars [5.3].

5.4.3.2 Lateral Ties (MSJC-08 Section 1.14.1.3) All longitudinal bars for columns are required to be enclosed by lateral ties in order to develop confinement of masonry as shown in Fig. 5.8. The MSJC *Code* requirements for lateral ties are modeled after those for reinforced concrete. Lateral ties perform three functions:

1. They provide the required support to prevent buckling of longitudinal bars acting in compression.

2. They provide resistance to diagonal tension for columns subjected to shear, an action similar to reinforced concrete columns [5.3].

3. They provide ductility.

Requirements for lateral ties for columns are modeled on those for reinforced concrete [5.3]. Except for permitting ¼-in. ties outside of Seismic Design Category D, E, or F, they reflect all applicable provisions of the reinforced concrete code (ACI-05). A summary of requirements for lateral ties for masonry columns as specified in MSJC-08 Section 1.14.1.3 follows:

1. Lateral ties shall be placed in either a mortar joint or in grout; the choice would depend on the type of masonry units used in construction. However, lateral ties in columns that form parts of a LFRS in Seismic Performance Category D shall be embedded in grout.

2. All vertical reinforcing bars, whether in isolated columns, wall columns, or pilasters, must be enclosed by lateral ties (bar or wire), which must be at least ¼ in. in diameter. Note that ¼-in. diameter bars are available only as *smooth bars*, not as deformed bars; however, they can be used for ties.

3. Vertical spacing of ties is to be the smallest of the following to comply with the code requirements:
   a. Sixteen times the diameter of the vertical bar
   b. Forty eight times lateral tie bar or wire diameter
   c. The least cross-sectional dimension of the column

Ties may be placed at desired spacings when using hollow column units (Fig. 5.4), in which case ties would be embedded in grout. Constructibility issues should be recognized when specifying spacing of lateral ties that would be placed in mortar joints. All masonry units (clay or concrete) are available in certain standard thicknesses. Therefore, tie spacing should preferably be a multiple of this dimension to match coursing; a different tie spacing might require cutting the units at the jobsite, involving unwarranted additional costs and time as well as resulting in an unsightly structure. For example, when using typical 8-in.

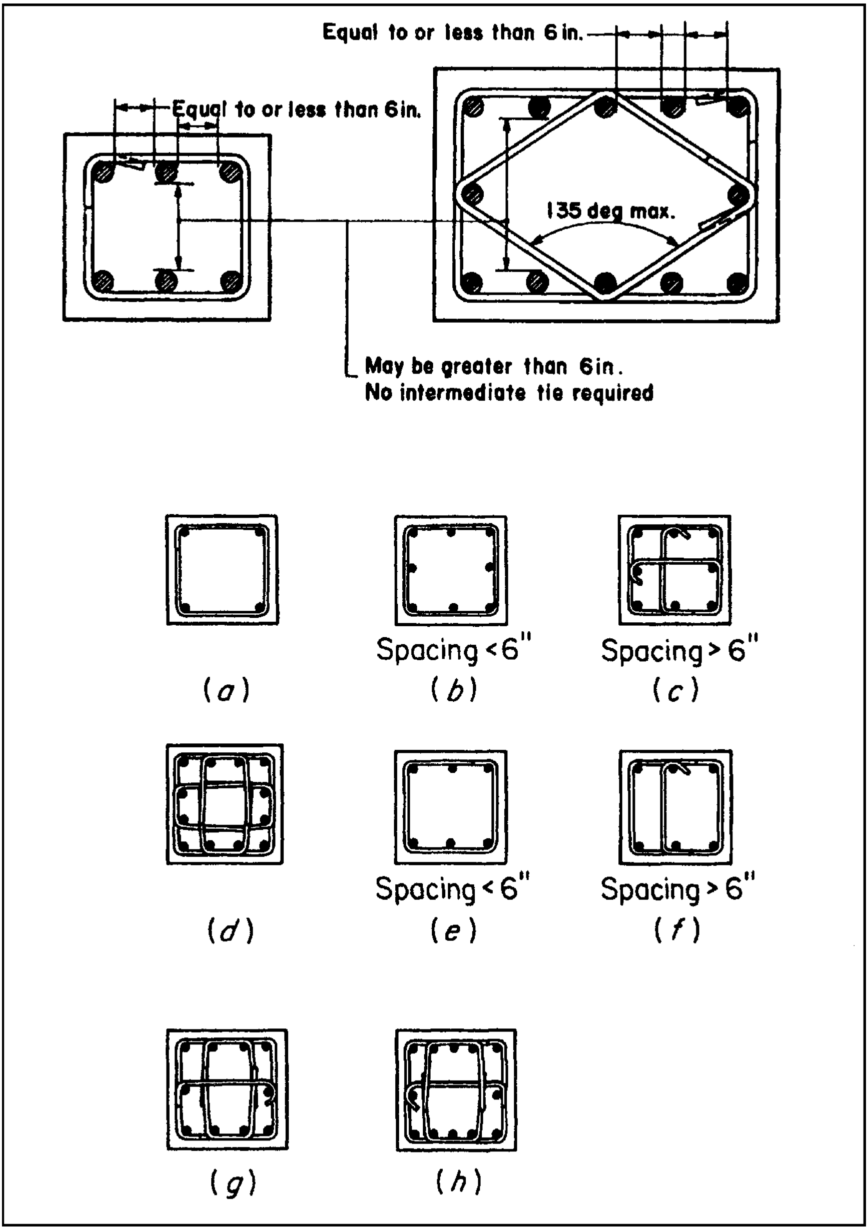


FIGURE 5.8 Configurations of lateral ties for masonry columns.

high concrete masonry units for a column and ¼ in. ties for the longitudinal reinforcement, the tie spacing should be 8 in. even though 48 times the tie diameter gives 48(¼) = 12 in. This is because 12 in. spacing would not match the coursing height of 8 in. Similarly, if one were to use 3/8-in.-diameter ties, the spacing should be 8 or 16 in. (24-in. tie spacing would exceed the 48 times the tie diameter when using a 3/8-in.-diameter tie). Any other tie spacing, say 10 or 15 in., would not match coursing, and might necessitate cutting of the masonry units at the jobsite, which would slow down the progress of work.

1. Lateral ties are to be arranged to comply with the following requirements:
   a. All corner bars and alternate longitudinal bars are laterally supported by the corner of a complete tie having an included angle of not more than 135°.
   b. No bar can be farther than 6 in. clear on each side along the lateral tie from such a laterally supported bar. See Fig. 5.6.
   c. Where longitudinal bars are placed around the perimeter of a circle, a complete circular tie is permitted. The required lap length for a circular tie is 48 tie diameters.

2. Lateral ties should be located vertically not more than one-half lateral tie spacing above the top of footing or slab in any story, and shall be spaced not more than one-half lateral tie spacing below the lowest horizontal reinforcement in beam, girder, slab, or drop panel above (Fig. 5.9).

3. Where beams or brackets frame into a column from four directions, there is a pocket of congested reinforcement formed by longitudinal reinforcement and stirrups provided in them. At such locations, lateral ties can be terminated not more than 3 in. below the lowest reinforcement in the shallowest of such beams or brackets.

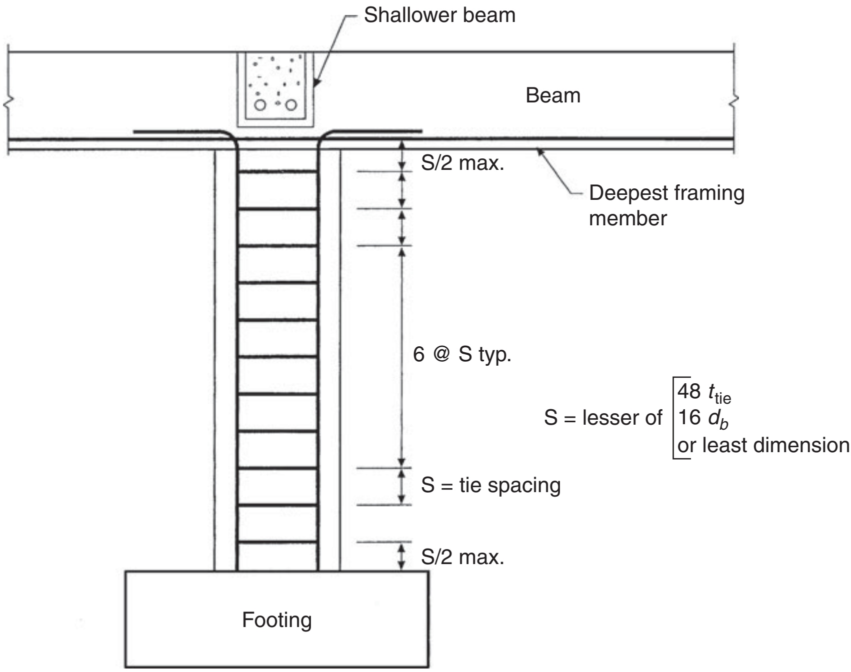


FIGURE 5.9 Placement of lateral ties in masonry columns—elevation.

Example 5.1 Maximum bar size for longitudinal reinforcement.
Determine the maximum size for longitudinal reinforcement that can be used for a column built from 8 × 8 × 16 standard CMU.

Solution
The cross section of an 8 × 8 × 16 CMU is shown in Fig. E5.1.

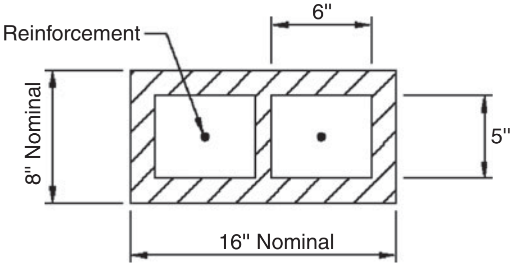


FIGURE E5.1

According to MSJC Section 3.3.3.1, the size of the longitudinal reinforcing bar would be limited to the smallest of the following:

1. No. 9.
2. 1/8 × 8 = 1 in.
3. ¼ × 5 = 1.25 in.

The diameter of No. 9 bar is 1.128 in. Therefore, the maximum permitted size for the longitudinal bar would be No. 8, which has a nominal diameter of 1 in., smallest of the three choices.

5.5 ANALYSIS OF REINFORCED MASONRY COLUMNS

Examples 5.2 to 5.4 present calculation procedures for determination of nominal strength of reinforced masonry columns based on the aforederived equations. In all cases, specified loads are axial unless stated otherwise.

In each example, the calculated value of ϕP_n for a given column is compared with the governing factored load P_u in order to determine the adequacy of the column to carry axial loads. Caution should be exercised when calculating the radius of gyration r, which is required to determine the h/r ratio. Note that for a square column, both sides are equal so that $b = t$ and the moment of inertia about either axis is the same. For a rectangular column, however, $b \neq t$; therefore, the moment of inertia should be calculated about its minor axis ($I_{\min} = bt^3/12$, $b > t$) and the smaller of the two dimensions should be taken to determine the radius of gyration (as in $r = 0.289t$). See Example 5.4.

As a rule, code compliance of column cross-sectional dimensions and h/t limitation (≤ 30) should be checked in each case as a first step in all problems involving both analysis and design of masonry columns. Note that

1. t = least *nominal* lateral dimension of column cross section when verifying that $h/t \leq 30$.
2. t = least *actual* lateral dimension of column cross section when calculating the moment of inertia of the column cross section for determining the h/r ratio of the column.

Example 5.2 Nominal strength of a square CMU column.

A nominal 24 × 24 in. CMU column having an effective height of 28 ft is reinforced with eight No. 9 Grade 60 bars (Fig. E5.2). It carries a service dead load of 200 kips and a service live load of 300 kips. $f'_m = 1500$ psi. Determine ϕP_n for the column and check if the column can support the imposed service loads.

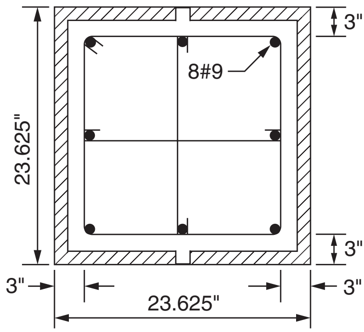


FIGURE E5.2 Columns cross section

Solution

Given: A nominal 24 × 24 in. CMU column, $h = 28$ ft, $f'_m = 1500$ psi, $A_{st} = 8.0$ in.^{2} (eight No. 9 Grade 60 bars), $D = 200$ kips, $L = 300$ kips.

Check dimensions and h/t ratio for code compliance.

$$\text{Nominal column width} = 24 > 8 \text{ in.} \qquad \text{OK}$$

$$\text{Nominal column depth} = 24 \text{ in.} < 3(24) = 72 \text{ in.} \qquad \text{OK}$$

$$h/t = 28(12)/24 = 14 < 30 \qquad \text{OK}$$

For a nominal 24 × 24 in. CMU,

$$A_n = (23.625)(23.625) = 558 \text{ in.}^2$$

$$A_{st} = 8.0 \text{ in.}^2 \text{ (eight No. 9 bars)}$$

$$h = 28 \text{ ft} \qquad b = t = 23.635 \text{ in.}$$

$$r = 0.289t = 0.289(23.625) = 6.83 \text{ in.}$$

$$\frac{h}{r} = \frac{(28)(12)}{6.83} = 49.19 < 99$$

For $\dfrac{h}{r} \le 99,$

$$C_P = \left[1 - \left(\frac{h}{140r}\right)^2\right] = 1 - \left(\frac{49.19}{140}\right)^2 = 0.876 \qquad \text{(5.13 repeated)}$$

Alternatively, C_P can be found directly from Table A.16 as follows:

$$\frac{h}{t} = \frac{(28)(12)}{23.625} = 14.22$$

By interpolation from Table A.16,

$$C_P = 0.880 - (0.880 - 0.862)(0.22) = 0.876$$

From Eq. (5.11)

$$\phi P_n = \phi(0.80)[0.80 f'_m (A_n - A_{st}) + f_y A_{st}]C_P$$
$$= 0.9(0.80)[0.80\,(1.5)(558 - 8) + (60)(8.0)](0.876)$$
$$= 719 \text{ kips}$$

Check loads on column. Calculate factored loads.
Load Combinations:

1. $U = 1.4D = 1.4(200) = 280$ kips
2. $U = 1.2D + 1.6L = 1.2(200) + 1.6(300) = 720$ kips > 280 kips
   $P_u = 720$ kips (governs)

$$\phi P_n = 719 \text{ kips} \approx 720 \text{ kips} \qquad \text{OK}$$

Check longitudinal reinforcement compliance with the code.

$$\rho = \frac{A_{st}}{A_n} = \frac{8.0}{558.0} = 0.0143$$

$$\rho_{\max} = 0.04 \qquad \rho_{\min} = 0.0025 \qquad \rho_{\text{provided}} = 0.0143$$
$$0.0025 < 0.0143 < 0.04 \qquad \text{OK}$$

The longitudinal reinforcement complies with the code requirements.

$$\phi P_n = 719 \text{ kips} \approx P_u = 720 \text{ kips}$$

The column can support the imposed service loads.

Example 5.3 Nominal strength of a square brick masonry column.

An 18 × 18 in. brick masonry column having an effective height of 20 ft is reinforced with eight No. 8 Grade 60 bars (Fig. E5.3). It carries a service dead load of 250 kips and a service live load of 200 kips. $f'_m = 2500$ psi. Determine ϕP_n for the column and check if the column can support the imposed service loads.

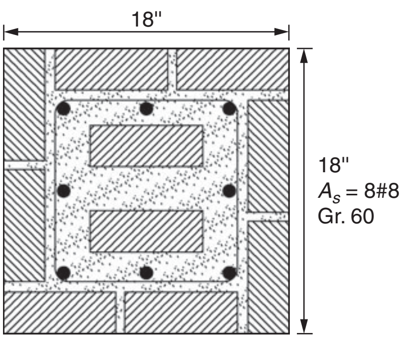


FIGURE E5.3

Solution

Given: An 18 × 18 in. brick column, $h = 20$ ft, $f'_m = 2500$ psi, $A_{st} = 6.28$ in.^{2} (eight No. 8 Grade 60 bars), $D = 250$ kips, $L = 200$ kips.

Check dimensions and h/t ratio for code compliance.

Nominal column width = 18 in. > 8 in. OK

$$\text{Nominal column depth} = 18 \text{ in.} < 3(18) = 54 \text{ in.} \qquad \text{OK}$$

$$h/t = 20(12)/18 = 13.33 < 30 \qquad \text{OK}$$

For an 18 × 18 in. brick column,

$$A_n = (18)(18) = 324 \text{ in.}^2$$
$$A_{st} = 6.28 \text{ in.}^2 \text{ (eight No. 8 bars)}$$

$$h = 20 \text{ ft} \qquad b = t = 18.0 \text{ in.}$$

$$r = 0.289t = 0.289(18.0) = 5.2 \text{ in.}$$

$$\frac{h}{r} = \frac{(20)(12)}{5.2} = 46.15 < 99$$

For $\dfrac{h}{r} \le 99,$

$$C_P = \left[1 - \left(\frac{h}{140r}\right)^2\right] = 1 - \left(\frac{46.15}{140}\right)^2 = 0.891 \qquad \text{(5.13 repeated)}$$

Alternatively, C_P can be found directly from Table A.16 as follows:

$$\frac{h}{t} = \frac{(20)(12)}{18} = 13.33$$

By interpolation from Table A.16,

$$C_P = 0.897 - (0.897 - 0.880)(0.33) = 0.891$$

From Eq. (5.11)

$$\begin{aligned}\phi P_n &= \phi(0.80)[0.80 f'_m(A_n - A_{st}) + f_y A_{st}]C_P \\ &= 0.9(0.80)[0.80\,(2.5)(324 - 6.28) + (60)(6.28)](0.891) \\ &= 649 \text{ kips}\end{aligned}$$

Check loads on column. Calculate factored loads.
Load Combinations:

1. $U = 1.4D = 1.4(250) = 350$ kips
2. $U = 1.2D + 1.6L = 1.2(250) + 1.6(200) = 620 > 350$ kips
   $P_u = 620$ kips (governs)

$$\phi P_n = 649 \text{ kips} > P_u = 620 \text{ kips} \qquad \text{OK}$$

Check the longitudinal reinforcement compliance with the code.

$$\rho = \frac{A_{st}}{A_n} = \frac{6.28}{324} = 0.0194$$

$$\rho_{\text{max}} = 0.04 \qquad \rho_{\text{min}} = 0.0025 \qquad \rho_{\text{provided}} = 0.0194$$

$$0.0025 < 0.0194 < 0.04 \qquad \text{OK}$$

The longitudinal reinforcement complies with the code requirements.

$$\phi P_n = 649 \text{ kips} > P_u = 620 \text{ kips}$$

The column can support the imposed service loads.

Example 5.4 Nominal strength of a rectangular CMU column.
A nominal 16×24 in. CMU column having an effective height of 20 ft is reinforced with six No. 9 Grade 60 bars (Fig. E5.4). It carries a service dead load of 150 kips and a service live load of 225 kips. $f'_m = 1800$ psi. Determine ϕP_n for the column and check if the column can support the imposed service loads.

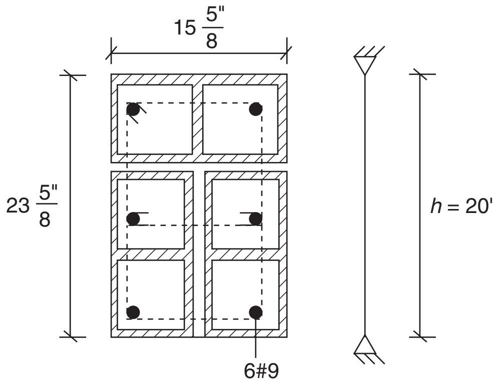


FIGURE E5.4

Solution

Given: CMU column nominal 16 × 24 in., $h = 20$ ft, $f'_m = 1800$ psi, $A_{st} = 6.0$ in.^{2} (six No. 9 Grade 60 bars), $D = 150$ kips, $L = 225$ kips.

Check dimensions and h/t ratio for code compliance.

Nominal column width = 16 in. > 8 in. OK

Nominal column depth = 24 in. < 3(16) = 48 in. OK

$h/t = 20(12)/16 = 15 < 30$ OK

For a nominal 16 × 24 in. CMU,

$$A_n = (15.625)(23.625) = 369 \text{ in.}^2$$

$$A_{st} = 6.0 \text{ in.}^2 \text{ (six No. 9 bars)}$$

$$h = 20 \text{ ft} \qquad t = 15.625 \text{ in.}$$

$$r = 0.289t = 0.289(15.625) = 4.515 \approx 4.52 \text{ in.}$$

$$\frac{h}{r} = \frac{(20)(12)}{4.52} = 53.1 < 99$$

For $\dfrac{h}{r} \le 99,$

$$C_P = \left[1 - \left(\frac{h}{140r}\right)^2\right] = 1 - \left(\frac{53.1}{140}\right)^2 = 0.856 \qquad \text{(5.13 repeated)}$$

Alternatively, C_P can be found directly from Table A.16 as follows:

$$\frac{h}{t} = \frac{(20)(12)}{15.625} = 15.36$$

By interpolation from Table A.16,

$$C_P = 0.862 - (0.862 - 0.843)(0.36) = 0.855$$

From Eq. (5.11)

$$\phi P_n = 0.80\phi[0.80 f'_m (A_n - A_{st}) + f_y A_{st}]C_P$$
$$= 0.80(0.9)[0.80(1.8)(369 - 6) + (60)(6.0)](0.856)]$$
$$= 544 \text{ kips}$$

Check loads on column. Calculate factored loads.
Load Combinations:

1. $U = 1.4D = 1.4(150) = 210$ kips
2. $U = 1.2D + 1.6L = 1.2(150) + 1.6(225) = 540 > 210$ kips
   $P_u = 540$ kips (governs)

$$\phi P_n = 544 > 540 \text{ kips} \qquad \text{OK}$$

Check longitudinal reinforcement compliance with the code.

$$\rho = \frac{A_{st}}{A_n} = \frac{6.0}{369.0} = 0.0163$$

$$\rho_{\max} = 0.04 \qquad \rho_{\min} = 0.0025 \qquad \rho_{\text{provided}} = 0.0163$$
$$0.0025 < 0.0163 < 0.04 \qquad \text{OK}$$

The longitudinal reinforcement complies with the code requirements.

$$\phi P_n = 544 > P_u = 540 \text{ kips}$$

The column can support the imposed service loads.

5.6 DESIGN PROCEDURE FOR REINFORCED MASONRY COLUMNS

5.6.1 Determination of Longitudinal Steel for Given Column Sizes

The design procedure for an axially loaded column of a given height is simple. Initially, the column dimensions are assumed, and the axial load reduction coefficient C_P is calculated for the given h/r (or h/t) ratio, from which ϕP_n is determined. For designing a column, it is required that

$$\phi P_n \geq P_u \tag{5.19}$$

The relationship $\phi P_n = P_u$ can be used in Eq. (5.11), the resulting expression being

$$P_u = \phi(0.80)[0.80 f'_m (A_n - A_{st}) + f_y A_s]C_P \tag{5.20}$$

The general procedure for sizing a structural member (e.g., beams, columns, slabs) requires assuming (guessing) a member size, and then checking its adequacy to carry design loads and compliance with code requirements. The same procedure is used for sizing a masonry column.

Equation (5.20) can be used to design a reinforced masonry column. It contains three unknowns: A_n, which depends on the column size (unknown), A_{st} (the area of longitudinal reinforcement), and C_P (the column stability factor) which is a function of both the column size and height. As a first step, a column size is assumed.

Because masonry construction consists of units (concrete and clay) of standard sizes (discussed in Chap. 2), a *nominal* column size is first guessed. This guessed size depends on the type of masonry (clay or concrete) and the type of cross section (square, rectangular, or circular, etc.). These sizes are usually, but not always, different for concrete and clay masonry because of differences in their standard sizes (e.g., a nominal 8 × 8 × 16 in. concrete masonry unit is much larger than a standard brick). In all cases of column design, cross-sectional dimensions should comply with code requirements discussed earlier. Note that *h*/*t* for the column must not exceed 30. Once a size is assumed, the next step is to determine C_P for the assumed cross section and the given column height. With these two parameters known, the required area of reinforcing steel, A_s, is easily calculated from Eq. (5.20). The required number of bars to satisfy A_s requirements can be found from Table A.9. Of course, one must ensure that the area of selected longitudinal reinforcement, A_{st}, conforms to the code's limits on the minimum and maximum percentage of longitudinal reinforcement as stated earlier. The number of bars selected should be *even* (minimum four bars) to permit symmetrical arrangement of bars in the column's rectangular cross section, to be in conformity with the assumption that all bars carry equal axial loads. Examples 5.6 to 5.8 illustrate the design procedure for axially loaded columns.

Reinforcement bars and their grades were discussed in Chap. 3. Grade 60 reinforcement bars are used more commonly than Grade 40 bars, which are generally available in size Nos. 3 to 6 only; Grade 60 bars must be used when larger size bars are required. Accordingly, it is a common practice to use Grade 60 bars for all column reinforcement. A major advantage with using Grade 60 bars instead of Grade 40 bars is that a smaller number of Grade 60 bars can be used to provide the same strength as a larger number of Grade 40 bars. A large number of bars would create crowding in the limited space available for placement of longitudinal reinforcement (cells of masonry units), particularly at location of splices. Such a condition would be detrimental to grouting operation as it would prevent free flow of grout.

Quite often, a column design problem narrows down to finding the area of longitudinal reinforcement (A_{st}) to carry loads for a known column cross section and height, and detailing of lateral ties. See Example 5.6.

Example 5.5 Determination for area of longitudinal reinforcement for a given column size.

Design a 24- × 24-in. CMU column to carry a service dead load of 200 kips and a service live load of 300 kips. The effective height of the column is 28 ft. $f'_m = 1500$ psi, $f_y = 60$ ksi.

Solution

Given: CMU column nominal 24 × 24 in., $D = 200$ kips, $L = 300$ kips, $f'_m = 1500$ psi. $f_y = 60$ ksi.

Check dimensions and *h*/*t* ratio for code compliance.

$$\text{Nominal column width} = 24 > 8 \text{ in.} \qquad \text{OK}$$

$$\text{Nominal column depth} = 24 \text{ in.} < 3(24) = 72 \text{ in.} \qquad \text{OK}$$

$$h/t = 28(12)/24 = 14 < 30 \qquad \text{OK}$$

Calculate factored loads.
Load Combinations:

1. $U = 1.4D = 1.4(200) = 280$ kips
2. $U = 1.2D + 1.6L = 1.2(200) + 1.6(300) = 720 > 280$ kips
   $P_u = 720$ kips (governs)

$$\phi P_n = 720 \text{ kips}$$

For a nominal 24 × 24 in. CMU,

$$A_n = 23.625 \times 23.625 = 558 \text{ in.}^2$$

$$h = 28 \text{ ft} \qquad b = t = 23.635 \text{ in.}$$

$$r = 0.289t = 0.289(23.625) = 6.83 \text{ in.}$$

$$\frac{h}{r} = \frac{(28)(12)}{6.83} = 40.19 < 99$$

For $\dfrac{h}{r} \le 99,$

$$C_P = \left[1 - \left(\frac{h}{140r}\right)^2\right] = 1 - \left(\frac{40.19}{140}\right)^2 = 0.876 \qquad \text{(5.13 repeated)}$$

Alternatively, C_P can be found directly from Table A.16 as follows:

$$\frac{h}{t} = \frac{(28)(12)}{23.625} = 14.22$$

By interpolation from Table A.16,

$$C_P = 0.880 - (0.880 - 0.862)(0.22) = 0.876$$

From Eq. (5.12)

$$\phi P_n = \phi(0.80)[0.80 f'_m (A_n - A_{st}) + f_y A_{st}]C_P$$

$$720 = (0.9)(0.08)[0.80\ (1.5)(558 - A_{st}) + (60)(A_{st})](0.876)$$

$$A_{st} = 8.03 \text{ in.}^2$$

Try eight No. 9 bars,

$$A_{st} = 8.0 \text{ in.}^2 \approx 8.03 \text{ in.}^2$$

Check longitudinal reinforcement compliance with the code.

$$\rho = \frac{A_{st}}{A_n} = \frac{8.0}{558.0} = 0.0143$$

$$\rho_{\text{max}} = 0.04 \qquad \rho_{\text{min}} = 0.0025 \qquad \rho_{\text{provided}} = 0.0143$$

$$0.0025 < 0.0143 < 0.04 \qquad \text{OK}$$

The longitudinal reinforcement complies with the code requirements.
Lateral ties: Provide 3/8-in.-diameter lateral ties. Tie spacing should be the smallest of

1. Sixteen times the of longitudinal bar diameter = 16 (1.0) = 16 in. (governs).
2. Forty eight times the lateral tie diameter = 48(3/8) = 18 in.
3. Least column cross-sectional dimension = 24 in.

Provide 3/8-in.-diameter lateral ties at 16 in. on center to match coursing. Code requires top and bottom ties to be within one-half of required spacing at top and bottom. This would require a tie spacing of 8 in. Provide the top and bottom tie at 8 in. to match coursing. See Fig. E5.5 for details of reinforcement.

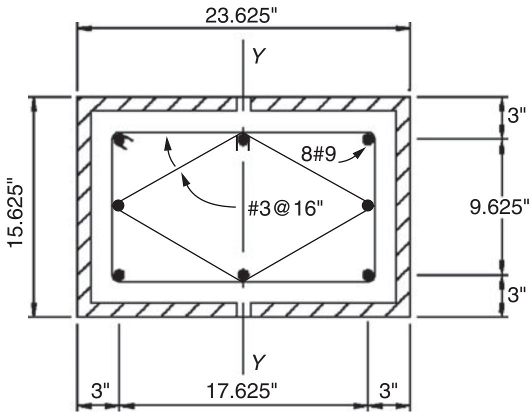


FIGURE E5.5 Details of reinforcement.

5.6.2 Determination of Size of a Masonry Column for Given Axial Loads

Sometimes, the first trial size assumed for the column design may not work out to be satisfactory. For example, the calculated area of longitudinal reinforcement may be greater than permitted by code. In such a case, a larger cross section should be assumed and the aforedescribed procedure repeated until code requirements are satisfied. See Example 5.5. Alternatively, higher strength masonry can be specified with which a smaller size column might be feasible. Usually, with some trial and error, a satisfactory solution can be found. See Example 5.7.

Example 5.6 Design of a CMU column

Design a square CMU column to carry a service dead load of 250 kips and a live load of 200 kips. The effective height of the column is 20 ft. $f'_m = 1800$ psi, $f_y = 60$ ksi.

Solution

Given: $D = 250$ kips, $L = 200$ kips, $h = 20$ ft, $f'_m = 1800$ psi, $f_y = 60$ ksi.
Calculate factored loads.
Load Combinations:

1. $U = 1.4D = 1.4(250) = 350$ kips
2. $U = 1.2D + 1.6L = 1.2(250) + 1.6(200) = 620$ kips > 350 kips
   $P_u = 620$ kips (controls)

$$\phi P_n = 620 \text{ kips}$$

Since a column size is not specified, we would try a nominal 16 × 16 in. concrete masonry column. Check dimensions and h/t ratio for code compliance.

Nominal column width = 16 in. > 8 in. OK

Nominal column depth = 16 in. < 3(16) = 48 in. OK

$$h/t = 20(12)/16 = 15 < 30 \qquad \text{OK}$$

$$A_n = 15.625\ (15.625) = 244\ \text{in.}^2$$

$$b = t = 15.625\ \text{in.}$$

$$r = 0.289t = 0.289(15.625) = 4.52\ \text{in.}$$

$$\frac{h}{r} = \frac{(20)(12)}{4.52} = 53.1 < 99$$

For $\dfrac{h}{r} \le 99,$

$$C_P = \left[1 - \left(\frac{h}{140r}\right)^2\right] = 1 - \left(\frac{53.1}{140}\right)^2 = 0.856 \qquad \text{(5.13 repeated)}$$

Alternatively, C_P can be found directly from Table A.16 as follows:

$$\frac{h}{t} = \frac{(20)(12)}{15.625} = 15.36$$

By interpolation from Table A.16,

$$C_P = 0.862 - (0.862 - 0.843)(0.36) = 0.855$$

From Eq. (5.12)

$$\phi P_n = \phi(0.80)[0.80 f'_m (A_n - A_{st}) + f_y A_{st}]C_P$$

$$620 = (0.9)(0.80)[0.80\ (1.8)(244 - A_{st}) + (60)(A_{st})](0.856)$$

$$A_{st} = 11.18\ \text{in.}^2$$

Try eight No. 11 bars,

$$A_{st} = 12.5\ \text{in.}^2 > A_{st,\ \text{reqd}} = 11.18\ \text{in.}^2 \qquad \text{OK}$$

(Note: Bars larger than No. 9 are not permitted by MSJC *Code* for strength design of masonry. However, No. 11 bars have been used here solely for illustrative purposes.

Check longitudinal reinforcement compliance with the code.

$$\rho = \frac{A_{st}}{A_n} = \frac{12.5}{244} = 0.0512 > \rho_{\max} = 0.04 \qquad \text{NG}$$

The longitudinal reinforcement provided is greater than permitted by the code. This indicates that a larger column size is required (or higher strength masonry needs to be specified for the column see Example 5.6). The next larger square CMU column would be a nominal 24 × 24 in. Calculate the area of longitudinal reinforcement required for this column.

For a nominal 24 × 24 in. CMU,

$$A_n = 23.625(23.625) = 558\ \text{in.}^2$$

$$h = 20\ \text{ft} \qquad b = t = 23.635\ \text{in.}$$

$$h/t = 20(12)/24 = 10 < 30 \qquad \text{OK}$$

$$r = 0.289t = 0.289(23.625) = 6.83\ \text{in.}$$

$$\frac{h}{r} = \frac{(20)(12)}{6.83} = 35.14 < 99$$

For $\frac{h}{r} \leq 99,$

$$C_P = \left[1 - \left(\frac{h}{140r}\right)^2\right] = 1 - \left(\frac{35.14}{140}\right)^2 = 0.937 \qquad \text{(5.13 repeated)}$$

Alternatively, C_P can be found directly from Table A.16 as follows:

$$\frac{h}{t} = \frac{(20)(12)}{23.625} = 10.16$$

By interpolation from Table A.16, for $h/t = 10.16$

$$C_P = 0.939 - (0.939 - 0.926)(0.16) = 0.937$$

$$\phi P_n = \phi(0.80)[0.80 f'_m(A_n - A_{st}) + f_y A_{st}]C_P \qquad \text{(5.12 repeated)}$$

$$620 = 0.9(0.80)[0.80\,(1.8)(558 - A_{st}) + (60)(A_{st})](0.937)$$

$$A_{st} = 1.97 \text{ in.}^2$$

Try four No. 7 bars,

$$A_{st} = 2.41 \text{ in.}^2 > A_{st,\ \text{reqd}} = 1.97 \text{ in.}^2 \qquad \text{OK}$$

(Alternatively, try eight No. 5 bars, $A_{st} = 2.45 \text{ in.}^2 > A_{st,\ \text{reqd}} = 1.97 \text{ in.}^2$ OK)

Check longitudinal reinforcement compliance with the code.

$$\rho = \frac{A_{st}}{A_n} = \frac{2.41}{558} = 0.0043$$

$$\rho_{\text{max}} = 0.04 \qquad \rho_{\text{min}} = 0.0025 \qquad \rho_{\text{provided}} = 0.0043$$

$$0.0025 < 0.0043 < 0.04 \qquad \text{OK}$$

The longitudinal reinforcement complies with the code requirements.

Lateral ties: Provide 3/8-in.-diameter lateral ties. Tie spacing should be the smallest of

1. Sixteen times the of longitudinal bar diameter = 16 (7/8) = 14 in. (governs).
2. Forty eight times the lateral tie diameter = 48(3/8) = 18 in.
3. Least column cross-sectional dimension = 24 in.

Provide No. 3 lateral ties spaced at 14 in. on center.

Final design: Provide a nominal 24 × 24 in. CMU column with four No. 7 Grade 60 vertical bars with 3/8-in.-diameter ties spaced at 14 in. on center. Code requires top and bottom ties to be within one-half of required spacing at top and bottom. Provide the top and bottom tie at 7 in. See Fig. E5.6 for details.

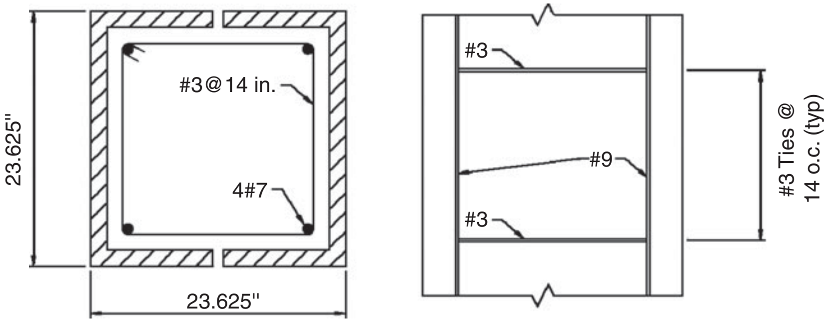


FIGURE E5.6 Details of reinforcement.

Example 5.7 Design of a CMU column.

For the data given in Example 5.6, specify a suitable higher strength masonry that can be used for a nominal 16 × 16 in. CMU column.

Solution

Given: $D = 250$ kips, $L = 200$ kips, $h = 20$ ft, $f_y = 60$ ksi.

$$P_u = 620 \text{ kips (see Example 5.5)}$$

$$\phi P_n = 620 \text{ kips}$$

For a nominal 16 × 16 in. CMU column (from Example 5.5):

$$h/t = 20(12)/16 = 15 < 30 \qquad \text{OK}$$

$$A_n = (15.625)(15.625) = 244 \text{ in.}^2$$

$$b = t = 15.625 \text{ in.}$$

$$r = 0.289t = 0.289(15.625) = 4.52 \text{ in.}$$

$$\frac{h}{r} = \frac{(20)(12)}{4.52} = 53.1 < 99$$

$$C_P = 0.937$$

Try $f'_m = 2000$ psi. Calculate required A_{st} from Eq. (5.20).

$$\phi P_n = \phi(0.80)[0.80 f'_m (A_n - A_{st}) + f_y A_s] C_P$$

$$620 = 0.9(0.80)[0.80\,(2.0)(244 - A_{st}) + (60)(A_{st})](0.937)$$

$$A_{st} = 9.05 \text{ in.}^2$$

$$\text{Maximum permissible } A_{st} = 0.04A_n = 0.04(244) = 9.76 \text{ in.}^2$$

$$\text{Try 12 No. 8 bars, } A_{st} = 8.64 \text{ in.}^2 < A_{st,\text{ reqd}} = 9.05 \text{ in.}^2 \qquad \text{NG}$$

$$\text{Try eight No. 9 bars, } A_{st} = 9.0 \text{ in.}^2 \approx A_{st,\text{ reqd}} = 9.05 \text{ in.}^2$$

Calculate ϕP_n assuming $f'_m = 2000$ psi, and $A_{st} = 9.0$ in.^{2} (Grade 60).

$$\begin{aligned}\phi P_n &= 0.80\phi[0.80 f'_m(A_n - A_{st}) + f_y A_{st}]\, C_P \\ &= 0.80(0.9)[0.80(2.0)(244 - 9.0) + 60(9.0)](0.937) \\ &= 618\,\text{kips}\end{aligned}$$

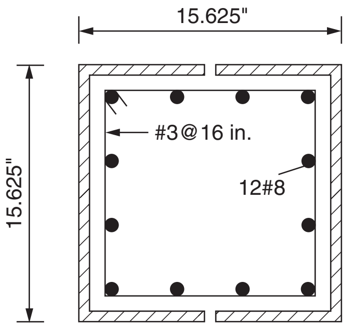


FIGURE E5.7

The calculated $\phi P_n = 618$ kips is just a little less than $P_u = 620$ kips. If this is not acceptable, try 2500-psi strength masonry and calculate $A_{st,\,\text{reqd}}$.

$$\phi P_n = 0.9(0.80)[0.80 f'_m(A_n - A_{st}) + f_y A_{st}]C_P$$

$$620 = 0.98(0.80)[0.80\,(2.5)(244 - A_{st}) + (60)(A_{st})](0.937)$$

$$A_{st} = 7.43\ \text{in.}^2$$

Try 12 No. 8 bars, $A_{st} = 8.64$ in.^{2} > $A_{st,\,\text{,reqd}} = 7.43$ in.^{2} and < $A_{st,,\,\max} = 9.76$ in.^{2} OK

Specify f'_m = 2500 psi for a nominal 16 × 16 in. CMU column and 12 No. 8 Grade 60 bars with #3ties@16 in. (Fig. E5.7).

5.7 COLUMNS UNDER COMBINED AXIAL LOAD AND BENDING

5.7.1 Analysis of Columns under Combined Axial load and Bending

Most columns are subjected to combined loads: axial and bending. The axial loads on columns result from gravity loads. The flexural loads on columns can result from

1. Construction imperfections and accidental eccentricity.
2. Eccentricity of the supported concentrated vertical load (such as reactions from supported beams being away from the vertical column axis).
3. Lateral loads transferred to the column.

In some cases, columns may possess excess capacity to carry axial loads in addition to the design loads. This can happen when

1. *Code*-prescribed minimum reinforcement is provided although masonry can carry the load without the reinforcement. See Example 5.9.
2. A minimum column size, which may be larger than the required size (A_n), must be provided.
3. A column can have excess capacity if the reinforcement provided is in excess of that required (reinforcing bars are available in certain sizes, and it is almost impossible to provide exact area of reinforcement, A_{st}). See Examples 5.8 and 5.9.

In such cases, columns can carry flexural loads (moments) in addition to the axial loads. The magnitudes of these moments depend on the column axis about which they would be applied. For a symmetrically reinforced square column, the magnitude of moment would be the same irrespective of the axes about which it is applied. In the case of rectangular columns, however, the magnitude of moments would be different for bending about the two

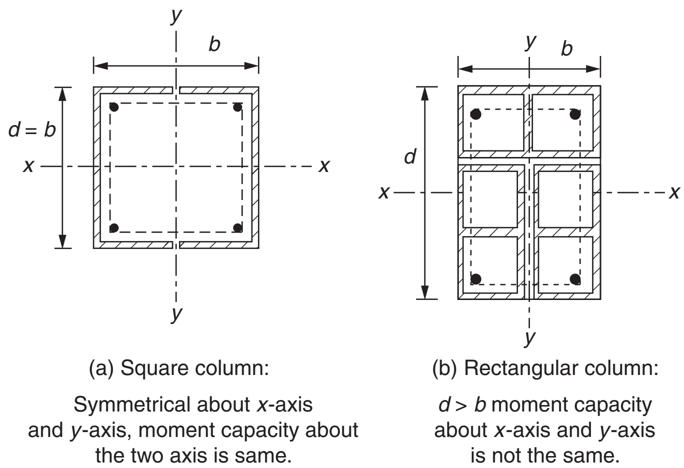


FIGURE 5.10 Masonry columns under combined axial load and bending: (*a*) square column, (*b*) rectangular column.

axes (i.e., axis parallel to the longer side of the column and the axis parallel to the shorter side of the column) because the perpendicular distances of the reinforcing bars with respect to the column axes would be different (Fig. 5.10). For clarity, Example 5.10 presents calculations for a column which can resist both axial and flexural loads simultaneously about an axis parallel to its *longer* side (*minor* axis), whereas Example 5.11 presents calculations for the same column to resist the same axial load, but bending moment about an axis parallel to its *shorter* side (*major* axis). Typically, everything else remaining the same (cross section, material strengths, area and arrangement of reinforcement bars, and axial load), a column would resist larger moment about an axis parallel to shorter side as compared to moment about the axis parallel to its longer side.

The axial strength of a column can be determined as discussed in Section 5.2. The flexural strength of a column can be determined as discussed in Chap. 4. A general method for designing a column subjected to simultaneous axial and flexural loads is presented in the next subsection.

The following notation is specific to Examples 5.8–5.11 in this chapter (for columns subjected to combined): axial load and flexure:

A_s = area of reinforcement in the tension zone of the column
A'_s = area of reinforcement in the compression zone of the column

Example 5.8 Column with code-prescribed minimum area of reinforcement (although not required for carrying imposed loads).

A nominal 24 × 24 in. CMU column (Fig. E5.8A) is required to carry service dead and live load of 100 and 175 kips, respectively. The effective height of the column is 20 ft. For this column (a) calculate the area of longitudinal steel reinforcement, (b) moment-carrying capacity ϕM_n. $f'_m = 1800$ psi and $f_y = 60$ ksi.

Solution

Given: A nominal 24 × 24 in. CMU column, $D = 100$ kips, $L = 175$ kips, $f'_m = 1800$ psi. $f_y = 60$ ksi, $h = 20$ ft.

a. Area of reinforcement:

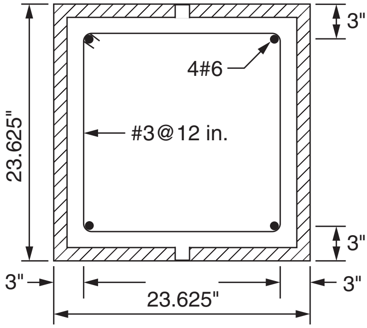


FIGURE E5.8A Column cross section.

Calculate factored loads.
Load Combinations:

1. $U = 1.4D = 1.4(10) = 140$ kips
2. $U = 1.2D + 1.6L = 1.2(100) + 1.6(175)$
$= 400 > 140$ kips

$$P_u = 400 \text{ kips (governs)}$$

$$\phi P_n = 400 \text{ kips}$$

Check dimensions and h/t ratio for code compliance.

Nominal column width = 24 in. > 8 in. OK

Nominal column depth = 24 in. < 3(24) = 72 in. OK

$h/t = 20(12)/24 = 10 < 30$ OK

For a nominal 24 × 24 in. CMU column,

$$A_n = (23.625)(23.625) = 558 \text{ in.}^2$$

$$h = 20 \text{ ft} \qquad t = 23.635 \text{ in.}$$

$$r = 0.289t = 0.289(23.625) = 6.83 \text{ in.}$$

$$\frac{h}{r} = \frac{(20)(12)}{6.83} = 35.14 < 99$$

For $\dfrac{h}{r} \leq 99,$

$$C_P = \left[1 - \left(\frac{h}{140r}\right)^2\right] = 1 - \left(\frac{35.14}{140}\right)^2 = 0.937 \qquad \text{(5.12 repeated)}$$

Alternatively, C_P can be found directly from Table A.16 as follows:

$$\frac{h}{t} = \frac{(20)(12)}{23.625} = 10.16$$

By interpolation from Table A.16,

$$C_P = 0.939 - (0.939 - 0.926)(0.16) = 0.937$$

From Eq. (5.12)

$$\phi P_n = \phi(0.80)[0.80 f'_m (A_n - A_{st}) + f_y A_{st}]C_P$$

$$400 = 0.9(0.80)[0.80\ (1.8)(558 - A_{st}) + (60)(A_{st})](0.937)$$

$$A_{st} = -3.60 \text{ in.}^2$$

Because the required area of reinforcement is negative, it is concluded that masonry alone can carry the imposed loads, and reinforcement is not required to carry the loads.

However, a minimum area of reinforcement must be provided to comply with the code requirements.

$$A_{st,\,\min} = 0.0025\,A_n = 0.0025\,(558) = 1.4\text{ in.}^2$$

Provide four No. 6 Grade 60 bars, $A_{st} = 1.77$ in.^{2}. Calculate ϕP_n from Eq. (5.12) with $A_{st} = 1.77$ in.^{2}.

$$\begin{aligned}\phi P_n &= \phi(0.80)[0.80 f'_m(A_n - A_{st}) + f_y A_{st}]\,C_P\\ &= 0.9(0.80)[0.80\,(1.8)(558 - 1.77) + (60)(1.77)](0.937)\\ &= 612\text{ kips} > P_u = 400\text{ kips}\end{aligned}$$

b. Moment capacity:

The column has an excess axial load capacity of 612–400 = 212 kips. Therefore, it can resist some bending moment. Because the column square, it is symmetrical about both axes so it would resist the same moment about either axis. As a result of bending, two No. 6 bars along one face of the column would be in compression, and the other two bars in tension. Figure E5.8B shows the strains and forces in the column due to bending.

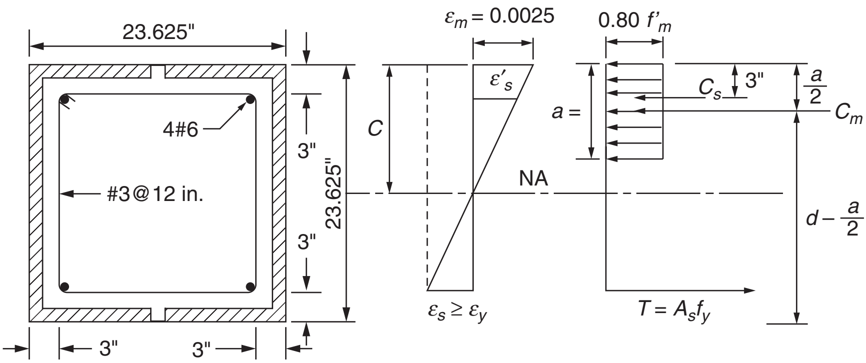


FIGURE E5.8B Strains and forces in the column due to bending.

Compression force in masonry [Eq. (4.6)]:

$$C_m = 0.80 f'_m ab = 0.80(1.8)(a)(23.625) = 34.02a\text{ kips}$$

(Note: Theoretically, the area of compression reinforcement should be subtracted from the masonry area in compression. However, this precision is ignored here as it would not affect the final result appreciably. This procedure is discussed in the next section).

By definition, $a = 0.8c$ [Eq. (4.5a)]; therefore,

$$C_m = 34.02a = 34.02(0.8c) = 27.216c\text{ kips}$$

Force in compression steel is calculated from strain (ε_s') in it, which is calculated from the strain distribution diagram (Fig. E5.8B).

$$\frac{\varepsilon_s'}{\varepsilon_m} = \frac{c-3}{c}$$

$$\varepsilon_s' = \left(\frac{c-3}{c}\right)\varepsilon_m$$

From Hooke's law, the stress in compression reinforcement f_s' is

$$f_s' = \varepsilon_s' E_s = \left(\frac{c-3}{c}\right)\varepsilon_m E_s = \left(\frac{c-3}{c}\right)(0.0025)(29{,}000) = 72.5\left(\frac{c-3}{c}\right)\text{kips}$$

Area of two No. 6 bars in compression, $A_s' = 0.88$ in.^{2}
Force in compression bars,

$$C_s = A_s' f_s' = (0.88)\left[72.5\left(\frac{c-3}{c}\right)\right] = 63.8\left(\frac{c-3}{c}\right)\text{kips}$$

The force in two No. 6 bars in tension (assuming yielding) is

$$T = A_s f_y = 0.88(60) = 52.8 \text{ kips}$$

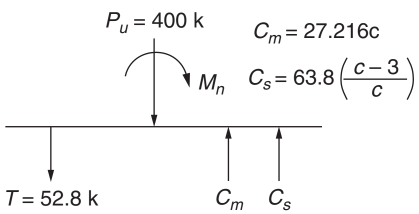


FIGURE E5.8C

All forces acting on the column including those due to bending are shown in Fig. E5.8C. For vertical equilibrium, $\Sigma F_y = 0$:

$$C_m + C_s - P_u - T_s = 0$$

$$27.216c + 63.8\left(\frac{c-3}{c}\right) - 400 - 52.8 = 0$$

$$27.216c^2 - 389c - 191.4 = 0$$

The above equation is a quadratic in c, the solution of which is

$$c = \frac{389 \pm \sqrt{(389)^2 + 4(27.216)(191.4)}}{2(27.216)} = \frac{389 \pm 414.92}{54.432} = 14.77\,\text{in.} - 0.48\,\text{in.}$$

The negative root is ignored because it is meaningless in this practical problem.

$$a = 0.8c = 0.8(14.77) = 11.82 \text{ in.}$$

The values of C_m and C_s can now be calculated from the calculated value of c. Thus, the forces acting on the column are

$$C_m = 27.216c = 27.216(14.77) = 402 \text{ kips}$$

$$C_s = 63.8\left(\frac{c-3}{c}\right) = 63.8\left(\frac{14.77-3}{14.77}\right) = 50.8\,\text{kips}$$

Check equilibrium:

$$C_m + C_s - P_u - T = 402 + 50.8 - 400 - 52.8 = 0$$

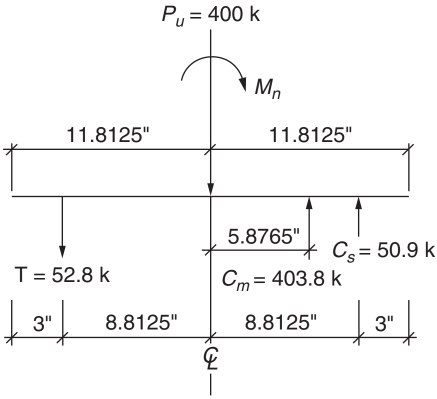


FIGURE E5-8D

Equilibrium is satisfied; all forces acting on the column include those due to bending are shown in Fig. E5.8D. The compression force in masonry C_m acts at $d/2–a/2 = ½(23.625 – 11.82) = 5.9$ in. from the centroidal axis of the column. Forces C_s and T act at 3 in. from the opposite faces of the column. Take moments of all forces about the centroidal axis of column:

$$M_n = 52.8(8.8125) + 402\ (5.9) + 50.8(8.8125)$$
$$= 3284.78 \text{ k-in} = 273.73 \text{ k-ft}$$
$$\phi M_n = (0.9)(273.73) = 246.4 \text{ k-ft}$$

The column can support a bending moment of 247.83 k-ft in addition to an axial load of 400 kips. It is noted that because the column cross section is square, it can resist a bending moment of 246.4 k-ft about either axes.

Example 5.9 A rectangular CMU column bending about minor axis of cross section.

A nominal 16 × 24 in. CMU column is reinforced with four No. 7 Grade 60 bars as shown in Fig. E5.9A. The effective height of the column is 24 ft. The service dead

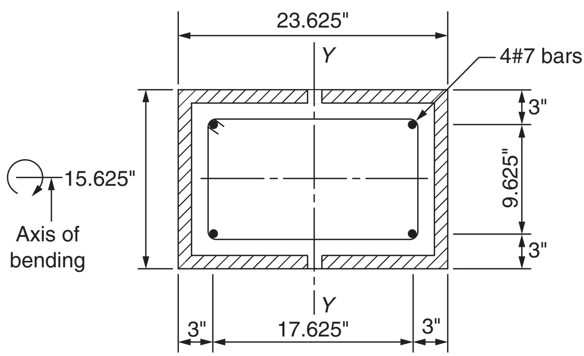


FIGURE 5.9A

and live load are 100 and 175 kips, respectively. Assuming $f'_m = 2500$ psi, calculate the moment-carrying capacity of the column about its minor axis (axis parallel to the *long* side of the column).

Solution

Given: CMU column nominal 16 × 24 in., $D = 100$ kips, $L = 175$ kips, $f'_m = 1800$ psi. $f_y = 60$ ksi, $A_{st} = 2.41$ in.^{2} (four No. 7 bars), $h = 24$ ft.

$$P_u = 400 \text{ kips (see calculations in Example 5.8).}$$

Check dimensions and h/t ratio for code compliance.

$$\text{Nominal column width} = 16 > 8 \text{ in.} \qquad \text{OK}$$

$$\text{Nominal column depth} = 24 \text{ in.} < 3(16) = 48 \text{ in.} \qquad \text{OK}$$

$$h/t = 24(12)/16 = 18 < 30 \qquad \text{OK}$$

For a nominal 16 × 24 in. CMU,

$$A_n = (15.625)(23.625) = 369.14 \text{ in.}^2$$

$$h = 24 \text{ ft} \qquad t = 15.635 \text{ in.}$$

$$r = 0.289t = 0.289(15.625) = 4.516 \text{ in.}$$

$$\frac{h}{r} = \frac{(24)(12)}{4.516} = 63.77 < 99$$

For $\dfrac{h}{r} \le 99,$

$$C_P = \left[1 - \left(\frac{h}{140r}\right)^2\right] = 1 - \left(\frac{63.77}{140}\right)^2 = 0.793 \qquad \text{(5.13 repeated)}$$

Alternatively, C_P can be found directly from Table A.16 as follows:

$$\frac{h}{t} = \frac{(24)(12)}{15.625} = 18.43$$

By interpolation from Table A.16,

$$C_P = 0.802 - (0.802 - 0.779)(0.43) = 0.792$$

From Eq. (5.12)

$$\phi P_n = \phi(0.80)[0.80 f'_m (A_n - A_{st}) + f_y A_{st}] C_P$$

$$= (0.9)(0.80)[0.80\ (2.5)(369.41 - 2.41) + (60)(2.41)](0.793)$$

$$= 501.3 \text{ kips} > P_u = 400 \text{ kips}$$

Because ϕP_n (= 501.3 kips) is greater than P_u (= 400 kips), the column can carry some bending moment.

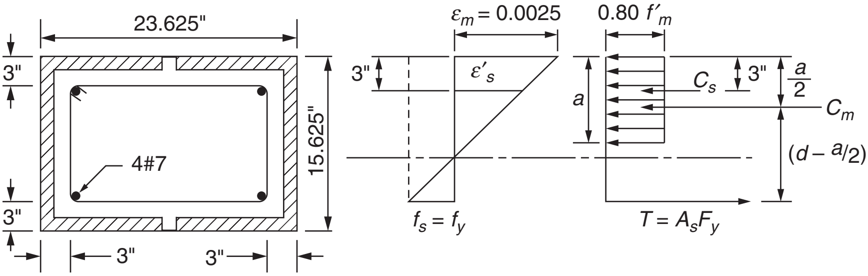


FIGURE E5.9B Strains and forces in the column due to bending.

Figure E5.9B shows strain and force in the column due to bending.

Compression force in masonry [Eq. (4.6)]:

$$C_m = 0.80 f'_m ab = 0.80(2.5)(a)(23.625) = 47.25a \text{ kips}$$

(Note: Theoretically, the area of compression reinforcement should be subtracted from the masonry area in compression. However, this precision is ignored here as it would not affect the final result appreciably).

By definition, $a = 0.8c$ [Eq. (4.5a) Chap. 4]; therefore,

$$C_m = 47.25a = 47.25(0.8c) = 37.8c \text{ kips}$$

Force in compression steel is calculated from strain (ε'_s) in it, which is calculated based on strain distribution diagram (Fig. E5.9B).

$$\frac{\varepsilon'_s}{\varepsilon_m} = \frac{c-3}{c}$$

$$\varepsilon'_s = \left(\frac{c-3}{c}\right)\varepsilon_m$$

From Hooke's law, the stress in compression reinforcement f'_s is

$$f'_s = \varepsilon'_s E_s = \left(\frac{c-3}{c}\right)\varepsilon_m E_s = \left(\frac{c-2}{c}\right)(0.0025)(29{,}000) = 72.5\left(\frac{c-3}{c}\right)\text{kips}$$

Area of two No. 6 bars in compression, $A'_{st} = 1.20$ in.^{2}

Force in compression bars,

$$C_s = A'_s f'_{st} = (1.20)\left[72.5\left(\frac{c-3}{c}\right)\right] = 87\left(\frac{c-3}{c}\right)\text{kips}$$

The force in two No. 6 bars in tension (assuming yielding) is

$$T = A_s f_y = 1.20(60) = 72.0 \text{ kips}$$

All forces acting on the column including those due to bending are shown in Fig. E5.9C.

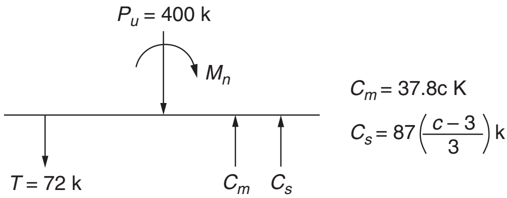


FIGURE E5.9C

For vertical equilibrium, $\Sigma Fy = 0$:

$$C_m + C_s - P_u - T = 0$$

$$37.8c + 87\left(\frac{c-3}{c}\right) - 400 - 72 = 0$$

$$37.8c^2 - 385c - 261 = 0$$

The solution of the above quadratic equation is: $c = 10.82$ in. (the negative root is ignored as it does not have any significance in this practical problem).

$$a = 0.8c = 0.8(10.82) = 8.66 \text{ in.}$$

The values of C_m and C_s can now be calculated from the calculated value of c. Thus, the forces acting on the column are

$$C_m = 37.8c = 37.8(10.82) = 409 \text{ kips}$$

$$C_s = 87\left(\frac{c-3}{c}\right) = 87\left(\frac{10.82-3}{10.82}\right) = 62.88 \text{ kips}$$

Check equilibrium: $\Sigma F_y = 0$:

$$C_m + C_s - P_u - T = 409 + 62.88 - 400 - 72$$
$$= -0.12 \text{ kips} \approx 0 \text{ (rounding off error)}$$

Equilibrium is satisfied; all forces acting on the column include those due to bending are shown in Fig. E5.9D. The compression force in masonry C_m acts at $d/2 - a/2 = ½(15.625–8.66) = 3.4825$ in. from the centroidal axis of the column. Forces C_s and T act at 3 in. from the opposite faces of the column. Take moments of all forces about the centroidal axis of column:

$$M_n = 72(4.8125) + 409\,(3.4825) + 62.88(4.8125)$$
$$= 2073.45 \text{ k-in.} = 172.79 \text{ k-ft}$$

$$\phi M_n = (0.9)(172.79) = 155.5 \text{ k-ft}$$

The column can support a bending moment of 155.5 k-ft about an axis parallel to its long side, along with an axial load of 400 kips.

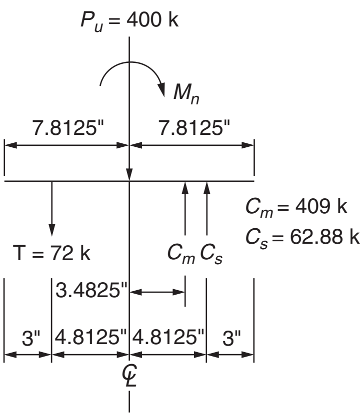


FIGURE E5.9D

Example 5.10 A rectangular CMU column bending about its major axis of cross section.
A nominal 16 × 24 in. CMU column having an effective height of 24 ft is reinforced with four No. 7 Grade 60 bars as shown in Fig. E5.10A. The service dead and live load are 100 and 175 kips, respectively. Assuming f'_m = 2500 psi, calculate the moment-carrying capacity of the column about its major axis (axis parallel to the *short* side of the cross section).

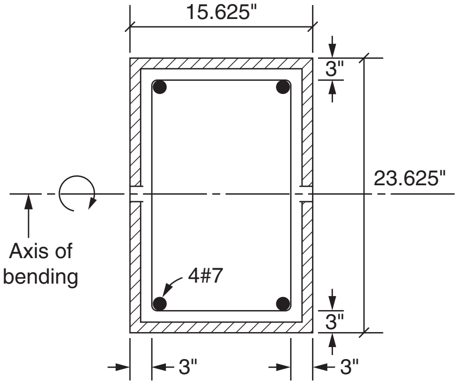


FIGURE E5.10A

Solution
Given: CMU column nominal 16 × 24 in., $D = 100$ kips, $L = 175$ kips, $f'_m = 1800$ psi. $f_y = 60$ ksi, $A_{st} = 2.41$ in.^{2} (four No. 7 bars), $h = 24$ ft.

The data in this example are the same as in Example 5.9 from which the following information is obtained (calculations not repeated):

$$P_u = 400 \text{ kips}, \qquad \phi P_n = 501.3 \text{ kips}$$

Because ϕP_n (= 501.3 kips) is greater than P_u (= 400 kips), the column can carry some bending moment. But the moment-carrying capacity in this example would be different from that in Example 5.10 because the column is bending about its major axis (in contrast to bending about the minor axis in Example 5.10).

Figure E5.10B shows strain and force in the column due to bending.

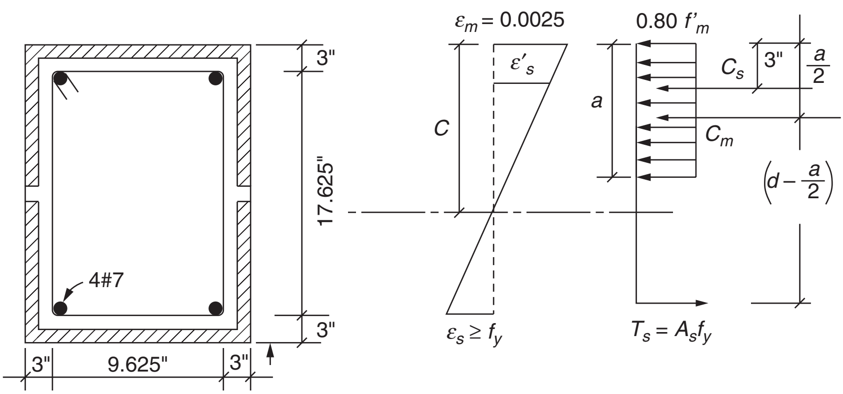


FIGURE E5.10B Strains and forces in the column due to bending.

Compression force in masonry [Eq. (4.6)],

$$C_m = 0.80 f'_m ab = 0.80(2.5)(a)(15.625) = 31.25a \text{ kips}$$

By definition, $a = 0.8c$ [Eq. (4.5a)], therefore,

$$C_m = 31.25a = 31.25(0.8c) = 25c \text{ kips}$$

Force in compression steel is calculated from strain (ε_s') in it, which is calculated based on strain distribution diagram (Fig. E5.11B).

$$\frac{\varepsilon_s'}{\varepsilon_m} = \frac{c-3}{c}$$

$$\varepsilon_s' = \left(\frac{c-3}{c}\right)\varepsilon_m$$

From Hooke's law, the stress in steel reinforcement f_s' is

$$f_s' = \varepsilon_s' E_s = \left(\frac{c-3}{c}\right)\varepsilon_m E_s = \left(\frac{c-3}{c}\right)(0.0025)(29{,}000) = 72.5\left(\frac{c-3}{c}\right)\text{kips}$$

Area of two No. 6 bars in compression, $A_s' = 1.20 \text{ in.}^2$

Force in compression bars,

$$C_s = A_s' f_s' = (1.20)\left[72.5\left(\frac{c-3}{c}\right)\right] = 87\left(\frac{c-3}{c}\right)\text{kips}$$

The force in two No. 6 bars in tension (assuming yielding) is

$$T = A_s f_y = 1.20(60) = 72.0 \text{ kips}$$

All forces acting on the column including those due to bending are shown in Fig. E5.10C.

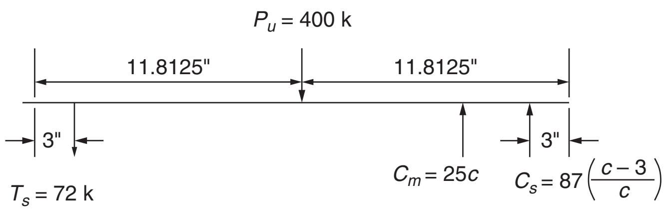


FIGURE E5.10C

For vertical equilibrium, $\Sigma F_y = 0$:

$$C_m + C_s - P_u - T_s = 0$$

$$25c + 87\left(\frac{c-3}{c}\right) - 400 - 72 = 0$$

$$25c^2 - 385c - 261 = 0$$

The solution of the above quadratic equation is: $c = 16.05$ in. (the negative root is ignored as it does not have any significance in this practical problem).

$$a = 0.8c = 0.8(16.05) = 12.84 \text{ in.}$$

The values of C_m and C_s can now be calculated from the calculated value of c. Thus, the forces acting on the column are

$$C_m = 25c = 25(16.05) = 401.25 \text{ kips}$$

$$C_s = 87\left(\frac{c-3}{c}\right) = 87\left(\frac{16.05-3}{16.05}\right) = 70.74 \text{ kips}$$

Check equilibrium: $\Sigma F_y = 0$:

$$C_m + C_s - P_u - T = 401.25 + 70.74 - 400 - 72 = -0.01 \text{ kips} \approx 0 \text{ (rounding off error)}$$

Equilibrium is satisfied (so all calculated values are correct). All forces acting on the column include those due to bending are shown in Fig. E5.10D. The compression force in masonry C_m acts at $d/2 - a/2 = ½(23.625–12.84) = 5.3925$ in. from the centroidal axis of the column. Forces C_s and T act at 3 in. from the opposite faces of the column. Take moments of all forces about the centroidal axis of column:

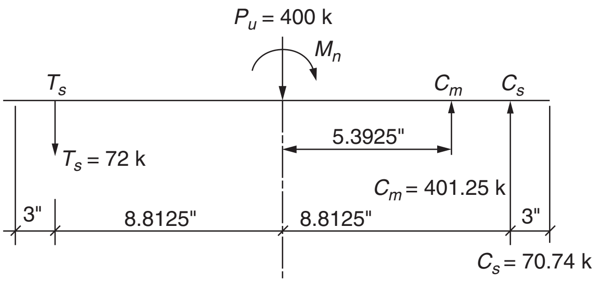


FIGURE E5.10D

$$M_n = 70.74(8.8125) + 401.25\,(5.3925) + 72(8.8125)$$

$$= 3421.64 \text{ k-in} = 285.1 \text{ k-ft}$$

$$\phi M_n = (0.9)(285.1) = 256.6 \text{ k-ft}$$

The column can support a bending moment of 256.6 k-ft about an axis parallel to its short side, along with an axial load of 400 kips.

5.7.2 Design of Columns under Combined Axial Load and Bending: Interaction Diagram

Examples 5.9 and 5.10 presented analysis of columns when the longitudinal reinforcement provided in a column was in excess of its exact requirement. As a result, the columns were able to resist some moment in addition to the imposed axial loads. In practice, there are cases when a column is subjected to both axial load and bending moment simultaneously. This section presents a discussion of a general method of determining the capacity of a column to resist both axial loads and bending moments simultaneously.

The design of a column subjected to simultaneous axial load and bending moment is performed with the help of an *interaction diagram*. Stated simply, an interaction diagram for a column is a curve that shows the flexural capacity of an axially loaded column. The coordinates of a point located on this curve indicate axial load capacity and corresponding moment-carrying capacity of the column when both forces are acting simultaneously. A column can resist a nominal axial load ϕP_n when there is no moment acting on the column (pure axial case). Similarly, it can resist a bending moment ϕM_n (pure flexural case) when there is no axial load present. In between these two values (ϕP_n and ϕM_n), a column can resist combinations of axial loads and bending moments such that the magnitude of each force is less than its corresponding nominal value. For a given axial load ($P < \phi P_n$), a column can resist a bending moment M such that $M < \phi M_n$. Obviously, there can be many such combinations of these two forces depending on their magnitudes. The interaction diagram is a plot of many such points, each of which represents a unique combination of P (on y-axis) and M (on x-axis), which a column can resist simultaneously.

First some fundamentals for plotting an interaction diagram. The axial strength of a column depends on the compressive strength of masonry, cross-sectional dimensions, area and yield strength of longitudinal reinforcement, and the h/r ratio. Therefore, each column would have a unique interaction diagram based on the unique values of these parameters. The flexural strength of a column depends also on the same parameters that control its axial strength (except the h/r ratio), but it is also influenced by the configuration of longitudinal reinforcement. Positions of longitudinal bars in the cross section influence the position of neural axis, depending on their distance from the face of the masonry.

Geometry of the column cross section influences its nominal moment strength. For a square column, the value of M_n would be the same for bending about either axis of the column, but it would be different for bending about the two axes in the case of a rectangular column (Fig. 5.11). Thus, an interaction diagram is a unique plot of a set of points for a columns defined by specific cross-sectional dimensions, compressive strength of masonry, area, configuration, and yield strength of longitudinal reinforcement, and the h/r ratio, and the axis about which bending is considered for a column having a rectangular cross section. A variation in any one of these parameters would require a different interaction diagram.

5.7.3 Calculations for Interaction Diagram

Generation of the many points for the interaction curve is time-consuming, which makes the procedure most suitable for a computer. However, a simple interaction diagram can be drawn if the values of at least the following three points can be determined:

1. Nominal axial strength P_n (pure axial case, no moment present)
2. Nominal moment strength, M_n (pure flexural case, no axial load present)

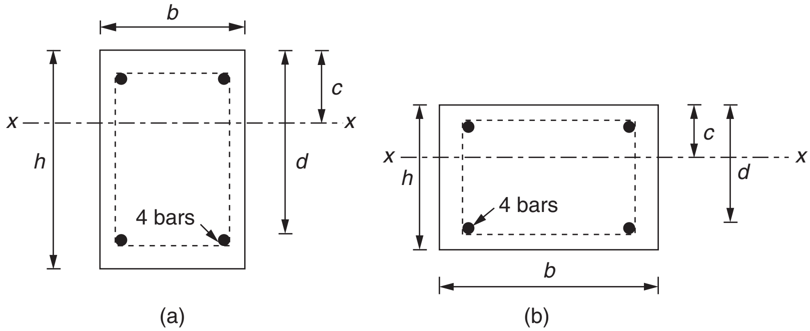


FIGURE 5.11 Moment capacity of a rectangular cross section about the two axes is different: *(a)* bending about the axis parallel to the short side of the column, *(b)* bending about the axis parallel to the long side of the column.

3. The balanced condition (when the strain in compression strain in extreme fibers of masonry equals ε_m (0.0025 for concrete masonry and 0.0035 for clay masonry) and the yield strain in tension reinforcement (f_y/E_s = 60/29,000 = 0.00207 for Grade 60 reinforcement) reach simultaneously.

Computationally, these three points are obtained as follows:

1. Nominal axial strength P_n

A reinforced masonry column can resist an axial load ϕP_n (pure axial capacity) if there is no bending moment present. The value of ϕP_n can be determined form Eq. (5.12) as illustrated earlier in several examples:

$$\phi P_n = \phi(0.80)[0.80 f'_m (A_n - A_{st}) + f_y A_{st}] C_P \qquad \text{(5.12 repeated)}$$

2. Nominal moment strength M_n

When there is no axial load present on the column, it can be treated as a beam. Determination of the nominal moment capacity, M_n, for a column cross section is similar to that for doubly reinforced beams (see Section 4.15). Unlike beams in which tension and compression reinforcing bars might be of different sizes, all longitudinal reinforcing bars in a column would typically be of the same size. Thus, in a masonry column reinforced with four symmetrically placed longitudinal bars (one in each corner), the area of bars in tension and compression (when the column acts as a beam) would be the same. This, of course, would not be the case when additional longitudinal bars (other than the corner bars) are present; in such a case, the area of tension and compression reinforcement might be different.

The forces in all longitudinal bars, whether in compression or tension, would need to be accounted for when calculating the nominal strength of the column cross section. See Example 4.15, which illustrates a detailed procedure for determining the nominal strength of a doubly reinforced beam.

Various equations presented in Section 4.5 would be used in the following example for determination of the nominal moment strength of a column cross section treating it as a doubly reinforced beam, with no axial load present. Figure 5.12 shows a rectangular column cross section reinforced with four longitudinal bars, along with the strain distribution and corresponding forces. The areas of reinforcement in compression and tension are shown, respectively, as A'_s and A_s (and $A'_s = A_s$). The stress in tension steel is assumed

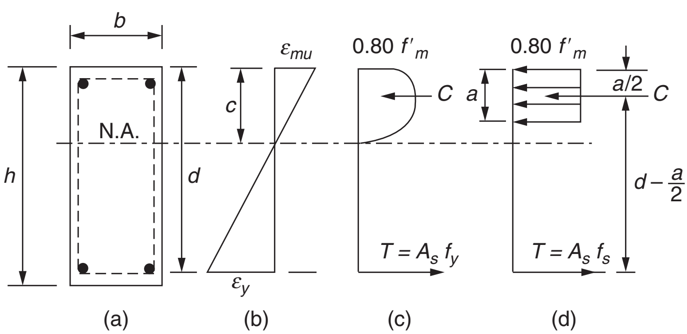


FIGURE 5.12 A rectangular column section under flexure at balanced conditions (axial load not present). (*a*) column corss section, (*b*) strain distribution diagram, (*c*) stress distribution at ultimate conditions, and (*d*) rectangular stress distribution in masonry at ultimate conditions.

equal to its yield strength f_y, whereas the stress in compression steel is determined from the strain distribution diagram assuming compressive strain in the face of the column equal to ε_m (0.0025 for concrete masonry and 0.0035 for clay masonry).

Referring to Fig. 5.12, the following values of forces acting on the column are obtained (governing equations from Chap. 4 are presented here for completeness; refer to Section 4.5 for the details of their derivations):

Compression force in masonry *Cm*:

$$C_m = 0.80 f'_m ab \qquad \text{(5.21, 4.6 repeated)}$$

where a = depth of the compression block in the column cross section.

Compression force in steel C_s:

$$C_s = A'_s f'_s \qquad \text{(5.22, 4.104 repeated)}$$

where A'_s = area of compression reinforcement

f'_s = stress in compression reinforcement ($\leq f_y$) as calculated from Hooke's law:

$$f'_s = \varepsilon'_s E_s \leq f_y \qquad \text{(5.23, 4.105 repeated)}$$

$$\varepsilon'_s = \left(1 - \frac{d'}{c}\right)\varepsilon_m \qquad \text{(5.24, 4.107 repeated)}$$

$$f'_s = \varepsilon'_s E_s = \left(1 - \frac{d'}{c}\right)\varepsilon_m E_s \qquad \text{(5.25, 4.108 repeated)}$$

$$C_s = A'_s\left(1 - \frac{d'}{c}\right)\varepsilon_m E_s \qquad \text{(5.26, 4.109 repeated)}$$

Equation (5.21) gives an overestimated value of the force in masonry in compression because the area of compression reinforcement, A'_s, was not deducted from the compression area of masonry, the actual area of masonry in compression being = $ab - A'_s$. To compensate for this overestimation, the force in compression reinforcement can be expressed as

$$C_s = A'_s\left[\left(1 - \frac{d'}{c}\right)\varepsilon_m E_s - 0.80 f'_m\right] \qquad \text{(5.27, 4.110 repeated)}$$

Tension force in tension reinforcement T:

$$T = A_s f_y \qquad \text{(5.28, 4.7 repeated)}$$

Equating sum of all horizontal forces to zero for equilibrium in the horizontal direction, we have,

$$C_m + C_s - T = 0 \qquad \text{(5.29, 4.111 repeated)}$$

Substitution of values of various parameters in Eq. (5.29) yields

$$0.80 f'_m ab + A'_s\left[\left(1 - \frac{d'}{c}\right)\varepsilon_m E_s - 0.80 f'_m\right] - A_s f_y = 0 \qquad \text{(5.30, 4.112 repeated)}$$

In Eq. (5.30), the value of a is unknown. By definition,

$$a = 0.80c \qquad \text{(5.31, 4.5a repeated)}$$

With the above substitution, Eq. (5.30) can be written as Eq. (5.32):

$$0.80f'_m(0.8c)b + A'_s\left[\left(1-\frac{d'}{c}\right)\varepsilon_m E_s - 0.80f'_m\right] - A_s f_y = 0 \quad \text{(5.32, 4.113 repeated)}$$

Equation (5.32) can be simplified and written as Eq. (5.33):

$$0.64f'_m bc^2 + A'_s\varepsilon_m E_s c - A'_s\varepsilon_m E_s d' - 0.80f'_m A'_s c - A_s f_y c = 0 \quad \text{(5.33, 4.114 repeated)}$$

Equation (5.33) can be expressed as a quadric in c:

$$(0.64f'_m b)c^2 + (A'_{st}\varepsilon_m E_s - A_s f_y - 0.80f'_m A'_s)c - A'_s\varepsilon_m E_s d' = 0 \quad \text{(5.34, 4.115 repeated)}$$

Equation (5.34) is a quadratic of the form: $Ax^2 + Bx + C = 0$, which can be solved for x [= c in Eq. (5.34)]:

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \quad \text{(5.35, 4.116 repeated)}$$

where $x = c$

$A = 0.64f'_m b$

$B = A'_s\varepsilon_m E_s - A_s f_y - 0.80 f_m A'_s$

$C = -A'_s\varepsilon_m E_s d'$

Note that Eq. (5.34) has two roots of x which are given by Eq. (5.35); the negative root should be ignored as it has no significance in this problem. Once c is known, $a = 0.8c$, and quantities C_m and C_s are easily determined. Finally, the magnitude of M_n can be determined by summing up moments due to C_m and C_s about T:

$$M_n = C_m\left(d - \frac{a}{2}\right) + C_s(d - d') \quad \text{(5.36, 4.117 repeated)}$$

It is noted that the strain in compression steel may be small, zero, or even tensile, depending on the distance of neutral axis from the compression reinforcement. If the neutral axis passes through the centroid of compression reinforcement ($c = d'$), the strain in compression reinforcement would be zero so that C_s would be zero. In case the strain in compression reinforcement is tensile ($c < d'$), the force in the compression reinforcement would be tensile, and C_s should be determined from Eq. (5.26).

3. The balanced condition

The balanced condition is defined as a load condition such that the maximum compressive strain in masonry (ε_m = 0.0025 for concrete masonry and 0.0035 for clay masonry) and tensile strain in steel (= 0.00207 for Grade 60 steel) occur simultaneously. The location of neutral axis (distance c from the compression face) and the strain in compression reinforcement (ε'_s) are determined, respectively, from Eqs. (5.37) and (5.24), based on the strain distribution diagram shown in Fig. 5.12. Thus,

$$c = \left(\frac{\varepsilon_m}{\varepsilon_m + \varepsilon_y}\right)d \quad \text{(5.37)}$$

$$\varepsilon'_s = \left(1 - \frac{d'}{c}\right)\varepsilon_m \quad \text{(5.24 repeated)}$$

With the known value of the strain in compression steel, the force in compression steel is determined from Eq. (5.27).

We can now write the equation for equilibrium by taking into account all vertical forces acting on the column, viz., the imposed axial load *P*, the compressive force in masonry C_m [Eq. (5.21)], the compressive force in steel C_s [Eqs. (5.22) and (5.28)], and the tensile force in steel *T* [Eq. (5.28)]. Thus,

$$\sum F_y = 0 \tag{5.38}$$

$$C_m + C_s - T - P = 0 \tag{5.39}$$

whence

$$P = C_m + C_s - T \tag{5.40}$$

Note that Eq. (5.40) is different from Eq. (5.29) because of the presence of axial load *P* on the column. This same equation was used in Examples 5.9 and 5.10. Once this equation is solved, the value of M_n can be determined by summing up moments due to all forces about the centroidal axis of the column as illustrated in Examples 5.9 and 5.10.

4. Other points in the interaction diagram

Generating a complete interaction diagram requires calculations for at least the aforedescribed three points and several other points on the curve. Values of *P* and *M* for other points in the interaction diagram may be calculated by varying strain distribution across the column cross section as suggested in the following step-by-step procedure. For any point on the interaction curve:

a) Set compressive strain in masonry ε_{mu} (= 0.0025 for concrete masonry and 0.0035 for clay masonry).
b) Assume a strain distribution in the column cross section (i.e., assume a value of *c*, the distance of neutral axis from the face of the column).
c) Calculate the compression forces in masonry (C_m) from Eq. (5.21), and forces in compression and tension steel (C_s, and *T*, respectively) based on strain distribution across the column cross section, as discussed earlier.
d) Calculate the value of the axial force *P* from equation of equilibrium [Eq. (5.40)].
e) Calculate the value of moment *M* by taking moments of all forces (C_s, C_m, and *T*) about the centroidal axis of column cross section (i.e., about the line of action of *P*).

In regard to item 2 above, it is often convenient to assume strain distribution by positioning neutral axis at several arbitrarily selected distances in the cross section and calculate the corresponding values of *P* and *M*. This procedure is illustrated in Example 5.11.

It is instructive to understand the variations in the strain distribution in the cross section of an axially loaded column subjected to bending. The influence of the two simultaneous loading conditions for a column—axial load and bending—can be determined from the principle of superposition. When under axial load only, the column experiences uniform compressive strain across its entire cross section. The maximum value of this axial load corresponds to the nominal strength (P_n) of the column, and it is assumed that the compressive strain in masonry is equal to ε_{mu} and the compressive strain in steel reinforcement equal to its yield strain.

When an axially loaded column experiences a bending moment, the latter introduces compressive strain on one face of the column (herein after referred to as the compression face) and tensile strain on the opposite face of the column (hereinafter referred to as the tension face), essentially the column behaving as a vertical beam. The net result of the combined axial load and bending moment is increased compressive strain on the compression face, and reduced compressive strain on the tension face of the column. Because the maximum compressive strain in masonry is limited to ε_{mu}, the combination of axial load and moment must be such that the sum of compressive strains on the compression face due to these two loads does not exceed ε_{mu}. Thus, an increase in bending moment is accompanied by a decrease in the axial load. At the same time, with increasing moment, the tension face of the column experiences increased tensile strain, so that strain on this face, which was initially compressive, transitions gradually to tensile. As the moment on the column increases, the position of the neutral axis in the column cross section also changes. Forces in reinforcement near the compression face and the tension face of the column are calculated from the strain distribution consistent with the position of the neutral axis. It is important to recognize that, depending on the position of the neutral axis, the strain in reinforcement near the opposite faces of the column could be tensile or compressive. In all cases, the maximum strain in the reinforcement, compressive, or tensile, is assumed limited to the yield strain.

Interaction diagrams involve time-consuming calculations; as such they are best generated by a computer. For illustrative purposes, Example 5.11 presents hand calculations for interaction diagram of a column. Complete calculations are presented for four points in the diagram (three points discussed above: Points 1, 12, and 6), and one randomly selected point on the curve (Point 8). Calculations for other points were performed on Excel spreadsheet. A summary of all calculations is presented in Table E5.11 and the corresponding interaction diagram shown in Fig. E5.11*f*. Various points in Table E5.11 were selected on the basis of assumed strain distribution as follows:

Point 1: Pure axial load case, zero moment

Point 2: Zero strain on tension side ($c = h$, the depth of the column cross section)

Point 3: Zero strain in tension steel ($c = d$)

Points 4 and 5, 7 to 11: Various values of c selected arbitrarily, decreasing gradually from $c = h$ (point 2) to a position close to compression face of the column (Point 11), as listed in Table E5.11.

Point 6: Balanced conditions

Point 12: Pure bending case, zero axial load

For an arbitrarily selected point on the interaction diagram (Points 4, 5, and 7 to 11), the location of neutral axis may be such that the strain in the compression reinforcement might be compressive (when $c > d'$) or tensile (when $c < d'$). In general, the neutral axis can have positions corresponding to the following strain distribution across the column cross section:

1. Zero strain on the tension face of the column, $c = h$ (Point 2, strain in tensile reinforcement is compressive)
2. Zero strain in tension reinforcement, $c = d$ (Point 3)
3. Neutral axis location such that $d' < c < d$ (strain in compression reinforcement is compressive, Points 4 to 6, and 7 to 11)
4. Neutral axis close to the compression face such that $c < d'$ (strain in compressive reinforcement is tensile)

For each of the neutral axis positions, forces in both compression and tension reinforcement, consistent with the strain in those reinforcements, need to be calculated. These

TABLE E5.11 Calculations for Values of Axial Load and Bending Moment for the Interaction Diagram for Column in Example 5.11 (Excel spreadsheet)

Point	c in.	ε_s'	ε_T	f_s' ksi	C_m kips	C_S kips	T kips	ϕP_n kips	ϕM_n k-in.
1	–	0.00207	0.00207					417.6	–
2	23.625	0.00210	–0.00040	–11.7	472.5	71.4	–14.0	398.2	1412
3	19.825	0.00202	0.00000	0.1	396.0	68.7	0.1	331.6	1877
4	17.0	0.00194	0.00042	12.0	340.0	66.0	14.5	279.4	2108
5	14.0	0.00182	0.00104	30.2	280.0	61.8	36.2	218.1	2267
6	10.8	0.00162	0.00208	60.0	216.6	54.9	72.0	142.4	2368
7	10.0	0.00155	0.00246	60.0	200.0	52.3	72.0	128.7	2298
8	8.0	0.00131	0.00370	60.0	160.0	44.1	72.0	94.3	2073
9	6.5	0.00104	0.00513	60.0	130.0	34.5	72.0	66.0	1843
10	5.0	0.00060	0.00741	60.0	100.0	19.3	72.0	33.7	1540
11	4.0	0.00013	0.00989	60.0	80.0	2.8	72.0	7.7	1274
12	3.67	–0.00009	0.01100	60.0	73.4	–3.1	72.0	–1.2	1181

strains, which might be compressive or tensile depending on the position of the neutral axis in various cases, can be calculated from strain distribution diagrams as follows:

Case 1: Zero strain on the tension face of the column, $c = h$ (Point 2 on the interaction diagram).

Figure 5.13 shows the column cross section and the strain distribution and force diagrams corresponding to zero strain on the tension face of the column. Strain is tension reinforcement (ε_T) in this case would be *compressive*, which can be calculated from similar triangles of strain distribution diagram:

$$\frac{\varepsilon_T}{\varepsilon_m} = \frac{c-d}{c} = \left(1 - \frac{d}{c}\right)$$

whence

$$\varepsilon_T = \left(1 - \frac{d}{c}\right)\varepsilon_m \text{ (compressive)} \tag{5.41}$$

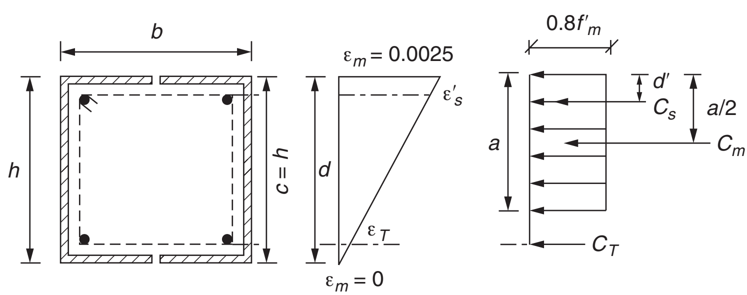


FIGURE 5.13 Strain distribution and force diagrams for the case of zero strain on the tension face of column.

The strain in compression reinforcement (ε_s') can be calculated similarly from similar triangles of strain distribution diagram:

$$\frac{\varepsilon_s'}{\varepsilon_m} = \frac{c-d'}{c} = \left(1 - \frac{d'}{c}\right)$$

whence

$$\varepsilon_s' = \left(1 - \frac{d'}{c}\right)\varepsilon_m \text{ (compressive)} \tag{5.42}$$

Case 2: Zero strain in tension reinforcement, $c = d$ (Point 3)

Figure 5.14 shows the column cross section, and the strain distribution and force diagrams corresponding to zero strain in the tension reinforcement. The strain in the compression reinforcement is obtained from the strain distribution diagram for which Eq. (5.42) is valid. Thus,

$$\varepsilon_s' = \left(1 - \frac{d'}{c}\right)\varepsilon_m \text{ (compressive)} \qquad \text{(5.42 repeated)}$$

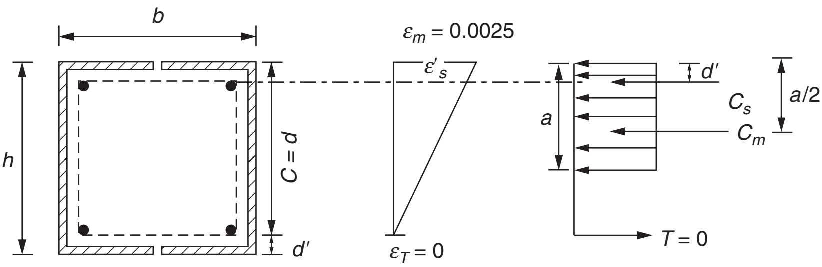


FIGURE 5.14 Strain distribution and force diagrams for the case of zero strain in tension reinforcement of the column.

Case 3: Neutral axis location such that $d' < c < d$ (strain in compression reinforcement is compressive, Points 4 to 6, 7 to 11).

Figure 5.15 shows the column cross section, and the strain distribution and force diagrams corresponding to neutral axis positioned anywhere between the compression and tension reinforcements. The strain in tension reinforcement is calculated from similar triangles of the strain distribution diagram:

$$\frac{\varepsilon_T}{\varepsilon_m} = \frac{d-c}{c} = \left(\frac{d}{c} - 1\right)$$

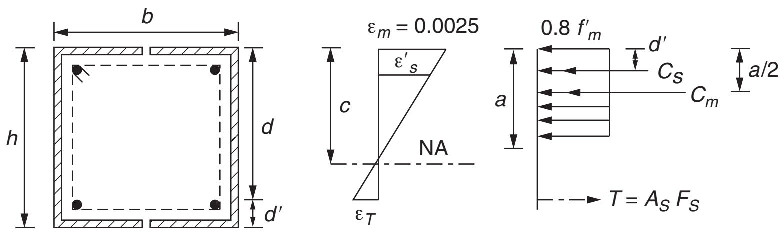


FIGURE 5.15 Strain distribution and force diagrams for the case of neutral axis positioned anywhere between the compression and tension reinforcements.

whence

$$\varepsilon_T = \left(\frac{d}{c} - 1\right)\varepsilon_m \text{ (tensile)} \tag{5.43}$$

Similar, the strain in the compression reinforcement is calculated as

$$\frac{\varepsilon_s'}{\varepsilon_m} = \frac{c - d'}{c} = \left(1 - \frac{d'}{c}\right)$$

whence

$$\varepsilon_s' = \left(1 - \frac{d'}{c}\right)\varepsilon_m \text{ (compressive)} \qquad \text{(5.42 repeated)}$$

Case 4: Neutral axis close to the compression face such that $c < d'$ (strain in compressive reinforcement is tensile).

Figure 5.16 shows the position of neutral axis close to the compression face of the column such that $c < d'$. The strain in the compressive reinforcement in this case would be *tensile*, which can be calculated from the similar triangles of strain distribution diagram:

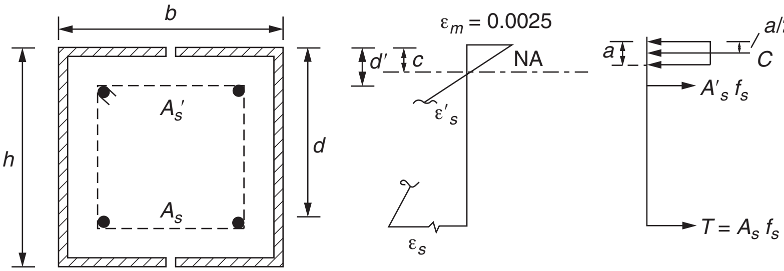


FIGURE 5.16 Strain distribution and force diagrams for the case of neutral axis positioned such that such that $c < d'$.

$$\frac{\varepsilon_s'}{\varepsilon_m} = \frac{d' - c}{c} = \left(\frac{d'}{c} - 1\right)$$

whence

$$\varepsilon_s' = \left(\frac{d'}{c} - 1\right)\varepsilon_m \text{ (tensile)} \tag{5.44}$$

The strain in the tension reinforcement is similarly calculated from the strain distribution diagram:

$$\frac{\varepsilon_T}{\varepsilon_m} = \frac{d - c}{c} = \left(\frac{d}{c} - 1\right)$$

whence

$$\varepsilon_T = \left(\frac{d}{c} - 1\right)\varepsilon_m \text{ (tensile)} \qquad \text{(5.43 repeated)}$$

In all cases, the stresses in the reinforcement, in compression or tension, are calculated from Hooke's law ($f = \varepsilon E$), but are limited to a maximum of the yield strength of the reinforcement. These stresses are then used to calculate forces in the reinforcements.

Example 5.11 presents calculations for axial load-bending moment interaction diagram for masonry columns. Example 5.11 uses the following two notations which are slightly different from those used earlier:

h' = effective height of a column (= h in previous examples)

h = nominal width of a column cross section (larger cross-sectional dimension, overall depth of a beam section)

Example 5.11 Axial load-bending moment interaction diagram for a rectangular CMU column bending about its major axis of cross section (axis parallel to the short side).

A nominal 16 × 24 in. CMU column having an effective height of 24 ft is reinforced with four No. 7 Grade 60 bars as shown in Fig. E5.11A, placed at 3.8 in. from the face of the column. Assuming f'_m = 2000 psi, plot the axial load-bending moment interaction diagram for this column. The bending of the column occurs about its major axis (axis parallel to the *short* side of the cross section).

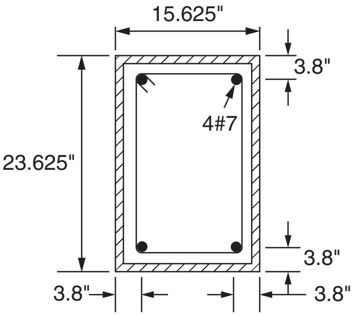


FIGURE E5.11A Column cross section.

Solution

Given: CMU column nominal 16 × 24 in., b = 15.625 in., h = 23.625 in., d = 19.825 in., d'= 3.8 in., A_{st} = 2.41 in.^{2} (four No. 7 bars), A'_s = 1.2 in.^{2} (two No. 7 bars), column height h' = 24 ft, f'_m = 2000 psi, f_y = 60 ksi.

Check code compliance with respect to dimensional limits, h'/t ratio and longitudinal reinforcement.

Nominal column width = 16 > 8 in. OK

Nominal column depth = 24 in. < $3t$ = 3(16) = 48 in. OK

$$h'/t = (24)(12)/16 = 18 < 30 \qquad \text{OK}$$

$$A_n = (15.625)(23.625) = 369.14 \text{ in.}^2$$

Longitudinal reinforcement, A_{st} = 2.41 in.^{2}

$$\rho = \frac{A_{st}}{A_n} = \frac{2.41}{369.0} = 0.0065$$

$$\rho_{\max} = 0.04 \qquad \rho_{\min} = 0.0025 \qquad \rho_{\text{provided}} = 0.0065$$

$$0.0025 < 0.0065 < 0.04 \qquad \text{OK}$$

Lateral ties: Tie diameter = 0.375 > 0.25 in. OK

s = 8 in. ≤ $16d_b$ = 16 (7/8) = 14 in. OK

≤ 48 tie diameter = 48(0.375) = 18 in. OK

≤ 16 in. OK

Point 1: Pure axial load, no bending moment

The axial load capacity can be determined from Eq. (5.12):

$$\phi P_n = 0.80\phi[0.80 f'_m (A_n - A_{st}) + f_y A_{st}]C_P$$

The value of C_P for a nominal 16 × 24 in. CMU column with an effective height of 24 ft was determined as 0.793 in Example 5.9 (calculations not repeated here).

For a nominal 16 × 24 in. CMU column, $A_n = (15.625)(23.625) = 369.14$ in.^{2}

$$\begin{aligned}\phi P_n &= 0.80\phi[0.80 f'_m (A_n - A_{st}) + f_y A_{st}]C_P \\ &= 0.80(0.9)[0.80(2.0)(369.14 - 2.41) + (60)(2.41)](0.793) \\ &= 417.6 \text{ kips}\end{aligned}$$

Point 2: Zero strain on tension side of column when a moment is applied

In this case, the neutral axis is assumed to be located on the tension face of the column so that $c = h = 23.625$ in. (depth of column). Figure E5.11B shows the strain distribution and forces acting on the column. The strain in the compression reinforcement is calculated from the strain diagram as:

Fig. E5.11B Strain distribution and forces acting on the column when strain is zero on the tension face of the column.

$$\varepsilon'_s = \left(1 - \frac{d'}{c}\right)\varepsilon_m = \left(1 - \frac{3.8}{23.625}\right)(0.0025) = 0.0021 > \varepsilon_y = 0.00207$$

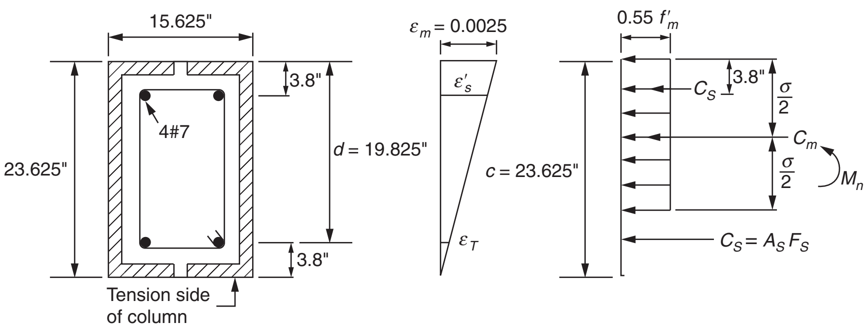


FIGURE E5.11B

Therefore, the stress in compression reinforcement, $f'_s = f_y = 60$ ksi

Force in compression reinforcement is calculated as

$$C_s = A'_s (f'_s - 0.80 f'_m) = 1.2[60 - 0.80(2.0)] = 70.08 \text{ kips}$$

Because $c = h = 23.625$ in. > $d = 19.825$ in., the tensile reinforcement is in slight compression. The strain in tensile reinforcement is given by

$$\varepsilon_T = \left(1 - \frac{d}{c}\right)\varepsilon_m = \left(1 - \frac{19.825}{23.625}\right)(0.0025) = 0.0004 < \varepsilon_y = 0.00207 \text{ (compressive)}$$

Therefore, stress in tensile reinforcement is calculated from Hooke's law:

$$f_T = \varepsilon_T E_s = (0.0004)(29{,}000) = 11.6 \text{ ksi (compressive)}$$

The force in the tensile reinforcement is

$$T = A_{st} f_T = 1.2(11.6) = 13.92 \text{ kips (compressive)}$$

The force in masonry in compression is given by Eq. (5.21):

$$C_m = 0.80 f'_m ab = 0.80(2.0)(a)(15.625) = 25a \text{ kips}$$

Substituting $a = 0.80c = 0.80(23.625) = 18.9$ in. in the above expression, we obtain

$$C_m = 25\ (18.9) = 472.5 \text{ kips}$$

For vertical equilibrium of forces, we must have $\Sigma F_y = 0$. Thus,

$$C_s + C_m + T - P = 0$$

$$70.08 + 472.5 + 13.92 - P = 0$$

whence

$$P = 556.5 \text{ kips}$$

Multiplying the above value of P by $C_P = 0.793$ (for slenderness), we have

$$P_n = PC_P = (556.5)(0.793) = 441.3 \text{ kips}$$

$$\phi P_n = 0.9(441.3) = 397.2 \text{ kips}$$

Taking moments of C_s, C_m, and T about the centroidal axis of the column, we obtain,

$$\begin{aligned}
M_n &= C_s\left(\frac{h}{2} - d'\right) + C_m\left(\frac{h}{2} - \frac{a}{2}\right) - T\left(d - \frac{h}{2}\right) \\
&= 70.08\left(\frac{23.625}{2} - 3.8\right) + 472.5\left(\frac{23.625}{2} - \frac{18.9}{2}\right) - 13.92\left(19.825 - \frac{23.625}{2}\right) \\
&= 561.5 + 1116.3 - 111.5 \\
&= 1566.3 \text{ k-in.} = 130.5 \text{ k-ft} \\
\phi M_n &= 0.9(1566.3) = 1409.7 \text{ k-in.} = 117.5 \text{ k-ft}
\end{aligned}$$

Point 3: Assume that the neutral axis passes through the tension reinforcement.

When the neutral axis passes through the tension reinforcement, the strain in this reinforcement is zero. Therefore, $T = 0$.

$$c = d = 23.625 - 3.8 = 19.825 \text{ in.}$$

$$a = 0.8c = 0.8(19.825) = 15.86 \text{ in.}$$

$$C_m = 0.80 f'_m ab = 0.80(2.0)(15.86)(15.625) = 396.5 \text{ kips}$$

The strain distribution would be as shown in Fig. E5.11C. The strain in compression reinforcement is given by

$$\varepsilon'_s = \left(1 - \frac{d'}{h}\right)\varepsilon_m = \left(1 - \frac{3.8}{19.825}\right)(0.0025) = 0.00202 < \varepsilon_y = 0.00207$$

Therefore, $\quad f'_s = \varepsilon'_s E_s = (0.00202)(29{,}000) = 58.58 \text{ ksi}$

$$C_s = A'_s(f'_s - 0.80 f'_m) = 1.2[58.58 - 0.8(2.0)] = 68.4 \text{ kips}$$

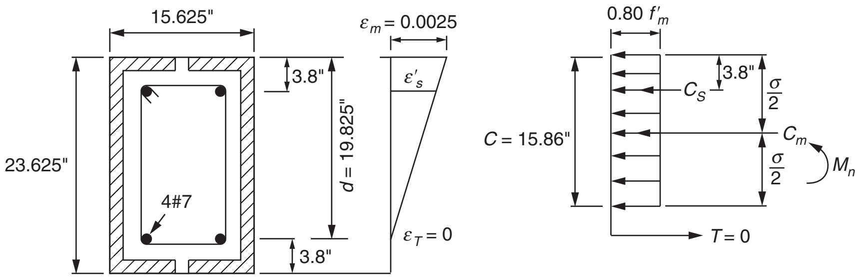


FIGURE E5.11C Strain distribution and forces acting on the column when strain is zero in the tension reinforcement.

For equilibrium in the vertical direction, we must have $\Sigma F_y = 0$. Thus, noting that $T = 0$,

$$C_s + C_m - T - P = 0 \qquad \text{(5.39 repeated)}$$

$$P = C_s + C_m - T = 68.4 + 396.5 + 0 = 464.9 \text{ kips}$$

$$P_n = PC_P = (464.9)(0.793) = 368.7 \text{ kips}$$

$$\phi P_n = 0.9(368.7) = 331.8 \text{ kips}$$

Taking moments of C_s, C_m, and T about the centroidal axis of the column, we obtain,

$$\begin{aligned} M_n &= C_s\left(\frac{h}{2} - d'\right) + C_m\left(\frac{h}{2} - \frac{a}{2}\right) \\ &= 68.4\left(\frac{23.625}{2} - 3.8\right) + 396.5\left(\frac{23.625}{2} - \frac{15.86}{2}\right) \\ &= 548.06 + 1539.41 \\ &= 2087.5 \text{ k-in.} = 174 \text{ k-ft} \\ \phi M_n &= 0.9(2087.5) = 1878.8 \text{ k-in.} = 156.6 \text{ k-ft} \end{aligned}$$

Points 4 to 6 and 7 to 10

Points 4 to 6 and 7 to 10 are arbitrarily selected at the following locations in the column cross section:

Point	c, in.
4	17.0
5	14.0
6	Balanced condition
7	10.0
8	8.0
9	6.5
10	5.0
11	4.0
12	Zero axial load

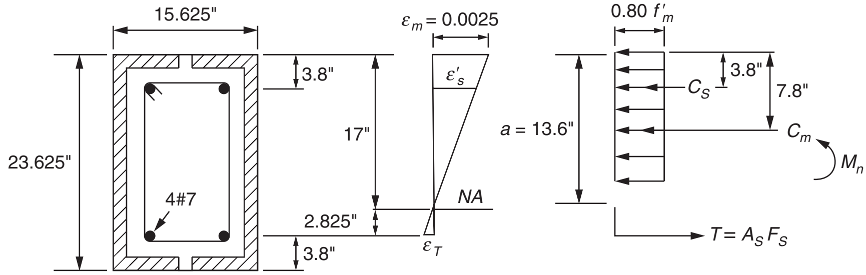


FIGURE E5.11D Strain distribution and forces acting on the column when the neutral axis is located at 17 in. from the compression face of the column.

The following calculations are presented for Point 4 which is located at 17 in. from the compression face of the column so that $c = 17$ in. This case is illustrated in Fig. E5.11D, which shows the strain distribution across the column cross section and the forces acting on the column.

$$c = 17.0 \text{ in.}$$

$$a = 0.8c = 0.8(17.0) = 13.6 \text{ in.}$$

$$C_m = 0.80 f'_m ab = 0.80(2.0)(13.6)(15.625) = 340 \text{ kips}$$

The strain in compression reinforcement is obtained from the strain distribution diagram as

$$\varepsilon'_s = \left(1 - \frac{d'}{c}\right)\varepsilon_m = \left(1 - \frac{3.8}{17.0}\right)(0.0025) = 0.00194 < \varepsilon_y = 0.00207$$

Therefore, $\quad f'_s = \varepsilon'_s E_s = (0.00194)(29{,}000) = 56.26 \text{ ksi}$

$$C_s = A'_s(f'_s - 0.80 f'_m) = 1.2[56.26 - 0.8(2.0)] = 65.59 \text{ kips}$$

The strain in tension reinforcement is determined from strain distribution diagram. The neutral axis is located at 23.625–17.0–3.8 = 2.825 in. from the tension reinforcement. Thus,

$$\varepsilon_T = \left(\frac{d}{c} - 1\right)\varepsilon_m = \left(\frac{19.825}{17.0} - 1\right)(0.0025) = 0.00042 < \varepsilon_y = 0.00207$$

Therefore, the stress in tensile reinforcement is

$$f_T = \varepsilon_T E_s = (0.00042)(29{,}000) = 12.18 \text{ ksi}$$

$$T = A_s f_T = 1.2(12.18) = 14.62 \text{ kips}$$

Equilibrium in the vertical direction requires that $\Sigma F_y = 0$. Thus,

$$C_s + C_m - T - P = 0 \qquad \text{(5.39 repeated)}$$

$$P = C_s + C_m - T = 65.59 + 340 - 14.62 = 391 \text{ kips}$$

$$P_n = PC_P = (391)(0.793) = 310.1 \text{ kips}$$

$$\phi P_n = 0.9(310.1) = 279.1 \text{ kips}$$

Taking moments of C_s, C_m, and T about the centroidal axis of the column, we obtain,

$$\begin{aligned}
M_n &= C_s\left(\frac{h}{2}-d'\right)+C_m\left(\frac{h}{2}-\frac{a}{2}\right)+T\left(d-\frac{h}{2}\right)\\
&= 65.59\left(\frac{23.625}{2}-3.8\right)+340\left(\frac{23.625}{2}-\frac{13.6}{2}\right)+14.62\left(19.825-\frac{23.625}{2}\right)\\
&= 525.54+1704.25+117.14\\
&= 2346.93\,\text{k-in.}=195.6\,\text{k-ft}\\
\phi M_n &= 0.9(2346.93)=2112.2\text{ k-in.}=176\text{ k-ft}
\end{aligned}$$

Calculations for other points in the interaction diagram are performed on Excel spreadsheet; so the results obtained are listed in Table E5.11. Calculations for Points 6 and 12 are presented as follows.

Point 6: Balanced conditions

Balanced condition implies a compressive strain of $\varepsilon_{mu} = 0.0025$ in masonry on one face of the column and yield strain (= 0.00207 in Grade 60 steel) in steel reinforcement on the opposite face of the column. Strain distribution and forces acting on the column under this condition are shown in Fig. E5.11E.

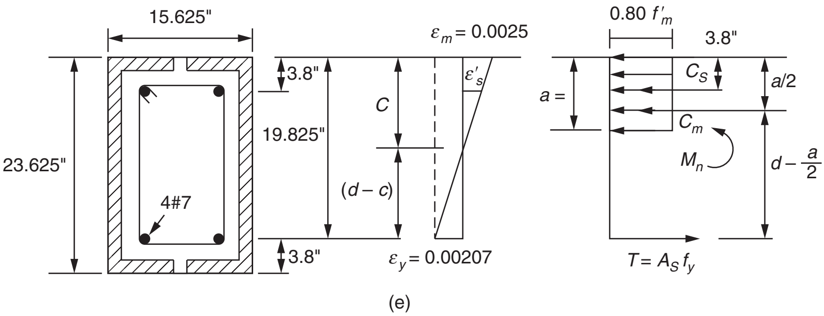


FIGURE E5.11E Strain distribution and forces acting on the column under balanced conditions in the column.

The position of the neutral axis is determined from the strain distribution diagram.

$$\begin{aligned}
\frac{d}{c} &= \frac{\varepsilon_y+\varepsilon_m}{\varepsilon_m}=\frac{0.00207+0.0025}{0.0025}=1.828\\
c &= \frac{d}{1.828}=\frac{19.825}{1.828}=10.85\,\text{in.}\\
a &= 0.8c = 0.8(10.85)=8.68\,\text{in.}\\
C_m &= 0.80 f'_m ab = 0.80(2.0)(8.68)(15.625)=217\,\text{kips}
\end{aligned}$$

The strain compression reinforcement is calculated as

$$\varepsilon'_s=\left(1-\frac{d'}{c}\right)\varepsilon_m=\left(1-\frac{3.8}{10.85}\right)(0.0025)=0.00162<\varepsilon_y=0.00207$$

Therefore, $\quad f'_s=\varepsilon'_s E_s=(0.00162)(29{,}000)=46.98\,\text{ksi}$

$$C_s=A'_s(f'_s-0.80f'_m)=1.2[46.98-0.80(2.0)]=54.46\,\text{kips}$$

The strain in the tensile reinforcement is equal to yield strain, therefore

$$T = A_s f_y = 1.2(60) = 72 \text{ kips}$$

Equilibrium in the vertical direction requires that $\Sigma F_y = 0$. Thus,

$$C_s + C_m - T - P = 0 \qquad \text{(5.39 repeated)}$$

$$P = C_s + C_m - T = 54.46 + 217 - 72.0 = 199.5 \text{ kips}$$

$$P_n = PC_P = (199.5)(0.793) = 158.2 \text{ kips}$$

$$\phi P_n = 0.9(158.2) = 142.4 \text{ kips}$$

Taking moments of C_s, C_m, and T about the centroidal axis of the column, we obtain,

$$\begin{aligned}
M_n &= C_s\left(\frac{h}{2} - d'\right) + C_m\left(\frac{h}{2} - \frac{a}{2}\right) + T\left(d - \frac{h}{2}\right) \\
&= 54.46\left(\frac{23.625}{2} - 3.8\right) + 217\left(\frac{23.625}{2} - \frac{8.6}{2}\right) + 72\left(19.825 - \frac{23.625}{2}\right) \\
&= 436.36 + 1630.21 + 576.9 \\
&= 2643.47 \text{ k-in.} = 220.3 \text{ k-ft} \\
\phi M_n &= 0.9(2643.47) = 2379.1 \text{ k-in.} = 198.3 \text{ k-in.}
\end{aligned}$$

Point 12: Zero axial load: $\phi P_n = 0$

In the absence of an axial load, the column behaves as a doubly reinforced beam. The location of neutral axis in this case is determined from Eq. (5.34):

$$(0.64 f'_m b)c^2 + (A'_s \varepsilon_m E_s - A_s f_y - 0.80 f'_m A'_s)c - A'_s \varepsilon_m E_s d' = 0 \qquad \text{(5.34 repeated)}$$

Equation (5.34) is a quadratic of the form: $Ax2 + Bx + C = 0$, which can be solved for x [= c in Eq. (5.34)]:

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

where $x = c$

$$\begin{aligned}
A &= 0.64 f'_m b = 0.64(2.0)(15.625) = 20 \text{ kips/in.} \\
B &= A'_s \varepsilon_m E_s - A_s f_y - 0.80 f'_m A'_s \\
&= 1.2(0.0025)(29{,}000) - 1.2(60) - 0.80(2.0)(1.2) \\
&= 13.08 \text{ kips} \\
C &= A'_s \varepsilon_m E_s d' = 1.2(0.0025)(29{,}000)(3.8) = 330.6 \text{ k-in.}
\end{aligned}$$

Substituting in the above equation, we obtain

$$20c^2 + 13.08c - 330.6 = 0$$

whence

$$\begin{aligned}
c &= \frac{-13.08 \pm \sqrt{(13.08)^2 + 4(20)(330.6)}}{2(20)} \\
&= +3.75 \text{ in.} - 4.41 \text{ in.}
\end{aligned}$$

The negative root (–4.41 in.) is ignored as it is meaningless in this problem. With c = 3.75 in., the compression reinforcement would be in tension because c = 3.75 in. < d' = 3.8 in. Therefore, c should be determined from the following equation [see Eq. (4.121), for derivation of this equation]:

$$(0.64 f'_m b)c^2 + (A'_s \varepsilon_m E_s - A_s f_y)c - A'_s \varepsilon_m E_s d' = 0 \qquad \text{(4.121 repeated)}$$

The above equation is a quadratic of the form: $Ax^2 + Bx + C = 0$, which can be solved for x (= c)

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

where $x = c$

$$A = 0.64 f'_m b = 0.64(2.0)(15.625) = 20 \text{ kips/in.}$$

$$\begin{aligned} B &= A'_s \varepsilon_m E_s - A_s f_y \\ &= 1.2(0.0025)(29{,}000) - 1.2(60) \\ &= 15 \text{ kips} \end{aligned}$$

$$C = A'_s \varepsilon_m E_s d' = 1.2(0.0025)(29{,}000)(3.8) = 330.6 \text{ k-in.}$$

Substituting in the above equation, we obtain

$$\begin{aligned} c &= \frac{-15 \pm \sqrt{(15)^2 + 4(20)(330.6)}}{2(20)} \\ &= +3.71 \text{ in.} - 4.46 \text{ in.} \\ c &= 3.71 \text{ in.} < d' = 3.8 \text{ in.} \qquad \text{OK} \\ a &= 0.8c = 0.8(3.71) = 2.97 \text{ in.} \\ C_m &= 0.80 f'_m ab = 0.80(2.0)(2.97)(15.625) = 74.25 \text{ kips} \end{aligned}$$

Because $c < d'$, the strain in compression reinforcement is tensile. From Eq. (5.44)

$$\varepsilon'_s = \left(\frac{d'}{c} - 1\right)\varepsilon_m = \left(\frac{3.8}{3.71} - 1\right)(0.0025) = 6.06(10^{-5}) < \varepsilon_y = 0.00207$$

Therefore, the strain in the compression reinforcement is calculated from Hooke's law as

$$f'_s = \varepsilon'_s E_s = (6.06 \times 10^{-5})(29{,}000) = 1.76 \text{ ksi (tensile)}$$

$$C_s = A'_s f'_s = 1.2(1.76) = 2.11 \text{ kips (tensile)}$$

$$T = 72 \text{ kips (as before)}$$

Equilibrium in the vertical direction requires that $\Sigma F_y = 0$. Thus,

$$C_s + C_m - T = 0$$

$$C_s + C_m - T = -2.11 + 74.25 - 72.0 = 0.14 \text{ kip} \approx 0$$

Taking moments of C_s, C_m, about T, we obtain,

$$
\begin{aligned}
M_n &= -C_s(d-d') + C_m\left(d - \frac{a}{2}\right) \\
&= -2.11(19.825 - 3.8) + 74\left(19.825 - \frac{2.97}{2}\right) \\
&= -33.81 + 1357.16 \\
&= 1323.53 \text{ k-in.} = 110.3 \text{ k-f}t \\
\phi M_n &= 0.9(1327.94) = 1195.1 \text{ k-in.} = 99.6 \text{ k-ft}
\end{aligned}
$$

Figure E5.11F shows a plot of the 12 points (with ϕM_n on the x-axis and ϕP_n on the y-axis for each point) listed in Table E5.11 (calculations performed on Excel spreadsheet).

Interaction diagram

ϕP_n (kips)	ϕM_n (k-in)
417.6	0
398.2	1412
331.6	1877
279.4	2108
218.1	2267
142.4	2368
128.7	2298
94.3	2073
66.0	1843
33.7	1540
7.7	1274
0	1181

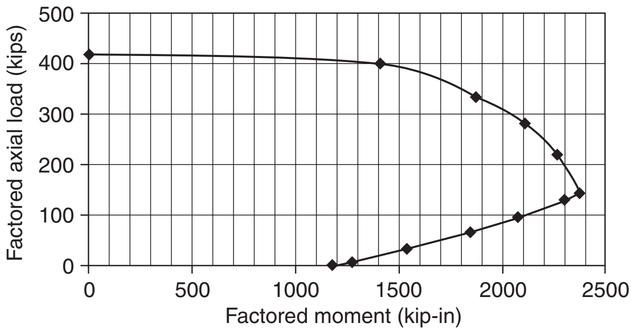


FIGURE E5.11F Interaction diagram.

5.8 DISCUSSION AND INTERPRETATION OF THE AXIAL LOAD-BENDING MOMENT INTERACTION DIAGRAMS

Calculations presented in the preceding section demonstrate that values of P_n and M_n for a given reinforced masonry column with a given set of strain conditions (ultimate compressive strain in masonry and yield strain in steel reinforcement) can be easily determined. However, hand calculations for a complete interaction diagram for columns are quite tedious as is evident from preceding calculations for just one column. Therefore, it is common practice to resort to computer programs to generate interaction diagrams for columns of various sizes, masonry strengths, and reinforcement ratios. As explained earlier, an interaction diagram is drawn for a column of a configuration, compressive strength of masonry, and amount of longitudinal steel reinforcement, as the load changes from a state of pure axial load to a pure bending moment with varying combinations of axial loads and bending moments in between.

The interaction diagram depicted in Fig. E5.11F was drawn for the specific column described in Example 5.11; however, interaction diagrams for other masonry columns would be typically

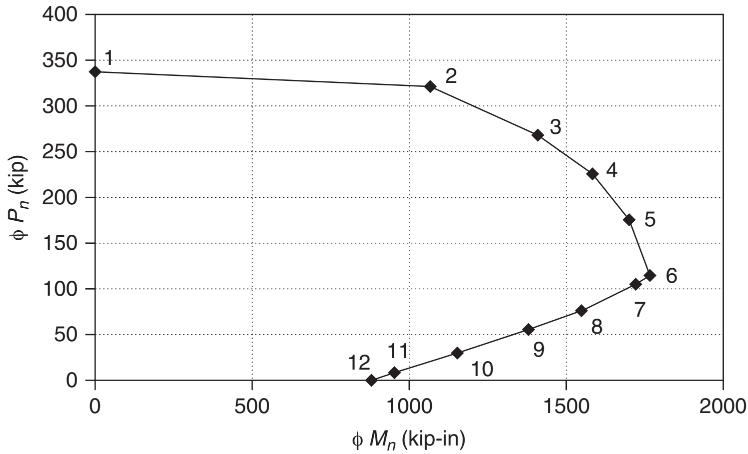


FIGURE 5.17 A typical interaction diagram for a reinforced masonry column (combined flexural and axial loads).

similar as shown in Fig. 5.17. Interaction diagrams are very useful in studying the strengths of masonry columns with varying proportions of axial loads and moments. A column can satisfactorily carry any combination of axial load and bending moment falling within the interaction curve; any combination of axial load and bending moment falling outside the interaction curve represents failure. For example, if a column were to fail under axial load only (no moment present), the failure would occur at point A on the diagram. All points on the curve between A and B (such as point D) represent a specific combination of axial load and bending moment which the column can resist at failure. Each point in between points A and B characterizes increase in bending moment accompanied by a decrease in axial load. Point C (on the interaction curve meeting the x-axis) represents the bending strength of the column; a column would fail under this bending moment, with no axial load present.

The lower part of the interaction curve (portion BC) is somewhat different from portion AB of the curve; it shows that the moment capacity increases as the axial load increases. This part of the curve represents tensile failures. Any compressive load in that range tends to reduce the stresses in tensile bars so that a larger moment can be resisted. Point B on the curve is called the *balanced point*; it represents the balanced loading case indicating a simultaneous occurrence of compression failure and tensile yielding.

5.9 INTERACTION DIAGRAM FOR A WALL UNDER COMBINED LOADING (AXIAL LOAD AND BENDING)

It is quite common in masonry buildings to have masonry walls subjected to combined axial load and in-plane bending, which occurs about the strong axis of the wall cross section (the out-of-plane bending occurs about the weak axis of the wall cross section). Such walls are called shear walls. A shear wall acts as a bearing wall when it carries gravity loads from roofs or floors it might be supporting, as well as lateral load (directed parallel to itself or in-plane) transferred to it from the diaphragm (roof or floor, or both in the case of a multistory building).

A complete treatment of shear walls is presented in Chap. 7, but because of its conceptual and behavioral similarity to columns under combined loading, a discussion of the axial load and bending moment interaction diagram for shear walls is presented in this section.

Calculations for the interaction diagram for a shear wall are very similar to those presented in the previous section for columns under combined loading. However, calculations for the interaction diagram for a shear wall require a few modifications in the calculations for the interaction diagram for columns presented in the preceding section. This is because of a few obvious differences between the configurations of a column and a shear wall. Masonry columns are most generally square or rectangular in plan, whereas the cross section of a shear wall is usually a narrow rectangle. As a result, the reinforcement configurations in columns and shear walls are quite different (see Fig. 5.18). A rectangular or a square column is most commonly reinforced with four longitudinal reinforcing bars, one in

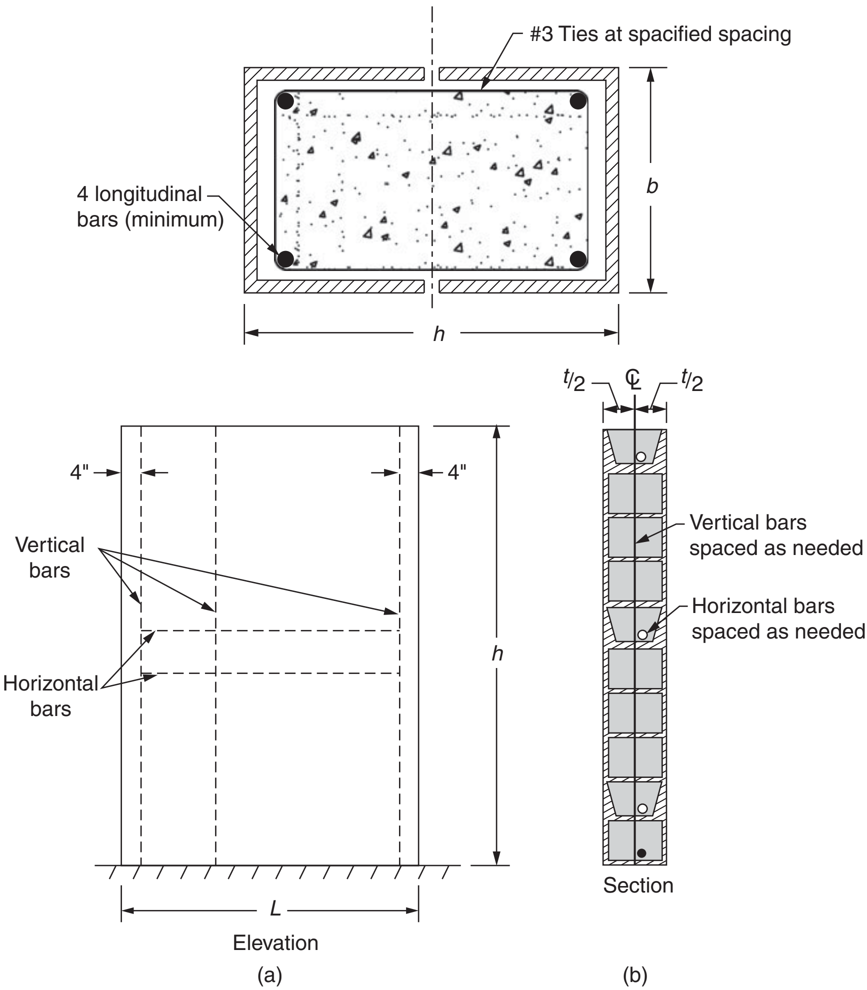


FIGURE 5.18 Comparison of reinforcement configurations for typical columns and shear walls. (*a*) shear wall reinforcement and (*b*) Column reinforcement.

each corner. Under flexure, the column behaves akin to a beam (oriented vertically), with two corner bars near the column face acting in tension or compression (depending on the direction of bending). A shear wall, on the other hand, acts as a deep beam; its reinforcement configuration consists of several equal-diameter reinforcing bars oriented vertically at equal intervals (at multiples of 8 in. in a wall constructed from 8 × 8 × 16 in. CMUs) along the length of the wall, and all cells are grouted solid. Due to bending caused by in-plane loads, some of those bars would be in tension and some in compression, depending on their distances from the neutral axis of the cross section—this is where a shear wall is different from a column. The forces in reinforcing bars calculated from strain distribution as illustrated in Sections 5.7 and 5.8.

5.10 SHEAR STRENGTH OF MASONRY COLUMNS

Shear strength of masonry columns depends on the shear span ratio, M_u/V_ud_v, and can be determined based on the same general principles as applied to beams (discussed in Section 4.10). Calculations for shear strength of masonry columns differ from those for beams because of the presence of axial load (beams are designed primarily as flexural members, may have very small axial loads, the factored axial load not to exceed $0.05A_nf'_m$, MSJC Section 3.3.4.2.1). The nominal shear strength, V_{nm}, of a reinforced masonry member is calculated as the sum of two component strengths: (1) nominal shear strength of masonry, V_m and (2) the nominal shear strength provided by reinforcement (i.e., lateral ties), V_{ns}:

$$V_n = V_{nm} + V_{ns} \tag{4.97 repeated}$$

where

$$V_{nm} = \left[4.0 - 1.75\left(\frac{M_u}{V_ud_v}\right)\right]A_n\sqrt{f'_m} + 0.25P_u \tag{4.91 repeated}$$

$$V_{ns} = 0.5\left(\frac{A_v}{s}\right)f_yd_v \tag{4.101 repeated}$$

The following limitations apply to the above three expressions:

1. The value of the shear span ratio, M_u/V_ud_v, in Eq. (4.91) is to be taken as positive number which need not be taken greater than 1.0.
2. The value of V_n in Eq. (4.97) is limited based on the shear span ratio, M_u/V_ud_v, as follows:

   (a) Where $M_u/V_ud_v \le 0.25$,

$$V_n \le 6A_n\sqrt{f'_m} \tag{4.92 repeated}$$

   (b) Where $M_u/V_ud_v \ge 1.0$,

$$V_n \le 4A_n\sqrt{f'_m} \tag{4.93 repeated}$$

   (c) The maximum value of V_n for M_u/V_ud_v between 0.25 and 1.0 is to be determined from linear interpolation.

Appropriate Load Combinations should be used when evaluating P_u for use in Eq. (4.91). When a column is subjected to a lateral seismic load, Seismic Load Combinations 5 and 7 as specified in ASCE 7-05 Section 12.4.2.3 should be used to calculate P_u:

$$\boldsymbol{U = (1.2 + 0.2S_{\mathrm{DS}})D + \rho Q_E + L + 0.2S} \quad \textbf{(5.45, ASCE 7-05 Load Combination 5)}$$

$$\boldsymbol{U = (0.9 - 0.2S_{\mathrm{DS}})D + \rho Q_E + 1.6H} \quad \textbf{(5.46, ASCE 7-05 Load Combination 7)}$$

where S_{DS} = design, 5 percent damped, spectral response acceleration parameter at short periods
D = effect of dead load
ρ = redundancy factor
Q_E = effect of horizontal seismic forces
$0.2S_{\mathrm{DS}}\,D = E_v$, effect of vertical seismic forces

The quantity $0.2S_{\mathrm{DS}}\,D$ in Eqs. (5.45) and (5.46) is included to account for the effects of vertical seismic forces.

A discussion on determination of seismic forces based on *equivalent lateral force procedure* [including determination of S_{DS}, the design, 5 percent damped, spectral response acceleration parameter at short periods used in Eqs. (5.45) and (5.46)] specified in ASCE 7-05 is presented in Chap. 7.

Example 5.12 illustrates calculations for the shear strength of a reinforced CMU column.

Example 5.12 Shear Strength of a CMU column.

A nominal 16 × 24-in. CMU column having an effective height of 24 ft is reinforced with four No. 7 Grade 60 bars as shown in Fig. E5.12. The service loads are: $D = 20$ kips, $L_r = 20$ kips. The seismic lateral load, $V_E = 2$ kips, $S_{\mathrm{DS}} = 1.25g$. The column is reinforced with four No. 7 Grade 60 bars and No. 3 Grade 60 ties at 8 in. on center. $f'_m = 2000$ psi.

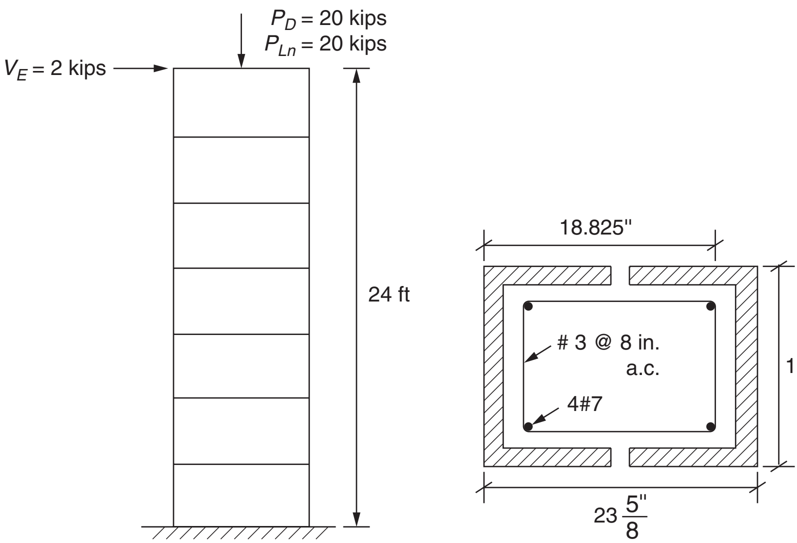


FIGURE E5.12 Shear strength of a reinforced masonry column.

The bending of the column occurs about its strong axis (axis parallel to the *short* side of the cross section). The design, 5 percent damped, spectral response acceleration parameter at short periods, $S_{DS} = 1.25g$. Calculate the shear strength of this column.

Solution

Given: CMU column nominal 16 × 24 in., $b = 15.625$ in., $h = 23.625$ in., $d = 19.825$ in., $d' = 3.8$ in., $A_{st} = 2.41$ in.^{2} (four No. 7 bars), column height $h' = 24$ ft, $f'_m = 2000$ psi, $f_y = 60$ ksi., $S_{DS} = 1.25g$.

(Note: The column described in this example is the same column as described in Example 5.11 for which interaction diagram was plotted. Based on that interaction diagram, it is evident that the column can support the imposed loads. These calculations are not repeated here. In general, for a column for which shear strength is to be determined, a designer must also ensure that it can support the imposed loads.)

Check code compliance with respect to dimensional limits, h'/t ratio and longitudinal reinforcement.

$$\text{Nominal column width} = 16 \text{ in.} > 8 \text{ in.} \qquad \text{OK}$$

$$\text{Nominal column depth} = 24 \text{ in.} < 3t = 3 \times 16 = 48 \text{ in.} \qquad \text{OK}$$

$$h'/t = (24)(12)/16 = 18 < 30 \qquad \text{OK}$$

$$A_n = (15.625)(23.625) = 369 \text{ in.}^2$$

Longitudinal reinforcement, $A_{st} = 2.41$ in.^{2}

$$\rho = \frac{A_{st}}{A_n} = \frac{2.41}{369.0} = 0.0065$$

$$\rho_{\max} = 0.04 \qquad \rho_{\min} = 0.0025 \qquad \rho_{\text{provided}} = 0.0065$$

$$0.0025 < 0.0065 < 0.04 \qquad \text{OK}$$

$$\text{Lateral ties: Tie diameter} = 0.375 \text{ in.} > 0.25 \text{ in.} \qquad \text{OK}$$

$$s = 8 \text{ in.} \le 16d_b = 16\,(7/8) = 14 \text{ in.} \qquad \text{OK}$$

$$\le 48 \text{ tie diameter} = 48(0.375)\ 18 \text{ in.} \qquad \text{OK}$$

$$\le 16 \text{ in.} \qquad \text{OK}$$

Load Combinations:

1. $U = 1.4D = 1.4(20) = 28$ kips
2. $U = 1.2D + 1.6L_r = 1.2\,(20) + 1.6\,(20) = 56$ kips
3. $U = (1.2 + 0.2S_{DS})D + \rho Q_E + L + 0.2S$
   $= [1.2 + 0.2(1.25)](20) = 29$ kips

   In Load Combination 3, the term ρQ_E represents horizontal force effects due to earthquake (= 0 for gravity load effects)
   [Note: Both live load (L) and snow load (S) are zero in this problem.]
4. $U = (0.9 - 0.2S_{DS})\,D + \rho Q_E + 1.6H$

   where H = load due to lateral pressure due to earth, ground water or bulk materials
   = 0 in this problem

Also, $\rho Q_E = 0$ as stated above. Therefore, the pertinent Load Combination can be expressed as

$$U = [0.9 - 0.2\,(1.25)]D$$
$$= [0.9 - 0.2\,(1.25)](20) = 13 \text{ kips (smallest value)}$$

Because P_u is additive to the shear strength of the column, V_{nm}, the smallest of the above four values of P_u would be used.

$$P_u = 13 \text{ kips}$$
$$Q_E = V_E = 2.0 \text{ kips}$$
$$\rho = 1.0 \text{ per ASCE-05 Section 12.3.4.1.}$$
$$M_u = 1.0\,(2)\,(24) = 48 \text{ k-ft}$$

The shear span ratio for the column is

$$\frac{M_u}{V_u d_v} = \frac{48(12)}{(1.5)(19.825)} = 19.37 > 1.0$$

$$\text{Use } \frac{M_u}{V_u d_v} = 1.0 \text{ (MSJC-08 Section 3.3.4.1.2.1)}$$

The nominal shear strength of the column, V_{nm}, is given by MSJC-08 Eq. (3.22) (discussed in Chap. 4, Section 4.10.1):

$$V_{nm} = \left[4 - 1.75\left(\frac{M_u}{V_u D_v}\right)\right] A_n \sqrt{f'_m} + 0.25 P_u$$

Using the Load Combination with minimum axial load ($P_u = 13$ kips), the shear strength contribution of masonry is

$$V_{nm} = [4 - 1.75(1.0)]\,(369)\,\sqrt{2000} + 0.25\,(13{,}000)$$
$$= 40{,}380 \text{ lb} = 40.38 \text{ kips}$$

The shear strength contributed by the shear reinforcement is given by MSJC-08 Eq. (3.23):

$$V_{ns} = 0.5\left(\frac{A_v}{s}\right) f_y d_v$$
$$= 0.5\left(\frac{0.11}{8}\right)(60)(19.825) = 8.18 \text{ kips}$$

The total nominal shear strength of the column is

$$\mathrm{V_n = V_{nm} + V_{ns}}$$
$$= 40.38 + 8.18 = 48.56 \text{ kips}$$

Since $M_u/V_ud_v \geq 1.0$, the nominal shear strength of the column is limited to

$$\begin{aligned} V_n &\leq 4A_n\sqrt{f'_m} \\ &\leq 4(369)(\sqrt{2000}) \\ &\leq 66{,}009\ lb \approx 66 \text{ kips.} > 48.56 \text{ kips} \end{aligned}$$

Therefore, $V_n = 48.56$ kips (the smaller value governs)

$$\phi_v V_n = 0.8\ (48.56) = 38.85 \text{ kips}$$

5.11 MASONRY PIERS

Piers are essentially columns that might have wall segments-like appearance. It is quite common for some masonry walls in a building to have openings; the wall segments between the openings are commonly referred to as piers. However, these wall segments are not isolated, and as such not considered piers under the MSJC-08 provisions. For design purposes, these structural elements are considered wall segments, which may be subjected to in-plane or out-of-plane loads.

For design purposes, a pier may be treated as a column under combined axial and flexural loads, discussed earlier in this chapter. According to MSJC-08 Section 3.3.4.3 [5.1], a pier is an isolated vertical member which meets the following prescribed dimensional limitations (Fig. 5.19):

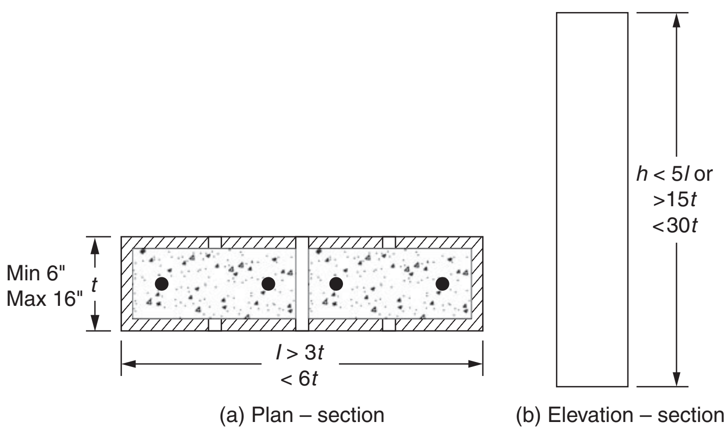


FIGURE 5.19 Definition of a pier. *(a)* Plan – section and *(b)* Elevation – section.

1. Horizontal dimension (b) measured at right angles to its thickness (t) is between 3 and 6 times its thickness ($3t \leq b \leq 6t$).
2. The nominal thickness (t) of the pier is not to exceed 16 in.
3. Clear height (h) is less than five times its length ($h < 5b$). However, if the factored axial load at the location of maximum moment is less than 0.05 $f'_m\ A_g$, the length of the pier can be equal to its thickness.
4. The distance between lateral supports of a pier is not to exceed 25 times its nominal thickness (t).

5. If the distance between the lateral supports of a pier exceeds 25 times its nominal thickness (t), then the pier is to be designed as a wall element (see MSJC-08 Section 3-5, discussed in Chaps. 6 and 7).

In addition to the abovestated dimensional limits, MSJC-08 specifies requirements for longitudinal reinforcement for piers subject to stress reversals as follows:

1. The piers are to be reinforced symmetrically about their neutral axes.
2. The longitudinal reinforcement should comply with the following:
  (a) One bar shall be provided at each end.
  (b) The minimum area of longitudinal reinforcement shall be $0.0007bd$.
  (c) Longitudinal reinforcement should uniformly be distributed throughout the depth of the pier.

It is noted that these requirements are predominantly seismic related and are intended to provide the greatest ductility economically. Additionally, if a pier is a part of a special reinforced masonry shear wall, it should satisfy strain compatibility requirements specified in MSJC-08 Section 3.3.3.5.1, which are as follows:

1. For in-plane loads, where $M_u/V_u d_v \geq 1.0$, the strain in the extreme tensile reinforcement must be at least equal to 4 times the yield strain.
2. For out-of-plane loads, the strain in the extreme reinforcement should be at least equal to 1.5 times the yield strain.

The factored axial compression on piers is not to exceed $0.3A_n\ f'_m$. According to the MSJC-08 *Commentary*, this is an arbitrary limit imposed due to the less severe requirements on the design of piers than for similar requirements for column. Indirectly, this provision dictates the cross section of the pier

PROBLEMS

In each of the following problems, check if the column reinforcement complies with code requirements with respect to size and longitudinal reinforcement. Provide 3/8-in.-diameter lateral ties in each case and show reinforcement details with suitable sketches.

Problems for analysis

5.1 A 24- × 24-in. CMU column having an effective height of 24 ft is reinforced with four No. 9 Grade 60 bars (one bar placed in each corner). Assume f'_m = 1500 psi. Determine ϕP_n for this column.

5.2 A 16- × 16-in. CMU column having an effective height of 20 ft is reinforced with four No. 8 Grade 60 bars. Assume $f'_m = 1800$ psi. Determine ϕP_n for this column. If the service dead load and service live load are 40 and 60 percent, respectively, of the applied load, calculate the total service load, dead load, and live load that can be supported by the column.

5.3 An 18- × 18-in. brick masonry column having an effective height of 22 ft is reinforced with six No. 9 Grade 60 bars. It carries a service dead load of 250 kips and a service live load of 200 kips. $f'_m = 2500$ psi. Determine ϕP_n for the column and check if the column can support the imposed service loads. Could this column carry the imposed loads if f'_m were to be lowered to 2000 from 2500 psi?

5.4 A 16- × 24-in. CMU column having an effective height of 24 ft is reinforced with four No. 9 Grade 60 bars. Assume $f'_m = 2000$ psi. Determine ϕP_n for this column. If the service dead and live loads are 30 and 70 percent, respectively, of the applied load, calculate the total service load, dead load, and live load that can be supported by the column.

5.5 A 16- × 16-in. brick masonry column has an effective height of 18 ft. It is reinforced with four No. 7 Grade 60 bars. Assume $f'_m = 2500$ psi. Determine ϕP_n for this column.

5.6 A nominal 16 × 24 in. CMU column having an effective height of 24 ft is reinforced with four No. 9 Grade 60 bars. It carries a service dead load of 100 kips and a service live load of 250 kips. $f'_m = 1800$ psi. Determine ϕP_n for the column and check if the column can support the imposed service loads.

Problems for design

5.7 Design a 24- × 24-in. CMU column to carry a service dead load of 250 kips and a service live load of 250 kips. The effective height of the column is 24 ft. $f'_m = 1800$ psi, $f_y = 60$ ksi.

5.8 Design a square CMU column to carry a service dead load of 200 kips and a live load of 250 kips. The effective height of the column is 16 ft. $f'_m = 1500$ psi, $f_y = 60$ ksi. Assume initially area of longitudinal reinforcement as 2 percent of the net cross-sectional area of masonry.

5.9 Design a 16- × 24-in. CMU column to carry a service dead load of 200 kips and a live load of 250 kips. The effective height of the column is 16 ft. $f'_m = 1800$ psi, $f_y = 60$ ksi.

5.10 Design an 18 × 18 in. brick masonry column to carry a service dead load of 175 kips and a live load of 225 kips. The effective height of the column is 16 ft. $f'_m = 1800$ psi, $f_y = 60$ ksi.

5.11 Design a square brick masonry column to carry a service dead load of 150 kips and a live load of 250 kips. The effective height of the column is 16 ft. $f'_m = 2500$ psi, $f_y = 60$ ksi. Assume initial area of longitudinal reinforcement as 2 percent of the net cross-sectional area of masonry.

5.12 A nominal 16- × 24-in. CMU column is reinforced with four No. 9 Grade 60 bars as shown in Fig. P5.12. The effective height of the column is 20 ft. The service dead and live loads are 150 and 200 kips, respectively. Assuming $f'_m = 2500$ psi, calculate the moment-carrying capacity of the column about its minor axis (axis parallel to the *long* side of the column).

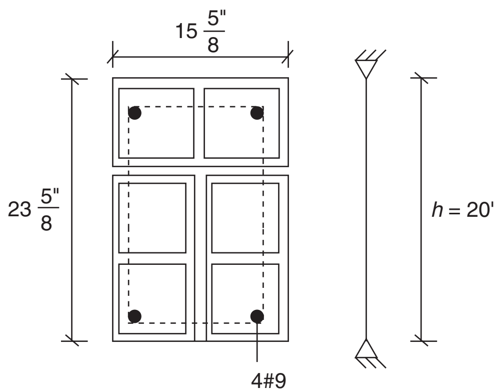


FIGURE P5.12

5.13 A nominal 16- × 24-in. CMU column is reinforced with four No. 9 Grade 60 bars as shown in Fig. P5.13. The effective height of the column is 20 ft. The service dead and live load are 150 and 200 kips, respectively. Assuming $f'_m = 2500$ psi, calculate the moment-carrying capacity of the column about its major axis (axis parallel to the *short* side of the column).

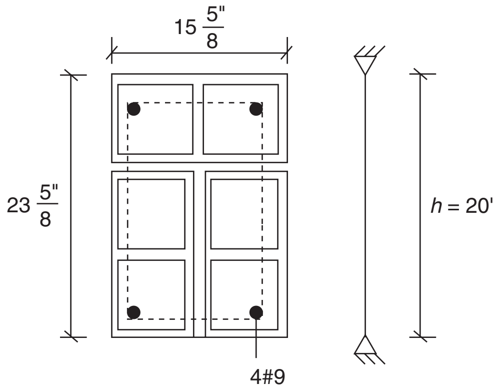


FIGURE P5.13

Problems for interaction diagram

5.14 A nominal 16- × 24-in. CMU column having an effective height of 20 ft is reinforced with four No. 6 Grade 60 bars as shown in Fig. P5.14. Assuming $f'_m = 1500$ psi, plot the axial load-bending moment interaction diagram for the column. The bending of the column occurs about its major axis (axis parallel to the short side of the cross section). (Hint: See Example 5.11 for sample calculations.)

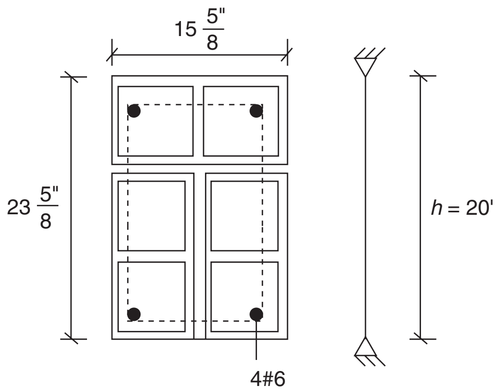


FIGURE P5.14

5.15 A nominal 16- × 24-in. CMU column having an effective height of 20 ft is reinforced with four No. 7 Grade 60 bars as shown in Fig. P5.15. Assuming $f'_m = 1500$ psi, plot the axial load-bending moment interaction diagram for the column. The bending of the column occurs about its minor axis (axis parallel to the *long* side of the cross section). (Hint: See Example 5.11 for sample calculations.)

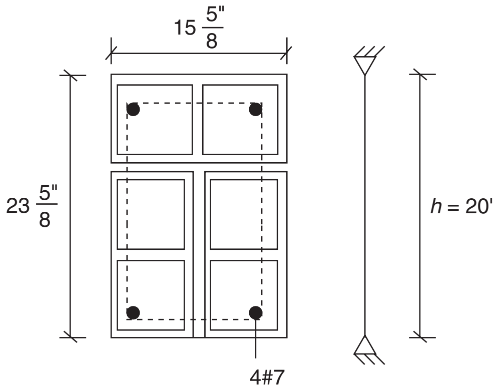


FIGURE P5.15

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CHAPTER 6
WALLS UNDER GRAVITY AND TRANSVERSE LOADS

6.1 INTRODUCTION

Walls form the most visible and functional components of masonry and other structures. They are provided to perform a multitude of functions. In *bearing wall structural systems*, masonry walls can be provided to carry both gravity and lateral loads (both *in-plane* and *out-of-plane*). Masonry walls are often provided as nonload-bearing elements in many forms such as curtain walls, panel walls, partition walls, in-fill walls, screen walls, fence walls, and highway sound barrier walls. Masonry walls are often used advantageously as means to impart architectural beauty to buildings and other structures. This is particularly true in case of brick walls and veneer walls. Available in many shades of red, brown, and gray colors, both bricks and veneers are often used to impart and/or enhance aesthetic qualities of masonry structures. For example, bricks can be used, for economic or architectural reasons, on the exterior of concrete masonry wall to form a *composite* wall.

Discussion in this chapter is focused on analysis and design of masonry (concrete and clay) subjected to both gravity *and* out-of-plane loads; the latter being those induced by wind and earthquakes. Walls may also be subjected to out-of-plane forces from other sources such as earth pressures (e.g., earth-retaining walls, basement walls, walls of buried structures, bridge abutments, etc.) and liquid pressures (on walls of liquid-retaining structures). Walls subjected to in-plane lateral loads, called shear walls, are discussed in Chap. 7. Walls subjected to lateral earth pressures (retaining walls and subterranean walls) are discussed in Chap. 8. Several new terms are introduced in this chapter in the context of masonry walls and related construction practices. These terms are defined in "Glossary" provided at the end of the last chapter in this book.

6.2 TYPES OF MASONRY WALLS

Masonry walls may be built from solid or hollow masonry units. Although common in past decades, solid masonry units are rarely used today, and consequently, not discussed in this book. Modern masonry walls are constructed from hollow masonry units or combined hollow and solid masonry units. Spurred by a combination of experience, research, and advances in producing various types of masonry units, modern masonry offers many types of construction practices for walls which may be provided to perform many different functions. Following are some of the many ways in which walls may be classified:

1. Concrete or clay masonry walls
2. Load-bearing and nonload-bearing walls
3. Single- or multiple-wythe walls
4. Solid- or partially grouted walls
5. Un-reinforced and reinforced walls
6. Composite walls
7. Cavity walls
8. Curtain walls, panel walls, and screen walls
9. Veneer walls
10. Empirically designed and engineered (or analytically designed) walls
11. Classification based on support conditions

Various types of walls are described as follows:

1. A wall can be described by the type of masonry units used in construction. Walls built from concrete masonry units (CMU) are called concrete block walls; those built from bricks are called brick walls.
2. From the standpoint of loads-carrying function, like walls made from other materials, all masonry walls can be characterized as *load-bearing* (or simply *bearing walls*) and *nonload-bearing* (or simply *nonbearing walls*). Walls provided to support gravity loads, such as from roofs and floors in buildings, are called *load-bearing* walls. Sometimes these types of walls have code-specific definitions. MSJC-08 (Section 1.6) [6.1] defines load-bearing walls as those that carry at least 200 lb per lineal foot of vertical load in addition to their own weight. Walls such as those used for fences, interior partitions or as exterior surfaces of a building (curtain walls and filler walls), highway sound barrier walls, etc., which do not carry vertical loads imposed by other elements of a building, are classified as the *nonload-bearing* type. Such walls are required to be designed to carry their own load only plus any superimposed finish and lateral forces (Fig. 6.1). Design of nonload-bearing walls is discussed in Section 6.11.

   Loads on a wall may be gravity loads or lateral loads, or both. Gravity loads on a wall may be concentrated, such as those transferred in the forms of reactions from supported trusses, beams, and girders, or they may be distributed, such as those transferred by a supported slab. Lateral loads may be those due to wind, earthquake, earth pressure, or water pressure. Again, lateral loads may be acting in the plane of the wall, commonly referred to as *in-plane forces*, as in the case of shear walls, or they may be perpendicular to the walls, commonly referred to as *out-of-plane forces* (Fig. 6.2). In cases where load-bearing walls also constitute lateral load-resisting system for a building, they are subjected to combined loads, that is, axial loads (gravity loads) and bending moment. Although the nonbearing walls, such as fences, curtain walls, and filler walls, do not carry any gravity load except their own weight, they must resist lateral loads induced by wind or earthquake.

   Codes contain specific provisions for material specifications for both load-bearing nonload-bearing masonry systems. Specifications for mortar and grout were discussed in Chap. 2. MSJC-08 Section 1.7.4.4 [6.1] prohibits use of Type N mortar and masonry cement for lateral force-resisting systems (LFRS) in Seismic Design Categories (SDC) D, E, and F structures (no such restrictions for structures in other Seismic Design Categories).

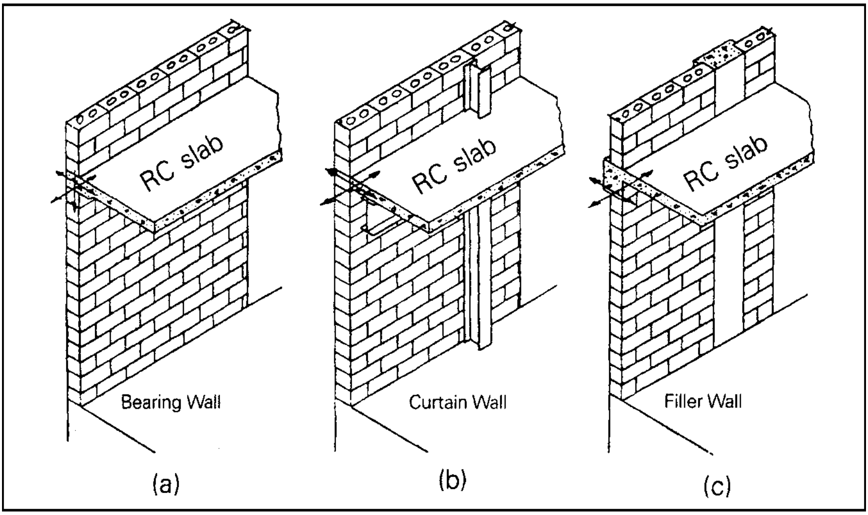


FIGURE 6.1 Load-bearing and nonload-bearing walls.

A brief discussion of Seismic Design Categories is presented in Chap. 7. Both MSJC-08 (Section 7.6) and 2009 IBC (Section 2103.8) [6.2] require use of Type S or N mortar for glass unit masonry.

3. Masonry walls can be described in terms of the number of wythes used to build them. A *wythe* is simply a continuous vertical section of masonry, one unit in thickness. A wall may be a single- (Fig. 6.3) or multiple-wythe (Fig. 6.4), depending on the number of wythes used (as seen in plan). In some older unreinforced multistory brick buildings, solid brick walls have been found to have as many as four wythes. Single-wythe walls consist of hollow masonry units laid with face-shell mortar bedding. Hollow units may

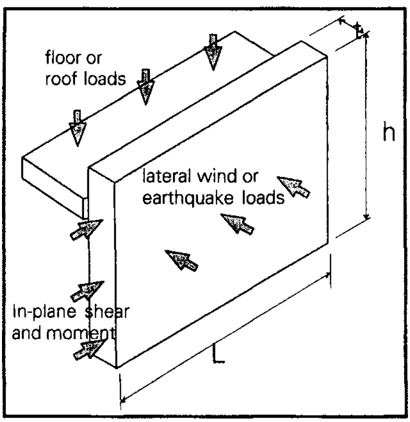


FIGURE 6.2 Typical gravity and lateral loads acting on a wall.

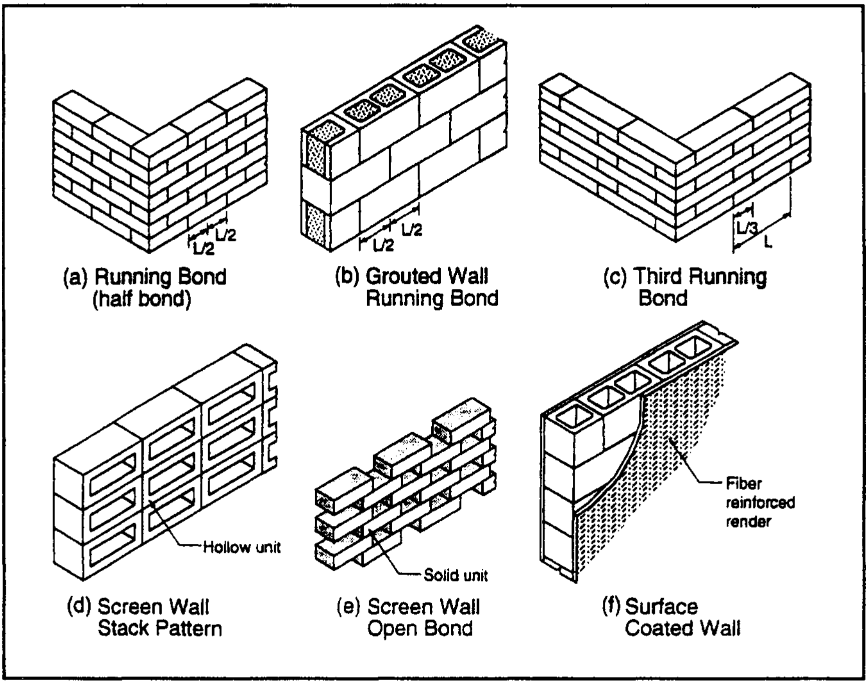


FIGURE 6.3 Single-wythe walls with various bond patterns. Walls are constructed from various kinds of brick or concrete masonry units. (*Courtesy: NCMA.*)

be two- or three-cored, with two-cored units being preferred. The units are laid so that vertical cores are aligned to provide vertical unobstructed space for placement of reinforcement and grout.

A single-wythe wall can be used both as a load-bearing or a nonload-bearing wall. Because of the limited resistance to moisture penetration and thermal transmission,

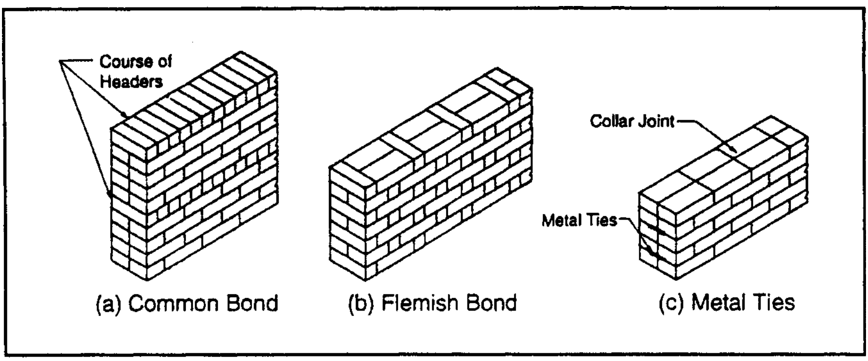


FIGURE 6.4 Two-wythe brick walls. (*Courtesy: BIA.*)

special precautions may be required when a single-wythe wall is used on the exterior of a building. The thickness t of a single-wythe wall is equal to the width of the masonry unit used, which can be solid, cored, or hollow.

4. Walls can be solid as constructed from solid bricks of concrete or clay or from hollow masonry units, which can be single- or multiple-wythe. When constructed from hollow masonry units, masonry walls can be partially or fully grouted. Before the development of modern masonry, solid wall consisted of two wythes of brickwork or stonework, with the space in-between filled with rubble or concrete. In modern masonry construction practice, solid walls are constructed with two or more closely spaced wythes. The intervening space between the wythes, generally, $^3/_8$ to $^3/_4$ in. wide, is filled with mortar or grout (Fig. 6.5) and referred to as the *collar joint*. In some cases, the intervening space may be 2 to 3 in. wide, referred to as the *cavity*, which is later filled with grout forming a *grouted wall*. Such walls are called the *cavity walls*. It important to construct two-wythe walls in such a manner that sufficient shear strength is developed at the interface, and the two separate wythes act integrally. This is accomplished by providing sufficient number of headers and metal ties that tie the two wythes together. These walls can be reinforced both horizontally and vertically. In cavity walls, it is common practice to provide horizontal and vertical reinforcement in the grouted cavity. In two-wythe walls with collar joints, the horizontal reinforcement is provided in mortar joints and vertical in the collar joints. Because of their greater thickness, the two-wythe walls offer greater strength and moisture-penetration resistance relative to a single-wythe wall.

   When walls are made from hollow masonry units, all or only a few selected cells may be grouted depending on the design requirements. When all the cells are grouted, a wall is rendered solid and referred to as a *solid* or *fully grouted* wall. Grouting only a few selected cells renders a wall *partially grouted* wall. Grouted cells might have reinforcing bars passing through them; such a wall is called a reinforced masonry wall. Reinforced masonry walls are usually single-wythe walls, unless two-wythe walls are used for some specific purposes.

5. Masonry walls can be built without any reinforcement, resulting in *unreinforced* masonry walls (not discussed in this book). Masonry walls built with reinforcement are called *reinforced* masonry walls. Both solid and single-wythe walls can be reinforced. Wall reinforcement can be horizontal or vertical. Horizontal reinforcement can be placed in bed joints, in collar joints, or in the grouted cavity between the wythes (Fig. 6.5). Vertical reinforcement can be provided in the grouted cores of hollow units or in the grouted pockets of walls constructed from solid units (Fig 6.6*a* and *b*). In single-wythe walls built

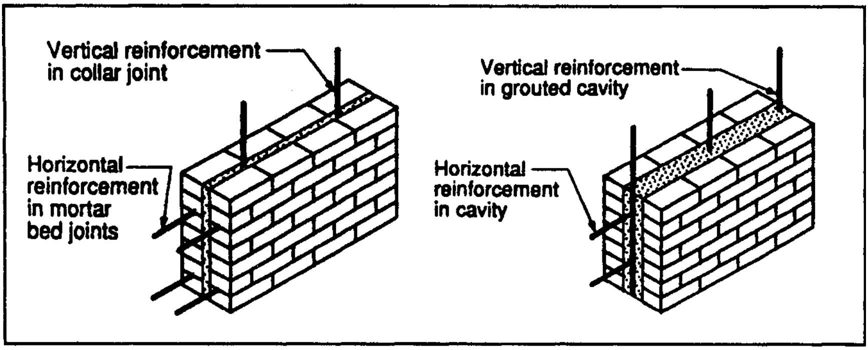


FIGURE 6.5 Cavity walls. (*Courtesy: BIA.*)

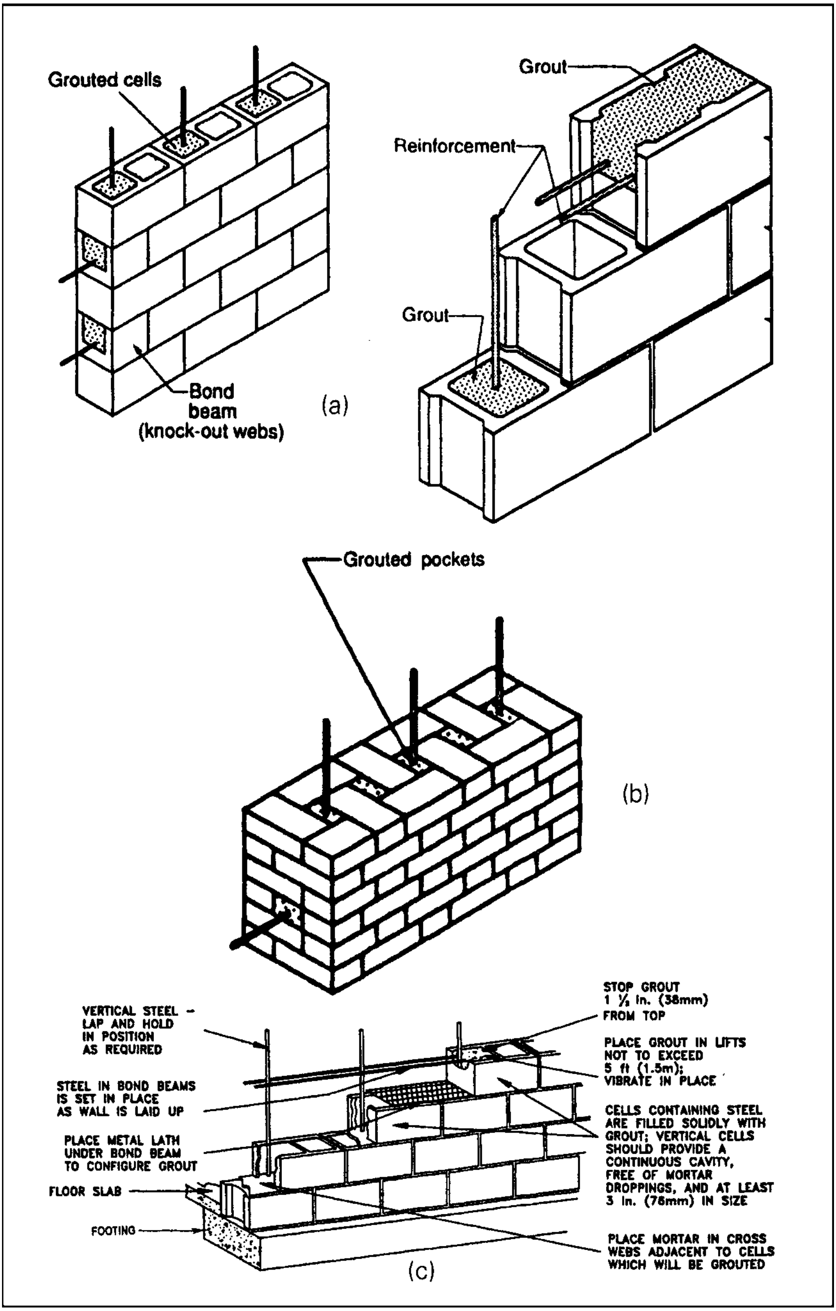
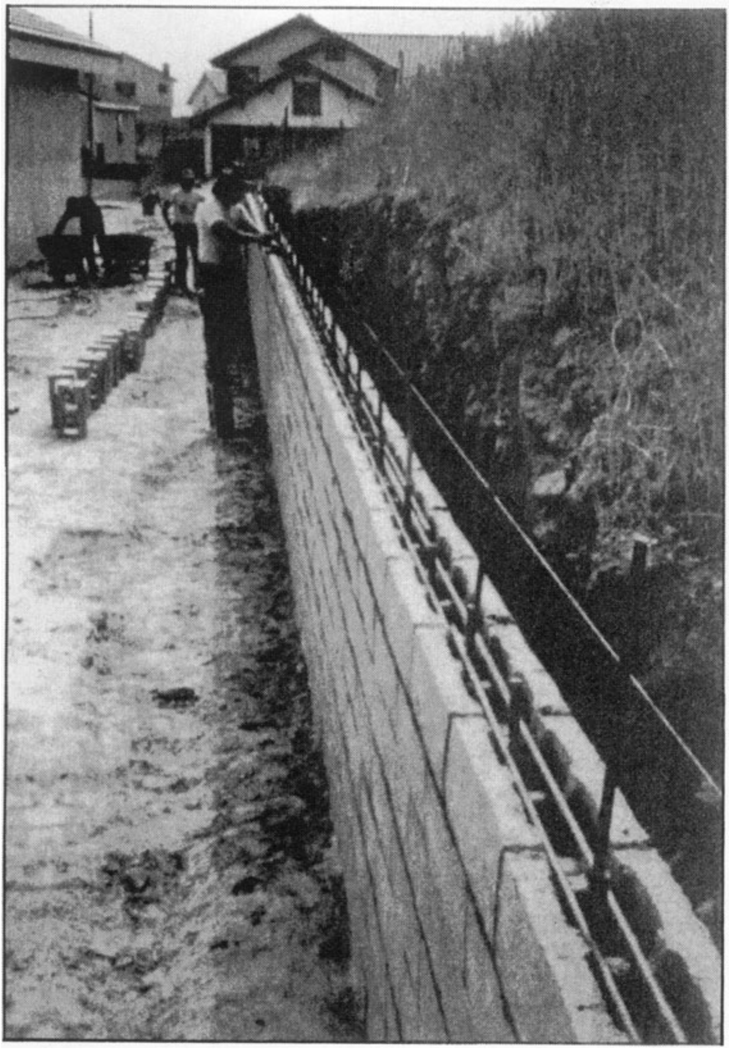


FIGURE 6.6 Reinforced masonry construction: (*a*) reinforcement in hollow masonry units, (*b*) reinforcement in pockets formed by masonry units, and (*c*) typical reinforced concrete masonry unit wall construction. (*Courtesy: NCMA*.)

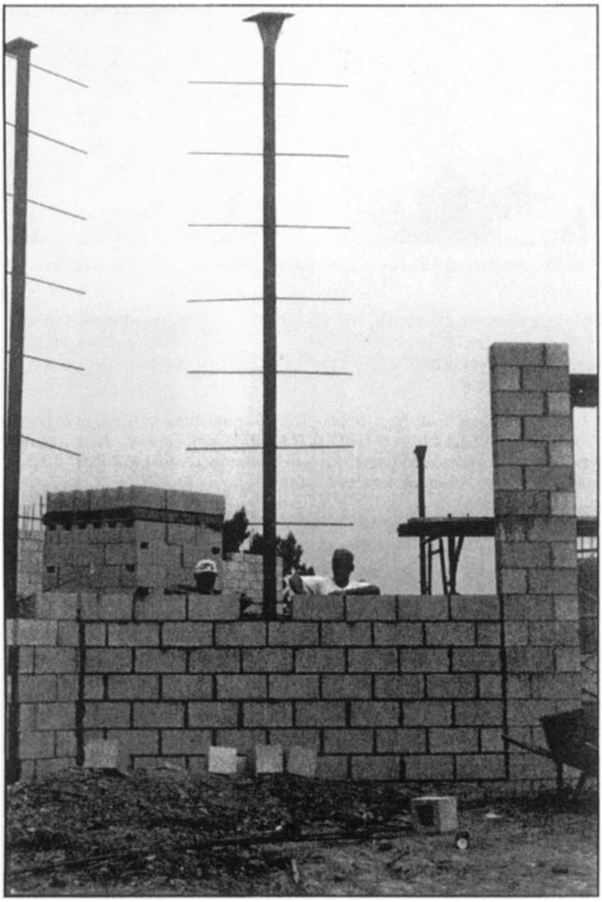
from hollow masonry units, the units are positioned so that their cores align vertically to permit placement of reinforcing bars in the cells and flow of grout (Fig. 6.6*c*). Structural considerations may necessitate grouting of only those cells that contain reinforcement, forming what is called a *partially grouted* wall. Figure 6.7 shows ways of providing vertical and horizontal reinforcement in masonry walls. Partial grouting is advantageous in that it affords saving of grout and, consequently, a lighter structure. A lighter structure offers the advantage of causing smaller inertia (or lateral seismic) forces, resulting in an economical design.



(a)

FIGURE 6.7 Methods of providing reinforcement in hollow concrete block walls. (*Courtesy: Author.*)

(b)



(c)

FIGURE 6.7 (*Continued*)

The vertical reinforcement in the walls is usually provided to develop the required flexural strength in the vertically spanning walls. However, this vertical reinforcement is not counted upon to carry any load in compression. This is because the vertical reinforcing bars are not easily laterally restrained by ties and are therefore vulnerable to buckling and consequent loss of their compression strength. Horizontal reinforcement is provided either to resist flexure in horizontally spanning walls, and as shear reinforcement in shear walls.

Both horizontal and vertical wall reinforcement are provided based on structural requirements. Walls with reinforcement spaced at large intervals are referred to as *lightly* or *partially reinforced* walls. In this type of construction, the walls are considered to consist of strips of reinforced masonry with un-reinforced masonry spanning between them. The reinforced masonry strips may be horizontal or vertical depending on the position of reinforcing bars. Some common forms of walls of this type are shown in Fig. 6.8. In Figure 6.8*a*, vertical reinforcement is grouted in some of its vertically aligned cells. The reinforcement, along with the grouted masonry units, acts as a vertical beam. The un-reinforced masonry spans horizontally between these strips to resist vertical loads. Similarly, in Fig. 6.8*b*, the bond beams provided at the top and bottom of the wall act as horizontal beams, and the un-reinforced masonry spans between these beams to resist lateral loads. Figure 6.8*c*, shows a system with both horizontally and vertically reinforced strips. In this case, the un-reinforced masonry is supported on all the four sides, resisting lateral loads by two-way flexural action similar to a plate supported on all four sides.

6. Multiple-wythe walls can be designed as *noncomposite* or *composite* walls. MSJC *Code* defines composite masonry as a multicomponent masonry member acting with composite action. Wythes in noncomposite walls act independently of each other in resisting loads. The term *composite action* implies an internal mechanism in a member so designed that transfer of stresses occurs between the components, so that the combined components act together as an integral unit in resisting loads. A masonry wall in which two or more wythes are bonded together so that it acts as a structural unit is often referred to as *bonded wall*. A composite wall is simply a multiple-wythe wall in which at least one of the wythes is different from the other wythe or wythes with respect to type or grade of masonry unit or mortar (i.e., strength or composition characteristics).

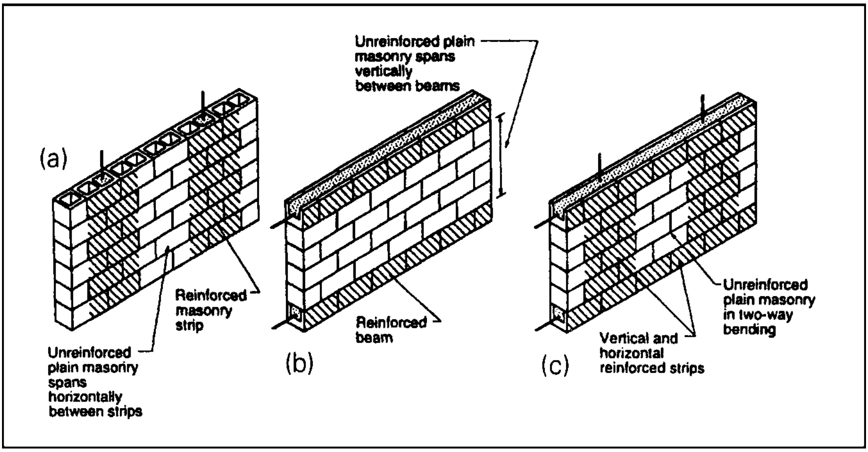


FIGURE 6.8 Vertical and horizontal reinforcing strips in the masonry walls. (*Courtesy: NCMA.*)

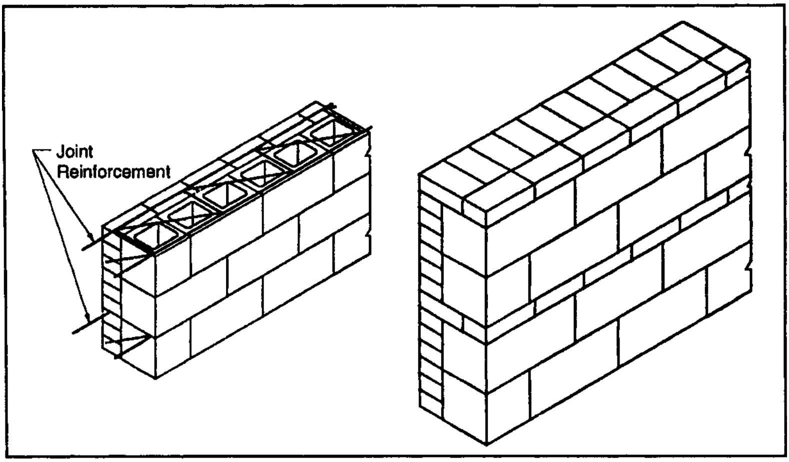


FIGURE 6.9 Composite walls. (*Courtesy: NCMA.*)

Composite masonry walls may consist of brick-to-brick, block-to-block, or brick-to-block wythes, with the collar joint filled with mortar or grout, and the wythes connected by metal ties. The width of collar joint may vary from $^3/_8$ to 4 in. Joints may be reinforced horizontally or vertically, or the reinforcement may be placed in either the brick or the block wythe. Alternatively, the two wythes may be connected by headers (Fig. 6.9).

Composite walls are particularly advantageous for resisting high loads, both in-plane or out-of plane. They may be preferred for architectural or economic reasons. Typically, composite walls are used as exterior walls in which the exterior-facing wythe consists of concrete brick or split block, and backing wythe is composed of hollow or solid concrete block. When built as load-bearing walls, axial loads from the roof or the floor slabs are applied on the inner wythes. The two wythes of the composite wall will act integrally, if adequate shear-transfer capacity is present at the interface between the two.

Composite masonry walls may be analyzed based on the elastic transformed area of the net section. Careful consideration should be given to the question of allowable stresses when two dissimilar materials are used to develop composite action in a masonry wall. The maximum stresses in composite walls or other structural members composed of different kinds or grades of units or mortars should not exceed the allowable stresses for the weakest of the combination of units and mortars of which the member is composed. Alternatively, the maximum compressive stress permitted in composite walls or other structural members may be based on prism tests.

Design of composite walls must comply with the pertinent provisions of the applicable code, such as the MSJC *Code* [6.1], which specify design requirements for masonry walls consisting of more than one wythe. According to MSJC *Code* Section 2.1.5.2 [6.1], some of the important requirements are as follows:

1. The Multiple-wythe walls designed for composite action shall have collar joints either:
   (a) Crossed by connecting headers, or
   (b) Filled with mortar or grout and connected by wall ties.

2. Shear stresses developed in the planes of interfaces between wythes and collar joints or thin headers shall not exceed the following:
   (a) Mortared collar joints, 5 psi
   (b) Grouted collar joints, 10 psi
   (c) Headers, $\sqrt{\text{unit compressive strength of header}}$, psi (over net area of header)

3. Headers of wythes bonded by headers shall meet the following requirements:
   (a) Headers shall be uniformly distributed and the sum of their cross-sectional areas shall be at least 4 percent of the wall surface area.
   (b) Headers connecting adjacent wythes shall be embedded a minimum of 3 in. in each wythe.

4. Spacing between metal ties shall be 36 in. horizontally and 24 in. vertically. Cross wires of joint reinforcement are permitted to be used as ties.

Research investigations have confirmed that the strength of the composite masonry wall systems depend not only on the strengths of the individual wythes used, but also on the manner in which these wythes interact. For monolithic action to be developed in composite masonry walls, shear forces acting between various wythes of the wall must be effectively resisted. This requires that the collar joint between various wythes must be completely filled with mortar or grout.

Composite walls are not discussed in this book. Readers are referred to *Commentary* [6.3] on the MSJC *Code* for a brief discussion on this topic. A discussion on this topic and examples can be found in Ref. 6.4.

When a multiple-wythe wall is designed as noncomposite, each wythe is designed to resist individually the effects of the load imposed on it. Loads acting parallel to the plane of a wall are to be carried only by the wythe on which it is applied. Transfer of stresses or forces between the two wythes is ignored.

5. Cavity wall was introduced earlier in Item 4. A cavity wall is a two-wythe wall, which derives its name from the *cavity* or *continuous air space* that separates the two wythes. The wythes can be of similar or dissimilar materials such as CMU and bricks (Fig. 6.5). Each wythe shares load proportional to its stiffness. Transfer of stresses from such loads between wythes is ignored. The two wythes can be joined together either through the use of horizontal joint reinforcement, or other acceptable metal system. According to MSJC *Code* Section 2.1.5.3.1(e) [6.1], the specified distances between wythes should be limited to $4\frac{1}{2}$ in. If the cavity width exceeds $4\frac{1}{2}$ in., the ties must be designed to carry the loads imposed on them based on a rational analysis taking into account buckling tension, pullout, and load distribution.

6. The many advantages attributed to the air space between the two wythes include energy conservation, increased fire protection, and noise reduction. Cavity walls provide significant resistance to moisture penetration or wind-driven rain. The cavity separating the two wythes essentially acts as a passage for any water penetrating the outer wythe, diverting it to flashing and weep holes at the bottom of the cavity to the exterior of the building. A discussion on these non-structural properties of cavity walls can be found in Ref. 6.5.

The design of a cavity wall is unlike that of other masonry walls. The cavity between the two wythes, which may vary from a minimum of 2 in. to a maximum of 3 to $4\frac{1}{2}$ in., presents a unique situation in design. Cavity wall design varies with different building codes. The effective thickness of the wall never includes the width of the cavity and varies with codes depending on the manner of loading. A discussion on cavity walls can be found in Refs. 6.6 and 6.7.

Curtain and panel (or filler) walls (Fig. 6.1) are nonload-bearing walls which have been used as building systems for decades. They are used to enclose structural frames of steel or concrete. Curtain and panel walls differ slightly in that a panel wall is wholly supported at each story, but a curtain wall is not. However, both must be capable of resisting lateral loads (due to wind or earthquakes) and transmitting these forces to adjacent structural members. They are designed to meet the requirements of exterior walls, which include providing a durable, attractive, and economical finish; control of heat flow; water penetration; noise; and fire.

Prevention of damage due to dimensional changes in walls and adjacent structural frames is one of the most important considerations in design of a building system. Differential movement occurs as a result of stresses caused by many factors such as temperature variations, moisture, loading, and the like. Temperatures vary throughout buildings depending on locations and time. Masonry walls may be exposed to wide variations in temperatures. An exterior wall may have an outside surface temperature as high as 150°F when exposed to hot summer sun, while the temperature on the interior wall may be about 70°F. Also, during winter, the temperature on an exterior wall me be as low as 20°F. Variations in moisture content also cause masonry materials to expand and contract. Possible damage to curtain and wall panels can be avoided by anticipating stresses that may be caused by these factors.

Screen walls (Fig. 6.10) are constructed from specially manufactured screen units. They are used in buildings for many purposes. They provide privacy with observation, interior light with shade and solar heat reduction, and airy comfort with wind control. Curtain walls, fence walls, sun screens, decorative veneers, and room dividers are some of the many applications of screen walls. Screen units are manufactured in a wide variety of shapes and sizes so that the screen walls can be advantageously used to enhance aesthetic qualities of a building. Screen walls are designed as nonload-bearing wall elements. As such, they are designed to support their own dead weight, and they should exhibit stability and safety.

Design of the screen walls depends on several factors such as function, location (exterior or interior), aesthetic requirements, and local building codes. They are extensively used for the following types of construction: (1) interior partitions, (2) free-standing walls supported on their foundations, (3) veneers, and (4) enclosed panels in masonry walls. As partitions, they are designed as nonload-bearing wall panels with primary

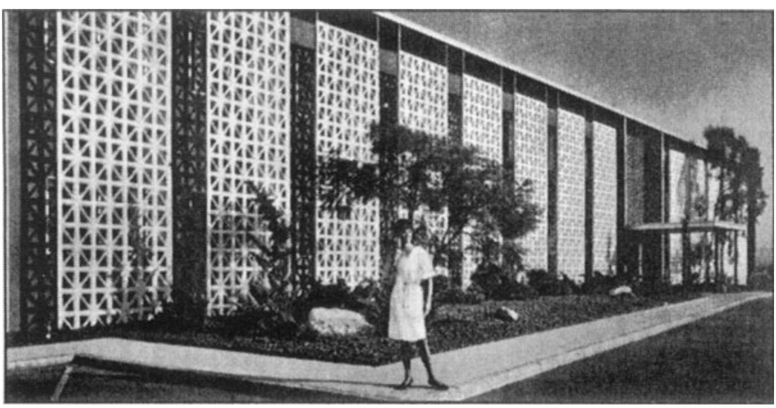


FIGURE 6.10 Screen walls [6.8].

consideration given to adequate anchorage at panel ends and/or top edge, depending on the type of lateral support furnished. A discussion on masonry screen walls can be found in Refs. 6.8 and 6.9.

7. Veneer walls (Fig. 6.11*a*) are characterized by their *nonstructural* cladding which enhances aesthetic qualities of a wall. A veneer is defined as a nonstructural facing of brick, concrete, stone, tile, metal, plastic, or any other material attached to a backing (or a backup wall) for the purpose of ornamentation, protection, or insulation. Veneer

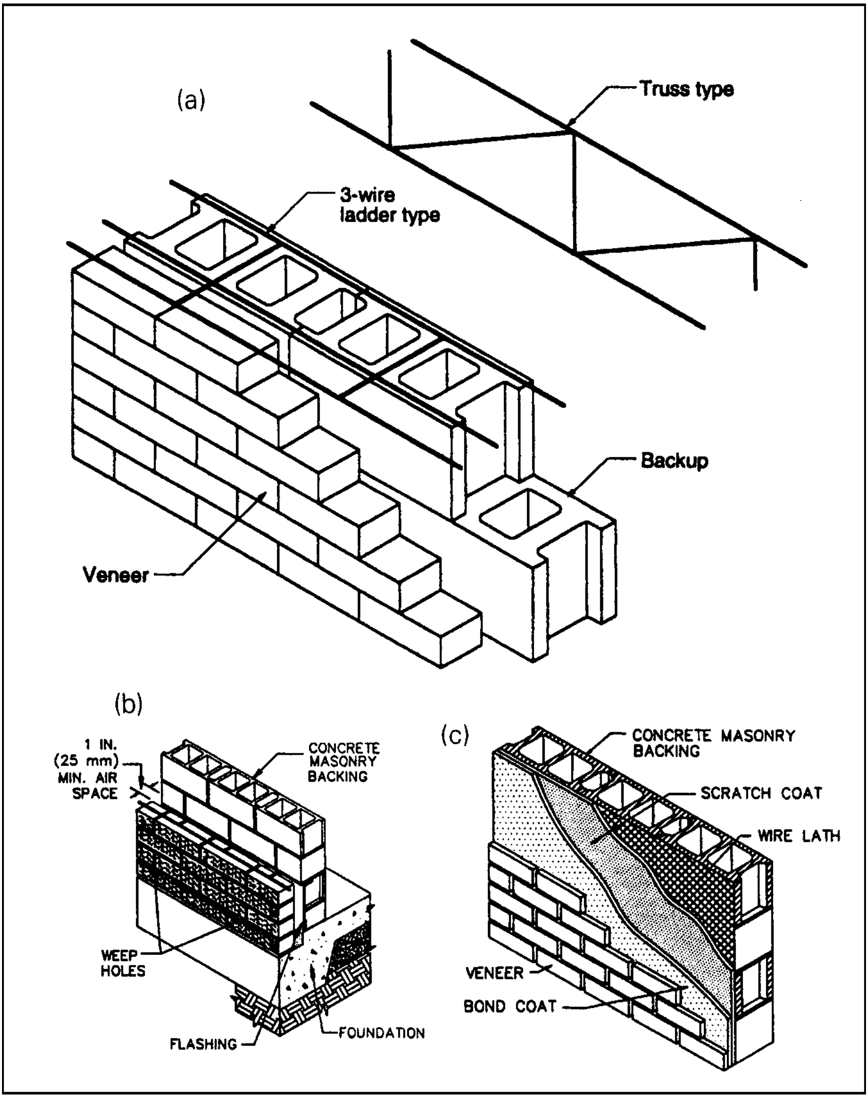


FIGURE 6.11 Types of veneer walls [6.10].

may be either anchored or adhered to what are referred to as *backup walls*. *Anchored veneer* (Fig. 6.11*b*) is secured to and supported by mechanical fasteners attached to an approved backing. A veneer secured and supported through adhesion to an approved bonding material is referred to as *adhered veneer* (Fig. 6.11*c*). A veneer may be *exterior* if applied to weather-exposed surfaces, or *interior* if applied to surfaces other than weather-exposed surfaces.

Veneers provide the exterior wall finish and transfer out-of-plane load directly to the backing. However, they are themselves not considered as contributing to the load-carrying capacity of the wall. For design purposes, a veneer is assumed to support no load other than its own weight plus the vertical load of the veneer above. In addition to structural requirements, differential movements between the veneer and its supports must be accommodated in design.

Examples of veneer applications can be found in residential building construction in which it is common practice to use masonry veneer as a nonload-bearing siding or facing material over a wood frame (wood frame carries the load). Designed to carry its own load, veneer is anchored to, but not bonded to, the backing material which is load-bearing such as masonry, concrete, wood studs, or metal studs. In commercial work, the exterior wythe may consist of the architectural units that may be bonded with joint reinforcement to a concrete masonry wall that is load-bearing. Masonry veneers may be thin or thick and are anchored to the backup walls through metal ties usually with an air space (1 to 2 in. minimum) to prevent water penetration. Where used, design of veneers forms a very important and specialized part of an overall project. Masonry veneers may be designed using engineered design methods to proportion the stiffness properties of the veneer and to provide the backing to limit stresses in veneer and achieve compatibility. Alternatively, prescriptive code requirements that have been developed based on judgment and successful past performance may be used. Design and detailing requirements for veneers are specified in MSJC-08 *Code* Chap. 6 [6.1]. A discussion on veneers may be found in several references [6.10–6.13].

8. Walls may be designed based on principles of mechanics and structural theories; such walls are referred to as *engineered* walls. Design of such walls is based on applicable building codes and standards. These walls are *designed* and *proportioned* for specific structural action under given loads. Historically, however, masonry structures had continued to be built long before the development of modern structural theories. These structures were built based on the rule of thumb and past experience in lieu of analytical methods. These structures are referred to as *empirically designed*. Empirical design provisions preceded the development of engineered masonry, and their evolution can be traced back several centuries. Most of these structures were rather massive and overdesigned by modern standards. Because of the satisfactory performance of those structures, empirical design is permitted by the modern codes.

Empirical design of masonry is permitted under certain conditions and limitations as specified in codes (MSJC-08 *Code* Chap. 5 [6.1], 2006 IBC Section 2109 [6.2]). Generally speaking, empirical design is permitted for buildings of limited height and low seismic exposure. Members not working integrally with the structure, such as partition or panel walls, or any member not absorbing or transmitting forces resulting from the behavior of structure under loads, may be designed empirically. Using empirical design provisions of the code [6.1], vertical and lateral loads resistance is governed by prescriptive criteria which include wall height-to-thickness ratios, shear wall length and spacing, minimum wall thickness, maximum building height, and other criteria which have proven to be conservative through years of experience and building performance.

9. For analytical purposes, walls are often described with reference to their support conditions and behavior. Walls may be of *cantilever type* (Fig. 6.12), such as retaining walls,

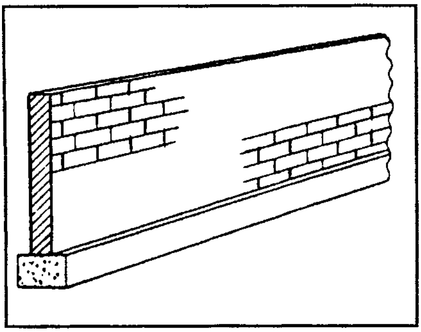


FIGURE 6.12 A cantilever wall.

fence walls, and highway noise barrier walls. Walls may be simply supported at both ends and may span vertically or horizontally. Vertically spanning walls may be supported between horizontal supports at top and bottom (Fig. 6.13). Horizontally spanning walls may span between vertical supports such as cross-walls or pilasters, in which case they may also have fixity at their supports (Fig. 6.14*a*). Figure 6.14*b* shows a wall with pilasters which carry gravity loads from truss joists supporting a roof or a floor. Pilasters may serve as columns, independently or as integral parts of a wall. In such type of construction, it is assumed that the pilasters provide lateral restraint to the wall, so that under lateral loads, the wall acts as a (horizontally supported) flexural member. Pilasters are discussed in Section 6.7.

In some cases, *wall panels*, which span horizontally between end supports (like beam supports) instead of functioning as load-bearing walls with continuous line support along the bottom, are referred to as "wall beams." These wall panels do act as beams; accordingly, they are designed according to the provisions applicable to flexural members and do not have to comply with the *h/t* requirements that are applicable to load-bearing walls. A discussion on concrete masonry wall beams can be found in several references [6.14, 6.15].

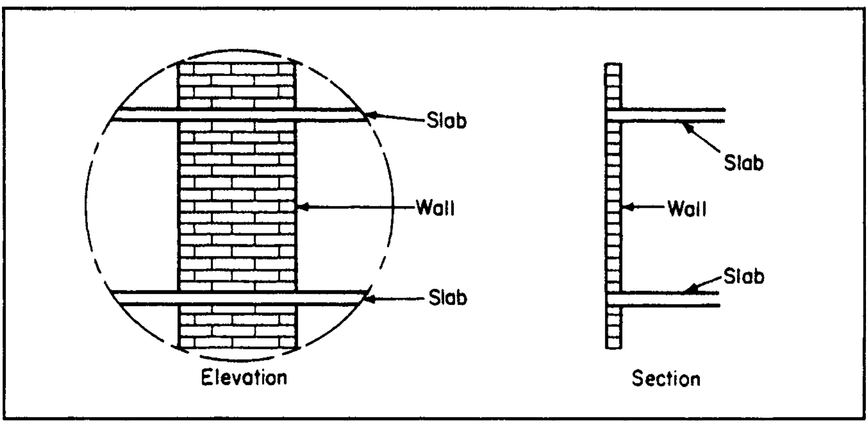


FIGURE 6.13 Vertically spanning walls.

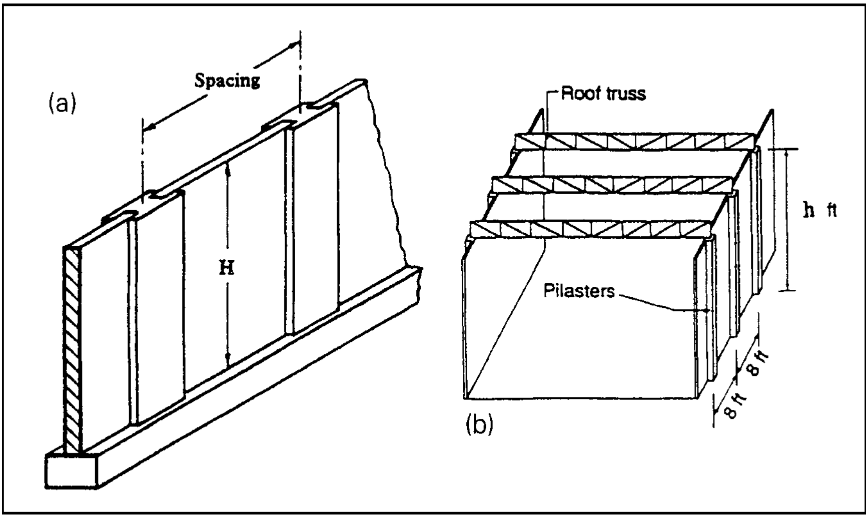


FIGURE 6.14 Horizontally spanning walls. (*Adapted from Ref. [6.6.]*)

Support conditions provided at the ends of vertically spanning walls define their *effective lengths*. Both ends may be simply supported or fixed, or one end simply supported and the other end fixed. Figure 6.15 shows the effective lengths of vertically spanning walls as defined by different support conditions.

In practically all buildings, exterior walls usually project above the roof level, usually 2 to 3 ft; these projections are called *parapets* and are provided for several reasons. 2009 IBC Section 705.11 [6.2] requires that parapets, with some exceptions, be provided on all exterior walls of buildings, and their heights be not less than 30 in. Parapets improve the aesthetics of a building and also give protection to people working or gathered on the roof. If built under empirical design provisions of the code, parapets are required to be at least 8-in. thick, not thinner than the wall below, and their height limited to three times their thickness [6.1 (Sections 5.6.2.6 and 5.6.4)], [6.2 (Sections 2109.5.2.6 and 2109.5.4.1)].

6.3 BOND PATTERNS IN MASONRY WALLS

Bond patterns refer to the *appearance of orientation of joints* in various courses of a wall as viewed in elevation. They can be classified based on patterns used in placing masonry units with respect to units in the adjacent courses. These joint orientations or patterns are called *bond patterns*. Various types of bond patterns used for masonry walls were discussed in Chap. 2 (Section 2.7, Figs. 2.34, 2.36-2.40). Variations in bond or joint patterns of masonry walls can be accomplished using standard units as well as sculptured-faced concrete masonry units and other architectural units. As a result, a wide variety of interesting and aesthetically pleasant wall appearances can be obtained. Bond patterns also have structural significance in that they affect the behavior and strength of walls. A general discussion on concrete masonry bond patterns is presented in Ref. 6.18, which is briefly summarized here.

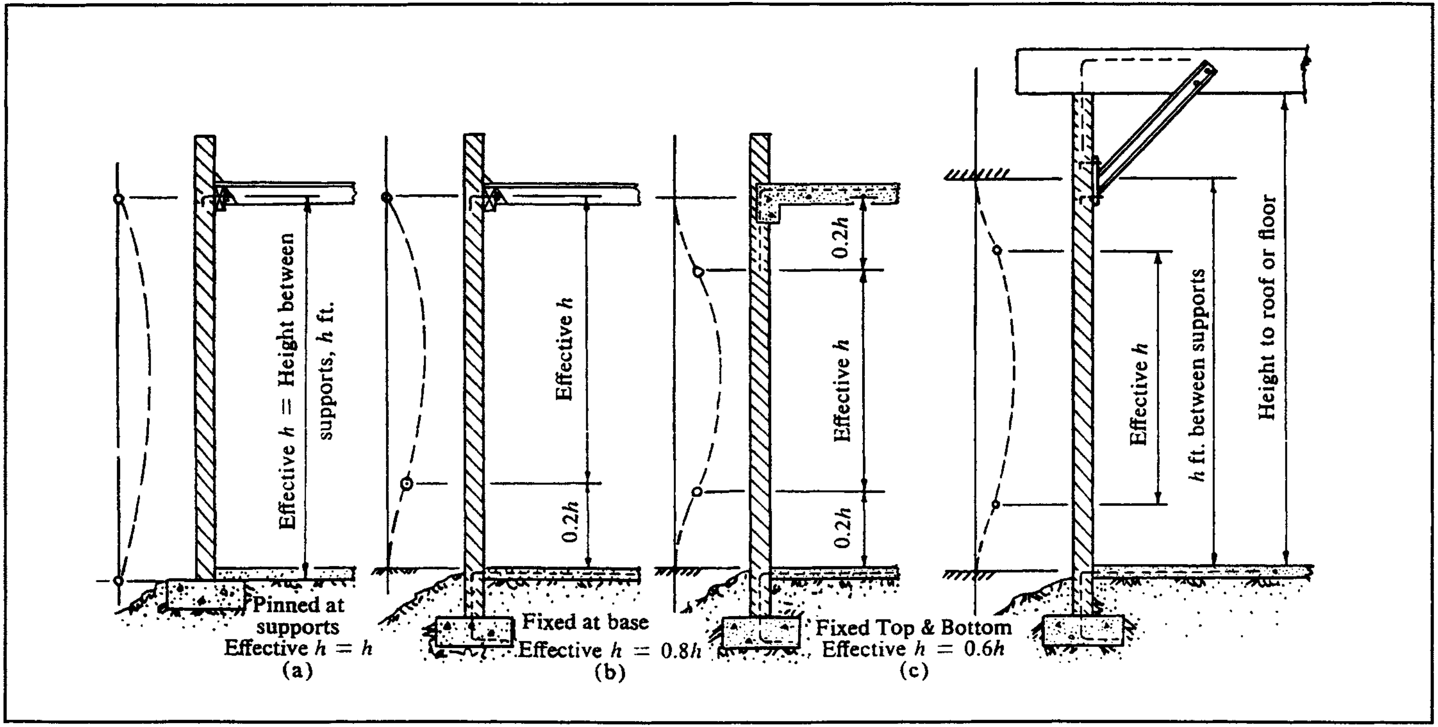


FIGURE 6.15 Effective lengths of vertically spanning walls with various end conditions. (*Adapted from Ref.* [*6.16*].)

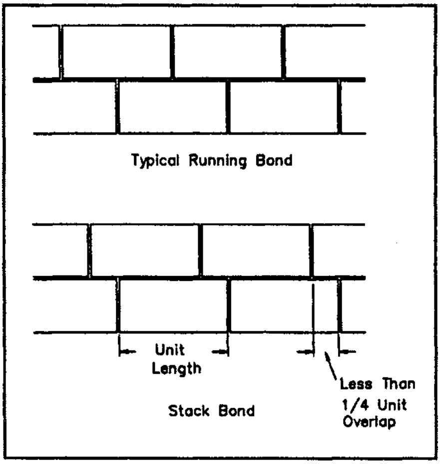


FIGURE 6.16 Running and stack bond patterns.

Two of the most common bond patterns used in masonry construction, in order of preference, are *running bond* and *stack bond* (Fig. 6.16). Design parameters, such as code-specified allowable stresses, strength formulas, lateral support and minimum thickness requirements, and maximum permitted reinforcement spacing for masonry are based primarily on structural testing and research on wall panels laid in running bond. Therefore, when a different bond pattern is used, its influence on the compressive and flexural strength of a wall should be properly taken into account. Specifically, the maximum permitted reinforcement spacing for walls in stack bond may be smaller than for walls in running bond. This aspect of masonry construction was discussed earlier in Chap. 2.

In running bond, the vertical joints are staggered in such a way that head joints in successive courses are horizontally offset at least one-quarter the unit length. In *half* running bond, each head joint in a course is positioned over the center of the masonry unit below, resulting in an attractive symmetrical pattern on the wall face in which vertical joints (i.e., head joints) in the alternate courses line up with each other. Since typical masonry units have their width equal to half their length, a running-bond pattern permits a corner to be constructed without cutting the units (Fig. 6.17*a*). When the width of a masonry unit is one-third of its length, running bond pattern can be obtained with one-third overlap of units in successive courses, so that the corners can be constructed without cutting the units (Fig. 6.17*b*). In this case also, the head joints in the alternate courses will line up resulting in a symmetrical pattern in the face of the wall. Figure 6.18 shows corner details of walls built from various size concrete masonry units laid in *half running* bond.

The MSJC *Code* [6.1] defines stack bond as a *bond pattern other than running bond*; usually the placement of units is such that the head joints in the successive courses are vertically aligned. This definition is in conformity with 2009 IBC [6.2], but differs slightly from the past practices which defined *stack bond*, a bond pattern in which less than 75 percent of the units in any transverse vertical plane lapped the ends of the units below either (a) a distance less than one-half the height of the unit below, or (b) less than one-fourth the length of the unit.

The fundamental difference in walls built in running bond and stack bond is in the pattern of vertical joints: staggered (discontinuous) in the former and (almost) continuous in the latter. Compressive strength is similar for stack and running bond construction. A wall

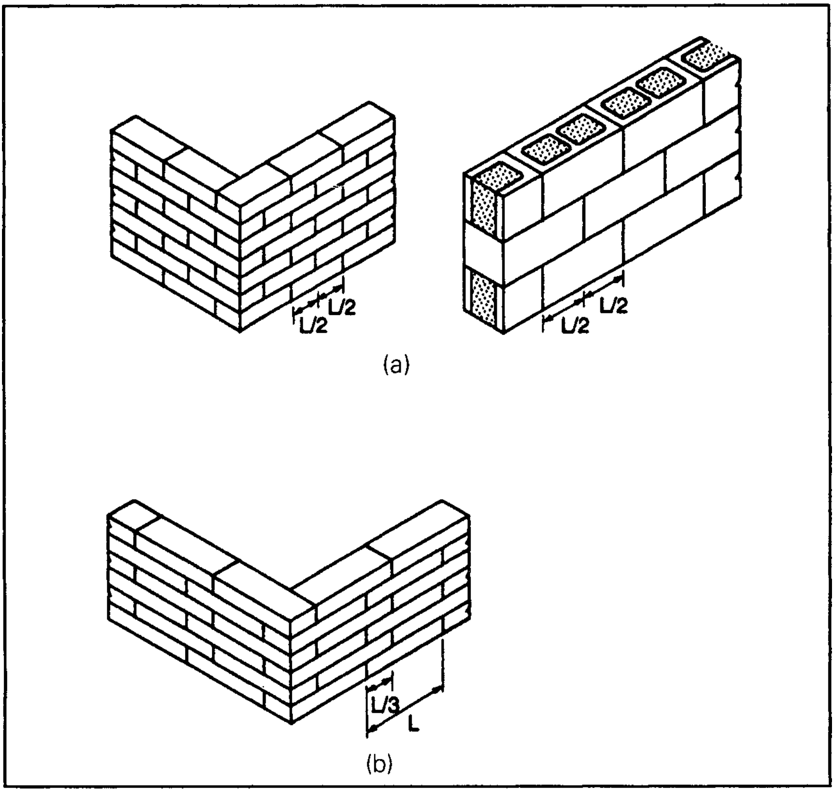


FIGURE 6.17 Corner details for walls laid in (*a*) *half* and (*b*) *one-third* running bond. (*Courtesy: NCMA.*)

with continuous vertical joints (stack bond) acts more like a series of adjacent vertical piers rather than a cohesive unit. In stack bond masonry, heavy concentrated loads will be carried down to the support by the particular tier or "column" of masonry directly under the load, with little distribution to the adjacent masonry. Accordingly, MSJC-08 Section 1.9.7.2 prohibits distribution of concentrated loads across head joints. Stability is not jeopardized if allowable stresses are not exceeded. Load distribution in walls built in other than running bond can be improved by providing reinforced bond beams. By contrast, the interlocking bond in walls with running bond has a merit of dispersing loads evenly (Fig. 6.19).

When stack bond is used in masonry walls that are subjected to loads, consideration must be given to requirements and restrictions consistent with local codes, experience, and engineering practice. MSJC-08 Section 1.11 [6.1] specifies the following general requirements for masonry other than in running bond:

1. The minimum area of horizontal reinforcement shall be 0.00028 multiplied by the gross vertical cross-sectional area of the wall using specified dimensions. (This is a prescriptive reinforcement requirement to provide continuity across the head joints.)
2. The maximum spacing of horizontal reinforcement shall not exceed 48 inches on center in horizontal mortar joints or in bond beams,

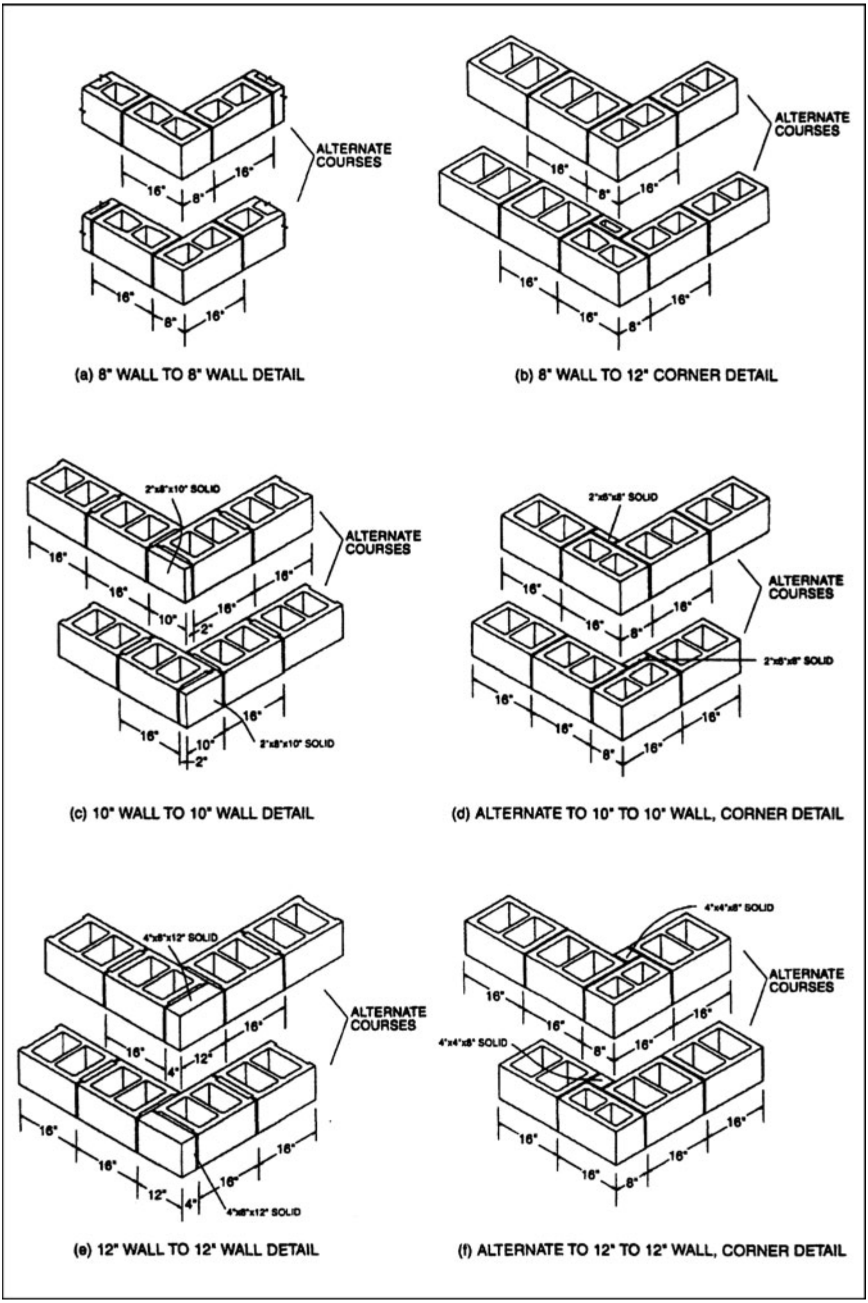


FIGURE 6.18 Standard corner layout details for walls in *half* running bond. (*Courtesy: NCMA.*)

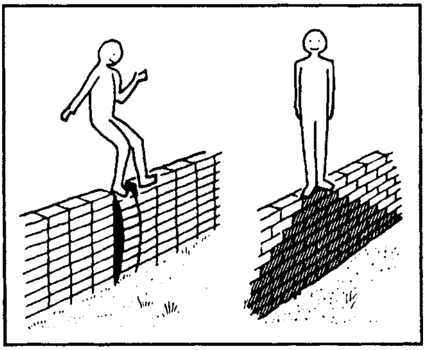


FIGURE 6.19 Stability and load dispersion in stack bond and running bond. Running bond provides good interlocking properties, much better dispersion, and greater stability than running bond.

The reinforcement requirements for masonry other than in running bond are, as a rule, more stringent than those for masonry in running bond. MSJC-08 Section 1.17.3.2.6 specifies the following requirements for masonry in stack bond when used for special reinforced masonry shear walls (discussed in Chap. 7):

1. The cross-sectional area of vertical reinforcement shall be not less than 0.007 times the gross cross-sectional area of the wall using specified dimensions.
2. The minimum area of horizontal reinforcement shall be not less than 0.0015 times the gross cross-sectional area of the wall using specified dimensions.
3. Masonry shall be solid grouted and shall be constructed of hollow open-end units or two wythes of solid units.

MSJC-08 Section 1.17.4.5.1[6.1] specifies the following minimum reinforcement requirements for nonparticipating masonry elements laid in other than running bond:

1. Stack bond masonry in Seismic Performance Category (SPC) E and F to be solidly grouted and constructed of hollow open-end units, fully grouted hollow units laid with full head units or solid units.
2. The maximum spacing of horizontal reinforcement is limited to 24 in. (48 in. for masonry in running bond) in this performance category.
3. For all masonry in stack bond, the minimum area of horizontal reinforcement is required to be at least 0.0015 times the gross sectional area of masonry.

It is instructive to know the relative strengths of walls laid in different bond patterns. To evaluate the strengths of walls in different bond patterns, a large number of concrete masonry panels were tested. Nine different bond patterns (Fig. 6.20) were used for structural tests [6.17]. Panels were composed of 8-in. hollow units with Type M or S mortar with face shell bedding. Panels for compressive tests were 4 ft wide by 8 ft high; those for flexural strength tests with wall spanning horizontally between supports were 8 ft wide by 4 ft high. For compressive tests, loading was applied at an eccentricity of one-sixth of the wall thickness. A summary of findings from these tests can be found in Ref. 6.18.

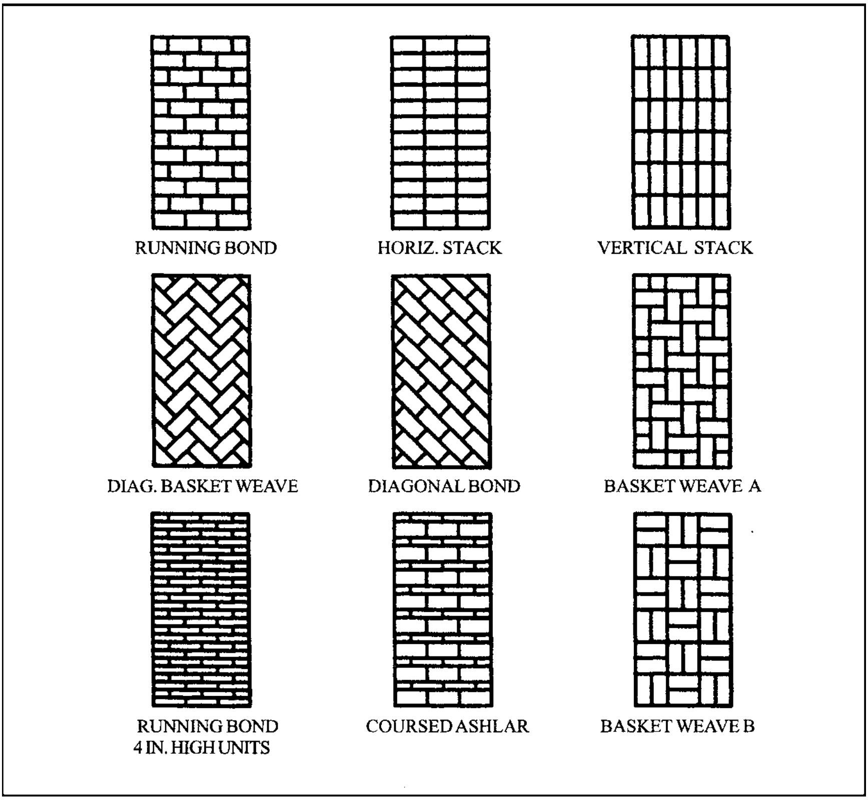


FIGURE 6.20 Concrete masonry bond patterns for structural tests [6.18].

Figure 6.21 shows the relative strengths of wall panels of different bond patterns using 8-in.-high units laid in running bond as the standard. It can be observed that there is no marked decrease in compressive strength of panels of different bond patterns. Units laid in vertical stack bond or diagonal bond generally exhibited 75 percent of the strength obtained from the running bond pattern. The reduction in strength of the vertical stack running bond can be attributed to the decrease in the net block area in compression. In vertical stack bond, the end walls and the web of the unit are so oriented with respect to the applied load that they do not contribute to the compressive strength of the panel. When blocks are laid in horizontal position, the end walls and the middle wall (i.e., the web) of the masonry units are parallel to the applied load, and they contribute to the strength of the wall panel.

With regard to the flexural strength of the walls spanning vertically, tests showed that failure from transverse loading occurred as bond failure between block and the mortar. Horizontal stack bond construction was 30 percent stronger in vertical span flexure, whereas the walls with units laid in diagonal position were 50 percent stronger. The latter higher strength is attributed to the fact that more mortar bond area is included in the "saw-tooth" line across the width of the panel. As for the flexural strength of the horizontally spanning wall panels, tests indicated that the relative strength of the stack bond walls is about 30 percent of the strength of the walls in running bond construction.

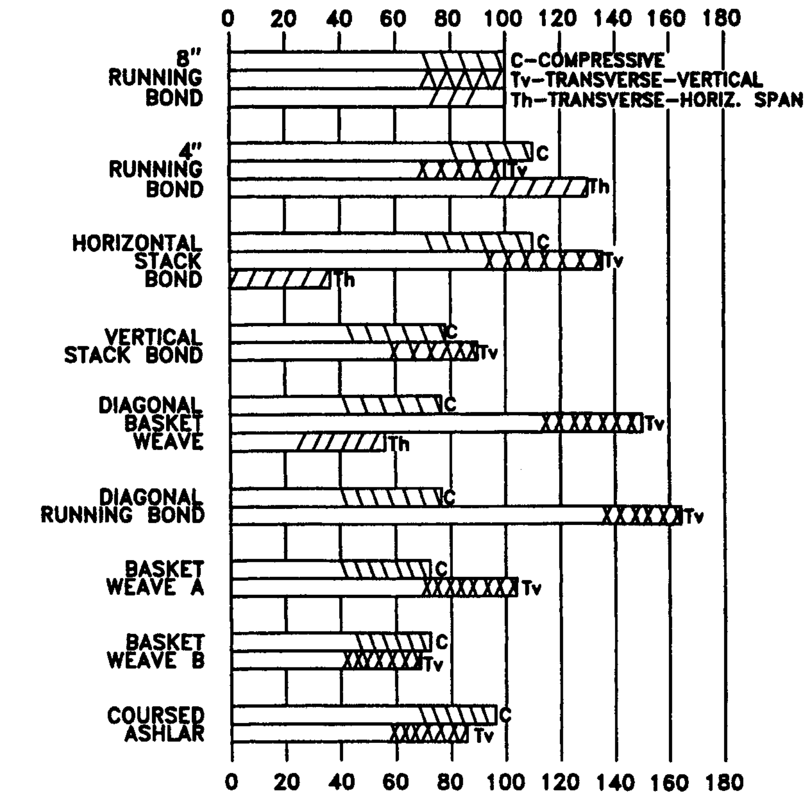


FIGURE 6.21 Relative strengths of walls laid in different bond patterns [6.18].

6.4 ANALYSIS OF WALLS UNDER GRAVITY AND TRANSVERSE LOADS

6.4.1 General Considerations

As pointed out earlier, from a structural design perspective, masonry walls (like all other wall) may be load-bearing or nonload-bearing. Generally speaking, all walls are subjected to combined loads—gravity and lateral. Load-bearing walls may be required to carry gravity loads from the roof or the floor, or both in addition to lateral loads. Nonload-bearing walls must also carry some gravity load (their own dead weight) as well as lateral loads (wind or seismic). In all cases, provisions must be made for those loads to be eventually transferred to foundation by providing a continuous load path.

The load path for loads supported by walls depends on their support conditions. Walls may span vertically between horizontal supports at top and bottom, for example, ground floor and the floor or the roof above. In that case, it is generally assumed that walls transfer their lateral load to the floor or the roof diaphragm above and the ground floor below. Analysis and design of vertically spanning walls is discussed in Sections 6.4 through 6.8. The walls may also span horizontally between lateral supports such as cross-walls, pilasters, buttresses, and the like. In such cases, walls transfer their lateral load to these vertical elements which eventually transfer those loads to the foundation. Discussion on horizontally spanning walls and pilasters is presented in Section 6.9.

Analysis of walls subjected to gravity loads alone or combined loads (i.e., gravity and lateral) is very similar to that for columns subjected to similar loading conditions as discussed in Chap. 5, albeit with some differences. Typically, a wall is analyzed by considering a 1-ft-wide strip between the supports; by contrast, a column is analyzed by considering its entire cross section.

For a wall supporting only gravity loads, this strip is assumed as a 1-ft-wide (i.e., $b = 12$ in.) vertical column, simply-supported at the bottom and the top; its thickness (t) being equal to the wall thickness. The gravity loads include superimposed dead and live loads (applied at the top of the wall) plus the dead weight of the wall itself, all loads being factored loads as required for strength design. The axial compressive strength of the wall should be checked at the bottom of the wall because the gravity loads would have maximum value at that point.

The lateral loads are the loads due to wind or earthquake; the more critical of the two should be considered for design. In this context, the following should be noted:

1. The wind loads calculated based on the wind load provisions of ASCE 7-05 [6.19] are ASD-level loads.
2. The seismic loads calculated based on seismic load provisions of ASCE 7-05 are LRFD (or strength-level loads).

Therefore, in order to compare wind and seismic loads and select the governing lateral load, the seismic loads should be multiplied by 0.7. The effect of lateral loads, which act transverse to the wall strip, should be checked against the maximum flexural strength of the vertical beam (12-in.-wide strip).

6.4.2 Dead Load of Masonry Walls

The dead weight of a wall can vary widely, depending on its width, unit weight of concrete and grout, and whether it is solid or partially grouted. A wall may need to be grouted solid in order to satisfy structural requirements or some fire safety requirements, or both. The dead weight of a partially grouted wall depends on the spacing of the grouted cells in addition to the above factors. The unit weight of hollow concrete blocks depends on whether they are made from lightweight, medium-weight or normal-weight concrete. Table A.18 provides information about the typical weight of 16-in.-long, 4-in.- and 8-in.-high units of various modular thicknesses. The grout may be of different unit weight as well: lightweight or normal weight.

Tables A.19 and A.20 provide information required to calculate dead load due to the dead weight of solid or partially grouted walls built from hollow concrete masonry blocks. The two tables are different in respect of the unit weight of grout (140 and 105 lb/ft^3, respectively). Each table lists dead load of three types of concrete masonry units by their unit weights: lightweight (103 lb/ft^3), medium weight (115 lb/ft^3), and normal weight (135 lb/ft^3). Unit weights of walls are given in terms of per square foot of the surface area measured in the plane of the wall. Dead load is tabulated for 6-, 8-, 10-, and 12-in.-wide hollow concrete block walls. Table A.21 provides dead weight of 4-, 6-, and 8-in.-wide clay masonry walls.

Dead load due to a wall of specific thickness and height can be computed simply by multiplying the tabulated value by the wall height. For example, consider an 8-in.-wide solid grouted wall built from normal weight, hollow concrete blocks and grout weight of 140 lb/ft^3. The dead load for such a wall is listed as 84 lb/ft^2 in Table A.19. For a wall 2 ft 8 in. (2.67 ft) high (e.g., a parapet), the dead load due to its own weight would be $2.67 \times 84 = 224$ lb per linear foot of the wall. As a second example, consider a 10-in.-wide wall made from medium-weight concrete blocks, and partially grouted (with a 140 lb/ft^3 grout) with vertical reinforcement at 32 in. on centers. The dead load for such a wall is listed as 68 lb/ft^2 in Table A.19. If this wall is 12 ft high, its dead weight would contribute a dead load of $68 \times 12 = 816$ lb per linear foot of the wall (i.e., a strip 1 ft wide and 12 ft high). If the grout unit weight for the same wall were 105 lb/ft^3 (instead of 140 lb/ft^3), Table A.20 lists the dead load for the same wall as 57 lb/ft^2. Therefore, the dead load of the wall would be $57 \times 12 = 684$ lb per linear ft of the wall, and so on.

6.5 OUT-OF-PLANE LOADS ON WALLS

6.5.1 Introduction

In addition to the gravity loads, walls are always subjected to out-of-plane (or lateral) loads. These lateral loads may be due to wind or earthquake, or earth pressure in the case of retaining walls and basement walls. As a result, walls are always subjected to combined loading condition, although it is common in some cases to ignore the influence of gravity loads, for example, in design of retaining walls.

Out-of-plane lateral loads are loads that act perpendicular to the plane of the wall and are discussed in this chapter. Lateral loads may also act in the plane of a wall; such forces are referred to as *in-plane forces*, which are discussed in Chap. 7. Out-of-plane loads may be induced by wind, earthquake, or earth retained against the wall. Lateral loads due to wind and earthquake acting normal to walls are briefly discussed in this section; walls subjected to lateral earth pressures are discussed in Chap. 8.

Which of the two loads—wind or seismic—would govern the design of a wall would depend on the location of the structure? Without a complete analysis for both types of lateral loads, it is difficult to predict the governing loads, although earthquake loads would probably govern in areas of high seismicity. Therefore, both types of loads should be determined and the wall should be analyzed/ designed for the governing loads.

Load calculations presented in this book are based on provisions contained in ASCE 7-05 [6.19] which are mirrored in 2009 IBC [6.2]. Users of earlier building codes are cautioned to note that the ASCE 7-05 contains sweeping changes in provisions for both wind and earthquake loads from their counterparts in earlier codes such as 97 UBC [6.16]. It is recommended that readers review these provisions thoroughly in order to follow discussion pertaining to loads on building structures, including provisions for wind and earthquake loads. It is incumbent upon readers to have a fairly good understanding of *main wind force–resisting systems* (MWFRS) and *seismic force–resisting systems* (SFRS, which are different from MWFRS) referred to in ASCE 7-05 Standard. A comprehensive discussion on structural loads and various *lateral force–resisting systems* (LFRS, which includes both MWFRS and LFRS) can be found in Ref. 6.20. A brief discussion on methods of determining out-of-plane wind and earthquake loads based on provisions in ASCE 7-05 follows.

6.5.2 Wind Loads

Provisions for determining design loads caused by wind are presented in Chap. 6 of ASCE 7-05 [6.19]. These provisions, although different in some respects, bear striking philosophical resemblance to their counterpart in ASCE 7-98 Standard [6.21] and discussed in Ref. 6.20. It is noted that design provisions for wind loads in 2009 IBC [6.2] direct readers to ASCE 7-05 Standard [6.19]; the requirements contained in the latter and used in this chapter are discussed as follows.

ASCE 7-05 Standard provides three methods for determining wind pressure on building structures: (1) simplified method, (2) analytical method, and (3) wind tunnel procedure. Readers are advised to review the philosophies underlying these methods discussed elsewhere [6.20]. In this book, the simplified method is used for determining wind pressure on building structures as needed.

The walls for which the out-of-plane loads due to wind are to be determined are referred to as *components and cladding* in ASCE 7-05 Standard, which essentially are elements of the building envelope that do not qualify as parts of its MWFRS. For simplified method to be applicable to components and cladding of a building, it must meet the following criteria:

1. The mean roof height h must not be greater than 60 ft ($h \leq 60$ ft).
2. The building is enclosed as defined in ASCE 7-05 Section 6.2 and conforms to wind-borne debris provisions of ASCE 7-05 Section 6.5.9.3.
3. The building is a rectangular-shaped building as defined in ASCE 7-05 Section 6.2.
4. The building does not have response characteristics making it subject to across-wind loading, vortex shedding, instability due to galloping or flutter, and does not have a site location for which channeling effects or buffeting in the wake of upwind obstructions warrant special consideration.
5. The building has either flat roof, a gable roof with a slope equal to less than 45° with the horizontal, or a hip roof with a slope equal to or less than 27° with the horizontal.

For components and cladding, the net wind pressure, p_{net}, to be applied to each building surface is given by Eq. (6.1):

$$p_{\text{net}} = \lambda K_{zt} I p_{\text{net30}} \tag{6.1}$$

where λ = adjustment factor for building height and exposure from Table 6.1 (ASCE 7-05 Fig. 6-3)
K_{zt} = topographic factor as defined in ASCE 7-05 Section 6.5.7, evaluated at mean roof height h
I = importance factor as defined in Table 6.2 (ASCE 7-05 Section 6.2)
p_{net30} = net design wind pressure for Exposure Category B, at h = 30 ft, and for I = 1.0 from Table 6.2 (ASCE 7-05 Fig. 6-3)

Values of λ, I, and p_{net30} are given, respectively, in Tables 6.1 through 6.3. The topographic factor, K_{zt}, is given by Eq. (6.2):

$$K_{zt} = (1 + K_1 K_2 K_3)^2 \tag{6.2}$$

where K_1, K_2, and K_3 are multipliers whose values are given in ASCE 7-05 Fig. 6-3, and apply only when a hill or escarpment relative to upwind terrain is present. Where such a topographical characteristic is not present, the value of K_{zt} can be taken as 1.0, as assumed for examples in this chapter.

TABLE 6.1 Adjustment Factor for Building Height and Exposure, λ; adapted from [6.19]

	Exposure		
Mean Roof Height (ft)	B	C	D
15	1.00	1.21	1.47
20	1.00	1.29	1.55
25	1.00	1.35	1.61
30	1.00	1.40	1.66
35	1.05	1.45	1.70
40	1.09	1.49	1.74
45	1.12	1.53	1.78
50	1.18	1.56	1.81
55	1.19	1.59	1.84
60	1.22	1.62	1.87

Unit conversion: 1 ft = 0.3048 m.

The importance factor, I, accounts for the degree of hazard to human life and damage to property. Its value, which varies between 0.77 and 1.15, depends on: (1) the occupancy category of a building or a structure as listed in Table 1-1 of ASCE 7-05, (2) whether the building or structure is located in a nonhurricane-prone region and hurricane-prone region with the basic wind speed V (defined as the 3-s gust speed at 33 ft above ground in Exposure Category C) of 85 to 100 mph and Alaska, or in hurricane prone regions with $V > 100$ mph. ASCE 7-05 classifies various occupancies in four categories: Category I through IV (see ASCE-05 Table 1-1):

1. *Category I:* Buildings in Category I are those that pose a low hazard to human life in the event of failure, such as agriculture facilities, certain temporary facilities, and minor storage facilities.
2. *Category II:* Buildings in this category are those that are not classified as buildings in categories I, III, and IV.
3. *Category III:* Included in this category are such buildings and structures that represent substantial hazard to human life in the event of failure. Such buildings include, amongst others: schools and colleges, hospitals, and buildings and other structures in which more than 300 people congregate in one area. Facilities that process; manufacture; handle; store; and/or use or dispose of such substances as hazardous fuels, chemicals, hazardous

TABLE 6.2 Importance Factors I for Wind Loads

	Occupancy Categories			
	I	II	III	IV
Qualifying Regions	Importance Factor, I_{PO}			
Nonhurricane-prone regions, and hurricane-prone regions with $V = 85$–100 mph	0.87	1.0	1.15	1.15
Hurricane-prone regions with $V = 85$–100 mph	0.87	1.0	1.15	1.15

Unit conversions: 1 mph = 1.61 kmph.
Source: Adapted from Ref. 6.19.

TABLE 6.3 Net Design Wind Pressure, p_{net30} (lb/ft^2) in Exposure B at $h = 30$ ft with $I = 1.0$ [6.19]

Zone	Effective Wind Area (ft^2)	Basic Wind Speed (mph)									
		85		90		100		110		120	
Interior Zone*	10	13.0	−14.1	14.6	−15.8	18.0	−19.5	21.8	−23.6	25.9	−28.1
	20	12.4	−13.5	13.9	−15.1	17.2	−18.7	20.8	−22.6	24.7	−26.9
	50	11.6	−12.7	13.0	−14.3	16.1	−17.6	19.5	−21.3	23.2	−25.4
	100	11.1	−12.2	12.4	−13.6	15.3	−16.8	18.5	−20.4	22.0	−24.2
	500	9.7	−10.8	10.9	−12.1	13.4	−14.9	16.2	−18.2	19.3	−21.5
End Zone*	10	13.0	−17.4	14.6	−19.5	18.0	−24.1	21.8	−29.1	25.9	−34.7
	20	12.4	−16.2	13.9	−18.2	17.2	−22.5	20.8	−27.2	24.7	−32.4
	50	11.6	−14.7	13.0	−16.5	16.1	−20.3	19.5	−24.6	23.2	−29.3
	100	11.1	−13.5	12.4	−15.1	15.3	−18.7	18.5	−22.6	22.0	−26.9
	500	9.7	−10.8	10.9	−12.1	13.4	−14.9	16.2	−18.1	19.3	−21.5
		130		140		145		150		170	
Interior Zone*	10	30.4	−33.0	35.3	−38.2	37.8	−41.1	40.5	−43.9	52.0	−56.4
	20	29.0	−31.6	33.7	−36.7	36.1	−39.3	38.7	−42.1	49.6	−54.1
	50	27.2	−29.8	31.6	−34.6	33.9	−37.1	36.3	−39.7	46.6	−51.0
	100	25.9	−28.4	30.0	−33.0	32.2	−35.4	34.4	−37.8	44.2	−48.6
	500	22.7	−25.2	26.3	−29.3	28.2	−31.4	30.2	−33.6	38.8	−43.2
End Zone*	10	30.4	−40.7	35.3	−47.2	37.8	−50.6	40.5	−54.2	52.0	−69.6
	20	29.0	−38.0	33.7	−44.0	36.1	−47.2	38.7	−50.5	49.6	−64.9
	50	27.2	−34.3	31.6	−39.8	33.9	−42.7	36.3	−45.7	46.6	−58.7
	100	25.9	−31.8	30.0	−36.7	32.2	−39.3	34.4	−42.1	44.2	−54.1
	500	22.7	−25.2	26.3	−29.3	28.2	−31.1	30.2	−33.6	38.8	−43.2

Unit conversions: 1.0 ft = 0.3048 m; 1 lb/ft^2 = 0.0479 kN/m^2.
Negative sign indicates outward pressure.
*Interior and end zones are denoted, respectively, as Zone 4 and Zone 5 in ASCE 7-05 Chap. 6.
***Source*:** Adapted from Ref. 6.22.

waste, or explosives are included in this category. Buildings and other structures, such as power-generating stations, water-treatment facilities, sewage-treatment facilities, and telecommunication centers, which have the potential of substantial economic impact in the event of failure, are also included in this category.

4. *Category IV:* This category includes buildings and other structures that are deemed to remain operational in the vent of extreme environmental loading from wind, earthquakes, or snow.

Readers should refer to ASCE 7-05 Table 1-1 for the expanded listing of various buildings and other structures in various occupancy categories. Table 6.2 lists values of importance factor, I, for the above described occupancy categories.

Table 6.3 lists values of p_{net30} (lb/ft^2) for the *interior zone* and the *end zone* of *walls* (pressures on the roof are different) of building. The end zone is essentially an area of discontinuity of the wall or abrupt change in the orientation of the wall (such as a wall corner), where the negative pressures (or suction) are much greater than on the wall area away from the area of discontinuity. ASCE 7-05 defines *end zones* as strips of vertical areas having width a equal to the smaller of 10 percent of least horizontal dimension

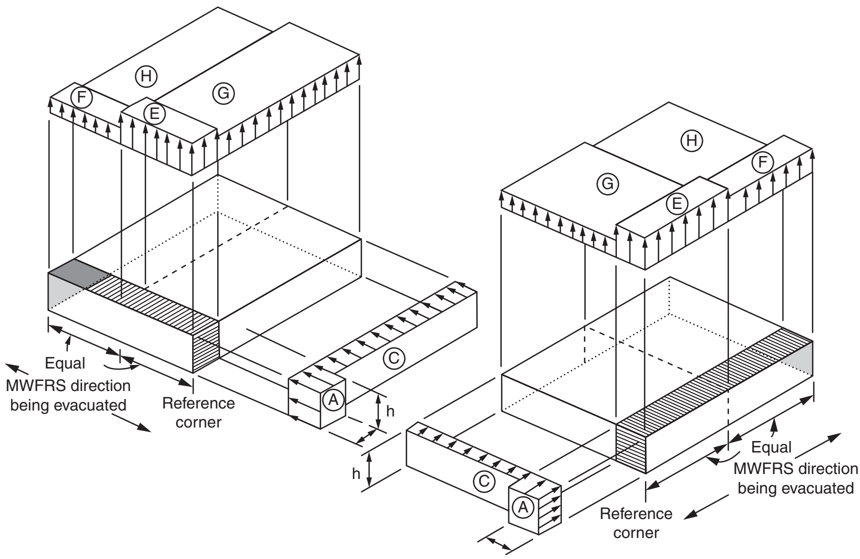


Figure 6.22 Wall *interior* and *end* zones. (*Adapted from Ref.* [*6.19*].)

or 0.4h (h = mean roof height of the building), but not less than either 4 percent of the least horizontal dimension or 3 ft (0.9 m). Both the interior zones and the end zones are shown in Fig. 6.22.

6.5.3 Earthquake Loads

Out-of-plane loads due to an earthquake can be determined based on ASCE 7-05 [6.19 (Chaps. 11, 12, and 13)] provisions. For analytical purposes, a wall may be considered as structural (or load-bearing) and nonstructural (or nonload-bearing). A wall may consist of two segments: (1) a structural wall that spans between floors or between a floor and the roof in the case of single-story building, and (2) a parapet that is an extension of the wall above the roof level, which is considered as a nonstructural component for design purposes. Lateral loads due to earthquake on these two segments are determined based on separate provisions in ASCE 7-05.

6.5.3.1 Out-of-Plane Earthquake Loads on Structural Walls Provisions for seismic design force normal to walls are given by ASCE 7-05 Section 12.11. Structural walls and their anchorage are required to be designed for a lateral seismic force, F_p, given by Eq. (6.3):

$$F_p = 0.4 S_{\mathrm{DS}} I W_c \tag{6.3}$$

where F_p = seismic force normal to the wall component
S_{DS} = design earthquake, 5 percent damped, spectral response period acceleration parameter at short period (0.2 s)
I = importance factor used for the seismic force–resisting system (SFRS) of the structure (Table 6.4)
W_c = gravity load of the component of the building

TABLE 6.4 Importance Factor for Seismic Loads (Adapted from Ref. 6.19)

*Occupancy category**	I	II	III	IV
Importance factor	1.0	1.0	1.25	1.5

*Refer to ASCE 7-5 [6.19] Table 1-1 for details of occupancy categorization.

The value of the lateral force F_p given by Eq. (6.3) is limited to the following values:

1. Force F_p may not be less than that given by Eq. (6.4)

$$F_{p,\min} = 0.1W_c \tag{6.4}$$

2. The anchorage of concrete or masonry walls to supporting construction shall provide a direct connection capable of resisting a force equal to or greater of the following to prevent the separation of these walls from the roof or floor diaphragms:
   (a) F_p given by Eq. (6.3)
   (b) A force equal to 400SDS I per linear foot of the wall
   (c) 280 lb per linear foot of the wall

The importance factor I is assigned three values: 1.0, 1.25, and 1.5 based on the occupancy category of the structure (see ASCE 7-05 Table 1-1), as listed in Table 6.4.

In Eq. (6.3), S_{DS} is determined from the Eq. (6.5):

$$S_{\mathrm{DS}} = {}^2\!/_3 S_{\mathrm{MS}} \tag{6.5}$$

where S_{MS} = the maximum considered earthquake, 5 percent damped, spectral response parameter at short periods (0.2 s) adjusted for site class effects. This adjustment is made from Eq. (6.6):

$$S_{\mathrm{MS}} = F_a S_s \tag{6.6}$$

where F_a = short period (0.2 s), coefficient adjusted for site class as listed in ASCE 7-05 Table 11.4-1.

The force F_p is assumed to act at the center of gravity of the wall component, and is to be distributed, for design purposes, over the height of the component, relative to the wall's mass distribution. For the case of a wall of uniform thickness (as is usually the case), the F_p force is to be distributed uniformly over the height of the wall (Fig. 6.23).

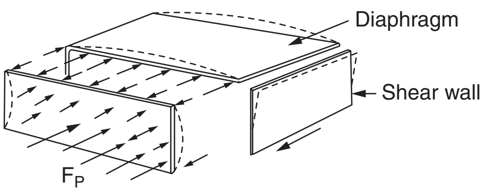


FIGURE 6.23 Distribution of out-of-plane seismic forces on walls.

For all design purposes, each structure can be categorized into one of the four *occupancy categories*: Categories I, II, III, and IV, which are based on the use of a building or structure. The seismic importance factor *I* depends on (1) the *Seismic Design Category* of a structure, which are designated as type A through F (ASCE 7-05 Section 11.6), and (2) the values of S_{DS} (defined above) and S_1, design earthquake, 5 percent damped, spectral response period acceleration parameter at 1 s. The values of spectral response acceleration parameters at short period (S_S) and at 1-s period (S_1) for a given site can be found from the internet. One such source is the California Geological Survey Web site (http://redirect.conservation.ca.gov/CGS/rghm/pshamap/pshamap.asp). Given the longitude and latitude of the site, the Web site provides values of these parameters. The site longitude and latitude can be obtained from an internet site such as http://www.gpsies.com/coordinate.do by simply inputting the address.

Table 6.5 summarizes, for various occupancy categories, the relationship between the values of short and 1-s spectral accelerations parameters, S_{DS} and S_1, respectively, and the SDC of a structure. The SDC of a structure is determined separately based on the its S_{DS} and S_1 values, which might be different; the structure is assigned the more severe of the SDCs so determined, irrespective of its fundamental period.

6.5.3.2 Earthquake Loads on Nonstructural Walls and Parapets For the purpose of calculating seismic force acting normal to a parapet, ASCE 7-05 considers it to be a *nonstructural* component. Poor performance and lack of redundancy requires parapets to be designed for higher design loads than those for walls.

Provisions for determination of forces on nonstructural walls and parapets are covered in ASCE 7-05 Section 13-5, which also covers seismic forces acting normal to a wide range of architectural, mechanical, and electrical equipment. *Architectural* components include a variety of components such as interior and exterior nonstructural walls, partitions, parapets, chimneys, veneer, ceilings, cabinets, signs and billboards, appendages, and ornamentations, etc.

For design purposes, nonstructural components are assigned the same SDC as the structures they occupy or are attached to. In higher seismic design categories, almost everything

TABLE 6.5 Seismic Design Category Based on Short Period and 1-s Period Spectral Response Acceleration Parameter (Adapted from Ref. [6.19]).

	Occupancy category		
Value of S_{DS}	I or II	III	IV
$S_{DS} < 0.167$	A	A	A
$0.167 \le S_{DS} < 0.33$	B	B	C
$0.33 \le S_{DS} < 0.50$	C	C	D
$0.50 \le S_{DS}$	D	D	D
Value of S_{D1}	I or II	III	IV
$S_{D1} < 0.067$	A	A	A
$0.067 \le S_{D1} < 0.133$	B	B	C
$0.133 \le S_{D1} < 0.20$	C	C	D
$0.20 \le S_{D1}$	D	D	D

on or attached to the structure requires design conforming to the provisions of ASCE 7-05 Chap. 13, with the following exceptions in five categories:

1. Architectural components in Seismic Design Category B, other than the parapets supported by bearing walls or shear walls provided that the component importance factor, I_p (discussed later), is equal to 1.0.
2. All mechanical and electrical equipment in Seismic Design Category B.
3. Mechanical and electrical equipment in Seismic Design Category C are exempt, provided that $I_p = 1.0$.
4. Mechanical and electrical equipment in Seismic Design Categories C, D, E, and F are exempt if $I_p = 1.0$, and either
   (a) Flexible connections between the components and ductwork, piping and conduit are provided.
   (b) Components are mounted at 4 ft (1.22 m) or less above a floor level and weigh 400 lb (1780 N) or less.
5. Mechanical and electrical equipment in Seismic Design Categories D, E, and F are exempt if $I_p = 1.0$, and
   (a) Flexible connections between the components and ductwork, piping and conduit are provided.
   (b) The components weigh 20 lb (89 N) or less, for distribution systems, weighing 5 lb/ft (73 N/m) or less.

The value of the component importance factor (I_p) alluded to in the preceding paragraph is either 1.0 or 1.5 depending on whether the structure to which it is attached has life-safety importance or hazard-safety importance. The value of I_p is assumed to be 1.5 when the following conditions apply:

1. The component is required to function for life-safety purposes after an earthquake, including fire-protection sprinkler systems.
2. The component contains hazardous materials.
3. The component is in or attached to Occupancy Category IV structure, and it is needed for the continued operation of the facility, or its failure would impair the continued operation of the facility.

   The horizontal seismic force normal to the component, F_p, is given by Eq. (6.7):

$$F_p = \frac{0.4 a_p S_{\mathrm{DS}}}{\left(R_p / I_p\right)}\left(1 + 2\frac{z}{h}\right) W_p \tag{6.7}$$

The value of F_p given by Eq. (6.7) is not required to be greater than

$$F_p = 1.6 S_{\mathrm{DS}} I_p W_p \tag{6.8}$$

or less than F_p given by Eq. (6.9):

$$F_p = 0.3 S_{\mathrm{DS}} I_p W_p \tag{6.9}$$

where F_p = seismic design force to be applied normal to the component at its center of gravity, and distributed over its height relative to the mass distribution of the component.

a_p = component amplification factor (varies from 1.0 to 2.5) as listed in ASCE 7-05 Table 13.5-1 or Table 13.6-1, as appropriate.
S_{DS} = short-period design earthquake spectral acceleration parameter.
W_p = component operating weight.
R_p = component response modification factor (varies from 1.0 to 12) as listed in ASCE 7-05 Table 13.5-1 or Table 13.6-1, as appropriate.
I_p = component importance factor.
z = height in structure of the point of attachment of the component as measured from the base. For items at or below the base, $z = 0$. The value of z/h need not exceed 1.0.
h = average roof height of the structure as measured from the base.

Eq. (6.7) was developed based on building acceleration data from research that studied the amplification of ground acceleration over the building height of instrumented buildings. The term $(1 + 2z/h)$ represents the amplification factor, which takes a value of unity for components anchored at the ground level ($z = 0$) and a value of 3 for components anchored at the roof level (at $z = h$). The response modification factor R_p represents the wall over-strength and ductility or energy-absorbing capability of a component. Note that $R_p = 2.5$ for reinforced masonry walls and 1.5 for un-reinforced masonry walls.

ASCE 7-05 Section 13.5.2 requires application of Eq. (6.7) for the design of parapets. But in determining design wall forces on a wall with a parapet or the design force for anchorage, Eq. (6.7) [ASCE 7-05 Eq. (13.3-1)] is not applied to the parapet. Instead, design forces determined per ASCE 7-05 Section 12.11.1 [Eq. (6.3) above] are applied to the entire wall including the parapet. Design forces calculated from Eq. (6.7) are much greater than those calculated from Eq. (6.3).

It is instructive to understand the philosophy that forms the basis of Eq. (6.7). The multipliers to the component weight W_p in Eqs. (6.7) through (6.9) may be thought of as seismic response coefficients for nonstructural elements [similar to C_s, seismic response coefficient for the seismic force-resisting systems (SFRS) as used in ASCE 7-05 Eq. (12.8-1)], which are independent of their position over the height of the structure. These equations recognize the unique dynamic and structural characteristics of components as compared to those of structures. Components typically lack desirable attributes of structures, such as ductility, toughness, and redundancy, which permit greatly reduced seismic lateral forces for their design. This is reflected in the lower values of R_p [in the denominator of Eq. (6.6)] as compared to the R-values for the structures (SFRS in ASCE 7-05 Table 12.14-1). In addition, the components may exhibit unique dynamic amplification characteristics; the term a_p in Eq. (6.7) reflects this attribute of components. The cumulative effects of parameters R_p and a_p in Eq. (6.7) is to require greater forces for the component integrity and connection to the supporting structure as a percentage of the component mass than are typically calculated for the main SFRS. This, in essence, is the philosophy underlying Eq. (6.7). A discussion on this topic can be found in Ref. 6.23.

Equations (6.7) through (6.9) would be used in this chapter to calculate the seismic lateral force normal to the parapets for which $R_p = 2.5$. For parapets, two values of a_p are assigned as follows, depending on whether they are unbraced or braced:

1. $a_p = 2.5$ if the parapet is unbraced or braced to the structural frame below its center of mass.

2. ap = 1.0 if the parapet is braced to the structural frame above its center of mass.

Table 6.6 lists values of a_p and R_p for some of the architectural elements.

ASCE 7-05 Section 13.3 mandates additional requirements for applying F_p in design. It requires that F_p be considered as acting independently in two orthogonal horizontal directions in combinations with the appropriate service loads associated with the component

TABLE 6.6 Values of a_p and R_p for Architectural Components for Use in Eq. (6.7)

	Architectural Component or Element	a_p^*	$R_p^\dagger$
1	Cantilever elements (unbraced or braced to structural frame below its center of mass)	2.5	2.5
	Parapets and cantilever interior nonstructural walls	2.5	2.5
	Chimneys and stacks whether laterally braced or supported by the structural frame		
2	Cantilever elements (braced to structural frame above its center of mass)	1.0	2.5
	Parapets	1.0	2.5
	Chimneys and stacks	1.0	2.5
	Exterior nonstructural walls†		
3	Interior nonstructural walls and partitions†	1.0	1.5
	Plain (unreinforced) masonry walls	1.0	2.5
	All other walls and partitions		
4	Exterior nonstructural wall elements and connections†	1.0	2.5
	Wall element	1.0	2.5
	Body of wall panel connections	1.25	1.0
	Fastener of the connecting system		

*A lower value for a_p shall not be used unless justified by dynamic analysis. Its value shall not be less than 1.0. The value of $a_p = 1$ is for rigid components or rigidly attached components. The value of $a_p = 2.5$ is for flexible components or flexibly attached components.

†Where flexible diaphragms provide lateral supports to concrete or masonry walls and partitions, the design forces for anchorage to the diaphragm shall be as specified in ASCE 7-05 Section 12.11.2.

Source: Adapted from Ref. [6.19]. See Chap. 1, Sections 1.8.4 and 1.8.5.

under consideration. However, if the component is a part of vertically cantilevered system, F_p needs to be considered as acting in *any horizontal direction.* In addition, the component is required to be designed for a concurrent vertical force of $\pm 0.2S_{DS}W_p$ as given by Eq. (6.10):

$$E = \rho Q_E \pm 0.2S_{DS}D \tag{6.10}$$

where ρ = redundancy factor
Q_E = effect of horizontal seismic forces
D = the effect of dead load (W_p)

When considering this force combination, the redundancy factor ρ is permitted to be taken as unity and the overstrength factor Ω_o does not apply. It was mentioned earlier that for the design of the walls and attachments, both wind (P_{net}) and seismic (F_p) lateral forces be calculated and the components be designed for more critical of the two loads. It must be noted that the lateral seismic force F_p, calculated from the above described procedure, is the *strength-level (or LFRD-level) force,* whereas the wind load (P_{nrt}) is the *allowable stress design-level (ASD-level) force.* To make a direct comparison of these two forces, the seismic force (F_p) should be multiplied by 0.7 to convert to the ASD-level force. Only the strength-level forces are considered for design in this book.

In cases where the seismic lateral force F_p is less than the nonseismic lateral load (i.e., wind load), the latter would govern the design of components and attachments; however, the detailing requirements and limitations prescribed in ASCE 7-05 Chap. 13 must be complied with irrespective of which lateral force governs the design. Example 6.1 presents calculations for out-of-plane wind and seismic forces on a reinforced masonry wall

Example 6.1 Calculate wind and seismic forces normal to the nominal 8-in. solid grouted reinforced masonry wall (unit weight = 84 lb/ft²) of a building that is 20 ft high to the roof level, and has a 2 ft 8 in. high unbraced parapet, as shown in Fig. E6.1A. The building is located in wind Exposure B terrain having a wind velocity of 90 mph, and $K_{zt} = 1.0$. For calculating seismic forces, the following information is provided:

Building occupancy category: III
Mapped short period (0.2 s) spectral acceleration parameter (S_S): 150%g
Mapped 1-s period spectral acceleration parameter (S_1): 90%g
Site class: Unknown

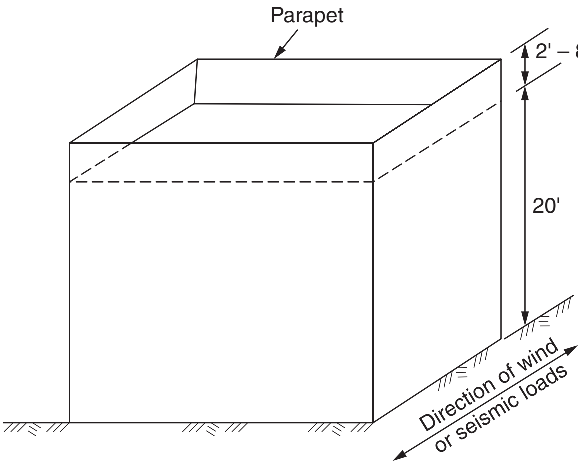


FIGURE E6.1A Reinforced masonry wall for Example 6.1.

Solution

a. Calculate wind load p_{net} from Eq. (6.1):

$$p_{\text{net}} = \lambda K_{zt} I p_{\text{net30}}$$

For Exposure B, $\lambda = 1.0$ for $h = 15$ to 30 ft (Table 6.1)
$I = 1.15$ for Occupancy Category III structure (Table 6.2)

$$K_{zt} = 1.0 \text{ (given)}$$

$$p_{\text{net}} = \lambda K_{zt} I p_{\text{net30}} = (1.0)(1.0)(1.15) p_{\text{net30}} = 1.15 p_{\text{net30}}$$

Wall forces—Interior Zone 4 for effective area of 10 ft² (Table 6.3):

$$p_{\text{net30}} = \begin{Bmatrix} 14.6 \text{ lb/ft}^2 \text{ (inward pressure)} \\ -15.8 \text{ lb/ft}^2 \text{ (outward pressure)} \end{Bmatrix}$$

Design wind pressure:

$$w = p_{\text{net}} = 1.15 p_{\text{net30}} = \begin{Bmatrix} 1.15(14.6) = 16.8 \text{ lb/ft}^2 \text{ (inward pressure)} \\ 1.15(-15.8) = -18.2 \text{ lb/ft}^2 \text{ (outward pressure)} \end{Bmatrix}$$

Wall forces—Interior Zone 5 for effective area of 10 ft² (Table 6.3):

$$p_{\text{net30}} = \begin{Bmatrix} 14.6 \text{ lb/ft}^2 & \text{(inward pressure)} \\ -19.5 \text{ lb/ft}^2 & \text{(outward pressure)} \end{Bmatrix}$$

Design wind pressure:

$$w = p_{\text{net}} = 1.15 p_{\text{net30}} = \begin{Bmatrix} 1.15(14.6) = 16.8 \text{ lb/ft}^2 & \text{(inward pressure)} \\ 1.15(-19.5) = -22.4 \text{ lb/ft}^2 & \text{(outward pressure)} \end{Bmatrix}$$

b. Calculate seismic load: Lateral seismic load on the wall would be calculated separately for the wall (w_{u1}) and for the parapet (w_{u2}).

$$w_{u1} = 0.4 S_{\text{DS}} I W_p$$

Determine short-period spectral acceleration parameter S_{DS}:

$$S_S = 150\%\text{g} = 1.5g$$

$$S_1 = 90\%\text{g} = 0.90g$$

$$S_{\text{DS}} = {}^2/_3\, S_{\text{MS}}$$

$$S_{\text{MS}} = F_a S_S$$

Site class is unknown; assume Site Class D.
For Site Class D and $S_S \geq 1.25g$,

$$F_a = 1.0 \text{ (ASCE 7-05 Table 11.4-1)}$$

Therefore,

$$S_{\text{MS}} = F_a S_S = (1.0)\,(1.5g) = 1.5g$$

$$S_{\text{DS}} = {}^2/_3\, S_{\text{MS}} = {}^2/_3\,(1.5g) = 1.0g$$

Determine 1-s period spectral acceleration parameter S_{D1}:
For Site Class D and $S_1 \geq 0.25g$,

$$F_v = 1.5$$

Therefore,

$$S_{M1} = F_v S_1 = (1.5)\,(0.9g) = 1.35g$$

$$S_{D1} = {}^2/_3\, S_{M1} = {}^2/_3\,(1.35g) = 0.90g$$

For Occupancy Category III,

$$I = 1.25 \text{ (Table 6.4)}$$

Determine the seismic design category: From ASCE 7-05 Table 11.6-1, for Occupancy Category III having $S_{\text{DS}} \geq 0.50g$, SDC is D. From ASCE 7-05 Table 11.6-2, for Occupancy Category III having $S_{D1} \geq 0.20g$, SDC is D. Therefore, SDC for the building is D.
The dead weight of the wall @ 84 lb/ft²,

$$W_p = (84)(22.67) = 1904 \text{ lb}$$

$$W_{u1} = 0.4 S_{\text{DS}} I W_p = 0.4(1.25)(1904) = 952 \text{ lb (acting at the center of gravity)}$$

Distribute 952 lb over the height of wall relative to its mass distribution.

$$w_{u1} = \frac{952}{22.67} = 42 \text{ lb/ft}^2$$

For parapets, the F_p force is given by Eq. (6.7):

$$F_p = \frac{0.4 a_p S_{\text{DS}}}{\left(R_p / I_p \right)} \left(1 + 2\frac{z}{h} \right) W_p R_p I_p$$

where $S_{\text{DS}} = 1.0$ (calculated above)
$a_p = 2.5$ (unbraced parapet, Table 6.6 or ASCE 7-05 Table 13.5-1)
$R_p = 2.5$ (unbraced parapet, Table 6.6 or ASCE 7-05 Table 13.5-1)
$I_p = 1.0$ for Occupancy Category III (ASCE 7-05 Section 13.1.3)
$z = \text{h} =$ roof height
$W_p = 2.67(84) = 224$ lb/ft length of wall

$$\begin{aligned} F_p &= \frac{0.4 a_p S_{\text{DS}}}{\left(R_p / I_p \right)} \left(1 + 2\frac{z}{h} \right) W_p \\ &= \frac{0.4(2.5)(1.0)}{\left(2.5 / 1.0 \right)} (1+2) \; W_p \\ &= 1.2 \; W_p \end{aligned}$$

Check Eqs. (6.8) and (6.9) (for maximum and minimum limits of F_p, respectively):

$$F_{p,\max} = 1.6 S_{\text{DS}} I_p W_p = 1.6\ (1.0)(1.0)\ W_p = 1.6 W_p$$

$$F_{p,\min} = 0.3 S_{\text{DS}} I_p W_p = 0.3\ (1.0)(1.0) = 0.30 W_p$$

$$0.30 W_p < 1.2 W_p < 1.6 W_p \qquad \text{OK}$$

$$F_p = 1.2 W_p = 1.2\ (224) = 269 \text{ lb/ft per linear foot of wall}$$

The 269-lb force acts at the center of gravity of the parapet and is to be distributed over its height (2 ft 8 in.). Therefore,

$$w_{u2} = \frac{269}{2.67} \approx 101 \text{ lb/ft}^2 \text{ per linear foot of wall}$$

The seismic forces on the wall and parapet are strength-level forces. Convert them to ASD-level force by multiplying with 0.7 in order to compare them with wind forces which are ASD-level forces. Thus,

$$0.70(w_{u1}) = (0.70)\ (42) = 29.2 \text{ lb/ft}^2$$

$$0.70(w_{u2}) = (0.70)\ (101) = 70.7 \text{ lb/ft}^2 \approx 71 \text{ lb/ft}^2$$

Both of the above forces are greater than wind forces. Thus, seismic loads govern (Fig. E6.1B).

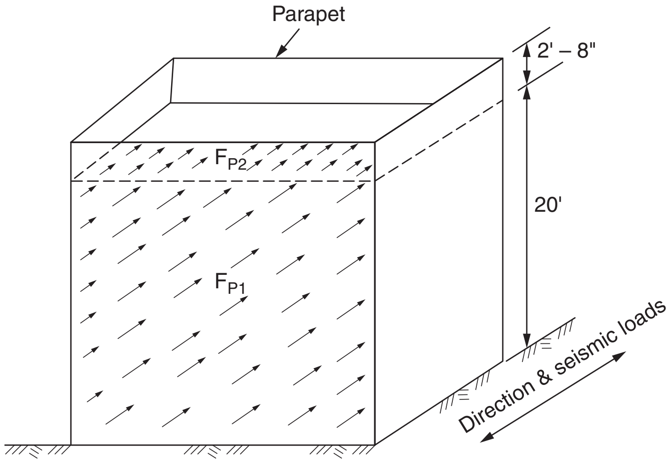


FIGURE E6.1B Seismic forces on the wall: Seismic load on the wall = 42 lb/ft^2, and seismic load on parapet = 101 lb/ft^2.

6.6 ANALYSIS OF MASONRY WALLS FOR OUT-OF-PLANE LOADS

6.6.1 Assumptions in Analysis

For analytical purposes, a wall may be visualized as a vertically oriented slab; multiple walls may be visualized as a series of slabs oriented vertically. It is common design practice to assume simple support conditions at locations of wall supports over the height of building. That is, a wall is analyzed as an element simply supported at the floors, or at the floor and the roof level (i.e., pinned-end conditions). This is a reasonable assumption considering the fact that the connections between the walls and their support elements (floors or roof) usually do not posses sufficient stiffness or strength to transfer the wall moments into the support elements to justify a rigid connection. Furthermore, because both wind and earthquake are dynamic phenomena, the assumption of pinned supports is consistent with the modal response of the walls subjected to these lateral loads [6.22].

MSJC-08 presents two separate procedures for analysis of reinforced masonry walls: one for the allowable stress design (ASD) and the other for strength design. Analysis and design of reinforced walls subjected to out-of-plane loads based on ASD philosophy is discussed in Ref. 6.24 and is not repeated here. The following discussion presents analysis and design of walls based on the strength design principles.

As the wall approaches ultimate load conditions, the lateral force on the wall (wind or seismic) causes an out-of-plane deflection, δ_u. As a result of this deflection, both factored gravity loads supported on the wall (from floor or the roof), P_{uf}, and the factored dead weight of the wall, P_{uw}, act eccentrically with respect to the midheight of the deflected wall, causing a moment in the wall. The values of these moments are maximum at the midheight of the wall where the moment due to the lateral loads is also maximum. Furthermore, the load from the roof or the floor, P_{uf}, might also be acting eccentrically with respect to the centerline of undeflected wall. All these loads and forces are assumed to be at the strength

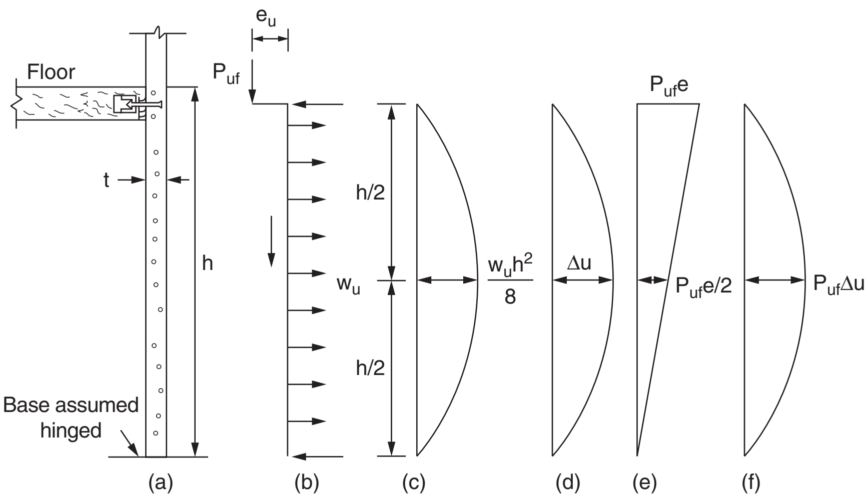


FIGURE 6.24 Forces acting on a wall.

level (i.e., these are factored or strength-level loads, denoted by subscript u) and act on the wall as shown in Fig. 6.24. These three moments can be summarized as follows:

$$\text{Lateral force on the wall (wind or seismic)} = w_u$$

$$\text{Moment in the wall due to } w_u,\ M_{u1} = \frac{w_u h^2}{8} \tag{6.11}$$

where w_u represents seismic load or the strength-level wind load normal to wall.

Equation (6.11) is similar to that for the maximum moment at the center of a uniformly loaded simply supported beam. For support conditions other than simply supported, Eq. (6.10) would be different.

$$\text{Gravity loads due to floor or roof supported on the wall} = P_{uf}$$

$$\text{Eccentricity of } P_{uf} \text{ from the support} = e_u$$

Moment due to P_{uf} varies linearly from zero at the base of the wall to $P_{uf}\,(e_u)$ at the top of the wall. Therefore, moment at the midheight of the wall,

$$M_{u2} = {}^1\!/_2(P_{uf}\,e_u) \tag{6.12}$$

Moment due to P_{uf} and P_{uw} is

$$M_{u3} = (P_{uw} + P_{uf})\ \delta_u \tag{6.13}$$

where δ_u = deflection of wall due to all factored loads, and

$$(P_{uw} + P_{uf}) = P_u \tag{6.14}$$

In Eq. (6.14), P_u is the sum of the factored superimposed loads and the factored dead load of the wall. The total moment is the sum of moments given by Eq. (6.11) through (6.13). Thus,

$$M_u = M_{u1} + M_{u2} + M_{u3} = \frac{w_u h^2}{8} + P_{uf}\left(\frac{e_u}{2}\right) + (P_{uw} + P_{uf})\delta_u \tag{6.15}$$

Eq. (6.15) is the same as MSJC-08 Eq. (3-25). The last term in Eq. (6.15) represents the moment due to deflection δ_u of the wall, and is called P-Δ effect (not considered in ASD). Note that the magnitude of this moment depends on δ_u, which, in turn depends on the magnitude of P_u. Therefore, calculating this moment is an iterative process as illustrated in the next example. Furthermore, the deflection of the wall depends on the extent of cracking of the wall (as the loads approach the strength level values), which should be accounted for in the analysis.

The preceding discussion suggested maximum deflection as occurring at the midheight of the wall, an assumption based on the simply supported conditions at both ends of the wall. Should the support conditions be such that maximum deflection dose not occur at the midheight of the wall, then the critical section where the maximum deflections occurs should be determined from analysis based on principles of structural mechanics.

6.6.2 Analysis Procedures

Flexural stresses in walls are computed based on the same general principles as discussed in Chap. 4. For flexural analysis, walls may be assumed simply supported between the two floors. Based on this assumption, the bending stress due to moment caused by lateral loads (wind or seismic) would be maximum at about the wall midheight. In addition, the eccentricity of the supported gravity loads (from the floor or the roof) would cause moment on the wall. These two types of moments would be additive. Therefore, for the combined load case (vertical dead load *plus* wind *or* seismic), the design loads should be computed at the wall midheight. However, the axial stress due to supported dead load and live loads would be maximum at the base of a wall, not at the midheight. Therefore, the wall should be checked at the bottom for maximum axial stress.

The manner in which the gravity loads are transmitted to a wall forms an important consideration in the analysis of a wall. A variety of connections for securing the roof or the beams to a wall are employed in practice. Essentially, these connection details depend on the floor or the roof system that is to be supported by the walls. These include plywood floor, precast reinforced concrete slab, prestressed concrete planks, etc. A few typical details are shown in Figs. 6.25 to 6.28 [6.24].

When beams or the roof diaphragms are supported on the interior walls (Fig. 6.25*a* and *d*), they transmit their gravity loads to walls without much eccentricity. However, when these gravity load-carrying components are supported on the exterior walls (Fig. 6.25*b* and *c*), they often transmit their gravity loads *eccentrically*, the amount of eccentricity depending on the constructional details.

A common construction practice is to support the roof diaphragm and the supporting beams on a *ledger* beam (Fig. 6.27). The latter is secured to the masonry wall by means of a number of anchor bolts, which are designed to resist the lateral load transferred by the roof diaphragm to the wall. The load path is quite simple. It may be assumed that the gravity loads are first transferred to the ledger beam, which, in turn, transfers them to the wall at the interface between the ledger and the wall. The eccentricity in this case may be assumed to be equal to half the wall thickness ($^1/_2 t$). In all cases, however, the value of eccentricity should be ascertained carefully from the actual connection details shown in the working drawings of the project.

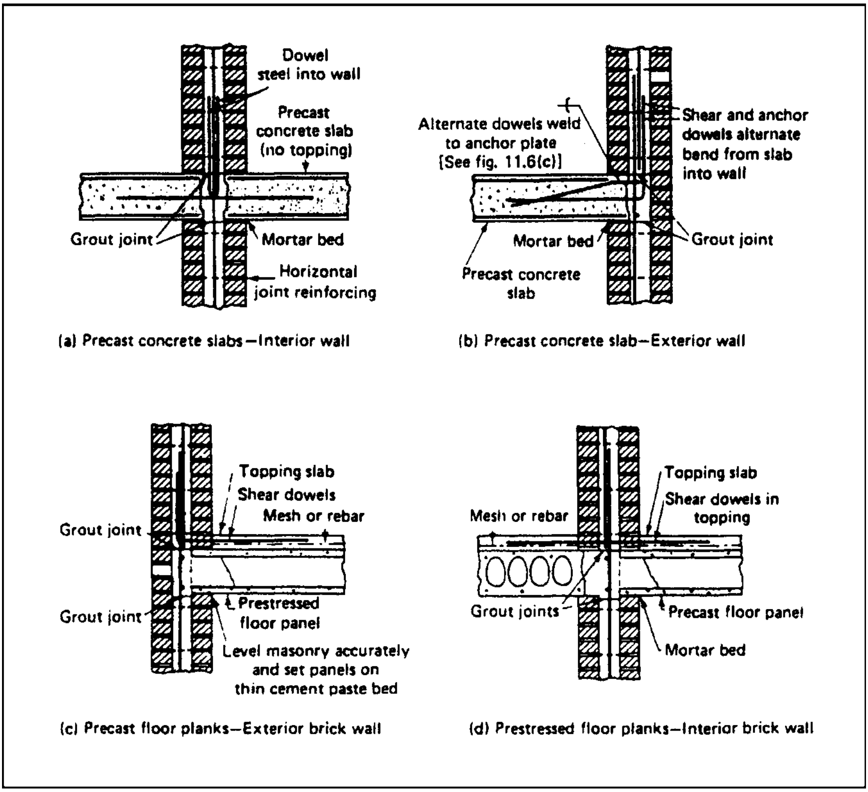


FIGURE 6.25 (*a*) and (*b*) Precast slab on the brick walls. (*c*) and (*d*) Precast floor planks on the brick walls. (*Courtesy: NCMA*.)

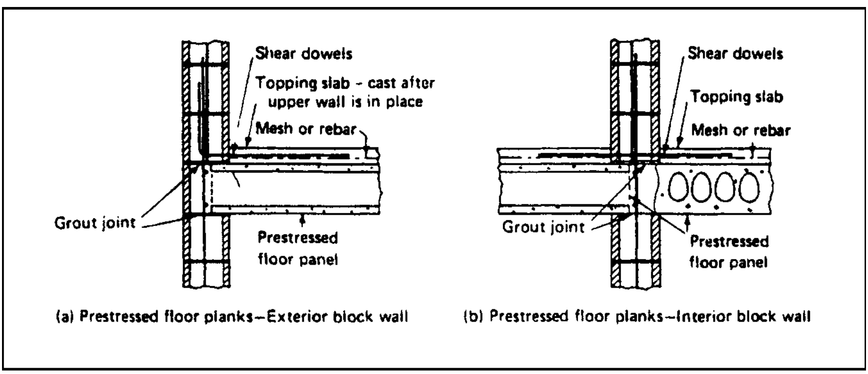


FIGURE 6.26 Prestressed planks on concrete block walls. (*Courtesy: NCMA*.)

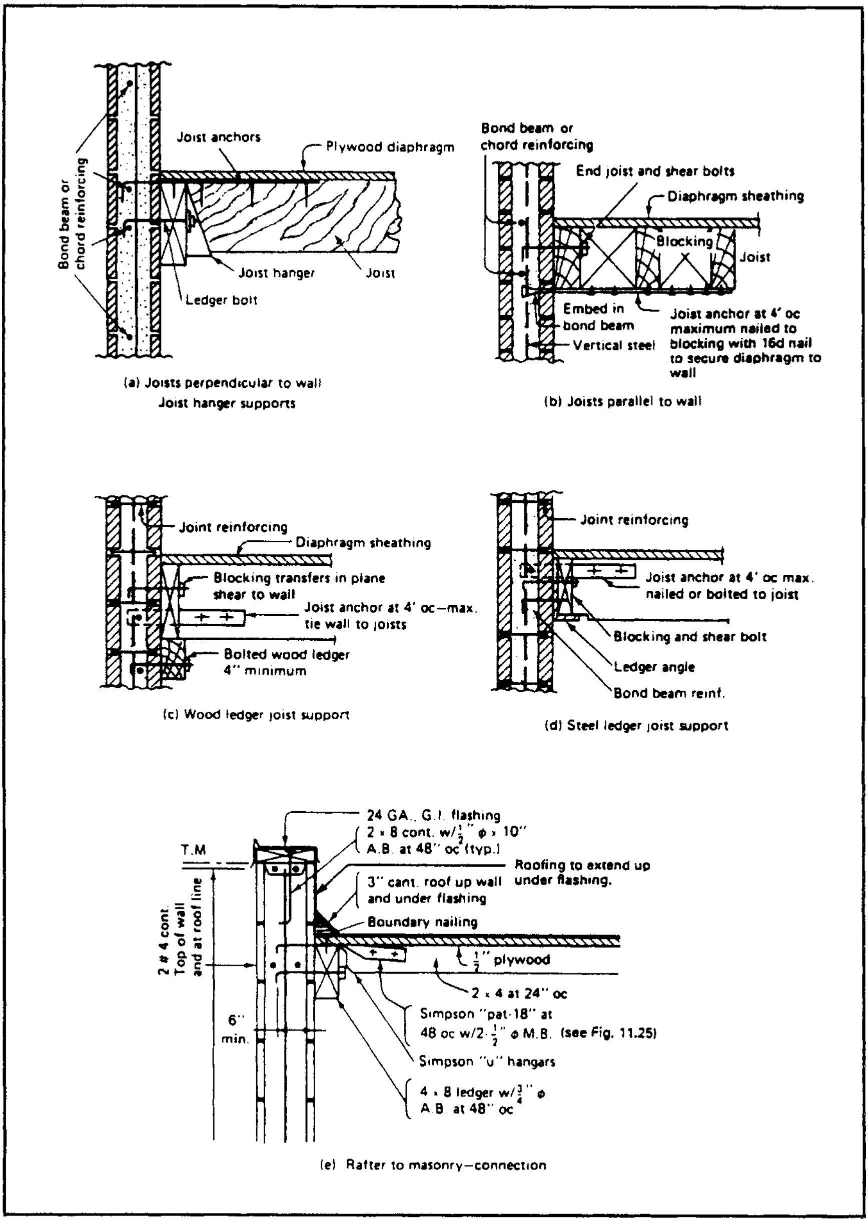


FIGURE 6.27 Bolted connections between plywood floor and masonry walls. (*Courtesy: NCMA.*)

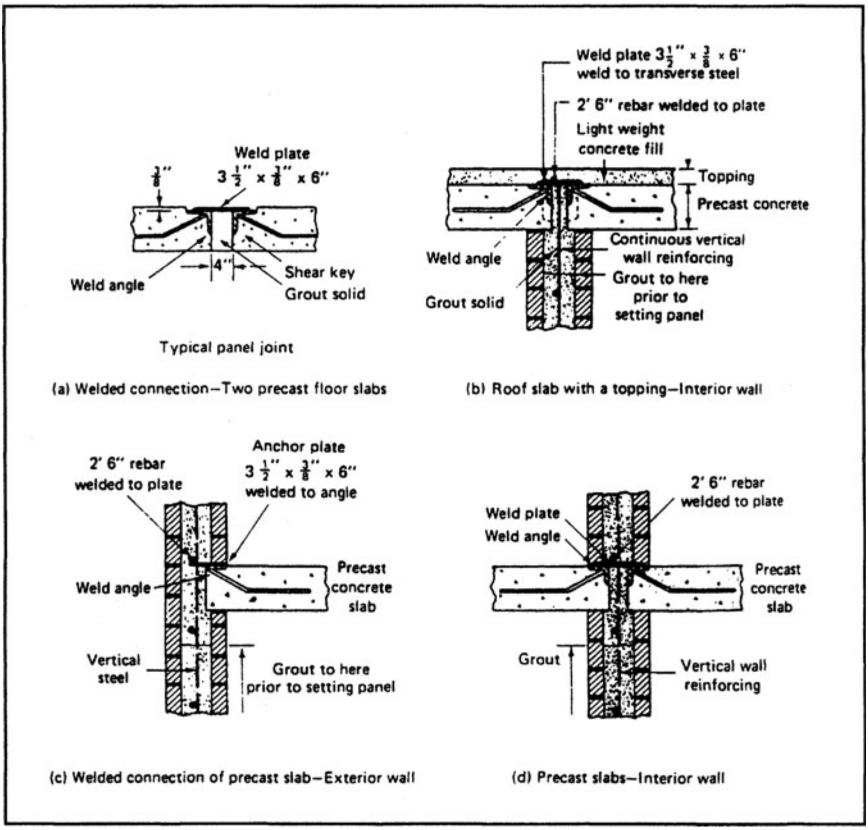


FIGURE 6.28 Weld plate connections. (*Courtesy: NCMA.*)

The eccentricity of the vertical load causes a moment which equals force *times* the eccentricity (i.e., $M = Pe$) at the top of the wall. This moment is assumed to vary linearly from maximum at the top of wall to zero at the bottom of the wall so that the moment at the wall midheight is $Pe/2$. Because the lateral loads can reverse in direction, the effect of this moment is *additive* to the wall moment due to the lateral loads, the maximum value of which is assumed to occur at the midheight of the wall (Fig. 6.29). Note that the moment due to eccentricity of *live* load is not to be combined with the lateral load. The total design moment at the wall midheight may be calculated from Eq. (6.16):

$$M = \frac{wL^2}{8} + \frac{Pe}{2} \tag{6.16}$$

where w = lateral load due to wind or earthquake
P = gravity load
e = eccentricity of gravity load
h = height of the wall

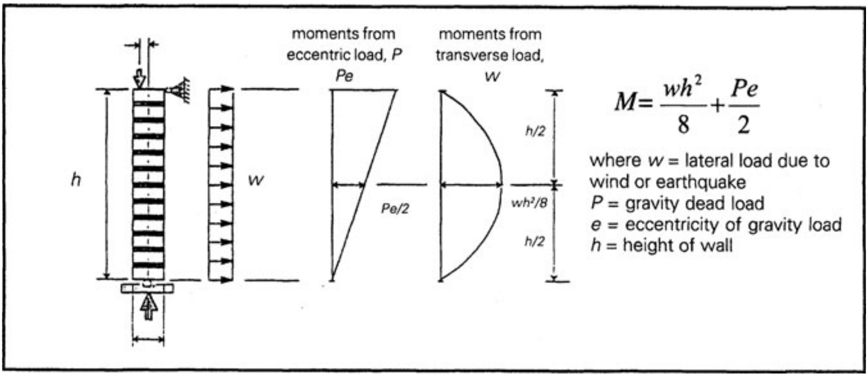


FIGURE 6.29 Moments in a wall due to lateral loads and eccentric vertical loads.

6.7 DESIGN OF WALLS FOR GRAVITY AND TRANSVERSE LOADS

6.7.1 Lateral Support for Walls

Walls may be subjected to both gravity and lateral loads. They must be supported laterally to restrain against displacement due to lateral loads such as wind and seismic forces. Walls may span horizontally or vertically. When spanning horizontally, lateral support to masonry walls may be provided in the form of cross-walls, columns, pilasters, counterforts, or buttresses. For walls spanning vertically, lateral support may be provided by floors, roofs, beams, or girts.

6.7.2 Distribution of Gravity Loads in Walls

Distribution of gravity loads on walls depends on the manner in which the wall supports the imposed loads. Several cases may be considered:

1. Roofs or floors made from reinforced concrete or prestressed concrete planks may be supported directly on top of the walls, in which case the axial load on the wall may be computed in terms of per linear foot of the wall.
2. When wood diaphragms are used, typically the connections are made to a ledger beam anchored to the walls. Gravity loads from floor or roof joists are transferred to wall through the anchor bolts. When these joists are closely spaced, the load from all joists may be distributed to wall on per linear foot basis.
3. In some cases, particularly when spans are large, joists may consist of glued-laminated or steel beams, which are supported directly on walls over bearing plates (Fig. 6.30). In such cases, the gravity loads act as concentrated loads. Distribution of such loads in walls depends on the *type* of bond pattern (i.e., running bond or other types) used to build the wall. Loads can only be transmitted through head joints of masonry laid in *running* bond. This is so because when running bond is used, the horizontal courses of bricks or blocks are staggered so that the vertical joints are not continuous. A wall with continuous vertical joints (e.g., *stack* bond) acts more like a series of adjacent vertical piers, rather than a cohesive unit. The interlocking bond has a further merit of dispersing loads evenly.

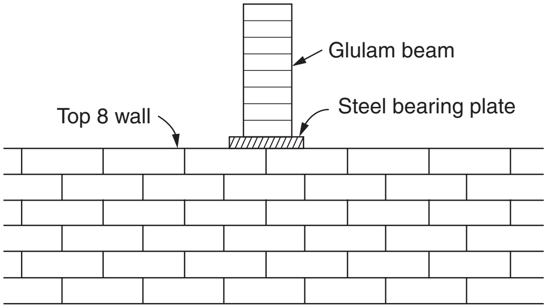


FIGURE 6.30 A concentrated load acting on a wall.

Determination of the effective length of the wall over which concentrated load(s) may be assumed to be distributed is necessary for design and analysis purposes. MSJC-08 Section 1.9.7 specifies the following limitations on the effective length of the wall (different from those in MSJC-05 *Code*) over which concentrated loads may be assumed distributed; various situations are illustrated in Fig. 6.31:

1. *Walls laid in running bond:* The effective length of the bearing area is to be limited to the smaller of the following:
   (a) The length of the bearing area plus the length determined by considering the concentrated load to be dispersed along a 2 vertical:1 horizontal line. The dispersion shall terminate at (1) half the wall height (measured from the point of application of the load to the foundation), (2) a movement joint, or (3) an opening, whichever provides the smallest length.
   (b) The center-to-center distance between the concentrated loads.
2. *Walls laid in other than running bond:* The concentrated load shall not be distributed across head joints. Where concentrated loads acting on such walls are applied to a bond beam, the concentrated load is permitted to be distributed through the bond beam, but shall not be distributed across the head joints below the bond beam.

6.7.3 Wall Reinforcement

Since walls are constructed from hollow masonry units, the vertical reinforcing bars can be placed only at intervals of spacing of cores in the units. It is common practice to use two-core hollow units for walls. When nominal 16-in.-long units (concrete or clay) are used, the cell spacings occur at 8-in. intervals so that reinforcing bars can be placed at 8-in. intervals or at multiples of 8 in. (e.g., 16, 24 in. on centers, etc.). When 12-in. nominal brick units are used, the cell spacings occur at 6-in. intervals so that reinforcing bars can be placed at 6-in. intervals or at multiples of 6 in. (e.g., at 12, 18 in. on centers, etc.).

In general, smaller bar spacings are highly desirable from a practical standpoint because of the following advantages:

1. Grouting of cells is easier when smaller diameter bars are provided in them.
2. Closely spaced bars reduce number of shrinkage cracks which minimizes water penetration.
3. Smaller bar spacings permit use of smaller diameter bars, which makes splicing of bars and grouting around the spliced bars easier.

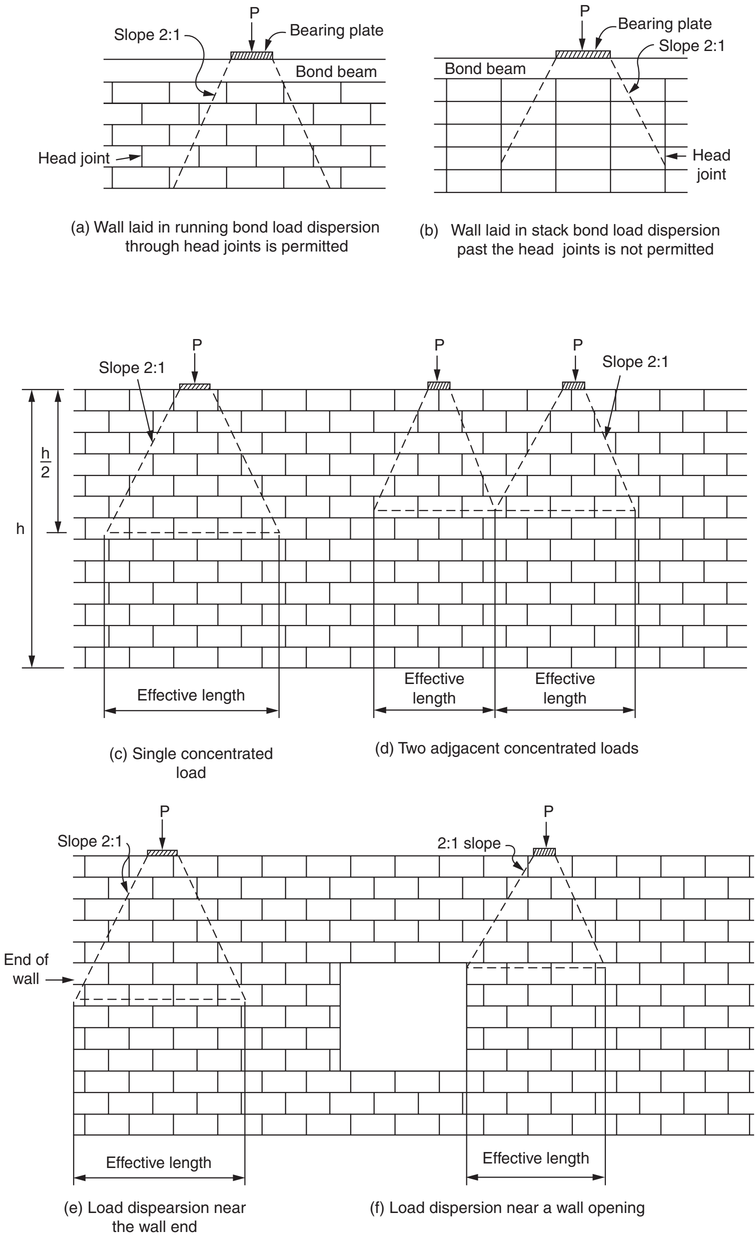


FIGURE 6.31 Various situations involving distribution of concentrated loads on walls: (*a*) a concentrated load in a wall laid in running bond, (*b*) a concentrated load in a wall laid in stack bond, (*c*) a single concentrated load on a wall laid in running bond, (*d*) two adjacent concentrated loads on a wall laid in running bond, (*e*) a concentrated load near the end of a wall laid in running bond, (*f*) load dispersion near a wall opening (*Adapted from Ref.* [*6.1*]).

A fundamental requirement of reinforced masonry construction is that all reinforcing bars shall be embedded in grout (MSJC-05 Section 1.13.1). Reinforcement should be completely surrounded by and bonded to masonry material so that they work in unison to resist loads. This is a basic assumption made for designing reinforced masonry. It follows that there should be ample space in the cell of a masonry unit for grout to effectively surround a reinforcing bar placed inside that cell.

Reinforcing bars must be placed in the cells prior to grouting, and they should be secured against displacement prior to grouting by wire positioners or other suitable devices at intervals not exceeding 200-bar diameters. However, construction practices vary somewhat. Horizontal reinforcing bars are positioned as the wall is erected. Vertical bars may be installed prior to laying masonry or may be inserted from the top after the masonry is placed to story height.

A construction situation may involve a large amount of closely spaced vertical reinforcement, or the reinforcement may be required to be in place prior to the installation of masonry units. In such a case, a variation of the vertical placement may be used. The vertical reinforcing bars may be secured in their proper position at the foundation or the base of the wall prior to laying up the units. Instead of threading hollow units down over the vertical reinforcing bars, open-ended units are typically used (Fig. 6.7*b*), which can be easily laid around the reinforcing bars as the wall progresses. These special units are manufactured with one or both end webs removed, resulting in “A” or “H”—shaped units (Fig. 6.32).

Pertinent reinforcing bar requirements are specified in MSJC-05 Section 1.13 and 3.3. The maximum size of reinforcing bar permitted is No. 9 for strength design of masonry. Furthermore, the maximum reinforcement area in a cell or in a course is limited to 4 percent of the cell area. Table 6.7 presents typical maximum bar size for the standard units. However, the cell sizes vary somewhat from one manufacturer to another; therefore, actual dimensions should be verified from the supplier.

All reinforcing bars, except joint reinforcement, shall be completely embedded in mortar or grout. The minimum cover requirements are as follows:

1. *Masonry face exposed t*o earth or weather: 2 in. (50.8 mm) for bars larger than No. 5 (M #16), $1^1/_2$ in. (38.1 mm) for No. 5 (M #16) bars or smaller
2. Masonr*y not exposed to weather:* $1^1/_2$ in. (38.1 mm)

6.7.4 Position of Vertical Reinforcement in Walls to Resist Flexure

The physical behavior of a masonry wall supported at top and bottom and subjected to flexure can be likened to that of a reinforced concrete slab spanning vertically (Fig. 6.33).

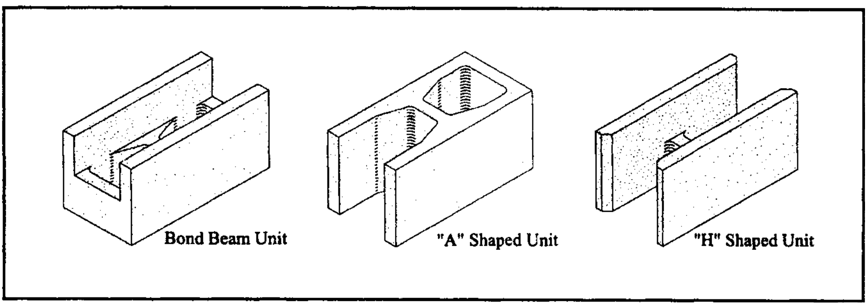


FIGURE 6.32 Block shapes for reinforced masonry construction. (*Courtesy: NCMA.*)

TABLE 6.7 Maximum Bar Size for Various Size Units

	12-in. Long Units		16-in. Long Units	
Unit Thickness (Nominal)	Cell Size (width × length)	Maximum Bar Size	Cell Size (width × length)	Maximum Bar Size
4 in.	$1^3/_4 \times 3^1/_2$	5	$1^3/_4 \times 3$	5
5 in.	$2^1/_2 \times 3^1/_2$	6	2×5	7
6 in.	$3^1/_2 \times 3^1/_2$	7	3×5	8
8 in.	$5 \times 3^1/_2$	9	5×5	9
10 in.	$6^3/_4 \times 3^1/_2$	10	$6½ \times 5$	9
12 in.	$8^1/_2 \times 3^1/_2$	11	85	9

In buildings, the windward face of the wall is subjected to compression and the back of the wall to tension. The same applies in the case of seismic lateral loads also—the face exposed to seismic loads is in flexural compression, and the opposite face in tension. Since the direction of wind and seismic forces can reverse, either face of the wall can be in tension or in compression. Therefore, it is common practice to place the vertical reinforcing bars at the center of the cells. However, based on structural requirements, bars can be placed on *both* faces of the cells (discussed in Section 6.7.5).

In retaining walls and basement walls, which are subjected to earth pressure from one side only, the stress conditions are somewhat different. In case of retaining wall, which acts as a cantilever, the face of the wall in contact with the soil is in tension. The opposite is true in case of basement wall. A basement wall may be assumed *simply supported* at the floor levels. Consequently, the face of the wall exposed to soil would be in compression and the opposite face in tension. In both cases, reinforcing bars will have to be placed near the tension face of the wall.

Since flexural stresses in a vertically spanning wall are directed in the *vertical* plane, the tension resisting reinforcement is oriented in the *vertical* direction also, extending from bottom to top, passing through the cells which are grouted. In a wall built from 8 × 8 × 16 in. standard concrete masonry units, the cell spacing is 8 in. on center. Therefore, the closest spacing of the reinforcement can be only 8 in. Such a close spacing is usually not required, however, and bars are provided only at intervals which are multiples of 8 in.

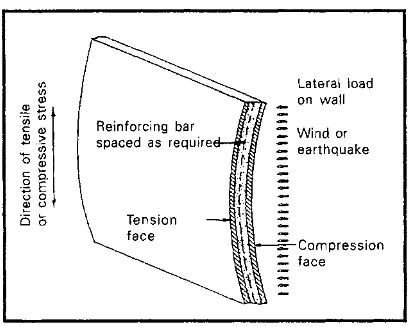


FIGURE 6.33 Behavior of vertically spanning walls in flexure.

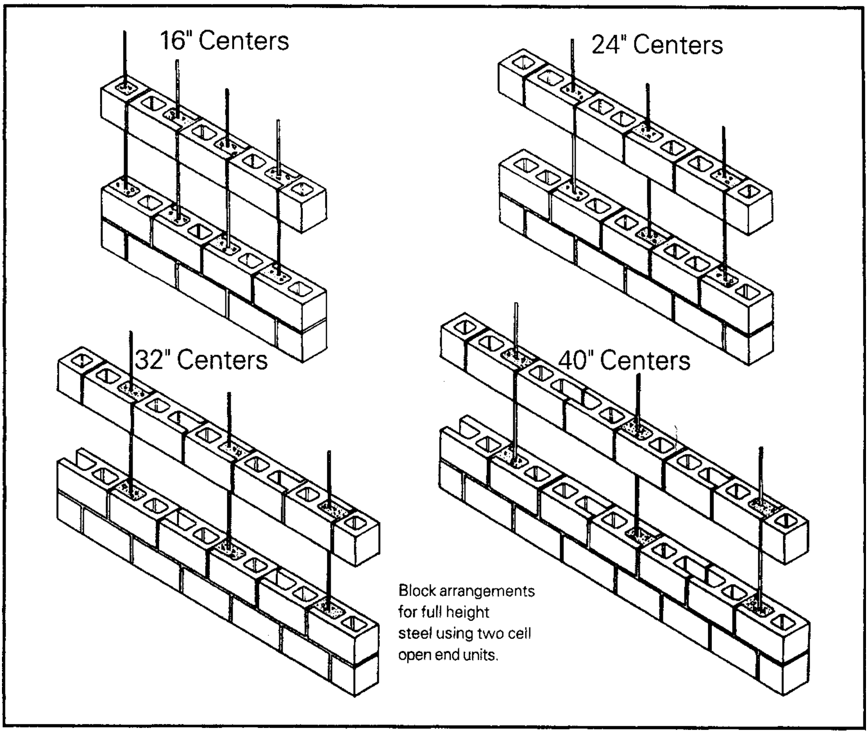


FIGURE 6.34 Vertical reinforcement in masonry walls. Note the positions and arrangement of open-ended units to permit reinforcing bars pass through the cells. (*Courtesy: NCMA.*)

(i.e., at 16, 24, 32, etc, with 48 in. on center maximum) as shown in Fig. 6.34. If all the cells of the wall were solidly grouted, the result would be a solid-grouted wall. Such a wall could be easily analyzed as a rectangular section. In partially grouted construction, only the cells (at certain spacings) containing reinforcing bars are grouted; the cells not containing reinforcing bars are left ungrouted (hollow). Such a wall resembles a vertical multiweb T-beam, where the loaded face of the wall (i.e., face shell) acts as *flange* and the cross-walls and the grouted cells as *web.* For analyzing such a wall, the principles of conventional T-beam analysis can be applied. Determination of the effective depth and the width of such a T-section are described in Section 6.7.6.

6.7.5 Effective Depth of Reinforced Masonry Walls

Effective depth of a wall section subjected to out-of-plane loads is measured from the centroid of the reinforcement to the compression face. Depending on the structural requirements, the reinforcing bars may be placed at the center of the cells (Fig. 6.35*a*) or near the inside face of the cells (Fig. 6.35*b*). In the latter case, two reinforcing bars are placed in the same cell, one each near the inside face of the shell. Although the latter arrangement of reinforcing bars gives the appearance of a doubly reinforced wall, it is really not a doubly reinforced section in a structural sense. As noted earlier, lateral loads acting on walls are

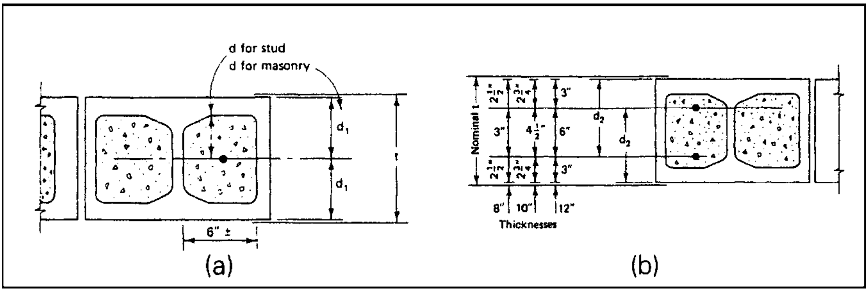


FIGURE 6.35 Position of reinforcement inside the cells of concrete masonry units: (*a*) reinforcing bars at the center of cells, (*b*) reinforcing bars near each face of cells.

reversible in direction, and any wall face can be subjected to tension. Therefore, only one reinforcing bar (near the tension face of the wall) is assumed effective at any given time; the contribution of reinforcing bar near the compression face in resisting flexure is ignored.

With the reinforcing bars at the center of the wall, as is generally the case, the effective depth of the wall section equals half the actual thickness of the masonry unit. For example, the nominal thickness of concrete masonry units is typically 6, 8, 10, 12, or 16 in., with actual thickness being $^3/_8$ in. smaller in each case. Thus, for a 12-in. nominal wall, the actual thickness would be $11^5/_8$ in. (= $12 - {}^3/_8 = 11.625$ in.), and the effective depth d for such a centrally reinforced wall, would be 5.81 in. (= $^1/_2 \times 11.625$ in.). On the other hand, if the reinforcement were placed near the tension face of the wall, the effective depth would be about 9 in. for the same 12-in. nominal wall, much larger than in the case of centrally reinforced-wall. This increased effective depth results in increased flexural resistance which can be used as an effective alternative to meet the higher moment strength requirements without increasing the wall thickness. However, this advantage may be offset by the cost of providing larger amount of reinforcing (on both faces of the cells). In general practice, for reasons of economy, it may be preferred to place the reinforcement at the center of the cell rather than near both faces of cells (one near each face of the same cell). Table 6.8 shows effective depths for walls of different nominal thicknesses for both alternatives.

TABLE 6.8 Dimensions to Calculate Effective Areas of Concrete Block (ASTM C 90, Ref. 6.25)

Width (in.)		Assumed Dimension* for Design (in.)					
Nominal	Actual	d_1†	d_2†	X‡	t_s	t_w	t_e
6	$5^5/_8$	2.81	—	6	1	1	1
8	$7^5/_8$	3.81	5.31	6	$1^1/_4$	1	$1^1/_4$
10	$9^5/_8$	4.81	7.06	6	$1^3/_8$	$1^1/_8$	$1^3/_8$
12	$11^5/_8$	5.81	8.81	6	$1^1/_2$	$1^1/_8$	$1^1/_8$

*See Fig. 6.21 for definitions of notations.

†$d_1 = d$ = effective depth for centrally reinforced wall, Fig. 6.35*a*.d_2 = d = effective depth for reinforcement placed near the far inside face of the cell, Fig. 6.35*b*.

‡x = length of void measured parallel to unit's length^2.

Brick, solid, cored, and hollow, are available in various sizes (modular and nonmodular) as discussed in Chap. 2. Pertinent details for brick sizes and related information can be found in the BIA Technical Notes [6.29 and 6.30]. The actual thickness of the wall will depend on the type of bricks used and the type of wall constructed (e.g., two-wythe wall with collar joint or a single-wythe wall, hollow or reinforced). Modular bricks are available in widths of 4, 6, and 8 in. Nonmodular bricks have their actual dimensions smaller than the nominal dimensions to allow for the mortar joint, generally $^3/_8$ to $^5/_8$ in; this permits even-dimensional modular coursing. Thus, a 4-in. nominal brick may be actually $3^1/_2$ to $3^5/_8$ in. wide, and the effective depth, d, for the centrally reinforced wall would be half the actual width of the brick.

It should be noted that dimensional variations may exist in the thicknesses of the web and the end walls of hollow masonry units of various sizes, as well as in the masonry units produced by different manufacturers. Therefore, it would be prudent to check with the material supplier regarding the availability and the actual size of the masonry units to be used for a project. Since these dimensions may not be known a priori or at the preliminary design stage, information given in Table 6.4 may be used as a guide.

6.7.6 Effective Width: T-Beam Analysis

If a wall under flexure due to out-of-plane loads were solid grouted, it could be analyzed as a (solid) rectangular section, using a 1-ft-wide vertical strip of the wall. But this analysis can not be used when such a wall is partially grouted. This is because in a partially grouted wall only those cells are grouted which contain reinforcing bars, leaving other cells as hollow. Cross section of such a wall to be used for analysis may be viewed akin to a T-section: the face shells of the units comprise the compression *flange* of the T-section, whereas the grouted cell and the cross-walls of the masonry units comprise the *web* of this T-beam section (Fig. 6.36), portions of which may be in compression. Therefore, analysis of such a wall depends on the depth "a" of the compression block (rectangular stress distribution, see discussion in Chap. 4). If the depth a happens to be smaller than or equal to the thickness of the face shell, the wall section can be analyzed as a rectangular section. However, in an unlikely case, if a happens to be greater than the thickness of the face shells, the section must be analyzed as a T-beam section (see Example 6.5).

The analysis of a partially grouted wall subjected to out-of-plane loads is similar to that for reinforced hollow core concrete beams or a multiweb concrete box girder although the two differ in appearance. Reinforced concrete hollow or box beams are positioned typically as horizontal elements; the slab acts as the flange and the stems act as web. The walls, on the other hand, are positioned as vertical elements; the face shells on the compression side of the wall act as the flange, and the cross-walls and the grouted cells act as web. Depending

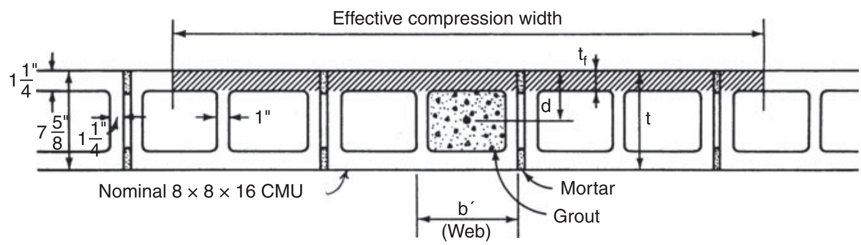


FIGURE 6.36 Cross section of a partially grouted wall as an equivalent T-Beam. Note the compression width of the section per reinforcing bar.

on the amount of reinforcing material present to resist tension, the compression block may lie entirely within the flange, or it may extend in the web. In the first case, the section can be analyzed as a rectangular section, whereas in the second case, the section is required to be analyzed as a T-section.

The analysis of a T-beam requires determining the effective width of flange based on consideration of *shear lag* phenomenon. What this means is that *uniform* compressive stress is assumed distributed only over a limited width of the compression flange of a T-beam instead of over the entire flange width that may be physically present in the wall cross section (because of the *shear lag* phenomenon). This limited width is called the *effective width,* b_e. The advantage of such an assumption is that it permits a designer to use the conventional flexural analysis methods which are applicable to rectangular beams, as presented in Chap. 4. A discussion on this topic can be found in references [6.26–6.27].

The *effective width* for reinforced concrete T-beams is specified in MSJC-08 Section 1.9.6: *Effective compressive width per bar*. The effective width b_e of compression area for flexural analysis is determined as follows (Fig. 6.36):

1. In *running bond* masonry and for masonry laid in other than running bond with bond beams spaced *not* more than 48 in. center-to-center, b_e is limited to the least of (a) center-to-center bar spacing, (b) six times the nominal thickness of the wall, and (c) 72 in.
2. For masonry laid in other than the running bond, with bond beams spaced *more* than 48 in. (1219 mm) center-to-center, b_e is limited to the length of the masonry unit (e.g., 16 in. for a nominal $8 \times 8 \times 16$ in. concrete masonry unit).

In Item 1 above, the center-to-center maximum spacing is a limit to keep areas of compressive stresses from overlapping each other. However, the 72-in. restriction is both empirical and arbitrary. In Item 2, the limited ability of head joints to transfer stresses when masonry is in stack bond is recognized by the requirements for bond beams.

Engineers familiar with reinforced concrete design would find it instructive to compare the provisions for the effective width of T-beams in the ACI *Code* [6.28]. For reinforced concrete T-beams, the effective width is limited to the least of the following:

1. One-quarter of the span length of the beam ($b_e \leq L/4$)
2. The effective overhanging flange width on each side of the web not to exceed eight times the slab thickness
3. The effective overhanging flange width on each side of the web not to exceed one-half the clear distance to the next web

Obviously, the effective width of a partially grouted wall section (a vertical T-beam) would depend on the spacing of reinforcing bars. For example, depending on the design requirements, the vertical reinforcement in a nominal 16-in. CMU wall may be spaced at 8 in. on centers or at multiples of 8 in. (i.e., at 16, 24, 32, 40, and 48 in. on centers, Fig. 6.36); the maximum permitted spacing being 48 in.

The aforestated limitations on the effective width of a partially grouted wall have important bearing on the flexural resistance of partially grouted walls designed to resist out-of-plane loads. For example, consider a hollow 8-in. (nominal, $t_{actual} = 7.625$ in.) CMU wall in running bond, with reinforcement at 32 in. center-to-center, subjected to out-of-plane loads. The effective width b_e of the wall will be the least of the following:

1. Center-to-center distance between the reinforcement = 32 in.
2. Six times the (nominal) wall thickness = (6)(8) = 48 in.
3. 72 in.

Therefore, in flexural computations, the value of b_e to be used for this 8-in. (nominal) wall would be 32 in. (the least of the above three values). As a second example, consider a 6-in. wall (nominal) reinforced at 48 in. on centers. The effective width b_e of the wall will be the least of the following:

1. Center-to-center distance between the reinforcement = 48 in.
2. Six times the (nominal) wall thickness = (6)(6) = 36 in.
3. 72 in.

Therefore, $b_e = 36$ in. (the least of the three values) will be used for all flexural calculations for this 6-in. (nominal) wall. The steel ratio, ρ ($= A_s/bd$), and the corresponding moment strength (M_u) will be computed with $b_e = 36$ in. However, the out-of-plane load (causing moment and shear in the wall) that must be carried by this T-beam would be computed for a tributary width of 48 in. (center-to-center distance between the reinforcing bars). That is, in essence, 36-in. width of the wall carries out-of-plane load on 48-in. width of the wall. This is the general principle used for designing T-beams.

It is noted that a wall constructed in stack bond is severely penalized. The reason for this is that masonry walls are stronger if the horizontal courses of masonry units are staggered so that the vertical joints are not continuous. A wall with continuous vertical joints (i.e., stack bond) acts more like a series of adjacent, vertical piers, rather than a cohesive unit. The interlocking bond has a further merit of dispersing loads evenly (Figs. 6.19 and 6.31).

Analysis of a T-section depends on the location of the neutral axis which may lie in the flange (i.e., within the face shell of the masonry unit) or in the web. Thus, two cases may be considered:

1. If the depth of compressions block a is less than or equal to the thickness of the face shell, the wall cross section can be analyzed as a rectangular section.
2. If the depth of compressions block a is greater than the thickness of the face shell so that it extends in the web, the section would have to be analyzed as a T-section. This would require that both the face shells *and* portions of webs be considered as contributing to compression area. The analysis in this case can be simplified by considering the compression stress resultant as the sum of two components: one contributed by the flange and the other by a portion of the web (the grouted cell).

It is noted that unless the wall is heavily reinforced, the depth of compression block a would be less than the face shell thickness of the masonry units, in which case the section can be analyzed as a rectangular unit. Table 6.9 gives face shell thicknesses of commonly used concrete masonry units.

T-beam analysis for partially grouted reinforced masonry is similar to that for reinforced concrete sections, which can be found in standard texts on reinforced concrete design.

TABLE 6.9 Face-Shell Thickness of Commonly Used Concrete Masonry Units

Nominal size of CMU (in.)	Actual thickness, t (in.)	Thickness of face shell t_s (in.)
6	5.625	1.0
8	7.625	1.25
10	9.625	1.375
12	11.625	1.5

Example 6.5 illustrates T-beam analysis of a masonry wall section built from nominal 8 × 8 × 16 in. concrete masonry units, which is subjected to out-of-plane loads.

6.7.7 MSJC-08 Provisions for Design of Masonry Walls for Out-of-Plane Loads

MSJC-08 Section 3.3.5.3 presents two different conditions for strength design of walls, although the design procedure under both conditions is identical:

1. When the axial stress due to factored loads is less than or equal to 0.20 f'_m.
2. When the slenderness ratio exceeds 30 (i.e., $h/t > 30$), the axial stress due to factored loads is required to be less than 0.05 f'_m.

The minimum required thickness of walls is 6 in. nominal. The design procedure suggested by the MSJC *Code* is as follows:

1. The design strength should satisfy the following condition [Eq. (6.17), MSJC-08/3-27]:

$$M_u \le \phi M_n \qquad (6.17, \text{MSJC-08/3-27})$$

where $\phi = 0.9$ is the strength reduction factor.

2. Calculate the nominal moment strength from Eq. (6.18):

$$M_n = (A_s f_y + P_u)\left(d - \frac{a}{2}\right) \qquad (6.18, \text{MSJC-08/3-28})$$

where

$$a = \frac{P_u + A_s f_y}{0.08 f'_m b} \qquad (6.19, \text{MSJC-08/3-29})$$

The quantity in the numerator of Eq. (6.18) may be considered as effective area of reinforcement given by Eq. (6.20):

$$A_{se} = \frac{P_u + A_s f_y}{f_y} \qquad (6.20)$$

3. The nominal shear strength of the wall should be determined as discussed in Chap. 4.

4. The horizontal midheight deflection δ_s, due to service lateral loads and service axial loads (without load factors), should be limited to value given by relationship in Eq. (6.21):

$$\delta_s \le 0.007h \qquad (6.21, \text{MSJC-08/3-30})$$

5. The midheight deflection (assuming that simple-support end conditions apply) is to be calculated from Eqs. (6.22) or (6.23) as applicable:

(a) When the moment due to service loads is less than the cracking moment (i.e., $M_{ser} < M_{cr}$)

$$\delta_{ser} = \frac{5M_{ser}h^2}{48E_mI_g} \qquad (6.22, \text{MSJC-08/3-31})$$

(b) When the service load moment is greater than the cracking moment but less than the nominal moment strength M_n (i.e., $M_{cr} < M_{ser} < M_n$)

$$\delta_{ser} = \frac{5M_{cr}h^2}{48E_mI_g} + \frac{5(M_{ser} - M_{cr})h^2}{48E_mI_{cr}} \qquad (6.23, \text{MSJC-08/3-32})$$

Note that the cracking moment M_{cr} is to be determined based on the modulus of rupture, f_r, as discussed in Chap. 4. Values of modulus of rupture are listed in MSJC-08 Table 3.1.8.2. Note that the moment of inertia of the cracked section would be based on the effective area of steel, A_{se}, given by Eq. (6.25). The expression for the moment of inertia of the cracked section can be expressed as

$$I_{cr} = \frac{bc^3}{3} + nA_{se}(d-c)^2 \qquad (6.24)$$

where $b = 12$ in. (unit width of wall)
c = depth of neutral axis from the compression face of the wall
d = depth of the centroid of tensile reinforcement
n = modular ratio, and

$$A_{se} = \frac{P_u + A_sf_y}{f_y} \qquad (6.20 \text{ repeated})$$

Examples 6.2 and 6.3 illustrate, respectively, analysis and design of reinforced masonry walls subjected to out-of-plane loads as discussed above. It would be noted that the basic procedure in both cases is identical. Typically, an analysis problem involves checking the adequacy of a wall for its capacity to carry gravity and lateral loads, and for deflection, all design details of the wall being known. In the case of a typical design problem, all loads (gravity and lateral) and eccentricity of gravity load would be known, and the wall thickness and its compressive strength dimensions would have been preselected, for example, an 8-in. nominal thick concrete masonry wall and the compressive strength (f'_m) of 2000 lb/in.^{2}. The only unknown to be determined would be the size and spacing of vertical reinforcing bars. One can proceed with design by assuming a vertical bar size (e.g., No. 5 or No. 6) and horizontal spacing of these bars (assuming that the wall is spanning vertically). The calculations then would be similar to those in Example 6.2. If the selected bar size and spacing do not prove to be adequate, one should try a larger or smaller bar at the same or different spacing as necessary. This step should be repeated until a satisfactory design is obtained.

Example 6.2 A 20-ft-high, 8-in.-nominal, solidly grouted, concrete masonry wall is centrally reinforced with No. 6 vertical bars (Grade 60) spaced horizontally at 24 in. on center. The wall weighs 84 lb/ft^2 per linear foot of the wall. It carries a superimposed dead roof load of 160 lb/ft and a roof live load of 60 lb/ft length of the wall. The roof is supported on a ledger beam attached to the wall, which creates an eccentricity of 4 in. from the center line of the wall (Fig. E6.2). The lateral load due to wind is 20 lb/ft^2 and due to earthquake 42 lb/ft^2. Check the adequacy of the wall to carry gravity and lateral loads, and deflection $f'_m = 2000$ lb/in.^{2}.

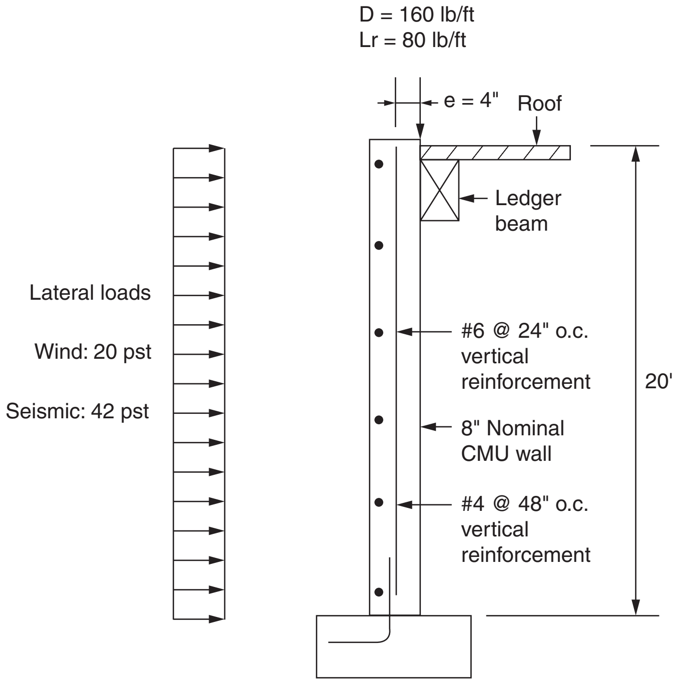


FIGURE E6.2

Solution

Commentary: The data given in this problem have been taken from Example 6.1 which illustrated calculations for lateral loads (wind and seismic) on the wall. A complete solution of a problem involving analysis of a wall would be combination of solution of Examples 6.1, and 6.2 presented as follows. The wall in this example does not have a parapet.

Given:

Wall height = 20 ft
Wall thickness = 8 in. nominal (7.625 in. actual)
Eccentricity of superimposed load = 4 in.
Roof dead load = 160 lb/ft of wall
Roof live load = 80 lb/ft of wall
Dead weight of wall at midheight = $84(^1/_2 \times 20) = 840$ lb
Wind load = 20 lb/ft^2 on the wall area (normal to wall) ASD-level load
Seismic load $F_p = 42$ lb/ft^2 (normal to wall) strength-level load
$f'_m = 2000$ lb/in^2
$E_m = (900)(f'_m) = (900)(2000) = 1{,}800{,}000$ lb lb/in.^{2} = 1800 kips/in.^{2}

Convert seismic load to ASD-level load to compare it with the wind load. ASD-level seismic load

$$w = 0.7(42) = 29.4 \text{ lb/ft}^2 > 20 \text{ lb/ft}^2 \text{ (wind)}$$

Hence, seismic load governs. Calculate service loads according to ASCE 7-05 provisions for ASD load combinations. Load combinations to be considered are:

1. Dead load + roof live load (ASD Load Combination 3)
2. Dead load + wind load (ASD Load Combination 5a)
3. Dead load + 0.7 earthquake (ASD Load Combination 5b)
4. Dead load + 0.75(0.7E) + 0.75Lr (ASD Load Combination 6)

Service loads:

$$\text{Superimposed loads } P_f = 0.16 + 0.08 = 0.24 \text{ k/ft of wall}$$

Factored loads:

$$P_{uf} = 1.2D + 1.6L_r = 1.2\,(0.16) + 1.6\,(0.08) = 0.32 \text{ k/ft of wall}$$

Wall weight at midheight $P_{uw} = 1.2\,(0.084)\,(10) = 1.008$ k/ft

$$P_u = P_{uf} + P_{uw} = 0.32 + 1.008 = 1.328 \text{ k/ft} = 1328 \text{ lb/ft}$$

Lateral load on wall due to wind $w_u = 1.6\,(20) = 32$ lb/ft^2 of wall area

Lateral load on wall due to earthquake $w_u = 42$ lb/ft^2 of wall area $>1.6 \times 20 = 32$ lb/ft^2 (wind)

Therefore, seismic lateral load governs.

Area of cross section of 1-ft length of wall:

Wall thickness $t = 7.625$ in. (8 in. nominal)

$$A_g = 12(7.625) = 91.5 \text{ in.}^2$$

Moment of inertia of gross section,

$$I_g = \frac{bt^3}{12} = \frac{(12)(7.625)^3}{12} = 443.3 \text{ in.}^4$$

Check P_u/A_g and compare it with $0.05f'_m$.

$$\frac{P_u}{A_g} = \frac{1328}{91.5} = 14.5 \text{ lb/in.}^2$$

$$0.05f'_m = 0.05(2000) = 100 \text{ lb/in.}^2$$

$P_u/A_g < 0.05f'_m$, hence, special conditions do not apply.

Calculate deflection, δ_{ser}. First, check if M_{ser} is less than or greater than M_{cr}.

$$M_{\text{ser}} = \frac{wL^2}{8} + P_u\left(\frac{e}{2}\right) = 0.294\left(\frac{20^2}{8}\right) + 0.24\left(\frac{4}{2}\right) = 14.7 + 0.48 = 15.18 \text{ k-in.}$$

$$M_{\text{cr}} = f_r S$$

$f_r = 163$ lb/in.^{2} (MSJC-08 Table 3.1.8.2)

$$S = \frac{bd^2}{6} = \frac{(12)(7.625)^2}{6} = 116.3 \text{ in.}^3$$

$$M_{\text{cr}} = (0.163)(116.3) = 18.96 \text{ k-in.} > M_{\text{ser}} = 15.18 \text{ k-in.}$$

Because $M_{\text{ser}} < M_{\text{cr}}$, the wall is not cracked at the service load condition. Therefore, use Eq. (6.22) to calculate deflection.

$$\delta_{\text{ser}} = \frac{5M_{\text{ser}}h^2}{48E_m I_g} = \frac{5(15.18)(20\times12)^2}{48(1800)(443.3)} = 0.114 \text{ in.}$$

Maximum allowable $\delta_s = 0.007h = 0.007(20 \times 12) = 1.68$ in. > 0.114 in.

The service load deflection (0.136 in.) is much smaller than the permissible deflection (1.68 in.). Hence, the service load deflection is OK without P-Δ effects. Calculate P-Δ effects.

Additional M_{ser} caused by $\delta_{\text{ser}} = P_u\,\delta_{\text{ser}} = 1.328(0.114) = 0.15$ k-in.

Calculate moments due to factored loads.

$$M_u = \frac{w_u L^2}{8} + P_u\left(\frac{e}{2}\right) = \frac{0.042(20^2(12)}{8} + 0.32\left(\frac{4}{2}\right)$$
$$= 25.2 + 0.64 = 25.84 \text{ k-in.} > M_{\text{cr}} = 18.96 \text{ k-in.}$$

Because $M_u > M_{\text{cr}}$, the wall is cracked; therefore, use Eq. (6.23) to calculate δ_u. Calculate the moment of inertia of the cracked section, I_{cr}, given by Eq. (6.24):

$$I_{\text{cr}} = \frac{bc^3}{3} + nA_{\text{se}}(d-c)^2$$

For No. 6 bar @ 24 in. on center, the reinforcement per unit width of wall,

$$A_s = 0.20 \text{ in.}^2 \text{ (Table A.23)}$$

$$A_{\text{se}} = \frac{P_u + A_s f_y}{f_y} = \frac{1.328 + 0.20(60)}{60} = 0.222 \text{ in.}^2$$

$$n = \frac{E_s}{E_m} = \frac{29{,}000}{1800} = 16.11 \text{ (or from Table A.14)}$$

$$\rho = \frac{A_{\text{se}}}{bd} = \frac{0.222}{(12)(3.81)} = 0.0048$$

$$\rho n = (0.0048)(16.11) = 0.078$$

$$k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n$$
$$= \sqrt{(0.078)^2 + 2(0.078)} - 0.078$$
$$= 0.325$$

(Alternatively, from Table A.15, $n\rho = 0.078$ and $k = 0.3246 \approx 0.325$ in.)

$$c = kd = (0.325)(3.81) = 1.24 \text{ in.} < 1.25 \text{ in. (shell thickness)}$$

Hence, the neutral axis lies in the shell, and the section can be treated as a rectangular section.

$$I_{\text{cr}} = \frac{bc^3}{3} + nA_{\text{se}}(d-c)^2$$

$$= \frac{(12)(1.24)^3}{3} + (16.11)(0.222)(3.81-1.24)^2$$

$$= 31.25 \ \text{in}^4$$

First iteration:

$$\delta_u = \frac{5M_{\text{cr}}h^2}{48E_m I_g} + \frac{5(M_{\text{ser}} - M_{\text{cr}})h^2}{48E_m I_{\text{cr}}}$$

$$= \frac{5(18.96)(20\times 12)^2}{48(1800)(443.3)} + \frac{5(25.84-18.96)(20\times 12)^2}{48(1800)(31.25)}$$

$$= 0.143 + 0.734$$

$$= 0.877 \text{ in.} < 0.00h = 1.68 \text{ in.} \qquad \text{OK}$$

Calculate moment due to new δ_u.

$$M_{u2} = 25.84 + 1.328(0.877) = 27.0 \text{ k-in.}$$

The new value of moment, 27.0 k-in., is very close to the first value of 25.84 k-in.; therefore, further iteration is not necessary. Calculate the nominal moment, M_n.

$$M_n = (A_s f_y + P_u)\left(d - \frac{a}{2}\right) \qquad \text{(6.18, MSJC-08/3-28)}$$

where

$$a = \frac{P_u + A_s f_y}{0.08 f'_m b} = \frac{1.328 + 0.20(60)}{0.80(2.0)(12)} = 0.694 \ \text{in.} < 1.25 \text{ in. (shell thickness)}$$

Hence, the compression block lies in the shell, and the section can be treated as a rectangular section.

$$M_n = (A_s f_y + P_u)\left(d - \frac{a}{2}\right)$$

$$= [(0.20)(60) + 1.328]\left(3.81 - \frac{0.694}{2}\right)$$

$$= 46.15 \text{ k-in.}$$

$$\phi M_n = (0.9)(46.15) = 41.54 \text{ k-in.} > M_{u2} = 27.0 \text{ k-in.} \qquad \text{OK}$$

Therefore, the wall is adequately reinforced.

Example 6.3 A 20-ft-high, 8-in.-nominal, solidly grouted, concrete masonry wall is to be designed to resist out-of-plane load due to wind = 20 lb/ft^2 and due to earthquake = 42 lb/ft^2 (Fig. E6.3A). The wall weighs 84 lb/ft^2 per linear foot and carries a superimposed dead roof load of 350 lb/ft and a roof live load of 240 lb/ft length of the wall. The roof is supported on a ledger beam attached to the wall, which creates an eccentricity of 4 in. from the center line of the wall. Determine the reinforcement (Grade 60) requirements for this wall. Assume that the wall is classified as SDC D and $f'_m = 2000$ lb/in.^{2}.

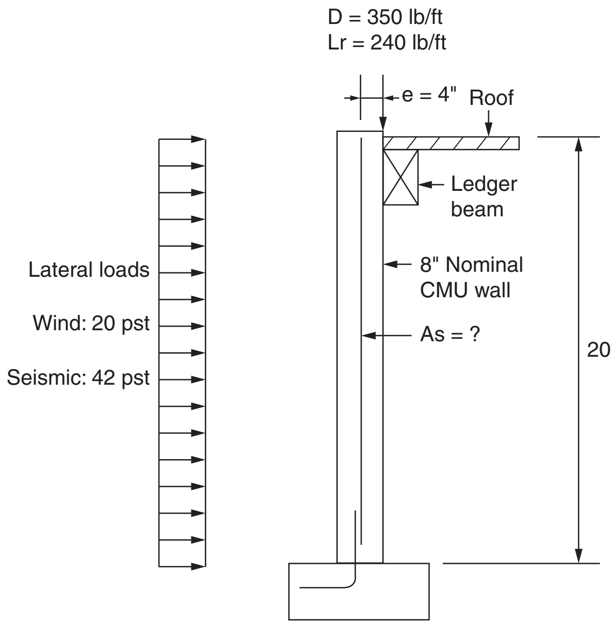


FIGURE E6.3A

Solution

Given:

Wall height = 20 ft
Wall thickness = 8 in. nominal (7.625 in. actual)
Eccentricity of superimposed load = 4 in.
Roof dead load = 350 lb/ft of wall
Roof live load = 240 lb/ft of wall
Dead weight of wall at midheight = $84(^1/_2 \times 20) = 840$ lb
Wind load = 20 lb/ft^2 on the wall area (normal to wall), ASD-level load
Seismic load $F_p = 42$ lb/ft^2 (normal to wall), strength-level load

$f'_m = 2000\ \text{lb/in.}^2$

$E_m = (900)(f'_m) = (900)(2000) = 1{,}800{,}000 \text{ lb lb/in.}^2 = 1800 \text{ kips/in.}^2$

Convert seismic load to ASD-level load to compare it with the wind load. ASD-level seismic load = 0.7(42) = 29.4 lb/ft^2 > 20 lb/ft^2 (wind).

Hence, seismic load governs.
Calculate service loads according to ASCE 7-05 provisions for ASD load combinations.
Load combinations to be considered are :

1. Dead load + roof live load (ASD Load Combination 3)
2. Dead load + wind load (ASD Load Combination 5a)
3. Dead load + 0.7 earthquake (ASD Load Combination 5b)
4. Dead load + 0.75(0.7E) + 0.75L_r (ASD Load Combination 6)

Service loads: Superimposed loads: $P_f = 0.35 + 0.24 = 0.59$ k/ft of wall

Factored loads:

$$P_f = 1.2D + 1.6L_r = 1.2\ (0.35) + 1.6\ (0.24) = 0.804 \approx 0.80 \text{ k/ft of wall}$$

Wall weight at midheight $P_{uw} = 1.2\ (0.084)\ (10) = 1.008$ k/ft

$$P_u = P_{uf} + P_{uw} = 0.80 + 1.008 = 1.81 \text{ k/ft} = 1810 \text{ lb/ft}$$

Lateral load on wall due to wind $w_u = 1.6\ (20) = 32$ lb/ft^2 of wall area
Lateral load on wall due to earthquake $w_u = 42$ lb/ft^2 of wall area > 32 lb/ft^2 (wind)
Therefore, seismic lateral load governs.
Area of cross section of 1-ft length of wall:
Wall thickness $t = 7.625$ in. (8 in. nominal)

$$A_g = 12(7.625) = 91.5 \text{ in.}^2$$

Moment of inertia of gross section,

$$I_g = \frac{bt^3}{12} = \frac{(12)(7.625)^3}{12} = 443.3 \text{ in.}^4$$

Check P_u/A_g and compare it with $0.05f'_m$.

$$\frac{P_u}{A_g} = \frac{1810}{91.5} = 20 \text{ lb/in.}^2$$

$$0.05f'_m = 0.05(2000) = 100 \text{ lb/in.}^2$$

$P_u/A_g < 0.05f'_m$, hence, special conditions do not apply.

Calculate deflection, δ_{ser}. First, check if M_{ser} is less than or greater than M_{cr}.

$$M_{\text{ser}} = \frac{w_u L^2}{8} + P_u\left(\frac{e}{2}\right) = 0.294\left(\frac{20^2}{8}\right) + 0.59\left(\frac{4}{2}\right) = 18.82 \text{ k-in.}$$

$$M_{\text{cr}} = f_r S$$

$$f_r = 163 \text{ lb/in.}^2 \qquad \text{(MSJC-08 Table 3.1.8.2)}$$

$$S = \frac{bd^2}{6} = \frac{(12)(7.625)^2}{6} = 116.3 \text{ in.}^3$$

$$M_{\text{cr}} = (0.163)(116.3) = 18.96 \text{ k-in.} > M_{\text{ser}} = 18.82 \text{ k-in.}$$

Because $M_{ser} < M_{cr}$, the wall is not cracked at the service load. Therefore, use Eq. (6.21) to calculate deflection.

$$\delta_{ser} = \frac{5M_{ser}h^2}{48E_m I_g} = \frac{5(18.82)(20\times 12)^2}{48(1800)(443.3)} = 0.14 \text{ in.}$$

Maximum allowable $\delta_s = 0.007h = 0.007(20 \times 12) = 1.68$ in. > 0.14 in.

The service load deflection (0.14 in.) is much smaller than the permissible deflection (1.68 in.). Hence, the service load deflection is OK without P-Δ effects.

Calculate P-Δ effects.

Additional M_{ser} caused by $\delta_{ser} = P_u\,\delta_{ser} = 1.81(0.14) = 0.252$ k-in.

Calculate moments due to factored loads.

$$M_u = \frac{w_u L^2}{8} + P_u\left(\frac{e}{2}\right) = \frac{0.042(20^2)(12)}{8} + 1.81\left(\frac{4}{2}\right)$$
$$= 28.82 \text{ k-in.} > M_{cr} = 18.96 \text{ k-in.}$$

Because $M_u > M_{cr}$, the wall is cracked; therefore, use Eq. (6.23) to calculate δ_u. Calculate the moment of inertia of the cracked section, where I_{cr} is given by Eq. (6.2).

$$I_{cr} = \frac{bc^3}{3} + nA_{se}(d-c)^2 \qquad \text{(6.23 repeated)}$$

Try No. 4 bar @ 24 in. on center, the reinforcement per unit width of wall,

$$A_s = 0.10 \text{ in.}^2 \text{ (Table A.23)}$$

$$A_{se} = \frac{P_u + A_s f_y}{f_y} = \frac{1.81 + 0.10(60)}{60} = 0.13 \text{ in.}^2$$

$$n = \frac{E_s}{E_m} = \frac{29{,}000}{1800} = 16.11 \qquad \text{(or from Table A.14)}$$

$$\rho = \frac{A_{se}}{bd} = \frac{0.13}{(12)(3.81)} = 0.00284$$

$$\rho n = (0.00284)(16.11) = 0.046$$

From Table A.15, for $n\rho = 0.046$, $k = 0.2608$

$$c = kd = (0.2608)(3.81) \approx 1.0 \text{ in.} < 1.25 \text{ in. (shell thickness)}$$

Hence, the centroidal axis lies in the shell and the section can be treated as a rectangular section.

$$I_{cr} = \frac{bc^3}{3} + nA_{se}(d-c)^2$$
$$= \frac{(12)(1.0)^3}{3} + (16.11)(0.13)(3.81 - 1.0)^2$$
$$= 20.54 \text{ in.}^4$$

First iteration:

$$\delta_{\text{ser}} = \frac{5M_{\text{cr}}h^2}{48E_m I_g} + \frac{5(M_{\text{ser}} - M_{\text{cr}})h^2}{48E_m I_{\text{cr}}}$$

$$= \frac{5(18.96)(20^2)(12)}{48(1800)(443.3)} + \frac{5(28.82 - 18.96)(20 \times 12)^2}{48(1800)(20.54)}$$

$$= 0.143 + 1.6$$

$$= 1.743 \text{ in.} > 0.00h = 1.68 \text{ in.} \qquad \text{NG}$$

Therefore, increase wall reinforcing so as to increase I_{cr} which would reduce δ_{ser}. Try No. 4 vertical bars @ 16 in. on center; $A_s = 0.15$ in.^{2} (Table A.23).

$$A_{\text{se}} = \frac{P_u + A_s f_y}{f_y} = \frac{1.81 + 0.15(60)}{60} = 0.18 \text{ in.}^2$$

$$n = 16.11 \text{ (Table A.14)}$$

$$\rho = \frac{A_{\text{se}}}{bd} = \frac{0.18}{(12)(3.81)} = 0.0039$$

$$\rho n = (0.0039)(16.11) = 0.063$$

From Table A.15, $n\rho = 0.063$, $k = 0.2975$

$$c = kd = (0.2975)(3.81) \approx 1.13 \text{ in.} < 1.25 \text{ in. (shell thickness)}$$

Therefore, the centroidal axis lies in the shell and the section can be treated as a rectangular section.

$$I_{\text{cr}} = \frac{bc^3}{3} + nA_{\text{se}}(d - c)^2$$

$$= \frac{(12)(1.13)^3}{3} + (16.11)(0.18)(3.81 - 1.13)^2$$

$$= 26.6 \text{ in.}^4$$

$$\delta_{\text{ser}} = \frac{5M_{\text{cr}}h^2}{48E_m I_g} + \frac{5(M_{\text{ser}} - M_{\text{cr}})h^2}{48E_m I_{\text{cr}}}$$

$$= \frac{5(18.96)(20^2)(12)}{48(1800)(443.3)} + \frac{5(28.82 - 18.96)(20 \times 12)^2}{48(1800)(26.6)}$$

$$= 0.143 + 1.236$$

$$= 1.379 \text{ in.} < 0.00h = 1.68 \text{ in.} \qquad \text{OK}$$

Calculate moment due to new δ_{u1}

$$M_{u2} = 28.82 + 1.81(1.379) = 31.32 \text{ k-in}$$

$$\delta_{se2} = \frac{5 M_{cr} h^2}{48 E_m I_g} + \frac{5(M_{ser} - M_{cr}) h^2}{48 E_m I_{cr}}$$

$$= \frac{5(18.96)(20^2)(12)}{48(1800)(443.3)} + \frac{5(31.32 - 18.96)(20 \times 12)^2}{48(1800)(26.6)}$$

$$= 0.143 + 1.549$$

$$= 1.692 \text{ in.} \approx 0.00h = 1.68 \text{ in.} \qquad \text{OK}$$

Calculate moment due to new δ_{u2}

$$M_{u2} = 28.82 + 1.81(1.692) = 31.88 \text{ k-in.}$$

The new value of moment, 31.88 k-in., is very close to the previous value of 31.32 k-in.; hence further iteration is not necessary. Use M_u = 31.88 k-in. Calculate the nominal moment, M_n, from Eq. (6.17):

$$M_n = (A_s f_y + P_u)\left(d - \frac{a}{2}\right)$$

where

$$a = \frac{P_u + A_s f_y}{0.08 f'_m b} = \frac{1.81 + 0.15(60)}{0.80(2.0)(12)} = 0.56 \text{ in.} < 1.25 \text{ in. (shell thickness)}$$

Hence, the compression block lies in the shell, and the section can be treated as a rectangular section.

$$M_n = (A_s f_y + P_u)\left(d - \frac{a}{2}\right)$$

$$= [1.81 + (0.15(60)]\left(3.81 - \frac{0.56}{2}\right)$$

$$= 38.16 \text{ k-in.}$$

$$\phi M_n = 0.9\ (38.16) = 34.34 \text{ k-in.} > M_u = 31.88 \text{ k-in.} \qquad \text{OK}$$

Therefore, No. 4 vertical bars spaced at 16 in. on center horizontally are adequate for resisting the seismic lateral load.

MSJC-08 (Section 1.17.3.2.6) requires a minimum reinforcement area (sum of vertical and horizontal reinforcement) of 0.002*bt* of the gross sectional area of the wall for structures in SDC D, with reinforcement no less than 0.0007*bt* of gross sectional area in any one direction.

$$A_{s,\,\min} = 0.002bt = 0.002(12)(7.625) = 0.183 \text{ in.}^2$$

$$0.0007bt = 0.0007(12)(7.625) = 0.064 \text{ in.}^2$$

Provide No. 4 horizontal bars spaced at 24 in. on center vertically; $A_s = 0.1$ in.^{2} > 0.064 in.^{2}.

$$\text{Total } A_s = 0.15 + 0.064 = 0.214 \text{ in.}^2 > A_{s,\min} = 0.183 \text{ in.}^2 \qquad \text{OK}$$

Reinforcement details are shown in Fig. E6.3B.

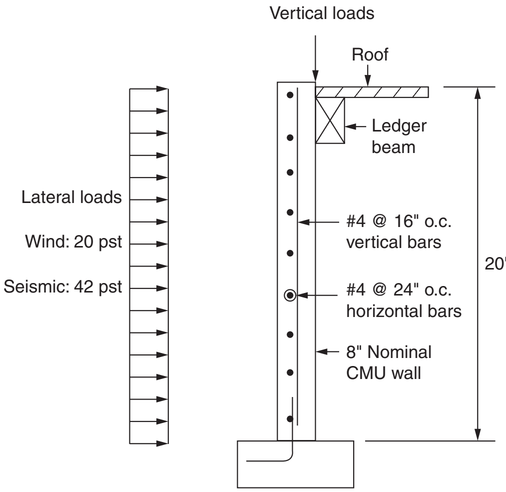


FIGURE E6.3B Reinforcement details for wall.

Example 6.4 Analysis of a T-section of a Wall under Flexure: Rectangular Beam Analysis.

A nominal 8-in.-wide concrete masonry wall is reinforced with No. 6 Grade 60 bars placed vertically at 32 in. center-to-center (Fig. E6.4). The masonry is running bond and $f'_m = 2000$ lb/in^2. Calculate the moment strength of wall which is subjected to out-of-plane loads.

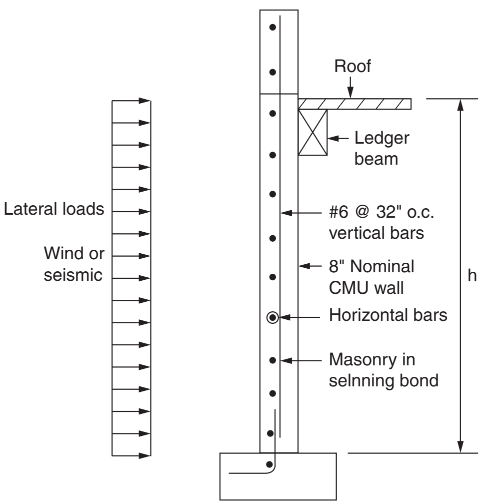


FIGURE E6.4

Solution

Estimate the area of masonry in compression. Assume that the depth of compression block equals the face-shell thickness of the masonry unit. For nominal 8 × 8 × 16 in. nominal concrete masonry unit, the face-shell is $1^1/_4$ in. thick.

$$t_s = 1^1/_4 \text{ in.} \qquad a = 1^1/_4 \text{ in.}$$

Effective width b_e is the smallest of

(a) Center-to-center of reinforcement = 32 in. (governs).
(b) Six times the nominal thickness of masonry unit = 6 × 8 = 48 in.
(c) 72 in.

Use $$b_e = 32 \text{ in.}$$

Area of masonry in compression = $b_e t_s = 32(1.25) = 40$ in.^{2}
Compression stress resultant:

$$C = 0.80 \quad f'_m ab = 0.80(2.0)(40) = 64.0 \text{ kips}$$

Tensile force resultant,

$$T = A_s f_y = 0.60(60) = 36.0 \text{ kips} < C = 64 \text{ kips}$$

Hence, the depth of the compression block is less than $1^1/_4$ in. (= t_s) as assumed, and it lies within the face-shell.

Accordingly, the section can be analyzed as a rectangular section. Check the capacity of the section:

$$M_n = \frac{P_u + A_s f_y}{0.80 f'_m b}$$

$$P_u = 1.2(200) + 1.6(80) = 368 \text{ lb/ft} = 0.368 \text{ k/ft}$$

On a width of 32 in. (effective width),

$$P_u = 0.368\left(\frac{32}{12}\right) = 0.98 \text{ kip}$$

$$A = \frac{0.98 + 0.60(60)}{0.80(2.0)(32)} = 0.72 \text{ in.}$$

From Eq. (6.18),

$$M_n = (A_s f_y + P_u)\left(d - \frac{a}{2}\right)$$

$$M_n = (0.6 \times 60 + 0.98)\left(3.81 - \frac{0.72}{2}\right)$$

$$= 127.58 \text{ k-in.} = 10.63 \text{ k-ft}$$

The 10.63 k-ft moment is resisted by wall length of 32 in. (effective width). Therefore, the nominal moment strength of wall per linear foot is

$$M_n = (10.63)\left(\frac{12}{32}\right) = 3.99 \text{ k-ft}$$

$$\phi M_n = 0.9(3.99) = 3.59 \text{ k-ft per linear ft of the wall.}$$

Example 6.5 Analysis of a T-Section of a Wall under Flexure: T-beam Analysis.

A nominal 8-in.-wide concrete masonry wall is reinforced with No. 8 Grade 60 bars placed vertically at 24 in. center-to-center (Fig. E6.5A). It carries a roof dead load of 400 lb/ft and live load of 250 lb/ft. The masonry is in running bond and $f'_m = 1500 \text{ lb/in}^2$. Calculate the moment strength of the wall which is subjected to out-of-plane loads.

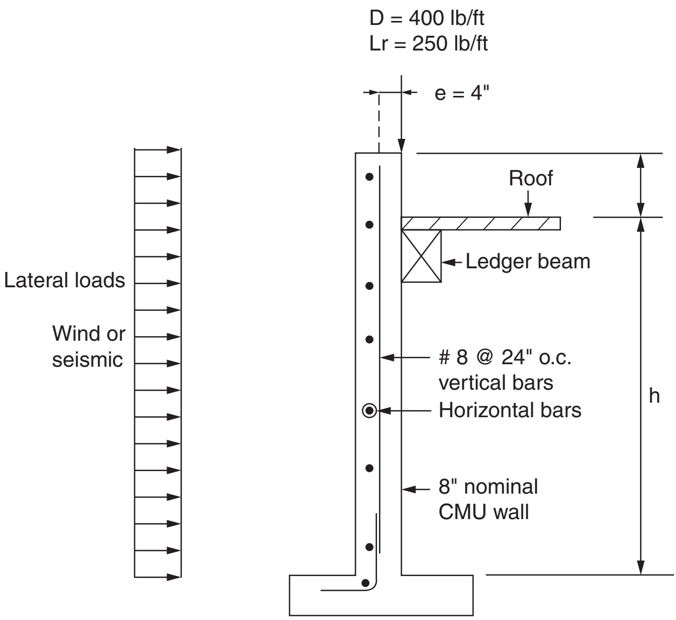


FIGURE E6.5A

Solution

Estimate the area of masonry in compression. Assume that depth of compression block equals the face-shell thickness, $t_s = 1.25$ in.

The effective width, b_e, is the smallest of

(a) Center-to-center of reinforcement = 24 in. (governs)
(b) Six times the nominal thickness of masonry unit = $6 \times 8 = 48$ in.
(c) 72 in.

$$\text{Use } b_e = 24 \text{ in.}$$

Area of masonry in compression = bets = 24 (1.25) = 30 in.^{2}
Compression stress resultant:

$$C = 0.80 \qquad f'_m ab = 0.80(1.5)(30) = 36 \text{ kips}$$

Tensile force resultant,

$$T = A_s f_y = 0.79(60) = 47.4 \text{ kips} > 36 \text{ kips}$$

Because $C < T$, additional compressive force is required to maintain equilibrium, which must be provided by additional masonry area in compression (in the web).

Additional compression force required (provided by the web)

$$C_w = 47.4 - 36 = 11.4 \text{ kips}$$

$$f'_m = 1.5/\text{in.}^2$$

$$\text{Additional area required} = \frac{C_w}{0.80 f'_m} = \frac{11.4}{0.80(1.5)} = 9.5 \text{ in.}^2$$

$$b_e = 24 \text{ in.}$$

In a length of $b_e = 24$ in., the total thickness of the cross-walls and the grouted cells = 6 in. (grouted cells) + 3 × 1.25 in. (cross-walls) + 1.5 in. (web) = 11.25 in. (see Fig. 6.36).

$$\text{Depth of web required} = \frac{9.5}{11.25} = 0.84 \text{ in.}$$

The entire masonry area in compression is shown shaded in Fig. E6.5B, the moment strength of the wall can be calculated by taking moments of compression stress resultants in the face-shell and the cross-walls and core about the centroid of reinforcing bars.

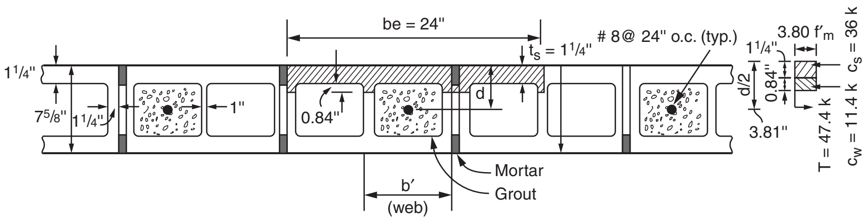


FIGURE E6.5B

$$M_{n1} = 0.80 f'_m ab\left(d - \frac{a}{2}\right)$$

$$= 0.80(1.5)(1.0)(24.0)\left(3.81 - \frac{1.25}{2}\right)$$

$$= 91.73 \text{ k-in.}$$

$$M_{n2} = 0.80(1.5)(11.25)(0.84)\left(3.81 - 1.25 - \frac{0.84}{2}\right)$$

$$= 24.27 \text{ k-in.}$$

$$M_n = M_{n1} + M_{n2} = 91.73 + 24.27 = 116 \text{ k-in.}$$

$$\phi M_n = 0.9(116) = 104.4 \text{ k-in.} = 8.7 \text{ k-ft.}$$

A moment of 8.7 k-ft is resisted by 24-in. (2 ft) length of wall.

$$\text{Moment per linear foot of wall} = \frac{8.7}{2} = 4.35 \text{ k-ft}$$

Commentary: In this example, the expression a = $(P_u + A_s f_y)/(0.80 f'_m b)$ has not been used as the compression depth of masonry extends beyond the face-shell thickness of masonry units. It is noted that contribution of the term P_u is very small as compared with the term $A_s f_y$; thus, the resulting error is negligible.

6.8 AXIAL LOADS ON WALLS SUBJECTED TO OUT-OF-PLANE LOADS

When walls are subjected to axial loads in addition to flexure due to out-of-plane loads, provisions discussed in Chap. 5 for columns apply. MSJC Section 3.3.5.3 requires that factored moment and axial loads to be used for design be determined at the midheight of the wall. The design procedure permitted under this provision (illustrated in forgoing examples) applies when the factored axial load stress at the location of maximum moment satisfies the requirement given by Eq. (6.25):

$$\left(\frac{P_u}{A_g}\right) = 0.20 f'_m \tag{6.25}$$

The term on the left side of Eq. (6.25) represents factored axial stress. When the slenderness ratio exceeds 30, the factored axial stress should not exceed 0.05 f'_m.

6.9 PILASTERS

6.9.1 Introduction

Analysis and design of reinforced masonry walls was discussed in several previous sections. When loads on walls are not too large, or when the walls themselves are not too tall, common types of masonry walls can be used to carry loads. However, for carrying large loads, or when the walls are especially tall, short segments of wall may be thickened at locations where concentrated gravity loads are applied. These short, thickened segments of walls are called *pilasters*. The increased thickness is achieved by projecting masonry from one side of the wall (single-projection or single-face pilaster, Fig. 6.37*a*) or both sides of the wall (double-projection or double-face pilaster, Fig.6.37*b*). Alternatively, pilasters may also be hidden *within* the wall, in which case their thickness being equal to the wall thickness. Such pilasters are variously referred to as *flush pilasters, in-wall pilasters,* or *internal pilaster* (Fig. 6.37*c*). Pilasters may be constructed from solid or hollow units, grouted hollow units or reinforced hollow units.

Several types and sizes of special concrete blocks that can be used in various combinations to build both the isolated columns and the pilasters are available. To achieve the desired cross-sectional configurations, special pilaster units (Fig. 2.11) are manufactured to give the required wall width along with the required wall projection. Typical modular construction schemes of single-projection and double-projection pilasters of several sizes are shown, respectively, in Figs. 6.38 and 6.39.

A pilaster can serve several functions:

- The thickened wall segment (i.e., the pilaster) provides larger cross-sectional area to carry loads. As such, it can be used as a column to support concentrated loads transmitted by structural members such as beams and trusses. If the loads are eccentric, the eccentricity is measured in a direction perpendicular to wall.

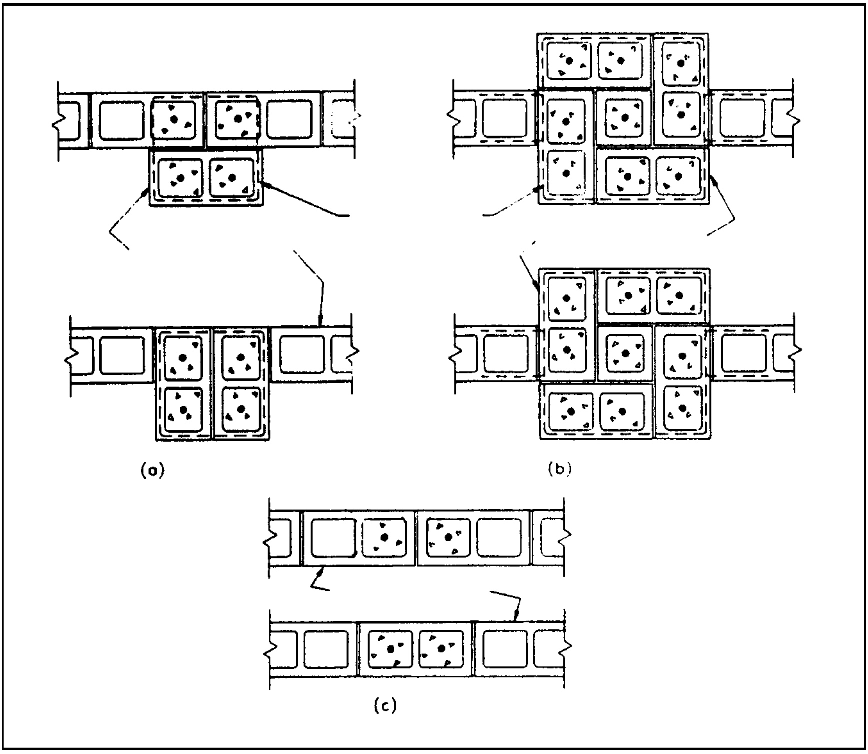


FIGURE 6.37 Pilaster configurations: (*a*) single-projection pilaster, (*b*) double-projection pilaster, (*c*) hidden (in-wall) pilaster [6.31].

- It increases wall's resistance to buckling. The increased stability helps better support uniform loads on the wall.
- A pilaster can act integrally with the wall and act as a vertical beam supported between the top and bottom horizontal supports. In conjunction with wall, it can support lateral loads acting on the wall.
- Pilasters can increase the stability of a freestanding wall such as a fence wall (Fig. 6.14*a*).

A pilaster may be constructed as an integral part of the wall. Alternatively, it may be constructed as an unbonded member in which case it provides lateral stability to wall through the use of suitable connections. The later alternative is used when control joints are provided for crack control, and which structurally isolate the pilaster from the wall. Typical construction details for 16-in. and 24-in. pilasters are shown in Figs. 6.40 and 6.41, respectively.

6.9.2 Design Considerations

Design methodology used for pilasters depends on the functions they are intended to serve. Often they are intended to carry gravity loads imposed on them from beams or other framing

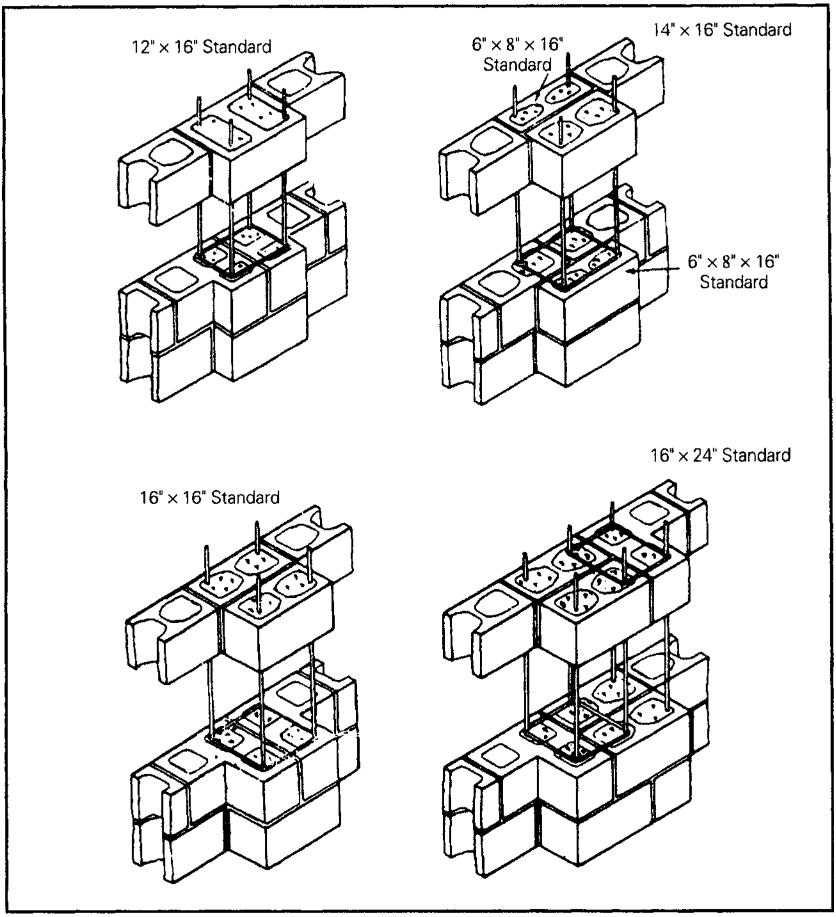


FIGURE 6.38 Standard concrete pilaster units for single-projection pilasters [6.31].

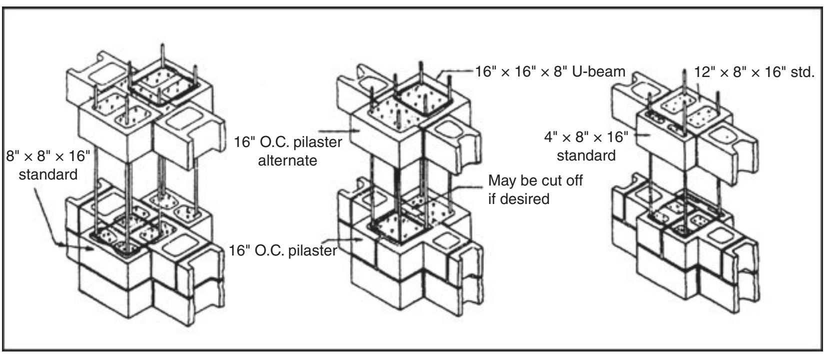


FIGURE 6.39 Standard concrete masonry units for double-projection pilasters [6.31].

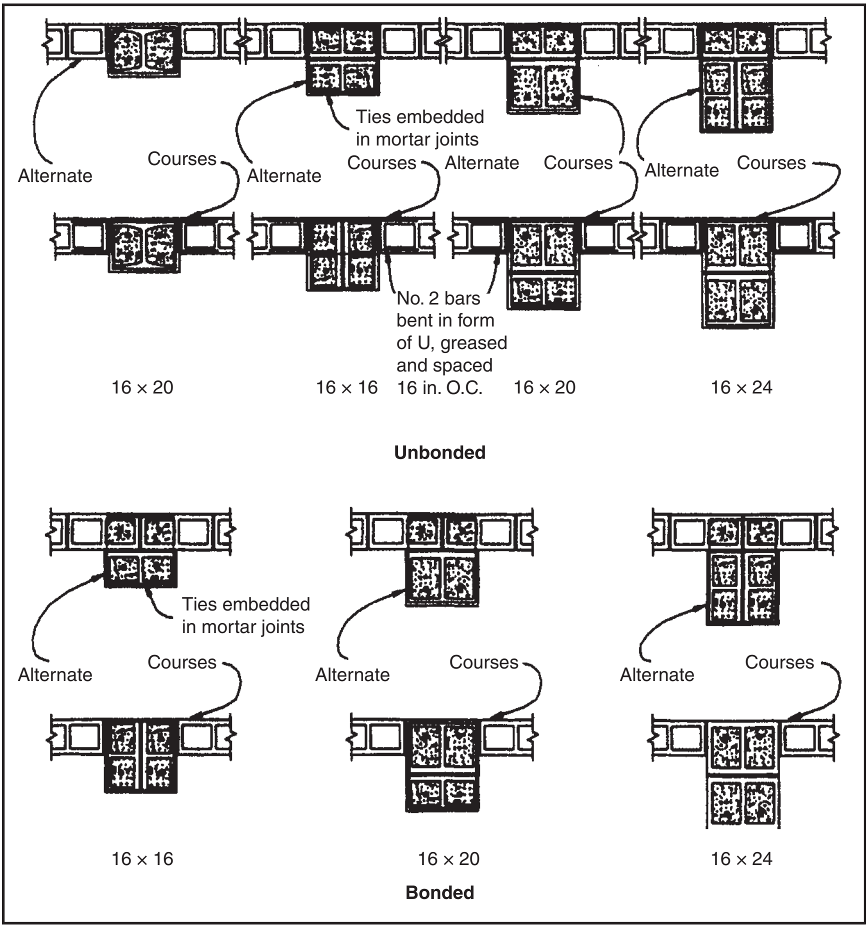


FIGURE 6.40 Unbonded and bonded 16-in. pilasters [6.31].

members. In that case, pilasters serve as columns and are designed as such. In other situations, when they are required to carry little or no vertical load other than their own self-weight, they function as vertical flexural members and must be designed to resist tension. They transfer their loads to foundation at the bottom, and to the floor or roof at the top. A discussion on design of pilasters can be found in several references [6.5, 6.8, 6.29].

When pilasters are required to carry vertical loads, they must comply with design requirements for columns. When vertical reinforcement is provided to resist axial compression, lateral ties (at least $^1/_4$ in. in diameter) meeting the requirements for columns must be provided as discussed in Chap. 5.

When a wall is subjected to lateral loads, as discussed in previous sections, it may be designed to span *vertically*, in which case it transmits its load to horizontal supports—roof or the floor above and the foundation below. Alternatively, a wall may be designed to span *horizontally* between the vertical supports such as pilasters and cross-walls. Design of

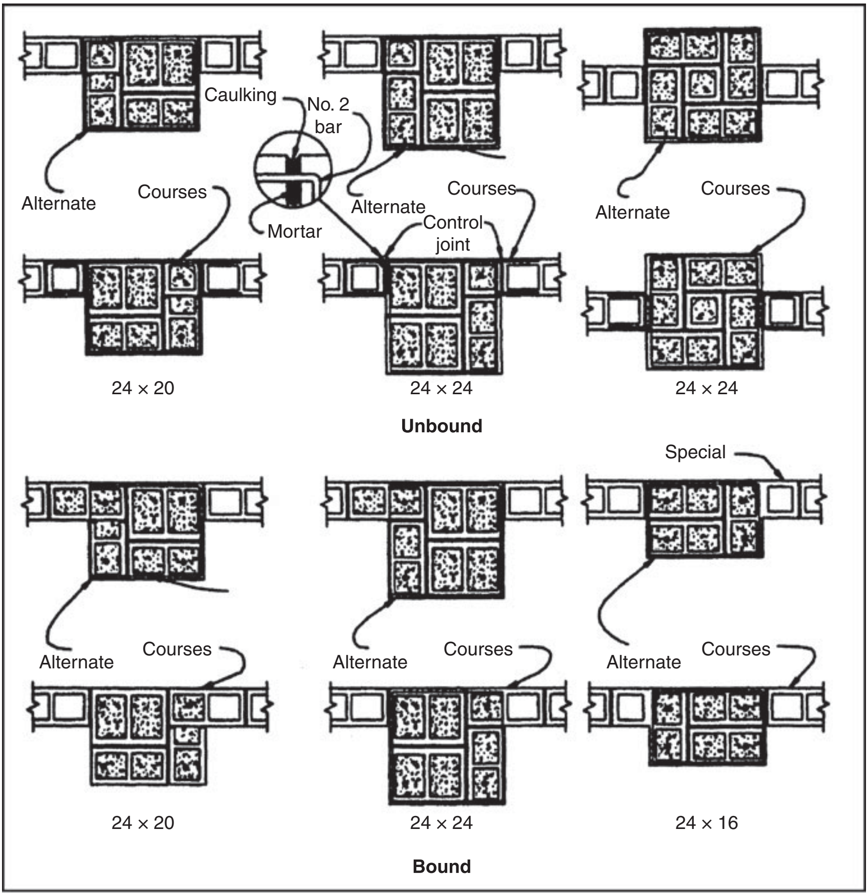


FIGURE 6.41 Unbonded and bonded 24-in. pilasters [6.31].

walls spanning between horizontal supports was discussed in previous sections. It was tacitly assumed that the wall was simply supported at top and bottom. While this approach is simple and straightforward from design standpoint, it ignores the influence of end-restraints at lateral supports when such supports (e.g., pilasters) are present. Also, such a procedure may be overly conservative if the ratio of horizontal distance between lateral supports and the height of wall (called the aspect ratio) is relatively small, and end-fixity is developed.

This section describes a method to determine lateral forces on walls supported on three or all four sides, and pilasters to which they may transfer their loads. In the following discussion, the wall segments between pilasters will be referred to as *wall panels*. From a structural design perspective, wall panels are assumed to function as thin plates or slabs. In the presence of pilasters, wall panels may be thought of as panels bounded by roof and foundation as horizontal supports, and pilasters as vertical supports. Under these support conditions, a wall can be assumed as a vertical two-way slab and designed for two-way bending. In some cases, gravity loads are not transmitted directly to wall; rather they are transmitted to roof trusses or beams, which are supported on columns or pilasters. In that case, the wall can be assumed as supported on three sides (pilasters as vertical supports and foundation as the horizontal support), and free at the top.

Design of wall panels can be simplified under certain simplifying assumptions. The support conditions must be idealized in order to determine design forces in wall panels. At the top, they may be assumed simply supported or free. At the other three edges, they can be assumed simply supported or fixed. The first step in design involves determining the proportion of lateral load transmitted to horizontal and vertical supports. Assuming that flexural resistance and stiffness in both horizontal and vertical directions are known, the lateral load capacity and the proportion of lateral load transmitted to both vertical and horizontal boundary members will depend on the aspect ratio of the wall panel and the restraint developed at the edges (i.e., at supports).

A simplified procedure for designing wall panels developed by NCMA is described in Ref. 6.32, which is summarized here. It involves using curves (Fig. 6.42) to determine approximately the proportion of wind load transmitted in the vertical and horizontal direction.

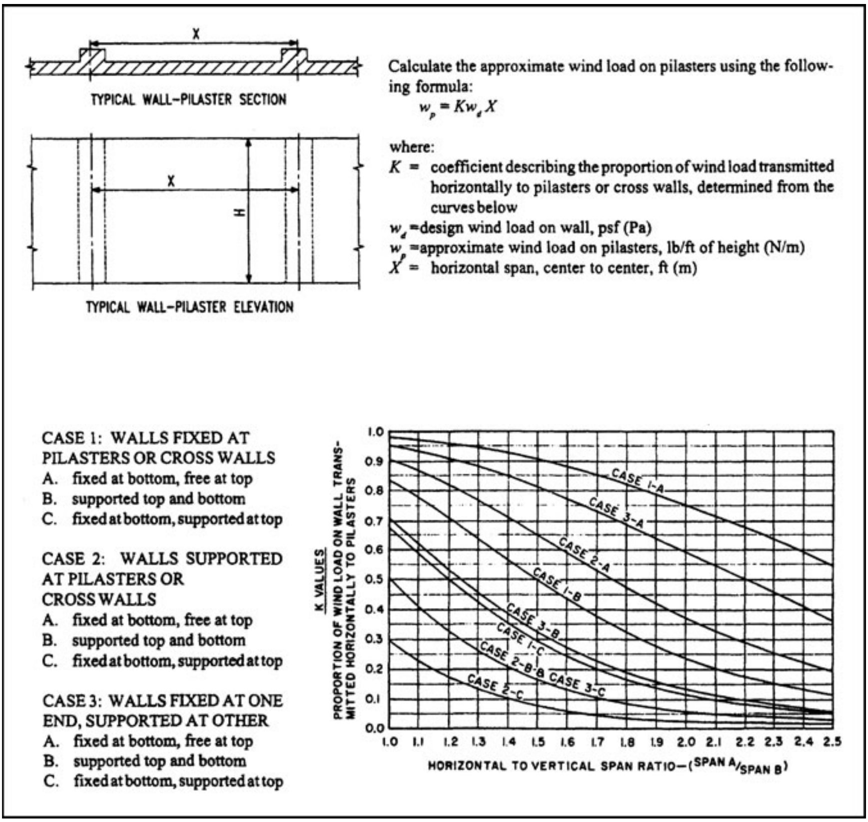


FIGURE 6.42 Approximate horizontal and vertical wind load distribution [6.32].

The curves were developed by equating the expressions for maximum deflection for a strip of wall bending in both directions, based on the following assumptions:

1. The moment of inertia and the modulus of elasticity of the wall are same in both directions.
2. There are no openings in the wall. If any openings are present, they are so located that their effect on the stiffness of the wall panel in both directions is the same.
3. Wall panels on each side of the support have identical configuration (i.e., length and height).

The curves in Fig. 6.45 are drawn with the aspect ratio of the wall panel as abscissa and a coefficient K as ordinate, corresponding to specific idealized support conditions at the edges of the wall panels. In all, eight curves are shown to represent three sets of *vertical* edge restraint conditions:

Case 1: Walls fixed at pilasters or cross-walls

Case 2: Walls simply supported at pilasters or cross-walls

Case 3: Walls fixed at one end and simply supported at the other

In each of the three cases, curves are given for three sets of *horizontal* edge restraint conditions:

1. Fixed at bottom, free at top
2. Simply supported at top and bottom
3. Fixed at bottom, simply supported at top

Knowing the value of K from the charts, the approximate wind load transmitted to wall panels and pilasters is calculated from Eqs. (6.26) and (6.27):

$$w_w = Kw_d \tag{6.26}$$

$$w_p = Kw_dX \tag{6.27}$$

where K = coefficient describing the proportion of wind load transmitted horizontally to pilasters or cross-walls
w_d = design wind load on wall (psf)
w_w = approximate wind load on the wall (lb/ft of width)
w_p = approximate wind load on pilasters (lb/ft of height)
X = horizontal span (distance) between pilasters (center-to-center, ft)

The load w_w is applied uniformly over the width of the wall, which approximates the actual wind load transmitted to wall panels, and which causes them to bend in a horizontal plane. Likewise, the load w_p is applied uniformly over the height of pilaster, which approximates the actual wind load transmitted to the pilaster by the wall panels. The lateral load so transferred to the pilaster causes it to bend in the vertical plane. Once these loads are known, the design forces (i.e., shears and moments) in the wall and pilaster can be easily determined.

The moments and shear developed in wall panels and the pilaster depend on their support conditions (Fig. 6.43). The maximum moment in the wall would be $wL^2/8$ at the midspan if the wall panels are assumed simply supported at the pilasters; the maximum moment would be $wL^2/12$ at the ends if fixed-end conditions are assumed at the pilasters. The moment in the pilaster can be similarly determined. If it is assumed simply supported at top and bottom, the maximum moment and shear in the pilaster due to lateral load would be, respectively, $wL^2/8$ (at the midspan) and $wL/2$. However, if the pilaster were assumed

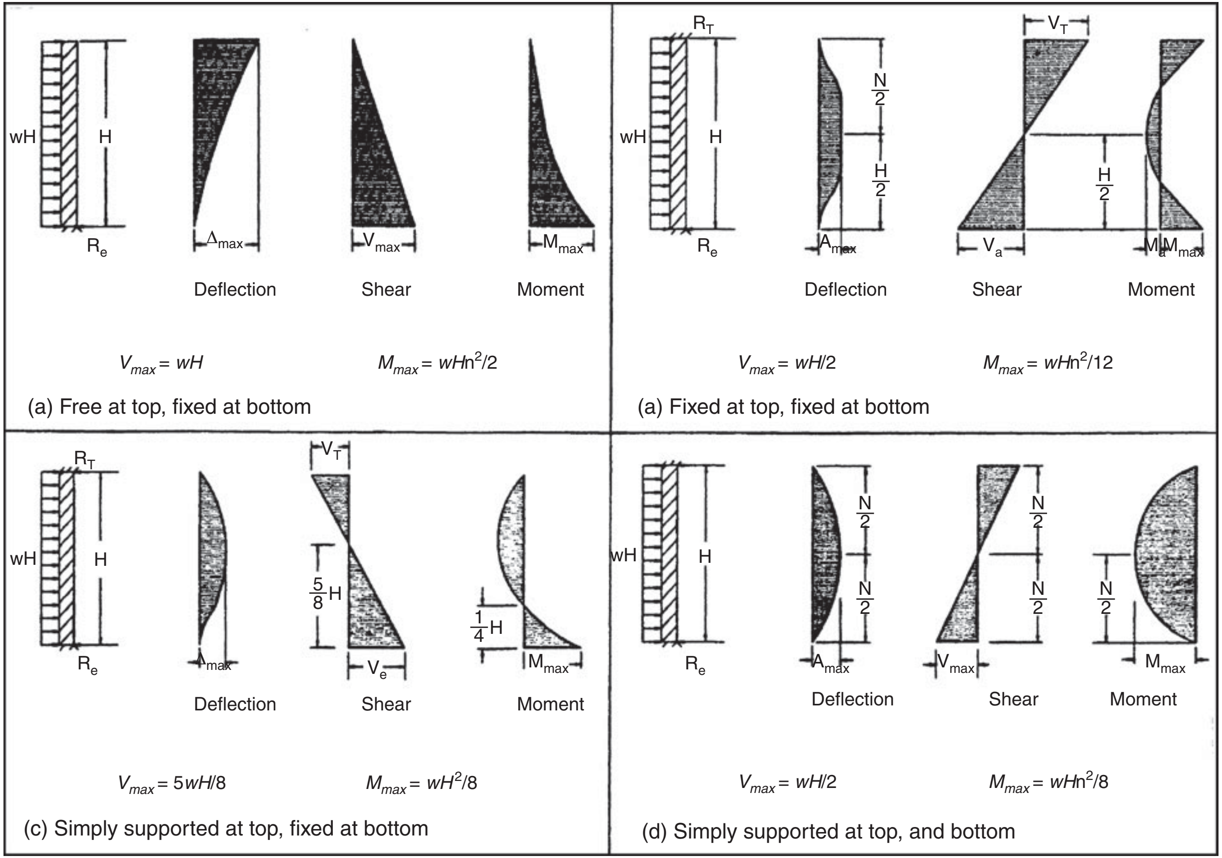


FIGURE 6.43 Formulas for maximum moments and shears for walls and pilasters subjected to uniform lateral loads [6.32].

fixed at the bottom and simply supported at the top (i.e., a vertical propped cantilever), the maximum moment and shear would be, respectively, $wL^2/8$ and $5wL/8$ (at the base).

Walls may or may not be built to act integrally with the pilaster. It the walls are not constructed to act integrally with the pilaster, the pilaster would have a square or a rectangular cross section. If the walls are constructed to act integrally with the pilaster, a short segment of the wall on either side of the pilaster would act as a flange, and the cross section of the pilaster would be a T-section.

No specific provisions are given in the IBC governing the cross-sectional dimensions of a T-section for a pilaster. MSJC-08 Section 2.1.7 [6.1] specifies requirements for a reinforced masonry pilaster. MSJC-08 Section 1.9.4.2.3 [6.1] specifies that the width of the flange considered effective on each side of the web shall be smallest of the actual flange on either side of the wall, or the following:

1. Six times the nominal flange thickness if the flange is in compression.
2. Six times the nominal flange thickness for unreinforced masonry when the flange is in flexural tension.
3. Three-fourths of floor-to-floor height for reinforced masonry when the flange is in flexure tension.

In all of the above cases, the effective width is not permitted to extend past the movement joint.The first of the above requirement essentially means that the effective width of flange of the T-beam not exceed 12 times the wall thickness plus the width of the web. It was pointed out earlier that the ACI *Code* has somewhat similar restrictions on the effective width of flange of a T-beam, but limits the maximum flange width to the center-to-center distance between the webs also.

In order to take advantage of T-beam action in a pilaster, the walls must act integrally with the pilaster. Accordingly, assumption of fixed-end conditions at the pilaster-wall intersection for determining design moments and shears would be justified. For the intersecting walls and the pilaster to act integrally, the two must be properly bonded. MSJC-08 Section 1.9.4.2 [6.1] specifies the following requirements for connection between the intersecting walls:

1. Masonry wall shall be in running bond.
2. Fifty percent of the masonry at the interface shall interlock.
3. Walls shall be regularly toothed with 8-in. maximum offsets and anchored by steel connectors meeting the following requirements:
   (a) Minimum size: $^1/_4$ in. × $1^1/_2$ in. × 28 in. including 2-in.-long 90° bend at each end to form a U- or Z-shape.
   (b) Maximum spacing 48 in.
4. Intersecting bond beams shall be provided in intersecting walls at a maximum spacing of 4 ft on centers. Bond beams shall be reinforced and the area of reinforcement shall not be less than 0.1 in.^{2} multiplied by the vertical spacing of the bond beams in feet of wall. Reinforcement shall be developed on each side of the intersection.

6.10 NONLOAD-BEARING WALLS

6.10.1 Introduction

Nonload-bearing (also referred to as nonbearing) walls were briefly described in Section 6.2. Such walls appear in buildings in many forms such as interior partitions, in-fill panels, or exterior walls such as fence walls and highway sound barrier walls. It was pointed out that these walls do not function as gravity load–carrying elements. For analytical purposes, MSJC-08 Section 1.6 [6.1] distinguishes nonload-bearing walls from load-bearing walls as the latter carrying superimposed load vertical load of at least 200 per lineal foot in addition to their own weight. However, nonload-bearing walls must be able to resist their own dead weights and lateral loads (e.g., due to wind and earthquake).

Design and analysis of nonload-bearing walls do not present any special difficulty. Principles and methods discussed in previous sections for analyzing and designing load-bearing walls are equally applicable to nonload-bearing walls. Since these walls do not carry any gravity load, except their own weight, the axial compressive stresses are very small. Design of fence walls and highway sound barrier walls are discussed in the next two sections. It should be noted that both types of walls are subjected to a wide range of loading conditions, temperatures, and moisture variations. Therefore, the selection of proper materials and proper workmanship is very important for satisfactory structural performance and durability of these walls.

6.10.2 Design of Fence Walls

A discussion on concrete masonry fence walls can be found in several references [6.8, 6.33, 6.34], which is briefly summarized here. Concrete masonry fence walls are constructed to serve many functions. They include:

1. Privacy
2. Security and protection
3. Ornamentation
4. Screening from unwelcome views
5. Excellent sound insulation
6. Shade
7. Wind protection

Availability of CMU in a wide range of colors and textures gives concrete masonry a unique advantage in that they can be used to enhance aesthetic qualities of the wall, complement adjacent architectural styles, or blend in with the natural landscape.

NCMA TEK 14-16A [6.34] recommends design of fence walls to ensure stability according to the following five methods:

1. As cantilever walls stabilized by continuous footings
2. As walls spanning between pilasters which are, in turn, stabilized by a footing or a caisson
3. As walls spanning between wall returns which are adequate to stabilize the wall
4. As curved walls with an arc to chord relationship that gives stability
5. As a combination of above methods

Fence walls must be deigned to carry their own weight and the governing lateral loads (due to wind or earthquake). The only dead load on fence walls is their own weight. Methods of calculating these loads were discussed in Section 6.5. The magnitude of the lateral seismic force F_p can be determined from Eq. (6.7):

$$F_p = \frac{0.4 a_p S_{\mathrm{DS}}}{\left(R_p \big/ I_p \right)} \left(1 + 2\frac{z}{h} \right) W_p \qquad \text{(6.7 repeated)}$$

In Eq. (6.7), F_p is not required to be greater than

$$F_p = 1.6 S_{DS} I_p W_p \quad \text{(6.8 repeated)}$$

or less than I_p given by Eq. (6.9):

$$F_p = 0.3 S_{DS} I_p W_p \quad \text{(6.9 repeated)}$$

When fence walls are designed as unbraced cantilever walls, the values of z and h in Eq. (6.7) should be taken equal to the height of the wall so that $z/h = 1$. The value of coefficients a_p and R_p should be taken, respectively, as 2.5 and 2.5 so that the resulting equation is

$$\begin{aligned} F_p &= \frac{0.4 a_p S_{DS}}{\left(R_p / I_p\right)}\left(1 + 2\frac{z}{h}\right) W_p \\ &= \frac{0.4(2.5) S_{DS}}{2.5 / I_p}\left(1 + 2\frac{h}{h}\right) W_p \\ &= 1.2 S_{DS} I_p W_p \end{aligned} \tag{6.28}$$

Since the value of F_p given by Eq. (6.28) is smaller than that given by Eq. (6.8), but larger than that given by Eq. (6.9), Eq. (6.28) will govern for seismic lateral load. Note that for cantilever conditions, the maximum moment and shear occur at the base of the wall. Alternatively, fence walls can be designed as panels supported by pilasters (discussed in Section 6.9.2).

Figure 6.44 shows a typical concrete masonry fence wall in a residential area and Fig. 6.45 shows details for a CMU for a fence wall. It is common practice to use 6 or 8 in. CMU for fence walls; 4-in. CMU may be used for walls of smaller heights, say 4 ft or

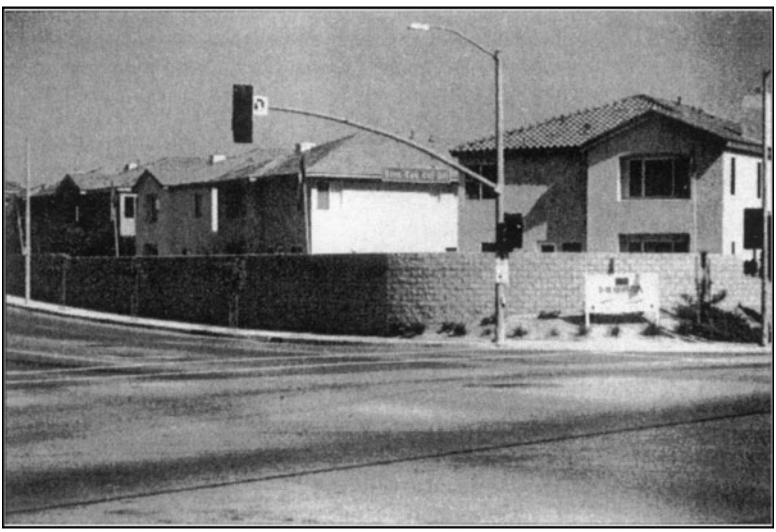


FIGURE 6.44 A typical fence wall in a residential area. (*Courtesy: Author*.)

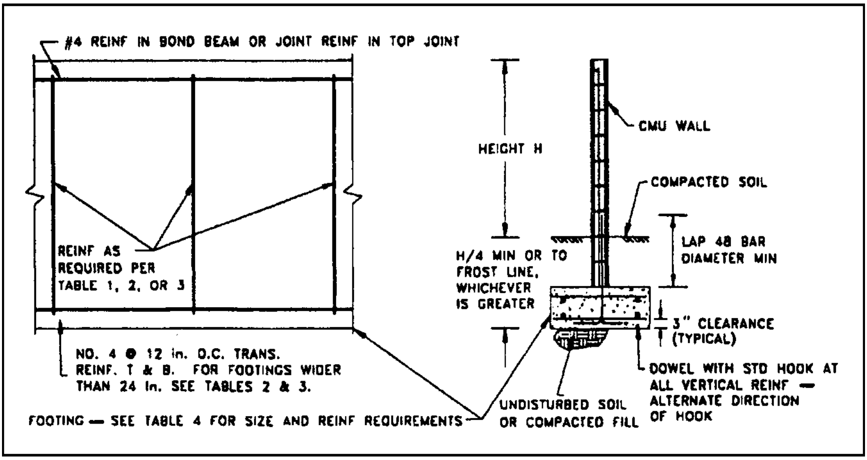


FIGURE 6.45 Typical details for a concrete masonry fence wall [6.34].

so. Depending on the magnitude of lateral loads and local code requirements, fence walls may be built hollow and unreinforced. Under the action of lateral loads, the wall can bend in either direction so that either face of the wall can be in tension. Therefore, if the wall is reinforced, it is advisable to provide reinforcing at the center of the wall. This results in the design depth of wall equal to half the actual thickness as discussed in Section 6.7.4. The amount and spacing of reinforcement in fence walls depends on load as well as on the minimum reinforcement requirements, which were discussed in Section 6.7.3.

Garden walls may have serpentine or “folded wall” profile which add interesting and pleasing character to the wall and enhance the landscape. From a structural standpoint, the curvature in the wall’s profile adds additional stability to the wall so that the wall can be built with a greater height than a wall with a straight profile in plan. Serpentine walls are built using empirical design guidelines based on historically proven performance over many years of experience. Reference 6.32 provides the following guidelines for design of serpentine or folded-plate walls, which are illustrated in Fig. 6.46. Similar details are also provided in Ref. 6.8.

1. The wall radius should not exceed twice the wall height.
2. Wall height should not exceed twice the width (or the depth of curvature, see Fig. 6.46).
3. Wall height should not exceed 15 times the wall thickness.
4. The free ends of the wall should have additional support such as a pilaster or a short-radius return.
5. To control shrinkage cracking, joint reinforcement at 16 in. on centers. should be provided and that control joints be provided in accordance with local practice.

Reference 6.34 provides design details for cantilever fence walls for three wall heights—4, 6, and 8 ft. In each case, reinforcing details are provided for three category of lateral loads, viz., low lateral load (9 to 10 psf), moderate lateral load (14 to 16 psf), and high lateral load (24 to 26 psf). Readers should consult this reference for additional details.

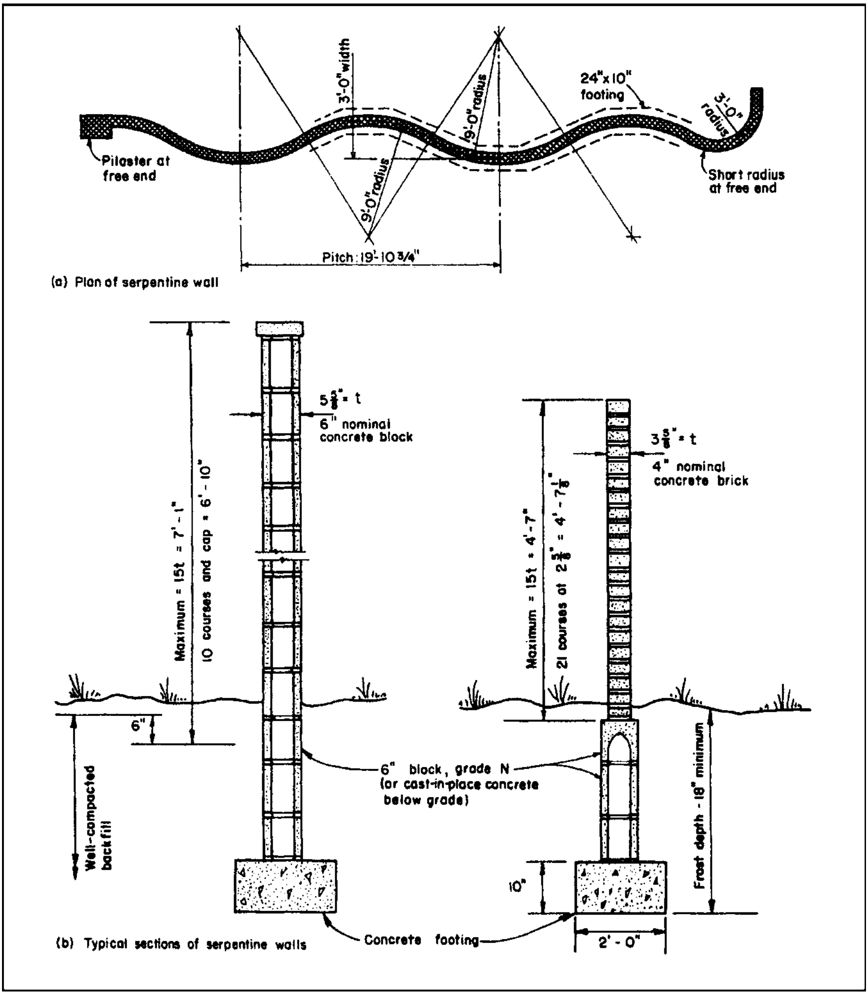


FIGURE 6.46 Guidelines for design of serpentine garden walls [6.34].

6.10.3 Highway Sound Barrier Walls

6.10.3.1 Purpose of Sound Barrier Walls The purpose of providing sound barrier walls is to reduce the high noise levels produced by the high volume, high speed traffic that flows through metropolitan areas, exposing a great number of people to this noise hazard. To reduce these noise levels, it is often necessary to provide noise abatement treatment in terms of acceptable transmission loss and diffraction of noise. This can be accomplished by taking several noise-abatement measures as suggested by the *Federal-Aid Highway Program Manual 7-7-3*:

1. Traffic management procedures
2. Alterations of horizontal and vertical alignments

3. Acquisition of property rights for installation or construction of noise-abatement barriers or devices
4. Installation or construction of noise barriers or devices (including landscapes for aesthetic purposes), whether within or outside the highway right-of-way
5. Acquisition of property to serve as a buffer zone to preempt development that would be adversely affected by the noise

Of these, installation and/or construction of noise barriers are the abatement measures most commonly used.

6.10.3.2 Advantages of Concrete Masonry Sound Barrier Walls Widespread use of concrete masonry as the most commonly used material for noise barrier walls can be credited to the many desirable properties and features that it possesses. These include design flexibility, structural capability, durability, and low initial and life cycle costs. A unique feature of concrete masonry is its unsurpassed aesthetic attribute in that it is available in many colors, shapes, and surface texture unlike any other material. Sound barriers constitute a very visible component of highway, and concrete masonry can be advantageously used with artistic treatment and patterns to enhance highway aesthetics.

But the major advantage of concrete masonry is its greater sound-absorbing capability than other materials. In other words, concrete masonry sound barriers are more effective in total noise reduction than those of other materials. Concept of noise reduction by a barrier is discussed in NCMA TEK-13-3 [6.35], which is summarized here. Effectiveness of a sound barrier is measured from a quantity referred to as “insertion loss” which is the difference between the sound level before and after a barrier placed next to a highway. Insertion loss has five components:

1. Barrier attenuation due to the diffraction of sound waves over and around a barrier placed in the line-of-sight plane between the source and the receiver
2. Transmission loss of sound through the barrier
3. Reductions in barrier attenuation resulting from multiple reflections caused by double barriers
4. Shielding attenuation from other barriers between the source and the receiver
5. Loss of excess attenuation already received from soft ground cover

Essentially, a sound barrier acts as an absorber of acoustical energy. Its effectiveness may be significantly compromised when this energy is permitted to transmit through it to the receiver. The amount of acoustical energy that can transmit through a barrier depends on several factors such as the density and stiffness of the barrier material, angle of incidence of sound, and the frequency spectrum of the sound. The ability of a material to transmit noise is commonly rated by a quantity called *transmission loss* (TL), which is related to the ratio of the incident acoustical energy to the transmitted acoustical energy. For highway noise sources and their typical spectral content, the TL of common barrier materials increases with increasing surface weight of the material. As a general rule, the TL should be at least 10 dBA above the attenuation resulting from diffraction over the top of the barrier to ensure that barrier noise reduction will not be significantly affected by transmission through the barrier (less than 0.5 dBA). Typical TL values of common materials are given in Table 6.10.

6.10.3.3 Design Considerations Highway sound barrier walls may be designed as either cantilever type as discussed in the previous section or as *pier and panel* walls. The

TABLE 6.10 Transmission Loss for Various Barrier Materials [6.35]

Material	Thickness (in.)	TL[a] (dBA)	Material	Thickness (in.)	TL[a] (dBA)
Woods[b]			Concrete, Masonry, etc.		
Fir	1/2	17	Light concrete	4	36
	1	20		6	39
	2	24			
			Dense concrete	4	40
Pine	1/2	18			
	1	19	Concrete block	4	32
	2	23		6	36
			Composites		
Redwood	1/2	16			
	1	19	Aluminum-faced plywood[e]	3/4	21–23
	2	23			
			Aluminum-faced particle board[e]	3/4	21–23
Cedar	1/2	15			
	1	18	Plastic lamina on plywood	3/4	21–23
	2	22			
			Plastic lamina on particle board	3/4	21–23
Plywood	1/2	20			
	1	23			
Particle board[c]	1/2	20			
Metals[d]			Miscellaneous		
Aluminum	1/16	23			
	1/8	25	Glass (safety glass)	1/4	22
	1/4	27	Plexiglass (shatterproof)	—	
			Masonite	1/2	22–25
Steel	24 ga	18	Fiberglass/Resin	1/4	20
	20 ga	22	Stucco on metal lath	1	32
	16 ga	25	Polyester with aggregate Surface[f]	3	20–30
Lead	1/16	28			

[a]A weighted TL is based on generalized truck spectrum.
[b]Tongue-and-groove boards recommended to avoid leaks (for fir, pine, redwood, and cedar).
[c]Should be treated for water resistance.
[d]May require treatment to reduce glare (for aluminum and steel).
[e]Aluminum is 0.01 in. thick. Special care is necessary to avoid delamination (for all composites).
[f]TL depends on surface density of the aggregate.

latter type is easy to build and economical due to the reduced thickness of the wall and intermittent pier foundations. The panel is designed to span horizontally between the piers. In both cases, design lateral loads on the wall can be determined as discussed also in the previous section. A discussion on pier and panel highway sound barrier walls can be found in NCMA TEK Notes [6.36, 6.37]. Reference 6.36 also provides *design tables* (based on allowable stress design method) for 6-, 8-, and 12-in.-thick concrete masonry cantilever highway noise barrier walls spanning 10 to 20 ft in 2-ft increments. Likewise, Ref. [6.37] provides allowable stress design (ASD) tables concrete masonry pier and panel sound barrier walls.

Problems

6.1 A 16 ft high, 8 in. nominal, solid grouted, concrete masonry wall is centrally reinforced with No. 6 vertical bars (Grade 60) spaced horizontally at 24 in. on center. The wall weighs 78 lb/ft² per linear foot of wall. It carries a superimposed dead roof load of 150 lb/ft and a roof live load of 80 lb/ft length of the wall. The roof is supported on a ledger beam attached to the wall, which creates an eccentricity of 4 in. from the center line of the wall (Fig. P6.1). The lateral load due to wind is 20 lb/ft² and due to earthquake is 42 lb/ft². Check the adequacy of the wall to carry gravity and lateral loads, and deflection. $f'_m = 1800$ lb/in.².

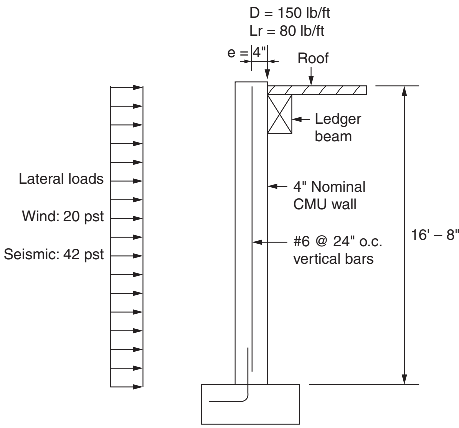


FIGURE P6.1

6.2 A 24 ft high, 8 in. nominal, solid grouted, concrete masonry wall is to be designed to resist out-of-plane load due to wind = 20 lb/ft² and due to earthquake = 42 lb/ft² (Fig. P6.2). It weighs 84 lb/ft² per linear foot and carries a superimposed dead roof load of 350 lb/ft and a roof live load of 240 lb/ft length of the wall. The roof is supported on a ledger beam attached to the wall, which creates an eccentricity of 4 in. from the center line of the wall. Determine the reinforcement (Grade 60) requirements for this wall. Assume that the wall is classified as Seismic Design Category D and $f'_m = 2500$ lb/in.².

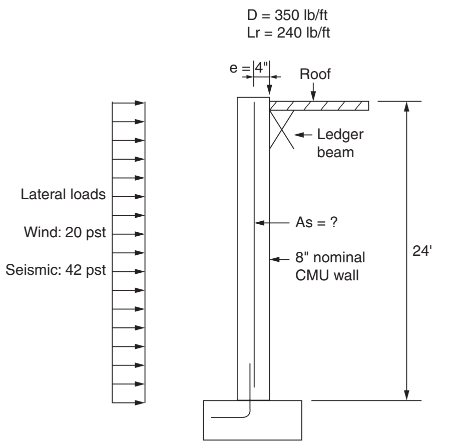


FIGURE P6.2

6.3 A nominal 8-in-wide concrete masonry wall is reinforced with No. 6 Grade 60 bars placed vertically at 32 in. center-to-center (Fig. P6.3). The masonry is running bond and $f'_m = 1500$ lb/in.^{2}. Calculate the moment strength of wall which is subjected to out-of-plane loads.

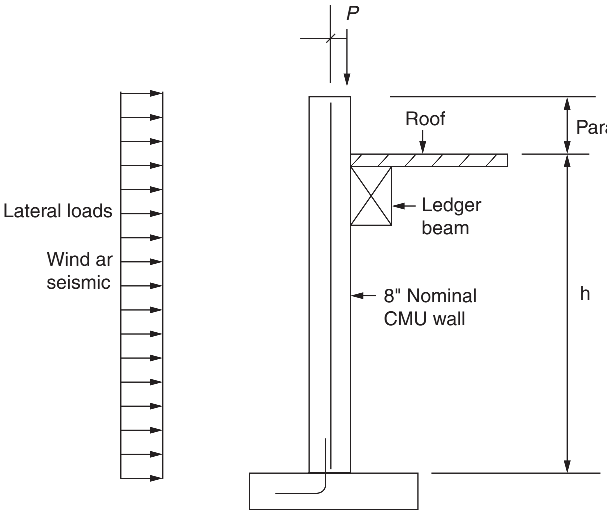


FIGURE P6.3

6.4 A nominal 8-in.-wide concrete masonry wall is reinforced with No. 6 Grade 60 bars placed vertically at 16 in. center-to-center (Fig. P6.4). It carries a roof dead load of 400 lb/ft and live load of 250 lb/ft. The masonry is in running bond and $f'_m = 1500$ lb/in.^{2}. Calculate the moment strength of the wall which is subjected to out-of-plane loads.

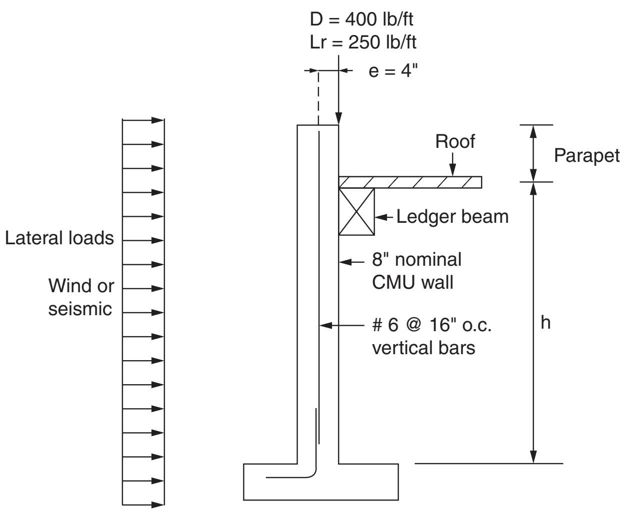


FIGURE P6.4

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CHAPTER 7
SHEAR WALLS

7.1 INTRODUCTION

All structures must have some means of resisting lateral forces, such as wind or seismic, and transfer them to the ground. In the vertical plane, several types of force-resisting systems are commonly used to resist lateral loads, such as *shear walls*, *braced frames*, *moment frames* (also called rigid frames), or a combination of these called *dual systems*. These systems are commonly referred to as *lateral force–resisting systems* (LFRS). When used to resist wind forces, these systems are referred to as *main wind force–resisting systems* (MWFRS); they are called *seismic force–resisting systems* (SFRS) when used to resist earthquake forces. Shear walls can be used in all types of buildings, from low-rise to high-rise, and from ordinary warehouse to multistory apartment buildings. They form an integral part of lateral load–carrying systems commonly known as *bearing wall systems* (known in the past as the *box system*), and are also used in conjunction with other seismic force–resisting systems referred to as *building frame systems* and *dual systems*. Essentially, a bearing wall system refers to a structural support system wherein major load-carrying columns are omitted and the walls and/or partitions are of sufficient strength to carry gravity loads. When used as SFRS, shear walls must be designed by engineered methods; this chapter presents analysis and design of such walls.

A bearing wall system offers unique advantages over other load-resisting systems and provides an economical alternative to structural frame system. It not only eliminates the need for a structural frame but also provides the enclosure walls for the building. Architectural units can be used to provide a durable, aesthetic finish for both exterior and interior walls thus eliminating the need for additional surface treatments. Additional costs of fireproofing and soundproofing are also eliminated. The walls and partitions supply in-plane lateral stiffness and stability to resist wind, earthquake, and any other lateral loads. Shear wall structures are inherently stiff and capable of limiting deformation and damage during extreme lateral loads. As such, shear walls form an important topic in building design and construction. Figure 7.1 shows a typical multistory shear wall system.

Braced and moment frames are generally constructed from steel or reinforced concrete. Shear walls may be constructed from a variety of materials like reinforced concrete, reinforced masonry, wood frame [with various kinds of sheathing material—plywood, gypsum wallboard, drywall, interior and exterior plaster (stucco), fiberboard, and lumber sheathing], and steel plates. In typical wood-frame buildings, the shear walls are built from sheathed wood panels, which are called *shear panels*. Shear walls made from masonry (clay or concrete units) may be unreinforced or reinforced. This chapter presents a discussion of analysis and design of reinforced concrete masonry shear walls.

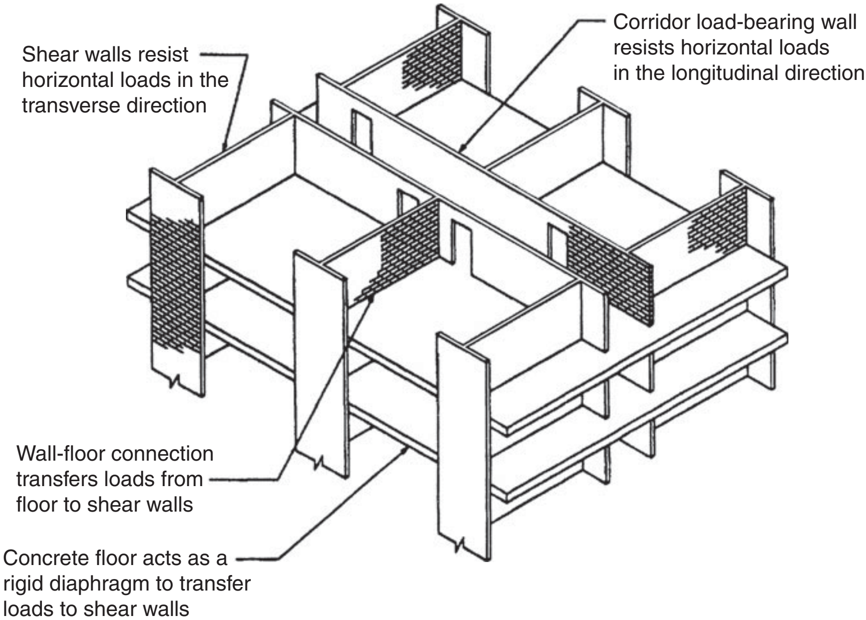


FIGURE 7.1 Multistory shear wall system. (*Adapted from Ref.* [*7.1*].)

7.2 FUNDAMENTAL CONCEPTS

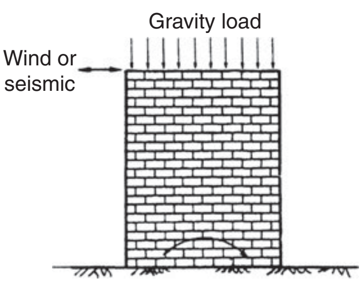


FIGURE 7.2 Gravity and lateral loads on a shear wall.

A shear wall is a wall designed to resist lateral forces parallel to the plane of the wall. These forces are commonly referred to as *in-plane forces*. A shear wall may be a bearing wall or a nonbearing wall, depending on its intended function. It would be a bearing wall if it were required to carry gravity loads from the supported elements (such as roof or floor) in addition to the lateral forces. As the name indicates, a shear wall provides resistance to in-plane forces by virtue of its strength in shear. The cross-sectional area of the wall that resists shear is taken as its length *times* thickness. Figure 7.2 shows gravity and lateral loads acting on a shear wall.

It is instructive to understand the load path for lateral loads in buildings with shear walls. Shear walls form the *LFRS* of such buildings. Essentially, shear walls receive their loads from the roof or the floor they support. Gravity loads are transferred from roof or floors to shear walls by bearing. Lateral loads are transferred from the same supported elements as inertial forces through connections between the supported elements (which act as diaphragms) and the shear walls. Thus, the floor and the shear walls act in unison as an assembly of structural elements to resist gravity and lateral forces.

The inertial forces generated in the roof and floor are oriented in the plane of these horizontal force–resisting elements and are called *diaphragm forces*. The term "diaphragm"

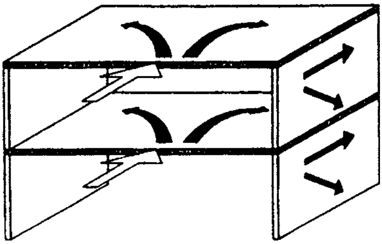


FIGURE 7.3 Load path in a shear wall building [7.2].

is used to identify horizontal force–resisting elements (generally floor or roofs of buildings) that act to transfer lateral loads to the vertical elements (shear walls or frames). Essentially, the diaphragm acts as a giant beam lying in a horizontal plane (see discussion in Chap. 4). The diaphragms transfer their inertial forces to shear walls, which act as vertical cantilevers and transfer these forces to the ground (Figs. 7.3 to 7.5).

The lateral load–resisting ability of a shear wall is characterized by its stiffness. The stiffness of a shear wall depends on its *aspect ratio*, defined as the ratio of its height *h* to length *d* (expressed as *h*/*d* ratio), the lower the *h*/*d* ratio, the higher the stiffness of the shear wall. Under the action of lateral loads, a shear wall acts as a deep vertical cantilever and is subjected to both shear and flexure. The height of a shear wall is analogous to the length of a cantilever beam, whereas length of the shear wall is analogous to the depth of the cantilever beam. For a given height of a shear wall, the longer the length, the greater is the stiffness. Figure 7.6 shows the approximate relationship between the aspect ratio and stiffness of shear walls. Walls with low *h*/*d* ratios, commonly referred to as *squat* walls, behave as shear elements, that is, deflection due to shear predominates. Squat walls with *h*/*d* ratios in the range of 2 to 3 find wide application

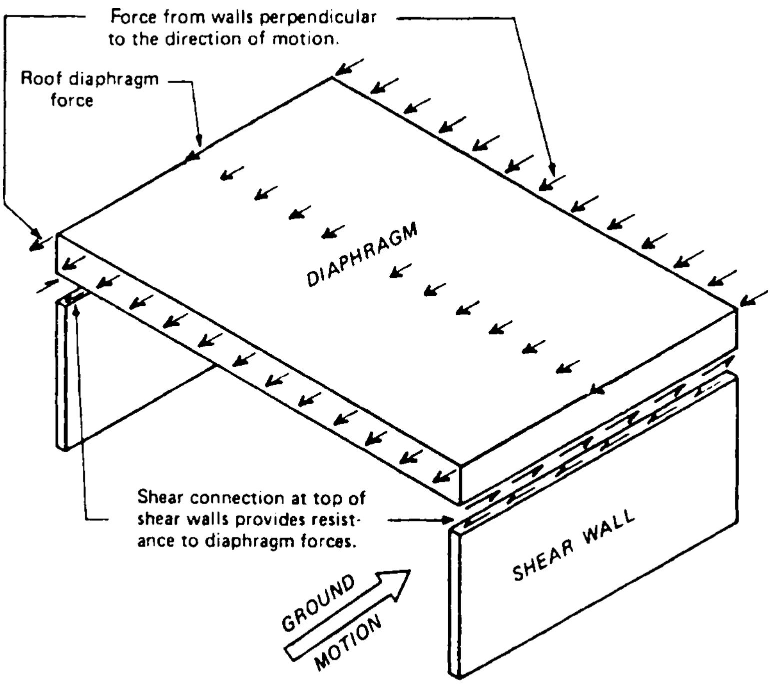


FIGURE 7.4 Inertial forces and resistance of roof or floor diaphragm [7.3].

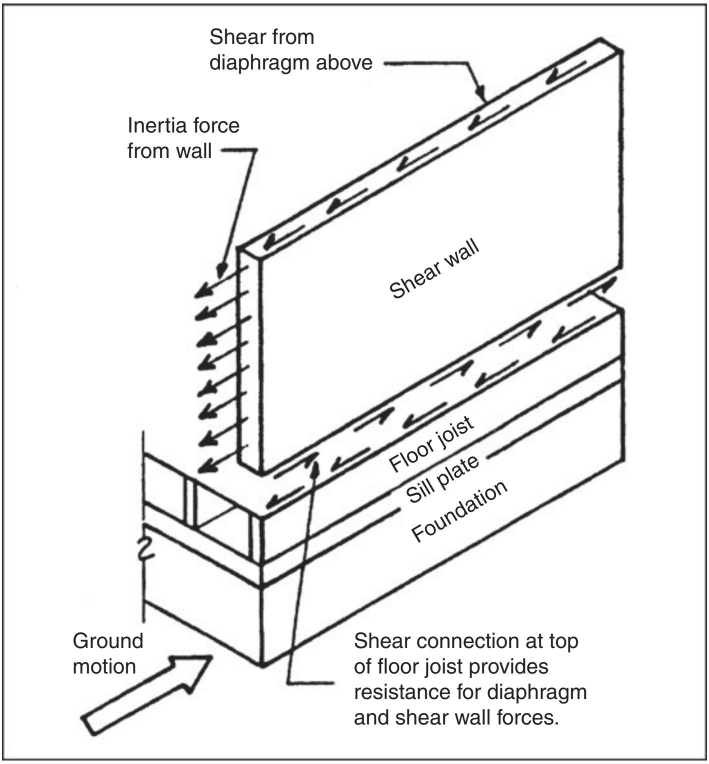


FIGURE 7.5 Inertial forces and resistance of walls parallel to direction of ground motion [7.3].

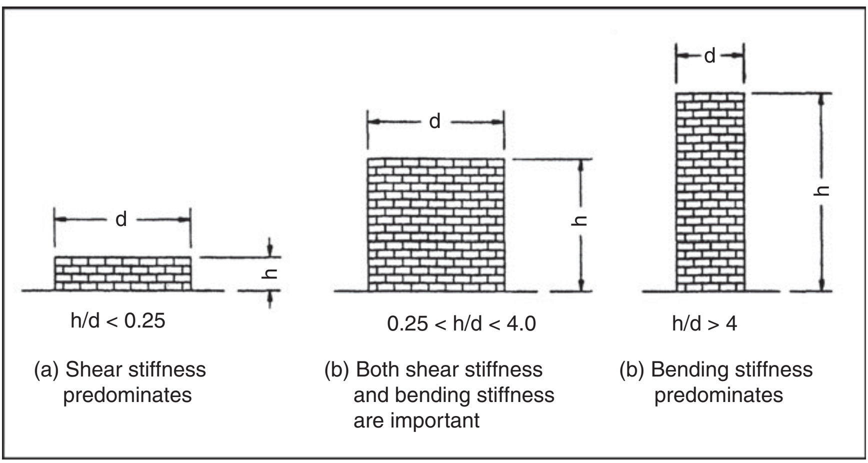


FIGURE 7.6 Aspect ratio and stiffness of shear walls [7.4].

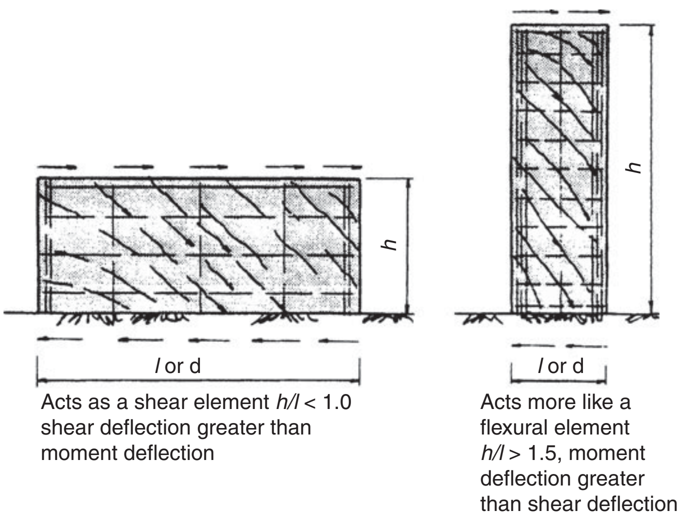


FIGURE 7.7 Behavior of shear walls based on aspect ratio [7.5].

in low-rise buildings. Walls with large *h*/*d* ratios behave as predominantly flexural elements, that is, deflection due to flexure is considerably larger than deflection due to shear (Fig. 7.7). This aspect of shear wall is discussed further in the next section.

The in-plane stiffness of a diaphragm plays a significant role in how the diaphragm transfers its in-plane forces to the shear walls. Diaphragms are classified as *flexible* or *rigid*. Diaphragms constructed from untopped steel decking, wood structural panels, or similar structural panels are classified as flexible diaphragms in structures having concrete or masonry shear walls. Reinforced concrete roofs or floors and composite concrete floors incorporating permanent metal decking are examples of rigid diaphragms.

When flexible diaphragms are used, lateral loads are transferred to the shear walls in proportion to the tributary areas they support. Wood diaphragms are considered to be flexible because they are capable of undergoing large in-plane deflections and transmit lateral loads to shear walls independent of their rigidities. They also are incapable of transmitting lateral forces by rotation. In case of rigid diaphragms, the lateral (or diaphragm) loads are transferred to shear walls in proportion to their relative rigidities. Unlike flexible diaphragms, the rigid diaphragms undergo very small in-plane deflections and are capable of transmitting lateral forces by rotation.

Due to the fact that both concrete and masonry shear walls offer significant bending and shear resistance, their analysis is different than that of the wood-frame shear walls. Unlike wood-frame shear walls, unit shear in any segment of the masonry shear wall depends on its relative stiffness. From this standpoint, a concrete or masonry shear wall can be considered as rigid and wood-frame shear wall (or shear panel) as flexible. The *unit shear* in a wall is simply the total shear divided by the net wall length (i.e., wall length *minus* the length of all openings).

Analytically, a shear wall may be considered as a deep vertical beam; its depth equals the wall length (i.e., measured parallel to floor) and its span (length of the cantilever) equals the height. In a one-story building, a shear wall acts as a vertical cantilever beam, fixed at the bottom and free to rotate at the top. In a multistory building, a shear wall between two stories may be considered fixed at both top and bottom (similar to a fixed-ended beam). It is common practice to consider shear walls in one- or two-story buildings as cantilevered, whereas in a multistory building, the segments of the walls between various stories are considered to be fixed both at the top and bottom.

While the discussion in this chapter is focused on the design of wall elements as shear walls (i.e., on their in-plane resistance), it is noted that these walls, as components of a bearing wall system, must posses adequate strength to resist the following forces:

1. Gravity loads from diaphragms (roofs or floors), which may be applied eccentrically
2. Lateral or in-plane loads received from the diaphragms
3. Out-of-plane loads (due to wind or earthquake) acting perpendicular to it

Loads in cases 2 and 3 are assumed not to act simultaneously.

7.3 TYPES OF SHEAR WALLS

Shear walls can be variously classified depending on the objective. They can be classified as load-bearing or nonload-bearing, depending on whether they carry gravity loads also in addition to lateral load. They can be classified based on the type of masonry and construction used, for example, brick or concrete, reinforced and unreinforced, single or multiple wythe, single story or multistory, solid or perforated, rectangular or flanged, cantilever or coupled, etc. Most commonly used shear walls have rectangular or flanged configuration. Several types of shear walls are shown in Fig. 7.8.

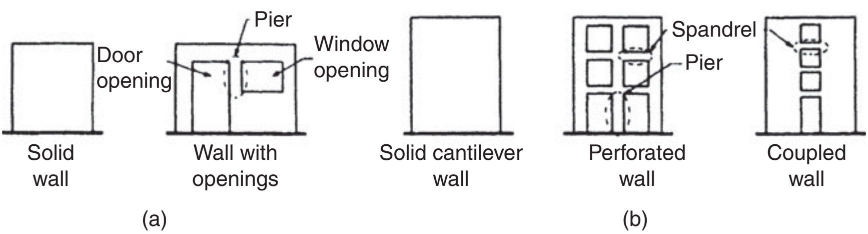


FIGURE 7.8 Types of shear walls: (*a*) single story, (*b*) multistory [7.1].

Efficiency of shear walls is best described in terms of their rigidities (or stiffnesses). As such, solid shear walls are the most efficient and, therefore, highly desirable. Often shear walls would have opening in them as a matter of functional necessity (e.g., doors and windows); such walls are sometimes referred to as *perforated* (i.e., wall with openings). The portion of a shear wall between two adjacent openings is called a *pier*, whereas, the segment of shear wall above the adjacent openings is called a *spandrel* or a *beam*. A shear wall with openings can be analyzed as a frame composed of short stiff wall segments (also called piers). In many shear walls, a regular pattern of windows or doors, or both, is required for functional considerations. In such cases, the walls between the openings may be interconnected by spandrels (or beams), resulting in *coupled shear walls*. The connecting elements (i.e., beams) between coupled shear walls typically require horizontal and vertical reinforcement to transfer shear from one segment of the wall to the other (Fig. 7.9). When the connecting elements are incapable of transferring shear from

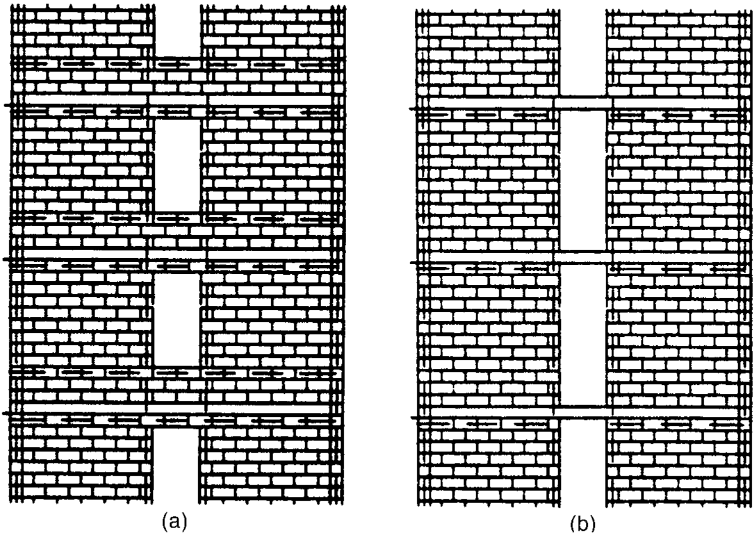


FIGURE 7.9 (*a*) Coupled and (*b*) noncoupled shear walls [7.4].

one shear wall to the other, the walls are referred to as *noncoupled* and can be analyzed as cantilevers fixed at the base.

Openings in shear walls can be provided in a regular and rational pattern to develop extremely efficient structural systems, particularly suited for ductile response with very good energy-dissipation characteristics [7.6]. However, the presence of openings reduces the rigidity of shear walls. Influence of openings on coupled shear walls is shown in Fig. 7.10.

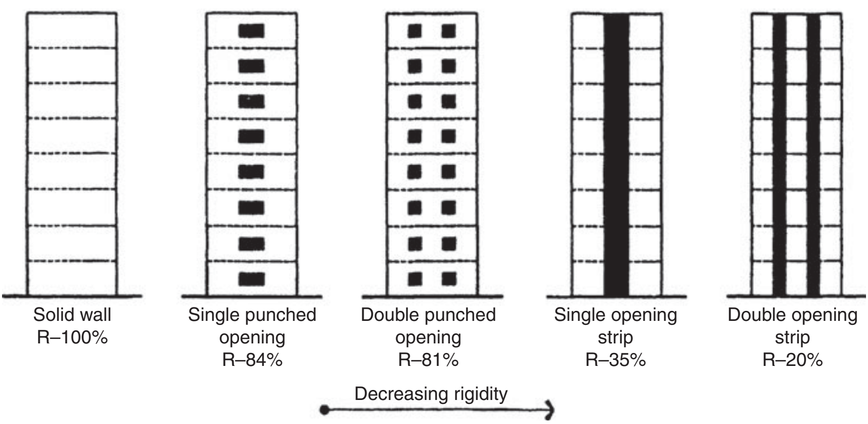


FIGURE 7.10 Influence of openings on the rigidity of shear walls [7.2].

Shear walls meeting each other at right angles (Fig. 7.1) result in flanged configurations and are referred to as *flanged walls*. In such cases, a portion of the intersecting wall can be treated as a flange of the shear wall (e.g., as an I-section or a T-section). Such walls are normally required to resist earthquake forces in both principal directions of the building. Several flanged configurations are shown in Fig. 7.11*a*. The designer must decide how much of the width of the flange should be considered to be effective. Effective width of the flange is an empirical requirement [7.4]. MSJC-08 1.9.4.2 [7.7] limits the width of the flange considered effective on each side of the web as the smallest of the following:

1. The actual flange on either side of the web wall
2. Six times the nominal flange thickness when the flange is in compression (Fig. 7.11*b*)
3. Three-fourths of the floor-to-floor height when the flange is in flexural tension

In addition to the above limitations, the effective flange width shall not extend past a movement joint. The area of shear wall used in horizontal shear calculation should not include the effective flange width.

For a shear wall to be effective as a flanged section, the two intersecting walls must be adequately bonded at the intersection. Connections of webs to flanges of shear walls may be accomplished by running bond, metal anchors, or bond beams. Running-bond geometry should be as shown in Fig. 7.12 with "T" geometry over their common intersection [7.4]. Alternatively, metal anchors may be provided as shown in Fig. 7.13, or bond beams may be used as shown in Fig. 7.14. Also shown in Fig. 7.14 is the detail that can be used to prevent bond between the two intersecting walls.

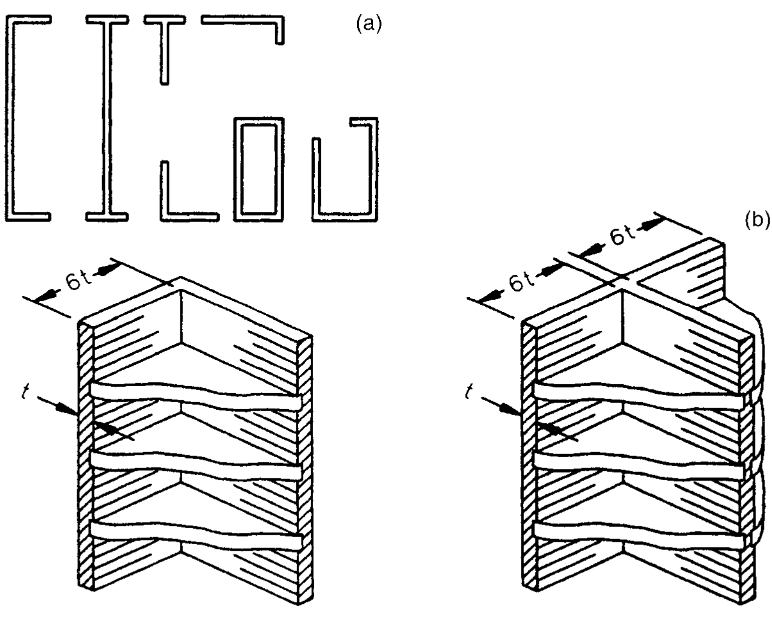


FIGURE 7.11 Flanged wall sections: (*a*) configurations, (*b*) effective width of flanges [7.5].

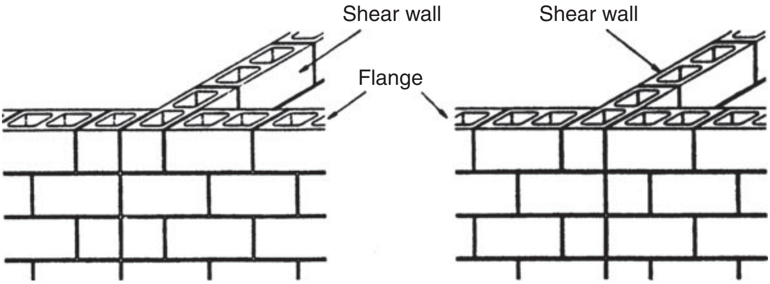


FIGURE 7.12 Running-bond lap at intersecting walls [7.4].

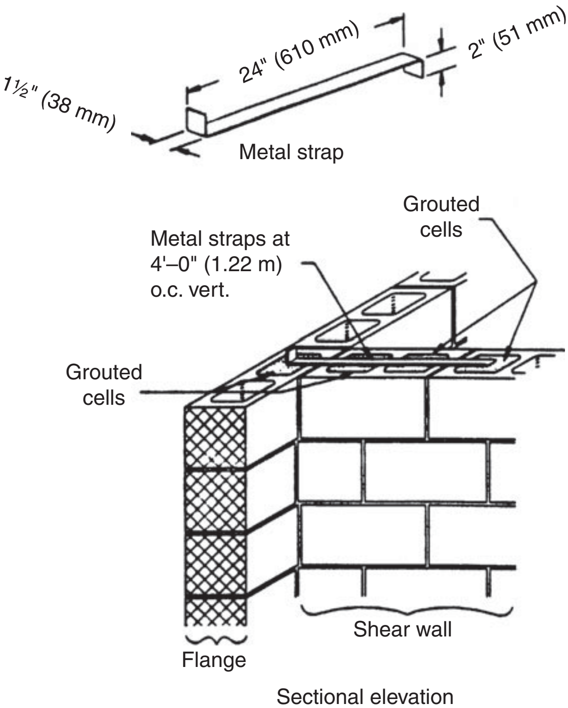


FIGURE 7.13 Metal straps and grouting at wall intersections [7.4].

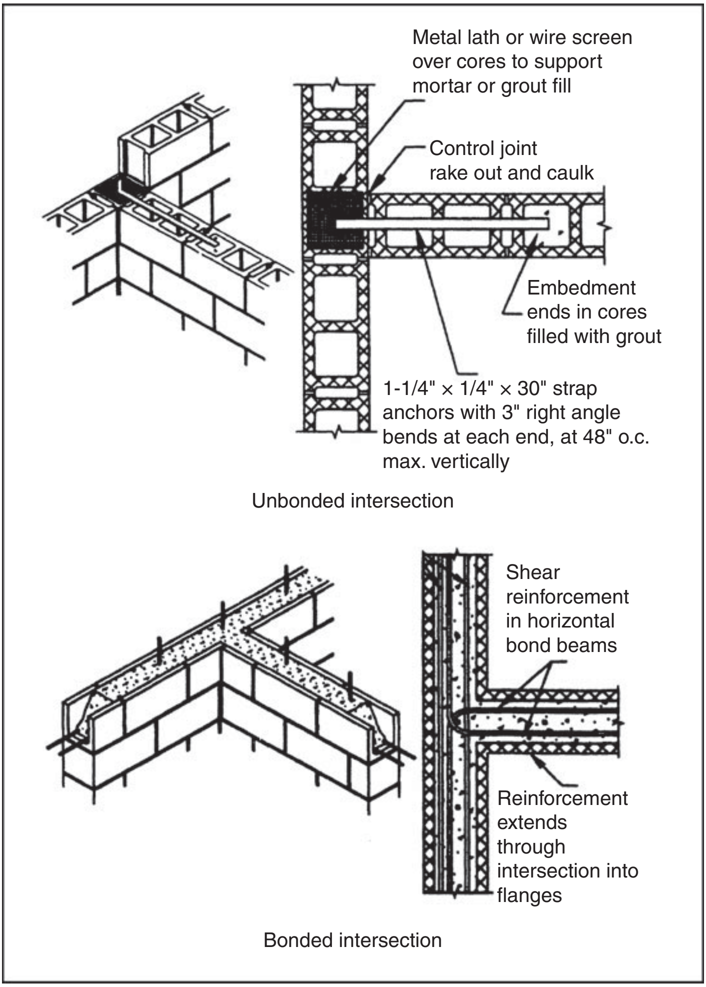


FIGURE 7.14 Reinforcement in bond beams at wall intersections [7.8].

7.4 RIGIDITY AND RELATIVE RIGIDITY OF A SHEAR WALL

7.4.1 General Concept

The most important parameter that describes the resistance of a shear wall is its rigidity (or stiffness). Analytically, rigidity of a shear wall is simply the force applied at one end of the wall to cause a unit deflection in the plane of the wall at that end while the other end is held fixed. When using standard English units, rigidity of a shear wall would usually be expressed in terms of kips per inch. For example, a rigidity of 100 kips/in. would mean that a force of 100 kips is required to cause the end of the wall (where the force is applied) to deflect 1 in. in the plane of the wall while the other end is held fixed.

The determination of rigidity of a shear wall is akin to determining the stiffness of a beam. For determining in-plane deflections, a shear wall may be visualized as vertical flexural element such that

1. Its length is analogous to the depth of a rectangular beam.
2. Its height is equal to the length of the beam.
3. The wall thickness is analogous to the width of the rectangular beam.

Because of the large depth, the deflection of a shear wall (a vertical beam) is the sum of two components: (1) deflection due to flexure and (2) deflection due to shear. *The reciprocal of the total deflection is defined as the rigidity of the wall*. Expressions for deflections are determined based on principles of structural mechanics (for deflections of beams), a topic well covered in texts on structural analysis and not discussed here. However, a brief review of these principles is presented in the next section.

7.4.2 Rigidity of a Cantilevered Shear Wall

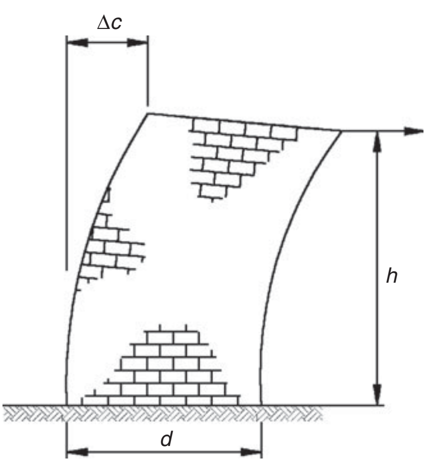


FIGURE 7.15 Deflection of a cantilevered wall.

Flexural deflection, Δ_F, at the free end of a cantilever wall due to a horizontal load P at free end of the wall (Fig. 7.15) is given by Eq. (7.1):

$$\Delta_F = \frac{Ph^3}{3E_m I} \tag{7.1}$$

Deflection Δ_V at the free end of a cantilever wall due to shear P can be computed from Eq. (7.2), similar to that for a rectangular beam:

$$\Delta_V = \frac{1.2Ph}{AE_V} \tag{7.2}$$

where P = lateral load at the free end of the wall
h = height of wall or pier
A = cross-sectional area of wall or pier
I = moment of inertia of wall or pier about the axis of bending
E_m = modulus of elasticity of masonry
E_v = modulus of rigidity for masonry (shear modulus) = $0.4E_m$

The 1.2 multiplier in numerator of Eq. (7.2) represents shape factor for a rectangular section of the wall. The total deflection at the free end of the cantilever wall is given by the sum of deflections given by Eqs. (7.1) and (7.2):

$$\Delta_c = \Delta_F + \Delta_V = \frac{Ph^3}{3E_m I} + \frac{1.2Ph}{AE_V} = \frac{Ph^3}{3E_m I} + \frac{3Ph}{AE_m} \tag{7.3}$$

Rigidity of a cantilevered pier, R_c, that is, the force required to cause a unit deflection at its free end can be expressed as

$$R_c = \frac{1}{\Delta_c} \tag{7.4}$$

For a wall of thickness t and width d, the section properties, viz., the sectional area A and the moment of inertia I can be expressed as

$$A = td \tag{7.5}$$

$$I = \frac{td^3}{12} \tag{7.6}$$

Substitution of Eqs. (7.5) and (7.6) in Eq. (7.3) yields

$$\Delta_c = \left(\frac{1}{E_m t}\right)\left[4\left(\frac{h}{d}\right)^3 + 3\left(\frac{h}{d}\right)\right] \tag{7.7}$$

The term h/d in Eq. (7.7) is called the *aspect ratio*. Substitution of Eq. (7.7) in Eq. (7.4) gives the rigidity R_c of a cantilevered wall:

$$R_c = \frac{E_m t}{\left[4\left(\frac{h}{d}\right)^3 + 3\left(\frac{h}{d}\right)\right]} \tag{7.8}$$

The units of rigidity are kips per inch (same as that for the stiffness of a spring, when E_m is expressed in kips per square inch and t in inches). In design of a shear wall structure, one is interested in only the *relative rigidity* of a shear wall. At a given level of a building, all shear walls would have (most generally) the same thickness, t, and the same masonry compressive strength, E_m. Therefore, the term $E_m t$ appearing in the numerator of Eq. (7.8) is dropped for computational implicity, and the resulting expression is used to express the relative rigidity of a cantilevered shear wall:

$$R_r = \frac{1}{\left[4\left(\frac{h}{d}\right)^3 + 3\left(\frac{h}{d}\right)\right]} \tag{7.9}$$

A comparison of Eqs. (7.8) and (7.9) shows that

$$R_r = \frac{R_c}{E_m t} \tag{7.10}$$

The relative rigidity, R_r, given by Eq. (7.9) is a function of h/d ratio only; therefore, this expression can be used to determine the relative rigidity of walls of any thickness and material (e.g., concrete). The values of relative rigidity given by Eq. (7.9) for large aspect ratios (i.e., large values of h/d ratio) are relatively small. Therefore, in order to preserve accuracy of these small values to three places of decimals, the right side of Eq. (7.9) is multiplied by an arbitrary factor of 10, the resulting expression being

$$R_r = \frac{1}{\left[0.4\left(\frac{h}{d}\right)^3 + 0.3\left(\frac{h}{d}\right)\right]} \tag{7.11}$$

Eq. (7.11) is commonly used to determine the relative rigidity. Values of relative rigidities of walls for various aspect ratios [determined from Eq. (7.11)] are listed in Table A.26. Note that the relative rigidity is dimensionless quantity (i.e., it has no units), which depends on only the dimensionless ratio *h*/*d*.

7.4.3 Rigidity of a Fixed-Ended Shear Wall

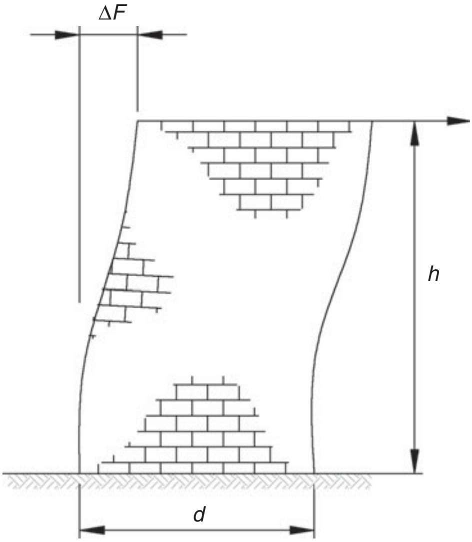


FIGURE 7.16 Deflection of a fixed-ended shear wall.

Flexural deflection Δ_F at the top of a fixed-ended wall due to a horizontal load *P* applied at top of that wall (Fig. 7.16) is given by Eq. (7.12):

$$\Delta_F = \frac{Ph^3}{12E_m I} \tag{7.12}$$

Deflection Δ_v at the end of a wall (or a beam) due to shear *P* is given by Eq. (7.2). Therefore, the total deflection, after substituting $E_v = 0.4E_m$, and values of *A* and *I*, respectively, from Eqs. (7.5) and (7.6), can be expressed as

$$\begin{aligned}\Delta_f &= \Delta_F + \Delta_V \\ &= \left(\frac{1}{E_m t}\right)\left[\left(\frac{h}{d}\right)^3 + 3\left(\frac{h}{d}\right)\right]\end{aligned} \tag{7.13}$$

The rigidity of a fixed-ended wall, R_f, can be calculated from the reciprocal of expression given by Eq. (7.13):

$$R_f = \frac{E_m t}{\left[\left(\frac{h}{d}\right)^3 + 3\left(\frac{h}{d}\right)\right]} \tag{7.14}$$

As explained earlier, the term $E_m t$ appearing in the numerator of Eq. (7.14) can be dropped and the resulting expression can be used to express the relative rigidity of a fixed-ended shear wall:

$$R_r = \frac{1}{\left[\left(\frac{h}{d}\right)^3 + 3\left(\frac{h}{d}\right)\right]} \tag{7.15}$$

As before, it is noted that the relative rigidity is a function of *h*/*d* ratio only, which is a dimensionless parameter; thus, the relative rigidity is a dimensionless quantity and has no units. A comparison of Eqs. (7.14) and (7.15) shows that

$$R_r = \frac{R_f}{E_m t} \tag{7.16}$$

Similarity between Eqs. (7.10) and (7.16) may be noted. In general both equations can be expressed in a general form as Eq. (7.17):

$$R_r = \frac{R}{E_m t} \tag{7.17}$$

where R = rigidity of the wall.

Note that relative rigidity, R_r, given by Eq. (7.15) is a function of h/d ratio only; therefore, as explained earlier, this expression can be used to determine the relative rigidity of walls of any thickness and material (e.g., concrete). The values of relative rigidity given by Eq. (7.15) for large aspect ratios (i.e., large values of h/d-ratio) are relatively small. Therefore, in order to preserve accuracy of these small values to three places of decimals, the right side of Eq. (7.15) is multiplied by an arbitrary factor of 10, the resulting expression being

$$R_r = \frac{1}{\left[0.1\left(\frac{h}{d}\right)^3 + 0.3\left(\frac{h}{d}\right)\right]} \tag{7.18}$$

Eq. (7.18) is commonly used to calculate the relative rigidity of fixed-ended shear walls. Values of relative rigidities of walls for various aspect ratios [determined from Eq. (7.18)] are listed in Table A.27.

Examples 7.1 and 7.2 illustrate calculations for determining the rigidity and relative rigidity for cantilevered and fixed-ended shear walls, respectively. In design problems, all masonry walls at a given level of a building are of the same compressive strength and thickness. Therefore, relative rigidity can be used for rigid diaphragm-shear wall analysis.

Example 7.1 Rigidity of a cantilevered shear wall.

Figure E7.1 shows a cantilevered masonry shear wall, which is 8 in. (nominal) thick. Calculate its rigidity if $f'_m = 2000$ lb/in.^{2}.

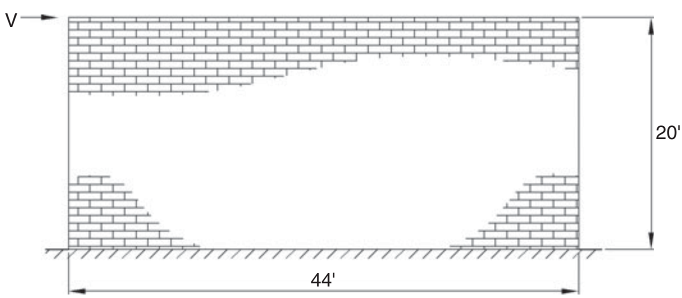


FIGURE E7.1 A cantilevered shear wall.

Solution

The subject wall is fixed at the base and free to rotate at the top. Therefore, it can be treated as a cantilevered wall and its deflection can be calculated from Eq. (7.7):

$$\Delta_c = \frac{1}{E_m t}\left[4\left(\frac{h}{d}\right)^3 + 3\left(\frac{h}{d}\right)\right] = \frac{20}{44} = 0.4545$$

$$E_m = 900\ f'_m = 900(2.0) = 1800 \text{ kips/in.}^2$$

$$t = 7.625 \text{ in. (8 in. nominal)}$$

$$\Delta_c = \frac{1}{(1800)(7.625)}[4(0.4545)^3 + 3(0.4545)]$$
$$= 1.2671\ (10^{-4})\ \text{in.}$$
$$R_c = \frac{1}{\Delta_c} = \frac{1}{1.2671(10^{-4})} = 7892\ \text{k/in.}$$

The rigidity of the wall is 7892 kips/in., that is, a lateral force of $V = 7892$ kips would be required to deflect the wall at the top by 1 in. in the plane of the wall.

Example 7.2 Relative rigidity of a cantilevered shear wall.
Calculate the relative rigidity of the cantilevered shear wall described in Example 7.1.

Solution
The relative rigidity of a cantilevered shear wall can be determined from Eq. (7.9):

$$R_r = \frac{1}{\left[4\left(\frac{h}{d}\right)^3 + 3\left(\frac{h}{d}\right)\right]}$$
$$\frac{h}{d} = \frac{20}{44} = 0.4545$$

Substituting for *h*/*d* in the above equation, we obtain

$$R_r = \frac{1}{\left[4\left(\frac{h}{d}\right)^3 + 3\left(\frac{h}{d}\right)\right]} = \frac{1}{[4(0.4545)^3 + 3(0.4545)]} = 0.575$$

Alternatively, we can determine the relative rigidity of a cantilevered shear wall from Eq. (7.10) if its rigidity is known a priori:

$$R_r = \frac{R_r}{E_m t} = \frac{7892}{(1800)(7.625)} = 0.575$$

For a shear wall with an *h*/*d* ratio of 0.4545, the relative rigidity is 0.575.
Commentary: The relative rigidity of a cantilevered shear wall can also be determined from Table A.26. For an h/d ratio of 0.4545, the value obtained from Table A.26 is 5.75 (by interpolation). As explained earlier, this value represents 10 times the actual value of relative rigidity, so that the relative rigidity is 0.575.

Example 7.3 Rigidity and relative rigidity of a fixed-ended masonry shear wall.
Calculate the rigidity and relative rigidity of the shear wall described in Example 7.1 assuming it to be fixed at both top and bottom. (Fig. E7.3.)

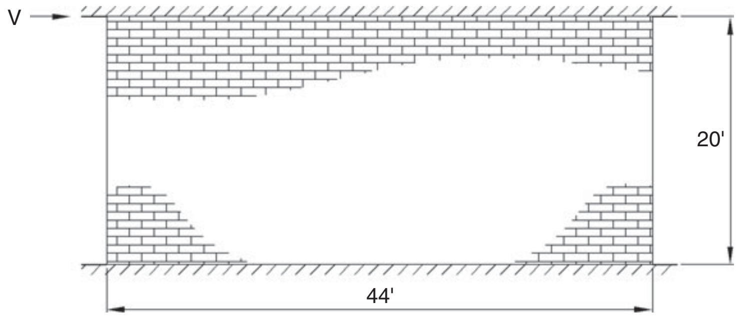


FIG. E7.3 Shear wall fixed at both top and bottom.

Solution

a. *Rigidity of the wall:* The subject wall is fixed both at top and bottom. Therefore, it can be treated as a fixed-ended shear wall and its deflection can be calculated from Eq. (7.13):

$$\Delta_f = \frac{1}{E_m t}\left[\left(\frac{h}{d}\right)^3 + 3\left(\frac{h}{d}\right)\right]$$

$$\frac{h}{d} = \frac{20}{44} = 0.4545$$

$$E_m = 900\ f'_m = 900(2.0) = 1800 \text{ kips/in.}^2$$

$$t = 7.625 \text{ in. (8-in. nominal)}$$

$$\Delta_f = \frac{1}{(1800)(7.625)}[(0.4545)^3 + 3(0.4545)]$$

$$= 1.06185(10^{-4}) \text{ in.}$$

$$R_f = \frac{1}{\Delta_f} = \frac{1}{1.06185(10^{-4})} = 9418 \text{ kips/in.}$$

The rigidity of the wall is 9418 kips/in.; that is a lateral force of $V = 9418$ kips would be required to deflect the wall by 1 in. in the plane of the wall.

b. *Relative rigidity of the wall:* The relative rigidity of the fixed-ended shear wall can be determined from Eq. (7.15):

$$R_r = \frac{1}{\left[\left(\frac{h}{d}\right)^3 + 3\left(\frac{h}{d}\right)\right]}$$

$$\frac{h}{d} = \frac{20}{44} = 0.4545$$

Substituting in the above equation, we obtain

$$R_r = \frac{1}{\left[\left(\frac{h}{d}\right)^3 + 3\left(\frac{h}{d}\right)\right]} = \frac{1}{[(0.4545)^3 + 3(0.4545)]} = 0.686$$

Alternatively, we can determine the relative rigidity of a fixed-ended shear wall from Eq. (7.16) if its rigidity is known a priori:

$$R_r = \frac{R_f}{E_m t} = \frac{9418}{(1800)(7.625)} = 0.686$$

For a shear wall with an *h*/*d* ratio of 0.4545, the relative rigidity is 0.686.

Commentary: The relative rigidity of a fixed-ended shear wall can also be determined from Table A.27. For an h/d ratio of 0.4545, the value obtained from Table A.27 is 6.86 (by interpolation). As explained earlier, this value represents 10 times the actual value of relative rigidity, so that the relative rigidity is 0.686.

7.4.4 A Comparison of Flexural and Shear Deflections in Shear Walls

The forgoing discussion shows that the deflection of a shear due to lateral loads consists of two components: deflection due to flexure and deflection due to shear. A perusal of Eq. (7.7) and (7.13) shows that for walls with large aspect ratios, the flexural component of the total deflection will predominate, whereas for squat walls (i.e., walls with small aspect ratios), the shear component of the total deflection will predominate. For very squat walls, say, with aspect ratio ≤ 0.25, the flexural deflection component is very small, roughly 8 percent and 2 percent, respectively, for cantilever and fixed-ended walls. Relative rigidities of such walls can be determined with reasonable accuracy based solely on their shear rigidity. For walls with aspect ratios in the range of 0.25 to 4, both shear and flexural deflection components contribute significantly to total deflection, and so both should be determined. Relative contribution of shear deflection as a fraction of total component can be determined from equations derived earlier. Dividing Eq. (7.2) by Eq. (7.3) and simplifying yields

Cantilever walls:

$$\Delta_{\text{shear}} = \left[\frac{1}{1.333(h/d)^2+1}\right]\Delta_{\text{total}} \tag{7.19}$$

Similarly, dividing Eq. (7.2) by Eq. (7.13) and simplifying yields
Fixed-ended walls:

$$\Delta_{\text{shear}} = \left[\frac{1}{0.333(h/d)^2+1}\right]\Delta_{\text{total}} \tag{7.20}$$

Values of shear deflection component as a fraction of total deflection for cantilever and fixed-ended walls [determined, respectively, from Eqs. (7.19) and (7.20)] for various aspect ratios are shown in Table 7.1. It is noted that relationships given by Eqs. (7.19) and (7.20) are valid for rectangular walls with lateral loads applied at the top of the walls. For other load configurations and end conditions, these relationships would be different.

7.5 RIGIDITY OF A SHEAR WALL WITH OPENINGS

It was pointed out earlier that openings are often provided in shear walls for functional purposes, for example, for providing doors and windows. Walls with openings are sometimes referred to as *perforated walls*. The presence of an opening in a wall increases its deflection

TABLE 7.1 In-plane Deflections of Walls due to Shear as a Percentage of Total Deflection

Aspect ratio, *h*/*d*	Cantilever wall	Fixed-ended wall
0.25	92.3	98.0
0.50	75.0	92.0
0.75	57.1	84.2
1.00	42.9	75.0
1.50	25.0	57.1
2.00	15.8	42.9
2.50	10.7	32.4
3.00	7.7	25.0
3.50	5.8	19.7
4.00	4.5	15.8
5.00	2.9	10.7
6.00	2.0	7.7
8.00	1.2	4.5

thereby reducing its rigidity. Therefore, the rigidity of a perforated wall would be *smaller* than that of a solid wall having the same overall dimensions. Figure 7.17 shows a shear wall with openings (shown crossed by diagonal lines).

In walls with openings, only the solid segments of the wall contribute shear resistance to applied lateral loads. The vertical wall segments between the openings (*b, c, d,* and *e* in Fig. 7.17) are called piers. The portions of the wall above and below the openings are called *spandrel beams* or simply *beams* (a and f in Fig. 7.17). The piers are characterized by their height-to-width ratios (*h*/*d* ratios), where *h* is the height of the pier (equal to the height of the shorter opening on either side) and *d* the horizontal distance between the openings on either side. Each pier is assumed tied at the top by the stiff spandrel beam located above the opening, and at the bottom by the foundation (e.g., under a door opening) or a beam (e.g., under a window). Under these assumptions, a pier acts like a fixed-ended vertical beam (restrained against rotation at top and bottom). Similarly, the beams (referred to as piers in examples to follow) are also described by their *h*/*d* ratios. The beams located in the

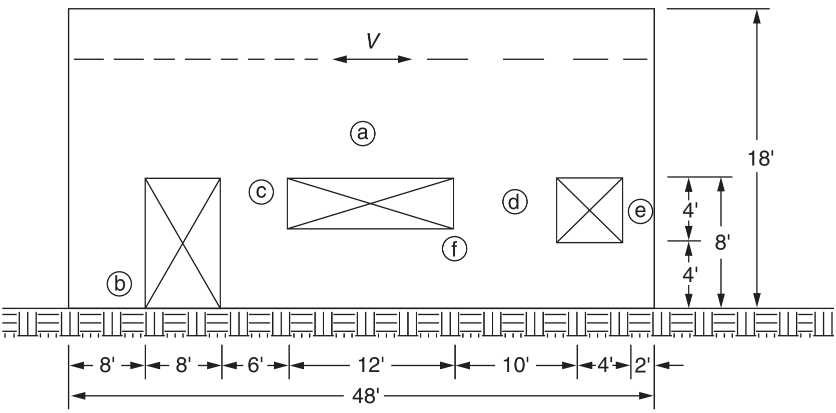


FIGURE 7.17 Shear wall with openings.

upper portion of a cantilevered shear wall (above the openings and extending to the top, *a* in Fig. 7.17) should be assumed fixed at the bottom and free to rotate at the top. However, a beam located between the openings and the floor below should be treated as fixed at both the top and bottom.

In calculating the relative rigidity of a shear wall, it is necessary to calculate the relative rigidity of the entire wall as well as that of its piers and beams. It is generally assumed that cantilevered conditions exist for walls in one- or two-story buildings taken in their entirety; piers and beams within the walls can be considered as cantilevered or fixed. For example, a shear wall in a single-story building taken as a whole would be considered as cantilevered, but piers within the wall may be considered as fixed at both top and bottom.

When lateral forces are transferred from a rigid diaphragm to shear walls with different rigidities, the shear walls share the lateral load in proportion to their rigidities. This is an important design principle: the wall with greater rigidity would attract larger share of the lateral load. Because the walls sharing lateral load from a diaphragm, in all likelihood, would be of the same thickness and masonry compressive strength, it is only necessary to determine the relative rigidity of a shear wall for design purposes.

When determining the relative rigidity of a perforated shear wall, the quantity of interest is its deflection; the rigidity of the wall is then determined as the reciprocal of its deflection. Whereas the deflection of a solid wall can be determined directly from Eqs. (7.7) or (7.13) (as applicable), determination of the deflection of a perforated wall is a bit tricky. The calculations get more involved as the number of openings increases; particularly when the heights of openings are different, or the openings are located at different elevations within the wall.

In a shear wall building, analysis for a shear wall with openings typically involves two important steps: (a) determination of shear from the diaphragm to the wall, and (b) distribution of the wall shear to its piers. In a masonry/concrete shear wall building with rigid diaphragms, the total diaphragm shear V is distributed to various shear walls in proportion to their relative rigidities. The shear in any individual pier of a wall is then determined by simply multiplying the total wall shear by a *distribution factor* (DF). The distribution factor is defined as the ratio of the rigidity of a particular pier, R_i, to the sum of the rigidities of all piers, ΣR_i, participating in resisting total shear in the wall, and can be expressed by Eq. (7.21):

$$\text{Distribution factor} = \frac{\text{rigidity of a pier}}{\text{sum of rigidities of all piers}} \tag{7.21}$$

or

$$\mathrm{DF} = \frac{R_i}{\sum_{i-1}^{n} R_i} \tag{7.22}$$

Equation (7.22) shows that the stiffer the element, the greater the force it would attract. Once the force in a pier is determined, it can be designed as a shear wall element.

Several methods of calculating rigidities of perforated walls are in vogue, all of which give slightly different answers. The magnitude of shear resistance offered by various piers in a perforated shear wall depends on the type of model used in analyses and underlying assumptions related to the material properties, dimensional characteristics of piers, and behavior of the wall. Analyses procedures, though formalized, are less than rigorous, and the accuracy in wall rigidity calculations is less than desirable. Therefore, values with more than three or four significant figures are unwarranted when calculating rigidities or relative rigidities.

Three simple methods for determining the rigidities and relative rigidities of perforated shear walls are presented in this section. It is important to recognize that these methods are approximate, and are not recommended for walls with larger openings or walls that should be treated as wall frames. These methods are discussed here with reference to Fig. 7.18.

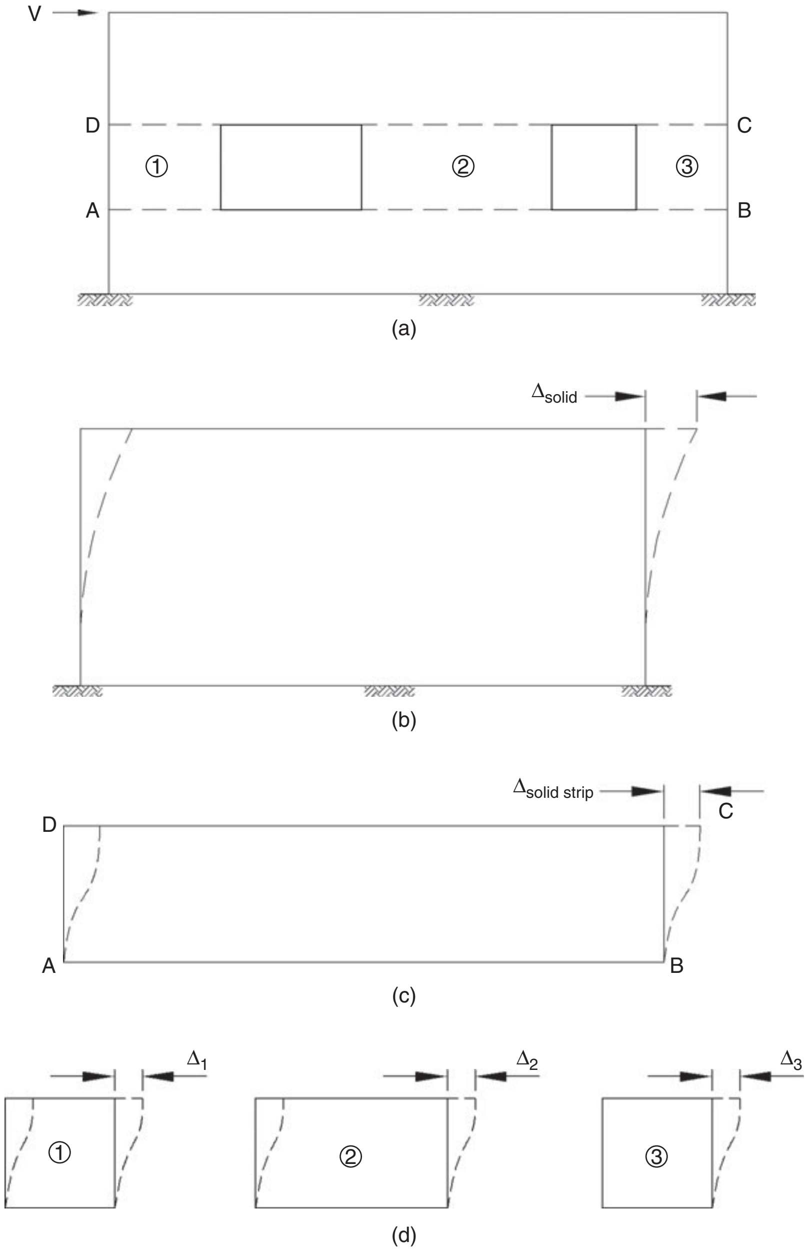


FIGURE 7.18 Rigidity (or relative rigidity) of a perforated shear wall.

1. *Method A:* This method involves a rather simplified approach and is also the easiest to use. It assumes that only the piers between the openings contribute to the rigidity of a wall with openings. Each pier is assumed to have width d equal to the width of the segment between the openings, and height h equal to the height of the shorter adjacent opening. Each pier is assumed tied at the top by a stiff spandrel beam and at the bottom by the foundation or a beam below the openings. Based on this assumption, each pier is considered as a fixed-ended vertical beam. Under the action of a lateral force, it is assumed that each pier will deflect the same amount. The rigidity of the wall is assumed to equal the sum of the rigidities of the individual piers. The percentage of the total wall shear distributed to an individual pier is computed from Eq. (7.22). The actual shear V_i in pier P_i can be determined from Eq. (7.23):

$$V_i = (DF_i)(V) \tag{7.23}$$

With reference to Fig. 7.18, the rigidity (or relative rigidity) of a perforated wall can be expressed by Eq. (7.24):

$$R_{\text{wall}} = R_1 + R_2 + R_3 \tag{7.24}$$

where R_1, R_2, and R_3 are the rigidities (or relative rigidities) of Piers 1, 2, and 3, respectively. The shear resistances of beams above and below the openings are ignored in this method. Therefore, the accuracy of this method is not great. Although fast, this method is very approximate and should be used for preliminary analysis/design only.

2. *Method B:* This method is described in the Tri-Services Publication, *Seismic Design for Buildings* [7.9] According to this method; first the cantilever deflection of the solid wall (ignoring all openings) is determined. From this deflection is subtracted the cantilever deflection of the interior strip of the wall containing the openings, and having a height equal to that of the highest opening. To this result is added the sum of the deflections of all the piers within the interior strip, to arrive at the deflection of the perforated wall. The reciprocal of this deflection gives the rigidity of that wall.

To explain this method, reference is made to Fig. 7.18*a*, which shows a perforated wall having two openings. The solid segments of the wall adjacent to openings are numbered 1, 2, and 3. A strip of the wall containing these openings is shown as ABCD; its length equals the length of the wall and the height equals the tallest opening in the strip. The net deflection of the wall may be obtained from the following steps:

Step 1: Calculate deflection of solid wall (ignore all openings, Fig. 7.18*b*) treating it as a cantilever. Let us call it gross deflection

Step 2: Calculate deflection of the solid strip ABCD assuming it as fixed-ended (Fig. 7.18*c*).

Step 3: Calculate deflections of all piers (Δ_1, Δ_2, and Δ_3, of Piers 1, 2, and 3, respectively, in Fig. 7.18*d*), assuming them as fixed-ended. The pier deflection correction is calculated as follows:

$$R_{1+2+3} = R_1 + R_2 + R_3 \tag{7.25}$$

$$\frac{1}{R_{1+2+3}} = \frac{1}{R_1 + R_2 + R_3} = \frac{1}{\dfrac{1}{\Delta_1} + \dfrac{1}{\Delta_2} + \dfrac{1}{\Delta_3}} \tag{7.26}$$

Because deflection equals the reciprocal of rigidity, we can write that

$$\frac{1}{R_{1+2+3}} = \Delta_{1+2+3} \tag{7.27}$$

Substitution of Eq. (7.27) in Eq. (7.26) yields Eq. (7.28):

$$\Delta_{1+2+3} = \frac{1}{\dfrac{1}{\Delta_1} + \dfrac{1}{\Delta_2} + \dfrac{1}{\Delta_3}} \tag{7.28}$$

Step 4: Calculate net deflection of the perforated shear wall as follows:

$$\Delta_{\text{net}} = \Delta_{\text{gross}} - \Delta_{\text{strip}} + \Delta_{\text{piers}} \tag{7.29}$$

$$\text{Relative rigidity } R = \frac{1}{\Delta_{\text{net}}} \tag{7.30}$$

3. *Method C*: It was noted in Method A that the presence of solid portions of wall above and below the openings is ignored; only the vertical segments between the openings are considered. In Method C, all solid portions of the wall—piers as well as beams—are considered for determining the wall rigidity. Each pier is assumed fixed at top and bottom. The method uses the principle of determining the stiffness of a system of springs in "parallel" or in "series." The following assumptions are made in this procedure:
(a) Vertical segments in between the openings are considered to be in parallel, similar to system of "springs in parallel." The height of such a segment is equal to the smaller of the height of the opening on each side of the segment.
(b) Segment of the wall (i.e., a beam) below the opening is assumed to be in "series" with the piers above it.
(c) The segment of the wall above the openings is assumed to be in "series" with the group of piers/beams below it.
(d) Rigidity of a segment of the wall containing piers and a beam (below) in the group is calculated based on the principles of springs in "parallel" and in "series."
(e) The rigidity of the entire wall is determined based on the springs in "series," considering the rigidities of the beam above and the group of piers below.

The principles involved in determining the stiffness of a system of springs in "parallel" or in series can be explained as follows:

(a) *Springs in parallel concept:* When a system of springs, each with a different stiffness, is in "parallel," all springs experience the same deformation, but the force in each spring is proportional to its stiffness (i.e., spring force = spring stiffness times the system displacement). Consider, for example, a system of three springs, having stiffnesses equal to k_1, k_2, and k_3. This system is subjected to a force P causing a displacement y as shown in Fig. 7.19. Force P is shared simultaneously by all three springs.

The force in each spring equals stiffness times displacement; thus, the force in the three springs can be expressed as k_1y, k_2y, and k_3y, respectively. For equilibrium, the sum of these three spring forces must equal P. Thus,

$$P = k_1y + k_2y + k_3y \tag{7.31}$$

Eq. (7.31) can be expressed as

$$\frac{P}{y} = k_1 + k_2 + k_3 \tag{7.32}$$

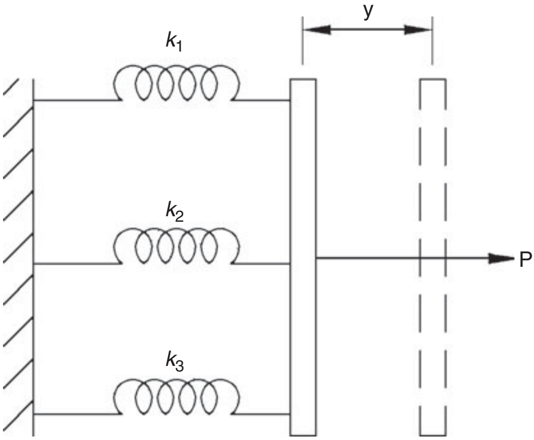


FIGURE 7.19 "Springs in parallel" concept.

The left side of Eq. (7.32) represents force per unit displacement, which is defined as rigidity. If we express $P/y = k$ = system stiffness, then

$$k = k_1 + k_2 + k_3 \tag{7.33}$$

In general, Eq. (7.33) can be expressed as Eq. (7.34):

$$k = \sum_{i=1}^{n} k_i \tag{7.34}$$

Equation (7.34) states that the rigidity of a system of springs in parallel equals the sum of the rigidities of individual springs. Method A is essentially based on this concept.

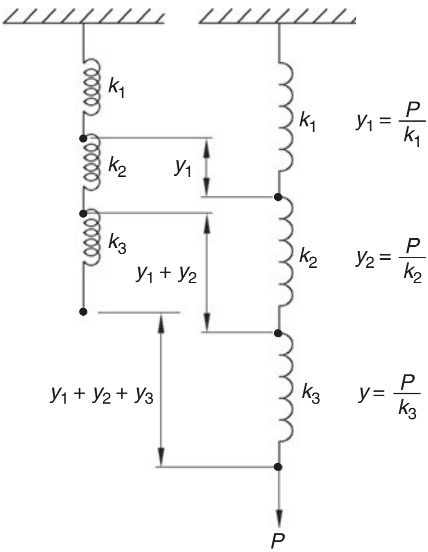


FIGURE 7.20 "Springs in series" concept.

(b) *Springs in series concept:* In a system of springs in a series, all springs experience the same spring force because the same force must be transferred from one spring to the other (Fig. 7.20). However, each spring undergoes different displacement depending on its stiffness. Consider again that three springs, having stiffnesses, k_1, k_2, and k_3, respectively, are connected to each other end-to-end. The displacements of these three springs can be expressed as spring force divided by spring stiffness. Thus,

$$y_1 = \frac{P}{k_1} \qquad y_2 = \frac{P}{k_2} \qquad y_3 = \frac{P}{k_3} \tag{7.35}$$

If the total displacement of the spring system is y, then y equals the sum of three quantities given by Eq. (7.36):

$$y = y_1 + y_2 + y_3 = \frac{P}{k_1} + \frac{P}{k_2} + \frac{P}{k_3} = P\left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}\right) \tag{7.36}$$

If we define $P/y = k$ = system stiffness, then Eq. (7.36) can be expressed as

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} \tag{7.37}$$

In general, Eq. (7.37) can be expressed as Eq. (7.38):

$$\frac{1}{k} = \sum_{i=1}^{n} \frac{1}{k_i} \tag{7.38}$$

Equation (7.38) states that the reciprocal of the system stiffness equals the sum of the reciprocals of stiffnesses of individual springs.

Application of the three above described methods is illustrated by Examples 7.4 through 7.8. In a masonry-building project where several perforated walls are encountered, calculation of rigidities of various piers becomes a time-consuming task. For computational efficiency, design aids (Tables A.26 and A.27) can be used.

Example 7.4 Rigidity of a perforated masonry shear wall. Calculate the rigidity of an 8-in.-thick (nominal) concrete masonry shear wall that has an opening as shown in Fig. E7.4, using Methods A, B, and C. Assume that f'_m = 2000 lb/in.^{2}.

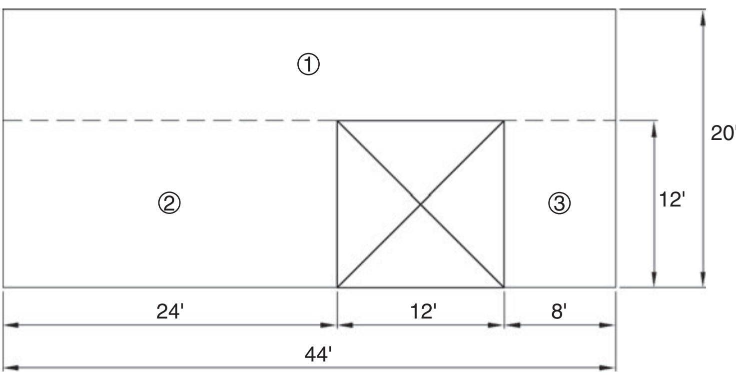


FIGURE E7.4 Rigidity of a shear wall with openings.

Solution

$f'_m = 2000$ lb/in.^{2} $\quad E_m = 900(2.0) = 1800$ kips/in.^{2} $\quad t = 7.625$ in. (8 in. nominal)

1. *Method A:* Calculate the rigidities of piers 2 and 3 assuming them as fixed-ended.
Pier 2:

$$\frac{h}{d}=\frac{12}{24}=0.5$$

$$\Delta_2=\frac{1}{E_m t}\left[\left(\frac{h}{d}\right)^3+3\left(\frac{h}{d}\right)\right]=\frac{1}{(1800)(7.625)}[(0.5)^3+3(0.5)]$$

$$=1.184(10^{-4})\text{ in.}$$

$$R_2=\frac{1}{\Delta_2}=\frac{1}{1.184(10^{-4})}=8446\text{ kips/in.}$$

Pier 3:

$$\frac{h}{d}=\frac{12}{8}=1.5$$

$$\Delta_3=\frac{1}{E_m t}\left[\left(\frac{h}{d}\right)^3+3\left(\frac{h}{d}\right)\right]=\frac{1}{(1800)(7.625)}[(1.5)^3+3(1.5)]$$

$$=5.738(10^{-4})\text{ in.}$$

$$R_3=\frac{1}{\Delta_2}=\frac{1}{5.738(10^{-4})}=1743\text{ kips/in.}$$

The rigidity of the wall is the sum of the rigidities of piers 1 and 2. Thus,

$$R_{\text{wall}}=R_2+R_3=8446+1743=10{,}189\text{ kips/in.}$$

2. *Method B:* Calculate the deflection of the entire wall assuming it as a solid cantilever (ignore all openings).

$$\frac{h}{d}=\frac{20}{44}=0.4545$$

$$\Delta_{\text{gross}}=\frac{1}{E_m t}\left[\left(\frac{h}{d}\right)^3+3\left(\frac{h}{d}\right)\right]=\frac{1}{(1800)(7.625)}[4(0.4545)^3+3(0.4545)]$$

$$=12.6706(10^{-5})\text{ in.}$$

Calculate the deflection of the interior strip containing the two openings, assuming it as solid and fixed-ended. This strip is 44 ft long and 12 ft high.

$$\frac{h}{d}=\frac{12}{44}=0.273$$

$$\Delta_{\text{solid strip}}=\frac{1}{E_m t}\left[\left(\frac{h}{d}\right)^3+3\left(\frac{h}{d}\right)\right]=\frac{1}{(1800)(7.625)}[(0.273)^3+3(0.273)]$$

$$=6.1155(10^{-5})\text{ in.}$$

Calculate deflections of piers 2 and 3 assuming each as fixed-ended.

$$\Delta_2 = (1.184)\ (10^{-4})\text{ in. (calculated for Method A)}$$

$$\Delta_3 = (5.738)\ (10^{-4})\text{ in. (calculated for Method A)}$$

Calculate the deflection of combined Pier 2-3:

$$\Delta_{2+3} = \frac{1}{\dfrac{1}{\Delta_2}+\dfrac{1}{\Delta_3}} = \frac{1}{\dfrac{1}{(1.184)(10^{-4})}+\dfrac{1}{(5.738)(10^{-4})}} = (9.8418)(10^{-4})\text{ in.}$$

Calculate the net deflection of the wall.

$$\Delta_{\text{net}} = \Delta_{\text{gross}} - \Delta_{\text{strip}} + \Delta_{\text{piers}} = (12.6707 - 6.1155 + 9.8148)\ (10^{-5})\text{ in.}$$

$$= (16.3699)\ (10^{-5})\text{ in.}$$

$$R_{\text{wall}} = \frac{1}{\Delta_{\text{net}}} = \frac{1}{(16.3699)(10^{-5})} = 6109\text{ kips/in.}$$

The rigidity of the wall is 6109 kips/in.

3. *Method C:* Calculate the rigidity of Pier 1 assuming it as a cantilever.

$$\frac{h}{d} = \frac{8}{44} = 0.182$$

$$\Delta_1 = \frac{1}{E_m t}\left[4\left(\frac{h}{d}\right)^3 + 3\left(\frac{h}{d}\right)\right] = \frac{1}{(1800)(7.625)}[4(0.182)^3 + 3(0.182)]$$

$$= 4.1538(10^{-5})\text{ in.}$$

$$R_1 = \frac{1}{\Delta_1} = \frac{1}{(4.1538)(10^{-5})} = 24{,}074\text{ kips/in.}$$

Calculate the rigidities of piers 2 and 3, assuming each one as fixed-ended. These were calculated for Method A. Thus,

$$R_2 = 8446\text{ kips/in.}$$

$$R_3 = 1743\text{ kips/in.}$$

Calculate the rigidity of Pier 2-3 combined. Piers 2 and 3 are in parallel. Therefore, the rigidity of Pier 2-3 is taken as the sum of their individual rigidities.

$$R_{2+3} = R_2 + R_3 = 8446 + 1743 = 10{,}189\text{ kips/in.}$$

Pier 1 and Pier 2-3 (combined) are in series. Therefore, the reciprocal of the rigidity of Pier 1-2-3 equals the sum of the reciprocals of rigidities of Pier 1 and Pier 2-3. Thus,

$$\frac{1}{R_{1+2+3}} = \frac{1}{R_1} + \frac{1}{R_{2+3}} = \frac{1}{24{,}074} + \frac{1}{10.189} = (1.3968)(10^{-5})$$

$$R_{1+2+3} = 7159 \text{ kips/in.}$$

The rigidity of the wall is 7159 kips/in.

Summary:

Method A: R = 10,189 kips/in.

Method B: R = 6109 kips/in.

Method C: R = 7159 kips/in.

Commentary: It is instructive to compare the rigidity of the perforated wall with that of the solid wall. The gross deflection of the solid wall was calculated for Method B:

$$\Delta_{\text{gross}} = (12.6706)(10^{-5}) \text{ in.}$$

$$R_{\text{solid}} = \frac{1}{\Delta_{\text{gross}}} = \frac{1}{(12.6706)(10^{-5})} = 7892 \text{ kips/in.}$$

The rigidity of the whole wall, 7892 kips/in., is smaller than the sum of the rigidities of the three component piers calculated by Method 1 (10,189 kips/in.), which obviously is absurd, and points out the approximation inherent in Method 1.

Example 7.5 Distribution of lateral loads to piers of a perforated shear wall.

The masonry wall shown in Figure E7.5 has a uniform nominal thickness of 8 in., and is a part of a one-story building. (a) Determine the relative rigidity of the wall by Method A; (b) describe the load path in the shear wall and shear force in each pier, where $f'_m = 2000$ lb/in.^{2}.

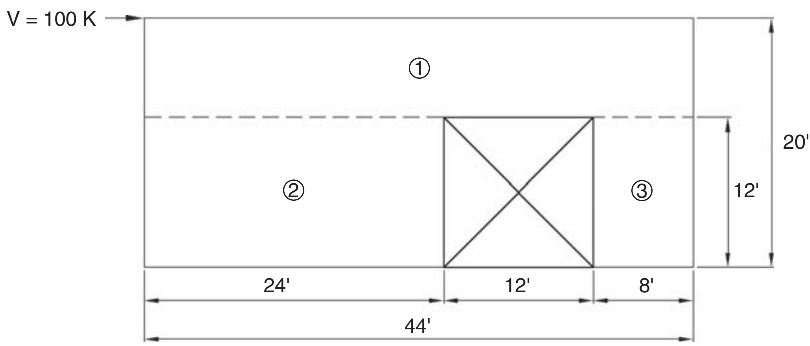


FIGURE E7.5

Solution

a. *Relative rigidity of wall by Method A:* The relative rigidity of the wall would be assumed as the sum of the relative rigidities of piers 2 and 3 assuming them as fixed-ended.

Pier 2:

$$\frac{h}{d} = \frac{12}{24} = 0.5$$

$$\mathrm{R}_2 = 6.154 \text{ (Table A.27)}$$

Pier 3:

$$\frac{h}{d} = \frac{12}{8} = 1.5$$

$$\mathrm{R}_3 = 1.27 \text{ (Table A.27)}$$

The relative rigidity wall is taken as the sum of the relative rigidities of piers 2 and 3 (springs in parallel). Thus,

$$\mathrm{R}_{\text{wall}} = \mathrm{R}_2 + \mathrm{R}_3 = 6.154 + 1.27 = 7.424$$

b. Load path: At the top of the wall, Pier 1 resists total shear force. At the door-opening level, the shear from Pier 1 is transferred to piers 2 and 3, in proportion to their relative rigidities calculated earlier:
Pier 2:

$$R_2 = 6.154$$

Pier 3:

$$R_3 = 1.27$$

$$\Sigma R = R_2 + R_3 = 6.154 + 1.27 = 7.424$$

$$V_1 = 100 \text{ kips}$$

$$V_2 = \left(\frac{R_2}{R_2 + R_3}\right) = \left(\frac{6.154}{7.424}\right)(100) = 82.9 \text{ kips}$$

$$V_3 = \left(\frac{R_3}{R_2 + R_3}\right) = \left(\frac{1.27}{7.424}\right)(100) = 17.1 \text{ kips}$$

Commentary: The rigidity of the entire wall assuming it a cantilever is

$$\frac{h}{d} = \frac{20}{44} = 0.455$$

$$R_c = 0.5(5.833 + 5.652) = 5.74 \text{ (by interpolation from Table A.26)}$$

Thus, it is seen that the relative rigidity of the entire wall (assuming it as a cantilever) is smaller than its relative rigidity calculated as the sum of the relative rigidities of piers 1 and 2, which is absurd. Let us now calculate the relative rigidity of the entire wall having a length of 44 ft and a height of 12 ft (same height as that of the piers).

$$\frac{h}{d} = \frac{12}{44} = 0.273$$

Considering the wall as fixed-ended (same as piers 2 and 3), the relative rigidity is

$$R = 12.053 + (12.053 - 11.602)(0.3)$$
$$= 11.92 > 7.424 \text{ (by interpolation from Table A.27)}$$

Thus, the relative rigidity of the entire wall is greater than the sum of the relative rigidities of piers 2 and 3.

Example 7.6 Rigidity of a perforated wall with two openings.
Figure E7.6 shows an 8-in.-thick (nominal) reinforced concrete masonry wall. Calculate its rigidity by the three methods discussed earlier. $f'_m = 2000$ lb/in.^{2}.

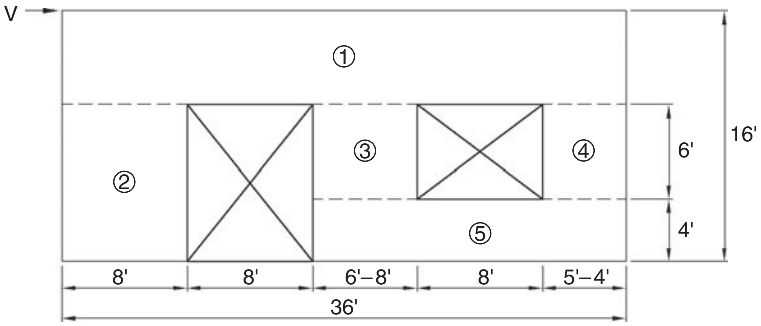


FIGURE E7.6 Rigidity of a perforated shear wall.

Solution

$$f'_m = 2000 \text{ lb/in}^2 \qquad E_m = 900\, f'_m = 900(2.0)$$
$$= 1800 \text{ kips/in}^2 \qquad t = 7.625 \text{ in. (8-in. nominal)}$$

Method A: The rigidity of the wall would be taken as the sum of the rigidities of piers 2, 3, and 4 (assumed as fixed-ended).
Pier 2:

$$\frac{h}{d} = \frac{10}{8} = 1.25$$

$$\Delta_2 = \frac{1}{E_m t} = \left[\left(\frac{h}{d}\right)^3 + 3\left(\frac{h}{d}\right)\right]$$

$$= \frac{1}{(1800)(7.625)}[(1.25)^3 + 3(1.25)]$$

$$= 4.1553(10^{-4}) \text{ in.}$$

$$R_2 = \frac{1}{\Delta_2} = \frac{1}{4.1553(10^{-4})} = 2407 \text{ kips/in.}$$

Pier 3:

$$\frac{h}{d} = \frac{6}{6.67} = 0.9$$

$$\Delta_3 = \frac{1}{E_m t}\left[\left(\frac{h}{d}\right)^3 + 3\left(\frac{h}{d}\right)\right]$$

$$= \frac{1}{(1800)(7.625)}[(0.9)^3 + 3(0.9)]$$

$$= 2.498\,(10^{-4})\text{ in.}$$

$$R_3 = \frac{1}{\Delta_3} = \frac{1}{2.498\,(10^{-4})} = 4003\text{ kips/in.}$$

Pier 4:

$$\frac{h}{d} = \frac{6}{5.333} = 1.125$$

$$\Delta_4 = \frac{1}{E_m t} = \left[\left(\frac{h}{d}\right)^3 + 3\left(\frac{h}{d}\right)\right]$$

$$= \frac{1}{(1800)(7.625)}[(1.125)^3 + 3(1.125)]$$

$$= 3.496\,(10^{-4})\text{ in.}$$

$$R_4 = \frac{1}{\Delta_4} = \frac{1}{3.496(10^{-4})} = 2860\text{ kips/in.}$$

$$R_{\text{wall}} = R_2 + R_3 + R_4 = 2407 + 4003 + 2860 = 9270\text{ kips/in.}$$

Method B: Calculate the deflection of the entire wall assuming it as a solid cantilever. Ignore all openings.

$$\frac{h}{d} = \frac{16}{36} = 0.4444$$

$$\Delta_3 = \frac{1}{E_m t}\left[4\left(\frac{h}{d}\right)^3 + 3\left(\frac{h}{d}\right)\right]$$

$$= \frac{1}{(1800)(7.625)}[4(0.4444)^3 + 3(0.4444)]$$

$$= 1.2271\,(10^{-4})\text{ in.}$$

Calculate the deflection of the solid strip containing the tallest opening (combined piers 2, 3, 4, and 5) assuming it as fixed-ended.

$$\frac{h}{d} = \frac{10}{36} = 0.2778$$

$$\Delta_{\text{solid strip }2+3+4+5} = \frac{1}{E_m t}\left[\left(\frac{h}{d}\right)^3 + 3\left(\frac{h}{d}\right)\right]$$

$$= \frac{1}{(1800)(7.625)}[(0.2778)^3 + 3(0.2778)]$$
$$= 6.2883(10^{-5})\text{ in.}$$

Calculate the deflection of Pier 2 assuming it as fixed-ended.

$$\Delta_2 = 4.1553\ (10^{-4})\text{ in. (calculated earlier)}$$

Calculate the deflection of the solid strip containing piers 3, 4, and 5 (ignore openings), assuming it as fixed-ended. This strip contains piers 3, 4, and 5.

$$\frac{h}{d} = \frac{10}{20} = 0.5$$
$$\Delta_{\text{solid strip }3+4+5} = \frac{1}{E_m t}\left[\left(\frac{h}{d}\right)^3 + 3\left(\frac{h}{d}\right)\right]$$
$$= \frac{1}{(1800)(7.625)}[(0.5)^3 + 3(0.5)]$$
$$= 1.184(10^{-4})\text{ in.}$$

Calculate the deflection of the solid strip containing piers 3 and 4, assuming it as fixed-ended.

$$\frac{h}{d} = \frac{6}{20} = 0.3$$
$$\Delta_{\text{solid striop }3+4} = \frac{1}{E_m t}\left[\left(\frac{h}{d}\right)^3 + 3\left(\frac{h}{d}\right)\right]$$
$$= \frac{1}{(1800)(7.625)}[(0.3)^3 + 3(0.3)]$$
$$= 6.7541(10^{-5})\text{ in.}$$

Calculate deflections of individual piers 3 and 4, assuming each as fixed-ended.

$$\Delta_3 = 2.4984\ (10^{-4})\text{ in. (calculated earlier)}$$

$$\Delta_4 = 2.4551\ (10^{-4})\text{ in. (calculated earlier)}$$

Calculate the net deflection of the Pier 3-4-5.

$$\Delta_{3+4} = \frac{1}{\frac{1}{\Delta_3} + \frac{1}{\Delta_4}} = \frac{1}{\frac{1}{2.4984(10^{-4})} + \frac{1}{2.4551(10^{-4})}} = 1.2381(10^{-4})\text{ in.}$$

$$\Delta_{3+4+5} = \Delta_{\text{solid}} - \Delta_{\text{solid }3+4} + \Delta_{3+4} = (1.184 - 0.6754 + 1.2381)(10^{-4}) = 1.7647(10^{-4})\text{ in.}$$

$$\Delta_{2+3+4+5} = \frac{1}{\frac{1}{\Delta_2} + \frac{1}{\Delta_{3+4+5}}} = \frac{1}{\frac{1}{4.1553(10^{-4})} + \frac{1}{1.7647(10^{-4})}} = 1.2387(10^{-4})\text{ in.}$$

$$\Delta_{\text{net}} = \Delta_{\text{gross}} - \Delta_{\text{solid}} + \Delta_{2+3+4+5} = (1.2271 - 0.6288 + 1.2387)(10^{-4}) = 1.837(10^{-4}) \text{ in.}$$

$$R = \frac{1}{\Delta_{\text{net}}} = \frac{1}{(1.837)(10^{-4})} = 5444 \text{ kips/in.}$$

Method C: In Fig. E7.6, all piers except Pier 1 can be considered as fixed against rotation at top and bottom. Pier 1 would be considered as a cantilever, fixed at the base and free to rotate at the top. With these assumptions, rigidities of various piers can be calculated as follows:

Pier 1:

$$\frac{h}{d} = \frac{6}{36} = 0.167$$

$$\Delta_1 = \frac{1}{E_m t\left[4\left(\frac{h}{d}\right)^3 + 3\left(\frac{h}{d}\right)\right]}$$

$$= \frac{1}{(1800)(7.625)}[4(0.167)^3 + 3(0.167)]$$

$$= 3.786(10^{-5}) \text{ in.}$$

$$R_1 = \frac{1}{\Delta_1} = \frac{1}{(3.786)(10^{-5})} = 26{,}413 \text{ kips/in.}$$

Pier 2:

$$\frac{h}{d} = \frac{10}{8} = 1.25$$

$$\Delta_2 = \frac{1}{E_m t} = \left[\left(\frac{h}{d}\right)^3 + 3\left(\frac{h}{d}\right)\right]$$

$$= \frac{1}{(1800)(7.625)}[(1.25)^3 + 3(1.25)]$$

$$= 4.1553(10^{-4}) \text{ in.}$$

$$R_2 = \frac{1}{\Delta_2} = \frac{1}{(4.1553)(10^{-4})} = 2407 \text{ kips/in.}$$

Pier 3:

$$\frac{h}{d} = \frac{6}{6.67} = 0.9$$

$$\Delta_3 = \frac{1}{E_m t}\left[\left(\frac{h}{d}\right)^3 + 3\left(\frac{h}{d}\right)\right]$$

$$= \frac{1}{(1800)(7.625)}[(0.9)^3 + 3(0.9)]$$

$$= 2.498(10^{-4}) \text{ in.}$$

$$R_3 = \frac{1}{\Delta_3} = \frac{1}{(2.498)(10^{-4})} = 4003 \text{ kips/in.}$$

Pier 4:

$$\frac{h}{d} = \frac{6}{5.333} = 1.125$$

$$\Delta_4 = \frac{1}{E_m t} = \left[\left(\frac{h}{d}\right)^3 + 3\left(\frac{h}{d}\right)\right]$$

$$= \frac{1}{(1800)(7.625)}[(1.125)^3 + 3(1.125)]$$

$$= 3.496\,(10^{-4})\text{ in.}$$

$$R_4 = \frac{1}{\Delta_4} = \frac{1}{(3.496)(10^{-4})} = 2860\text{ kips/in.}$$

Pier 5:

$$\frac{h}{d} = \frac{4}{20} = 0.2$$

$$\Delta_5 = \frac{1}{E_m t} = \left[\left(\frac{h}{d}\right)^3 + 3\left(\frac{h}{d}\right)\right]$$

$$= \frac{1}{(1800)(7.625)}[(0.2)^3 + 3(0.2)]$$

$$= 4.43\,(10^{-5})\text{ in.}$$

$$R_5 = \frac{1}{\Delta_5} = \frac{1}{(4.43)(10^{-5})} = 22{,}573\text{ kips/in.}$$

Piers 3 and 4 are in parallel. Therefore, the combined rigidity of these two piers is the sum of their rigidities. Thus,

$$R_{3+4} = R_3 + R_4 = 4003 + 2860 = 6863\text{ kips/in.}$$

Piers 3-4 and 5 are in series. Therefore, the reciprocal of the rigidity of Pier 3-4-5 is given by the sum of the reciprocals of the rigidities of piers 3-4 and 5. Thus,

$$R_{3+4+5} = 5263\text{ kips/in.}$$

$$\frac{1}{R_{3+4+5}} = \frac{1}{R_{3+4}} + \frac{1}{R_5} = \frac{1}{6863} + \frac{1}{22{,}573} = 1.900\,(10^{-4})$$

Piers 2 and 3-4-5 are in parallel. Therefore, their combined rigidity is given by the sum of their rigidities. Thus,

$$R_{2+3+4+5} = R_2 + R_{3+4+5} = 2407 + 5263 = 7670\text{ kips/in.}$$

Pier 1 and Pier 2-3-4-5 are in series. Thus, the combined rigidity of these piers is given by

$$\frac{1}{R_{1+2+3+4+5}} = \frac{1}{R_1} + \frac{1}{R_{2+3+4+5}}$$

$$= \frac{1}{26{,}413} + \frac{1}{7670} = 1.6842\,(10^{-4})$$

$$R_{1+2+3+4+5} = \frac{1}{1.6842(10^{-4})} = 5944\text{ kips/in.}$$

Summary:

Method A: R = 9270 kips/in.

Method B: R = 5444 kips/in.

Method C: R = 5944 kips/in.

Commentary: It is instructive to compare the rigidity of the perforated shear wall with that of the solid shear wall. For the solid shear wall,

$$\frac{h}{d} = \frac{16}{36} = 0.444$$

Considering the solid wall as a cantilever,

$$\begin{aligned}\Delta_{\text{solid}} &= \frac{1}{E_m t} = \left[4\left(\frac{h}{d}\right)^3 + 3\left(\frac{h}{d}\right)\right] \\ &= \frac{1}{(1800)(7.625)}[4(0.444)^3 + 3(0.444)] \\ &= 1.2256(10^{-4})\,\text{in.}\end{aligned}$$

$$R_{\text{solid}} = \frac{1}{\Delta_{\text{solid}}} = \frac{1}{1.2256(10^{-4})} = 8159\,\text{kips/in}$$

Thus, the rigidity of the perforated wall (using the smallest calculated value, Method B) is

$$\left(\frac{5444}{8159}\right)(100) = 66.7\,\text{percent of the rigidity of the solid wall}$$

The rigidity of the perforated wall based on Method C is

$$\left(\frac{5944}{8159}\right)(100) = 72.8\text{ percent of the rigidity of the solid wall}$$

It is also instructive to compare the rigidity of the solid wall, 8159 kips/in., with the rigidity of the perforated wall given by Method A, 9270 kips/in. (>8159 kips/in.). The reason for this discrepancy was explained earlier.

Example 7.7 Relative rigidity of a perforated masonry shear wall and distribution of shear to various piers within the wall.

The masonry shear wall shown in Fig. E7.6 is 8 in. thick (nominal) and subjected to a shear force V = 100 kips. Using Method C, calculate (a) the relative rigidity of the wall, (b) distribution of shear force in various piers.

Solution

a. Observe that Pier 1 is fixed at the base and free to rotate at the top; accordingly, it would be treated as a cantilever. By the virtue of their positions, Piers 2, 3, 4, and 5 would be treated as fixed at both ends. Relative rigidities of all five piers are shown in Table E7.7 as determined from Tables A.26 and A.27.

TABLE E7.7 Relative Rigidities of Piers

Pier	h(ft)	D(ft)	h/d	R_r	Comments
1	6	36	0.167	19.248	Cantilever
2	10	8	1.25	1.753	Fixed
3	6	6.67	0.9	2.916	Fixed
4	6	5.333	1.125	2.084	Fixed
5	4	20	0.2	16.447	Fixed

Piers 3 and 4 are in parallel. Their combined relative rigidity is equal to the sum of the relative rigidities of individual piers. Thus,

$$R_{3+4} = R_3 + R_4 = 2.916 + 2.084 = 5.0$$

Piers 3-4 and 5 are in series. Therefore, the relative rigidity is given by the sum of the reciprocals:

$$R_{3+4+5} = \frac{1}{0.261} = 3.831$$

$$\frac{1}{R_{3+4+5}} = \frac{1}{R_{3+4}} + \frac{1}{R_5} = \frac{1}{5.0} + \frac{1}{16.447} = 0.261$$

Piers 2 and 3-4-5 are in parallel. Therefore, their combined relative rigidity is given by the sum of their relative rigidities:

$$R_{2+3+4+5} = R_2 + \mathrm{R}_{3+4+5} = 1.753 + 3.831 = 5.584$$

Piers 1 and 2-3-4-5 are in series. Therefore, their combined relative rigidity is given by

$$\frac{1}{R_{1+2+3+4+5}} = \frac{1}{R_1} + \frac{1}{R_{2+3+4+5}}$$

$$= \frac{1}{19.248} + \frac{1}{5.584} = 0.231$$

$$R_{\text{wall}} = \frac{1}{0.231} = 4.33$$

The relative rigidity of the solid wall can be calculated.

$$\left(\frac{h}{d}\right)_{\text{solid}} = \frac{16}{36} = 0.444$$

$R_{\text{solid}} = 6.021 - (6.021\text{-}5.833)(0.4) = 5.946$ (by interpolation from Table A.26)

The relative rigidity of the perforated wall is

$$\left(\frac{4.33}{5.946}\right)(100) \approx 72.8\% \text{ of relative rigidity the solid wall}$$

This is the same as the result obtained in Example 7.6.

b. Distribution of shear in piers:

$$V_1 = 100 \text{ kips}$$

$$V_2 = \left(\frac{1.753}{1.753 + 3.831}\right)(100) = 31.39 \text{ kips}$$

$$V_{3+4+5} = \left(\frac{3.831}{1.753 + 3.831}\right)(100) = 68.61 \text{ kips}$$

$$V_3 = \left(\frac{2.916}{2.916 + 2.084}\right)(68.61) = 40.01 \text{ kips}$$

$$V_4 = \left(\frac{2.084}{2.916 + 2.084}\right)(68.61) = 28.60 \text{ kips}$$

$$V_5 = V_3 + V_4 = 40.01 + 28.60 = 68.61 \text{ kips}$$

Example 7.8 Relative rigidity of a perforated wall with several openings. Figure E7.8 shows an 8-in.-thick (nominal) concrete masonry shear wall that has several openings. Calculate (a) relative rigidity of the wall by Methods 1, 2, and 3, (b) distribution of a shear force of 100 kips applied at the top of the wall to various piers, (c) relative rigidity of perforated wall as a percentage of the relative rigidity of the solid wall. $f'_m = 2000$ lb/in.^{2}.

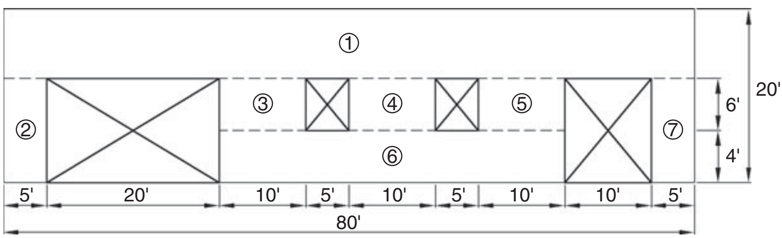


FIGURE E7.8 Relative rigidity of a shear wall with multiple openings.

Solution

a. Relative rigidity of wall: Relative rigidities of various piers shown in Fig. E7.8 are obtained from Tables A.26 and A.27 based on their respective *h*/*d* ratios, and are listed in Table E7.8. Pier 1 is considered as a cantilever because it is fixed at its base and free to rotate at the top. All other piers are considered fixed at both top and bottom.

TABLE E7.8 Relative rigidities of piers.

Pier	*h*(ft)	*d*(ft)	*h*/*d*	R_r	Comments
1	10	80	0.125	26.122	Cantilever
2	10	5	2.0	0.714	Fixed
3	6	10	0.6	4.96	Fixed
4	6	10	0.6	4.96	Fixed
5	6	10	0.6	4.96	Fixed
6	4	40	0.1	33.223	Fixed
7	10	5	2.0	0.714	Fixed

Method A: The relative rigidity of the entire wall is taken as the sum of the relative rigidities of individual piers. Thus,

$$R = \sum_{i=1}^{n} R_i = 0.714 + 4.96 + 4.96 + 4.96 + 0.714 = 16.31$$

The relative rigidity of the perforated shear wall is 16.31.
Method B: Calculate the deflection of the entire wall assuming it as a solid cantilever.

$$\frac{h}{d} = \frac{20}{80} = 0.25$$

$$R = 12.308 \text{ (Table A.26)}$$

Calculate the deflection of the solid strip of the wall having length equal to length of the entire wall and height equal to that of the tallest opening, assuming it as fixed-ended. It contains piers 2 through 7.

$$\frac{h}{d} = \frac{10}{80} = 0.125$$

$$R_{\text{solid strip } 2+3+4+5+6+7} = 26.528 \text{ (Table A.27)}$$

Calculate the deflection of the solid strip containing piers 3 through 6, assuming it as fixed-ended.

$$\frac{h}{d} = \frac{10}{40} = 0.25 \quad R_{3+4+5+6} = 13.061$$

$$\Delta_{3+4+5+6} = \frac{1}{R_{3+4+5+6}} = \frac{1}{13.061} = 7.6564(10^{-2})$$

Calculate the deflections of piers 2 and 7, assuming them as fixed-ended.

$$R_2 = R_7 = 0.714 \text{ (calculated earlier)}$$

Therefore, $$\Delta_2 = \Delta_7 = \frac{1}{0.714} = 1.401$$

Calculate deflections of piers 3, 4, and 5 individually, assuming them as fixed-ended.

$$R_3 = R_4 = R_5 = 4.96 \text{ (calculated earlier)}$$

Therefore, $$\Delta_3 = \Delta_4 = \Delta_5 = \frac{1}{4.96} = 0.2061$$

Calculate deflection of Pier 3-4-5.

$$\Delta_{3+4+5} = \frac{1}{\dfrac{1}{\Delta_2} + \dfrac{1}{\Delta_{3+4+5+6}} + \dfrac{1}{\Delta_7}} = \frac{1}{\dfrac{1}{1.401} + \dfrac{1}{0.0984}\,\dfrac{1}{1.401}} = 0.08628$$

For the entire wall,

$$\Delta_{\text{net, wall}} = \Delta_{\text{gross}} - \Delta_{\text{solid strip}} + \Delta_{\text{piers}} = 0.08125 - 0.0377 + 0.08628 = 0.12983$$

$$R_{\text{wall}} = \frac{1}{\Delta_{\text{net}}} = \frac{1}{0.12983} = 7.70$$

The relative rigidity of the perforated shear wall is 7.70.

Method C: Piers 3, 4, and 5 are in parallel. Therefore, their combined relative rigidity equals the sum of their relative rigidities:

$$\begin{aligned} R_{3+4+5} &= R_3 + R_4 + R_5 \\ &= 3(4.96) = 14.88 \end{aligned}$$

Piers 3-4-5 and 6 are in series. Therefore, the reciprocal of the relative rigidity of Pier 3-4-5-6 equals the sum of the reciprocals of the relative rigidities of piers 3-4-5 and 6:

$$\begin{aligned} \frac{1}{R_{3+4+5+6}} &= \frac{1}{R_{3+4+5}} + \frac{1}{R_6} \\ &= \frac{1}{14.88} + \frac{1}{33.223} \\ &= 0.0973 \\ R_{3+4+5+6} &= \frac{1}{0.0973} = 10.28 \end{aligned}$$

Now, piers 2, 3-4-5-6, and 7 are in parallel. Therefore, their combined relative rigidity is given by the sum of their relative rigidities.

$$R_{2+3+4+5+6+7} = 0.714 + 10.28 + 0.714 = 11.71$$

Pier 1 is in series with all other piers combined. Therefore, the reciprocal of the relative rigidity of the entire wall equals the sum of the reciprocals of the relative rigidity of Pier 1 and the pier group 2-3-4-5-6-7:

$$\frac{1}{R_{\text{wall}}} = \frac{1}{R_1} + \frac{1}{R_{2-7}} = \frac{1}{26.122} + \frac{1}{11.71} = 0.1237$$

$$R_{\text{wall}} = \frac{1}{0.1237} = 8.08$$

The relative rigidity of the perforated shear wall is 8.08.

b. Distribution of shear to various piers:

Method 3:

$$\begin{aligned} V_1 &= 100 \text{ kips} \\ V_2 &= \left(\frac{0.714}{11.71}\right)(100) = 6.1 \text{ kips} \\ V_{3+4+5} &= \left(\frac{10.28}{11.71}\right)(100) = 87.8 \text{ kips} \\ V_7 &= \left(\frac{0.714}{11.71}\right)(100) = 6.1 \text{ kips} \end{aligned}$$

Check: $V_2 + V_{3+4+5} + V_7 = 6.1 + 87.8 + 6.1 = 100 \text{ kips} = V_1$

$$V_3 = V_4 = V_5 = \frac{1}{3}(87.8)(100) = 29.27 \text{ kips}$$

$$V_6 = 87.8 \text{ kips}$$

c. For the solid wall,

$$\frac{h}{d} = \frac{20}{80} = 0.25$$

As a cantilevered wall,

$$R = 12.308 \text{ (Table A.26)}$$

$$R_{\text{perforated wall}} = \left(\frac{8.08}{12.308}\right)(100) = 65.6\% \text{ of } R_{\text{solid}}$$

7.6 DETERMINATION OF SEISMIC LATERAL FORCES IN SHEAR WALLS

7.6.1 Introduction

Determination of horizontal shear to be carried by shear walls involves determination of lateral loads due to wind and earthquake. Depending on the geographical location of the structure, either wind or earthquake loading condition may govern the design of a structure for which shear walls may be used to function as the seismic force–resisting system. Both lateral loads need to be calculated, and the shear walls designed for the more critical of the two loading conditions.

Criteria for construction and design of buildings and other structures subject to earthquake ground motions are presented in Ch. 11 of ASCE 7-05 [7.11]. The earthquake forces specified in that reference are based on post-elastic energy dissipation in the structure, and because of this fact, the requirements for design, detailing, and construction shall be satisfied even for structures and members for which load combinations that do not contain earthquake loads indicate larger demands than combinations that include earthquake loads (ASCE 7-05 Section 11.1.1).

Provisions for determining seismic loads specified in 2009 *International Building Code* [7.10] are based on ASCE 7-05 [7.11], which are new and complex; they are also more expanded and entirely different in content and format from their earlier versions. Background information and discussion on these provisions and other code requirements pertaining to seismic design of buildings can be found in the SEAOC *Blue Book* [7.12] and the NEHRP documents—*Recommended Provisions* [7.13] and *Commentary* [7.14]. It is highly recommended that readers develop a thorough understanding of these provisions.

7.6.2 Design Forces for Shear Walls—Seismic Analysis Methods

Design forces for shear walls in a structure are determined through a procedure that involves several steps; these forces cannot be determined directly. Whether due to wind or earthquake, lateral forces must be first determined for the structure of which shear walls form a part, then these forces must be transferred to horizontal diaphragms, which, in turn, transfer lateral forces to supporting vertical seismic force–resisting elements such as shear walls or braced frames. In all cases of force transfer, a complete, continuous load path must be followed in order to preserve structural safety.

Minimum design seismic forces acting on a structure can be determined by methods commonly referred to as *equivalent lateral force procedure* and *dynamic analysis*. The latter method can be further classified into response spectrum analysis and the time-history analysis. The *equivalent lateral force procedure* is covered by ASCE 7-05 Section 12.8, which is briefly reviewed in this section. ASCE 7-05 Section 12.14.8 also permits a *simplified lateral force analysis procedure*, which is much simpler than the equivalent lateral force procedure. Readers should refer to Refs. 7.13 and 7.14 for a comprehensive discussion on this topic; the following is a summary from these references and Ref. 7.11.

7.6.3 Seismic Load Effects and Load Combinations

In accordance with MSJC-08 Section 3.1.2, design loads for strength design of masonry are to be determined based on load combinations given by ASCE 7-05 Section 2.3 (unless provided for differently by the local building codes). Of the seven load combinations specified in Section 2.3, the two that include seismic load are load combinations 5 and 7, which can be expressed as follows:

$$U = 1.2D + 1.0E + f_1 L + 0.2S \tag{7.39}$$

$$U = 0.9D + 1.0E + 1.6H \tag{7.40}$$

where D = dead load
E = earthquake load
H = load due to lateral earth pressure, ground water pressure, or pressure of bulk materials
L = live load
S = snow load
f_1 = load factor, which is equal to 1.0 for live loads for garages and areas occupied as places of public assembly, and equal to 0.5 for all other live loads

The earthquake load E in Eqs. (7.39) and (7.40) is a function of both horizontal and vertical earthquake-induced forces as specified in ASCE 7-05 Section 12.4, and is expressed as Eq. 7.41:

$$E = E_h \pm E_v \tag{7.41}$$

where E_h and E_v, respectively, are effects of horizontal and vertical earthquake-induced forces defined by Eqs. (7.42) and (7.43):

$$E_h = \rho Q_E \tag{7.42}$$

$$E_v = 0.2\, S_{DS} D \tag{7.43}$$

where Q_E = effects from horizontal earthquake forces from V or F_p
V = total design seismic lateral force or base shear
F_p = seismic force acting on a component of the structure
ρ = redundancy factor
D = effects of dead load
S_{DS} = 5 percent damped, design spectral acceleration parameter for short periods (0.2-s period)

In Eq. (7.41), the *plus* sign is to be associated with Eq. (7.39), that is, when the gravity and seismic loads produce more critical effects, whereas the *minus* sign is to be associated with Eq. (7.40), that is, when the seismic load effects counteract the dead load effects. Accordingly, Eqs. (7.39) and (7.40) can be expressed, respectively, as

$$U = (1.2 + 0.2\,S_{\mathrm{DS}})D + \rho Q_E + f_1 L + 0.2S \tag{7.44}$$

$$U = (0.9 - 0.2\,S_{\mathrm{DS}})D + \rho Q_E + 1.6H \tag{7.45}$$

For gravity loads only, the following load combinations (as applicable) are to be used:

$$U = 1.4(D + F) \tag{7.46}$$

$$U = 1.2(D + F + T) + 1.6(L + H) + 0.5(L_r \text{ or } S \text{ or } R) \tag{7.47}$$

$$U = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (L \text{ or } 0.8W) \tag{7.48}$$

7.6.4 The Equivalent Lateral Force Procedure

In shear wall buildings, lateral loads are transferred to shear walls through diaphragms. Determination of forces for seismic design of a building is typically performed in three steps:

1. Determination of base shear V, the horizontal force that must be transferred to the foundation at the base of a building.

2. Determination of appropriate percentages of the base shear V, which must be assigned to various story levels throughout the height of the building (these forces are referred to as *story forces*).

3 Determination of element forces (or forces on particular elements) which develop as result of story forces (structural elements are designed for these forces).

7.6.4.1 Seismic Response Coefficient

The seismic base shear V acting in a given direction is determined from Eq. (7.49):

$$V = C_s W \tag{7.49}$$

where C_s = seismic response coefficient
W = the effective seismic weight of the structure.

The effective seismic weight W of a structure includes total dead load and other loads as listed below:

1. A minimum of 25 percent of the floor live load in areas used for storage, except that floor live load in public garages and open parking structures need not to be included.

2. Where partition load is to be included in design, the actual partition load or a minimum weight of 10 lb/ft^2 (0.48 kN/m^2) of floor area, whichever is greater.

3. The operating weight of permanent equipment.

4. Where the flat roof snow load P_f exceeds 30 lb/ft^2 (1.44 kN/m^2), 20 percent of the uniform design snow load regardless of the roof slope.

The seismic response coefficient C_s is determined from Eq. (7.50):

$$C_s = \frac{S_{DS}}{\left(\frac{R}{I}\right)} \tag{7.50}$$

where S_{DS} = the design spectral response acceleration parameter in the short period range
R = structural response modification factor for the seismic force–resisting system as listed in ASCE 7-05 Table 12.2-1 (discussed in the next section).
I = importance determined based on the occupancy category of the building
= 1.0 for buildings in Occupancy Category I and II
= 1.25 for buildings in Occupancy Category III
= 1.5 for buildings in Occupancy Category IV.

The value of the seismic response coefficient C_s determined from Eq. (7.50) is subject to an upper limit and a lower limit. Its value need not exceed the value given by Eqs. (7.51) or (7.52) as applicable:

$$C_s = \frac{S_{D1}}{T\left(\frac{R}{I}\right)} \qquad \text{for } T \le T_L \tag{7.51}$$

$$C_s = \frac{S_{D1}T_L}{T^2\left(\frac{R}{I}\right)} \qquad \text{for } T > T_L \tag{7.52}$$

The value of C_s given by Eq. (7.50) may not be less than the value given by Eq. (7.53) (ASCE 7-05 Suppl. 2):

$$C_s = 0.044S_{DS} \ge 0.01 \tag{7.53}$$

In addition to the lower limit for C_s given by Eq. (7.53), there is an additional lower limit for C_s for structures located where the spectral parameter S_1 is equal to or greater than 0.6g:

$$C_s = \frac{0.5S_1}{\left(\frac{R}{I}\right)} \tag{7.54}$$

where S_{D1} = the design spectral response acceleration parameter at a period of 1 s
T = fundamental period of the structure, to be determined as follows
T_L = long-period transition period (seconds) as given in Figs. 22-15 to 22-20 of ASCE 7-05
S_1 = the mapped maximum considered earthquake spectral parameter at a period of 1 s as given Figs. 22-2, 22-4, 22-6, 22-8, 22-10, 22-12, and 22-14 of ASCE 7-05

The design spectral parameters S_{DS} and S_{D1} are determined from the Eqs. (7.55) and (7.56):

$$S_{DS} = 2/3(S_{MS}) \tag{7.55}$$

$$S_{D1} = 2/3(S_{M1}) \tag{7.56}$$

where S_{MS} = the maximum considered earthquake (MCE), 5 percent damped, spectral acceleration parameter at short periods adjusted for the site class effects

S_{M1} = the maximum considered earthquake (MCE), 5 percent damped, spectral acceleration parameter at a period of 1 s adjusted for the site class effects

Spectral parameters S_{MS} and S_{M1} in Eqs. (7.55) and (7.56) are determined from Eqs. (7.57) and (7.58):

$$S_{MS} = F_a S_s \tag{7.57}$$

$$S_{M1} = F_v S_1 \tag{7.58}$$

where S_S = the mapped maximum considered earthquake (MCE) spectral acceleration at short periods (0.2-s period)

S_1 = the mapped maximum considered earthquake (MCE) spectral acceleration at a period of 1 s

F_a = short period (0.2 s) site coefficient (Table 7.2)

F_v = long period (1.0 s) site coefficient (Table 7.3)

Values of S_S and S_1 for various regions of the United States are listed in ASCE 7-05 Figs. 22-1 through 22-14. The values of coefficients F_a and F_v vary with site class and the values of S_s and S_1. The values of both coefficients are 1.0 for Site Class B for all values of S_S and S_1.

TABLE 7.2 Values of Site Coefficient F_a [7.13]

	Mapped MCE spectral response acceleration parameter at 0.2-s period[a]				
Site class	$S_s \le 0.25$	$S_s = 0.50$	$S_s = 0.75$	$S_s = 1.00$	$S_s \ge 1.25$
A	0.8	0.8	0.8	0.8	0.8
B	1.0	1.0	1.0	1.0	1.0
C	1.2	1.2	1.1	1.0	1.0
D	1.6	1.4	1.2	1.1	1.0
E	2.5	1.7	1.2	0.9	0.9
F	-[b]	-[b]	-[b]	-[b]	-[b]

[a]Use straight-line interpolation for intermediate values of S_S.
[b]Site-specific geotechnical investigation and dynamic site response analyses shall be performed.

TABLE 7.3 Values of Site Coefficient F_v [7.13]

	Mapped MCE spectral response acceleration parameter at 1-s period[a]				
Site class	$S_1 \le 0.1$	$S_1 = 0.2$	$S_1 = 0.3$	$S_1 = 0.4$	$S_1 \ge 0.5$
A	0.8	0.8	0.8	0.8	0.8
B	1.0	1.0	1.0	1.0	1.0
C	1.7	1.6	1.5	14	1.3
D	2.4	2.0	1.8	1.6	1.5
E	3.5	3.2	2.8	2.4	2.4
F	-[b]	-[b]	-[b]	-[b]	-[b]

[a]Use straight-line interpolation for intermediate values of S_1.
[b]Site-specific geotechnical investigation and dynamic site response analyses shall be performed.

The value of fundamental period, T, appearing in Eqs. (7.51) and (7.52) should be determined from structural properties and deformational characteristics of the seismic force–resisting system by a properly substantiated analysis. Although this analysis cannot be performed until *after* the structure has been designed and member properties are completely known, its value must be known a priori to use Eqs. (7.51) or (7.52). To obviate this difficulty, an approximate value of the fundamental period, T_a, is determined from Eq. (7.59):

$$T_a = C_t (h_n)^x \tag{7.59}$$

where h_n = height of the structure from the base to the highest level (level n)
C_t, x = coefficients listed in Table 7.4

The height h_n to be used in Eq. (7.59) need not include heights of penthouses and other rooftop-supported structures weighing less than 25 percent of the weight at the roof level.

TABLE 7.4 Values of Approximate Period Parameters C_t and x [7.13]

Structure type	C_t	x
Moment-resisting frame systems of steel in which the frames resist 100 percent of the required seismic force and are not enclosed or adjoined by more rigid components that will prevent the frames from deflecting where subjected to seismic forces.	0.028 (metric 0.0724)	0.8
Moment-resisting frame systems of reinforced concrete in which the frames resist 100 percent of the required seismic force and are not enclosed or adjoined by more rigid components that will prevent the frames from deflecting where subjected to seismic forces.	0.016 (metric 0.0466)	0.9
Eccentrically braced steel frames and buckling restrained braced frames	0.03 (metric 0.0731)	0.75
All other structural systems	0.02 (metric 0.0488)	0.75

The approximation involved in Eq. (7.59) for the value of fundamental period should be evident from the fact that it does not take into account the nature of building configuration (e.g., length-to-width ratio, squat versus slender structure, flexible roof versus rigid roof, etc.), particularly in the "all other structural systems" category.

Alternatively, for structures not exceeding 12 stories in height with each story being at least 10 ft (3 m) high, and the seismic force–resisting system consisting entirely of steel or concrete moment-resisting frames, the approximate fundamental period can be determined from Eq. (7.60):

$$T_a = 0.1N \tag{7.60}$$

where N = number of stories

ASCE 7-05 makes an exception for concrete or masonry shear wall structures for which T_a can be determined from Eq. (7.61):

$$T_a = \frac{0.0019}{\sqrt{C_w}} h_n \tag{7.61}$$

where coefficient C_w of Eq. (7.61) is determined from Eq. (7.62):

$$C_w = \frac{100}{A_B} \sum_{i=1}^{x} \left(\frac{h_n}{h_i} \right)^2 \frac{A_i}{\left[1 + 0.83 \left(\frac{h_i}{D_i} \right)^2 \right]} \tag{7.62}$$

where A_B = area of base of structure, ft^2
A_i = web area of shear wall "i" in ft^2
D_i = length of shear wall "i" in ft
h_i = height of shear wall "i" in ft
x = number of shear walls in the building effective in resisting lateral forces in the direction under consideration

7.6.4.2 Response Modification Factor R for Shear Wall Buildings

The numerical value of response modification coefficient R which appears in Eqs. (7.51) through (7.52), and in Eq. (7.54) depends on the type of seismic force–resisting system employed in a building. The seismic force–resisting system refers to that portion of the structure, which provides load path for transmitting lateral seismic force from a building (or any structure) to the ground. Various types of seismic force–resisting systems and the corresponding values of response modification coefficients are given in ASCE 7-05 Table 12.2-1. The structural systems in which shear walls may be used as the seismic force–resisting systems are *bearing wall systems*, *building frame systems*, and *dual systems* (there are many subcategories within these systems as listed in ASCE 7-05 Table 12.2-1). The values of response modification coefficients applicable to the specific seismic force–resisting systems in which shear walls are permitted (per ASCE 7-05) are listed in Table 7.5. Also shown are the overstrength factors, Ω_o, and the height limitations applicable for these systems (for buildings assigned to seismic design categories B, C, D, E, and F).

Noteworthy in Table 7.5 is the fact that the values of response modification coefficients are smaller for the bearing wall systems than those for the building frame systems and dual systems; the reason for which lies in the way these structural systems resist seismic forces. In a bearing wall system, the walls serve both as vertical (or gravity) load–carrying elements as well as seismic force–resisting elements. Failure of such an element during an earthquake would compromise the ability of the structural system to support gravity loads. By contrast, in building frame systems and dual systems, separate elements are employed to carry gravity and seismic loads. Hence, failure of a seismic force–resisting system does not necessarily compromise the capability of the building frame to carry gravity loads. Thus, a better performance is expected of a building frame system and a dual system than that of a bearing wall system; for this reason lower values of response modification coefficients R are assigned to the latter.

In a typical SFRS with shear walls, the load path (or structural hierarchy) for lateral seismic loads is described as follows:

1. Lateral seismic loads act on walls as out-of-plane loads.
2. The walls transfer these out-of-plane loads to the roof or the floor diaphragms (loads are determined on tributary height basis).
3. The diaphragms transfer their loads including those due to their own inertia, to the shear walls.
4. The shear walls transfer these forces to ground through foundation (Fig. 7.5).

All elements of this hierarchy—the walls receiving seismic forces, the diaphragms, the shear walls, and connections between them—have to be designed to resist their share of seismic forces.

TABLE 7.5 Shear Wall Building Systems [Adapted from Ref. 7.11].

Seismic force–resisting system	Response modification factor, R	System overstrength factor, Ω_o	Deflection amplification factor, C_d	Structural system limitation and building height (ft)				
				Seismic design category				
				B	C	D	E	F
Bearing wall systems								
1 Special reinforced masonry shear walls	5	2½	3½	NL	NL	160	160	100
2 Intermediate reinforced masonry shear walls	3½	2½	2¼	NL	NL	NP	NP	NP
3 Ordinary reinforced masonry shear walls	2	2½	1¾	NL	160	NP	NP	NP
4 Detailed plain masonry shear walls	2	2½	1¾	NL	NP	NP	NP	NP
5 Ordinary plain masonry shear walls	1½	2½	1¼	NL	NP	NP	NP	NP
6 Prestressed masonry shear walls	1½	2½	1¾	NL	NP	NP	NP	NP
Building frame systems								
1 Special reinforced masonry shear walls	5½	2½	4	NL	NL	160	160	100
2 Intermediate reinforced masonry shear walls	4	2½	4	NL	NL	NP	NP	NP
3 Ordinary reinforced masonry shear walls	2	2½	2	NL	160	NP	NP	NP
4 Detailed plain masonry shear walls	2	2½	2	NL	NP	NP	NP	NP
5 Ordinary plain masonry shear walls	1½	2½	1¼	NL	NP	NP	NP	NP

(*Continued*)

Dual systems with special moment frames capable of resisting at least 25% of prescribed seismic forces									
1	Special reinforced masonry shear walls	5½	3	5	NL	NL	NL	NL	NL
2	Intermediate reinforced masonry shear walls	4	3	3½	NL	NL	NP	NP	NP
Dual systems with intermediate moment frames capable of resisting at least 25% of prescribed seismic forces									
1	Ordinary reinforced masonry shear walls	3	3	2½	NL	160	NP	NP	NP
2	Intermediate reinforced masonry shear walls	3½	3	3	NL	NL	NP	NP	NP

Legend: NL = not limited (in height), NP = not permitted (to be built)
See ASCE 7-05 Table 12.2-1 for additional information and footnotes to this table.
Refer to ASCE 7-05 Section 11.6 for seismic design category of structures.

7.6.5 Distribution of Base Shear over the Height of a Building—F_x-Story Forces

The lateral seismic forces induced at any level over the height of a building are referred to as F_x forces (kip or kN). These forces are determined from Eqs. (7.63) and (7.64):

$$F_x = C_{vx} V \tag{7.63}$$

$$C_{vx} = \frac{w_x h_x^k}{\sum_{i=i}^{n} w_i h_i^k} \tag{7.64}$$

where C_{vx} = vertical shear distribution factor
V = total design lateral force or shear at the base of the structure (kip or kN)
w_i *and* w_x = the portion of the total effective seismic weight of the structure (W) located or assigned to Level i or x.
h_i and h_x = the height (ft or m) from the base to level i or x.
k = an exponent related to the fundamental period of the structure, which varies as follows:
$k = 1$ for structures having period of 0.5 s or less
$k = 2$ for structures having period 2 s or more
$k = 2$ for structures having period between 0.5 and 2 s, or can be determined by linear interpolation between $k = 1$ and 2.

Example 7.9 illustrates calculations for the base shear and its distribution over the height of a building.

Example 7.9 Design base shear and distribution of the base shear over the building height.

Determine the base shear and its distribution over the height of a building that consists of a building frame system with special reinforced masonry shear walls (Fig. E7.9). The building is assigned Seismic Design Category D and importance factor $I = 1.0$. It is located in a seismic region for which spectral acceleration parameters S_S and S_1 have been determined, respectively, as $0.90g$ and $0.45g$. The site class for building site has been determined as C. The seismic weight assigned to various levels is as follows:

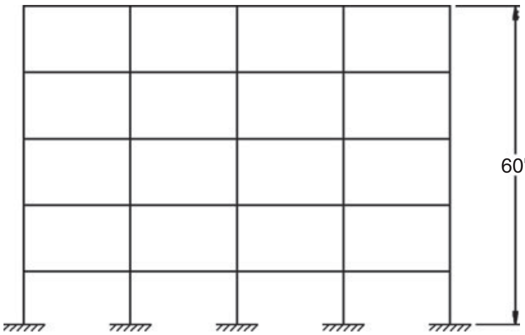


FIGURE E7.9

Level 1: 650 kips
Level 2: 600 kips
Level 3: 500 kips
Level 4: 450 kips
Level 5: 400 kips

Solution

Given:

$S_S = 0.90g$
$S_1 = 0.45g$
Seismic design category = D
Site class = C
I = 1.0

1. Determine S_{DS}

$$S_{DS} = \frac{2}{3} S_{MS}$$

From Table 7.2, $S_S = 0.98g$, $F_a \approx 1.0$

$$S_{MS} = F_a S_s = 1.0(0.90) = 0.90$$

$$S_{DS} = \frac{2}{3}(0.90) = 0.60$$

$$S_{D1} = \frac{2}{3} S_{M1}$$

$$S_{M1} = F_v S_1 = (1.35)(0.45) = 0.61$$

($F_v = 1.35$ found by interpolation for $S_1 = 0.4$ and 0.5 from Table 7.3)

$$S_{D1} = \frac{2}{3}(0.61) = 0.41$$

2. Calculate fundamental period T. Use approximate period, T_a, for T.

$$T_a = C_t\,(h_n)^x$$

$C_t = 0.02$ (all other structural systems) $\qquad h_n = 60$ ft $\qquad x = 0.75$

$$T_a = 0.02\,(60)^{0.75} = 0.43 \text{ s}$$

3. Calculate the seismic response coefficient, C_s.

For a building frame system with special reinforced masonry shear walls, $R = 5.5$ (Table 7.5)

$$C_s = \frac{S_{DS}}{\left(\frac{R}{I}\right)} = \frac{0.60}{\left(\frac{5.5}{1.0}\right)} = 0.11$$

C_s need not exceed

$$C_{s,\max} = \frac{S_{D1}}{T\left(\frac{R}{I}\right)} = \frac{0.41}{(0.43)\left(\frac{5.5}{1.0}\right)} = 0.17 \qquad \text{for } T \le T_L$$

$T_{L,\min} = 4$ s (ASCE 7-05 Fig. 22.15)

In this example $T = 0.43$ s $< T_{L,\min} = 4$ s. Therefore, $C_{s,\max} = 0.17$ [Eq. (7.52) does not apply because $T < T_L$. C_s shall not be less than

$$C_s = 0.044 S_{DS} \ge 0.01$$

$$0.44 S_{DS} = 0.044(0.6) = 0.0264 > 0.01$$

Therefore, $\qquad C_{S,\min} = 0.0264$

For the given structure, $S_1 = 0.45g < 0.60g$; therefore Eq. (7.54) does not apply.

Therefore, $\qquad C_s = 0.11$ governs

$$V = C_s W = 0.11\text{W}$$

$$W = 400 + 450 + 500 + 600 + 650 = 2600 \text{ kips}$$

$$V = 0.11W = 0.11(2600) = 286 \text{ kips}$$

$$F_x = C_{vx} V$$

$$C_{vx} = \frac{w_x h_x^k}{\sum_{i=1}^{n} w_i h_i^k}$$

For the example building, $T = 0.43$ s < 0.5 s, therefore, $k = 1$ so that

$$C_{vx} = \frac{w_x h_x}{\sum^n w_i h_i}$$

Calculations for vertical distribution of base shear are shown in Table E7.9.

TABLE E7.9 Vertical Distribution of Shear ($k = 1.0$)

Level x	h_x (ft)	h_x^k	w_x (kips)	$w_x h_x^k$ (kip-ft)	$C_{vx} = \frac{w_x h_x^k}{\Sigma w_i h_i}$	$F_x = C_{vx} V$ (kips)	Story shear (kips)
5	60	60	400	24,000	0.280	80.1	80.1
4	48	48	450	21,600	0.252	72.1	152.2
3	36	36	500	18,000	0.210	60.1	212.3
2	24	24	600	14,400	0.168	48.0	260.3
1	12	12	650	7,800	0.091	26.0	286.3
Σ			2600	85,800	1.001	286.3	

7.6.6 Horizontal Distribution of Seismic Forces

The seismic design story shear in any story, V_x (kip or kN), shall be determined from Eq. (7.65):

$$V_x = \sum_{i=x}^{n} F_i \tag{7.65}$$

where F_i = the portion of the seismic base shear (V_x) (kip or kN) induced at Level i.

The seismic design story shear, V_x (kip or kN), shall be distributed to the various vertical elements of the seismic force–resisting system in the story under consideration based on the relative lateral stiffness of the vertical resisting elements and the diaphragm.

7.7 HORIZONTAL DIAPHRAGMS

7.7.1 Flexible and Rigid Diaphragms

Diaphragms transfer horizontal forces to vertical elements of seismic force–resisting systems as listed in ASCE 7-05 Table 12.2.1. While diaphragms can also be sloped, the discussion presented herein pertains to floors and roof of buildings or building-like structures, which are generally horizontal. In a building, both roof and floors act as horizontal diaphragms which transfer lateral seismic forces to shear walls. Design requirements for diaphragms are specified in ASCE 7-05 Section 12.3.

Diaphragms are typically classified as *flexible* or *rigid*, depending on how they are constructed. Diaphragms constructed from untopped steel decking or wood structural panels are usually (and permitted as such by design codes) considered as flexible diaphragms in structures in which the vertical elements are steel or composite steel and concrete braced frames or concrete, masonry, steel, or composite shear walls. Diaphragms constructed of wood structural panels or untopped steel decks in one- or two-family residential buildings of light frame construction are also considered as flexible. Diaphragms of concrete slab or concrete-filled metal deck with span-to-depth ratio of 3 or less in structures that have no horizontal irregularities are considered as rigid (ASCE 7-05 Section 12.3.1.2).

For design purposes, ASCE 7-05 Section 12.3.1.3 defines a diaphragm as rigid or flexible based on considerations of drift of the supporting vertical elements (e.g., shear wall or braced frames). A diaphragm can be idealized as flexible where its calculated maximum in-plane deflection under lateral load is more than two times the average story drift of the adjoining vertical elements of the seismic force–resisting system of the associated story under equivalent tributary lateral load (Fig. 7.21).

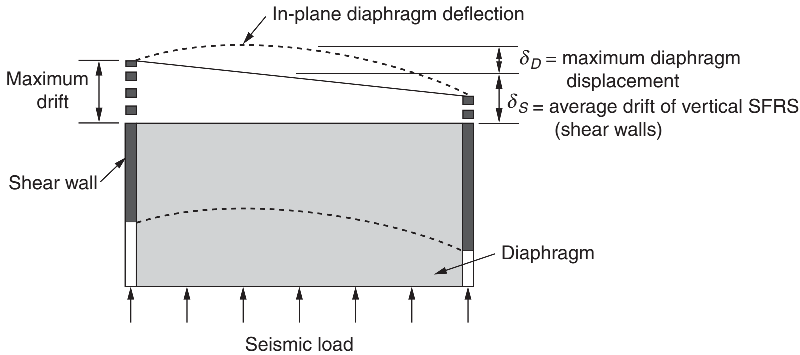


FIGURE 7.21 Definition of a flexible diaphragm. A diaphragm is flexible if $\delta_D/\delta_S > 2.0$.

7.7.2 Design Forces for Diaphragms

Diaphragms are to be designed for both the shear and bending stresses resulting from design forces. At diaphragm discontinuities, such as openings and reentrant corners, the design shall ensure that dissipation or transfer of edge (chord) forces combined with other forces in the diaphragm are within shear and tension capacity of the diaphragm.

Floor and roof diaphragms shall be designed to resist design seismic forces from the structural analysis, but shall be not less than those determined in accordance with Eq. (7.66) [ASCE 7-05 Eq. (12.10-1)] as follows:

$$F_{px} = \frac{\sum_{i=x}^{n} F_i}{\sum_{i=x}^{n} w_i} w_{\mathrm{px}} \tag{7.66}$$

where F_{px} = the diaphragm design force
F_i = the design force applied to level i
w_i = the weight tributary to level i
w_{px} = the weight tributary to the diaphragm at level x

The force determined from Eq. (7.66) is subject to upper and lower limits as follows:

$$F_{px,\,\max} = 04\mathrm{S}_{\mathrm{DS}}\, Iw_{\mathrm{px}} \tag{7.67}$$

$$F_{p,\,\min} = 0.2\mathrm{S}_{\mathrm{DS}}\, Iw_{\mathrm{px}} \tag{7.68}$$

where the diaphragm is required to transfer design seismic force from the vertical resisting elements above the diaphragm to other vertical resisting elements below the diaphragm due to offsets in the placement of the elements or to changes in relative lateral stiffness in the vertical elements; these forces shall be added to those determined from Eq. (7.66). The redundancy factor ρ applies to the design of diaphragms in structures assigned to Seismic

Design Category D, E, or F. For inertial forces calculated in accordance with Eq. (7.66), the redundancy factor shall equal 1.0. For transfer forces, the redundancy factor shall be the same as that used for the structure. For structures having horizontal or vertical structural irregularities (of the types discussed in Section 7.8), the requirements indicated in AISC 7-05 Section 12.3.3.4 shall also apply.

7.7.3 Design Forces for Chords and Collector Elements

Shear walls receive their loads from horizontal diaphragms through connections between the two elements. A shear wall supporting a diaphragm may be continuous running full width of the diaphragm, or it may be discontinuous because of the presence of openings. When the shear wall is discontinuous, collector elements (Fig. 7.22) are required to be provided such that they are capable of transferring the seismic forces originating in other portions of the structure to the element providing the resistance to those forces.

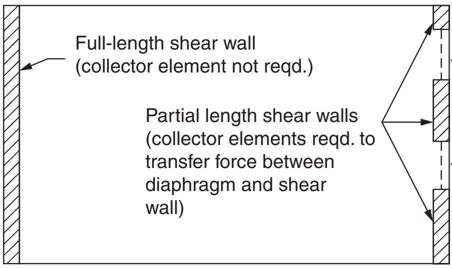


FIGURE 7.22 Collector elements in discontinuous shear walls.

In structures assigned to Seismic Design Category C, D, E, or F, collector elements, splices, and their connections to resisting elements shall resist the load combinations with overstrength specified in ASCE 7-05 Section 12.4.3.2. Example 7.10 illustrates calculations for the five story building of Example 7.9.

Exception: In structures or portions thereof braced entirely by light-frame shear walls, collector elements, splices, and connections to resisting elements need only be designed to resist forces in accordance with ASCE 0-5 Section 12.10.1.1.

Example 7.10 Determination of diaphragm forces.
Calculate diaphragm forces for the five-story building of Example 7.9.

Solution
The following information is obtained from Example 7.9.

TABLE E7.9 Vertical Distribution of Shear (k = 1.0)

Level x	h_x (ft)	h_x^k	w_x (kips)	$w_x h_x^k$ (kip-ft)	$C_{vx} = \dfrac{w_x h_x^k}{\Sigma w_i h_i}$	$F_x = C_{vx}V$ (kips)	Story shear (kips)
5	60	60	400	24,000	0.280	80.1	80.1
4	48	48	450	21,600	0.252	72.1	152.2
3	36	36	500	18,000	0.210	60.1	212.3
2	24	24	600	14,400	0.168	48.0	260.3
1	12	12	650	7,800	0.091	26.0	286.3
Σ			2600	85,800	1.001	286.3	

Design diaphragm at level x is F_{px}, given by Eq. (7.66):

$$F_{px} = \frac{\sum_{i=x}^{n} F_i}{\sum_{i=x}^{n} w_i} w_{px}$$

Value given by the above equation has the following limits on it:

$$0.2\ S_{DS} I w_{px} \leq F_{px} \leq 0.4\ S_{DS} I w_{px}$$

Level 5:

$$F_{px} = \left(\frac{80.1}{400}\right)(400) = 80.1 \text{ kips}$$

$$0.2 S_{DS} I w_{px} = 0.2(0.6)(1.0)(400) = 48 \text{ kips}$$

$$0.4 S_{DS} I w_{px} = 0.4(0.6)(1.0)(400) = 96 \text{ kips}$$

$$48 < 80.1 < 96 \text{ kips}$$

Therefore, $F_{px5} = 80.1$ kips

Level 4:

$$F_{px} = \left(\frac{80.1 + 72.1}{400 + 450}\right)(450) = 80.6 \text{ kips}$$

$$0.2 S_{DS} I w_{px} = 0.2(0.6)(1.0)(450) = 54 \text{ kips}$$

$$0.4 S_{DS} I w_{px} = 0.4(0.6)(1.0)(450) = 108 \text{ kips}$$

$$54 < 80.6 < 108 \text{ kips}$$

Therefore, $F_{px4} = 80.6$ kips

Level 3:

$$F_{px3} = \left(\frac{80.1 + 72.1 + 60.1}{400 + 450 + 500}\right)(500) = 78.6 \text{ kips}$$

$$0.2 S_{DS} I w_{px} = 0.2(0.6)(1.0)(500) = 60 \text{ kips}$$

$$0.4 S_{DS} I w_{px} = 0.4(0.6)(1.0)(500) = 120 \text{ kips}$$

$$60 < 78.6 < 120 \text{ kips}$$

Therefore, $F_{px3} = 78.6$ kips

Level 2:

$$F_{px2} = \left(\frac{80.1 + 72.1 + 60.1 + 48.0}{400 + 450 + 500 + 600}\right)(600) = 80.1 \text{ kips}$$

$$0.2S_{\mathrm{DS}}Iw_{px} = 0.2(0.6)(1.0)(600) = 72 \text{ kips}$$

$$0.4S_{\mathrm{DS}}Iw_{px} = 0.4(0.6)(1.0)(600) = 114 \text{ kips}$$

$$72 < 80.1 < 144 \text{ kips}$$

Therefore, $F_{px2} = 80.1$ kips
Level 1:

$$F_{px1} = \left(\frac{80.1 + 72.1 + 60.1 + 48.0 + 26.0}{400 + 450 + 500 + 600 + 650}\right)(650) = 71.6 \text{ kips}$$

$$0.2S_{\mathrm{DS}}Iw_{px} = 0.2(0.6)(1.0)(650) = 78 \text{ kips}$$

$$0.4S_{\mathrm{DS}}Iw_{px} = 0.4(0.6)(1.0)(650) = 156 \text{ kips}$$

$$71 < 78 < 156 \text{ kips}$$

Therefore, $F_{px1} = 78$ kips.

The above calculations are detailed to demonstrate the procedure in completeness. The calculations are easily performed on a handheld calculator and results shown in a spreadsheet as shown in Table E7.10.

TABLE E7.10 Diaphragm Forces

Level x	w_{px} (kips)	Σw_{px} (kips)	F_x (kips)	ΣF_x (kips)	$F_{px} = \frac{\Sigma F_x}{\Sigma w_{px}} w_{px}$ (kips)	$0.2S_{\mathrm{DS}}Iw_{px} = 0.12\,w_{px}$ (kips)	$0.4S_{\mathrm{DS}}Iw_{px} = 0.24w_{px}$ (kips)	F_{px} (kips)
5	400	400	80.1	80.1	80.1	48	96	80.1
4	450	850	72.1	152.2	80.6	54	108	80.6
3	500	1350	60.1	212.3	78.6	60	120	78.6
2	600	1950	48.0	260.3	80.1	72	144	80.1
1	650	2600	26.0	286.3	71.6	78	156	78.0

Note: The redundancy factor in this example, $\rho = 1.3$, is to be applied to the load Q_E due to F_{px} forces, such as chord forces and floor/roof to frame connections in conformance to ASCE 7-05 Section 12.3.4.2a (each story not resisting more than 35 percent of the base shear). $\rho = 1.0$ if each story resists more than 35 percent of the base shear.

7.7.4 Diaphragm Force for a One-Story Building

Determination of diaphragm force for a one-story building is considerably simpler than that for a multistory building. In the case of a single-story building, $i = x = n = 1$ in Eq. (7.66), so that

$$\sum_{i=x}^{i=n} F_i = V$$

$$\sum_{i=x}^{i=n} w_i = w_{px} = W$$

Substituting the above values in Eq. (7.66) yields

$$F_{px} = \frac{\sum_{i=x}^{i=n} F_x}{\sum_{i=x}^{i=n} w_i} w_{px} = \left(\frac{V}{W}\right) W = V \tag{7.69}$$

Thus, for a single-story building

$$V = F_x = C_s w_{px} \tag{7.70}$$

That is, the base shear, story shear, and the diaphragm force are all equal, with the limits of

$$0.2\ S_{DS} I w_{px} \le F_x \le 0.4\ S_{DS}\, w_{px}$$

Example 7.11 presents calculations for this important concept. It was stated earlier that

$$F_x = C_{vx} V \qquad \text{(7.63 repeated)}$$

and

$$C_{vx} = \frac{w_x h_x^k}{\Sigma w_i h_i^k} \qquad \text{(7.64 repeated)}$$

For structures having periods of 0.5 s or less, $k = 1$ in Eq. (7.64). Therefore, for such one-story buildings, Eq. (7.64) can be expressed as Eq. (7.69):

$$C_{vx} = \frac{w_x h_i^k}{\Sigma w_i h_i^k} = \frac{w_1 h_1}{w_1 h_1} = 1.0$$

Therefore,

$$F_x = C_{vx} V = V \tag{7.71}$$

Example 7.11 Determination of the diaphragm force for a single-story building.

A single-story building having special reinforced masonry shear walls and a panelized wood roof is shown in Fig. E7.11. The building is classified as Occupancy Category 1 and is assigned Seismic Design Category D. The site class for the building is *D* and S_S and S_1 for the site have been determined, respectively, as 55.2%*g* and 24.3%*g*. Dead weight of the roof and the masonry walls may be assumed, respectively, as 16 lb/ft^2 and 84 lb/ft^2. Calculate the diaphragm design force at the roof level.

Solution

Given information

$S_s = 55.2\%g = 0.552g$
$S_1 = 24.3\%g = 0.243g$
$I = 1.0$
SDC = D
Site class = *D*
Bearing wall system with special reinforced masonry shear walls: $R = 5$ (Table 7.5)
$\rho = 1.0$ (assumed, explained later)
Diaphragm weight = 16 lb/ft^2
Wall weight = 84 lb/ft^2

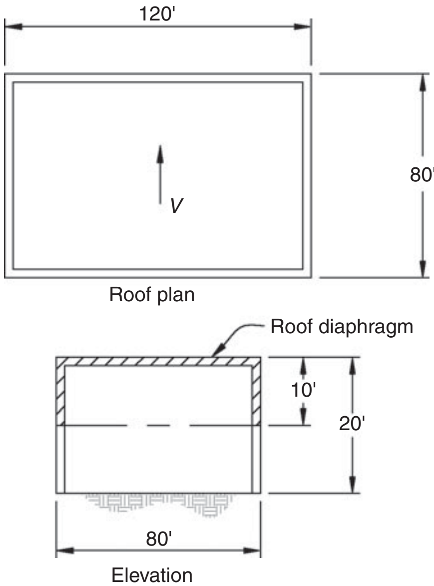


FIGURE E7.11

1. Calculate S_{DS} and S_{D1}:

$$S_{DS} = \frac{2}{3} S_{MS} = \frac{2}{3} F_a S_s$$

$$S_{D1} = \frac{2}{3} S_{Mi} = \frac{2}{3} F_v S_1$$

From Table 7.2, by interpolation for Site Class D, for $S_S = 0.552$

$$F_a = 1.4 + \left(\frac{1.4 - 1.2}{0.75 - 0.5}\right)(0.552 - 0.5) = 1.358$$

$$S_{DS} = \frac{2}{3} F_a S_s = \frac{2}{3}(1.358)(0.552) = 0.5$$

From Table 7.3 by interpolation for Site Class D, for $S_1 = 1.914$

$$F_v = 2.0 - \left(\frac{2.0 - 1.8}{0.3 - 0.2}\right)(0.243 - 0.2) = 1.914$$

$$S_{D1} = \frac{2}{3} F_a S_1 = \frac{2}{3}(1.914)(0.243) = 0.31$$

2. Calculate seismic response coefficient C_s

$$C_s = \frac{S_{DS}}{(R/I)} = \frac{0.5}{(5/1.0)} = 0.10$$

$$C_{s\max} = \frac{S_{D1}}{T(R/I)} \qquad \text{for } T < T_L \ (= 4 \text{ s minimum})$$

$$T_a = C_t (h_x)^n = 0.02(20)^{0.75} = 0.19 \text{ s} < T_L = 4 \text{ s (minimum)}$$

$$C_{S,\max} = \frac{S_{D1}}{T(R/I)} = \frac{0.31}{0.19(5/1.0)} = 0.326 > 0.10$$

$$C_{s,\min} = 0.01$$

Therefore, $C_s = 0.1$ (governs)

$$V = C_s W = 0.1W$$

3. Calculate the tributary weight W for a 1-ft wide strip of wall and the roof diaphragm.

$$\text{Tributary wall height} = \frac{20}{2} = 10 \text{ ft}$$

$$\text{Tributary length of roof diaphragm} = 80 \text{ ft}$$

$$W_{p1} = \text{weight of tributary walls} + \text{weight of diaphragm}$$
$$= (2 \text{ walls})(10)(84) + 80(16) = 2960 \text{ lb}$$

For a one-story building,

$$F_{p1} = V = 0.1W = 0.1(2960) = 296 \text{ lb/ft}$$

Check limits,

$$0.2\, S_{DS} I w_{px} = 0.2(0.5)(2960) = 296 \text{ lb/ft}$$

$$0.4 S_{DS} I w_{px} = 0.4(0.5)(2960) = 592 \text{ lb/ft}$$

Therefore, $F_{px} = 296$ lb/ft

Commentary: The redundancy factor $\rho = 1.0$ can be used in this example as this building (assigned to Seismic Design Category D) satisfies requirements of ASCE 7-05 Section 12.3.4 2a (story resists 100 percent of base shear, more than 35 percent required).

7.8 INFLUENCE OF BUILDING CONFIGURATION ON LATERAL FORCE DISTRIBUTION IN SHEAR WALLS

7.8.1 General Considerations

Building configuration can be described as *regular* or *irregular*. Regular buildings are those that are symmetrical or almost symmetrical about both axes in plan, and having load-resisting elements so placed that they provide a *continuous* load path for both gravity and lateral loads. The assumption is that regular structures provide a reasonably uniform distribution of inelastic behavior in elements throughout the seismic force–resisting system.

Building configurations can be considered with respect to plan or elevation. A building that lacks symmetry in terms of location of its force-resisting elements is said to be irregular. A building can have *horizontal structural irregularities* or *vertical structural irregularities*. Horizontal structural irregularities result from diaphragms with offsets, with reentrant corners, or with any kind of discontinuity (Figs. 7.23 and 7.24). Vertical irregularities result from factors such as distribution of mass, stiffness, or strength, which can result in seismic forces and/or deformations over the height of the structure that are significantly different from the linearly varying distribution assumed in design. These irregularities are referred to as *dynamic force irregularities* and include stiffness (soft story), weight (mass), and vertical geometric irregularities (Figs. 7.25 and 7.26). Horizontal or diaphragm characteristics that result in significant amounts of torsional response, diaphragm deformations, or diaphragm stress concentrations are called *horizontal structural irregularities*.

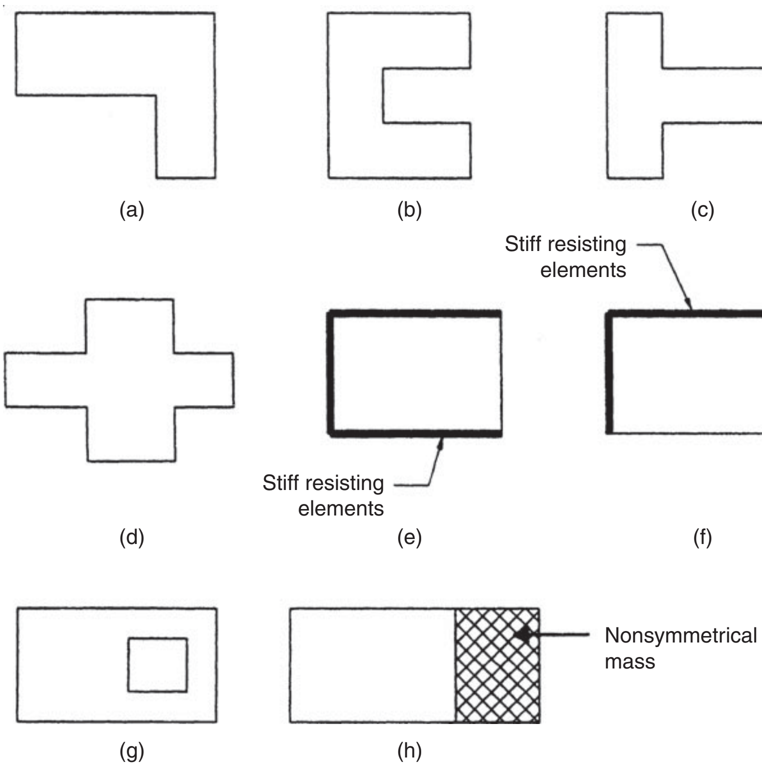


FIGURE 7.23 Horizontal irregularities: (a) – (c) nonsymmetric and symmetric plans, (d) nonsymmetric plan with reentrant corners, (e) – (g) nonsymmetric distribution of lateral rigidities, (h) nonsymmetric mass ditribution [7.12].

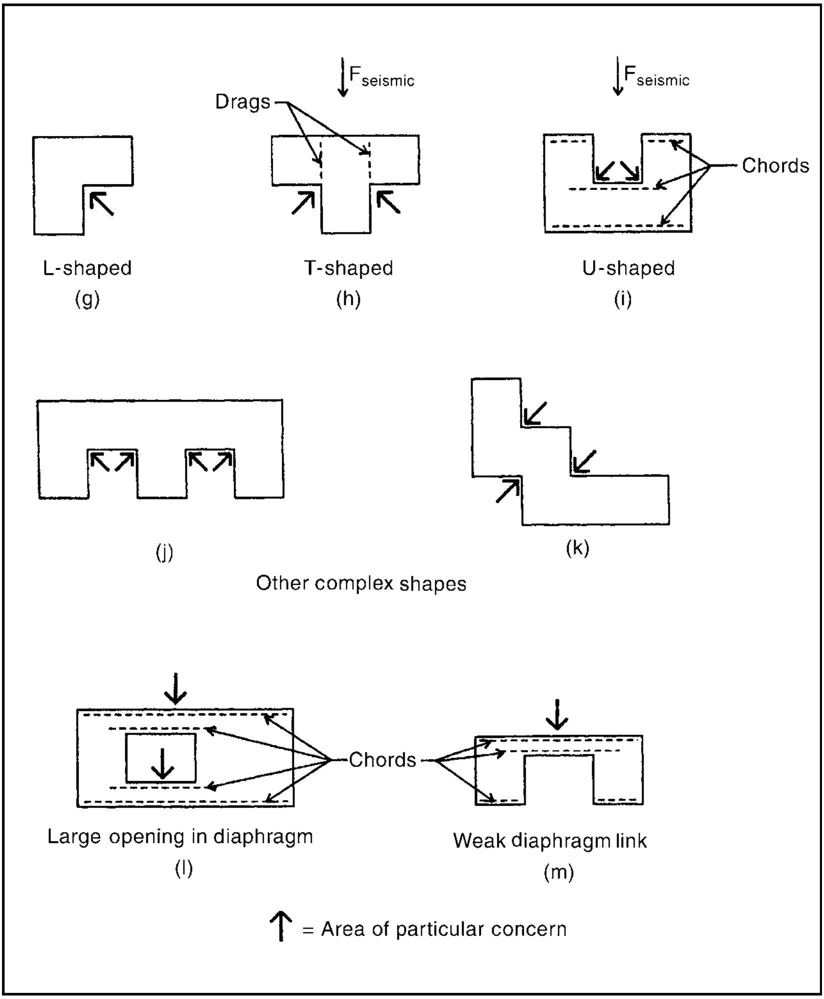


FIGURE 7.24 Plan irregularities (reentrant corners).

7.8.2 Horizontal Structural Irregularities

In many cases, diaphragms are not regularly shaped (e.g., diaphragms with offsets, with reentrant corners, or with any kind of discontinuity), or they may be supported by vertical elements that have different rigidities. In such cases, structures are said to have *horizontal structural irregularities*. ASCE 7-05 Section 12.3.2.1 recognizes five types of irregularities as listed in Table 7.6:

1. (a) Torsional irregularity, (b) extreme torsional irregularity
2. Reentrant corners irregularity
3. Diaphragm discontinuity irregularity

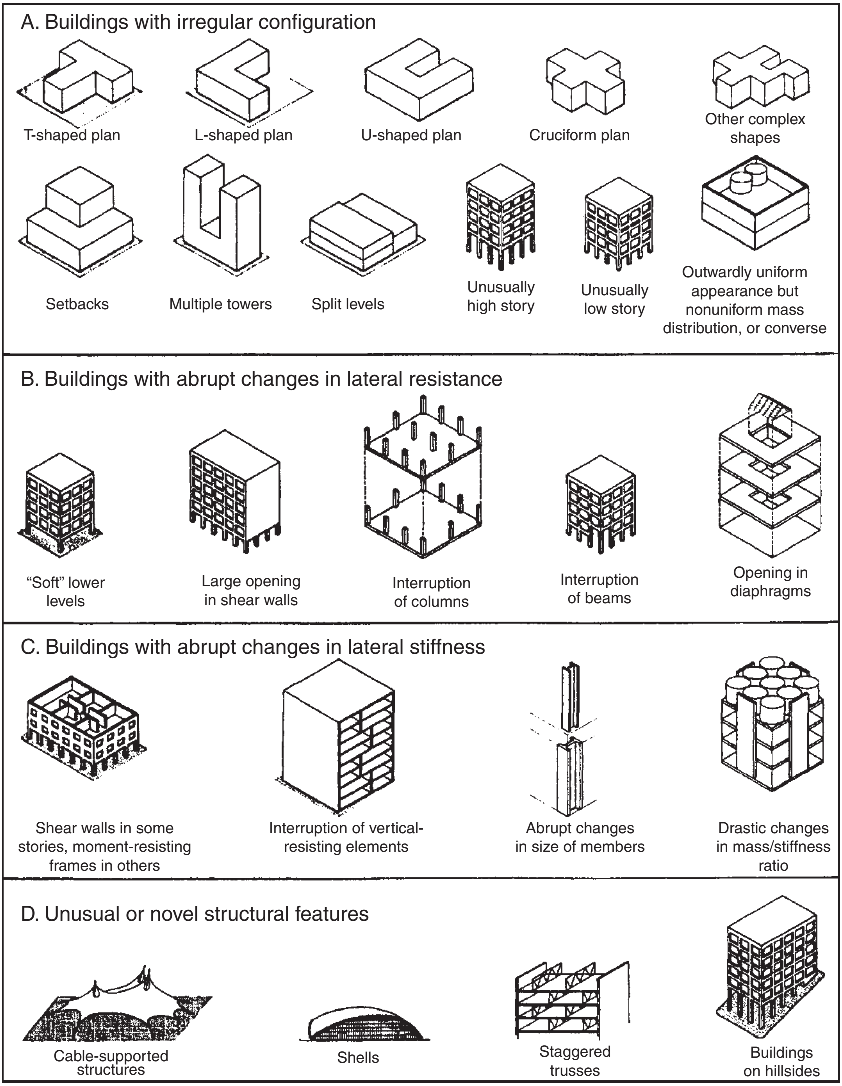


FIGURE 7.25 Varieties of vertical irregularities [7.15].

4. Out-of-plane offsets irregularity

5. Nonparallel systems irregularity

Each of these irregularities carries certain design requirements as listed Table 12.3-1 of ASCE 7-05 Standard, specified by various sections in that table. Those sections refer to seismic design categories of structures (categories A through F) as specified in Section 11.6 of ASCE 7-05 Standard. The seismic design category of a structure is determined based on its occupancy category listed in Table 1-1 of the Standard, as well as on the design spectral

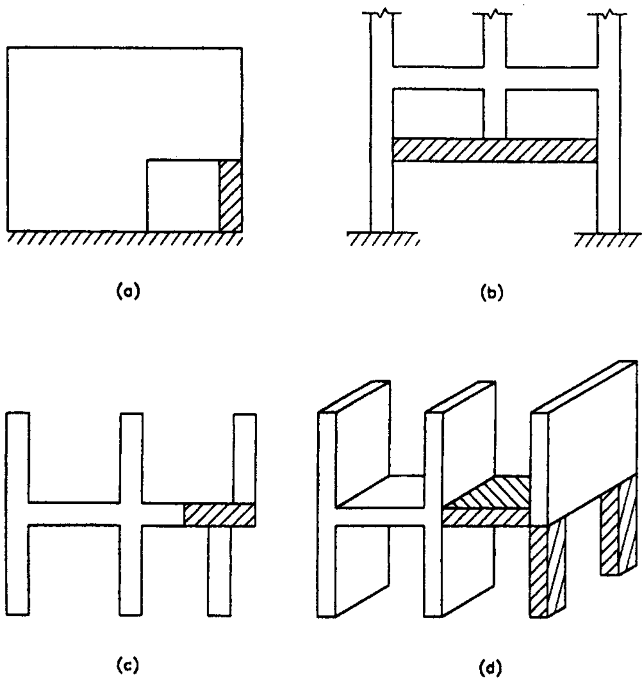


FIGURE 7.26 Irregularities in force transfer

TABLE 7.6 Types of Horizontal Structural Irregularities [7.13]

	Irregularity type	Description
1a	Torsional irregularity	Torsional irregularity is defined to exist where the maximum story drift, computed including accidental torsion, at one end of the structure transverse to an axis is more than 1.2 times the average of the story drifts at the two ends of the structure. Torsional irregularity requirements in the reference sections (of ASCE 7-5 Table 12.3-1) apply only in structures in which the diaphragms are rigid or semirigid.
1b	Extreme torsional irregularity	Extreme torsional irregularity is defined to exist where the maximum story drift, computed including accidental torsion, at one end of the structure transverse to an axis is more than 1.4 times the average of the story drifts at the two ends of the structure. Extreme torsional irregularity requirements in the reference sections (of ASCE 7-5 Table 12.3-1) apply only to structures in which the diaphragms are rigid or semirigid.
2	Reentrant corner irregularity	Reentrant corner irregularity is defined to exist where both plan projections of the structure beyond a reentrant corner are greater than 15% of the plan dimension of the structure in the given direction.
3	Diaphragm discontinuity irregularity	Diaphragm discontinuity irregularity is defined to exist where there are diaphragms with abrupt discontinuities or variations in stiffness, including those having cutout or open areas greater than 50% of the gross enclosed diaphragm area, or changes in effective diaphragm stiffness of more than 50% from one story to the next.
4	Out-of-offsets irregularity	Out-of-offsets irregularity is defined to exist where there are discontinuities in a lateral force resistance path, such as out-of-plane offsets of the vertical elements.
5	Nonparallel systems irregularity	Nonparallel systems irregularity is defined to exist where the vertical lateral force–resisting elements are not parallel to or symmetric about the major orthogonal axes of the seismic force–resisting system.

acceleration parameter, S_{DS} (short period response spectral acceleration parameter) and S_{D1} (1-s period response spectral acceleration parameter) as listed, respectively, in Tables 11.6-1 and 11.6-2 of ASCE 7-05 Standard.

The 2003-NEHRP Provisions [7.13] and ASCE 7-05 [7.11] classify torsional irregularity into two types when rigid diaphragms are used, based on the ratio of shear wall displacements in a story. Torsional irregularity Type 1*a* is said to exist when the maximum *story* drift, computed including accidental torsion, at one end of the *structure* transverse to an axis is more than 1.2 times the average of the *story* drifts at the two ends of the structure. The other type, classified as torsional irregularity Type 1*b* (*extreme torsional irregularity*) is said to exist when the maximum *story* drift, computed including accidental torsion, at one end of the *structure* transverse to an axis is more than 1.4 times the average of the *story* drifts at the two ends of the *structure*. Extreme torsional irregularities are prohibited for structures located very close to major active faults and should be avoided in all structures whenever possible. Example 7.12 shows calculations for shear wall supporting a diaphragm without torsional irregularities. Examples 7.13 and 7.14 present calculations for shear wall systems supporting diaphragms that involve horizontal torsional irregularities Type 1*a* and Type 1*b*, respectively.

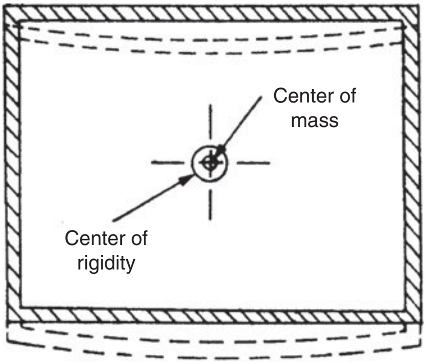
7.8.3 Effects of Horizontal Torsional Eccentricity

Buildings that use shear walls as SFRS receive their loads from the roof or the floor diaphragms. These diaphragms may be *flexible* or *rigid*. The magnitude of lateral forces that are transferred to shear walls depends on the type of diaphragm, that is, flexible or rigid. In the case of a flexible diaphragm, the lateral loads are distributed to shear walls in proportion to the tributary areas that are assigned to the individual shear walls. Diaphragms are considered flexible for the purposes of distributing story shear and torsional moment when the maximum lateral deformation of the diaphragm is more than 2 times the average story drift of the associated story. This may be determined by comparing the computed midpoint in-plane deflection of the diaphragm itself under lateral load with the story drift of the adjoining vertical-resisting elements under equivalent tributary lateral load (Fig. 7.22).

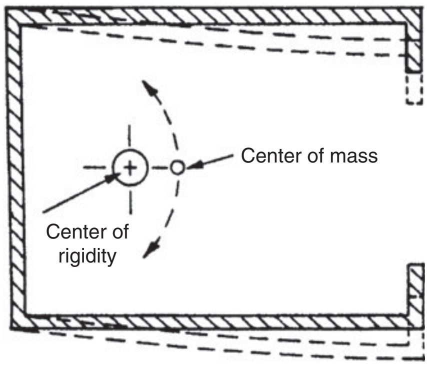
When a diaphragm is rigid, the lateral seismic forces are distributed to shear walls in proportion to their rigidities. If the shear walls have equal rigidities and are symmetrically placed (in plan) with respect to the *center of mass* (the point where the resultant of the masses of the lateral force distributing diaphragm and the tributary areas of shear walls is located), they will share lateral forces equally. But if shear walls have unequal rigidities, or if they are not symmetrically located with respect to the center of mass, they will not be subjected to the same magnitude of lateral force. As a result of this unequal distribution of lateral forces in shear walls, their resultant is located at a point called the *center of rigidity*, which is different than the center of mass. The distance between the center of mass and the center of rigidity is called the *horizontal torsional eccentricity* or simply *torsional eccentricity*. The presence of torsional eccentricity in a building causes the center of mass to rotate about an axis passing through the center of rigidity, causing horizontal torsional moments which must be resisted by shear walls (Fig. 7.27*a* and *b*). As a result of this torsion, shear walls are subjected to additional lateral forces which must be accounted for in design.

7.8.4 Vertical Structural Irregularity

In addition to the horizontal structural irregularities listed in Table 7.6, a structure might have *vertical structural irregularities*. Examples of vertical irregularities include such



(a) Equal deflection of walls



(b) Unequal deflection of walls due to torsion

FIGURE 7.27 Center of mass and center of rigidity in a building with (*a*) symmetrical walls (no torsion), (*b*) unsymmetrical walls (torsional moments present) [7.5].

configurations as discontinuous shear wall or frames (in a building elevation), uneven distribution of mass on various levels of a building, and significant differences in story stiffnesses (See Figs. 7.24–7.26). For design purposes, five types of vertical structural irregularities are recognized (ASCE 7-05 Section 12.3.2.2) as listed in Table 7.7.

1. (a) Stiffness-soft story irregularity, (b) stiffness-extreme soft story irregularity
2. Weight (mass) irregularity
3. Vertical geometric irregularity
4. In-plane discontinuity in vertical lateral force–resisting element irregularity
5. (a) Discontinuity in lateral strength-weak story irregularity, (b) discontinuity in lateral strength-extreme weak story irregularity

A type of vertical structural irregularity that might cause poor seismic performance of buildings results from the lack of direct load path for load transfer. Such an irregularity is referred to as *irregularity in force transfer* [7.12]. This occurs when the lateral load resistance path is not continuous (Fig. 7.26). Force transfer irregularities include configurations such as discontinuous columns (Fig. 7.26*a* to *c*), discontinuous or offset shear walls or bracing (Fig. 7.26*d*), weak stories in shear and/or torsion, large openings in horizontal framing systems at any level, and the presence of stiff nonstructural systems [7.12]. The most critical of the discontinuities to be considered is the out-of-plane offset of vertical elements of the seismic force–resisting elements. Such offsets impose vertical and lateral loads on horizontal elements that are, at the least, difficult to provide for adequately [7.16]. In general, a discontinuous lateral load resistance path can cause a concentration of inelastic demand, and can occur even when no plan or vertical irregularities are present.

7.8.5 Inherent Torsion in Diaphragms: Direct and Torsional Shears

It was noted earlier that when flexible diaphragms transfer lateral forces to supporting vertical SFRS elements such as shear walls, the lateral force is distributed to the latter in proportion to the tributary areas of the diaphragm supported by those elements. When the diaphragm is rigid, the supporting vertical SFRS elements share the lateral force in proportion to their rigidities. This section presents a discussion on how to determine these lateral forces to shear walls.

TABLE 7.7 Types of Vertical Structural Irregularities (Adapted from Ref. 7.13)

	Irregularity type	Description
1a	Stiffness-soft story irregularity	Stiffness-soft story irregularity is defined to exist where there is a story in which the lateral stiffness is less than 70% of that in the story above or less than 80% of the average stiffness of the three stories above.
1b	Stiffness-extreme soft story irregularity	Stiffness-extreme soft story irregularity is defined to exist where there is a story in which the lateral stiffness is less than 60% of that in the story above or less than 70% of the average stiffness of the three stories above.
2	Weight (mass) irregularity	Weight (mass) irregularity is defined to exist where the effective mass of any story is more than 150% of the effective mass of an adjacent story. A roof that is lighter than the floor below need not be considered.
3	Vertical geometric irregularity	Vertical geometric irregularity is defined to exist where the horizontal dimension of the seismic force–resisting system in any story is more than 130% of that in an adjacent story.
4	In-plane discontinuity in vertical lateral force–resisting element irregularity	In-plane discontinuity in vertical lateral force–resisting element irregularity is defined to exist where an in-plane offset of the lateral force–resisting elements is greater than the length of those elements or there exists a reduction in stiffness of the resisting element in the story below.
5A	Discontinuity in lateral strength-weak story irregularity	Discontinuity in lateral strength-weak story. Irregularity is defined to exist where the story lateral strength is less than 80% of that in the story above. The story lateral strength is the total lateral strength of all seismic-resisting elements sharing the story shear for the direction under consideration.
5B	Discontinuity in lateral strength-extreme weak story irregularity	Discontinuity in lateral strength-extreme weak story irregularity is defined to exist where the story lateral strength is less than 65% of that in the story above. The story strength is the total strength of all seismic-resisting elements sharing the story shear for the direction under consideration.

When analyzing a seismic force–resisting system, it would be generally found that the *center of mass* (CM) and the *center of rigidity* (CR) are not coincident. The center of mass is located at the resultant of the centers of gravity of masses of all structural and nonstructural elements comprising the system, which can be determined from statics. The center of rigidity, on the other hand, is located at the location of the resultant of the centers of rigidities of the vertical seismic force–resisting elements (e.g., shear walls or braced frames) of the system. Its position can be determined by taking moments of the rigidities of walls parallel to the applied force, about one of those walls or any reference point. The lateral seismic force acting on the system is distributed to these elements in proportion to their rigidities (or relative rigidities), thus, the element closer to the center of rigidity attracts a larger share of the lateral seismic force. This is an important concept in the design of shear walls. These element forces are referred to as *direct shears*.

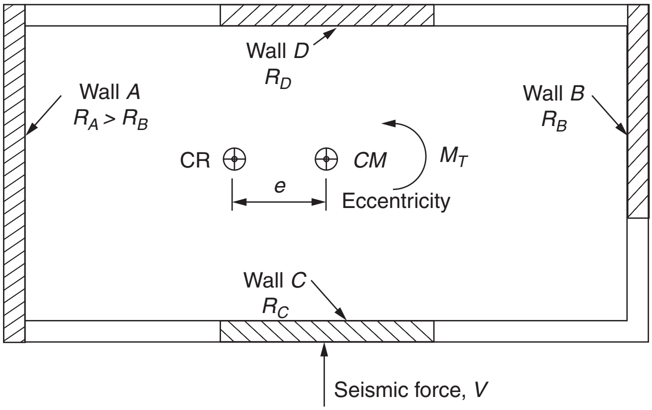


FIGURE 7.28 Torsional shears caused by eccentricity, which exist because center of mass and center of rigidity are at different locations.

When the center of mass and the center of rigidity are not coincident, horizontal torsional moment is caused because the seismic force is assumed to act through the center of mass, whereas the seismic resistance of the system acts through the center of rigidity. The distance between the center of mass and the center of rigidity measured perpendicular to the seismic force–resisting elements is called *horizontal eccentricity, e*, and the magnitude of the torsional moment, M_t, is taken equal to the applied seismic force *V times* the eccentricity, $M_t = Ve$. The effect of the torsional moment is to introduce *torsional shears* in the vertical seismic force–resisting elements (i.e., in shear walls). Depending on the location of the center of rigidity with respect to the center of mass, the torsional moment can be clockwise or counterclockwise (Fig. 7.28). Analysis of shear walls subjected to torsional moments is discussed in Section 7.9.

7.8.6 Accidental Eccentricity and Accidental Torsion in Diaphragms

In analyzing an SFRS, it is tacitly assumed that lateral seismic force acts through the center of mass of the system, which is easily determined from statics. The location and distribution of mass at each level required to be considered for earthquake motion response cannot be determined with precision because of the uncertainties involved in calculation of dead loads and centers of gravity of various structural and nonstructural elements that comprise the mass. To account for this uncertainty, the mass at each level is assumed to be displaced from the calculated center of mass in each direction a distance equal to 5 percent of the building dimension at that level perpendicular to the direction of the force under consideration (ASCE 7-05 Section 12.8.4.2). This distance is referred to as *accidental eccentricity* and the corresponding moment as *accidental horizontal torsional moment*, M_{ta}. According to SEOAC *Bluebook* [7.12], M_{ta} is specified to account for uncertainties arising from factors such as

1. Differences between the analytical model and actual structure
2. The real nonuniform distribution of both dead and live load

3. The eccentricities in the structural stiffness due to nonstructural elements such as stairs and interior partitions

4. Torsional input loading on the structure due to differences in seismic ground motion over the extent of foundation

Accidental torsional moment is a force that is assumed to act in addition to the torsional moment due to actual eccentricity (inherent torsion). Thus, total torsional moment acting on the system would be equal to torsional moment given by Eq. (7.72):

$$V(e \pm e_{\text{acc}}) \tag{7.72}$$

where V = lateral seismic force
e = eccentricity
e_{acc} = accidental eccentricity
= ± 5 percent of structure dimension perpendicular to the applied seismic force.

The *minus* sign in Eq. (7.72) should be used when the torsional shear would act to reduce the effects of direct shear; *plus* sign should be used when the torsional shear would act in the same direction as the direct shear. In some cases of analyses, the seismic forces may be applied in two orthogonal directions concurrently. In such cases, it is not necessary to apply the required 5 percent displacement of the center of mass in both of the orthogonal directions concurrently. However, this 5 percent required displacement must be applied in the direction that causes more critical effect.

7.8.7 Amplification of Accidental Torsional Moments

When structures assigned to seismic design categories C, D, E, and F have horizontal structural irregularity Type 1*a* or 1*b* listed in Table 7.6, the effects of accidental torsional eccentricity are required to be accounted for by multiplying M_{ta} by a torsional amplification factor given by Eq. (7.73):

$$A_x = \left(\frac{\delta_{\max}}{1.2\delta_{\text{avg}}}\right)^2 \tag{7.73}$$

where A_x = amplification factor ≤ 3.0
$\delta_{\max}$ = the maximum displacement at level x computed assuming $A_x = 1$
δ_{avg} = average of the displacements at extreme points of structure at level x computed assuming $A_x = 1$

The torsional amplification factor A_x need not exceed 3. The accidental torsion is not required to be amplified for structures of light frame construction.

As discussed in SEAOC *Blue Book* [7.12], the eccentricity amplification factor is intended to represent the increases caused by yielding of the perimeter elements. This factor provides a simple yet effective control on systems that might otherwise have excessive torsional yield in a given story. This requirement is intended to avoid potential torsional mechanism failure by ensuring that the structure has the stiffness and strength to resist both calculated and accidental torsional effects (discussed later). See Examples 7.13 and 7.14 which illustrate the application of eccentricity amplification factor, A_x.

7.8.8 Ramifications of Horizontal and Vertical Structural irregularities

Structural irregularities, both horizontal and vertical, can result in forces and deformations significantly different from those assumed when using equivalent static procedures. As such they create great uncertainties in the ability of a structure to meet the design objective. Experience from past earthquakes has shown that buildings having irregularities suffer greater damage than buildings having regular configurations. This situation prevails even with good design and construction.

Poor seismic performance of irregular buildings can be attributed to the manner in which such buildings respond to ground shaking. In a regular structure, inelastic demands produced by strong ground shaking tend to be well distributed throughout the structure, which results in dispersion of energy dissipation and damage. By contrast, in irregular structures, inelastic response can concentrate in the zones of irregularity, which can cause rapid failure of structural elements in those areas. Other factors can contribute to the poor performance of irregular structures as well. For example, irregularities can also introduce unanticipated stresses into the structure, which the designers may overlook when detailing the structural system. The elastic analysis methods typically employed in design of structures, such as equivalent static procedure, often cannot predict the distribution of earthquake demands in an irregular structure very well, leading to poor design in the zones of irregularity.

The configuration of shear walls in a building affects building configuration which, in turn, can significantly influence its performance in an earthquake. Figure 7.29 shows some typical shear wall layouts with potential problems leading to poor performance. A building having a regular configuration can be square, rectangular, or circular. Buildings with minor reentrant corners are generally acceptable as regular buildings. However, buildings having large reentrant corners that may create a crucifix form would be classified as irregular even though they have geometrical symmetry about both axes. This is because the response of the wings of this type of building is generally different from the response of the building as a whole. Other plan configurations such as H-shapes that have geometrical symmetry would be considered as irregular also because of the response of the wings [7.13, 7.14].

In general, irregularities in structural configurations and in load paths are known to be major contributors to structural damage due to strong earthquake ground motion [7.12]. Therefore, it is extremely important for designers to recognize the presence of irregularities in a structure. In some cases, even seemingly regular buildings may not perform well during strong ground shaking. An example of such a building is a core-type building with the vertical components of the seismic force–resisting system, symmetrically placed, but concentrated near the center of the building (Fig. 7.29*c*). Better performance has been observed when the vertical components are distributed near the perimeter of the buildings Fig. 7.29*f*).

Structural design involves serious ramifications whenever seismic forces cause a structure to have horizontal or vertical structural irregularity:

1. When accidental torsional moments cause horizontal structural irregularity Type 1*a* or Type 1*b* in a structure, it is required to be analyzed using a three-dimensional model. When using such a model, it is required that a minimum of three dynamic degrees of freedom consisting of translation in two orthogonal plan directions and torsional rotation about the vertical axis of rotation at each level of the structure be used for analysis (ASCE 7-05 Section 12.7.3).
2. For structures assigned to Seismic Design Category D, E, or F and having a horizontal structural irregularity Type 1*a*, 1*b*, 2, 3 or 4 (see Table 7.6), or a vertical structural irregularity Type 4 (Table 7.6), design seismic forces are required to be *increased* 25 percent for connections of diaphragms to vertical elements and to collectors, and for connections of collectors to vertical elements (ASCE 7-05 Section 12.3.3.4).

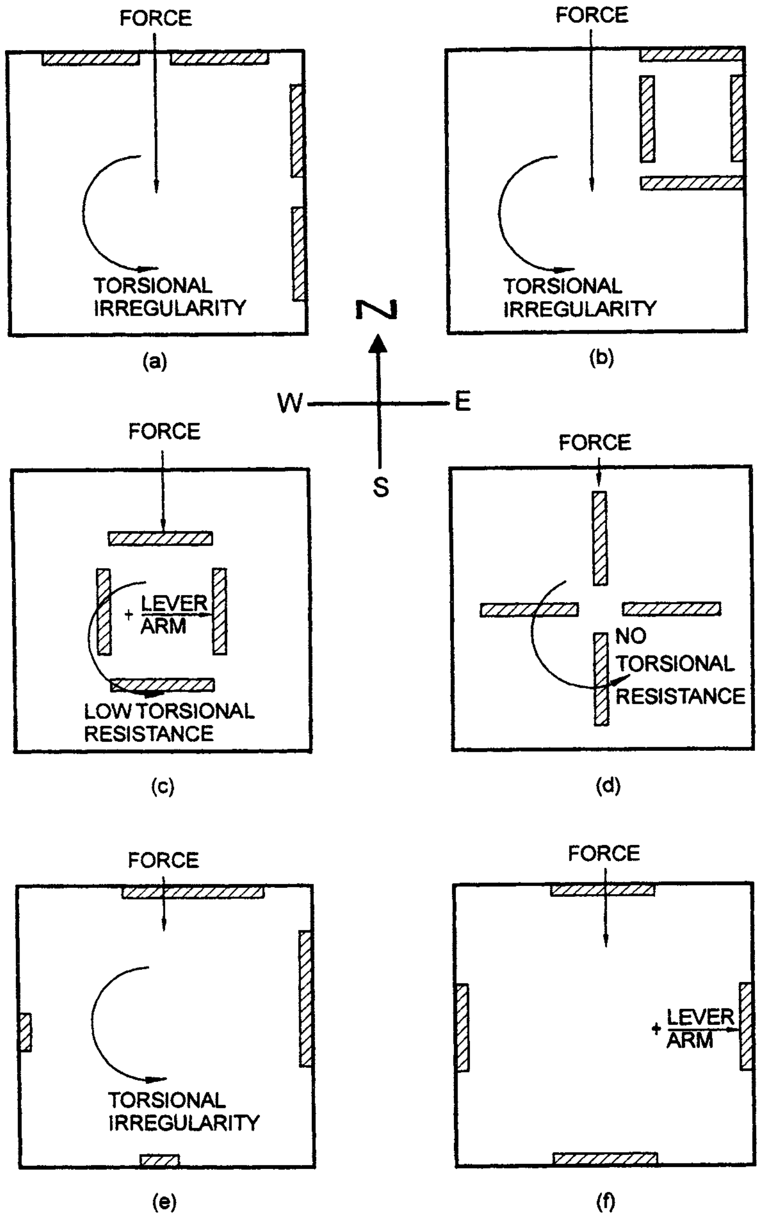


FIGURE 7.29 Shear wall configurations.

3. Structures assigned to certain seismic design categories and having certain types of structural irregularities are not permitted to be built:
(a) Structures assigned to Seismic Design Category D and having vertical structural irregularity Type 5*b* are not permitted to be built.
(b) Structures assigned to seismic design categories E and F and having horizontal structural irregularity Type 1*b* or vertical structural irregularity Type 1*b*, 5*a*, or 5*b* are not permitted to be built.

7.9 ANALYSIS OF SHEAR WALLS AND DIAPHRAGMS UNDER DIRECT SHEAR AND TORSIONAL MOMENTS

When a rigid diaphragm transmits a seismic force to two parallel shear walls of equal rigidities and the center of mass is coincident with the center of rigidity, the seismic force would be transmitted equally to both shear walls. But when the center of mass is not coincident with the center of rigidity, which happens when the shear wall have unequal rigidities, a torsional moment would be caused that must be resisted by shear walls. In such a case, forces in shear wall can be determined from superposition of two cases: (a) direct shear, (b) torsional shear.

Figure 7.30 shows a rigid horizontal diaphragm supported by shear walls A and B, which are of unequal rigidity such that rigidity of Wall A is greater than that of Wall B ($R_A > R_B$); as a result the center of rigidity is closer to Wall A than Wall B. The center of mass of the seismic force–resisting system can be determined from statics, assumed located to the right of the center of rigidity for this discussion. The applied seismic force, *V*, acts through the center of mass, whereas the resultant of shear forces in walls acts through the center of rigidity, the distance between the centers of mass and rigidity (measured perpendicular to the lateral seismic force) being the eccentricity. The analysis is carried out by introducing two equal and opposite forces (each equal to the applied seismic force *V*) at the center of rigidity as shown in Fig. 7.30; this system of forces is statically equivalent to the original system (with only *V* acting at the center of mass). The influence of the force applied at the center of rigidity is to cause direct shear in walls A and B, in direct proportion to their rigidities (or relative rigidities) given by Eq. (7.74):

$$V_A = \left(\frac{R_A}{R_A + R_B}\right)V \qquad V_B = \left(\frac{R_B}{R_A + R_B}\right)V \tag{7.74}$$

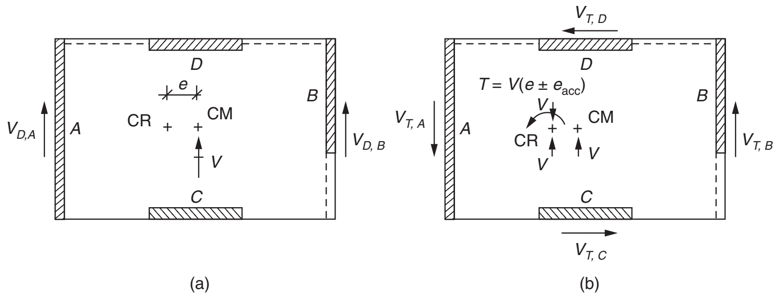


FIGURE 7.30 (*a*) Direct and (*b*) torsional shears in shear walls.

The two equal and opposite forces (acting at the center of mass and at the center of rigidity) cause a counterclockwise torsional moment which causes shear forces in all four walls (A, B, C,. and D) as shown in Fig. 7.30. These shear forces can be determined from Eq. (7.75)

$$V_{\text{wall i}} = \frac{M_t r_i R_i}{J} \tag{7.75}$$

where M_t = horizontal torsional moment = Ve
r_i = perpendicular distance of wall i from the center of rigidity
R_i = rigidity of wall i
J = polar moment of inertia of walls taken about the center of rigidity = $\sum R_i r_i^2$

When R_i is expressed in kips/in. and r_i in ft, the units of J would be (kips/in.)ft^2. When R_i is expressed in terms of relative rigidity, the units of J would be ft.^{2}.

The total shear in walls A and B would be equal to the algebraic sum of direct shear and torsional shears. It is noted that horizontal torsional moment causes shears in walls C and D also, which are perpendicular to the applied seismic force; however, these force are inconsequential in this analysis as these shear walls do not resist the direct shear.

Equation (7.75) is based on the same principles as used for elastic analysis of bolt forces in a multibolt joint subjected to torsional moment M_t; quantity J is analogous to the polar of moment of inertia used to calculate bolt forces in that analysis. The same equation can be used for analyzing the influence of accidental torsional moment by substituting M_{ta} for M_t.

Examples 7.12 through 7.14 present calculations for analysis for forces in shear walls transmitted from a rigid diaphragm. Example 7.12 presents calculations for analysis of shear wall forces that do not involve any type of horizontal structural irregularity. Examples 7.13 and 7.14 present calculations for shear wall forces that involve horizontal structural irregularities (horizontal torsional irregularities Type 1*a* and 1*b*).

Example 7.12 Distribution of lateral forces from a rigid diaphragm to shear walls involving no horizontal structural irregularity.

Figure E7.12A shows the roof plan of a single-story shear wall building with rigid roof diaphragm. It is supported by four walls A, B, C, and D as shown in the figure. The relative rigidities of these walls are 4.5, 5.5, 4, and 4, respectively. The building is subjected to a lateral force of 100 kips in the northwardly direction. Determine design shears in walls A and B. The building is classified as a Seismic Design Category D structure. Assume that the mass of the building is uniformly distributed with respect to the geometric center of the roof diaphragm.

Solution

1. Calculate the eccentricity and the rigidity properties. The relative rigidities of the four walls are

$$R_A = 4.5 \qquad R_B = 5.5 \qquad R_C = 4 \qquad R_D = 4$$

Mass is uniformly distributed with respect to plan, so the center of mass (CM) is located at the geometrical center of the roof, which is rigid. Therefore,

$$x_M = ½\,(90) = 45 \text{ ft}$$

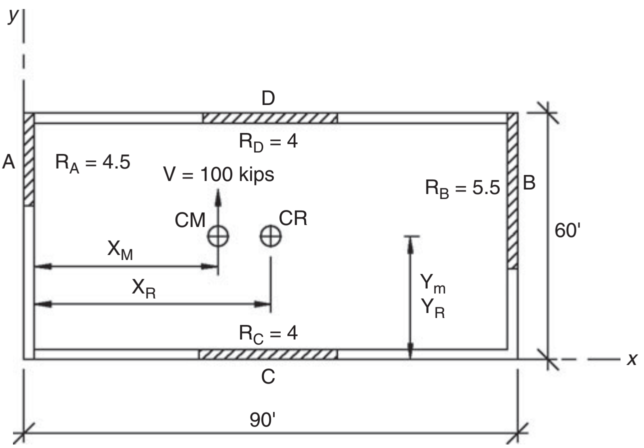


FIGURE E7.12A Roof plan of the shear wall building.

Center of Rigidity (CR): Summing up moments of relative rigidities of walls A and B (parallel to the applied force) about Wall A:

$$x_R = \left(\frac{R_B}{R_A + R_B}\right)(90) = \left(\frac{5.5}{4.5+5.5}\right)(90) = 49.5 \text{ ft}$$

Summing up moments of relative rigidities of walls C and D about Wall C:

$$y_R = \left(\frac{R_D}{R_C + R_D}\right)(90) = \left(\frac{4}{4+4}\right)(60) = 30 \text{ ft}$$

(or $y_R = 30$ ft from symmetry as $R_C = R_D = 4.0$). Because of uniform distribution of mass in plan, the center of mass is located at $x_M = 45$ ft from either wall.

$$\text{Eccentricity } e = x_M - x_R = 45 - 49.5 = -4.5 \text{ ft}$$

Calculate the polar moment of inertia of shear walls, J, about the center of rigidity.

$$J = \Sigma R_i d_i^2$$

where R_i = relative rigidity of Wall i
d_i = perpendicular distance between Wall i and the center of rigidity

$$J = R_A\,(x_R)^2 + R_B\,(60 - x_R)^2 + R_C\,(y_R)^2 + R_D\,(40 - y_R)^2$$

$$= 4.5\,(49.5)^2 + 5.5\,(90 - 49.5)^2 + 4\,(30)^2 + 4\,(60 - 30)^2$$

$$= 27{,}248 \text{ ft}^2$$

2. Direct shear in walls A and B: The relative rigidity of the building in the direction of the force equals the sum of relative rigidities of walls parallel to the force. In this example, shear walls A and B are parallel, and are oriented in the direction of applied lateral force. Therefore, they would share the applied lateral force in proportion to their relative rigidities.

$$V_{D,A} = \left(\frac{R_A}{R_A + R_B}\right)(V) = \left(\frac{4.5}{4.5+5.5}\right)(100) = 45 \text{ kips}$$

$$V_{D,B} = \left(\frac{R_B}{R_A + R_B}\right)(V) = \left(\frac{5.5}{4.5+5.5}\right)(100) = 55 \text{ kips}$$

3. Plan irregularity considerations: The influence of torsional irregularity needs to be determined as specified in Table 12.3-1 (ASCE 7-05). It requires determination of story drifts in walls A and B. This evaluation must include accidental torsion caused by an eccentricity of 5 percent of the building dimension perpendicular to the applied force each way (ASCE 7-05, Section 12.8.4.2), 90 ft in this example.

$$\text{Additional eccentricity} \pm e_{\text{acc}} = 0.05\ (90) = 4.5 \text{ ft}$$

$$\text{Total eccentricity } e_{\text{total}} = (4.5 \pm 4.5) \text{ ft}$$

Initial torsional shear: Calculate from Eq. (7.75).

$$M_{\text{ta}} = Ve_{\text{acc}}$$

Based on $V = 100$ kips, and $e = (4.5 + 4.5)$ ft

$$V'_{T,A} = \frac{V(e+e_{\text{acc}})(x_R)(R_A)}{J} = \frac{100(4.5+4.5)(49.5)(4.5)}{27,248} = 7.36 \text{ kips}$$

$$V'_{T,B} = \frac{V(e+e_{\text{acc}})(60-x_R)(R_B)}{J} = \frac{(100)4.5+4.5(90-49.5)(5.5)}{27,248} = 7.36 \text{ kips}$$

These shear forces are shown in Fig. E7.12B.

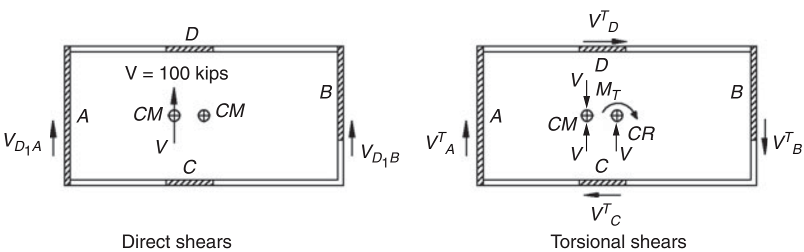


FIGURE E7.12B Direct and torsional shears.

Initial total shear:

$$V'_A = V_{D,A} - V'_{T,A} = 45 + 7.36 = 52.36 \text{ kips}$$

$$V'_B = V_{D,B} + V'_{T,B} = 55 - 7.36 = 47.64 \text{ kips}$$

(Check: Total shear, $V'_A + V'_B = 52.36 + 47.64 = 100$ kips)

Note: $V = 47.64$ kips in Wall B, smaller than the direct shear value of 55 kips, is not the design shear for this wall as the accidental eccentricity has been used here to *reduce* the force.

4. Calculate displacements of shear walls A and B due to shear forces calculated above. The resulting displacements are:

$$\delta_A = \frac{V'_A}{R_A} = \frac{52.36}{4.5} = 11.69$$

$$\delta_B = \frac{V'_B}{R_B} = \frac{47.64}{5.5} = 8.66$$

$$\delta_{\text{avg}} = 0.5(\delta_A + \delta_B) = 0.5(11.69 + 8.66) = 10.18$$

$$\delta_{\text{max}} = 11.69$$

$$\frac{\delta_{\text{max}}}{\delta_{\text{avg}}} = \frac{11.69}{10.18} = 1.14 < 1.2$$

Note: Units of shear wall displacement, δ, are not given here because *relative* rigidity rather than *actual* rigidity of the wall has been used to for its calculation. Because we use ratio of wall displacements in this example, actual units of δ are not required here.

The above calculation shows that horizontal torsional irregularity does not exist for the roof diagram of this building, and amplification factor, A_x, does not apply.

Most severe torsional shears: The most severe total torsional shears result from the use of torsional moment equal to $V(e + A_x e_{\text{acc}})$ for wall A (*plus* sign is used as this force acts in the same direction as the direct shear $V_{D,A}$), and $V(e - A_x e_{\text{acc}})$ (*minus* sign is used as this force acts opposite the direct shear $V_{D,B}$) for wall B. Thus,

$$V_{T,A} = \frac{V(e + A_X e_{\text{acc}})(x_R)(R_A)}{J} = \frac{100(4.5 + 4.5)(49.5)(4.5)}{27{,}248} = 6.36 \text{ kips}$$

$$V_{T,B} = \frac{V(e - A_X e_{\text{acc}})(90 - x_R)(R_B)}{J} = \frac{100(4.5 - 4.5)(90 - 49.5)(5.5)}{27{,}248} = 0$$

5. Total design shears in walls: Total shear in walls A and B equals the sum of direct and torsional shears (Fig. E7.12B):

$$V'_A = V_{D,A} + V_{T,A} = 45 + 17.36 = 52.36 \text{ kips}$$

$$V'_B = V_{D,B} + V_{T,B} = 55 + 0 = 55 \text{ kips}$$

Commentary:

1. To get an overall understanding of the influence of torsional moment, it is noted that total lateral force to be resisted by walls A and B equals $52.36 + 55 = 107.37$ kips versus applied lateral force of 100 kips.

2. The torsional moment also introduces shear force in walls C and D, but they are not considered in this example as these walls are perpendicular to the applied shear force and do not participate in resisting the applied shear force.

Example 7.13 Distribution of lateral forces from a rigid diaphragm to shear walls involving horizontal structural irregularity (torsional irregularity Type 1*a*).

Figure E7.13A shows the roof plan of a single-story shear wall building. The roof is supported by four walls A, B, C, and D as shown in the figure. The relative rigidities of these walls, which also act as shear walls, are 6, 4, 4, and 4, respectively. The center of mass of the roof is uniformly distributed over its area. The building, classified as a Seismic Design Category D structure, is subjected to a lateral force of 100 kips in the northwardly direction. Determine design shears in walls A and B.

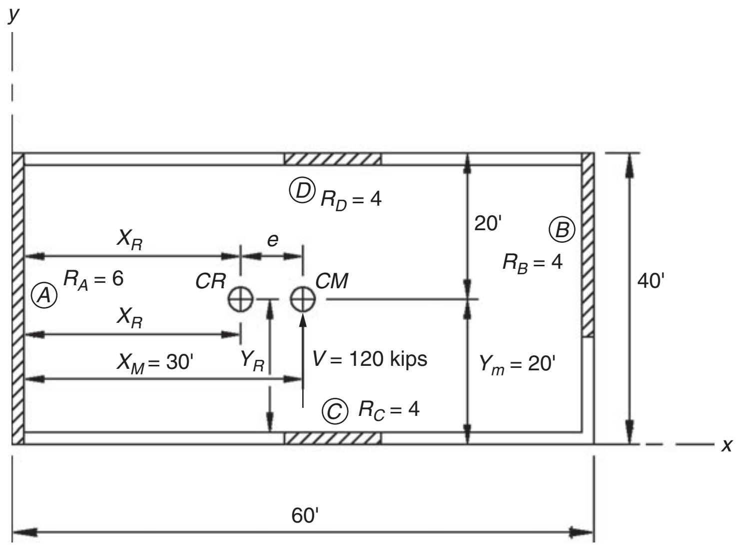


FIGURE E7.13A Roof plan of the shear wall building

Solution

1. Calculate the eccentricity and the rigidity properties.

$$R_A = 6 \qquad R_B = 4 \qquad R_C = 4 \qquad R_D = 4$$

$$V = 100 \text{ kips}$$

The mass of the roof is uniformly distributed over its area, so the center of mass (CM) is located at the geometrical center of the roof, which is rigid.

Center of rigidity (CR): Assume the origin at the bottom left corner of the roof. Summing moments of relative rigidities of walls A and B (parallel to the applied force) about Wall A:

$$x_R = \left(\frac{R_B}{R_A + R_B}\right)(60) = \left(\frac{4}{6+4}\right)(60) = 24 \text{ ft}$$

Summing up moments of relative rigidities of walls C and D about Wall C:

$$y_R = \left(\frac{R_D}{R_C + R_D}\right)(40) = \left(\frac{4}{4+4}\right)(40) = 20 \text{ ft}$$

(or from symmetry, $y_R = ½\,(40) = 20$ ft)
The center of mass is located at $x_M = ½(60) = 30$ ft from either wall.
Eccentricity $e = x_M - x_R = 30 - 24 = 6$ ft
Calculate the polar moment of inertia of shear walls, J, about the center of rigidity.

$$J = R_A\,(x_R)^2 + R_B\,(60 - x_R)^2 + R_C\,(y_R)^2 + R_D\,(40 - y_R)^2$$

$$= 6\,(24)^2 + 4\,(60 - 24)^2 + 4\,(24)^2 + 4\,(48 - 24)^2$$

$$= 11,840 \text{ ft}^2$$

2. Direct shear in walls A and B: The rigidity of the building in the direction of the force equals the sum of rigidities of walls parallel to the force. In this example, shear walls A and B are parallel, and are oriented in the direction of applied lateral force. Therefore, they would share the applied lateral force in proportion to their rigidities. Accordingly,

$$V_{D,A} = \left(\frac{R_A}{R_A + R_B}\right)(V) = \left(\frac{6}{6+4}\right)(100) = 60 \text{ kips}$$

$$V_{D,B} = \left(\frac{R_B}{R_A + R_B}\right)(V) = \left(\frac{4}{6+6}\right)(100) = 40 \text{ kips}$$

3. Torsional moments due to eccentricity: The applied lateral force V acts at the center of mass of the system as shown in Fig. E7.13B. Because the center of mass and the center of rigidity are not coincident, eccentricity is created as shown in Fig. E7.13B. As a result, the applied shear force is transmitted to the wall eccentrically. This situation is analyzed by introducing two equal and opposite forces at the center of rigidity as shown in Fig. E7.13C causing torsional moments. The net result of eccentricity is to cause direct shear forces in walls A and B, and torsional shears in all four walls as shown in Fig. E7.13D. However, we are concerned with

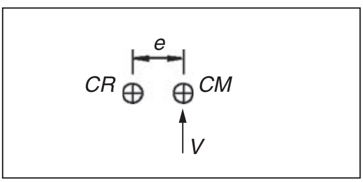


FIGURE E7.13B Direct shears.

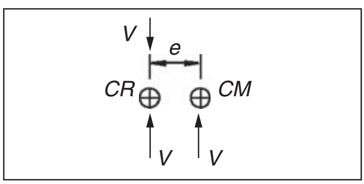


FIGURE E7.13C Torsional shears.

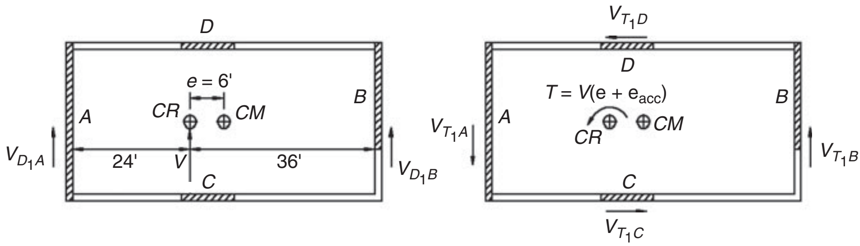


FIGURE E7.13D Direct and torsional shear in all shear walls.

torsional shears in walls A and B only, which are parallel to the applied shear force V.

4. Plan irregularity considerations: The influence of torsional irregularity needs to be determined as specified in Table 12.3-1 (ASCE 7-05). It requires determination of story drifts in walls A and B. This evaluation must include accidental torsion caused by an eccentricity of 5 percent of the building dimension perpendicular to the applied force (96 ft in this example) each way (ASCE 7-05, Section 12.8.4.2).

$$\text{Additional eccentricity } e_{\text{acc}} = \pm\, 0.05\,(60) = \pm\, 3 \text{ ft}$$

$$\text{Total eccentricity } e_{\text{total}} = (6 \pm 3) \text{ ft}$$

Initial torsional shear: Calculate from Eq. (7.75).
For the determination of torsional irregularity, the initial most severe torsional shears, V' and corresponding story drifts (so as to produce the lowest value of average story drift) would result from the largest eccentricity, $e + e_{\text{acc}}$. Thus, based on $V = 100$ kips, and $e = (6 + 3)$ ft, $M_{\text{ta}} = Ve_{\text{acc}}$, the torsional shear are

$$V'_{T,A} = \frac{V(e+e_{\text{acc}})(x_R)(R_A)}{J} = \frac{100(6+3)(24)(6)}{11{,}840} = 10.95 \text{ kips}$$

$$V'_{T,B} = \frac{V(e+e_{\text{acc}})(60-x_R)(R_B)}{J} = \frac{100(6+3)(60-24)(4)}{11{,}840} = 10.95 \text{ kips}$$

These shear forces are shown in Fig. E7.13D.
Initial total shears in walls A and B are:

$$V'_A = V_{D,A} - V'_{T,A} = 60 - 10.95 = 49.05 \text{ kips}$$
$$V'_B = B_{D,B} + V'_{T,B} = 40 + 10.95 = 50.95 \text{ kips}$$

(Check: total shear, $V'_A + V'_B = 49.05 + 50.95 = 100$ kips)

Note: V = 49.05 kips in Wall A, which is smaller than the direct shear value of 60 kips, is not the design shear for this wall as the accidental eccentricity, and has been used here to reduce the force.

Calculate displacements of shear walls A and B due to shear forces calculated above. The resulting displacements are

$$\delta_A = \frac{V'_A}{R_A} = \frac{49.05}{6} = 8.18$$

$$\delta_B = \frac{V'_B}{R_B} = \frac{50.95}{4} = 12.74$$

$$\delta_{\text{avg}} = 0.5(\delta_A + \delta_B) = 0.5(8.18 + 12.74) = 10.46$$

$$\delta_{\text{max}} = 12.74$$

$$\frac{\delta_{\text{max}}}{\delta_{\text{avg}}} = \frac{12.74}{10.46} - 1.22 > 1.2 \qquad \text{but} \qquad < 1.4$$

Note: Units of shear wall displacement, δ, are not given here because *relative* rigidity rather than *actual* rigidity of the wall has been used to for its calculation. Because we use ratio of wall displacements in this example, actual units of δ are not required here.

The above calculation shows that torsional irregularity Type 1*a* (ASCE 7-05 Table 12.3-1, Item 1*a*) exists. Because the seismic design category (SDC) for this building is D, structural modeling would require a three-dimensional model (ASCE 7-05 Section 12.7.3). In addition, the diaphragm shear forces to collectors must be increased 25 percent (ASCE 7-05 Section 12.3.3.4).

ASCE 7-05 Section 12.8.4.3 requires the evaluation and application of torsional amplification factor, A_x.

$$A_X = \left(\frac{\delta_{\text{max}}}{1.2\delta_{\text{avg}}}\right)^2 = \left(\frac{12.74}{1.2 \times 10.46}\right)^2 = 1.03 < 3.0$$

Therefore, use $A_x = 1.03$

5. Most severe torsional shears: To account for the effects of torsional irregularity, ASCE 7-05 Section 12.8.4.3 requires that accidental torsional moment, Ve_{acc}, be multiplied by the torsional amplification factor, A_x.
The most severe total torsional shears result from the use of torsional moment equal to $V(e - A_x\, e_{\text{acc}})$ for wall A (*minus* sign is used for $A_x\, e_{\text{acc}}$ as this force acts opposite to the direct shear $V_{D,A}$), and $V(e + A_x\, e_{\text{acc}})$ (*plus* sign is used for $A_x\, e_{\text{acc}}$ as this force acts with the direct shear $V_{D,B}$) for wall B. Thus,

$$V_{T,A} = \frac{V(e - A_X e_{\text{acc}})(x_R)(R_A)}{J} = \frac{100(6 - 1.03 \times 3)(24)(6)}{11{,}840} = 1.1 \text{ kips}$$

$$V_{T,B} = \frac{V(e + A_X e_{\text{acc}})(60 - x_R)(R_B)}{J} = \frac{150(6 + 1.03 \times 3)(36)(4)}{11{,}840} = 11.1 \text{ kips}$$

6. Total shear in walls: Total shear in walls A and B equals the sum of direct and torsional shears.

$$V'_A = V_{D,A} - V_{T,A} = 60 - 1.1 = 58.9 \text{ kips}$$

$$V'_B = V_{D,B} + V_{T,B} = 40.5 + 11.1 = 51.1 \text{ kips}$$

Commentary:

1. To get an overall understanding of the influence of torsional moment, it is noted that total lateral force that must be resisted by walls A and B equals 58.9 + 51.1 = 110 kips versus applied lateral force of 100 kips.
2. The torsional moment also introduces shear force in walls C and D, but they are not considered in this example as these walls are perpendicular to the applied lateral force and do not participate in resisting the applied lateral force.

Example 7.14 Distribution of lateral forces from a rigid diaphragm to shear walls involving *extreme torsional irregularity* (horizontal structural irregularity Type 1*b*).

Figure E7.14*a* shows the floor plan of a one-story shear wall building with a rigid roof diaphragm. The center of mass of the building is located at the geometric center of the rigid diaphragm (roof). The rigidities of shear walls A and B are 600 kips/in. and 200 kips/in., respectively; those of shear walls C and D are 300 kips/in. The building, classified as a Seismic Design Category D structure, is subjected to a lateral force of 150 kips acting northwardly. Determine design shear forces in walls A and B.

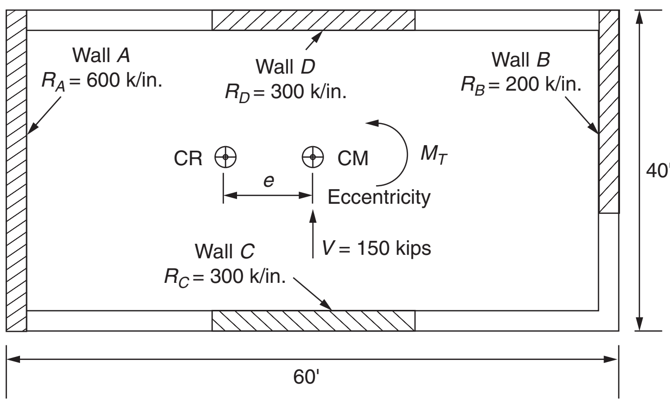


FIGURE E7.14A Floor plan of a one-story shear wall building.

Solution

Commentary: This example is similar to Example 7.13 except for the fact that *actual* wall rigidities are given instead of their *relative* rigidities; the analysis procedure, however, is essentially the similar. All calculations are presented in a step-by-step manner as follows.

The lateral force of 150 kips would be resisted by shear walls A and B which are parallel to the applied lateral force. Therefore, the building would be analyzed for seismic forces in the north-south direction (in the direction of the applied force). The analysis procedure for the problem is presented in a step-by-step manner as follows:

1. Calculate the eccentricity and the rigidity properties:

$$R_A = 600 \text{ kips/in.} \qquad R_B = 200 \text{ kips/in.} \qquad R_C = R_D = 300 \text{ kips/in.}$$

$$V = 150 \text{ kips}$$

Mass is uniformly distributed so the center of mass is located at the geometrical center of the roof, which is rigid.

Center of rigidity : Summing moments of rigidities of walls A and B (parallel to the applied force) about Wall A:

$$x_R = \left(\frac{R_B}{R_A + R_B}\right)(96) = \left(\frac{200}{600+200}\right)(96) = 24 \text{ ft}$$

Summing up moments of rigidities of walls C and D about Wall C:

$$y_R = \left(\frac{R_D}{R_C + R_D}\right)(48) = \left(\frac{300}{300+300}\right)(48) = 24 \text{ ft}$$

(or from symmetry, $y_R = ½\,(48) = 24$ ft)
The center of mass is located at $x_M = ½(96) = 48$ ft from either wall.
Eccentricity $e = x_M - x_R = 48 - 24 = 24$ ft
Calculate the polar moment of inertia of shear walls, J, about the center of rigidity.

$$\begin{aligned} J &= R_A\,(x_R)^2 + R_B\,(96 - x_R)^2 + R_C\,(y_R)^2 + R_D\,(48 - y_R)^2 \\ &= 600\,(24)^2 + 200\,(96-24)^2 + 300\,(24)^2 + 300\,(48-24)^2 \\ &= 1{,}728{,}000 \text{ (k/in.) ft}^2 \end{aligned}$$

2. Direct shear in walls A and B: The rigidity of the building in the direction of the force equals the sum of rigidities of walls parallel to the force. In this example, shear walls A and B are parallel, and are oriented in the direction of applied lateral force. Therefore, they would share the applied lateral force in proportion to their rigidities.

$$V_{D,A} = \left(\frac{R_A}{R_A + R_B}\right)V = \left(\frac{600}{600+200}\right)150 = 112.5 \text{ kips}$$

$$V_{D,B} = \left(\frac{R_B}{R_A + R_B}\right)V = \left(\frac{200}{600+200}\right)150 = 37.5 \text{ kips}$$

3. Plan irregularity considerations: The influence of torsional irregularity needs to be determined as specified in Table 12.3-1 (ASCE 7-05). It requires determination of story drifts in walls A and B. This evaluation must include accidental torsion caused by an eccentricity of 5 percent of the building dimension perpendicular to the applied force, 96 ft in this example (ASCE 7-05, Section 12.8.4.2).

$$\text{Additional eccentricity } e_{\text{acc}} = \pm\,0.05\,(96) = \pm\,4.8 \text{ ft}$$

$$\text{Total eccentricity } e_{\text{total}} = 24 + 4.8 = 28.8 \text{ ft}$$

Initial torsional shear: Based on $V = 150$ kips, and $e = (24 + 4.8)$ ft

$$V'_{T,A} = \frac{V(e + e_{\text{acc}})(x_R)(R_A)}{J} = \frac{150(24+4.8)(24)(600)}{1{,}728{,}000} = 36 \text{ kips}$$

$$V'_{T,B} = \frac{V(e + e_{\text{acc}})(96 - x_R)(R_B)}{J} = \frac{150(24+4.8)(72)(200)}{1{,}728{,}000} = 36 \text{ kips}$$

These shear forces are shown in Fig. E7.14B

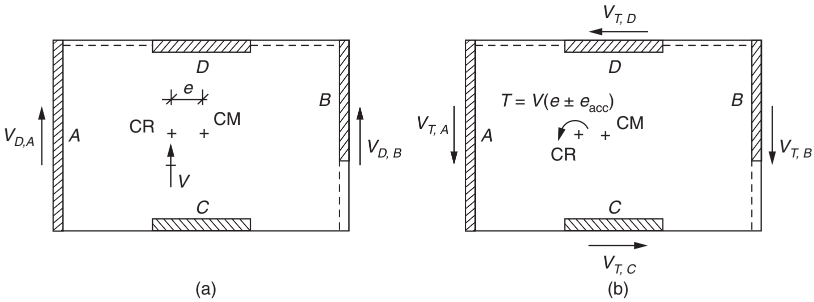


FIGURE E7.14B (*a*) Direct and (*b*) torsional shears.

Initial total shear:

$$V'_A = V_{D,A} - V'_{T,A} = 112.5 - 36 = 76.5 \text{ kips}$$
$$V'_B = B_{D,B} + V'_{T,B} = 37.5 + 36 = 73.5 \text{ kips}$$

(Check: Total shear, $V'_A + V'_B = 76.5 + 73.5 = 150$ kips)

Note: V = 76.5 kips in Wall A, which is smaller than the direct shear value of 112.5 kips, is not the design shear for this wall as the accidental eccentricity has been used here to *reduce* the force.

Calculate displacements of shear walls A and B due to shear forces calculated above. The resulting displacements are:

$$\delta_A = \frac{V'_A}{R_A} = \frac{76.5}{600} = 0.13 \text{ in.}$$
$$\delta_B = \frac{V'_B}{R_B} = \frac{73.5}{200} = 0.37 \text{ in.}$$
$$\delta_{\text{avg}} = 0.5(\delta_A + \delta_B) = 0.5(0.13 + 0.37) = 0.25 \text{ in.}$$
$$\delta_{\max} = 0.37 \text{ in.}$$
$$\frac{\delta_{\max}}{\delta_{\text{avg}}} = \frac{0.37}{0.25} = 1.48 > 1.4$$

The above calculation shows that extreme torsional irregularity Type 1*b* (ASCE 7-05 Table 12.3-1, Item 1*b*) exists. Because the structural design category for this building is D, structural modeling would require a three-dimensional model (ASCE 7-05 Section 12.7.3). In addition, the diaphragm shear forces to collectors must be increased 25 percent (ASCE 7-05 Section 12.3.3.4).
ASCE 7-05 Section 12.8.4.3 requires the evaluation and application of torsional amplification factor, A_x.

$$A_X = \left(\frac{\delta_{\max}}{1.2\delta_{\text{avg}}}\right)^2 = \left(\frac{0.37}{1.2 \times 0.25}\right)^2 = 1.52 < 3.0$$

4. Most severe torsional shears: To account for the effects of torsional irregularity, ASCE 7-05 Section 12.8.4.3 requires that accidental torsional moment, Ve_{acc}, be multiplied by the torsional amplification factor, A_x.

The most severe total torsional shears result from the use of torsional moment equal to $V(e - A_x\, e_{\text{acc}})$ for Wall A (*minus* sign is used for $A_x\, e_{\text{acc}}$ because this force acts opposite to the direct shear $V_{D,A}$), and $V(e + A_x\, e_{\text{acc}})$ (*plus* sign is used for $A_x\, e_{\text{acc}}$ because this force acts with the direct shear $V_{D,B}$) for wall B. Thus,

$$V_{T,A} = \frac{V(e - A_X e_{\text{acc}})(x_R)(R_A)}{J} = \frac{150(24 - 1.52 \times 4.8)(24)(600)}{1{,}728{,}000} = 20.88 \text{ kips}$$

$$V_{T,B} = \frac{V(e + A_X e_{\text{acc}})(96 - x_R)(R_B)}{J} = \frac{150(24 + 1.52 \times 4.8)(72)(200)}{1{,}728{,}000} = 39.12 \text{ kips}$$

5. Total shear (Fig. E7.14B)

$$V'_A = V_{D,A} - V_{T,A} = 112.5 - 20.88 = 91.62 \text{ kips}$$

$$V'_B = V_{D,B} + V_{T,B} = 37.5 + 39.12 = 76.62 \text{ kips}$$

Commentary:

1. To get an overall understanding of the influence of torsional moment, it is noted that total lateral force to be resisted by walls A and B equals 91.62 + 76.62 = 168.24 kips versus applied lateral force of 150 kips.

2. The torsional moment also introduces shear force in walls C and D, but they are not considered in this example as these walls are perpendicular to the applied shear force and do not participate in resisting the applied shear force.

7.10 DESIGN CONSIDERATIONS FOR SHEAR WALLS

7.10.1 Types of Reinforced Masonry Shear Walls

Masonry shear walls may be unreinforced or reinforced. As mentioned in previous chapters, use of Type *N* mortar or masonry cement mortar is prohibited in Seismic Design Category (SDC) D, E, and F structures (no such restriction for SFRS in other seismic design categories).

Based on MSJC-05 *Code*, 2006 IBC (Section 2106) defined five types of masonry shear walls (including both unreinforced and reinforced) for purposes of seismic design as follows:

1. Ordinary plain (or unreinforced) masonry shear walls
2. Detailed plain masonry shear walls
3. Ordinary reinforced masonry walls
4. Intermediate reinforced masonry walls
5. Special reinforced masonry walls

These shear walls can be used as parts of SFRS in buildings as noted in Table 7.5. The MSJC-08 *Code* introduced a few more types (a total of 12, including the above 5 types) of masonry shear walls, which provides considerable flexibility in designing masonry shear wall to resist lateral forces. The main differences between these different types of shear walls are characterized by the following design considerations:

1. Masonry is reinforced, prestressed, or unreinforced.
2. Masonry uses autoclaved aerated concrete (AAC) units.
3. Minimum reinforcement is provided, but their resistance to shear resistance is neglected.
4. Stresses in reinforcement (provided to resist shear) are considered, and walls satisfy specific minimum reinforcement and connection requirements.
5. Stresses in reinforcement (provided to resist shear) are considered, and walls satisfy prescriptive reinforcement and connection requirements.
6. Seismic design category (A through F).

MSJC-08 *Code* defines the 12 types of shear walls as follows:

1. *Detailed plain (unreinforced) AAC masonry shear wall:* An AAC masonry shear wall designed to resist lateral forces while neglecting stresses in reinforcement, although provided with minimum reinforcement and connections. Permitted in Seismic Design Category A and B structures.
2. *Detailed plain (unreinforced) masonry shear wall:* A masonry shear wall designed to resist lateral forces while neglecting stresses in reinforcement, although provided with minimum reinforcement and connections. Permitted in Seismic Design Category A and B structures.
3. *Intermediate reinforced masonry shear wall:* A masonry shear wall designed to resist lateral forces while considering stresses in reinforcement and satisfying specific minimum reinforcement and connection requirements. Permitted in Seismic Design Category A, B, and C structures.
4. *Intermediate reinforced prestressed masonry shear wall:* A prestressed masonry shear wall designed to resist lateral forces while considering stresses in reinforcement and satisfying specific minimum reinforcement and connection requirements. Permitted in Seismic Design Category A, B, and C structures.
5. *Ordinary plain (unreinforced) AAC masonry shear wall:* An AAC masonry shear wall designed to resist lateral forces while neglecting stresses in reinforcement. Permitted in Seismic Design Category A and B structures.
6. *Ordinary plain (unreinforced) masonry shear wall:* A masonry shear wall designed to resist lateral forces while neglecting stresses in reinforcement. Permitted in Seismic Design Category A and B structures.
7. *Ordinary plain (unreinforced) prestressed masonry shear wall:* A prestressed masonry shear wall designed to resist lateral forces while neglecting stresses in reinforcement. Permitted in Seismic Design Category A and B structures.
8. *Ordinary reinforced AAC masonry shear wall:* An AAC masonry shear wall designed to resist lateral forces while considering stresses in reinforcement and satisfying prescriptive reinforcement and connection requirements. Permitted in all Seismic Design Category structures (A through F).
9. *Ordinary reinforced masonry wall:* A masonry shear wall designed to resist lateral forces while considering stresses in reinforcement and satisfying prescriptive reinforcement and connection requirements. Permitted in Seismic Design Category A, B, and C structures.

10. *Special reinforced masonry wall:* A masonry shear wall designed to resist lateral forces while considering stresses in reinforcement and satisfying special reinforcement and connection requirements. Permitted in all seismic design category structures (A through F).

11. *Special reinforced prestressed masonry wall:* A prestressed masonry shear wall designed to resist lateral forces while considering stresses in reinforcement and satisfying special reinforcement and connection requirements. Permitted in all seismic design category structures (A through F).

12. *Empirically designed shear wall:* A masonry shear wall designed under Provisions of Chap. 5 of MSJC-08 *Code* and assigned to Seismic Design Category A structures.

Of the above list of 12 types, only 6 (listed as No. 2, 3, 6, 9, 10, and 12) were covered in MSJC-05 *Code*; all other types are new to the MSJC-08 *Code*, and are not referenced in ASCE 7-05 for use as shear wall building systems. 2009 IBC (Section 2102) recognizes eight of types of shear walls (listed as No. 2, 3, 4, 6, 7, 9, 10, and 11).

Discussion and design of prestressed masonry and AAC masonry walls are outside the scope of this book. Pertinent design provisions for these types of walls can be found in MSJC-08 *Code* Chap. 4 and App. A, respectively. A brief description, design methods, and requirements of the above described shear walls types, and pertinent seismic design categories can be found in the Commentary to the MSJC-08 *Code* (Table CC-1.17.3.2: Requirements for Masonry Shear Walls Based on Shear Wall Designation).

In general, classification of shear walls is based on their ability to act as parts of SFRS for buildings in seismic regions. Table 7.5 lists various building systems in which these different types of shear walls are permitted or not permitted to act as lateral force–resisting elements in buildings assigned to various seismic design categories (B through F, discussed in the next section), pertinent height limits, response modification factors (R-factors), system overstrength factors (D_o), and deflection amplification factors (C_{od}).

The relevance of this classification lies in the fact that when used as parts of SFRS in buildings, they are subjected to certain restrictions and height limitations as listed in Table 7.5 as well as certain mandatory detailing and reinforcement requirements for the purposes of seismic design (MSJC-08 Section 1.17). The intent is for the shear walls to have adequate capacity for inelastic response and energy dissipation during a seismic event. They are assigned different values of seismic response modification factors, R, and overstrength factors, Ω_o. The ductility levels of various types of shear walls are indicated by values of their seismic response medication factors, R (Table 7.5). The higher the value of R, the greater the ductility of shear walls; consequently, the smaller the design seismic forces they are required to resist.

Regardless of the type of shear walls used as parts of SFRS in a building, a designer must provide proper load path for transmission of seismic forces from diaphragms or other parts of a building to shear walls. Load path connections and minimum anchorage forces should comply with the legally adopted building code. When a legally adopted building code (e.g., 2009 IBC) does not provide minimum load path connection requirements and anchorage design forces, designers should use provisions of ASCE 7-05 Section 12.11.

Ordinary plain masonry walls are simply unreinforced masonry walls that are permitted to be used as SFRS elements in SDC A or B structures. These walls are assigned an R value of 1½, and are designed in accordance with the requirements specified in MSJC-08 Section 2.2 or 3.2, for allowable stress design (ASD) and strength design (SD), respectively. The tensile strength of masonry is taken into account and walls are so designed that tensile strength of masonry is not exceeded, and thus preclude the possibility of cracking.

Detailed plain masonry shear walls are masonry walls designed to comply with the requirements of MSJC-08 Secs. 2.2 or 3.2 for ASD and SD, respectively. These walls are assigned an R value of 2 and an overstrength factor $\Omega_o = 2.5$, and are permitted to be used as SFRS elements in Seismic Design Category A and B structures. Although classified as *unreinforced*, these walls are subject to certain reinforcement detailing (both vertical and horizontal) requirements (MSJC-08 Section 1.17.3.2.3.1) as follows:

1. Vertical reinforcement of 0.2 in.^{2} in cross-sectional area at
   (a) All corners
   (b) Within 16 in. of each openings
   (c) Within 8 in. of each side of movement joints
   (d) Within 8 in. of ends of wall
   (e) At a maximum spacing of 10 ft

2. Horizontal reinforcement of at least 0.2 in.^{2} in cross-sectional area of bond beam reinforcement spaced not more than 10 ft on center, or at least two wires of W 1.7 joint reinforcement spaced not more than 16 in. at
   (a) Top and bottom wall openings extending at least 24-in. or 40-bar diameters (whichever is greater) past openings
   (b) Continuously at structurally connected roofs and floors
   (c) Within 16 in. of top of walls

Ordinary reinforced masonry shear walls are shear walls in which the tensile strength of masonry is neglected and tension at any cross section is assumed to be resisted by the reinforcement. For seismic design, they are permitted as parts of SFRS in Seismic Design Category A, B, and C structures (limited to a height of 160 ft in Seismic Design Category C); they are assigned an R value of 2 and an overstrength factor $\Omega_0 = 2.5$. They are designed to comply with requirements of MSJC-08 Sections 2.3 and 3.3 for ASD and SD, respectively. Additionally, these walls must also comply with detailing requirements for detailed plain masonry shear walls specified above (MSJC-08 Section 1.17.3.2.3.1).

Minimum reinforcement requirements for detailed plain and ordinary reinforced masonry shear walls are shown in Fig. 7.31.

Intermediate reinforced masonry shear walls are designed to comply with requirements of MSJC-08 Sections 2.3 and 3.3 for ASD and SD, respectively. They must be provided with detailing requirements for detailed plain masonry shear walls (described above) except

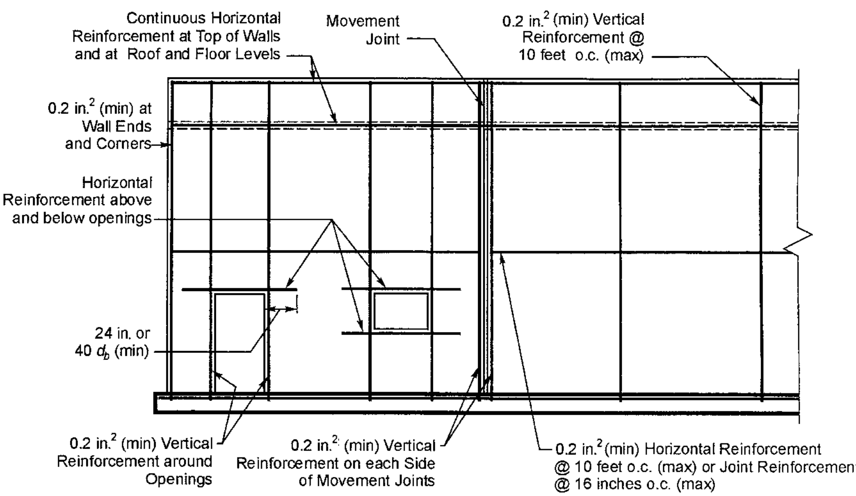


FIGURE 7.31 Minimum detailing requirements for detailed plain and ordinary reinforced masonry shear walls [7.16]. (*Courtesy: CMACN.*)

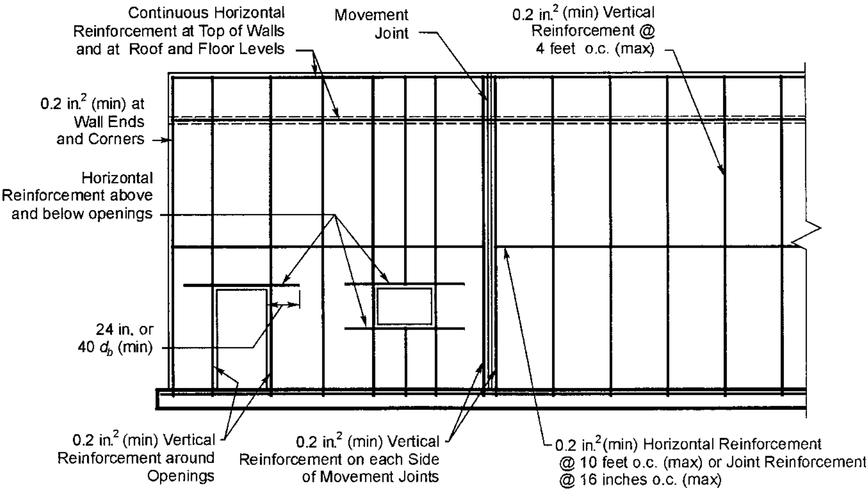


FIGURE 7.32 Minimum reinforcement requirements for intermediate reinforced masonry shear walls [7.16]. (*Courtesy: CMACN.*)

that spacing of reinforcement must not exceed 4 ft (MSJC-08 Section 1.17.3.2.5). They are permitted for structures in Seismic Design Category A, B, and C (not permitted in SDC D, E, and F) without any height limitations. These walls are assigned an R value of 3½ and overstrength factor $\Omega_0 = 2.5$.

Figure 7.32 shows minimum reinforcement requirements for intermediate reinforced masonry shear walls.

Special reinforced masonry shear walls are so called because they are specially reinforced to provide large ductility which enables them to deform sufficiently without failure during large earthquakes. To ensure adequate ductility in these walls, it is required that design shear strength, ϕV_n, shall exceed the shear corresponding to the development of 1.25 times the nominal flexural strength, M_n, of the member, except that the nominal shear strength of these walls, V_n, need not exceed 2.5 times the required shear strength, V_u (MSJC-08 Section 1.17.3.2.6.1.1). As a result, they are assigned an R value of 5 (highest of all types of masonry shear walls) and overstrength factor $\Omega_0 = 2.5$. They are designed according to requirements of MSJC-08 Sections 2.3 and 3.3 for ASD and SD, respectively.

Special reinforced masonry shear walls are permitted as lateral force–resisting elements in structures of all seismic design categories (A through F). Note that only special reinforced masonry shear walls are permitted for Seismic Design Category C, D, E, and F structures (i.e., structures in high seismic regions) because of their high ductile capacity. Shear walls in Seismic Design Category C, D, and E structures are restricted to a height of 160 ft, but only 100 ft is permitted for Seismic Design Category F structures.

Design of special reinforced masonry wall should comply with requirements specified in MSJC-08 Secs. 2.3 and 3.3, and detailing requirements for minimum reinforcement specified in Section 1.17.3.2.6 as follows. These minimum reinforcement requirements are intended to improve ductile behavior of these shear walls under earthquake loading, and to assist in crack control. Because the minimum required reinforcement may be provided to satisfy design requirements, at least one-third of the required reinforcement amount is reserved for the lesser stressed direction to ensure appropriate distribution of reinforcement in both directions.

1. Walls should be reinforced in both horizontal and vertical directions.
2. The reinforcement should be uniformly distributed.

3. Minimum area of sum of horizontal and vertical reinforcement should be 0.002 times the gross cross-sectional area of the wall.
4. Minimum area of reinforcement in each direction should be 0.0007 times the gross cross-sectional area of the wall.
5. The minimum area of vertical reinforcement should be one-third of required shear reinforcement.
6. Maximum spacing of both horizontal and vertical reinforcement should be limited to the smallest of
   (a) One-third of the height of wall
   (b) One third the length of wall
   (c) 48 in.
7. Shear reinforcement should be anchored around vertical reinforcing bars with a standard hook (MSJC-08 Section 3.3.3.3.2.1).

MSJC specifies additional special requirements when shear wall masonry is laid in other than running bond (e.g., in stack bond):

1. The maximum spacing of reinforcement is 24 in.
2. Wythes of stack bond masonry shall be constructed of fully grouted hollow open-end units, or two wythes of solid units.

Figure 7.33 shows minimum detailing requirements for special reinforced masonry shear walls.

In a building, often there are other walls that are not a part of SFRS of the building. These include such walls as partition walls, screen walls, and other masonry elements not designed to resist gravity or lateral forces other than those induced by their own gravity loads and inertia forces. Such masonry elements must be isolated from the building so that gravity and lateral forces from the building are not imparted to these elements. Isolation joints and connectors between these elements and the structure must be designed to accommodate the design story drift.

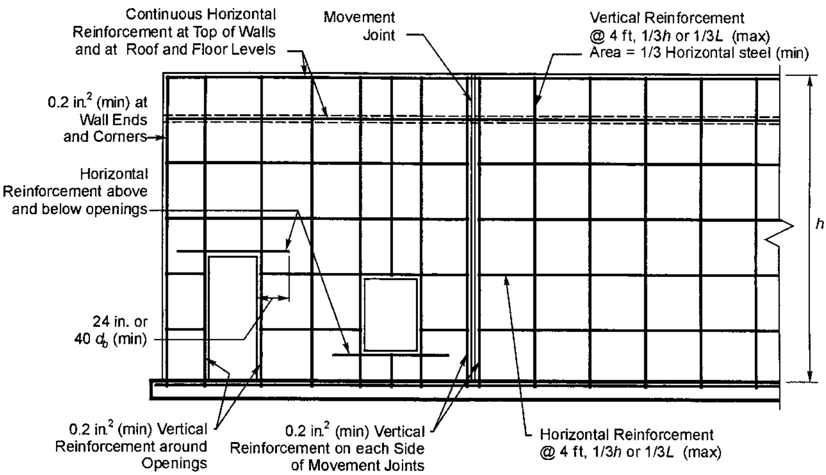


FIGURE 7.33 Minimum detailing requirements for special reinforced masonry shear walls [7.16]. (*Courtesy: CMACN.*)

7.10.2 Seismic Design Categories

Every structures in an occupancy category (Occupancy Category I, II, III, and IV) is assigned a SDC A through F based on consideration of following seismic design parameters (both must be considered simultaneously):

1. Design, 5 percent damped, spectral acceleration parameter at short periods (0.2s), S_{DS}
2. Design, 5 percent damped, spectral acceleration parameter at 1-s period, S_{D1}.

Values of S_{DS} and S_{D1} for various occupancy categories are listed in Tables 7.8 and 7.9; each building and structure is assigned more severe of the categories so determined.

TABLE 7.8 Seismic Design Category Based on Design, 5 Percent Damped, Spectral Acceleration Parameter at Short Periods, S_{DS} [7.13]

	Occupancy category		
Value of S_{DS}	I or II	III	IV
$S_{DS} < 0.167$	A	A	A
$0.167 \le S_{DS} < 0.33$	B	B	C
$0.33 \le S_{DS} < 0.5$	C	C	D
$0.5 \le S_{DS}$	D	D	D

TABLE 7.9 Seismic Design Category Based on Design, 5 Percent Damped, Spectral Acceleration Parameter at a Period of 1 s, S_{D1} [7.13]

	Occupancy category		
Value of S_{DS}	I or II	III	IV
$S_{D1} < 0.067$	A	A	A
$0.067 \le S_{D1} < 0.133$	B	B	C
$0.133 \le S_{D1} < 0.20$	C	C	D
$0.20 \le S_{D1}$	D	D	D

The following exceptions apply to determination of seismic design category as determined from Tables 7.8 and 7.9:

1. Occupancy Category I, II, and III structures located where the mapped spectral acceleration parameter at 1-s period, S_1, is greater than or equal to 0.75 shall be assigned Seismic Design Category E.
2. Occupancy Category IV structures located where the mapped spectral acceleration parameter at 1-s period, S_1, is equal to or greater than 0.75 (i.e., $S_1 \ge 0.75$) shall be assigned Seismic Design Category F.
3. Where S_1 is greater than 0.75, seismic design category is permitted to be determined based on considerations of only S_{DS} as listed in Table 7.8 where all of the following apply:
   (a) In each of the two orthogonal directions, the approximate fundamental period of the structure, T_a, is less than $0.8T_s$.
   (b) In each of the two orthogonal directions, the fundamental period of the structure used to calculate the story drift is less than T_s.

(c) Eq. (7.50) [ASCE 7-05 Eq. (12.8-2)] is used to calculate the seismic response coefficient, C_s.
(d) The diaphragms are rigid; or for diaphragms that are flexible, the distance between the vertical elements of the seismic force–resisting system does not exceed 40 ft.

It is important to understand the significance of SDC of a building. ASCE 7-05 Table 12.2-1 (see Table 7.5 for shear wall SFRS) specifies the following compliance requirements for different SFRS to be used in buildings:

1. No limit in height (NL) for building in specified SDCs.
2. Specific height limitations for buildings in specific SDCs.
3. Restrictions (not permitted) on using certain SFRS in buildings having different SDC (NP).

Example 7.15 presents the procedure for determination of seismic design category of buildings. It illustrates and highlights the point that buildings of different occupancy categories but having the same site class and the same short-period and 1-s spectral acceleration parameters (Ss and S_1, respectively) can have different seismic design category.

It is noted that occupancy categories are defined in ASCE 7-05 Section 1.5 (Table 1-1) *and* in 2009 IBC Table 1604.5. There are, however, some differences between the two tables. In the following example, provisions of 2009 IBC Table 1604.5 have been used (legal building code).

Example 7.15 Determination of seismic design category of buildings.
Determine the Seismic Design Category for
(a) A daycare facility with capacity greater than 250
(b) A police station
The following information applies to both structures: Site Class C, $S_s = 1.76g$, $S_1 = 0.81g$.

Solution:

1. Determine the occupancy category for each building based on 2009 IBC Table 1604.5.
(a) Daycare facility with capacity greater than 250: Occupancy Category III
(b) Police station: Occupancy Category IV
2. Determine S_{DS} and S_{D1} for the given buildings.

$$S_s = 1.76g$$

For Site Class C and $S_s \geq 1.25g$, $F_a = 1.0$

$$S_{MS} = F_a S_s = (1.0)(1.76) = 1.76$$

$$S_{DS} = 2/3\, S_{MS} = 2/3\,(1.76) = 1.17$$

$$S_1 = 0.81g$$

For Site Class C and $S_1 \geq 0.5g$, $F_v = 1.3$

$$S_{M1} = F_v S_1 = (1.3)(0.81) = 1.05$$

$$S_{D1} = 2/3\, S_{M1} = 2/3\,(1.05) = 0.70$$

3. Determine seismic design category.

(a) Daycare facility: Occupancy Category III, based on $S_{DS} = 1.17 > 0.5$, S_{DC} is D
Based on $S_{D1} = 0.7 > 0.2$, SDC is D.
But $S_1 = 0.81 > 0.75$; therefore, for this Occupancy Category III building, SDC is E.

(b) Police station: Occupancy Category IV, based on $S_{DS} = 1.17 > 0.5$, S_{DC} is D
Based on $S_{D1} = 0.7 > 0.2$, SDC is D.
But $S_1 = 0.81 > 0.75$; therefore, for this Occupancy Category IV building, SDC is F.

7.10.3 Design for Flexure

A shear wall is necessarily subjected to forces in two perpendicular planes. It receives lateral loads perpendicular to wall (the out-of-plane lateral loads) that cause the wall to bend out-of-plane, a topic discussed in Chap. 6. Additionally, it receives in-plane lateral loads which it resists as a shear wall. Procedure for designing a wall as a shear wall is discussed in this section. Although these two sets of forces are not assumed to act concurrently, the wall must be adequate to resist these two sets of forces independently.

A shear wall is analyzed as a vertical beam. Under the action of in-plane lateral force acting on the wall, the wall is subjected to shear as well as moment. For a single-story wall, the magnitude of this moment equals the lateral force times the wall height. In addition, the wall is subjected to a seismic lateral force at the midheight due to its own inertia. This inertial force equals the seismic coefficient times the self-weight of the wall, and the moment due to this lateral force equals this force times the distance from the center of gravity of mass of the wall to the base. In a single-story building, this moment is added to the moment caused by the in-plane lateral force applied at the top of wall. See Example 7.16. In multistory shear walls, the self-weight of walls is lumped the with the masses tributary at various floor levels. The shear wall is subjected to in-plane lateral force at each diaphragm level as well as inertial forces due to the self-weight of shear walls themselves, which act at the centers of gravity of their tributary masses. The wall must be designed for all forces that it must resist.

The total moment on the wall is resisted by the rectangular section of the wall (as seen in plan). Often, it is easier to visualize the wall as a cantilever beam by simply rotating the wall by 90° (Fig. 7.34). Moment causes flexural stresses which are tensile and compressive, and vary linearly between the two vertical ends of the wall. In addition, the wall may be subjected to axial loads in addition of the self-weight of the wall. Axial loads on the wall, when concentric, cause uniform compressive stresses over the wall cross section. Depending on the magnitude of the moment, the section of the wall may be uncracked or cracked. This is determined by comparing the axial compressive stress with the flexural tensile stress in the masonry.

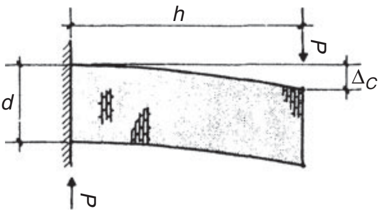


FIGURE 7.34 Visualization of shear wall as a cantilever. Tension and compression on the ends of the wall will reverse upon the reversal of the applied load.

7.10.4 Minimum Thickness Requirements for Shear Walls

There are no specific code requirements for minimum thickness of shear walls, but 6 in. is considered as a practical minimum to satisfy grouting requirements. In bearing wall systems, load-bearing walls might also be acting as shear walls. Typical wall thickness would be 8-in. nominal. Larger thickness might be required for tall structures. A discussion on available thicknesses of masonry hollow units is presented in Chap. 3. It might be preferable and economical to use higher-strength masonry rather than larger size units, which would be heavier and require more space to build. Nominal 8-in. walls have been used for examples in this chapter.

7.10.5 Design Shear Strength

The shear strength of shear walls can be determined in a manner similar to that used for beams, columns, and piers (see examples in Chaps. 4 and 5). However, to minimize the possibility of brittle failure of shear walls in structures built in high seismic regions, the design strength of such shear walls is determined a bit differently. The procedure of determining the nominal shear strength of such shear walls is specified in MSJC-08 Section 3.3.6, and explained in MSJC-08 Commentary Section 3.3.6.5.3. The model presented in the following discussion is based on 2006 IBC Section 2106.5.2. The strength reduction factor, ϕ, to be used in conjunction with nominal shear strength is 0.8 (MSJC-08 Section 3.1.4.3).

A shear wall may fail in flexure or shear. For shear walls failing in flexure but having shear strength exceeding that corresponding to the nominal flexural strength, it is assumed that a plastic hinge forms at the base of wall, which extends vertically a distance equal to the length of the wall, L_w, as shown in Fig. 7.35. As a result of this cracking, the portion of wall within this height is assumed to offer no shear resistance to the applied forces, and steel reinforcement must be designed to resist the entire shear so that $V_n = V_s$. For shear walls whose nominal shear strength exceeds the shear strength corresponding to the development of its nominal moment strength, two shear regions exist: (1) a region of plastic hinging,

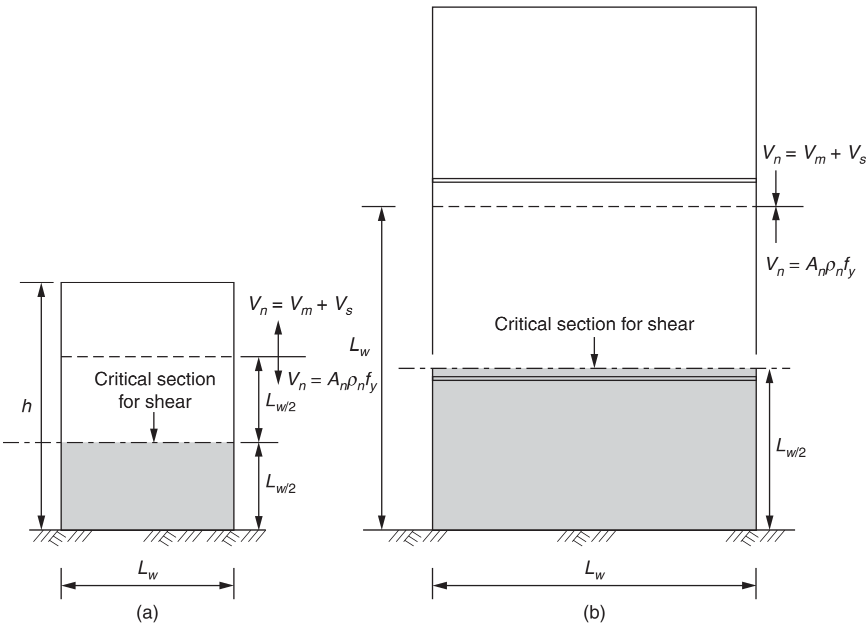


FIGURE 7.35 Determination of shear strength of masonry walls according to 2006 IBC Section 2106.5.2: (*a*) single-story and (*b*) multistory shear wall.

and (2) the other above this region. Shear strengths of these two regions are determined separately as follows:

1. For all cross sections within a region defined by the base of the shear wall and a plane at a distance L_w (wall length) above the base, the nominal shear strength is to be determined from Eq. (7.76):

$$V_n = A_n \rho_n f_y \tag{7.76}$$

where V_n = nominal shear strength, lb (kN)
$A_n = L_w t$ = net cross-sectional area of masonry, in.^{2} (mm^2)
ρ_n = ratio of distributed shear reinforcement on plane perpendicular to plane of A_n
f_y = yield strength of steel reinforcement

The value of ρ_n in Eq. (7.76) can be expressed as

$$\rho_n = \frac{\text{total area horizontal shear reinforcement}}{\text{net area of masonry}} = \frac{L_w A_v / s}{L_w t} \tag{7.77}$$

where L_w = length of wall, in. (mm)
A_v = cross-sectional area of one horizontal shear reinforcement bar
s = spacing of horizontal shear reinforcement

Substitution for ρ_n from Eq. (7.77) into Eq. (7.76) yields

$$V_n = \frac{L_w f_y A_v}{s} \tag{7.78}$$

The total factored shear force for designing the region of plastic hinging is determined at a distance equal to the *smaller* of $L_w/2$ or one-half the story height above the base of the shear wall.

2. For the region above that of plastic hinging, the nominal shear strength is determined as specified in MSJC-08 Section 3.3.4.1.2 [Eqs. (3-19) through (3-22)] and is discussed in Chap. 4. The nominal shear strength *above* the plastic hinge region, V_n, is obtained as the sum of the nominal shear strengths of masonry, V_m, and the nominal strength of reinforcement, V_s, as follows:

$$V_n = V_m + V_s \tag{7.79}$$

The nominal shear strength V_n in Eq. (7.79) is subject to the following limitations based on $M_u/V_u d_v$ ratio:

(a) Where $M_u/V_u d_v \leq 0.25$

$$V_n = 6 A_n \sqrt{f'_m} \tag{7.80}$$

(b) Where $M_u/V_u d_v \geq 1.0$

$$V_n = 4 A_n \sqrt{f'_m} \tag{7.81}$$

(c) The value of V_n may be interpolated for the condition: $0.25 < M_u/V_u d_v < 1.0$

The nominal strength of masonry, V_m, is determined from Eq. (7.82):

$$V_m = \left[4.0 - 1.75\left(\frac{M_u}{V_u d_v}\right)\right] A_n \sqrt{f'_m} + 0.25 P_u \tag{7.82}$$

In Eq. (7.82), the value of $M_u/V_u d$ must be positive and need not be taken greater than 1.0.

(d) The nominal shear strength provided by steel reinforcement is calculated from Eq. (7.83):

$$V_s = 0.5\left(\frac{A_v}{s}\right) f_y d_v \tag{7.83}$$

In the above equations $d_v = L_w$ = length of wall for all practical purposes and P_u = axial force associated with V_u

7.10.6 Other Reinforcement Requirements for Shear Walls

Various types of masonry shear walls were discussed in Section 7.10.1. All walls should be reinforced as required by design. However, it is reiterated that, with the exception of unreinforced masonry shear walls, MSJC-08 specifies minimum amount of reinforcement and detailing requirements (both are mandatory, see Figs. 7.31 through 7.33) for detailed plain reinforced masonry walls and ordinary reinforced masonry shear walls, intermediate reinforced masonry shear walls, and special reinforced masonry shear walls.

Example 7.16 Design of a shear wall.
The shear wall shown in Figure E7.16A is located in a reinforced masonry building assigned to Seismic Design Category D. It carries the following loads from the roof diaphragm: dead load = 500 lb/ft, live load = 400 lb/ft, design strength level lateral force = 15 kips at the top of the wall. The 5 percent, damped spectral acceleration parameter, S_{DS}, and the seismic response coefficient for the building, C_S, have been determined to be, respectively, 0.925 and 0.18. Assume the redundancy factor $\rho = 1.0$ and that the design shear strength of the wall is not less than 1.25 times the shear corresponding to the nominal flexural strength. Assume dead weight of wall = 84 lb/ft^2,

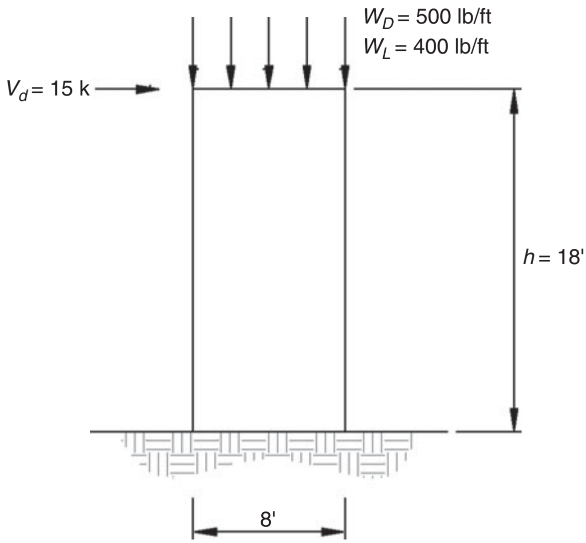


FIGURE E7.16A Loads on shear wall of Example 7.16

$f'_m = 2000$ lb/ft^2, and $f_y = 60$ ksi. Determine the horizontal and vertical reinforcement requirements for this wall.

Solution

Given:
Length of the shear wall, $L_w = 8$ ft = 96 in.
Thickness of shear wall $t = 7.625$ in. (8-in. nominal)
Story height = 18 ft

According to IBC 2006 Section 2106.5.2, the segment of the wall from its base to $L_w = 8$ ft above the base is to be provided entirely by the reinforcement. The nominal shear strength in this region is given by

$$V_n = V_s = A_n \rho_n f_y$$

V_s = total design factored shear calculated at a height above the base equal to the smaller of

$$h_v = \frac{h_s}{2} = \frac{18}{2} = 9\,\text{ft}$$

or

$$h_v = \frac{L_w}{2} = \frac{8}{2} = 4\,\text{ft}$$

$$h_v = 4 \text{ ft (governs)}$$

Weight of the wall above $h_v = 4$ ft:

$$W_{\text{wall}} = (18 - 4)(8)(84) = 9408 \approx 9.4 \text{ kips}$$

The seismic force due to weight of wall

$$V_w = C_s W_w = 0.18\ (9.4) = 1.69 \text{ kips}$$
$$V_w = V_d + V_w = 15 + 1.69 = 16.69 \text{ kips}$$

Provide No. 3 Grade 60 bars at 16-in. on centers vertically. From Eq. (7.79)

$$\begin{aligned} V_n &= A_n \rho_n f_y \\ &= L_w f_y A_v / s \\ &= (96)(60)(0.11)/16 \\ &= 39.6 \text{ kips} \end{aligned}$$

The design shear strength of the wall is not less than 1.25 times the shear corresponding to its nominal flexural strength. Therefore, the strength reduction factor for shear, $\phi = 0.8$

$$\phi V_n = (0.8)(39.6) = 31.68 \text{ kips} > V_u = 16.69 \text{ kips (satisfactory)}$$

The minimum horizontal steel area is specified to be $0.0007A_g$ (MSJC-08 Section 1.17.3.2.6).

$$\begin{aligned} A_{v,\min} &= 0.0007A_g \\ &= 0.0007(7.625)(12) = 0.064 \text{ in.}^2/\text{ft} \end{aligned}$$

With No. 3 bars at 16 in. on centers,

$$A_v = \frac{(0.11)(12)}{16} = 0.0825 \text{ in.}^2 > 0.064 \text{ in.}^2\text{/ft (required)}$$

Therefore, horizontal reinforcement provided is satisfactory.

Check maximum allowable bar spacing: Maximum spacing of horizontal reinforcement is specified to be (MSJC-08 Section 1.17.3.2.6) as the smaller of

1. One-third of the wall length,

$$s = L_w/3 = \frac{(8)(12)}{3} = 32 \text{ in.}$$

2. One-third of the wall height,

$$s = \frac{h_w}{3} = \frac{(18)(12)}{3} = 72 \text{ in.}$$

$$s = 48 \text{ in. (governs)}$$

Spacing provided = 16 in., satisfactory.

Shear above the plastic hinge zone: The strength-level shear on the shear wall at a height h_v

$$V_u = 16.69 \text{ kips (calculated earlier)}$$

The strength-level moment associated with V_u is

$$M_u = V_u(h - h_v)$$
$$= 16.69\ (18 - 4) = 234 \text{ kip-ft}$$

The strength-level axial load associated with V_u is

$$P_u = (1.2 + 0.2S_D)D$$

where D = diaphragm dead load plus weight of wall

$$P_u = [1.2 + 0.2\ (0.925)](0.5 \times 8 + 9.4)$$
$$= 18.6 \text{ kips}$$

$$\frac{M_u}{V_u d_v} = \frac{234}{(16.69)(8)} = 1.75 > 1.0 \qquad \text{use 1.0 (maximum)}$$

In the wall segment above the plastic region, the shear strength equals the sum of the contributions from masonry and reinforcement,

$$V_n = V_m + V_s$$

$$V_n = \left[4 - 1.75\left(\frac{M_u}{V_u d_v}\right)\right] A_n \sqrt{f'_m} + 0.25P_u$$

$$= (4 - 1.75 \times 1.0)(7.63 \times 96)(\sqrt{2000}) + 0.25(18{,}600)$$

$$= 78{,}354 \text{ lb}$$

$$\phi \mathrm{V_m} = 0.8(78{,}354) = 62{,}683 \text{ lb} > V_u = 16{,}690 \text{ lb} \qquad \text{satisfactory}$$

Provide only minimum horizontal reinforcement

$$A_{v,\min} = 0.0007Ag$$
$$= 0.0007\ (7.625)(12) = 0.064\ \text{in.}^2/\text{ft}$$

Provide No. 3 bars at 16 in. on centers vertically

$$A_v = \frac{(0.11)(12)}{16} = 0.0825\ \text{in}^2/\text{ft} > 0.064\ \text{in.}^2 \qquad \text{satisfactory}$$

Requirements for maximum permissible spacing are the same as determined earlier for the plastic hinge segment of the wall.

$$s_{\max} = 48\ \text{in.}$$
$$s_{\text{provided}} = 16\ \text{in.} < 48\ \text{in.} \qquad \text{satisfactory}$$

Reinforcement requirements are shown in Fig. E7.16B.

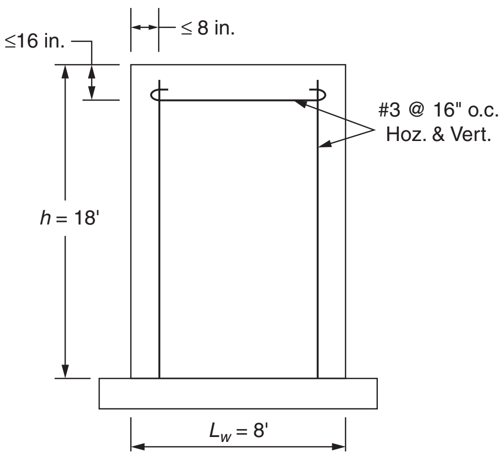


FIGURE E7.16B Reinforcement requirements.

7.11 ANALYSIS OF SHEAR WALLS UNDER FLEXURE AND AXIAL LOADS

7.11.1 Analysis of Shear Walls under Flexure

Shear walls constitute the lateral load–resisting elements in both masonry and concrete shear wall buildings (referred to as bearing wall systems in design codes), and dual systems and shear wall–frame interactive systems. By their very nature shear walls are subjected to the following concurrent forces:

1. Gravity loads, which cause axial compressive stresses in the wall.
2. Lateral loads, which cause shear stresses in the wall as well as bending moment and overturning moment in the wall, the latter being the maximum moment at the bottom of the shear wall.

In design, strength of shear walls under the two kinds of forces—axial compression and flexure—is determined separately. A wall subjected to flexure can be treated based on provisions for flexural design (discussed in Chap. 4). The fundamental assumptions underlying flexural analysis of reinforced concrete masonry elements are as follows:

1. Maximum compressive strain in masonry = 0.0025.
2. Maximum compressive stress in masonry = 0.80 f'_m.
3. Depth of equivalent compression block $a = 0.80c$ (c = distance of neutral axis from the compression force).

There are additional requirements that reinforced masonry shear walls must satisfy.

1. MSJC-08 Section 3.1.2 requires that members subject to axial load be designed for the maximum moment that can accompany the axial load. The required strength is to be determined from the factored load combination of ASCE 7-05 Section 2.3.3 (basic load combinations for strength design).
2. The flexural capacity of the wall *must* exceed the cracking moment of the section to preclude the possibility of a sudden brittle failure (masonry is a brittle material). To this effect, MSJC-08 Section 3.3.4.2.2.2 requires that the nominal flexural strength of the wall, M_n, be not less than 1.3 times of its cracking strength, M_{cr} (i.e., $M_n \geq 1.3M_{cr}$; see discussion in Chap. 4). A shear wall is treated as a vertical beam for this purpose. The cracking moment of the wall section can be determined from Eq. (7.84):

$$M_{cr} = S_n f_r \tag{7.84}$$

where S_n = section modulus of the wall section, in.^{2} (mm^2)
f_r = modulus of rupture as specified in MSJC-08 Table 3.1.8.2 (for flexural tensile stress normal to bed joints)

For a shear wall, the section modulus can be expressed as

$$S_n = \frac{tL_w^2}{6} \tag{7.85}$$

3. The amount of tensile reinforcement in a reinforced masonry shear wall is limited as specified in MSJC-08 Section 3.3.3.5.1. For masonry members where $M_u/V_u d_v \geq 1$ (typical for shear walls), the cross-sectional area of flexural tensile reinforcement shall not exceed the area required to maintain equilibrium under the following conditions:
   (a) Strain in extreme tensile reinforcement is 1.5 times the strain associated with the reinforcement yield stress f_y for the wall subjected to out-of-plane forces.
   (b) In intermediate reinforced masonry shear walls, the strain in extreme tensile reinforcement is 3 times the strain associated with the reinforcement yield stress f_y for the wall subjected to in-plane forces.
   (c) In special reinforced masonry shear walls, the strain in extreme tensile reinforcement is 4 times the strain associated with the reinforcement yield stress f_y for the wall subjected to in-plane forces.
   (d) Axial forces shall be taken from the loading combination given by Eq. (7.86) (MSJC-08 Section 3.3.3.5.1d):

$$P_u = D + 0.75L + 0.525Q_E \tag{7.86}$$

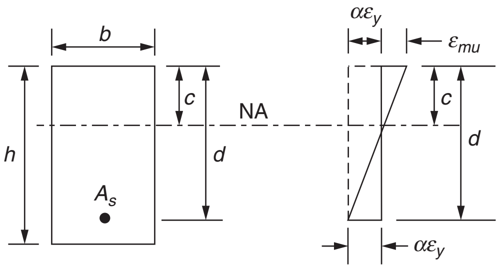


FIGURE 7.36 Strain gradient.

(e) The effect of compression reinforcement, with or without lateral restraining reinforcement, shall be permitted to be included for purposes of calculating maximum flexural tensile reinforcement.

Simple expressions can be derived to determine strains in the extreme tensile reinforcement in terms of *c*/*d* ratios. The relationship for a strain gradient corresponding to a strain equal to 1.5 times the yield strain in extreme tensile reinforcement was derived in Chap. 4 [Eqs. (4.28) through (4.36)]. Referring to similar triangles in Fig. 7.36, we obtain

$$\frac{c}{d}=\frac{\varepsilon_{\text{mu}}}{\varepsilon_{\text{mu}}+\alpha\varepsilon_y} \tag{7.87}$$

where α = strain gradient.

Substituting $\varepsilon_{\text{mu}} = 0.0025$ for concrete masonry, $\alpha = 3$, and $\varepsilon_y = 0.00207$ for Grade 60 reinforcement in Eq. (7.87), we obtain

$$\frac{c}{d}=\frac{0.0025}{0.0025+3(0.00207)}=0.287 \tag{7.88}$$

whence

$$c = 0.287d \tag{7.89}$$

Therefore, for a strain gradient corresponding to 3 times the yield strain in the extreme tensile reinforcement, $c_{\max} = 0.287d$. Similarly, substitution of $\alpha = 4$ in Eq. (7.87) yields

$$\frac{c}{d}=\frac{0.0025}{0.0025+4(0.00207)}=0.232 \tag{7.90}$$

whence

$$c = 0.232d \tag{7.91}$$

Therefore, for a strain gradient corresponding to 4 times the yield strain in the extreme tensile reinforcement, $c_{\max} = 0.232d$. An important step in determining the flexural strength of a shear wall is to find the location of neutral axis first. The procedure is akin to that for beams. However, because of many vertical bars present in a shear wall, some of which may be in compression and others in tension, locating neutral axis requires an iterative procedure based on both strain compatibility and force equilibrium. Under flexure, one end of the shear wall would be in compression and the other end in tension. It can be reasonably assumed that the bar nearest the compression end of the wall would be in compression. As a first approximation, assume that all bars, except the one in the compression zone of the wall, have yielded (in tension) so that stress in each of those bars equals f_y. Accordingly, the maximum tensile force can be expressed as given by Eq. (7.92):

$$T_{\max} = A_s f_y \tag{7.92}$$

Ignoring the concrete displaced by the reinforcing bars in the compression zone as well as the compression force in the bar in the compression zone of the wall, the maximum compression force in masonry can be expressed as given by Eq. (7.93):

$$C_{\max} = 0.80\, f'_m\, at \tag{7.93}$$

where a = depth of compression zone in masonry. Equating $C_{\max}$ to $T_{\max}$, one obtains the first approximate value of a from Eq. (7.94):

$$a = \frac{A_s t_y}{0.80 f'_m t} \tag{7.94}$$

Note that the value of a given by Eq. (7.94) is approximate because the compression force in the reinforcing bar located in the compression zone of the wall has not been accounted for in Eq. (7.93). As a result, the actual value of a would be *smaller* than that given by Eq. (7.94). Therefore, for the second trial, a smaller value of a than the one given by Eq. (7.94) should be assumed in order to determine the compression force in masonry, C, compression force in the reinforcing bar, C_s, and the tensile force resultant in tension bars, T. Note that the forces in compression and tension bars would depend on stresses in those bars which can be determined from strain distribution across the wall cross section, which is defined by ultimate strain in masonry (0.0025 and 0.0035 for concrete and clay masonry, respectively) and the position of neutral axis given by the relationship: $a = 0.80c$ [Chap. 4, Eq. (4-5*a*)]. Stresses in various bars can be determined from strains in those bars (stresses in compression and tension bars nearest the neutral axis, in all likelihood, would be less than yield stress, f_y). The correct value of a must satisfy the force equilibrium condition expressed by Eq. (7.95):

$$\sum F_y = 0$$

$$C + C_s - T = 0 \tag{7.95}$$

Eq. (7.95) cannot be solved directly because

1. The total compression force $(C + C_s)$ depends on the value of a which, in turn, depends on the value of c (because $a = 0.80c$). Any variation in the value of a affects both C and C_s.
2. The total tensile force T also depends on the value of a. Note that the strain in the tension bar closest to neutral axis might be smaller than the yield strain.

Accordingly, a few iterations might be required before force equilibrium condition is satisfied. See Example 7.17. Readers should review Examples 5.9 through 5.11 (Chap. 5) for an overview of this procedure.

Walls under axial loads can be treated using provisions discussed in Chap. 5. It is reiterated that shear walls are invariably subjected to out-of-plane loads. Such walls are subject to the following restrictions where the factored axial stress exceeds $0.05 f'_m$ (MSJC-08 Section 3.3.5.3).

1. Factored axial load stress ≤ 0.2 f'_m
2. Slenderness ratio (h/t ratio) ≤30
3. Minimum nominal thickness ≥6 in. (Not a code requirement, but a practical minimum.)

It is, therefore, only logical that a limit to h/t = 30 be maintained for shear walls. Expressions for axial loads on compression elements were derived in Chap. 5. It was shown that for rectangular sections,

$$r = \frac{t}{\sqrt{12}} \tag{5.16 repeated}$$

so that

$$t = r\sqrt{12} \tag{7.96}$$

Therefore, the condition $h/t = 30$ can be expressed in terms of h/r ratio as follows:

$$\frac{h}{t} = \frac{h}{r\sqrt{12}} = 30$$

so that

$$\frac{h}{r} = 30\sqrt{12} = 103.923 \tag{7.97}$$

Equation (7.97) can be treated as the upper limit for h/r ratio for a shear wall. Accordingly, the nominal axial strength of a shear wall can be expressed, based on MSCJC-08 Section 3.3.4.1, Eq. (3.17) [see Chap. 5, Eq. (5.11)]:

$$\begin{aligned} P_n &= 0.80[0.80f'_m(A_n - A_s) + f_yA_s]\left(\frac{70r}{h}\right)^2 \\ &= 0.80[0.80f'_m(A_n - A_s) + f_yA_s]\left(\frac{70}{103.923}\right)^2 \\ &= 0.363[0.80f'_m(A_n - A_s) + f_yA_s] \end{aligned} \tag{7.98}$$

Equation (7.98) can be treated as the limiting value of nominal axial capacity of reinforced masonry walls.

The strength factor ϕ to be used for axial loads, axial loads with flexure, or flexure, is 0.9 (MSJC-08 Section 3.1.4.1). Example 7.17 presents analysis of a shear wall under flexure.

Example 7.17

Calculate the flexural capacity of the 8-in. (nominal) shear wall having cross section as shown in Fig. E7.17A. The wall constructed from 8 × 8 × 16 (nominal) concrete masonary units (CMUs) measures 18 ft. 8 in. end-to-end and reinforced with 10 No 6 Grade 60 bars spaced at 24 in. on centers. The compressive strength of masonry is 2000 lb/in.^{2}.

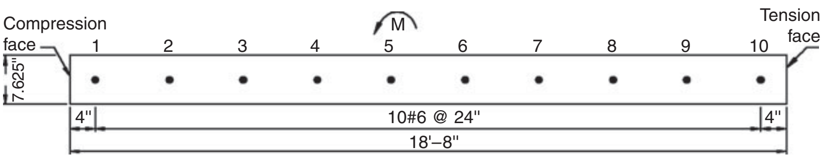


FIGURE E7.17A Reinforced masonry shear wall subjected to flexure.

Solution

Given:

Length of the shear wall L_w = 18 ft. 8 in. = 224 in.

Thickness of shear wall t = 7.625 in. (8-in. nominal)

The depth of compression block a would be determined by an iterative procedure. There are 10 vertical bars in the wall. Assume that one bar (nearest the compression face of wall) is in compression and other nine bars are in tension, and assume also that all nine bars have yielded so that the stress in each is f_y (= 60 ksi).

Trial 1: Ignore compression force in the reinforcing bar in the compression zone of the wall as well as the area displaced by the compression reinforcing bars.

$$C_{\max} = 0.80 f_m' at$$
$$= 0.80(2.0)(7.625)a = 12.2\,a \text{ kips}$$

For 9 No. 6 bars,

$$A_s = 3.98 \text{ in.}^2$$
$$T_{\max} = A_s f_y = 3.98\ (60)$$
$$= 3.98\ (60) = 238.8 \text{ kips}$$

Equating $C_{\max}$ to $T_{\max}$, we have

$$12.2a = 238.8 \text{ kips}$$
$$a = \frac{238.8}{12.2} = 19.57 \text{ kips}$$

The calculated value of a = 19.5 in. ignores the compression force in the bar nearest the compression face. Obviously, the correct value of a would be smaller than 19.57 in.

Trial 2: Assume a = 18 in.

$$c = \frac{a}{0.80} = \frac{18}{0.80} = 22.5 \text{ in.}$$

Calculate forces in masonry and reinforcing bars based on compatibility of strains as shown in Fig. E7.17B.

$$\varepsilon_{s1} = \left(\frac{18.5}{22.5}\right)(0.0025) = 0.00206 < \varepsilon_y = 0.00207\ (f_s < f_y)$$

$$\varepsilon_{s2} = \left(\frac{1.5}{22.5}\right)(0.0025) = 0.00017 < \varepsilon_y = 0.00207\ (f_s < f_y)$$

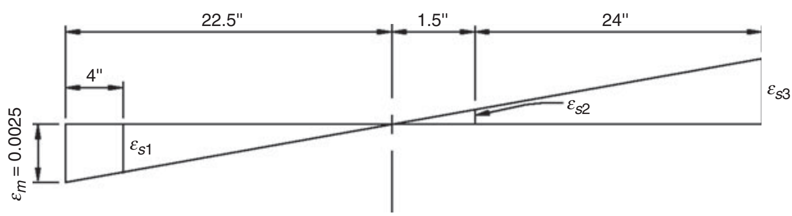


FIGURE E7.17B Strain distribution for Trial 2 iteration.

From observation,

$$\varepsilon_{s3} > \varepsilon_m = 0.0025 > \varepsilon_y = 0.00207$$

Strains in bars 3 through 10 have strains greater than the yield strain. Therefore, stress in each of those bars equals f_y (= 60 ksi). Calculate forces in bars 1 and 2 based on strains in those bars.

Bar 1:

$$f_s' = \varepsilon_s E_s = (0.00206)(29{,}000) = 59.74 \text{ ksi (compressive)}$$
$$C_s' = A_s' f_s' = 0.44(59.74) = 26.29 \text{ kips (compression)}$$

Bar 2:

$$f_{s2} = \varepsilon_{s2} E_s = (0.00017)(29{,}000) = 4.93 \text{ ksi (tensile)}$$
$$T = 0.44\ (4.93) = 2.2 \text{ kips (tension)}$$

Compression force in masonry:

$$C = 0.80\ f_m' at = 0.80\ (2.0)(18)(7.625) = 219.6 \text{ kips}$$

For 8 No. 6 bars in tension, $A_s = 3.53$ in.^{2}.

$$T = 3.53(60) = 211.8 \text{ kips}$$

Total force in tension, $\Sigma T = 2.2 + 211.8 = 214$ kips
Check equilibrium: $\Sigma F_y = 0$

$$C + C_s - T = 219.6 + 26.29 - 214 = 31.9 \text{ kips} \neq 0$$

Therefore, a should be smaller than 18 in. as assumed.
Trial 3: Assume $a = 14$ in.

$$c = \frac{a}{0.80} = \frac{14}{0.80} = 17.5 \text{ in.}$$

Calculate forces in masonry and reinforcing bars (as in Trial 2) based on strain compatibility. The strain distribution is shown in Fig. E7.17C.

$$\varepsilon_{s1} = \left(\frac{13.5}{17.5}\right)(0.0025) = 0.00193 < \varepsilon_y = 0.00207\ (f_s < f_y)$$

$$\varepsilon_{s2} = \left(\frac{6.5}{17.5}\right)(0.00205) = 0.00093 < \varepsilon_y = 0.00207\ (f_s < f_y)$$

From observation,

$$\varepsilon_{s3} > 0.0025 > \varepsilon_y = 0.00207\ (f_s = f_y)$$

Stresses in all remaining bars in tension would be equal to $f_y = 60$ ksi.

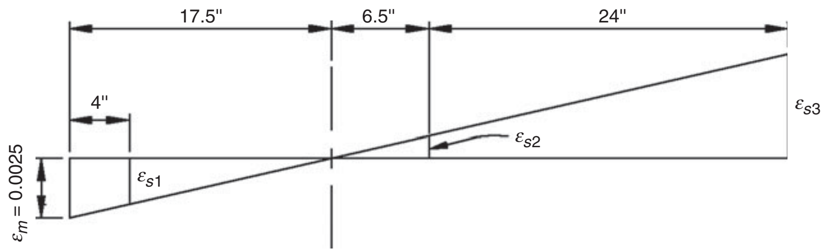


FIGURE E7.17C Strain distribution for Trial 3 iteration.

Bar 1:

$$f_s' = \varepsilon_{s1} E_s = (0.00193)(29{,}000) = 55.97 \text{ ksi (compression)}$$

$$C_s = A_s' f_s' = 0.44(55.97) = 24.63 \text{ kips (compression)}$$

Bar 2:

$$f_{s2} = \varepsilon_{s2} E_s = (0.00093)(29{,}000) = 26.97 \text{ ksi (tension)}$$

$$\text{Tensile force in Bar 2} = A_s f_{s2} = 0.44(26.97) = 11.87 \text{ kips (tension)}$$

Compression force in masonry:

$$C = 0.80\, f_m' at = 0.80\,(2.0)(14)(7.625) = 170.80 \text{ kips}$$

$$\text{Total compression force} = C + C_s = 170.80 + 24.63 = 195.43 \text{ kips.}$$

$$\text{Total tensile force} = 11.87 + 3.53(60) = 223.67 \text{ kips} > (C + C_s)$$

Check equilibrium: $\Sigma F_y = 0$

$$C + C_s - T = 170.80 + 24.63 - 223.67 = -28.24 \text{ kips} \neq 0$$

Therefore, $14 \text{ in.} < a < 18 \text{ in.}$

Trial 4: Assume $a = 16$ in.

$$c = \frac{a}{0.80} = \frac{16}{0.80} = 20 \text{ in.}$$

Calculate forces in masonry and reinforcing bars as before. The strain distribution diagram is shown in Fig. E7.17D

$$\varepsilon_{s1} = \left(\frac{16}{20}\right)(0.0025) = 0.0020 < \varepsilon_y = 0.00207 \;(f_s < f_y)$$

$$\varepsilon_{s2} = \frac{4}{20}(0.00205) = 0.0005 < \varepsilon_y = 0.00207 \;(f_s < f_y)$$

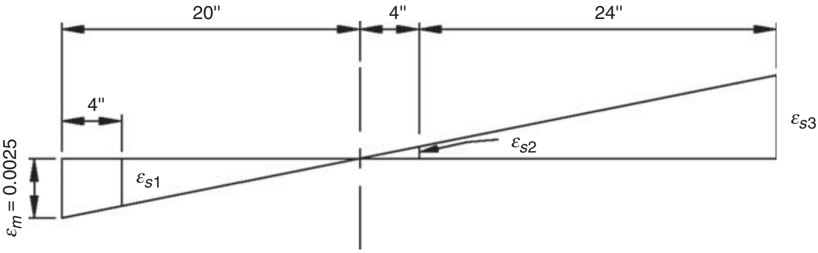


FIGURE E7.17D Strain distribution for Trial 4 iteration.

From observation,

$$\varepsilon_{s3} > 0.0025 > \varepsilon_y = 0.00207 \ (f_s = f_y)$$

Stresses in all remaining bars in tension would be equal to $f_y = 60$ ksi.
Bar 1:

$$f_s' = \varepsilon_{s1} E_s = 0.0020(29{,}000) = 58 \text{ ksi (compressive)}$$

$$C_s = A_s' f_s' = 0.44(58) = 25.52 \text{ kips (compression)}$$

Bar 2:

$$f_{s2} = \varepsilon_{s2} E_s = 0.0005(29{,}000) = 14.5 \text{ ksi (tensile)}$$

$$\text{Tensile force in the bar} = 0.44(14.5) = 6.38 \text{ kips}$$

Tensile force in bars 3 through 10,

$$T = 3.53(60.0) = 211.8 \text{ kips}$$

$$C = 0.80\ f_m' at = 0.80\ (2.0)(16)(7.625) = 195.2 \text{ kips}$$

$$\text{Total compression force} = C + C_s = 195.2 + 25.52 = 220.72 \text{ kips.}$$

$$\text{Total tensile force } \sum T = 6.38 + 211.8 = 218.18 \text{ kips}$$

Check force equilibrium: $\sum F_y = 0$

$$C + C_s - T = 220.72 - 218.18 = 2.54 \text{ kips} \neq 0$$

The difference, 2.54 kips, is negligible (about 1 percent). Therefore, a = 16 in. is acceptable, so that c = 20 in. as assumed. The nominal moment strength of the wall, M_n, can be determined by taking moments about the center line of the wall as shown in the force diagram (Fig. E7.17E). Thus,

$$\begin{aligned} M_n &= 25.52(108) + 195.2(104) - 6.38(84) - 26.4(60 + 36 + 12) \\ &\quad + 26.4(12 + 36 + 6 + 84 + 108) \\ &= 27569.04 \text{ k-in.} \approx 2297 \text{ k-ft} \end{aligned}$$

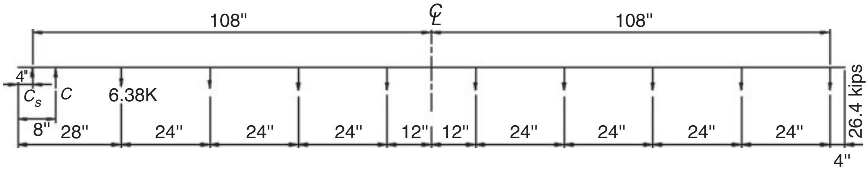


FIGURE E7.17E Force diagram.

Check that $M_n \geq 1.3\ M_{cr}$

$$M_{cr} = S_n f_r$$

Thickness of wall = 7.625 in.

Length of wall L_w = 18 ft 8 in. = 224 in.

$$S_n = \frac{tL_w^2}{6} = \frac{(7.625)(224)^2}{6} = 63{,}675 \text{ in.}^3$$

f_r = 163 lb/in.^{2} (MSJC-08 Table 3.1.8.2, Type M or S Portland cement/lime or mortar cement for flexural tensile stress normal to bed joints in fully grouted masonry in running or stack bond)

$$M_{cr} = \frac{(63{,}765)(163)}{12{,}000} = 860.8 \text{ k-ft}$$

$$1.3M_{cr} = 1.3(860.8) = 1119 \text{ k-ft}$$

$$M_n = 2297 \text{ k-ft} > 1.3M_{cr} = 1119 \text{ k-ft}$$

$$\phi M_n = 0.9(2297) = 2067 \text{ k-ft}$$

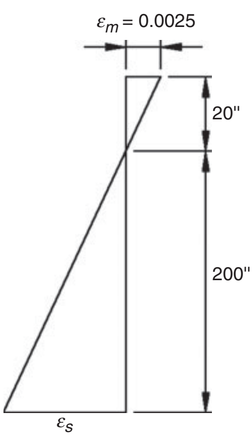


FIGURE E7.17F Strain distribution diagram.

Check strain in the extreme tension reinforcing bars.

$$c = 20 \text{ in.} \qquad d - c = 220 - 20 \text{ in.}$$

From strain distribution diagram (Fig. E7.17F),

$$\frac{\varepsilon_s}{\varepsilon_m} = \frac{d-c}{c} = \frac{200}{20} = 10$$

$$\varepsilon_s = 10\ \varepsilon_m = 10\ (0.0025) = 0.025$$

$$4\varepsilon_y = 4(0.00207) = 0.00828$$

$$\varepsilon_s > 4\varepsilon_y$$

The percentage of steel in the wall is

$$\rho = \left(\frac{4.42}{(224)(7.65)}\right)(100) = 0.259 \text{ percent}$$

7.11.2 Flexural Capacity of Axially Loaded Shear Walls

Flexural capacity of axially loaded shear walls can be determined in the same manner as the flexural capacity of shear walls without axial loads (as in Example 7.17). The effect of an axial load on a shear wall can be taken into account as in the case of flexural capacity of columns (see discussion and Examples 5.9 to 5.11 in Chap. 5).

Determination of the flexural capacity of an axially loaded shear wall requires first determining the location of neutral axis in the wall cross section. This is an iterative procedure as illustrated in Example 7.18. However, it is noted that the presence of the axial load on the wall would cause the depth of compression block a to be larger than if the axial load were not present. The calculated value of a is correct when the equilibrium condition given by Eq. (7.99) is satisfied:

$$\sum F_y = 0$$

$$C + C_s - T - P_u = 0 \tag{7.99}$$

where C = compression force in masonry
C_s = compression force in reinforcing bars
T = tension force in reinforcing bars
P_u = factored axial load on shear wall

The factored axial load, P_u, is to be taken from the loading combination given by Eq. (7.86)

$$D + 0.75L + 0.525Q_E, \text{ (MSJC-08 Section 3.3.3.5.1d)}$$

where Q_E = effect from horizontal earthquake force, V or F_P [see Eq. (7.42)].

Example 7.18 Flexural capacity of an axially loaded shear wall. Calculate the flexural capacity of the shear wall described in Example 7.17 when it carries a factored axial load of 465 kips.

Solution

The depth of compression block a would be determined by an iterative procedure.

Trial 1: Based on the fact that some bars (closer to the compression face of the wall) would be in compression, whereas the remaining bars would be in tension, assume that the neutral axis passes through the third bar from the compression face. Therefore, strain (and, hence, the force) in that bar would be zero. Calculate forces in masonry and reinforcing bars based on strain distribution across the wall cross section (Fig. E 7.18A)

$$c = 52 \text{ in.}$$

$$\varepsilon_{s1} = \left(\frac{48}{52}\right)(0.0025) = 0.00231 > \varepsilon_y = 0.00207 (f_s = f_y)$$

$$\varepsilon_{s2} = \left(\frac{24}{52}\right)(0.0025) = 0.00115 < \varepsilon_y = 0.00207 (f_s < f_y)$$

$$f_{s2} = \varepsilon_{s2} E_s = (0.00115)(29{,}000) = 33.35 \text{ ksi} \quad \text{(compression)}$$

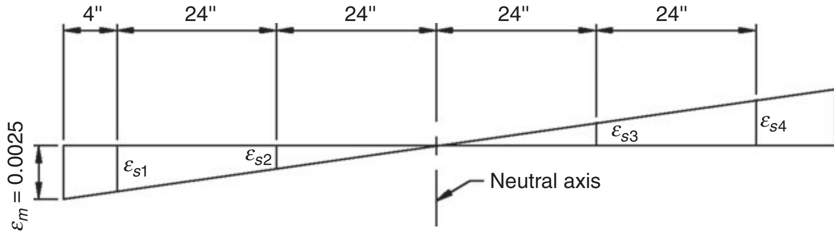


FIGURE E7.18A Flexural strength of an axially loaded shear wall.

All bars are equally spaced at 24 in. on center. Therefore, strain in Bars 2 and 3 are equal:

$$\varepsilon_{s3} = \varepsilon_{s2} = 0.00115$$
$$f_{s3} = f_{s2} = 33.35 \text{ ksi}$$

From strain distribution diagram:

$$\varepsilon_{s4} = 2\varepsilon_2 = 2(0.00115) = 0.0023 > \varepsilon_y = 0.00207 \ (f_s = f_y)$$

Compression force in Bar 1,

$$C_{s1} = A_s' f_y = 0.44(60) = 26.4 \text{ kips}$$

Compression force in Bar 2,

$$C_{s2} = A_s' f_{s2} = 0.44(33.35) = 14.67 \text{ kips}$$

Depth of compression block,

$$a = 0.80c = 0.80(52) = 41.6 \text{ in.}$$

Compression force in masonry,

$$\begin{aligned} C &= 0.80 f_m' at \\ &= 0.80(2.0)(41.6)(7.625) = 507.52 \text{ kips} \end{aligned}$$

Tension force in Bar 3,

$$T_3 = A_s f_{s3} = 0.44(33.35) = 14.67 \text{ kips}$$

Tension force in Bars 4 through 10,

$$T = 0.44(60) = 26.4 \text{ kips (each bar)}$$

Check force equilibrium: $\Sigma F_y = 0$

$$C_s + C - T - P = 0$$

$$26.4 + 14.67 + 507.52 - 14.67 - 7(26.4) - 465 \cong -115.88 \text{ kips} \neq 0$$

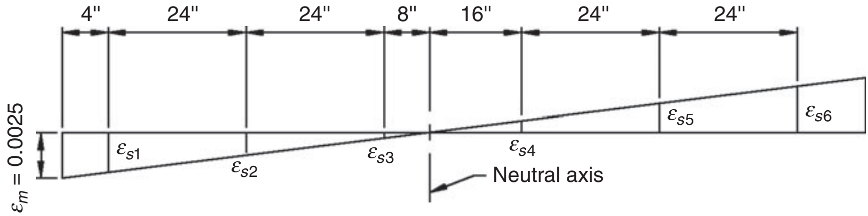


FIGURE E7.18B Strain distribution for Trial 2 iteration.

Trial 2: The result of Trial 1 shows that *a* should be increased (i.e., compression force should be increased to achieve equilibrium). Assume that c = 60 in. (was 52 in. in Trial 1); the corresponding strain distribution across the wall cross section is shown in Fig. E7.18B. Calculate forces in masonry and reinforcing bars based on strain distribution across the cross section of wall.

$$\varepsilon_{s1} = \left(\frac{56}{60}\right)(0.0025) = 0.00233 > \varepsilon_y = 0.00207\ (f_s = f_y)$$

$$f_{s1} = f_y = 60 \text{ ksi (compressive)}$$

$$C'_{s1} = A'_s f_{S1} = 0.44(60) = 26.4 \text{ kips (compression)}$$

$$\varepsilon_{s2} = \left(\frac{32}{60}\right)(0.0025) = 0.00133 < \varepsilon_y = 0.00207\ (f_{s2} < f_y)$$

$$f_{s2} = \varepsilon_{s2} E_s = (0.001133)(29{,}000) = 38.57 \text{ ksi (compressive)}$$

$$C'_{s2} = A'_s f_{S2} = 0.44(38.57) = 16.97 \text{ kips (compression)}$$

$$\varepsilon_{s3} = \left(\frac{8}{60}\right)(0.0025) = 0.00033 < \varepsilon_y = 0.00207\,(f_s < f_y)$$

$$F_{s3} = \varepsilon_{s3} = E_s = (0.00033)(29{,}000) = 9.57 \text{ ksi (compression)}$$

$$C'_{s3} = A'_S f_{S3} = 0.44(9.57) = 4.21 \text{ kips}$$

$$\varepsilon_{s4} = \left(\frac{16}{60}\right)(0.0025) = 0.00067 < 0.00207\,(f_{s4} < f_y)$$

$$f_{s4} = \varepsilon_{s4} E_s = (0.00067)(29{,}000) = 19.43 \text{ ksi} \quad \text{(tensile)}$$

$$\varepsilon_{s5} = \left(\frac{64}{60}\right)(0.0025) = 0.00267 > 0.00207\,(f_{s4} = f_y)$$

Compression force in masonry,

$$C = 0.80 f'_m at = 0.80(2.0)(46)(7.625) = 585.6 \text{ kips}$$

$$\text{Tension in Bar 4} = A_s f_s = 0.44(19.43) = 8.55 \text{ kips}$$

$$\text{Tension in Bar 5 through 10} = A_s f_y = 0.44(60) = 26.4 \text{ kips (each bar)}$$

Check equilibrium:

$$\sum F_y = 0$$

$$C_s + C - T - P = 0$$

$$26.4 + 16.97 + 4.21 + 585.6 - 8.55 - 6(26.4) - 465 = 1.23 \text{ kips}$$

The difference of 1.23 kips is negligible; therefore, c = 60 in. is acceptable. Calculate the moment capacity by taking moments of all forces (see Fig. E7.18C) about the centroid of the section (so that moment due to axial load is zero).

$$M_n = 26.4(108) + 16.97(84) + 4.21(60) - 8.55(36) - 24.6(12)$$

$$+ 26.4(12 + 36 + 60 + 84 + 108) + 585.6(112 - 24)$$

$$= 62{,}830.28 \text{ k-in.}$$

$$\phi M_n = 0.9(62{,}830.28) = 56{,}547.25 \text{ k-in.} \approx 4712 \text{ k-ft}$$

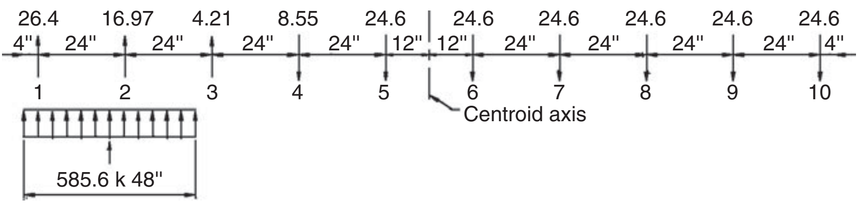


FIGURE E7.18C Force diagram.

Commentary: It is noted that the moment capacity of the wall is 4712 k-ft when an axial load of 465 kips is applied to it, which is considerably greater than the moment capacity of the same wall without the axial load (2067 k-ft).

7.12 DESIGN OF MULTISTORY SHEAR WALLS

Multistory shear walls are designed using the same procedures as discussed in Section 7.11 for one-story shear walls except that shear and overturning and moments have to be calculated for every floor level. Both shear and overturning moments, as well as axial load on the wall, increase progressively as one proceeds from Level *n* of a building to the base. The overturning moment at the base can be easily calculated once the story forces are determined as discussed earlier. The overturning moment at the base can be calculated from Eq. (7.100):

$$\text{OTM}_{\text{base}} = \sum_{i=1}^{n} F_i h_i \tag{7.100}$$

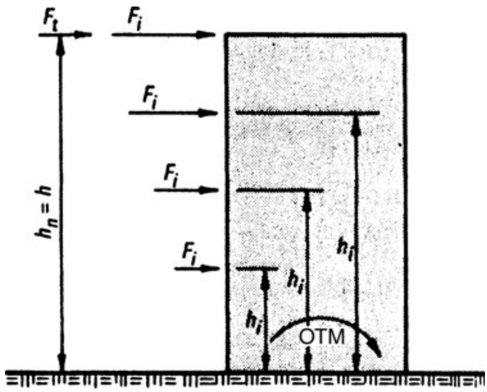


FIGURE 7.37 Overturning moment at base of a multistory building.

The lever arms for all these forces are measured from the base (see Fig. 7.37). The overturning moment at any level x above the base can be obtained in a similar manner, except that the lever arms for various story forces above level x are measured form level x (Fig. 7.38). Thus,

$$\mathrm{OTM}_{\text{level}\,x} = \sum_{i=1}^{n} F_i (h_i - x) \tag{7.101}$$

Since shear, moment, and axial loads on shear walls become progressively greater at lower levels, so do shear, flexural, and axial stresses in walls at those levels. To keep flexural and/or shear stresses within the allowable limits, it may be necessary to increase the wall thickness, use high-strength masonry, or both, for the lower-level shear walls.

For a wall of a given length, shear, flexural, and compressive strength are functions of the wall thickness. Consequently, these strengths can be increased by increasing the wall

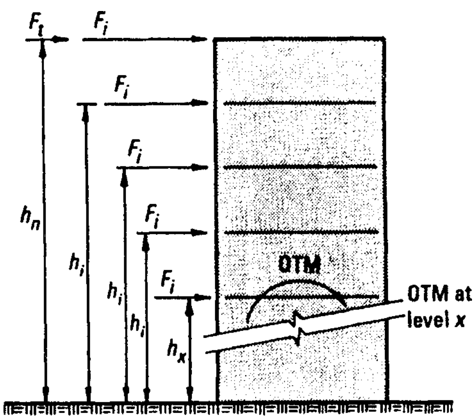


FIGURE 7.38 Overturning moment at any level x in a multistory building.

thickness. The flexural strength of shear walls can be greatly enhanced by planning them as flanged sections. This can be accomplished easily by considering returns at the ends of the wall (e.g., near doors) as flanges. Addition of flanges to a rectangular section of a shear wall (and thus resulting in a flanged section) increases its section modulus resulting in considerably reduced flexural stresses due to overturning moment. Flanges facilitate placement of tensile reinforcing steel (all of which can be placed in the flange) that results in increased lever arm which, in turn, results in reduced amount of tension steel. They also provide the wall with increased cross-sectional area to resist compressive loads. The concept of shear walls with flanged section can be utilized for longitudinal as well as cross walls. The flanges can be single return or double returns, resulting in L, T, I, or channel shapes (Fig. 7.11*a*). Note that the effective length of flanges on either side of the web is limited by general provisions specified in MSJC-08 Section 1.9.4.2 as follows:

1. The masonry shall be in running bond.
2. The width of flange considered effective on each side of the web shall be smaller of the actual flange on either side of the web wall or the following:
   (a) 6 times the nominal flange thickness, when the flange is in compression
   (b) 6 times the nominal flange thickness for unreinforced masonry, when the flange is in flexural tension.
   (c) 0.75 times multiplied by the floor-to-floor wall height for reinforced masonry, when the flange is in flexural tension.
3. The effective flange width shall not extend past a movement joint.

While the flanged sections provide enhanced flexural capacity to resist overturning moments, it should be recognized that a combination of shear walls with rectangular and flanged sections may result in horizontal torsional eccentricity as shown in Fig. 7.39, which must be accounted for in design as discussed earlier.

7.13 FAILURE MODES OF SHEAR WALLS

The behavior and failure modes of shear walls depend on many factors which include the type of loading, aspect ratio, type of construction (e.g., single wythe or multiwythe, reinforced or unreinforced, single story or multistory, coupled and uncoupled), properties of materials, and reinforcement detailing, to name a few. Each of these plays a part in the behavior of shear walls under loads and consequent failure modes. Masonry, reinforced or unreinforced, is inherently brittle in nature. The brittle failure modes in masonry structures are manifested by bed-joint slip, loss of anchorage, diagonal tension failure, and crushing at toe of wall.

In general, four types of failure modes may be identified: shear failure, flexural failure, rocking failure, and sliding failure. The shear failure is indicated by typical diagonal-tension cracks as seen in the elevation of a shear wall. Because of the reversible nature of seismic loads, these diagonal cracks are bidirectional (Figs. 7.40, 7.41). Flexural cracks are manifested by cracking along the bed joints, yielding of vertical tensile reinforcement, and crushing of the toe of the shear wall (Fig. 7.40).

Failure may be caused by uplift at one end of a shear wall (also called *rocking*) due to combined action of axial compressive loads and overturning moment. Failure of shear walls may also occur by sliding across the construction joints or the bed joints, which can occur when the applied shear exceeds the shear-slip resistance along the bed joints. Each of these failure types can be prevented by proper design and detailing, both of which are key to developing sufficient ductility in shear walls that would ensure good dissipation of

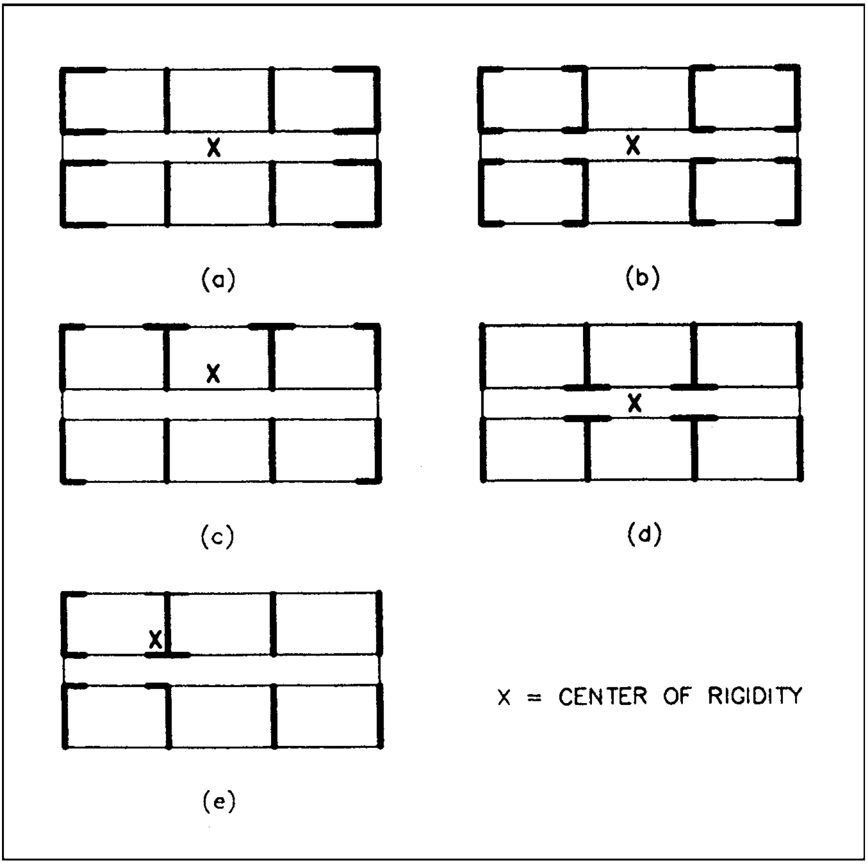


FIGURE 7.39 Approximate positions of center of rigidity for shear walls of mixed cross sections [7.3].

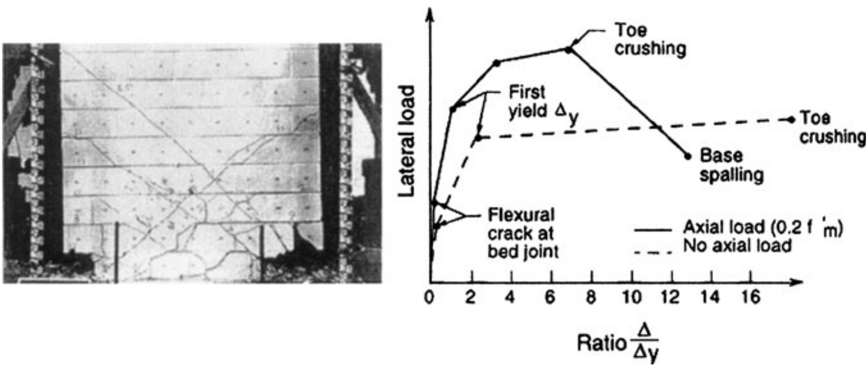


FIGURE 7.40 Flexure failure mode (*Courtesy of P. Shing*) [7.17].

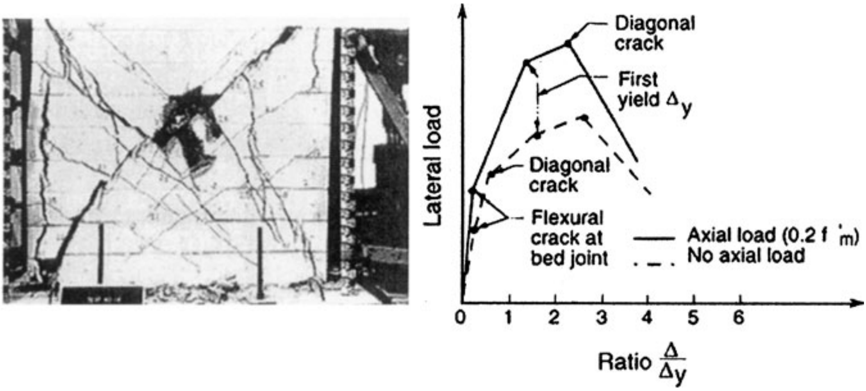


FIGURE 7.41 Shear failure mode (*Courtesy of P. Shing*) [7.18].

energy during seismic ground shaking. Tests on reinforced masonry shear walls conducted at University of Colorado as a part of the TCCMAR program [7.17–7.19] concluded that the amount of horizontal reinforcement (which acts as shear reinforcement) has significant influence on the cracking pattern, shear strength, and ductility.

Failure modes in multistory shear walls depend on the type of construction, that is, whether they are solid, perforated, linked, or coupled (Fig. 7.9). Figure 7.42 shows response of multistory reinforced masonry shear walls subjected to seismic lateral loads. As discussed in Ref. 7.20, the ductility capacity of a shear wall will depend primarily on the structural form and the ultimate capacity of the wall. To ensure adequate ductility, the preferred structural form is the simple cantilever wall. When two or more walls occur in the same plane, linkage between them should be provided by flexible floor slabs (links) to ensure that moment transfer between the connected shear walls is minimized. It is necessary to keep the openings in the wall rather small so as not to affect the behavior of the wall as a basic cantilever. Energy dissipation, very important for shear walls in seismic zones, occurs only in carefully detailed plastic hinges at the base of each wall (Fig. 7.42*a*).

It is common to have openings for doors and windows in the peripheral masonry shear walls of multistory buildings (Fig. 7.10). Such openings affect the rigidity of these walls. Under inelastic response to seismic loading, hinging may initiate in piers (the vertical elements, Fig. 7.42*b*) or in spandrels (the horizontal elements, Fig. 7.38*c*). In the former,

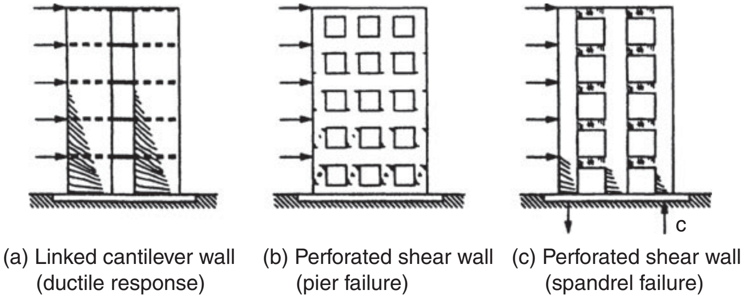


FIGURE 7.42 Failure modes of multistory shear walls subjected to seismic lateral loads: (*a*) linked shear wall (ductile response), (*b*) perforated shear wall (pier failure), (*c*) perforated shear wall (spandrel failure) [7.1].

plastic displacement will inevitably be concentrated in the piers of one story, generally the lowest. The strength degradation of piers will affect the stability of the entire shear wall. Therefore, the piers should be designed to exhibit substantial ductility and stronger than spandrels to avoid this type of failure mode. The energy dissipation will cause the spandrels to sustain damage through cracking as shown in Fig. 7.43*c* without compromising the stability of the wall. Walls with a regular pattern of large openings may be designed as a moment-resisting frame using the strength design concept.

(a)

(b)

FIGURE 7.43 Failure of shear walls in the January 17, 1994, M 6.7 Northridge, California earthquake: (*a*) torsional failure due to horizontal torsional eccentricity, (*b*) crushing of toe of a shear wall, (*c*) failure of piers in a multistory shear wall. (*Courtesy: Author.*)

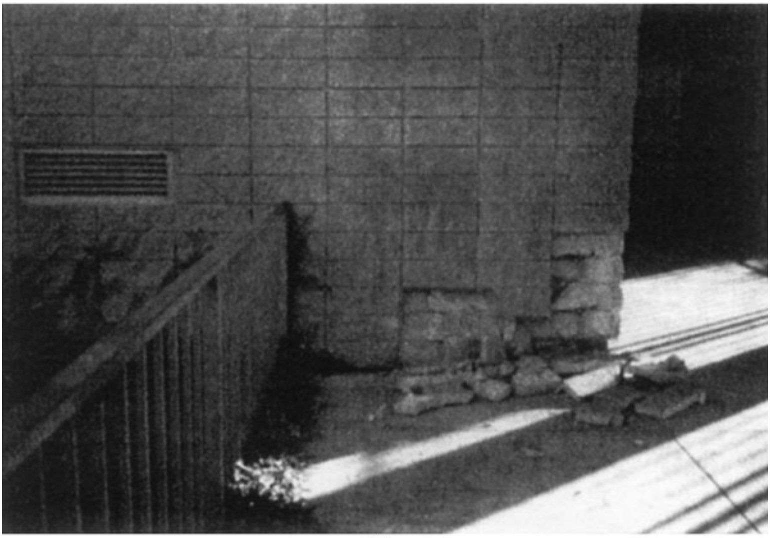

(c)

FIGURE 7.43 (*Continued*)

Considerable insight in the failure modes of multistory shear walls was gained from the TCCMAR test on a five-story, full-scale, reinforced masonry building subjected to simulated seismic loading [7.21]. The building consisted of flanged walls, with door openings and prestressed concrete hollow core planks with 2 in. concrete topping. The door openings were spanned by nonstructural lintels that were separated from the adjacent walls by movement joints. Walls were linked by the concrete floor slabs. The building was designed according to the limit states design approach so as to have, under the most severe earthquake loading, a displacement ductility of at least 4, dissipate energy through inelastic flexural deformation, and avoid all brittle modes of failures such as diagonal tension and shear slip. The building behaved and failed in a ductile manner. The inelastic deformations were limited to designated zones. The failure occurred as intended, through formation of plastic hinges at the bases of the first-story piers and at the coupling slab ends. Several significantly important lessons were learnt from this test:

1. Desirable ductile structural response of reinforced masonry buildings suitable in areas of high seismic ground shaking can be ensured by proper design and detailing.
2. The key to achieving ductile response is to provide sufficient shear reinforcement to preclude shear failures in piers, coupling elements, and joints.
3. Bond failure of reinforcement can be eliminated by proper selection of splice locations, development length, joint dimensions, and reinforcement size.

A discussion on various types of failure modes can be found in the literature [7.1, 7.6, 7.17–7.22]. Many of the failure modes of reinforced and unreinforced masonry shear walls were evident in the January 17, 1994, M 6.7 Northridge, California earthquake which was one of the most damaging earthquakes in the history of the Unites States. Figure 7.43 illustrates some typical failures of shear walls in that earthquake.

Problems

7.1 Figure P7.1 shows a cantilevered masonry shear wall which is 8 in. (nominal) thick. Calculate its rigidity if $f'_m = 1500$ lb/in.^{2}.

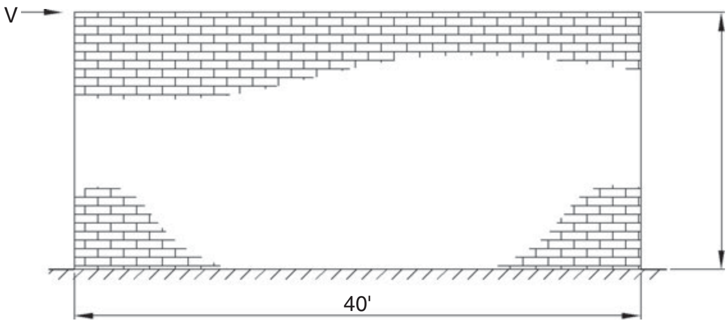


FIGURE P7.1

7.2 Calculate the relative rigidity of the cantilevered shear wall described in Prob. 7.1.

7.3 Calculate the rigidity and relative rigidity of the shear wall described in Fig. P7.1 assuming it to be fixed at both top and bottom.

7.4 Calculate the rigidity of an 8-in. (nominal) thick concrete masonry shear wall that has an opening as shown in Fig. P7.4. Assume that $f'_m = 1500$ lb/in.^{2}.

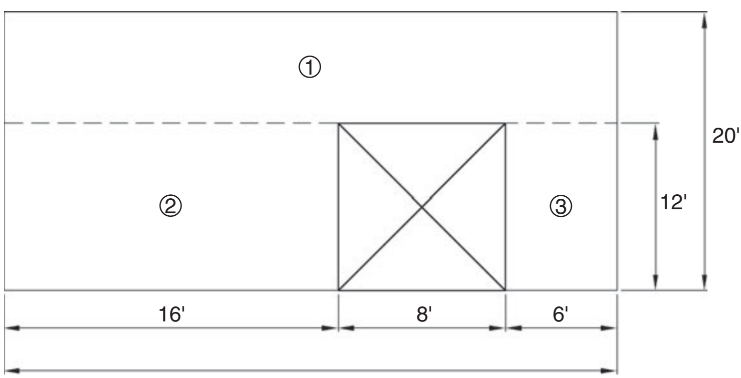


FIGURE P7.4

7.5 The masonry wall shown in Fig. P7.5 has a uniform nominal thickness of 8 in., and is a part of a one-story building. (a) Determine the relative rigidity of the wall by Method A; (b) describe the load path in the shear wall and shear force in each pier. Assume the compressive strength of masonry, $f'_m = 1500$ lb/in.^{2}.

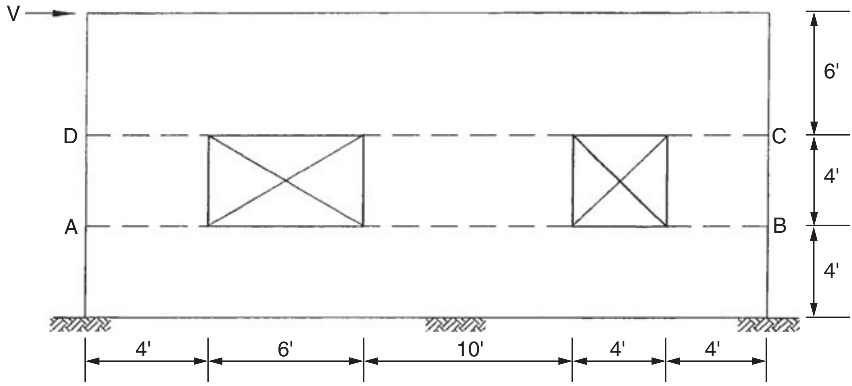


FIGURE P7.5

7.6 Figure P7.6 shows an 8-in.-thick (nominal) reinforced concrete masonry wall. Calculate its rigidity by the three methods discussed earlier. Assume the compressive strength of masonry, $f'_m = 1800$ lb/in.^{2}.

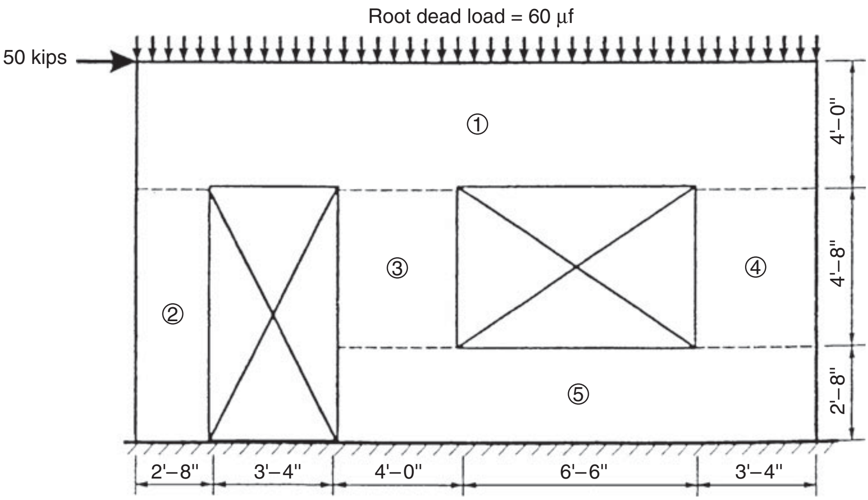


FIGURE P7.6

7.7 The masonry shear wall shown in Fig. P7.6 is 8 in. thick (nominal) and subjected to a shear force $V = 100$ kips. Using Method C, calculate (a) the relative rigidity of the wall, (b) distribution of shear force in various piers.

7.8 Figure P7.8 shows an 8-in.-thick (nominal) concrete masonry shear wall that has several openings. Calculate (a) relative rigidity of the wall by Methods 1, 2, and 3, (b) distribution of a shear force of 100 kips applied at the top of the wall to various piers, (c) relative rigidity of perforated wall as a percentage of the relative rigidity of the solid wall. Assume the compressive strength of masonry $f'_m = 1500$ lb/in.^{2}.

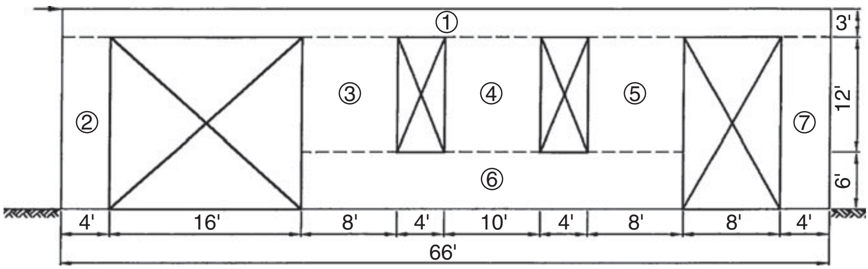


FIGURE P7.8

7.9 Determine the base shear and its distribution over the height of a building that consists of a building frame system with special reinforced masonry shear walls (Fig. P7.9). The building is assigned Seismic Design Category D. It is located in a seismic region for which spectral acceleration parameters S_S and S_1 have been determined, respectively, as $1.25g$ and $0.60g$. The site class for building site has been determined as C. Assume importance factor $I = 1.0$. The seismic weight at each floor is 350 kips.

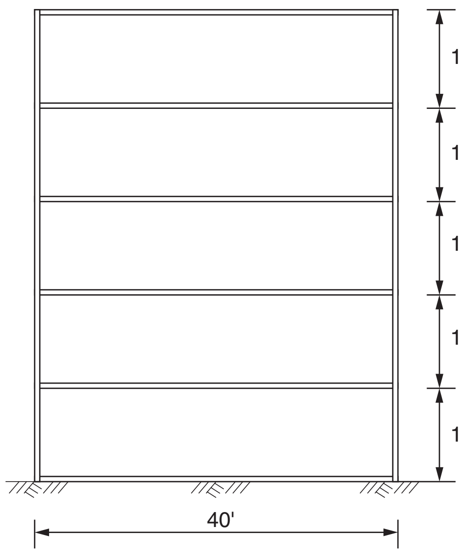


FIGURE P7.9

7.10 Calculate diaphragm forces for the five-story building of Prob. 7.9.

7.11 A single-story building with special reinforced masonry shear walls and a panelized wood roof is shown in Fig. P7.11. The building is classified as Occupancy Category 1 and is assigned Seismic Design Category D. The site class for the building is D and S_S and S_1 for the site have been determined, respectively, as $95.2\%g$ and $42.5\%g$. Dead weight of the roof and the masonry walls may be assumed, respectively, as 16 lb/ft^2 and 84 lb/ft^2. Calculate the diaphragm design force at the roof level.

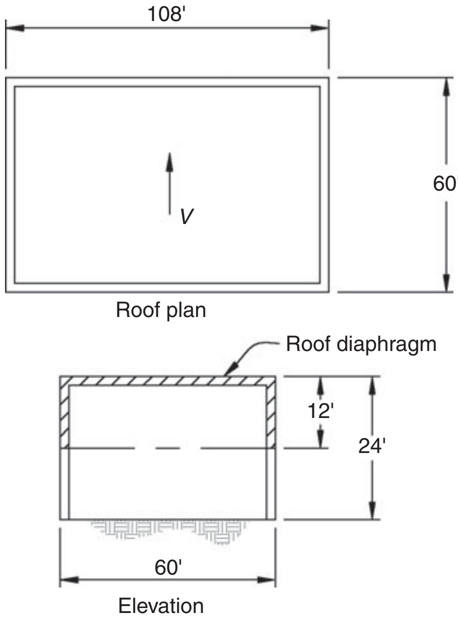


FIGURE P7.11

7.12 Figure P7.12 shows the roof plan of a single-story shear wall building with rigid roof diaphragm. It is supported by four walls A, B, C, and D as shown in the figure. The relative rigidities of these walls are 4.5, 5.5, 4, and 4, respectively. The building is subjected to a lateral force of 150 kips in the northwardly direction. Determine design shears in walls A and B. The building is classified as a Seismic Design Category D structure. Assume that the mass of the building is uniformly distributed with respect to the geometric center of the roof diaphragm.

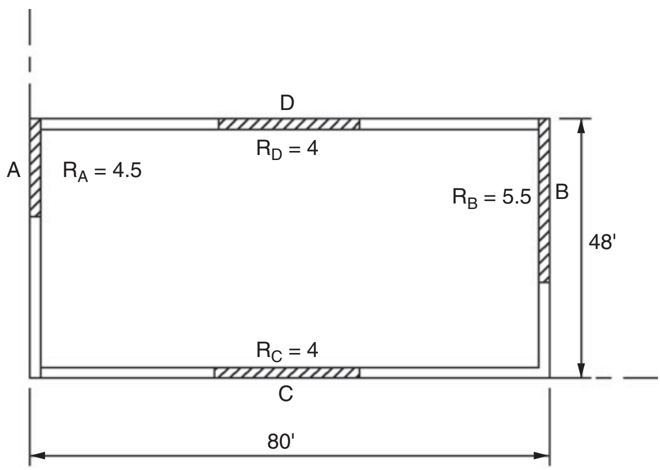


FIGURE P7.12

7.13 Figure P7.13 shows the roof plan of a single-story shear wall building. It is supported by four walls A, B, C, and D as shown in the figure. The relative rigidities of these walls are 6, 4, 4, and 4, respectively. The building is subjected to a lateral force of 100 kips in the northwardly direction. Determine design shears in walls A and B. The building is classified as a Seismic Design Category C structure.

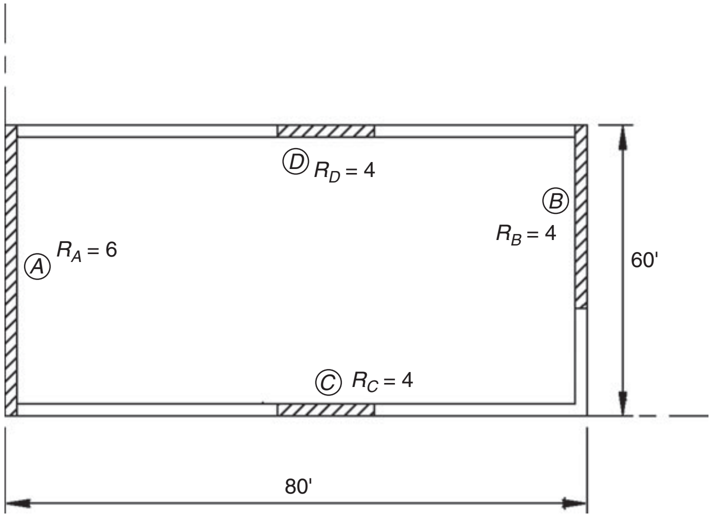


FIGURE P7.13

7.14 Figure P7.14 shows the floor plan of a one-story shear wall building with a rigid roof diaphragm. The center of mass of the building is located at the geometric center of the rigid diaphragm (roof). The rigidities of shear walls A and B are 500 and 250 kips/in., respectively; those of shear walls C and D are 300 kips/in. The building, classified as a Seismic Design Category D structure, is subjected to a lateral force of 150 kips acting northwardly. Determine design shear forces in walls A and B.

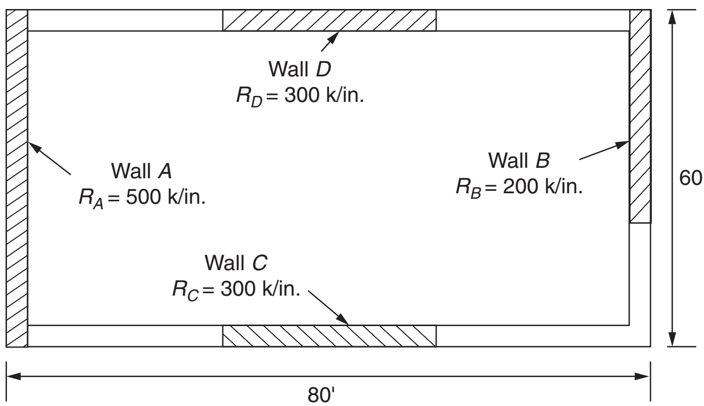


FIGURE P7.14

7.15 Determine the seismic design category for (a) a water treatment facility, (b) a hospital having emergency treatment facilities, for which the following information is provided: For both structures, use Site Class D, $S_s = 1.56g$, $S_1 = 0.76g$.

7.16 The shear wall shown in Fig. P7.16 is located in a reinforced masonry building assigned to Seismic Design Category D. It carries the following loads from the roof diaphragm: dead load = 400 lb/ft, live load = 350 lb/ft, design strength level lateral force = 12 kips at the top of the wall. The 5 percent, damped spectral acceleration parameter, S_{DS}, and the seismic response coefficient for the building, C_S, have been determined to be, respectively, 0.825 and 0.16. Assume the redundancy factor $\rho = 1.0$ and that the design shear strength of the wall is not less than 1.25 times the shear corresponding to the nominal flexural strength. Assume dead weight of wall = 84 lb/ft², f'_m = 1500 lb/ft², and f_y = 60 ksi.

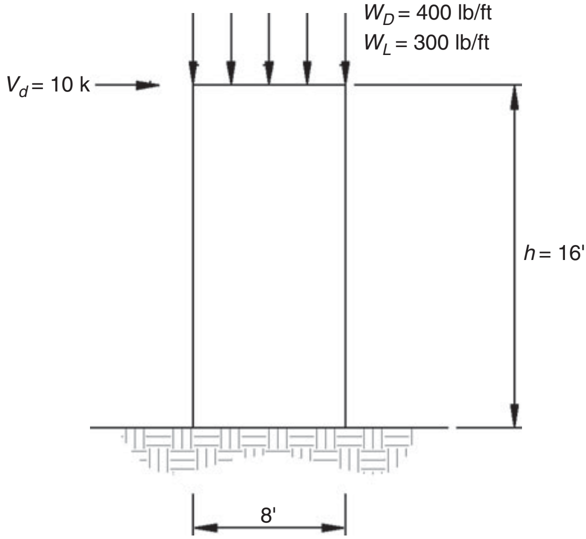


FIGURE P7.16

7.17 Calculate the flexural capacity of the 8-in. (nominal) shear wall having cross section as shown in Fig. P7.17. The wall constructed from 8 × 8 × 16 (nominal) CMUs measures 22 ft 8 in. end-to-end and is reinforced with 12 No 6 Grade 60 bars spaced at 24 in. on centers. The compressive strength of masonry is 1500 lb/in.².

7.18 Calculate the flexural capacity of the shear wall described in Prob. 7.17 when it carries a factored axial load of 300 kips.

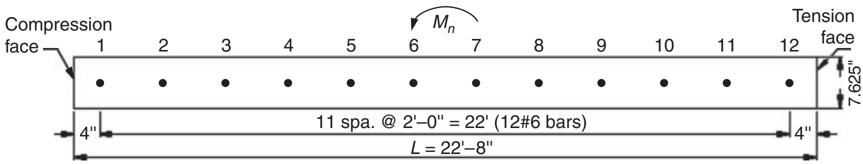


FIGURE P7.17

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CHAPTER 8
RETAINING AND SUBTERRANEAN WALLS

8.1 INTRODUCTION

Retaining walls and abutments form one of the oldest forms of structures. Their main function is to support soil that would otherwise spread and stabilize at angle of repose. Conceptually, retaining walls may be characterized as *earth retaining systems* that support earth or backfill at slopes steeper than their angle of repose. Some of the many applications of earth retaining structures are

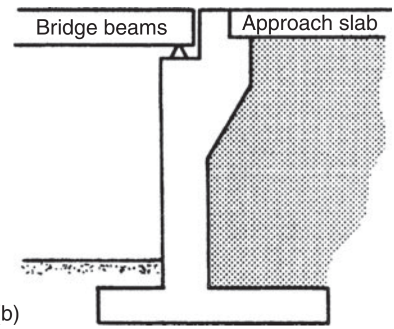
1. Structural walls such as those used for retaining highway and other embankments
2. Bridge abutments that support the superstructure
3. Wing walls
4. Landscaping walls
5. Waterfront structures
6. Parking area support
7. Stream channelization
8. Tunnel access walls

A few of the aforelisted structures are shown in Fig. 8.1.

Before the advent of reinforced concrete, retaining walls were constructed from stone masonry or mass concrete, both unreinforced. Such walls provide resistance to earth pressure by virtue of their own weight. Accordingly, they are called gravity retaining walls. These walls derive their strength or resistance from the compressive strength of the material used. Modern masonry retaining walls are constructed also from concrete or block masonry, but both reinforced. Design of reinforced concrete retaining walls can be found in texts on design of reinforced concrete structures. This chapter discusses design of retaining walls, and subterranean or basement walls constructed from reinforced concrete block masonry.

Generally speaking, no gravity loads are applied to retaining walls. Thus, they have to be designed to resist lateral pressures due to the retained material, and their own weight. Earth retaining walls, masonry and concrete dams, walls of water tanks and grain silos, etc., fall into this category. These structures have to resist pressures due to the retained earth, water, or grains, as the case may be. There are exceptions, however. For example, a bridge abutment retains earth behind it but also supports gravity and lateral loads from the supported superstructure. Thus, an abutment is designed to resist both gravity and lateral loads. Similarly, residential basement walls are required to resist lateral load due to earth as well as the gravity

(a)



(b)

FIGURE 8.1 (*a*) A masonry wall retaining embankment, under construction (*b*) a bridge abutment. (*Courtsey: Author*.)

load they receive from the superstructure above. Accordingly, basement and subterranean walls are essentially retaining walls that are called upon to also support gravity loads.

Irrespective of the presence or absence of gravity loads, lateral loads induced by the supported backfill render retaining walls highly vulnerable to failure by sliding and/or overturning. Consequently, retaining walls have to be designed for safety against sliding and overturning (that is, designed for stability) in addition to providing the required strength.

Because only one face of retaining walls is subjected to tension, it is common practice to provide reinforcement near the face of the wall that is in tension. This practice is different from providing reinforcement at the center of load-bearing walls discussed in Chap. 6.

8.2 PRINCIPAL TYPES OF RETAINING WALLS

Although described as retaining walls in the past, the modern approach to describing various types of earth retaining structures is to characterize them as *earth retaining systems*. Whether built from concrete or masonry, retaining walls may be classified into three basic types, depending on whether they can stay stable by themselves while retaining the earth or not. These are: *gravity retaining walls, cantilever retaining walls,* and *segmental retaining walls.* The gravity and cantilever retaining walls are undoubtedly of the oldest forms; however, the development of segmental retaining walls (SRWs) is of fairly recent origin. In addition, there are other structural forms of retaining walls such as bridge abutments, and basement and subterranean walls. All are discussed in the following sections.

8.2.1 Gravity Walls

Gravity retaining walls, so called because they achieve their stability (against sliding and overturning) from their massiveness, form the earliest type of construction. The massiveness of these walls permits them to be designed as unreinforced walls. However, due to the inherent weakness of masonry in tension, gravity retaining walls are so proportioned that the resultant of the foundation pressure passes through the *kern* of the section (i.e., through the middle-third of the cross section) so that the entire wall section remains in compression. This requires the wall cross section to be trapezoidal (Fig. 8.2*a*). These types of walls are built for heights in the 10- to 12-ft range.

With the advent of modern reinforced masonry, use of gravity retaining walls has declined considerably. However, use of unreinforced stone masonry for dams, reservoirs, and spillways remains popular in many parts of the world for economical reasons, which include locally available, relatively unskilled labor and material (savings in material transportation/hauling costs).

Another form of gravity walls is called *semigravity wall* which, unlike the gravity retaining wall, depends for its stability on its own weight plus the weight of some soil behind it (Fig. 8.2*b*). These walls also have some light reinforcement in the stem and the slab to reduce the section. They are suitable for the same height range as the gravity retaining walls.

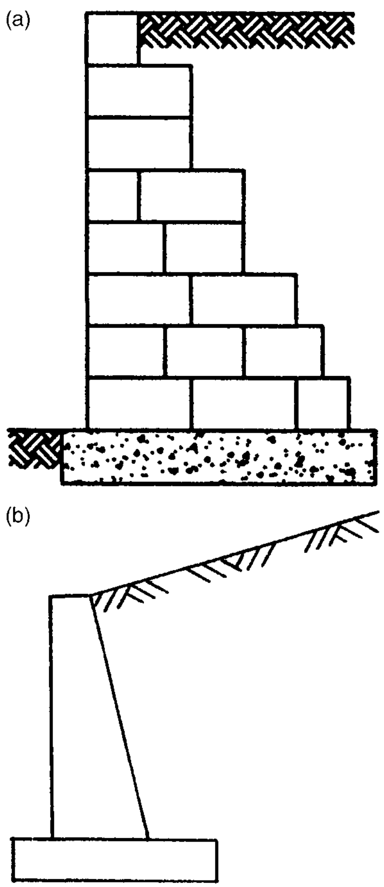


FIGURE 8.2 (*a*) Gravity and (*b*) semigravity retaining walls.

8.2.2 Cantilever Walls

Cantilever walls, so called because of their vertical cantilever profile, form the most common type of retaining walls. Depending on the site requirement, its cross section has the shape of letter "L" (called *L-type* cantilever, Fig. 8.3*a*) or an inverted-T (Fig. 8.3*b*). The vertical portion of the wall, referred to as *stem*, provides lateral support to earth and behaves like a vertical cantilever fixed at the base. The base slab, buried under the backfill, provides stability against both sliding and overturning. The portion of the base

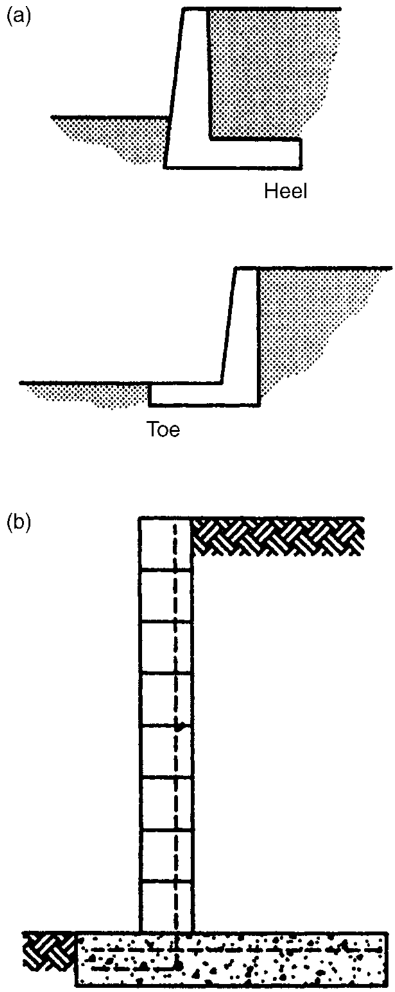


FIGURE 8.3 Cantilever retaining walls: (*a*) L-shaped, (*b*) inverted T-shaped.

slab under the backfill is called the *heel*, and the portion projecting from the front face of stem is called the *toe*. Retaining walls with the inverted T-section are the most commonly used type. The L-shaped cross sections are used where the walls may be required to retain soil, but the site restrictions do not allow for footing projection beyond the face of the wall. The horizontal slab of the L-shaped wall can be either under the backfill or on the other side of the backfill, as the case may be. Ordinarily, cantilever retaining walls are suitable for heights in the 10- to 25-ft range, but are most economical below 20 ft.

8.2.3 Counterforted and Buttressed Retaining Walls

When a cantilever wall has to be built for large heights, say above 25 ft or so, the moments at the base of stem become so large that the wall becomes uneconomical. To reduce these moments, vertical cross-walls are introduced, which are oriented perpendicular to the wall. They are usually spaced at approximately half the height of the wall. When these cross-walls are located such that they support the wall on the same side as the backfill (i.e., on the heel), they are called *counterforts* which are covered by the backfill, and the wall is called a *counterforted retaining wall* (Fig. 8.4*a*). On the other hand, if the cross-walls are located on the side opposite the backfill (i.e., on the toe), they are called *buttresses* which are visible, and the wall is called a *buttressed wall* (Fig. 8.4*b*). In both cases, the wall is continuously supported by the counterforts or buttresses in the vertical plane, and by the bottom slab in the horizontal plane. Thus, structurally speaking, the wall acts like a huge plate supported on three sides and free on the fourth side (i.e., at the top). Counterforted or buttressed walls are also used when minimum wall deflection is desired.

Both the counterforts and the buttresses have certain advantages and disadvantages. Because the counterforts are hidden underneath backfill, the wall has an attractive appearance. Also, the space in front of the stem can be used to some advantage. For these reasons, they are preferred over the buttressed walls. The disadvantage with the counterforts is that they are subjected to tension as the wall is pushed away from the backfill, which requires them to be adequately tied to the heel with stirrups. Buttresses, on the other hand, are subjected to compression as the wall pushes against them under the pressure of the backfill, a condition in which both masonry and concrete can be used more efficiently.

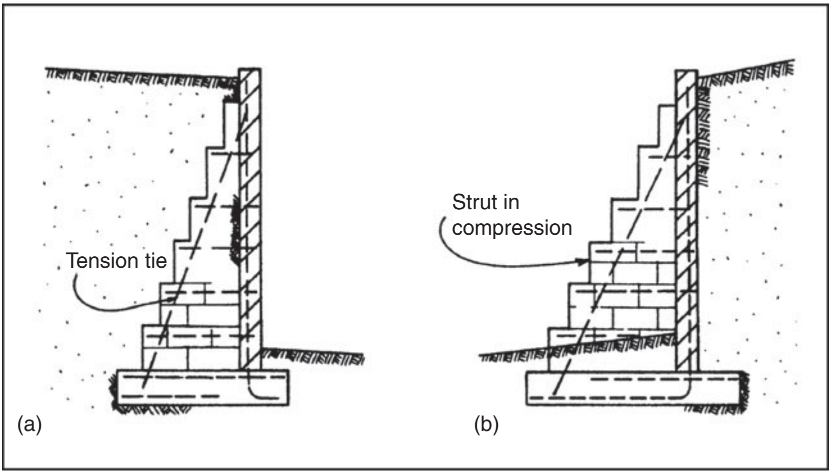


FIGURE 8.4 (*a*) Counterforted wall, (*b*) buttressed wall.

The disadvantage is that buttresses remain visible over the toe, which gives the wall a poor and often objectionable appearance. Also, the presence of buttresses at certain intervals uses up the valuable space in front of the wall, which could have been otherwise put to some productive use.

8.2.4 Bridge Abutments

A bridge abutment is essentially a retaining wall that supports a bridge deck at the top (Fig. 8.1*b*). In the presence of the bridge superstructure, the abutment may be assumed fixed at the base and simply supported at the top. Construction procedures play an important role in design of an abutment. During construction, a situation may arise when the abutment is complete but the superstructure not yet installed. In such a situation, the abutment will behave as a cantilever wall. However, this may be avoided by installing the superstructure before placing the backfill. A similar condition may exist during bridge rehabilitation also. Bridge superstructures often deteriorate needing replacement. In such cases, the entire superstructure may be removed, leaving no support to the abutment at the top, in which case the abutment will behave as a cantilever wall. Therefore, the stability of abutments must be carefully checked for both conditions and designed accordingly.

8.2.5 Basement or Subterranean Walls

As the terms suggest, these walls are provided below grade, typically provided underneath residential and other buildings below the first floor, and for subterranean parking structures under office buildings (Fig. 8.5). These walls span vertically between the basement-floor slab and the first-floor slab. However, it must be recognized that this condition does not exist until the first-floor slab is in place. In the absence of the first-floor slab, the basement wall acts as a cantilever, and is usually braced during construction as discussed in Section 8.7. Unlike the conventional earth retaining walls, the basement walls have to resist gravity load

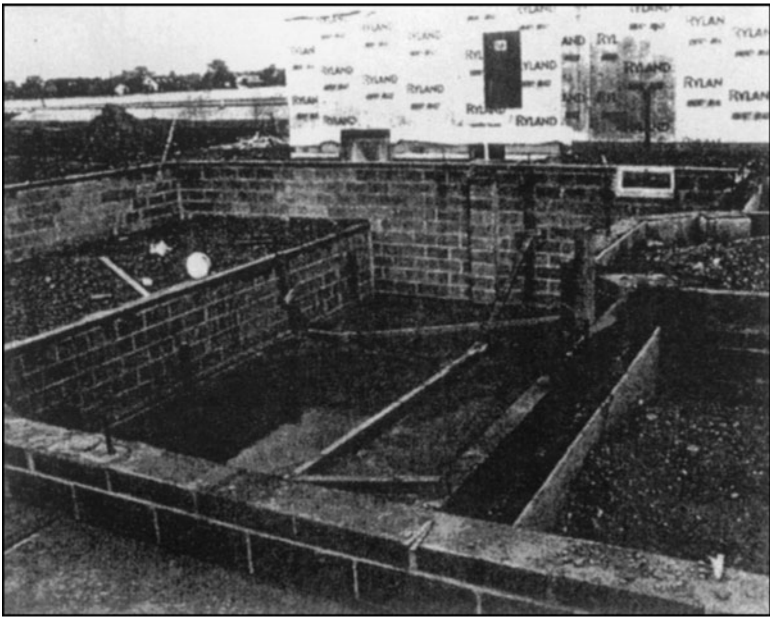


FIGURE 8.5 A subterranean wall. (*Courtesy: NCMA.*)

from the superstructure above as well as the earth pressure from the outside, a condition similar to that for bridge abutments.

8.2.6 Segmental Retaining Walls

The types of retaining walls discussed in the forgoing paragraphs are the conventional types that are in wide use. Of fairly recent origin are the segmental retaining walls which can be classified into two types of gravity retaining walls: *conventional* segmental retaining walls and *reinforced soil* segmental retaining walls. Both conventional and reinforced soil SRWs function as gravity structures by relying on self-weight to resist the destabilizing forces induced by the retained soil and surcharge loading on the structure.

8.2.6.1 Conventional SRWs Conventional SRWs are structures that resist destabilizing forces induced by the retained soil solely through the self-weight and batter of the SRW units. These walls are classified into two categories: single depth (similar to single-wythe walls) and multiple depths (similar to multiwythe walls) (Fig. 8.6*a*).

8.2.6.2 Reinforced Soil SRWs Reinforced soil SRWs can be characterized as composite earth retaining systems that use SRW units in combination with a mass of reinforced soil stabilized by horizontal layers of geosynthetic reinforcing materials (Fig. 8.6*b*). Wall heights of SRWs are controlled mainly by bearing capacity of the foundation materials. Wall heights in excess of 60 ft are feasible for some soil-reinforcing systems where

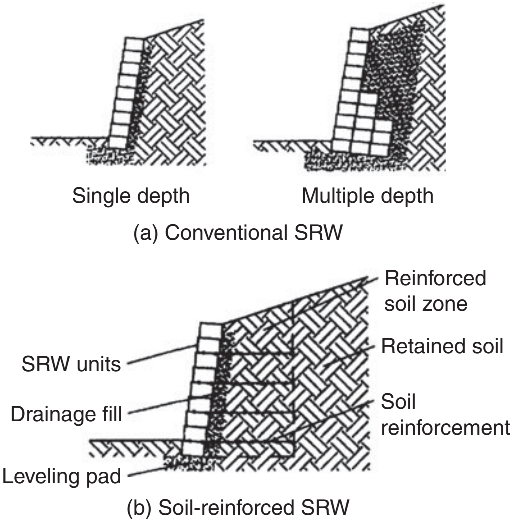


FIGURE 8.6 Segmental retaining wall (SRW) systems: (*a*) conventional SRW, (*b*) reinforced soil SRW. (*Courtesy: NCMA.*)

foundation conditions permit. Foundation investigations for soil reinforcement systems are similar to those for conventional retaining walls.

Essentially, the system consists of *reinforced soil*, a composite material in which the soil and reinforcement form a stable unit capable of resisting the backfill pressures and transferring them to the foundation. The soil is compacted in layers between which are placed the geosynthetic or metallic reinforcing elements such as steel bars or grids, or sheets of various polymers. Typically, a facing attached to reinforcement is required to prevent the fill from raveling but is subject to virtually no pressure [8.1]. Many types and arrangements of facing and reinforcement, including some proprietary, have evolved since the introduction of the original patented system, called *reinforced earth*, in France [8.2].

An increasing number of retaining walls have been built using this system since the mid-1960s when they were first introduced. The units that constitute the facing system of the wall are architecturally acceptable and are machine made or cast from concrete without any internal steel reinforcement. Significant use of SRW units is reported to have begun around 1984, and the use of reinforced soil SRW systems around 1986. Since then, the popularity of these earth retaining systems really took off, with over 100,000 completed wall installations in North America as of 1996, 25 percent of these being reinforced soil SRWs [8.3].

Common to both types SRWs are dry-stacked *segmental units* that are arranged in a running bond pattern (Fig. 8.6). The majority of SRW units commercially available are dry-cast, machine-produced concrete, without reinforcement. A variety of proprietary segmental units are available. A few samples are shown in Fig. 8.7 to illustrate the variety in size, shape, and interlocking mechanism. Although there are no restrictions to size, most proprietary SRW units are 3 to 24 in. in height, 6 to 30 in. in width, and 6 to 72 in. in length.

Geosynthetic reinforcing elements are made from high-tensile-strength polymeric sheet materials. These reinforcing elements may be in the form of geogrids or geotextiles, though

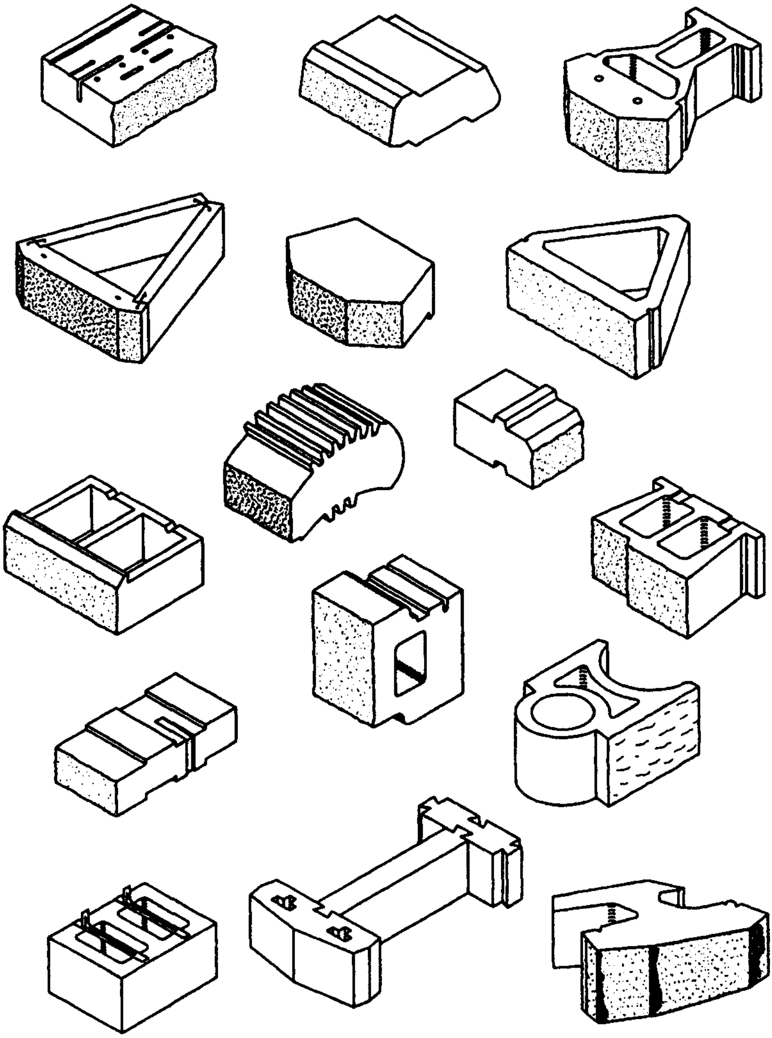


FIGURE 8.7 Examples of commercially available segmental units. (*Courtesy: NCMA.*)

most SRW construction to date has used geogrids. Geosynthetic reinforcements extend through the interface between the SRW units and into the soil to create a composite gravity mass structure. This enlarged composite gravity mass system, comprised of the SRW units and a reinforced soil mass, offers the required resistance to lateral loads from soil pressure and the surcharge loads [8.3].

Reinforced soil SRWs are also referred to as *mechanically stabilized earth* (MSE) system. This term is often used to describe all forms of fill-type reinforced soil structures [8.4–8.10].

It is interesting to note that while the widespread use of the SRWs began around 1986, the concept of segmental retaining walls and reinforced soil is surprisingly old. The Ziggurats of Babylonia (i.e., Tower of Babel[1]) were built some 2500 to 3000 years ago using soil reinforcing methods similar to the modern methods [8.4, 8.7]. Similarly, the Great Wall of China was built using an early version of MSE system, consisting of a mixture of clay and gravel reinforced with tamarisk branches [8.4, 8.7].

Segmental retaining walls are not discussed further in this book. Design methods for SRWs can be found in several references [8.4–8.10]. Specifications for SRWs for highways and highway bridge structures can be found in Refs. 8.11 and 8.12.

8.2.7 Curbs

Curbs are the shortest earth retaining structures used for retaining backfill of about 2 ft in height or less. The two common forms are shown in Fig. 8.8. The one shown in Fig. 8.8*b* is used when it is necessary to have a gutter on the low side of the curb.

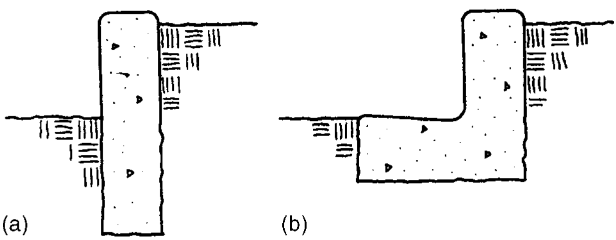


FIGURE 8.8 Curbs: (*a*) freestanding curb, (*b*) curb with gutter.

8.3 LATERAL PRESSURES ON RETAINING WALLS

8.3.1 Basic Concepts of Earth Pressure

A detailed discussion of earth pressure theories is beyond the scope of this book. However, a brief overview of the earth pressure theories used in practice and code requirements are presented in this section.

Generally speaking, earth pressure is a function of the following parameters:

1. Type and unit weight of soil
2. Water content
3. Soil creep characteristics
4. Degree of compaction

[1]People of Babylon began building this tower intended to reach heaven but were forced to abandon their work upon the confusion of their languages by God. Gen. 11:4-9.

5. Location of ground water table
6. Soil-structure interaction
7. Amount of surcharge
8. Seismic effects
9. Back slope angle
10. Wall inclination

If a retaining wall were built against a solid rock face, the wall will not be subjected to any lateral pressure from the rock. This is because rock is a rigid material, and unless subjected to external force, will retain its form and shape. The principle forces acting on retaining structures arise from retaining loose materials, such as earth, grains, etc., or liquids, which are not capable of supporting or retaining themselves in a desired stable shape. In the absence of supporting walls, these materials will simply flow and spread into their natural shape. For example, loose earth, sand, and grains, if not retained by a wall, will simply spread in a pyramid shape. Liquids will simply flow. Consequently, the principal force that retaining walls have to resist is the pressure exerted on them by the retained materials. All walls that retain materials or liquids are designed based on the same general principles, that is, they all must be able to resist lateral pressure exerted by the supported materials. For example, basement walls in buildings, subterranean walls in buried structures and underground parking structures must be designed to resist earth pressure.

From an engineering standpoint, it is appropriate to introduce the concept of strain in soils. Loose soil poured freely on a flat surface will spread into a heap of pyramid shape freely because of unlimited strain it can have. When the same soil is retained by a wall, the amount strain (expansion or contraction) the soil can have is severely limited. The amount and distribution of lateral pressure due to the retained soil depends largely on the relative lateral strain it can have which, in turn, depends on the rigidity of the retaining structure.

If the retaining structure is unyielding and highly rigid, it will be subjected to lateral pressure called *earth pressure at rest*. Lateral pressure against basement walls is generally in this category. On the other hand, if a retaining wall is permitted to yield (i.e., move away from the retained soil) allowing a lateral expansion of the soil, the earth pressure decreases with increasing expansion. This continues until a stage is reached when further expansion causes a shear failure of the soil, characterized by a sliding wedge moving forward (toward the wall) and downward with respect to the original position (Fig. 8.9*a*). At this stage of failure, the value of lateral pressure on the wall is the smallest; additional deformation in the soil does not cause any further reduction in lateral pressure. This minimum earth pressure is called *active earth pressure*.

A situation may exist where the retaining structure may have a mass of soil in the front which will be pushed as the wall moves away from the retained soil. In that case, the retained soil is pushed causing it to contract laterally. A larger force is required to move the soil

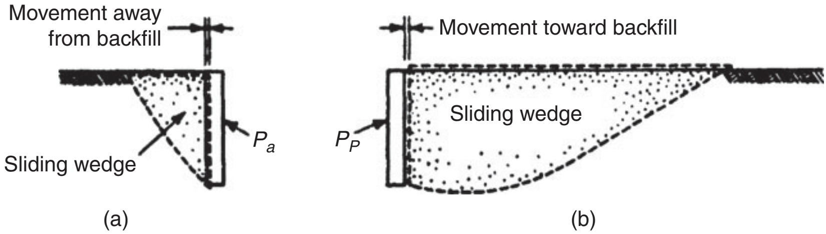


FIGURE 8.9 Concept of lateral earth pressure: (*a*) active pressure, (*b*) passive pressure.

behind the retaining structure. This soil movement continues until a stage is reached when further contraction causes a shear failure of the soil, characterized by a sliding wedge moving backward and upward with respect to it original position (Fig. 8.9*b*). The lateral pressure experienced by the wall at this stage is the largest, and no additional force is required to cause further movement of the wedge. This maximum lateral pressure is called *passive pressure*. In addition, in the event of an earthquake, the wall will be subjected to additional earth pressure called *seismic* or *dynamic* earth pressure.

Several methods of estimating pressure due to backfill are in use, all of which are associated with some degree of uncertainty. Various factors that cause these uncertainties include the type of soil (i.e., dry granular soil, clayey soil, etc.), moisture content, density, and the presence or absence of surcharge, etc. As a result, accurate determination of earth pressure is difficult and often it can be estimated for only specific conditions of backfill material and other assumptions. In addition to conventional *active* and *passive* earth pressures that act on retaining walls, there are *seismic* active and passive pressures that act on retaining walls in seismically active areas.

Methods of determining earth pressures on retaining walls based on classical earth-pressure theories can be found in several references [8.1, 8.3–8.16]. References 8.11 and 8.12 present design method and specifications for determining earth pressure on abutments and retaining walls for highways bridges; Ref. 8.15 discusses the topic for railway bridges. Discussion on computational methods for lateral pressures in walls, bins, and grain elevators can be found in Ref. 8.17. The following is a brief overview of current practices of determining earth pressures on retaining walls. Various methods of determining earth pressures on retaining walls are based on three assumptions [8.1]:

1. The pressure in the pore water of the backfill is negligible.
2. The soil properties appearing in the earth-pressure equations have definite values that can be determined reliably.
3. The wall can yield by tilting, deforming, or sliding through a distance sufficient to develop the full shearing resistance of the backfill.

These assumptions are very significant in that if they are not satisfied, the calculated earth pressures will be invalid. For example, if the first and the second assumptions are not satisfied, the retaining wall will be acted upon by agents and forces beyond the scope of any earth pressure theory [8.1]. Properties of loosely deposited or inadequately drained backfill change from season to season so that the backfill passes through states of partial or total saturation alternating with states of drainage or even partial desiccation. Changes in earth pressures caused by these phenomena are not accounted for in the classical earth-pressure theories. Pressure cell measurements on the back of a 10-m-high retaining wall by McNary [8.18] indicated that within 1 year the pressure varied from the average value by ±30 percent [8.1].

The third assumption is extremely significant because its implications have led and still lead to considerable confusion among designers. This involves uncertainties regarding the movements of retaining walls when acted upon by the earth pressure. These uncertainties were first investigated by Terzaghi through model tests [8.19–8.22]. His findings indicated that neither small- nor large-scale tests nor even filed measurements could furnish consistent results, unless the walls moved far enough to establish the active state. For granular soils placed under controlled conditions, the required movements were relatively small. Therefore, it was concluded "that most conventional walls, if not restrained at their tops, could move far enough without objectionable consequences to reduce the earth pressure to the active value and that the stability of the walls could, therefore, properly be investigated on the basis of that pressure. This conclusion is subject to several limitations not always realized by designers" [8.1, p. 333]. Readers are referred to Ref. 8.1 (Art. 45.4) for a detailed discussion on this important topic.

8.3.2 Theories of Earth Pressures

Irrespective of the type of retaining wall used, two basic requirements must be satisfied:

1. The retaining wall must be stable and have an adequate factor of safety against sliding, overturning, and settlement.
2. The retaining wall must provide sufficient strength to resist forces to which it is subjected (i.e., structural design must be adequate).

These two requirements are sometimes referred to, respectively, as *external stability* and *internal stability*. The magnitude and direction of earth pressure on a retaining wall depends on many factors, chiefly on the nature of the backfill and other factors (e.g., wall movements) that cannot be determined as accurately as the gravity loads. Although the distribution of pressure on the back of the retaining wall is complex, for simplicity it is common to assume hydrostatic or linear pressure distribution so that the earth pressure increases with the height of the wall. The pressure p at any height of wall is only a function of wall height H below the surface of the backfill and the unit weight of soil γ, and can be expressed as

$$p = K\gamma H \tag{8.1}$$

The value of coefficient K in Eq. (8.1) depends on the physical properties of soil, varying from about 0.3 for loose granular soil (e.g., dry sand) to about 1.0 for cohesive soils such as wet clays. Both active and passive pressures against a retaining wall are assumed to have linear distribution. The nature of the backfill plays a defining role in the distribution of pressure. Noncohesive, granular materials, such as dry sand, behave differently from cohesive materials, such as clay, silt, or any soil containing these soils as constituents. When the backfill consists of dry granular material, the assumption of hydrostatic or linear distribution of earth pressure is fairly satisfactory. However, cohesive soils or saturated sands behave in nonlinear manner which is not well defined. For this reason, it is common practice to *specify* granular material, such as dry sand, as backfill material, and also to provide adequate means for the drainage of water from the back of the wall so that linear pressure distribution can be justifiably used.

The two most commonly used theories for computing earth pressures are: Rankine's theory and Coulomb's theory. Both theories are based on the three assumptions discussed in the previous section. Another method to calculate earth pressures, called the *log spiral method*, was developed by Ohde [8.24] and is discussed in Ref. 8.1.

8.3.2.1 Rankine's Earth Pressure Theory Rankine's theory, published in 1857 [8.24], gives the value of earth pressure against a retaining wall with a perfectly smooth surface (i.e., friction between the soil and the back of the wall is neglected). For a wall with horizontal backfill, the magnitude of active earth pressure at a depth H below the top of the backfill is given by Eq. (8.2):

$$p_a = K_a\gamma H \tag{8.2}$$

where K_a is the coefficient of active earth pressure having a value given by Eq. (8.3*a*):

$$K_a = \left(\frac{1-\sin\phi}{1+\sin\phi}\right) \tag{8.3a}$$

where ϕ = angle of internal friction of soil. Alternatively, Eq. (8.3*a*) can be expressed as

$$K_a = \tan^2(45-\phi/2) \tag{8.3b}$$

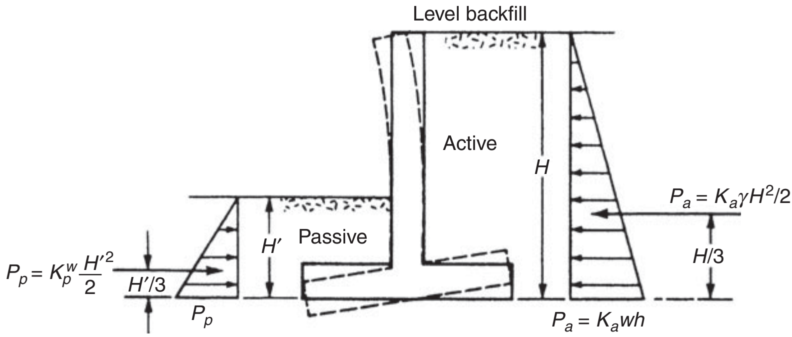


FIGURE 8.10 Active and passive earth pressure on retaining walls.

Assuming a linear pressure distribution (Fig. 8.10), the resultant of the active earth pressure is given by Eq. (8.4):

$$P_a = \frac{\gamma H^2}{2}\left(\frac{1-\sin\phi}{1+\sin\phi}\right) \tag{8.4}$$

The resultant of the active earth pressure, P_a, acts at the centroid of the triangle, that is, at $H/3$ from the base. When the backfill is surcharged at an angle α to the horizontal, the coefficient K_a in Eq. (8.2) is given by Eq. (8.5):

$$K_a = \cos\alpha\left(\frac{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi}}{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi}}\right) \tag{8.5}$$

The resultant in this case still acts at $H/3$ from the base, but is inclined at an angle α to the horizontal (Fig. 8.11). Note that when $\alpha = 0$, Eq. (8.5) simplifies to Eq. (8.3*a*). The value of passive soil pressure p_p is given by Eq. (8.6):

$$p_p = K_p \gamma H' \tag{8.6}$$

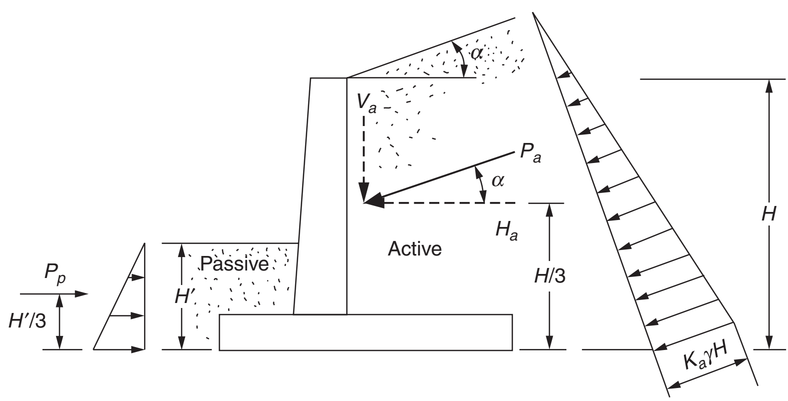


FIGURE 8.11 Active earth pressure due to backfill with surcharge.

where K_p is the coefficient of passive earth pressure having a value given by Eq. (8.7*a*):

$$K_p = \left(\frac{1+\sin\phi}{1-\sin\phi}\right) \tag{8.7a}$$

Alternatively, Eq. (8.7*a*) can be expressed as

$$K_p = \tan^2(45+\phi/2) \tag{8.7b}$$

Assuming that the pressure distribution is linear, the resultant of passive pressure can be computed from Eq. (8.8):

$$P_p = \frac{\gamma H'^2}{2}\left(\frac{1+\sin\phi}{1-\sin\phi}\right) \tag{8.8}$$

The resultant of the passive earth pressure, P_p, acts at the centroid of the triangle, that is, at $H'/3$ from the base. When the backfill is surcharged at an angle α to the horizontal, the coefficient K_p is given by Eq. (8.9):

$$K_p = \cos\alpha\left(\frac{\cos\alpha+\sqrt{\cos^2\alpha-\cos^2\phi}}{\cos\alpha-\sqrt{\cos^2\alpha-\cos^2\phi}}\right) \tag{8.9}$$

The resultant in this case still acts at $H'/3$ from the base, but is inclined at an angle α to the horizontal (Fig. 8.11). Note that when $\alpha = 0$, Eq. (8.9) simplifies to Eq. (8.7). The important inverse relationship between the coefficients of active pressure and passive pressure, K_a and K_p, respectively, is noteworthy:

$$K_p = \frac{1}{K_a} \tag{8.10}$$

Values of K_a and K_p for some typical values of angle of surcharge α and the angle of internal friction ϕ are given, respectively, in Tables 8.1 and 8.2. Table 8.3 lists properties of a few selected soils. For well-drained sands and gravel backfills, the angle of internal friction is often assumed to be equal to the angle of repose.

The backfill is always placed after the wall is built. During the placement of the backfill, the wall yields somewhat under pressure induced by the backfill. The ultimate value of the

TABLE 8.1 Values of Coefficient of Active Earth Pressure, K_a

	Angle of internal fraction, ϕ						
Angle of surcharge α	28°	30°	32°	34°	36°	38°	40°
0°	0.361	0.333	0.307	0.283	0.260	0.238	0.217
10°	0.380	0.350	0.321	0.294	0.270	0.246	0.225
20°	0.461	0.414	0.374	0.338	0.306	0.277	0.250
25°	0.573	0.494	0.434	0.385	0.343	0.307	0.275
30°	0	0.866	0.574	0.478	0.411	0.358	0.315

TABLE 8.2 Values of Coefficient of Passive Earth Pressure, K_p

Angle of surcharge α	Angle of internal fraction, ϕ						
	28°	30°	32°	34°	36°	38°	40°
0°	2.77	3.00	3.25	3.54	3.85	4.20	4.60
10°	2.55	2.78	3.02	3.30	3.60	3.94	4.32
20°	1.92	2.13	2.36	2.61	2.89	3.19	3.53
25°	1.43	1.66	1.90	2.14	2.40	2.68	3.00
30°	0	0.87	1.31	1.57	1.83	2.10	2.38

TABLE 8.3 Values of γ and ϕ for selected soils [8.15]

Type of backfill	Unit weight, γ		Angle of internal friction ϕ
	pcf	Kg/m³	
Soft clay	90–120	1440–1920	0°–15°
Medium clay	100–120	1600–1920	15°–30°
Dry loose silt	100–120	1600–1920	27°–30°
Dry dense silt	110–120	1760–1920	30°–35°
Loose sand and gravel	100–130	1600–2100	30°–40°
Dense sand and gravel	120–130	1920–2100	25°–35°
Dry loose sand, graded	115–130	1840–2100	33°–35°
Dry dense sand, graded	120–130	1920–2100	42°–46°

pressure depends not only on the unit weight of soil and the wall height, but also on the amount of yield. If the wall were rigid (i.e., remain fixed and unyielding), the earth pressure would likely to retain a value forever close to *earth pressure at rest*. However, as soon as the wall yields far enough, the deformation conditions for the transition of the adjoining soil mass from the state of rest into an active state of plastic equilibrium is satisfied [8.1]. This assumption is most fundamental to earth pressure computation methods commonly used in practice.

It was noted earlier that Rankine's theory assumes the back of the retaining wall to be perfectly smooth, an ideal condition that is hardly encountered in practice because there are no perfectly smooth surfaces. The roughness of the back of a wall plays a role in that it reduces the active earth pressure and increases the passive earth pressure. Therefore, the error associated with assumption of Rankine's conditions is on the safe side; and because of their simplicity, Rankine's equations are commonly used for determining earth pressures against retaining walls.

The equations presented in this section were developed for cohesionless soils. For cohesive soils containing clay and/or silt, empirical charts such as shown in Figs. 8.12 and 8.13 may be used for estimating earth pressures for retaining walls less than 20 ft high. The charts in Figs. 8.12 and 8.13 are for the types of soils characterized in Table 8.4.

It will be noted that the charts in Figs. 8.12 and 8.13 give values of coefficients k_h and k_v for different types of soils and backfill configuration. It was pointed out earlier that the resultant earth pressure due to a backfill sloping at an angle α is directed also at angle α with the horizontal. For analysis, it is convenient to resolve this resultant into horizontal and vertical

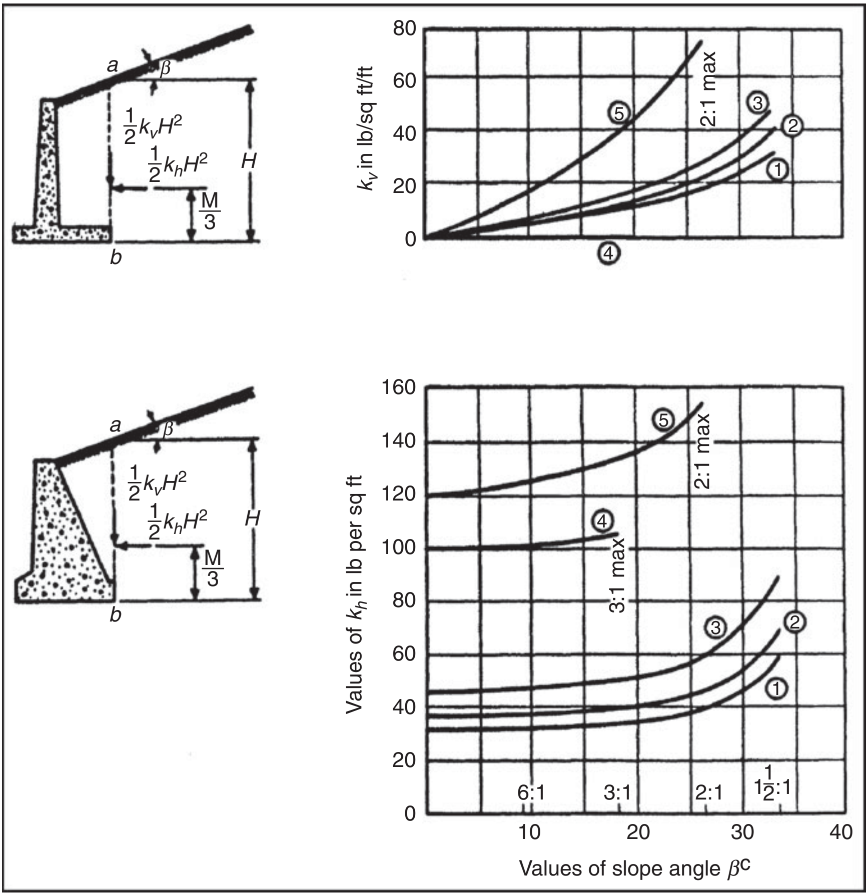


FIGURE 8.12 Earth pressure charts for retaining walls less than 20 ft high supporting backfill with plane surface [8.15].

components. The coefficients k_h and k_v shown in the charts simply represent horizontal and vertical components, P_h and P_v, respectively, of the resultant of the earth pressure:

$$P_h = \frac{1}{2}k_h H^2 \tag{8.11}$$

$$P_v = \frac{1}{2}k_v H^2 \tag{8.12}$$

The value of the unit weight of soil γ should be obtained through geotechnical investigations. However, it is common practice to design minor retaining walls (less than 20 ft high) using unit soil weight specified in applicable codes/specifications. For simplicity in earth pressure calculations, it is convenient to think of the lateral pressure on walls as being caused by some *equivalent fluid* instead of actual soil (e.g., pressure due to a water column

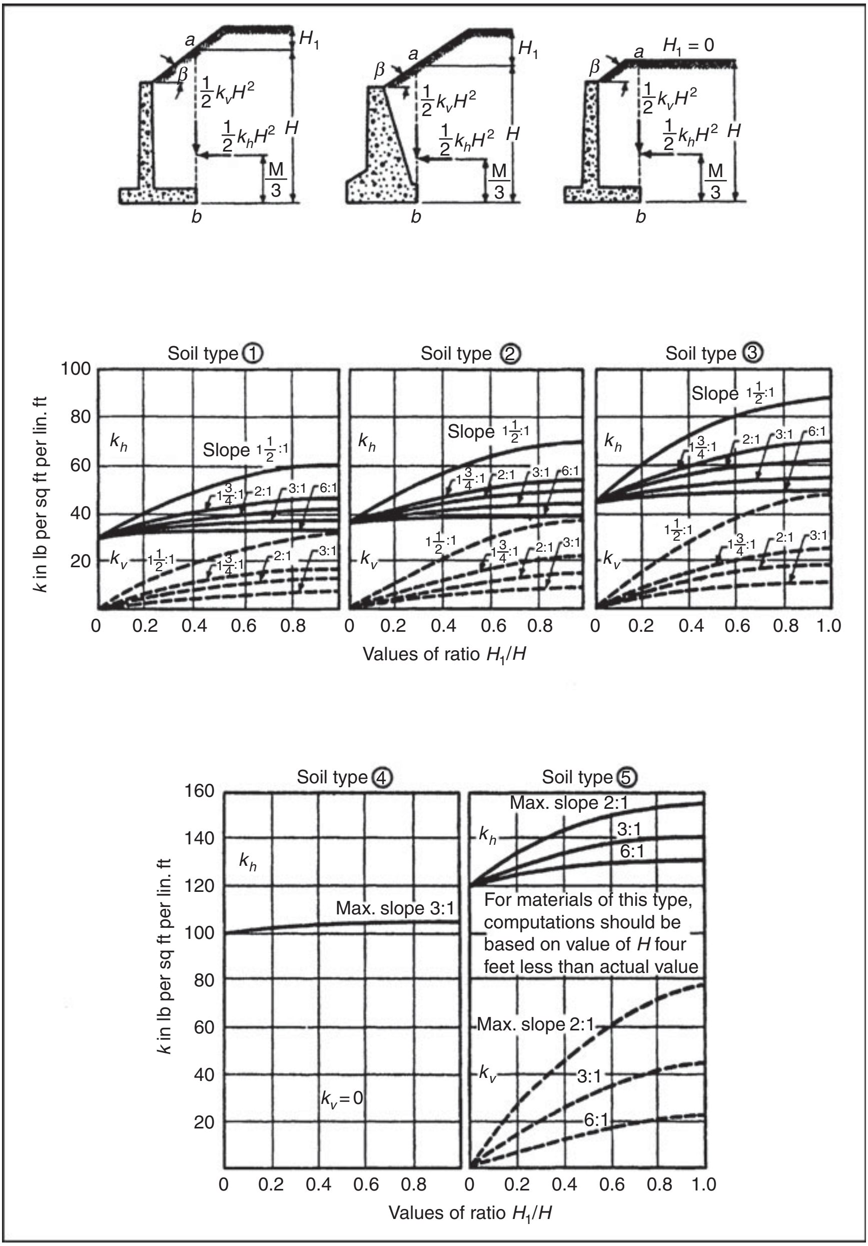


FIGURE 8.13 Earth pressure charts for retaining walls less than 20 ft high supporting backfill with plane surface sloping upward from the top of the wall to some limited distance and becoming horizontal [8.15].

TABLE 8.4 Types of Backfill for Retaining Walls [8.15]

Type of backfill	Unit weight, γ(lb/ft^3)	Cohesion, C (lb/ft^2)	Angle of internal friction, ϕ
Coarse-grained soil without admixture of fine soil particles, very free draining (clean sand, gravel, or broken stone).	105	0	33° 42′ (38° for broken stone)
Coarse-grained soil of low permeability due to admixture of particles of silt size.	110	0	30°
Fine silty sand, granular materials with conspicuous clay content, or residual soil with stones.	125	0	28°
Soft or very soft clay, organic silt, or soft silty clay.	100	0	0
Medium or stiff clay that may be placed in such a way that a negligible amount of water will enter the spaces between the chunks during floods or heavy rains.	120	240	0

of depth H), a method called the *equivalent fluid method*. The weight of this equivalent fluid w_e, is assumed to be equal to the term $K_a\gamma$ in Eq. (8.2). With this definition, the lateral pressure on the wall can be expressed in terms of pressure due to an equivalent fluid rather than the actual soil so that pressure at depth H is equal to w_eH instead of $K_a\gamma H$. The chief advantage of this procedure is that ordinary retaining walls can be designed for standardized soil conditions instead of using soil data that must be obtained through geotechnical investigations. The Rankine theory is the basis of the equivalent fluid method. Therefore, this method may be used where the Rankine earth pressure is applicable, and only when the backfill is free-draining.

Codes and specifications have traditionally required that earth pressure be determined based on specific values of equivalent fluid pressure based on the properties of soils. These requirements are based on the assumption that ideal conditions for active pressure would develop. These conditions were discussed in previous paragraphs. For example, a common practice in the past required that wall retaining drained soil, where the surface of the retained soil is level, be designed for a load, H, equivalent to that exerted by a fluid weighing not less than 30 lb/ft^2 per foot of depth and having depth equal to that of the retained soil.

The 2009 IBC (Section 1610.1) [8.26] specifies the values of minimum equivalent fluid weight (referred to as soil lateral load) as shown in Table 8.5, which can be used for preliminary design of retaining structures in lieu of actual data. However, as pointed out in the footnote to that table, data from geotechnical investigation would govern. This is particularly true when the backfill has a slope. Also listed in this table are the soil types considered unsuitable as backfill material. Section 1610.1 also mandates the following considerations for determining earth pressures for specific conditions:

1. Foundation walls and other walls in which horizontal movement is restricted at the top shall be designed for at-rest pressure.
2. Foundation walls extending not more than 8 ft (2438 mm) below grade and laterally supported by the top by flexible diaphragms shall be permitted to be designed for active earth pressure.
3. Retaining walls free to move and rotate at the top shall be permitted to be designed for active earth pressure.

TABLE 8.5 Soil Lateral Loads (adapted from Ref. [8.26])

Description of the backfill material[e]	Unified soil classification	Design soil lateral load[a] (psf per foot of depth) active pressure	At-rest pressure
Well graded, clean gravels, gravel-sand mixes	GW	30[c]	60
Poorly graded clean gravels, gravel-sand mixes	GP	30[c]	60
Silty gravels, poorly graded gravel and clay mixes	GM	40[c]	60
Well-graded clean sands, gravelly sand mixes	GC	45[c]	60
Clayey gravels, poorly graded gravel and clay mixes	SW	30[c]	60
Poorly-graded clean sands, sand-gravel	SP	30[c]	60
Silty sands, poorly graded sand-silt mixes	SM	45[c]	60
Sand-silt clay mix with plastic fines	SM-SC	45[d]	100
Clayey sands, poorly graded sand-clay mixes	SC	60[d]	100
Inorganic silts and clayey silts	ML	45[d]	100
Mixture of inorganic silt and clay	ML-CL	60[d]	100
Inorganic clays of low to medium plasticity	CL	60[d]	100
Organic silt and silt clays, low plasticity	OL	See footnote b.	See footnote b.
Inorganic clayey silts, elastic silts	MH	See footnote b.	See footnote b.
Inorganic clays of high plasticity	CH	See footnote b.	See footnote b.
Organic clays and silty clays	OH	See footnote b.	See footnote b.

[a]Design lateral soil loads are given for moist conditions for the specified soils at their optimum densities. Actual filed conditions shall govern. Submerged or saturated soils pressures shall include the weight of the buoyant soil plus the hydrostatic loads.

[b]Unsuitable as backfill material.

[c]For relatively rigid walls, as when braced by floors, the design lateral soil load shall be increased for sand and gravel type soils to 60 psf/ft of depth. Basement walls extending not more than 8 ft below grade and supporting flexible floor systems are not considered as being relatively rigid walls.

[d]For relatively rigid walls, as when braced by floors, the design lateral load shall be increased for silt and clay type soils to 100 psf /ft of depth. Basement walls extending more than 8 ft below grade and supporting flexible floor systems are not considered as being relatively rigid walls.

[e]The definition and classification of soil materials shall be in accordance with ASTM D2487.

4. Design lateral pressure from surcharge loads shall be added to the lateral earth pressure load.
5. Design lateral pressure shall be increased if soils with expansion potential are present at the site.

8.3.2.2 Coulomb's Earth Pressure Theory Coulomb's earth pressure theory, published in 1776 [8.27], takes into consideration the friction between the back of the wall and the soil, and assumes that active earth pressure results from the tendency of a wedge of soil to slide against the surface of the retaining wall. Therefore, this earth pressure theory is sometimes called *wedge theory*. The surface of sliding in the backfill of a retaining wall is slightly curved but, in order to simplify earth pressure calculations, the plane of sliding is assumed to be plane. The error in disregarding the curvature is quite small, however [8.1]. The expression for coefficient of active pressure according to Coulomb's theory is given by Eq. (8.13):

$$K_a = \frac{\sin^2(\beta+\phi)}{\sin^2\beta\sin(\beta-\delta)\left[1+\sqrt{\dfrac{\sin(\phi+\delta)\sin(\phi-\alpha)}{\sin(\beta-\delta)\sin(\alpha+\beta)}}\right]^2} \tag{8.13}$$

where β = angle between the wall and the backfill
ϕ = angle of internal friction
δ = angle of friction between the back of wall and the soil
α = angle of surcharge

Various angles referred to in Eq. (8.13) are defined in Fig. 8.12. Interestingly, if the wall friction is ignored (i.e., if $\delta = 0°$), for vertical wall back ($\beta = 90°$) and level backfill ($\alpha = 0°$), Eq. (8.13) reduces to

$$K_a = \frac{\cos^2\phi}{(1+\sin\phi)^2} = \frac{1-\sin\phi}{1+\sin\phi} \tag{8.14}$$

which is the same as Eq. (8.3). That is, under certain conditions, both Rankine's and Coulomb's theories give the same results.

Retaining walls are generally constructed from concrete or masonry. Table 8.6 [8.14] gives the range of wall friction angle δ for a few selected types of soils.

TABLE 8.6 General Range of Wall Friction Angles for Selected Soils [8.14]

Type of soil	Range of δ (deg)
Gravel	27–30
Coarse sand	20–28
Fine sand	15–25
Stif clay	15–20
Silty clay	12–16

8.3.3 Selection of Applicable Earth Pressure Theory

For practical design of retaining walls, applicable deign codes/specifications should be referred to regarding the particular earth pressure theory that may be used. For example, AASHTO Specifications [8.11] specify that "for yielding walls, lateral pressures shall be computed assuming active stress conditions and wedge theory using a planer surface of sliding defined by Coulomb Theory." However, in most cases, the angle of wall friction, δ is difficult to ascertain, and the value of the angle of internal friction, ϕ, is only approximately assessed. Therefore, for design of cantilever walls of ordinary proportions, assumption of $\delta = 0$ is justified, and Rankine theory is used. This gives a slightly conservative result [8.23].

Development of a state of stress in the backfill behind a retaining wall requires an outward rotation of the wall about its toe. The amount of movement of a retaining structure is often called the *yield*. Walls that can tolerate little or no movement away from the backfill should be designed for at-rest pressure condition. Walls which can move away from the backfill should be designed for pressures between at-rest and active conditions. Movement required to reach minimum active pressure or the maximum passive pressure is a function of the wall height and the soil type. It was noted earlier that actual retaining walls usually yield sufficiently to develop the active state of earth pressure. Table 8.7 gives approximate values of relative movements, for different types of soil backfill, to reach minimum active or maximum passive pressure conditions behind retaining walls. It can be seen from Table 8.7 that a 20-ft high wall retaining dense sand backfill would require a forward movement of a mere two-hundredths of an inch to mobilize active earth pressure.

TABLE 8.7 Approximate Values of Relative Movements Required to Attain Active and Passive Pressure Conditions [8.12]

	Wall rotation, Δ/H^*	
Soil type and condition	Active	Passive
Dense sand	0.001	0.01
Medium dense sand	0.002	0.02
Loose sand	0.004	0.04
Compacted silt	0.002	0.02
Compacted lean clay	0.010	0.05
Compacted fat clay	0.010	0.05

*Δ = movement of top of wall required to reach minimum active or maximum passive pressure by tilting or lateral translation
H = height of wall

8.3.4 Effects of Backfill Surcharge

Often the surface of the backfill in the vicinity of a retaining wall is subjected to various types of vertical loads. Presence of this loading is referred to as *surcharge*, and it induces additional pressure on the back of the wall. Two common examples of surcharge loads are the surface of the backfill behind a bridge abutment which is always subjected to vehicular loading (i.e., vehicular live load), and load due to vehicles on an embankment (e.g., parking area) that is supported by a retaining wall. When surcharge loads are applied, earth pressure computations are often made by substituting the load by an equivalent surcharge layer, h_{eq}. According to AASHTO-LRFD Specification [8.12], the increase in horizontal earth pressure due to high live load surcharge can be determined from Eq. (8.15):

$$\Delta_p = k\lambda_s h_{\text{eq}} \tag{8.15}$$

where Δ_p = constant horizontal earth pressure due to live load surcharge
γ_s = unit weight of soil
k = coefficient of lateral earth pressure
h_{eq} = equivalent height of soil for vehicular load

TABLE 8.8 Equivalent Height of Soil, h_{eq}, for Vehicular Loading on Highway Bridge Abutments and Retaining Walls [8.12]

Abutment or retaining wall height	Equivalent height of soil, h_{eq} (ft)		
	Abutment	Retaining walls: Distance from wall back face to edge of traffic	
		0.0 ft	1.0 ft or further
5.0	4.0	5.0	2.0
10.0	3.0	3.5	2.0
≥20 ft	2.0	2.0	2.0

Magnitudes of surcharge loads are often specified by applicable design codes/specifications. For example, for highway bridge abutments and retaining walls, AASHTO Specification provides the equivalent heights of soil for vehicular loading on abutments (perpendicular to traffic) and retaining walls (parallel to traffic) as listed in Table 8.8. In both cases, the wall height is taken as the distance between the surface of the backfill and bottom of the footing along the pressure surface being considered. In the case of retaining walls, the earth pressure due to live load surcharge depends on the distance from the wall back face to the edge of the traffic.

Another factor akin to surcharge that may cause increased lateral pressure is the effect of compacting backfill in confined areas behind retraining walls. As a result of backfill compaction in the vicinity of the wall, lateral pressures greater than active or at-rest values may develop. Such possibilities should be properly accounted for in design.

For a surcharge that is uniform over the backfill, the effects of surcharge are accounted for by assuming an equivalent soil height h_s acting on the wall in addition to the actual backfill height H. The horizontal pressure due to surcharge is assumed uniform over the height of the retaining wall (Fig. 8.14). The lateral pressure due to surcharge is given by Eq. (8.16):

$$p_s = K_a \gamma_s \tag{8.16}$$

The resultant earth pressure due to surcharge, which is assumed to act at wall midheight (i.e., at 0.5H) is given by Eq. (8.17):

$$P_s = K_a \gamma_s H_s \tag{8.17}$$

For partial uniform surcharge loads acting at a distance from the back of the wall, the effect of surcharge is as shown in Fig. 8.15. In this case, only a portion of the surcharge load affects the wall. The effective height of pressure H' is the vertical distance between point B on the wall and the base of the wall, which is based on load dispersion at an angle of 45° with the horizontal. For the cases when surcharge consists of a *line load* or a *strip load*, different models are used to calculate increase in earth pressures due to their presence. A discussion on this topic can be found in texts on foundation engineering [8.14, 8.23], and is not discussed here.

8.3.5 Seismic Earth Pressure

For earth retaining structures that are likely to be subjected to seismic forces, the effects of wall inertia and probable amplification and/or mobilization of passive masses by earthquakes should be accounted for in design.

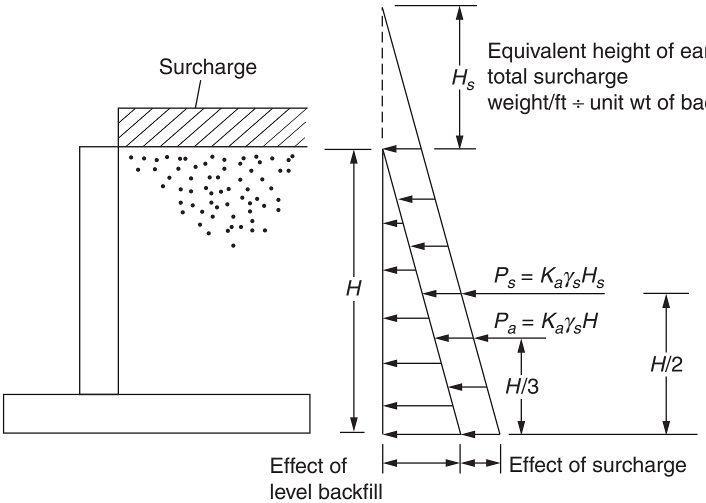


FIGURE 8.14 Pressure distribution due to surcharge from uniform load close to the wall.

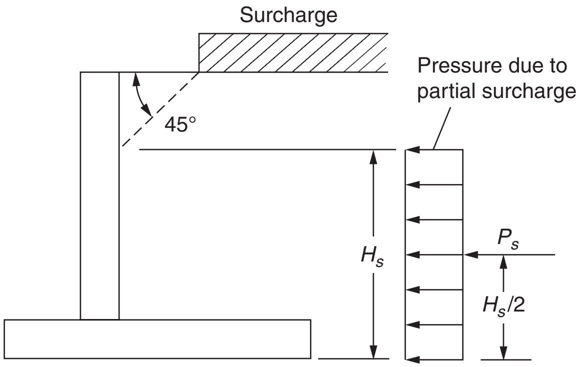


FIGURE 8.15 Pressure distribution due to surcharge under partial uniform load at a distance from the back of a wall.

In seismic events, the backfill behind a retaining wall responds dynamically, resulting in additional lateral pressure called *seismic earth pressure*. Several studies [8.28–8.41] have been carried out to study the response of cantilever retaining walls. These studies may be classified into two groups [8.41]: (1) elastic analysis in which the wall is considered to be fixed against both deflection and rotation at the base, and the backfill is presumed to respond as a linearly elastic or viscoelastic material, and (2) limit-state analyses in which the wall is considered to displace sufficiently at the base to mobilize the full shearing strength of the backfill.

The most widely used methods for estimating seismic active and passive pressures are based on the pseudo-static approach developed in 1920s by Mononabe [8.28] and Okabe [8.29] commonly known as the Mononabe-Okabe (M-O) analysis. It is based on an extension of the Coulomb sliding wedge theory (discussed earlier) taking into account horizontal and vertical inertia forces acting on the soil. However, the effects of wall inertia are neglected in the analysis. A detailed discussion on this method has been provided by Seed and Whitman [8.30] and summarized by Elms and Martin [8.32]. The M-O analysis or methodology is now a part of AASHTO-LRFD Specification [8.12] which is summarized as follows. The following assumptions are made in the M-O analysis:

1. The abutment is free to move sufficiently so that the soil strength will be mobilized. If the abutment is rigidly fixed and unable to move, the soil forces will be very much higher than those predicted by the M-O analysis.
2. The backfill is cohesionless, with a friction angle of ϕ.
3. The backfill is unsaturated and thus, not subject to liquefaction.

The seismic active earth pressure can be estimated by considering the equilibrium of the soil wedge behind the retaining wall when the abutment is at the point of failure (Fig. 8.16). Under this condition, the resultant active earth pressure, E_{AE}, is given by Eq. (8.18):

$$E_{AE} = \frac{1}{2}\gamma H^2(1-k_v)K_{AE} \tag{8.18}$$

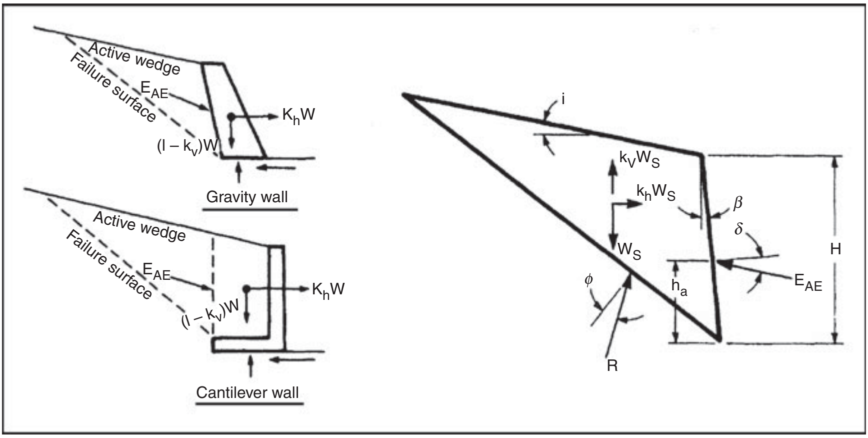


FIGURE 8.16 Active earth pressure due to earthquake.

where the seismic active earth pressure coefficient, K_{AE}, is given by Eq. (8.19):

$$K_{AE} = \left[\frac{\cos^2(\phi-\theta-\beta)}{\cos\theta\cos^2\beta\cos(\delta+\beta+\theta)}\right]\left[1+\sqrt{\frac{\sin(\phi+\delta)\sin(\phi-\theta-i)}{\cos(\delta+\beta+\theta)\cos(i-\beta)}}\right]^{-2} \tag{8.19}$$

where γ = unit weight of soil
H = height of backfill
ϕ = angle of internal friction of soil (°)
θ = arc tan $(k_h/(1-k_v))$ (°)
δ = angle of friction between the backfill and the wall
i = backfill slope angle
β = slope of wall to vertical (negative as shown) (°)
k_h = horizontal acceleration coefficient
k_v = vertical acceleration coefficient

An examination of the trigonometric terms in Eq. (8.19) shows that solutions are only possible for the case $\phi \leq (\theta + i)$. The distribution of total earth pressure including dynamic effects is different than that of purely static active earth pressure which is linear and the resultant acting at $H/3$ from the base of the wall. The resultant of the dynamic component of the earth pressure is assumed to act at $0.6H$ from the base of the wall [8.31]. As suggested by Elms and Martin [8.32] and specified in AASHTO-LRFD Specifications [8.12], it would be sufficient to assume distribution of total earth pressure (including dynamic effects) as uniform over the height of the wall, with the resultant acting at $H/2$. The equivalent expression for the passive force if the wall is being pushed into the backfill is given by Eq. (8.20):

$$E_{\mathrm{PE}} = \frac{1}{2}\gamma H^2 (1 - k_v) K_{\mathrm{PE}} \tag{8.20}$$

where the seismic passive earth pressure coefficient, K_{PE}, is given by Eq. (8.21):

$$K_{AE} = \left[\frac{\cos^2(\phi - \theta + \beta)}{\cos\theta \cos^2\beta \cos(\delta - \beta + \theta)}\right]\left[1 - \sqrt{\frac{\sin(\phi + \delta)\sin(\phi - \theta + i)}{\cos(\delta - \beta + \theta)\cos(i - \beta)}}\right]^{-2} \tag{8.21}$$

It is pointed out that as the seismic inertial angle θ increases, the values of seismic earth pressure coefficients, K_{AE} and K_{PE}, approach each other and, for vertical backfill, become equal when $\theta = \phi$. Further discussion of dynamic earth pressure is beyond the scope of this book and is not discussed further. A discussion on seismic earth pressure calculations and examples of SRW can be found in Ref. 8.42.

8.4 EXTERNAL STABILITY OF A RETAINING WALL

8.4.1 Stability Considerations

2009 IBC Section 1806.2 [8.26] requires that retaining wall be deigned to ensure stability against overturning, sliding, excessive foundation pressure, and water uplift. Analysis for external (or structural) stability of a retaining wall involves five components [8.14]:

1. Stability against overturning
2. Stability against sliding
3. Bearing capacity failure of the base
4. Settlement failure
5. Overall stability

The first three aspects of stability analysis are discussed in this section. For the last two aspects of stability analysis, readers are referred to texts on foundation engineering [8.14, 8.23]. Analysis for *overall stability* involves checking for *shallow shear failure* and *deep shear failure* [8.14]. The following factors of safety (FS) are generally required to ensure the safety of a retaining wall [8.14]:

(a) Factor of safety against overturning = 1.5 (for granular backfill) = 2.0 (for cohesive backfill)
(b) Factor of safety against sliding = 1.5
(c) Factor of safety against bearing failure = 3.0

In addition, it must be ensured that the bearing pressure under the footing does not exceed the allowable bearing capacity of the underlying soil. All three aspects of stability are discussed in the following sections.

8.4.2 Stability against Overturning

A retaining wall is likely to overturn about its toe as result of overturning moment produced by the lateral pressure from the backfill and surcharge (if any). This overturning moment is resisted by the stabilizing moment produced by the dead weight of the soil above the base of the wall and the self-weight of the wall. Most design specifications require that the stabilizing moment be at least 50 percent greater than the overturning moment. For example, 2009 IBC Section 1807.2.3 requires that design of retaining walls must be based on a safety factor of at least 1.5. The factor of safety against overturning, FS_{OT}, can be expressed by Eq. (8.22):

$$FS_{OT} = \frac{\text{total stabilizing moment}}{\text{total overturning moment}} = 1.5 \text{ to } 2.0 \tag{8.22}$$

It is common practice to neglect, in stability analysis, the passive resistance of the soil in front of the wall that may be present. If the contribution of the passive pressure is included in the total stabilizing moment, for example, in cases where the toe is covered by a large soil depth, a reduced depth of soil should be considered to account for seasonal influence, erosion, possible future excavation, and tension cracks in cohesive soils [8.23]. AASHTO Specifications [8.12] specify that "the resistance due to passive earth pressure in front of the wall shall be neglected unless the wall extends well below the depth of frost penetration, scour, or other types of disturbance (e.g., a utility trench excavation in front of the wall)." An interesting discussion on stability of retaining walls against overturning has been presented by Greco [8.43].

8.4.3 Stability against Sliding

The lateral pressure due to the backfill and surcharge tend to push the wall forward horizontally, causing the wall to slide along the base of the wall. This tendency to slide is resisted by the passive resistance of soil present in front of the wall and the frictional resistance between the soil and the wall footing. The frictional resistance between the wall footing and the foundation soil depends on the physical conditions at their interface. For example, if the bottom of the footing is rough (as in the case of concrete footing poured directly on soil), the coefficient of friction is taken as being equal to $\tan\phi$ (ϕ = the angle of internal friction of soil). A smaller value of ϕ may be used if the original soil is relatively dense.

Values of coefficient of friction between wall footing and the underlying soil are often provided by design codes and specifications. Designers should obtain the values of coefficient of friction from pertinent sources as applicable as these values might be different in different codes. Values of coefficients of friction specified in AASHTO and AREA Specifications are given in Tables 8.9 and 8.10, respectively.

The factor of safety against sliding can be calculated from Eq. (8.23):

$$FS_{\text{Sliding}} = \frac{\text{horizontal resistance}}{\text{sliding force}} \geq 1.5 \tag{8.23}$$

TABLE 8.9 Coefficients of Friction Between Wall Footing and Soil from AASHTO-LRFD Specifications [8.12]

Interface materials	Friction factor, f (tan δ)	Friction angle, δ (degrees)
Mass concrete on the following materials:		
Clean sound rock	0.70	35
Clean gravel, gravel-sand mixtures, coarse sand	0.55–0.60	29–31
Clean fine to medium sand, silty medium to coarse sand, silty, or clayey gravel	0.45–0.55	24–29
Clean fine sand, silty or clayey fine to medium sand	0.35–0.45	19–24
Fine sandy silt, nonplastic silt	0.30–0.35	17–19
Very stiff and hard residual or preconsolidated clay	0.40–0.50	22–26
Medium stiff and stiff clay and silty clay	0.30–0.35	17–19
(Masonry on foundation materials has same friction factors)		

TABLE 8.10 Coefficients of Friction Between Wall Footing and Soil from AREA Specifications [8.15]

Type of foundation soil	Coefficient of friction, μ
Coarse-grained soils (without silt)	0.55
Coarse-grained soils (with silt)	0.45
Silt	0.35
Sound rock	0.60

It is common practice to provide a factor of safety against sliding of at least 1.5 as required by 2009 IBC Section 1807.2.3 [8.26]. The horizontal resistance in Eq. (8.23) is computed as the product of the gravity loads (i.e., the weight of the backfill and the self-weight of the wall) and the coefficient of friction, μ, between the concrete footing and the supporting soil. The values of coefficient of friction would depend on the type of foundation soil. Typical values of μ used in practice vary from 0.45 to 0.55, with the lower value used if some silt is present; a value as high as 0.6 is used when the footing is supported on sound rock or rough surface.

In the course of the stability analysis of a retaining wall, the calculated factor of safety against sliding might sometimes be less than 1.5. This can happen because the stability of a wall can be checked only after the wall has been sized (i.e., preliminary sizes of the footing and the wall would have been assumed). In such cases, the most common practice is to increase the mass supported by the footing. This can be attained by increasing width of the heel, which results in increased mass of the soil supporting directly on it, thus increasing the gravity loads. Thus, sizing or proportioning a retaining wall is an iterative process which must be completed before engaging in structural design.

An alternative sometimes used by designers is to provide a shear key in the footing, with its front face cast against undisturbed soil, to increase the resistance to sliding forces. The purpose of a shear key is to develop passive pressure in front of and below the base of the footing, which opposes the sliding forces. It recommended that the resistance due to passive pressure due to soil in front of the shear key only may be considered in stability analysis, and that too as a measure of last resort only. A shear key is typically square in cross section, with depth varying between two-thirds and full depth of the footing. It is generally located under the stem so that the vertical reinforcing bars from the stem may be extended into the key, with no other reinforcement provided. It should be noted that the benefit of providing a shear key is generally small unless the key is embedded in a rock or hard soil.

8.4.4 Bearing Pressure under the Footing

Computation of bearing pressure under the footing is an integral design component for design of retaining walls. The foundation pressure must not exceed the allowable bearing capacity of the soil or the footing should be proportioned so as to provide a safety factor of at least three over the ultimate bearing capacity [8.14, 8.44, 8.45]. Because in a retaining wall there is net moment about the toe, the foundation pressure results from the combined loading condition, that is, due to gravity load (i.e., total vertical load) as well as moment (caused by horizontal earth pressure). The foundation pressure can be calculated from Eq. (8.24):

$$p = \frac{P}{A} \pm \frac{Mc}{I} = \frac{P}{bd} \pm \frac{6Pe}{bd^2} \tag{8.24}$$

where p = foundation pressure in the soil
P = total vertical load
A = area of footing = bd
b = unit length of footing = 1 ft
d = width of footing
I = moment of inertia of footing of unit length
M = net moment about the toe of the footing = $M_R - M_O$
M_R = resisting moment
M_o = overturning moment
e = eccentricity of the resultant force at the base

In Eq. (8.24), the "+" sign gives the maximum foundation pressure (at the end of toe) and "–" sign gives the minimum foundation pressure (at the end of heel). The eccentricity e is measured as the distance between the point of application of the resultant at the base and the midpoint of the base. The point of application of the resultant, located at a distance $\bar{x}$ from the toe, can be determined from Eq. (8.25):

$$\bar{x} = \frac{M_{\text{net}}}{\sum V} = \frac{\sum M_R - \sum M_O}{\sum V} \tag{8.25}$$

Knowing the value of $\bar{x}$ from Eq. (8.25), the eccentricity can be determined from Eq. (8.26):

$$e = \frac{d}{2} - \bar{x} \tag{8.26}$$

Under normal conditions, the maximum foundation pressure, $p_{\max}$, occurs under the toe and the minimum foundation pressure, $p_{\min}$, occurs under the heel. It should be ensured that $p_{\max}$ does not exceed the allowable bearing capacity of the soil. Also, the point of application of the resultant force must lie in the middle-third of the base to preclude the possibility of tension in the base.

8.5 DESIGN PROCEDURE FOR MASONRY RETAINING WALLS

Example 8.1 illustrates design of a cantilever retaining wall to be constructed from concrete masonry units. The general design principles remain the same as discussed in Chaps. 4 and 6. Readers are urged to familiarize themselves with the material presented in those chapters. The first step in designing a retaining wall is to develop a trial cross section of the wall (i.e., the width and thickness of concrete footing and the thickness of the masonry wall). The second step is to check the stability of wall against overturning and sliding as discussed in the preceding section. Earth pressure behind the wall should be determined as discussed in earlier sections. The minimum factor of safety in each case should be at least 1.5. In the following examples, the walls are grouted solid in order to increase the mass of the wall so as to increase resistance to overturning and sliding. In addition, the bearing pressure under the footing should not exceed the allowable soil pressure, or should provide a safety factor of at least three with respect to the ultimate bearing capacity [8.14, 8.44].

It would be recalled that lateral loads acting on a parapet due to wind or earthquake (discussed in Chap. 6) were assumed uniform over its height. By contrast, earth pressure loading on a retaining wall has a linear variation (i.e., the loading diagram has a triangular profile)—maximum pressure at the base and zero at the top (except in the case of surcharge loads). Thus, for a cantilever wall of height *H*, the maximum moment at the base, *M*, can be expressed as given by Eq. (8.27):

$$M = \frac{w_e H^3}{6} \tag{8.27}$$

Similarly, maximum shear, *V*, at the base due to soil having an equivalent fluid weight w_e can be expressed as given by Eq. (8.28)

$$V = \frac{w_e H^2}{2} \tag{8.28}$$

Retaining walls, as a rule, are not subjected to any external gravity loads. The dead load of the wall itself is rather small which, although must be taken into account for stability analysis, is ignored as far as axial stresses in the wall are concerned. Thus, the wall is designed for flexure only and checking for combined loading condition is not required.

The moment and shear determined from Eqs. (8.27) and (8.28), respectively, correspond to service load-level forces. For strength design of the retaining wall, these forces must be recalculated at the strength level by using appropriate load combinations listed in ASCE 7-05 Section 2.3. The following load combinations are pertinent to lateral loads due to earth pressure *H*:

$$U = 1.2\,(D + F + T) + 1.6(L + H) + 0.5(L_r \text{ or } S \text{ or } R) \tag{8.29}$$

$$U = 0.9D + 1.6W + 1.6H \tag{8.30}$$

$$U = 0.9D + 1.0E + 1.6H \tag{8.31}$$

Based on Eq. (8.29), the factored moment and shear corresponding to service load-level values given by Eqs. (8.27) and (8.28) can be expressed as follows:

$$M_u = (1.6)\frac{w_e H^3}{6} \tag{8.32}$$

$$V_u = (1.6)\frac{w_e H^2}{2} \tag{8.33}$$

Eqs. (8.32) and (8.33) would be used for the strength design of retaining wall. It is noted that while the stem of a masonry retaining wall will consist of masonry units, the footing will be constructed from reinforced concrete. Design of concrete footings should comply with the requirements of the ACI *Code* [8.46], but it is not included in the example presented here. Readers should refer to texts on design of reinforced concrete for the treatment of reinforced concrete footings.

Example 8.1 Design a concrete masonry retaining wall for a backfill that is 8 ft high from the top of the footing. Assume equivalent fluid pressure as 45 lb/ft^3. Use f'_m =1800 lb/in.^{2} and Grade 60 reinforcement. A factor of safety of at least 2.0 must be provided against overturning and 1.5 against sliding. The allowable bearing capacity of the soil is 2500 lb/ft^2. The coefficient of friction between concrete and soil may be taken as 0.5.

Solution

Step 1: Choose approximate dimensions of the retaining wall: The height of the retained soil above the ground is 6 ft. The bottom of the footing will be placed 3 ft below the ground level on firm and undisturbed soil. This gives the total height of backfill as 9 ft from the bottom of footing. A 12-in.-thick footing (as a minimum) is selected, which gives the height of stem as 8 ft above the top of footing. The passive resistance of the soil is ignored (see discussion in the preceding section), consequently, a slightly wider than normal footing will be required. Assuming a footing width equal to three-fourths of the total height of the wall, $b = 0.75H = (0.75)\ (9.0) = 6.75$ ft. The preliminary cross section of the wall is shown in Fig. E8.1A. It is pointed out that the other wall cross sections were tried with $b = 5$ ft 4 in. and $b = 6$ ft, respectively, both were found to be unsafe against sliding. A 10-in.-thick CMU stem has been selected (normal weight CMU, solid grouted, weighing 104 lb/ft^2) for the preliminary design. An 8-in.-thick CMU stem was also tried, but was found to be structurally unsafe. Calculations for the trial size are not shown here.

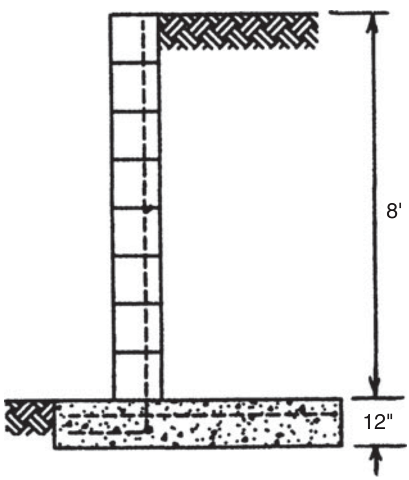


FIGURE E8.1A Preliminary cross section of retaining wall.

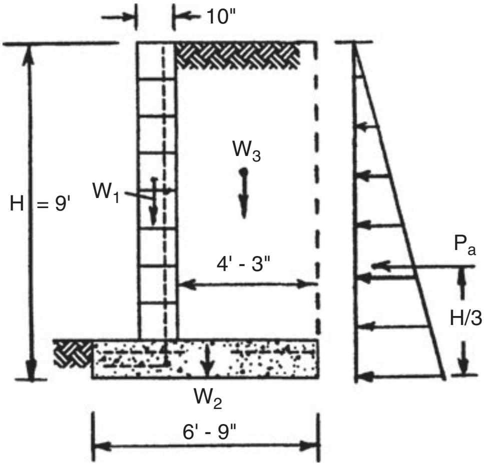


FIGURE E8.1B Forces acting on the retaining.

Step 2: Check the stability of the wall: Calculate various forces acting on the wall (Fig. E8.1B) and moments about the toe. These calculations are organized in the following table:

Force	Lever arm (ft)	Stabilizing moment (lb-ft)
Stem: $W_1 = (8)(104) = 832$	2.1	1747
Footing: $W_2 = (6.75)(1.0)(150) = 1013$	3.375	3419
Backfill: $W_3 = (4.25)(8.0)\ (110) = 3740$	4.625	17,298
$\Sigma V = 5585$		$\Sigma M_R = 22{,}464$

The resultant of active earth pressure, P_a, is given by

$$P_a = \frac{1}{2} w_e H^2 = \left(\frac{1}{2}\right)(45)(9)^2 = 1823\text{ lb}$$

which acts at 9/3 = 3 ft from the bottom of the footing. Therefore, the overturning moment is

$$M_O = (P_a)\ (H/3) = (1823)\ (3.0) = 5469 \text{ lb-ft}$$

(a) *Factor of safety against overturning:* From Eq. (8.20), the factor of safety against overturning is

$$(FS)_{\text{OT}} = \frac{\text{total stabilizing moment about toe}}{\text{total overturning moment}} = \frac{M_R}{M_O} = \frac{22{,}464}{5469} = 4.11 > 2.0 \text{ OK}$$

(b) *Factor of safety against sliding:* The factor of safety against sliding is computed as the ratio of total resisting force to total sliding force [Eq. (8.23)]:

$$\text{Total resisting force} = \mu\Sigma V = (0.5)\ (5585) = 2793 \text{ lb}$$

$$\text{Total sliding force } P_a = 1823 \text{ lb}$$

$$(FS)_{\mathrm{SL}} = \frac{\text{total resisting force}}{\text{total sliding force}} = \frac{2793}{1823} = 1.53 > 1.5 \text{ OK}$$

(c) *Check the bearing pressure:* The maximum and minimum bearing foundation pressures are given by Eq. (8.24)

$$p = \frac{P}{A} \pm \frac{Mc}{I} = \frac{P}{bd} \pm \frac{6Pe}{bd^2}$$

Locate the point of application of resultant force at the base from Eq. (8.25). It is located at $\bar{x}$ from toe:

$$\bar{x} = \frac{M_{\text{net}}}{\sum V} = \frac{22{,}464 - 5469}{5585} = 3.04 \text{ ft}$$

The eccentricity at the base, e, is given by Eq. (8.26):

$$e = \frac{d}{2} - \bar{x} = \frac{6.75}{2} - 3.04 = 3.375 - 3.04 = 0.335 \text{ ft}$$

The maximum and minimum foundation pressures are computed from Eq. (8.24) as

$$p_{\max} = \frac{5585}{(1.0)(6.75)} + \frac{(6)(5585)(0.335)}{(1.0)(6.75)^2} = 827.4 + 246.4 = 1073.8 \text{ lb/ft}^2$$

$$p_{\min} = \frac{5585}{(1.0)(6.75)} - \frac{(6)(5585)(0.335)}{(1.0)(6.75)^2} = 827.4 - 246.4 = 581 \text{ lb/ft}^2$$

The allowable bearing capacity is, $q = 2500$ lb/ft$^2 > p_{max} = 1073.8$ lb/ft^2 OK.

Step 3. Structural design of wall: The structural design of the footing is omitted here. Readers should refer to standard texts on reinforced concrete design for design methods for footings. Structural design of CMU stem follows:

Height of the stem, $H = 8$ ft
Equivalent fluid pressure, $w_e = 45$ lb/ft^3
The design moment, M is given by Eq. (8.27):

$$M = \frac{w_e H^3}{6} = \frac{(45)(8.0)^3}{6} = 3840 \text{ lb-ft}$$

Factored bending moment on the wall $M_u = 1.6(3840) = 6144$ lb-ft
Calculate the nominal moment strength of the wall section:

$$M_n = A_s f_y \left(d - \frac{a}{2}\right)$$

In the above expression, there are two unknowns: A_s and a. Try No. 5 bars at 16 in. on center in the wall, $A_s = 0.23$ in.^{2}. For a 10-in.-nominal wall, the face shell thickness is 1.375 in. Assuming a ½-in thickness of grout between the reinforcement and the inside face of the masonry units, the design depth d is calculated to be

$$d = 9.625 - 1.375 - 0.5 - 0.5(0.625) = 7.44 \text{ in.}$$

Calculate the depth of the compression block from the following expression [as used in Eq. (4.8)]:

$$0.80 f'_m ab = A_s f_y$$

$$a = \frac{A_s f_y}{0.80 f'_m b} = \frac{(0.23)(60)}{0.80(1.8)(12)} = 0.8 \text{ in.}$$

Because $a = 0.8$ in. < 1.375 in. (face shell thickness); the compression stress block lies within the face shell and the wall section can be analyzed as a rectangular section. Thus, from Eq. (4.12),

$$\begin{aligned} M_n &= A_s f_y \left(d - \frac{a}{2}\right) \\ &= 0.23(60)\left(7.44 - \frac{0.8}{2}\right) = 97.2 \text{ k-in.} = 8096 \text{ lb-ft} \end{aligned}$$

$$\phi M_n = 0.9(8096) = 7285 \text{ lb-ft} > M_u = 6144 \text{ lb-ft}$$

Therefore, the flexural strength of the wall is satisfactory. Check the shear strength. The factored shear in the wall is given by Eq. (8.33):

$$V = (1.6)\frac{w_e H^2}{2} = (1.6)\frac{(45)(8)^2}{2} = 2304 \text{ lb}$$

The nominal shear strength of a masonry member is given by [Eq. (4.97)]

$$V_u = V_m + V_s$$

From Eq. (4.91),

$$V_m = \left[4 - 1.75\left(\frac{M_u}{V_u d_v}\right)\right] A_v \sqrt{f'_m} + 0.25 P_u$$

$$\frac{M_u}{V_u d_v} = \frac{(6144)(12)}{(2304)(9.625)} = 3.32 > 1.0 \qquad \text{(use 1.0.)}$$

In this example, $P_u = 0$ (no gravity load), $d_v = 9.625$ in. (10 in. nominal). Therefore,

$$V_m = (4 - 1.75) A_n \sqrt{f'_m} = (4 - 1.75)(9.625)(12)\sqrt{1800} = 11{,}026 \text{ lb}$$

$$\phi V_m = 0.8(11{,}026) = 8821 \text{ lb} > V_u = 2304 \text{ lb}$$

Hence, the section is safe in shear, and shear reinforcement is not required.

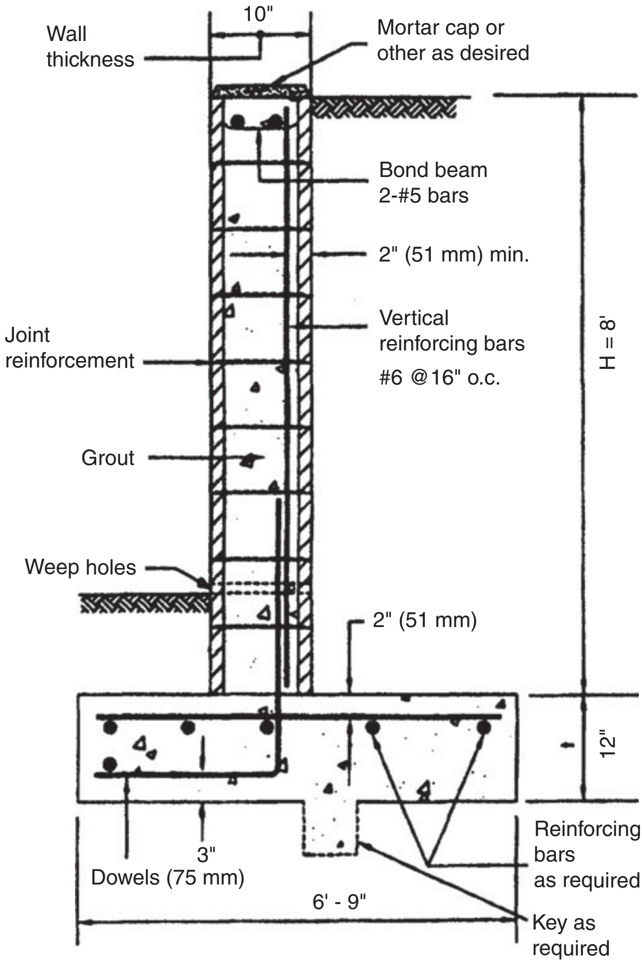


FIGURE E8.1C Details of the reinforced CMU cantilever wall.

Provide No. 5 Grade 60 bars @ 16 in. on center on the rear face of the wall (Fig. E8.1C) and solid grout the wall. Also, provide a bond beam with 2 No. 5 bars as the top course of the wall as shown in the figure. The top of the wall will be capped or otherwise protected to prevent ingress of moisture into the wall. In addition, provision should be made to prevent the accumulation of water behind the wall to reduce the effects of possible frost action. It is common practice to provide weep holes spaced 4 to 6 ft apart along the base of the wall. Alternatively or additionally, a continuous longitudinal drain may be provided along the back of the wall.

It should be noted that bottom of the wall is assumed to have a fixed-end condition at the interface with the footing. Therefore, it is extremely important that flexural reinforcement be properly spliced with the bars extending as dowels from the footing as required by the ACI *Code* [8.46]. The amount of lap of lapped splices must be sufficient to transfer the stress in the reinforcement as required by MSJC-08 Sections 3.3.3.3 and 3.3.3.4. The minimum length of lap for bars is greater of 12 in. or the development length as required by MSJC-08 Eq. (3-16).

8.6 SUBTERRANEAN OR BASEMENT WALLS

8.6.1 Purpose and Advantages of Subterranean Walls

Walls discussed in Chap. 6 were built *above grade*, and designed for gravity loads and lateral loads induced by wind or earthquakes. In many types of construction, walls must extend *below grade* in which case they are subjected to lateral soil pressures. Walls encountered in basements and subbasements of buildings and in subterranean garages fall into this category. Essentially, these walls function as retaining walls which resist lateral soil pressure in addition to supporting some form of gravity loads from the upper floors. This section presents a discussion on design of these types of walls.

Subterranean garages are commonly built as integral parts of individual parking facilities or as integral parts of multistory residential, commercial, hotel, and office buildings. Basements form integral part of many residential buildings, particularly in cold climates. They are built for good reasons. Typically, footings for buildings in these areas must be placed several feet below the frost line to prevent freezing damage. This requires extensive excavation for footing and walls. Also, the problem of snow accumulation at grade level necessitates providing the first floor of a building a few feet above grade. These two factors together require several feet of walls below the first floor of a building. A few extra feet of excavation provides enough height of walls, which is advantageously used to create living space called *basements*. Several potential benefits of basement construction include

1. Increased living space for recreation rooms, workshops, exercise rooms, office, storage, and mechanical equipment rooms
2. Additional main floor living space by moving mechanical equipment and stored items in the basement
3. Safe havens during hurricanes and tornadoes since the surrounding earth protects the basement area from high winds and flying debris
4. Comfortable living space which is free from external noise, cool in the summer, and free of drafts in the winter
5. Easy access to ductwork, plumbing, electrical wiring, and mechanical equipment
6. Easy access to underfloor structural members such as floor joists, sill plates, and other wood members for inspection of possible termite damage

8.6.2 Plain and Reinforced Subterranean Walls

Strength, durability, economy, and resistance to fire and termites makes masonry construction well suited for basement construction. Traditionally, residential basement walls have been constructed of plain or unreinforced masonry. Most unreinforced concrete masonry basement walls can easily support typical earth pressures which they must withstand. These walls depend on the tensile strength of masonry to resist the lateral loads. However, when extra strength is required, these walls can be easily and economically reinforced. Reinforced masonry also provides ductility and enables thinner walls to resist larger backfill pressures. Furthermore, in seismic regions, unreinforced masonry construction is simply not permitted by the applicable building codes, and thus the necessity of providing reinforced masonry construction.

8.6.3 Design Loads for Subterranean Walls

Subterranean walls are required to serve two functions simultaneously: to resist lateral pressure from the soil, and to support the floor above. The dominant loads on basement walls are exerted by the lateral soil pressure. The gravity loads are caused by the dead weight of the structure above the ground and its contents. These loads are usually relatively lighter in comparison to the lateral soil pressures exerted on subterranean walls. Consequently, the size of common subterranean walls is governed by lateral earth pressure. In fact, the compressive action of gravity loads on subterranean walls can be advantageously used to counteract tensile stresses produced by the lateral loads. However, influence of gravity loads must be considered for the case of combined loading (i.e., axial load plus flexure).

As for retaining walls, lateral soil pressures on basement walls vary depending on the type of soil, the degree of saturation, and any surcharge loads that may be present. Variation of soil pressure along the wall height is assumed to be linear. Granular soils with good draining properties constitute ideal backfill material since they exert the lowest lateral pressures. In contrast, soils containing large amounts of clay drain poorly, and can exert relatively high lateral pressures. Table 8.5 lists various types of soils and the equivalent fluid pressures which can be used for design purposes. For certain types of soils, particularly those containing certain types of clays, silts, silt-clays, and clays having high plasticity, the equivalent fluid pressure to be used in design should be based on geological investigations.

8.6.4 Design Considerations for Subterranean Walls

The shears and moments developed in basement or subterranean walls depend on the edge restraint conditions. The behavior of such walls is analogous to transversely loaded rectangular flat plates with various types of boundary conditions at the edges. Where several floors exist in a building (as in the case of basements and subbasements), walls may be treated as spanning vertically as continuous slabs supported by the floors. On the other hand, in a building with a single-level basement, the walls would span between the foundation and the floor above. Moments in the walls causing bending in the vertical plane may be positive, or both positive and negative depending on the nature of edge restraints at top and bottom of the wall (provided by the floors above and below). Figure 8.17 shows lateral loading conditions for subterranean walls.

1. If fixed-end conditions are assumed at the top and bottom, the portion of wall near these supports would develop negative moments at the top and bottom (meaning tension on the face of walls against the backfill). Away from these supports, the wall would be subjected to tension on the inside face of the wall (Fig. 8.17*a*).
2. If the wall is assumed fixed at the bottom and simply supported at top, akin to a propped cantilever, the maximum negative moment will occur at the bottom and the maximum positive moment will occur somewhere between the point of inflection and the simple support at the top (Fig. 8.17*b*). This support condition is commonly assumed for reinforced concrete subterranean walls.
3. If the wall is assumed simply supported at both top and bottom, the maximum moment will occur close to the midspan, and the face of the wall against the backfill will be in compression (Fig. 8.17*c*). This support condition is commonly assumed for masonry subterranean walls.

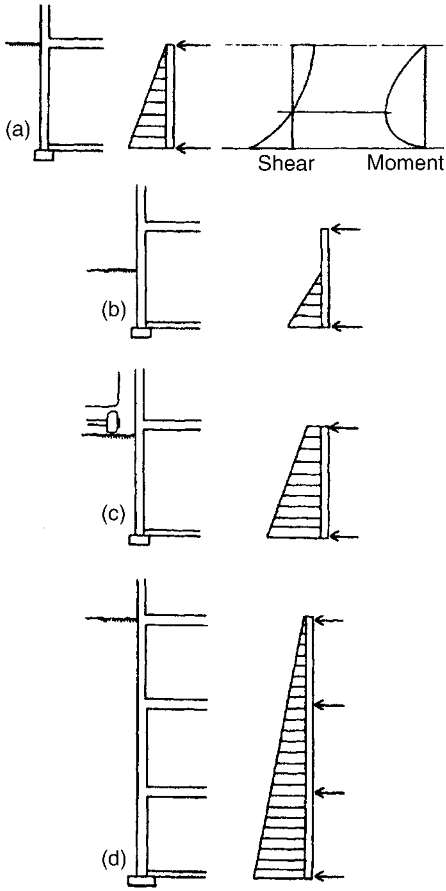


FIGURE 8.17 Various support conditions for subterranean walls. (*Courtesy: NCMA.*)

For single-level basement walls, maximum moments may be calculated based on the assumption that the walls act as simple vertical slabs laterally supported at the top and bottom. It should be noted that in some cases the height of backfill may be smaller than the height of the wall, a condition that occurs quite commonly (Fig. 8.18). In such cases, the upper part of the wall is subjected to wind or seismic loads, whereas the lower part is subjected to earth pressure. In some cases, the walls may span horizontally between intersecting walls, in which case the wall would need to be reinforced horizontally.

The reinforcing requirements for basement and subterranean walls are governed by the height and the type of backfill, the height of wall, any accompanying surcharge loads, and the support conditions of the wall. The soil pressures may produce tension on the inside or the outside face of the wall depending on the edge restraint conditions. In all cases, the reinforcing bars must be placed as close to the tension face of the wall as possible to afford the largest effective depth. A space of at least ½ in. should be maintained between the bar and the inside of the face shell to permit flow of grout completely around the bar.

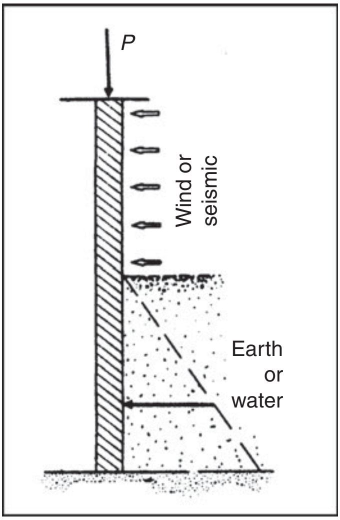


FIGURE 8.18 Equivalent lateral forces on basement walls due to lateral soil pressure [8.50].

To prevent the bars from moving out of their intended position, they should be secured in position with bar positioners at the top and bottom of the wall.

The design procedure for a subterranean wall is similar to that for a wall subjected to lateral load and axial load as discussed in Chap. 6 and not repeated here. Example 8.2 presents calculation procedure for a basement wall. References 8.48 through 8.50 provide useful information on the design and construction of residential masonry walls.

Example 8.2 Design of a subterranean (or basement) wall.

The basement wall in a building measures 11 ft 4 in. from the top of its footing to the top of floor that it supports as shown in Fig. E8.2A. The construction sequence requires the back fill to be placed behind the wall before the construction of the ground floor slab that the wall supports. Geotechnical investigation has determined earth pressure of 35 lb/ft^2/ft for the unrestrained condition (i.e., before the slab is constructed) and a uniform lateral pressure of 140 lb/ft^2 for the restrained condition (i.e., after the slab has been poured at the top of the wall). P-Δ effects may be ignored for designing the wall as the wall height is not large compared to its thickness. Use $f'_m = 1800$ psi and Grade 60 reinforcement.

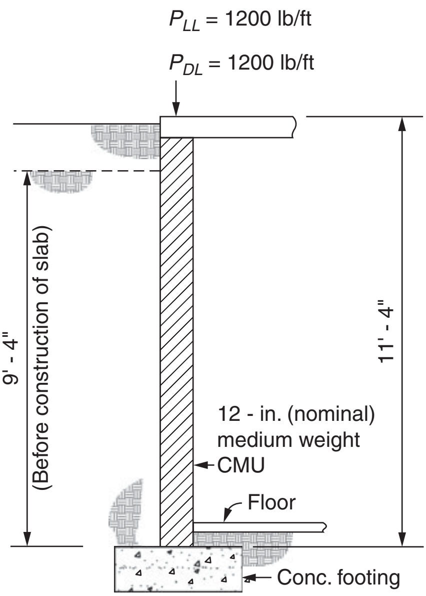


FIGURE E8.2A Preliminary cross section of the basement wall.

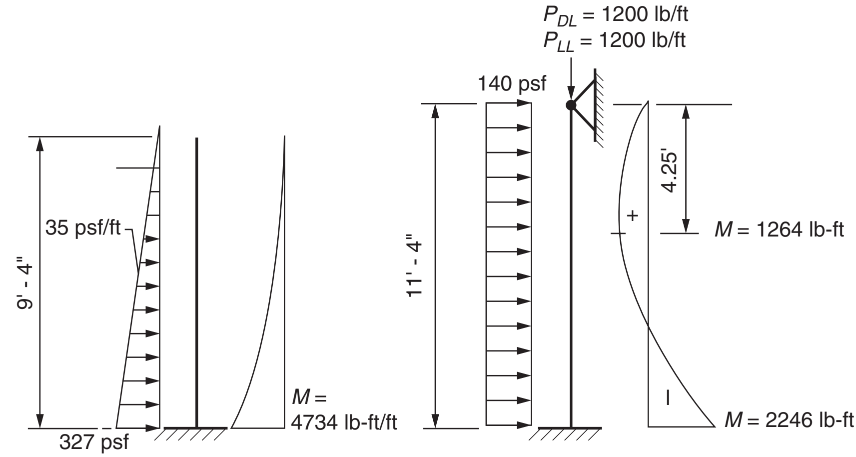


FIGURE E8.2B Forces on the basement wall.

Solution

The basement wall would be subject to two different loading conditions during the construction phase as follows:

1. Before the ground floor slab (which the wall supports at the top) is poured, the wall acts as a vertical cantilever subjected of linearly verifying earth pressure as shown in Fig. E8.2B.
2. After the slab is poured, the slab provides restraint at the top of the wall. Under this condition, the wall may be assumed to act as a vertical, propped cantilever, that is, fixed at the base and simply supported at the top. The forces acting on the wall under this condition are
   (a) Gravity load (dead load and live load) transmitted by the slab
   (b) A uniform lateral earth pressure of 140 lb/ft^2/ft

This second condition subjects the wall to maximum positive moment which causes tension on the inside face of the upper portion of the wall.

Because we do not know the thickness of the wall, let us begin with an assumed nominal wall thickness of 12 in. Assume also using medium weight CMU, grout weight of 140 lb/ft^3, and reinforcing bars at 16 in. on center.

Load condition 1: The wall acts as a vertical cantilever. Axial load (at the base of the wall) due to dead weight of wall, D = 94 lb/ft^2 for a nominal 12-in CMU wall grouted at 16-in. on center (Table A.19).

$$\text{Wall height} = 9.33 \text{ ft}$$

$$P_D = 94\,(9.33) = 877 \text{ lb}$$

The governing load combination is ASCE 7-05 Seismic Load Combination 7:

$$U = 0.9D + 1.6H$$

$$P_u = 0.9(877) = 789 \text{ lb}$$

$$M_u = 1.6H = 1.6\left(\frac{wh^3}{6}\right) = 1.6\left[\frac{(35)(9.33)^3}{6}\right] = 7580 \text{ lb-ft/ft}$$

Assume that reinforcement would be provided at the center of the wall. Therefore,

$$d = \frac{11.63}{2} = 5.81 \text{ in.}$$

The required amount of tensile reinforcement is calculated as

$$A_{S\text{reqd.}} = \frac{M_u - P_u \dfrac{t}{2}}{\phi f_y (0.9) d} = \frac{7580(12) - \dfrac{789(11.63)}{2}}{(0.9)(60{,}000)(0.9)(5.81)} = 0.31 \text{ in.}^2/\text{ft}$$

Provide No. 6 Grade 60 bars @ 16 in. on center, A_s = 0.33 in^2/ft (Table A.23). From MSJC Eq. (3.28), the depth of compression block is

$$\begin{aligned} a &= \frac{P_u + A_s f_y}{0.80 f'_m b} \\ &= \frac{789 + 0.33(60{,}000)}{0.80(1800)(12)} \\ &= 1.19 \text{ in.} < 1.5 \text{ in.(face-shell thickness)} \end{aligned}$$

The nominal strength of wall is determined from MSJC-08 Eq. (3-27):

$$\begin{aligned} M_n &= (A_s f_y + P_u)\left(d - \frac{a}{2}\right) \\ &= [(0.33)(60{,}000) + 789]\left(5.81 - \frac{1.19}{2}\right) \\ &= 8948 \text{ lb-ft/ft} \\ \phi M_n &= 0.9(8948) = 8053 \text{ lb-ft/ft} > M_u = 7580 \text{ lb-ft/ft OK} \end{aligned}$$

Load condition 2: H = 11.33 ft.

The wall would be assumed as fixed at the base and simply supported at the top. For these support conditions, the maximum negative moment at the base is

$$M_{\max} = -\frac{wH^2}{8} = -\frac{(140)(11.33)^2}{128} = -2246 \text{ lb-ft/ft}$$

This value is much smaller than maximum moment calculated for load condition 1; therefore, no need to check further. The maximum positive moment occurs at

$$\frac{3H}{8} = \frac{3(11.33)}{8} = 4.25 \text{ ft}$$

from the top of the floor slab; its magnitude is

$$\begin{aligned} M &= \frac{9wH^2}{128} = \frac{9(140)(11.33)^2}{128} = 1264 \text{ lb-ft/ft} \\ M_u &= 1.6H = 1.6(1264) = 2022 \text{ lb-ft/ft} \end{aligned}$$

The axial load at the point of maximum positive moment ($H = 4.25$ ft) is

$$P_D = 94\ (4.25) + 1200 = 1600 \text{ lb}$$

$$P_u = 0.9\text{D} = 0.9\ (1600) = 1440 \text{ lb}$$

With No. 6 bars @ 16 in. on center, $A_s = 0.33$ in.^{2},

$$a = \frac{A_s f_y + P_u}{0.8 f'_m b} = \frac{0.33(60{,}000) + 1440}{0.8(1800)(12)} = 1.23 \text{ in.} < 1.5 \text{ in.(face shell thickness)}$$

$$\begin{aligned} M_n &= (A_s f_y + P_u)\left(d - \frac{a}{2}\right) \\ &= \frac{1}{12}[(0.33)(60{,}000) + 1440]\left(5.81 - \frac{1.23}{2}\right) \\ &= 9195 \text{ lb-ft/ft} \end{aligned}$$

$$\phi M_n = (0.9)\ (9195) = 8276 \text{ lb-ft/ft}$$

Use 12-in. × 8-in. × 16-in. medium weight concrete masonry units. Provide No. 6 Grade 60 reinforcing bars at 16-in. on center placed at the center of the wall. See Fig. E8.2C.

Commentary: The wall width of 12 in. could be reduced to 8-in. nominal at 4 to 5 ft above the footing and continuing to the top (because of substantial reduction in earth pressure), with smaller size vertical reinforcing bars, say No. 5 @16 in. o.c. This option has not been used in this example for constructional simplicity. However, it is noted that the reduced width might result in some cost savings. Readers are urged to try out this option as an exercise.

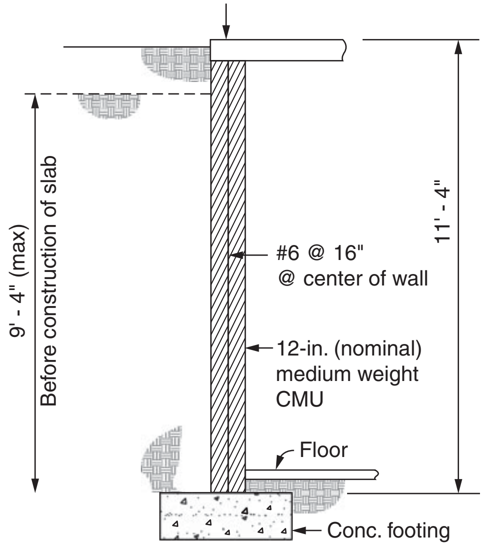


FIGURE E8.2C Reinforcement details.

TABLE 8.11 Maximum Depth of Backfill for Various Plain Concrete Masonry Basement Walls [8.47]

Wall construction	Nominal wall thickness (in.)	Maximum depth of unbalanced backfill (ft)
Hollow unit masonry construction	8	5
	10	6
	12	7
Solid unit masonry construction	8	5
	10	7
	12	7
Fully grouted masonry construction	8	7
	10	8
	12	8

8.6.5 Empirical Design of Basement Walls

The empirical design of basement walls is based on experience to proportion and size masonry elements. Empirical design method is popular and often used to design foundation walls due to its simplicity and proven satisfactory performance. Table 8.11 presents details of empirical design of foundation walls as provided in MSJC-08 Section 5.6.3, Table 5.6.3.1 [8.47]. These walls are suitable for heights up to 8 ft under the following conditions:

1. The height of the wall does not exceed 8 ft (2.44 m) between the lateral supports.
2. The terrain surrounding the foundation wall is graded so as to drain surface water away from the foundation walls.
3. Backfill is drained to remove ground water away from foundation walls.
4. Lateral support is provided at the top of foundation walls prior to backfilling.
5. The length of foundation walls between perpendicular masonry walls or pilasters is a maximum of three times the basement wall height.
6. The backfill is granular and soil conditions in the area are nonexpansive.
7. Masonry is laid in running bond using Type M or S mortar.

8.7 CONSTRUCTION CONSIDERATIONS

8.7.1 Footings for Retaining and Basement Walls

Footings for basement walls are typically strip concrete or solid masonry footings. These footings must extend well below the frost line to prevent damage and heaving caused by cyclical freezing and thawing of water in the soil.

8.7.2 Selection of Backfill Material

Care should be exercised in the selection of the backfill material for retaining walls and basement walls. Generally speaking, the requirements specified for backfill materials specified for retaining walls also apply to basement walls. Most backfill materials can be assigned to one of the following five categories [8.1]:

1. Coarse-grained soil without admixture of fine soil particles, very permeable (clean sand or gravel)
2. Coarse-grained soil of low permeability due to admixture of particles of silt size
3. Residual soil with stones, fine silty sand, and granular materials with conspicuous clay content
4. Very soft or soft clay, organic silts, or silty clays
5. Medium or stiff clay

Backfill material should preferably be free-draining soil without large stones, construction debris, organic materials, and frozen earth. The most favorable backfill materials are coarse-grained soils, preferably with little or no silt or clay content. Design codes/specifications commonly provide general specifications for the selection of a suitable backfill such as follows [8.11 Section 5.5.7]:

> The backfill material behind all retaining walls shall be free draining, nonexpansive, noncorrosive material and shall be drained by weep holes with French drains, placed at suitable intervals and elevations . . . In counterfort walls, there shall be at least one drain in each pocket formed by the counterforts. Silts and clays shall not be used for backfill unless suitable design procedures are followed and construction control measures are incorporated in the construction documents to account for their presence.

However, such ideal backfill materials may be either unavailable or too expensive. Use of less suitable materials is associated with large earth pressures and increases the cost of the wall. For example, backfill materials such as soft clays, silty clays, or organic silts can be expected to lead to large pressures and progressive wall movements. Stiff clays, as a rule, should not be used. This is because they are certain to experience an increase in water content over the years and to exert *swelling pressures* that may greatly exceed the lateral resistance of the wall [8.1]. At the very least, it should be ensured that the properties of the selected backfill material match, as closely as possible, the soil properties used in design. Saturated backfill material, such as saturated clays, should be avoided as the wet backfill materials significantly increase the hydrostatic pressure on the walls.

8.7.3 Backfilling Operation

Backfilling plays an important role in the construction phase of retaining walls. Construction and backfilling practices should comply with the requirements of *Specification for Masonry Structures* [8.51] or applicable codes and specifications. Footings should be placed on firm, undisturbed soil. In areas exposed to freezing temperatures, the base of the footing should be placed below the frost line. It is recommended that backfill against a retaining wall be not placed earlier than at least seven days after placing the grout, to enable the grout to achieve desired strength [8.44]. During backfilling, heavy earth-moving equipment should not approach the wall closer than a distance equal to height of wall in order to minimize the influence of forces caused by vibrations and impact. Dumping a large mass of earth close to the wall during backfilling should be avoided as it could cause large impact forces.

Backfilling is also one of the most crucial aspects of basement and subterranean wall construction. This is because walls are designed for soil pressures which are determined based on specific support conditions. It is essential that walls be properly braced or the

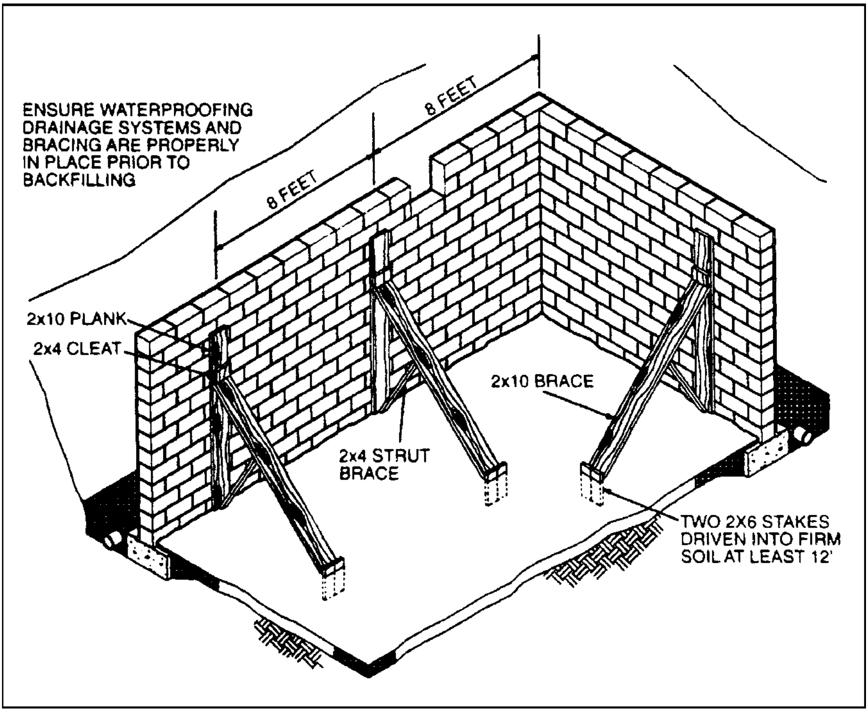


FIGURE 8.19 Typical bracings for concrete masonry basement walls [8.54].

first floor diaphragm be in place prior to backfilling. In the absence of the floor diaphragm, the bracing should be so placed as to ensure that they will resist lateral load as assumed in design. If neither is done, a wall which was designed as simply supported at the top will act as a cantilever, and may crack or even fail due to large moments generated at the bottom. Figure 8.19 shows a typical bracing scheme that has been widely used for residential basement walls. Higher walls or walls subjected to large backfill pressures would require more substantial bracing schemes.

Equally important consideration is the avoidance of surcharge loads close to the wall if the same have not been considered in design. Presence of surcharge loads very close to the wall, such as bulldozers or cranes in use during backfilling, cause higher earth pressures on the wall, which may be detrimental to the safety of wall if the same have not been accounted for in design.

Another important aspect of backfilling involves practical conditions that might be encountered in practice. Backfill material should not be placed when any appreciable amount of water is standing in the excavation. The need for special bracing of partially completed walls should be carefully considered whenever the construction is halted due to inclement weather. In some instances, heavy rains caused collapse of the side slopes of excavation or earth slides, resulting in filling the excavation adjacent to basement walls with earth and water. This, in turn, caused lateral pressures on the unbraced walls large enough to cause failure. Special precautions should be taken in advance of such occurrences. For example, in areas of heavy rains where considerable surface water would channel toward the foundation walls, some contractors purposely omit an occasional block close to the bottom of the wall. In the event of flooding of the area adjacent to the walls, these openings prevent the water level from rising to excessive heights on one side of the wall. The passage of water through the opening equalizes the pressure on both sides of the wall, thus minimizing the danger of collapse of wall due to excessive lateral pressures on one side only. The blocks are installed in the wall after the wall is completed to the top and prior to the placement of the backfill material [8.52, 8.53].

8.7.4 Provisions for Drainage

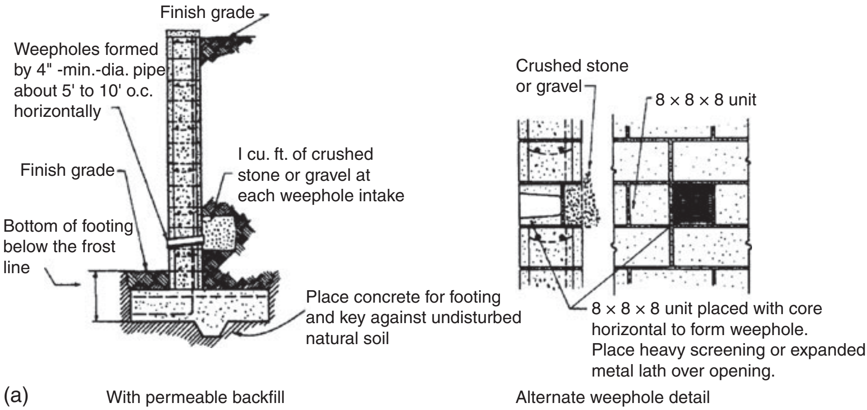
The earth pressure, which constitutes the principal force for design of retaining walls and subterranean walls discussed in this chapter, was based on the assumption that the backfill was dry and granular; consequently, they did not include any hydrostatic pressure. When a situation exists such that rain water may infiltrate the backfill, the water pressure must be included in design in addition to the earth pressure. The added lateral pressure would result in an uneconomical design which can be avoided. To avoid the hydrostatic pressure, adequate drainage should be provided behind the wall. A discussion on the several types of systems that can be used for drainage of backfill can be found in texts on foundation engineering such Refs. as 8.14 and 8.23. The two commonly used methods (Fig. 8.20) to provide a drainage system are

1. Weep holes 4 to 6 in. in diameter may be provided at 5 to 15 ft on center horizontally and 5 ft on center vertically (Fig. 8.20*a*). Immediately behind the wall, approximately 12-in.-thick blanket of pervious material should be provided so that only clear water would drain out of the backfill and through the weep holes. In the absence of such a provision, the danger is that backfill material may be washed into weep holes and drainage pipes and clog up the drainage system.
2. A perforated 8-in. pipe may be laid along the base of the wall and surrounded by selected filter material such as gravel to preclude the possibility of clogging and thus preventing the soil particles to be carried by water (Fig. 8.20*b*).

8.7.5 Provisions for Prevention of Water Penetration in Basement Walls

Basement walls are required to be dampproofed for conditions where hydrostatic pressure will not occur, and waterproofed where hydrostatic pressure may exist. Methods to provide dampproofing and waterproofing are discussed in publications such as Refs. 8.54 and 8.55. Essentially, the purpose of providing water-penetration resistance systems is to prevent entry of moisture and water into the drier space in the basement. This is accomplished by providing a barrier to water and water vapor. Dampproofing is appropriate where groundwater drainage is good through granular backfill into a subsoil drainage system. Waterproofing is required where ground-water investigation indicates that a hydrostatic condition exists, and the design does not include a ground-water control system. Provisions for damproofing and waterproofing are specified in 2009 IBC Section 1805 [8.26]. Readers should refer to these provisions for details of methods of dampproofing and waterproofing.

Some of the important considerations in choosing a waterproof or dampproof systems include the degree of resistance to hydrostatic head of water, absorption characteristics, elasticity, and stability in moist soil, resistance to mildew and algae, impact or puncture resistance, and abrasion resistance [8.54, 8.55].



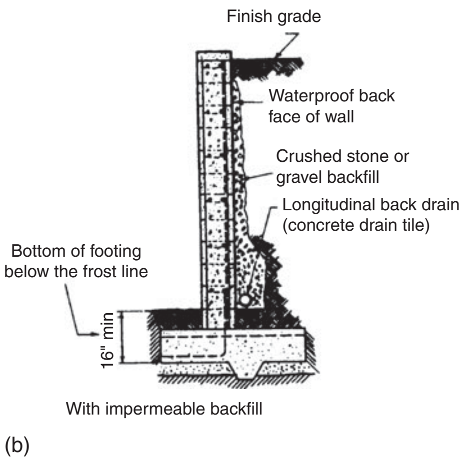


FIGURE 8.20 Common types of retaining wall drainage: (*a*) weep holes, (*b*) longitudinal drain pipe. (*Courtesy: NCMA.*)

It was pointed out in the preceding section that draining water away from retaining walls significantly reduces the lateral pressure the walls must resist. The same logic applies to basement walls also. Draining water away from basement walls reduces the potential for cracking and possibility of water penetration into the basement in case of failure of the waterproof or dampproof system.

Problems

8.1 Design a concrete masonry retaining wall to support a backfill that is 6 ft high from the top of the footing. Assume equivalent fluid pressure as 35 lb/ft^3. Use $f'_m = 1500$ lb/in.^{2} and Grade 60 reinforcement. A factor of safety of at least 2.0 must be provided against overturning and 1.5 against sliding. The allowable bearing capacity of the soil is 2000 lb/ft^2. The coefficient of friction between concrete and soil may be taken as 0.5.

8.2 Design a concrete masonry retaining wall for a backfill that is 9 ft high from the top of the footing. Assume equivalent fluid pressure as 50 lb/ft^3. Use f'_m =1800 lb/in.^{2} and Grade 60 reinforcement. A factor of safety of at least 2.0 must be provided against overturning and 1.5 against sliding. The allowable bearing capacity of the soil is 2000 lb/ft^2. The coefficient of friction between concrete and soil may be taken as 0.5.

Design reinforced concrete masonry basement walls in Problems 8.3 and 8.4.

8.3 The basement wall in a building measures 10 ft 4 in. from the top of its footing to the top of ground floor that it supports as shown in Fig. P8.3. The construction sequence requires the back-fill to be placed behind the wall to a height of 8 ft 4 in. from the top of the footing before pouring of the ground floor slab that the wall supports. Geotechnical investigation has determined earth pressure of 40 lb/ft^2/ft for the unrestrained condition (i.e., before the slab is poured) and a uniform lateral pressure of 140 lb/ft^2 for the restrained condition (i.e., after the slab has been poured at the top of the wall). P-Δ effects may be ignored for designing the wall as the wall height is not large compared to its thickness. Use f'_m = 1500 psi and Grade 60 reinforcement.

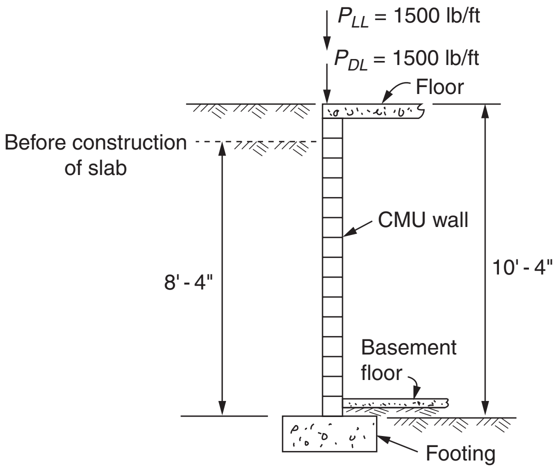


FIGURE P8.3

8.4 The basement wall in a building measures 12 ft from the top of its footing to the top of ground floor that it supports as shown in Fig. P8.4. The construction sequence requires the back-fill to be placed behind the wall to a height of 9 ft before pouring of the ground floor slab that the wall supports. Geotechnical investigation has determined earth pressure of 40 lb/ft^2/ft for the unrestrained condition (i.e. before the slab is constructed) and a uniform lateral pressure of 140 lb/ft^2 for the restrained condition (i.e. after the slab has been poured at the top of the wall). P-Δ effects may be ignored for designing the wall as the wall height is not large compared to its thickness. Use f'_m = 1800 psi and Grade 60 reinforcement.

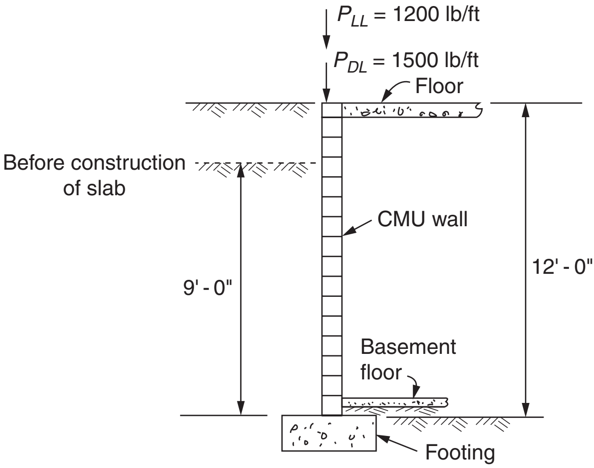


FIGURE P8.4

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CHAPTER 9
CONSTRUCTION ASPECTS

9.1 INTRODUCTION

Conceptualizing, planning, feasibility study, design, and construction are five main phases of a project that turn it into reality. Some projects are more involved than others, depending on the complexity of design and construction. In all cases, however, the role of construction is of paramount importance, as demonstrated by many failed buildings or portions thereof, for even a well-designed building will not withstand the applied loads if not constructed properly. Indeed, the importance of construction lies in the fact that not only it must progress and be completed in ways as to satisfy assumptions made in designing the structure, but that the end result is a structure that is capable of resisting design loads, functional for the intended occupancy, and free of maintenance problems and distress such as cracking and settlement of walls. The topic of "construction of reinforced masonry" is worthy of a book by itself, and it is difficult to present all aspects of construction in a chapter. Nevertheless, a brief discussion on several aspects of reinforced masonry construction is presented in this chapter.

Masonry remains as one of the oldest form of construction used by humans. A brief historical development of masonry construction was presented in Chap. 1. Design and analysis of various reinforced masonry members were discussed in preceding chapters. Reinforced masonry consists of four essential elements: masonry units (concrete or clay), mortar, grout, and reinforcement. Stated simply, masonry is a solid mass of individual units bonded together with mortar and grout. Masonry is inherently strong in compression but weak in tension and shear. To improve and enhance load-carrying capacity, masonry is reinforced horizontally and vertically, allowing the two materials to complement each other and resulting in an excellent structural material. Each of these four elements participates in resisting loads. Craftsmanship of masons plays an extremely important role in putting these elements together so that the structure would act as an integral body of these four elements in resisting loads. Thus, masonry construction is an art that entwines structural engineering with masons' craftsmanship resulting in a safe masonry structure that will perform as expected and anticipated in design. Materials of masonry construction were discussed in Chaps. 2 and 3. This chapter presents but a brief discussion of some important phases of masonry construction. Because of the nature of the topics covered in this chapter, considerable information presented herein has been taken from numerous references including building codes, specifications for masonry structures, and industry publications [9.1–9.11].

Weather and temperature conditions can have significant influence on the performance of constructed masonry. Readers should refer to 2009 IBC (Sections 2104.3 and 2104.4) for construction requirements for a range of temperatures and weather conditions.

9.2 PLACEMENT OF STEEL REINFORCEMENT

9.2.1 General Considerations

Reinforcement is the backbone of *reinforced* masonry structures. It is the element of construction that provides masonry strength beyond its strength in unreinforced state, which is not significant when it comes to resisting seismic loads. Only deformed bars are to be used for reinforcing purposes.

Reinforcement may be vertical or horizontal, and may be placed in cells of single-wythe walls (Fig. 9.1*a*) or into the space between the wythes of multiwythe walls (Figs. 9.1*b* and 9.2). Hollow units in single-wythe walls are laid so that the cells are aligned vertically, providing continuous, unobstructed space that facilitate placement of vertical bars (Figs. 9.3 and 9.4) and grout. All reinforcing bars should be placed in accordance with the type, size, and location as indicated in construction drawings, and as specified. Placement of steel reinforcement in its specified position is critical to the performance of reinforced masonry, especially in flexure. This is because the flexure strength of masonry elements [$M = A_s f_y(d - 0.5a)$] is based on the effective depth d of the element, which is measured from the compression face of the element to the centroid of tensile reinforcement.

The maximum reinforcement size is limited by building codes. The MSJC-08 Section 1.15.2 [9.4] limits the maximum reinforcement to No. 11 for allowable stress design, and to No. 9 for the strength design (MSJC-08 Section 3.3.3.1). These limits are arbitrary and are based on accepted practice, research, and experience with successful performance in construction. The limit of using a No. 9 bar is motivated by the desire to use a large number of smaller diameter bars to transfer stresses rather than a fewer number of larger diameter bars. Testing by the National Concrete Masonry Association reveled concerns with the large size reinforcing bars and low bound of masonry strength. Accordingly, use of larger size bars is not recommended. When vertical reinforcement is spliced in a grout space between the wythes or within an individual cell, the two bars should be put in contact and wired together. For strength design of masonry, the nominal diameter of a bar is not to exceed one-eighth of the nominal member thickness, and also it should not exceed one-quarter of the least clear dimension of the cell, course, or collar joint in which the bar is placed. Maximum reinforcement area in cells is limited to 4 percent of the cell area without splices and 8 percent with splices (MSJC-08 Section 3.3.3.1). The limitations on clear

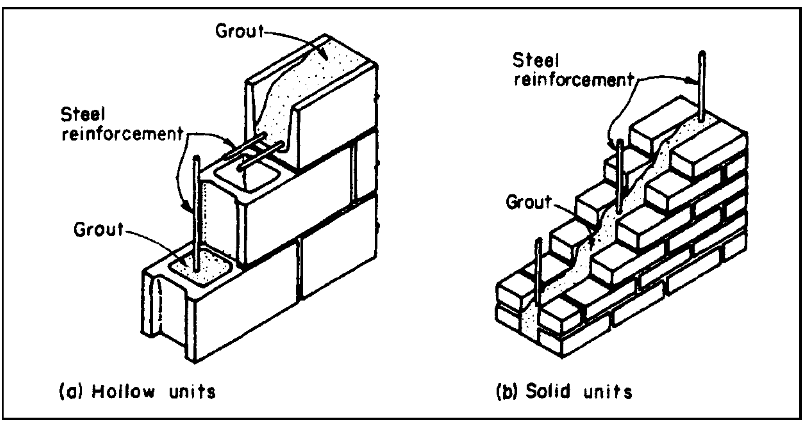


FIGURE 9.1 Vertical and horizontal reinforcement in reinforced masonry walls [9.1]. (*Courtesy: Portland Cement Association.*)

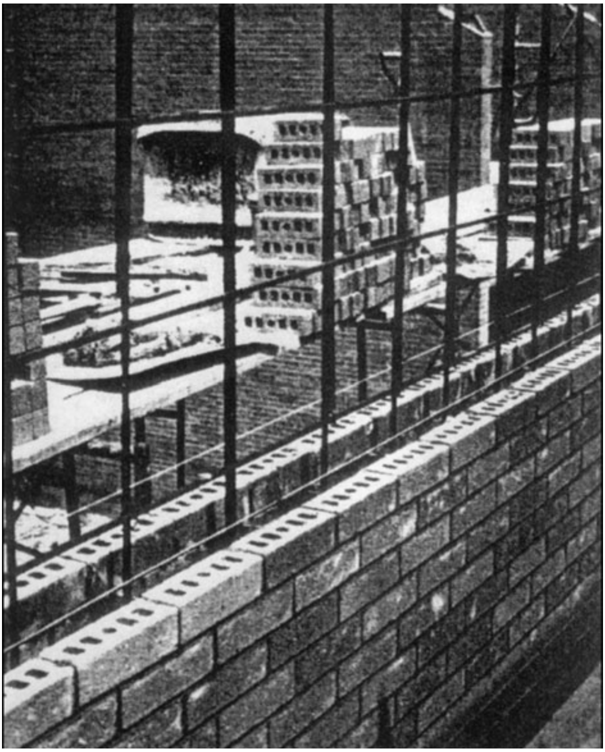


FIGURE 9.2 Vertical reinforcement in a two-wythe cored brick masonry wall [9.3]. (*Courtesy: BIA*.)

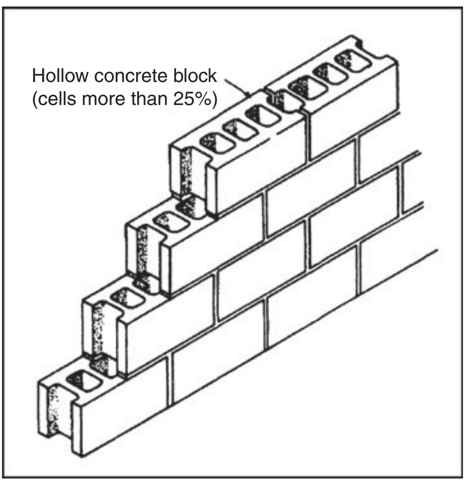


FIGURE 9.3 Cells in hollow masonry units aligned vertically for placement of vertical reinforcement [9.1]. (*Courtesy: Portland Cement Association*.)

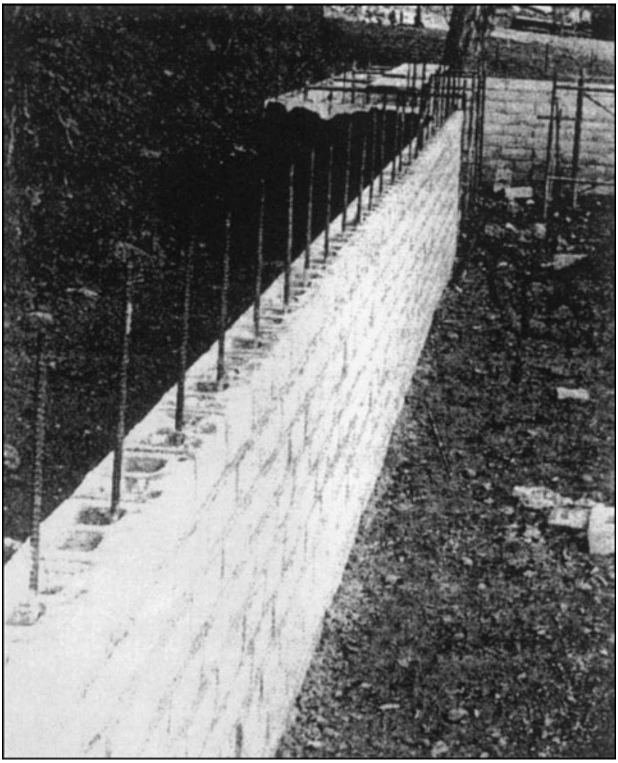


FIGURE 9.4 Vertical reinforcement in concrete masonry wall [9.2]. (*Courtesy: Portland Cement Association*.)

spacing and percentage of cell area are indirect methods of preventing problems associated with overreinforcing and grout consolidation.

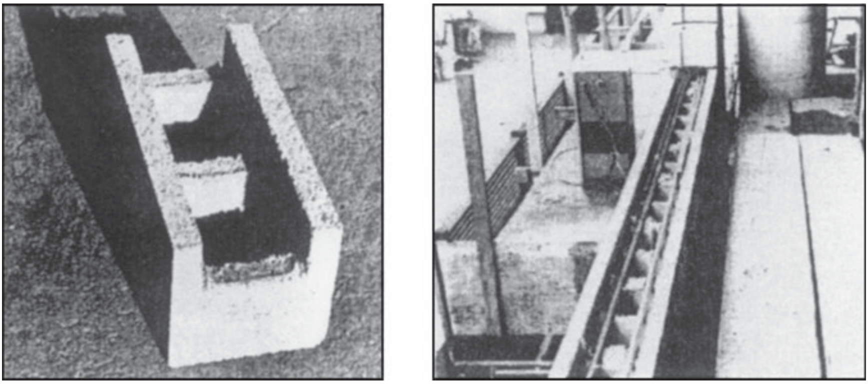
Bundling of reinforcing bars is not permitted at the present time due to lack of research on their behavior and performance (MSJC-08 Section 3.3.3.6).

9.2.2 Installation of Reinforcement

Reinforcement required for reinforced masonry construction is installed as required by codes. Vertical reinforcement in walls is placed in one of the two ways. Some codes require reinforcement to be installed prior to laying units. Alternatively, reinforcement can be installed as the wall is constructed.

When the reinforcement is to be placed prior to laying masonry, as in concrete masonry construction, it is convenient to use open-ended (A or H) blocks (see discussion in Chap. 2). These units can be conveniently placed around the reinforcing steel. This eliminates the need for the units to be lifted up and placed over the reinforcement (Fig. 2.19, Chap. 2), which is quite cumbersome.

When code allows for reinforcement to be installed after the wall erected, one of the two methods can be used. Full-length reinforcing bars can be installed after the units are laid. It is common to use steel spacer ties or other devices to hold reinforcement at the specified location.



(a) (b)

FIGURE 9.5 Horizontal reinforcement in walls: (*a*) low-web bond beam, (*b*) construction of a bond beam for wall reinforcement [9.1]. (*Courtesy: Portland Cement Association*.)

Vertical reinforcement should be braced at the top and bottom of the element. Additional bar positioners may be necessary to facilitate proper placement of the bars. Alternatively, bars can be installed in shorter lengths as the wall is constructed. In this method, masonry can be constructed to a height of 4 ft and allowed to cure properly. Then a 6-ft long bar can be placed into the masonry and grouted to a height of about 4 ft, leaving an adequate length of reinforcement exposed to provide a lap splice. This process is repeated until the wall is completed.

Horizontal reinforcement is usually placed in the mortar bed joints as the work progresses, or in continuous bond beams at the completion of the bond beam course. Bond beams are formed by using special bond beam units (e.g., lintel or channel units) when feasible (i.e., when vertical reinforcing is not required), by removing portions of webs that connect the face shells, or using special low-web bond beams. In fully grouted walls where both horizontal and vertical reinforcing bars are required, the horizontal reinforcing bars may be supported on cross-webs of hollow masonry units (Fig. 9.5). In partially grouted walls, the bond beam should be grouted prior to further construction of masonry on top of the bond beam. For two-wythe, solid brick masonry walls, the horizontal reinforcement may be placed in the collar joint. All horizontal bars should be placed on the same side of the vertical reinforcing bars to facilitate consolidation of the grout.

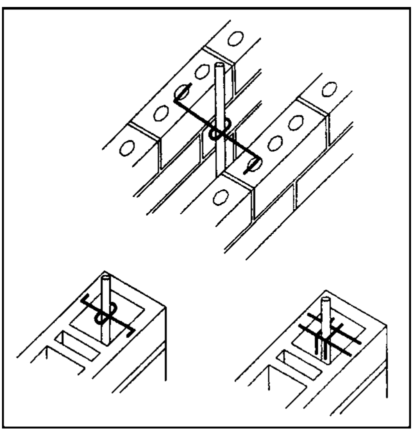


FIGURE 9.6 Bar spacing devices used in reinforced brick walls [9.5]. (*Courtesy: BIA*.)

Both vertical and horizontal reinforcement should be located accurately and held in place by bar positioners or other acceptable means to prevent displacement during grouting. To ensure proper positioning of reinforcing bars at the designated locations, they are required to be secured against displacement prior to grouting by bar positioners or other suitable devices at intervals as specified in project specifications. Figures 9.6 and 9.7 show some commonly used bar securing devices.

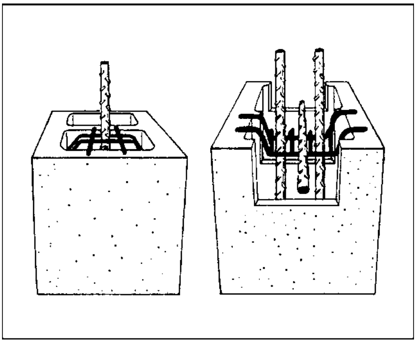


FIGURE 9.7 Bar spacing devises used in reinforced concrete masonry walls [9.1]. (*Courtesy: Portland Cement Association.*)

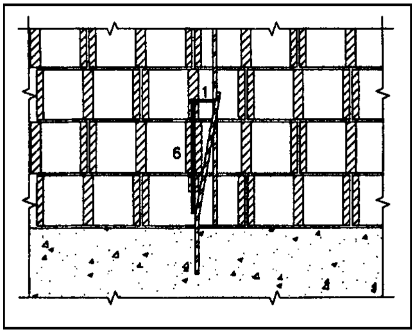


FIGURE 9.8 Foundation dowel clearance [9.6]. (*Courtesy: NCMA.*)

In reinforced masonry wall construction, it is a common practice for vertical starter bars to be cast into the foundation (or footing) and lap them with the main wall reinforcement, which may be placed after laying the units for a full height. When these foundation dowels are present, they should align with the cores of the masonry units. If a few dowels interfere with the placement of units, they may be bent slightly—a maximum of one horizontally to six vertically (Fig. 9.8) [9.6]. In the case of fully grouted walls, chipping away a portion of the web to better accommodate dowels may be acceptable. The engineer should be notified if a substantial dowel alignment problem exists.

Splice requirements for reinforcing bars are mandated by applicable building codes. MSJC-08 Section 3.3.3.4 specifies the following requirements:

1. In no case shall the length of the lapped splice be less than 12 in. or the calculated development length of the bar in accordance with MSJC-08 Section 3.3.3.3 (the greater of the two lengths governs).
2. Both welded and mechanical splices are permitted. A welded splice is required to have bars butted and be capable of developing at least 125 percent of the yield strength of the bar in tension or compression, as required.
3. Mechanical splices are permitted; they are required to have bars connected to develop at least 125 percent of the yield strength of the bar in tension or compression, as required.
4. Development length of epoxy-coated reinforcing bars shall be taken as 150 percent of the development length determined by MSJC-08 Eq. (3.16) (see discussion in Chap. 4).
5. Reinforcing bars may be spliced by placing them in the adjacent cells. Such a splicing arrangement is called a *noncontact* splice. Bars spliced by noncontact lap splices shall not be spaced transversely farther apart more than one-fifth the required length of lap, nor more than 8 in. (MSJC-08 Section 3.3.3.3.1 [9.4]).

9.2.3 Reinforcement Placement Requirements

Table 9.1 presents reinforcement placement requirements per MSJC *Code* [9.4] excerpted from Ref. 9.5. According to the *Commentary* to MSJC-08 [9.4], placement limits for reinforcement are based on successful construction practices over many years. The limits are intended to facilitate the flow of grout between bars. A minimum spacing between bars in a

TABLE 9.1 Reinforcement Placement Requirements [9.5]

Spacing requirements				
Minimum clear distance between parallel bars and between a contact lap splice and adjacent splices or bars				Diameter of larger bar, but at least 1 in.
Minimum clear distance between vertical bars and between a contact lap splice and adjacent splices of bars in columns and pilasters.				1½ times the diameter of the larger bar, but at least 1½ in.
Minimum grout thickness between reinforcement and surrounding masonry			Fine grout	¼ in.
			Coarse grout	½ in.
Minimum masonry cover	Bars	Masonry face exposed to earth or weather	No. 6 or larger	2 in.
			No. 5 or smaller	1½ in.
		Masonry face not exposed to weather		1½ in.
	Joint reinforcement wires	Masonry face exposed to earth or weather		5/8 in.
		Masonry face not exposed to weather		½ in.
Erection tolerances				
Maximum variation of vertical bar position along wall length from location indicated on project drawings				2 in.
Tolerance from d dimension indicated on project drawings or as specified*			$d \le 8$ in.	±½ in.
			8 in. $< d \le 24$ in.	±1in.
			$d > 24$ in.	±1¼ in.

*In flexural members, the "d" dimension is the distance between the extreme compression face to the centroid of the tensile reinforcement.

layer prevents longitudinal splitting of the masonry in the plane of the bars. Use of bundled bars in masonry is rarely required.

9.3 GROUTING

While reinforcement enhances the strength of masonry beyond its capacity in unreinforced state, grout is the medium that makes it possible. Information on requirements for grout and grouting practices can be found in building codes [9.4, 9.7], industry publications [9.1, 9.3, 9.5, 9.6, 9.8–9.12], and many other references [9.13, 9.14]. Provisions for masonry construction are given in 2009 IBC Section 2104 [9.7] and in *Specification for Masonry Structures* [9.16]. This section presents a summary from these references pertaining to the grouting of reinforced masonry.

9.3.1 General Considerations

Grouting is one of the most important phases of modern masonry construction. Grout is simply a mixture of cementitious material, aggregate, and enough water to cause the mixture to flow readily without segregation into the cores or cavities in the masonry. The purpose of providing grout and general requirements for grouting were discussed in Chap. 3 and are not repeated here. However, from a practical standpoint, it is pointed out that

1. Grouting is used for both unreinforced and reinforced masonry construction.
2. In an unreinforced masonry construction, grouting is sometimes used to enhance the strength of loadbearing masonry walls. This is accomplished by filling some or all of the cores, for example, filling the cores of the units forming the jamb of a door. Grout may be used to fill bond beam cavities and occasionally to fill the collar joint of a double-wythe wall.
3. In reinforced masonry construction, grout may be placed only in spaces or cells containing reinforcement (e.g., in hollow unit masonry), which is commonly referred to as partial grouting. The grout bonds masonry units and reinforcement so they act integrally to resist loads. Grout acts as the medium through which reinforcement participates in resisting forces imposed on masonry structures. Poor grouting of masonry can cause the reinforcement to function ineffectively, which can compromise the safety of the structure.
4. In some reinforced masonry construction using hollow units, all cores, with and without reinforcement, are grouted to enhance the strength of the walls.
5. Reinforced masonry can be advantageously used to build taller, thinner, and more economical walls; this cannot be accomplished without the benefit of grouting.
6. Walls constructed in certain seismic zones are required to be reinforced and grouted to resist the dynamic seismic forces.

Historically, the earliest form of placing reinforcement in brick masonry involved placing reinforcement in mortar joints as bricks were laid. Later, the reinforcement was placed in collar joints between two masonry wythes and surrounded by mortar or fine grout. Eventually, the space between the wythes was increased and filled with grout. Horizontal reinforcement and grout were placed as the outer wythes were completed. This procedure came to be commonly referred to as "low-lift" grouting. The next development was the "high-lift grouting system," a masonry construction technique developed in the San Francisco area during the 1950s [9.9]. Using this method, the brick masonry wythes are built around the reinforcement and allowed to set for a minimum period of three days, and then grout is pumped into the space containing the reinforcement (Fig. 9.9). Figure 9.2 shows a two-wythe reinforced brick construction in which the space between the two wythes is grouted. Figure 9.10 shows plan and elevation of a typical wall constructed by the high-lift grouting system.

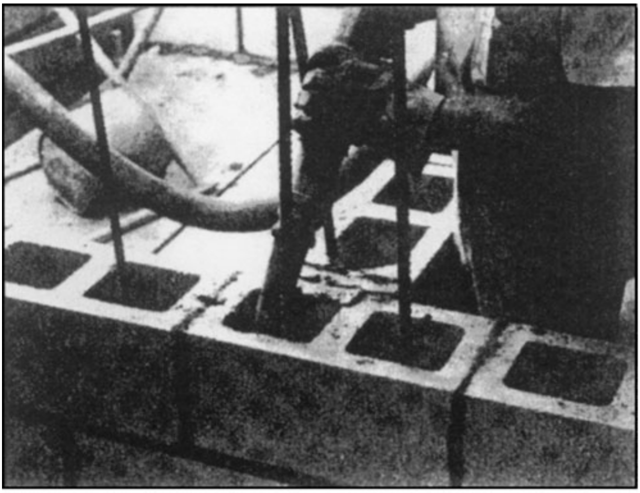
As discussed in Chap. 3, both brick and concrete masonry units are manufactured with large open cells which align vertically when the units are laid. Vertical (or longitudinal) reinforcement is placed in these cells as specified in design and the cells are grouted. In all cases involving reinforced masonry construction, the following fundamental requirements must be met in order to develop the design strength:

Units must be laid up so that the cores or cells are reasonably aligned to form an unobstructed, continuous series of vertical spaces within the wall to permit the placement of vertical reinforcement and to allow proper flow of grout.

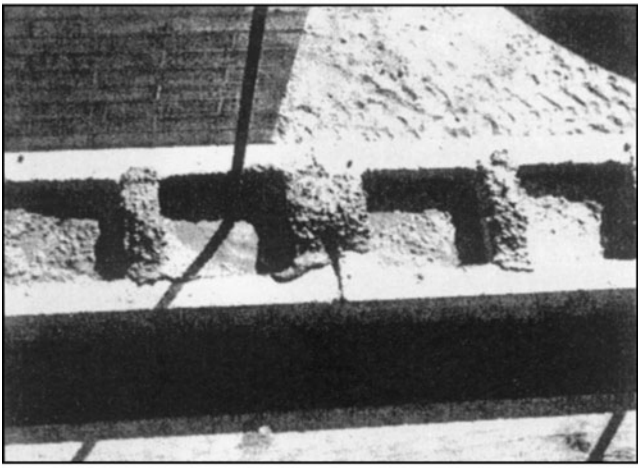
Reinforcement shall be placed prior to grouting (Fig. 9.11).

Metal reinforcement shall be located in accordance with the plans and specifications. Reinforcement shall be secured against displacement prior to grouting by wire positioners or other suitable devices as discussed in Section 9.2. Placement of steel reinforcement in its specified position is critical to the intended performance of reinforced masonry.

All cells and spaces containing reinforcement shall be filled with grout.



(a)



(b)

FIGURE 9.9 (*a*) View of grout shooting from the hose into a cell of a concrete masonry unit, (*b*) grouting is stopped at least 1½ in. below the top of the block to form a shear key within the next lift [9.1]. (*Courtesy: Portland Cement Association*.)

9.3.2 Placing Grout

Grout is placed in lifts either between two wythes of masonry or into the cells of masonry units. A *lift* is the layer of grout placed in a single continuous operation. Grout lifts are poured in increments not exceeding 5 ft. However, this limit may be exceeded if it can be demonstrated that grout space can be properly filled. One or more lifts constitute a grout *pour,*

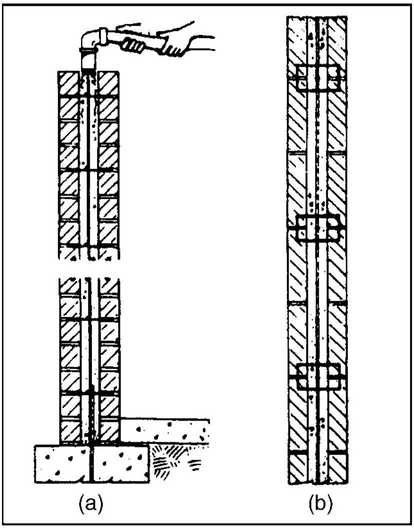


FIGURE 9.10 High-lift grouting of a typical two-wythe brick wall: (*a*) elevation, (*b*) plan [9.9]. (*Courtesy: BIA*.)

which is the total height of grout placed in a masonry wall prior to the construction of additional masonry. It may be the entire height of the grout placed in one day and may be composed of several lifts (Fig. 9.12). For example, if a wall is constructed to a height of 10 ft, the grout will be placed in two 5-ft lifts, and the total grout pour will be 10 ft.

The maximum height of a grout pour is restricted by (1) the type of size and type of the grout space (e.g., space between the two wythes, or space dimensions of grouting cells in hollow units) and (2) the type of grout mix. Table 9.2 presents grout pour heights and space requirements per MSJC *Code* as well as cleanout requirements as suggested by the International Masonry Institute [9.11]. Higher grout pours or smaller cavity widths or cell sizes can be used when approved, and if it is demonstrated that grout spaces will be properly filled. Grouting of any section of the wall must be completed in one day with no interruptions greater than one hour.

9.3.3 Grouting Methods

Grouting process is the most crucial aspect of reinforced masonry construction. Grout provides integrity between the elements of masonry assembly consisting of masonry units, mortar, and reinforcement. Since the strength of a reinforced masonry structure is based on the fundamental premise that portions or whole of the masonry will be grouted as specified, improper grouting will result in a weak structure, and thus compromise its performance and safety. Seemingly a simple matter of filling the cells of masonry or a grout space, this is one aspect of reinforced masonry construction that can cause most problems. The most common problems associated with grouting include creation of voids in the grout space due to stiff

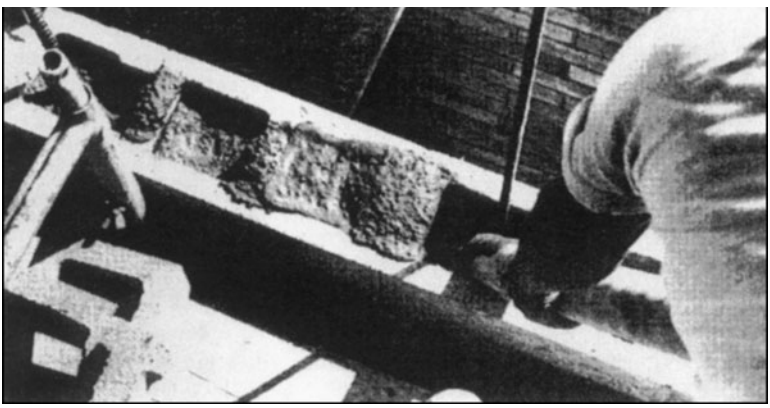


FIGURE 9.11 Reinforcing bars are placed in the cells prior to filling them with grout [9.1]. (*Courtesy: Portland Cement Association.*)

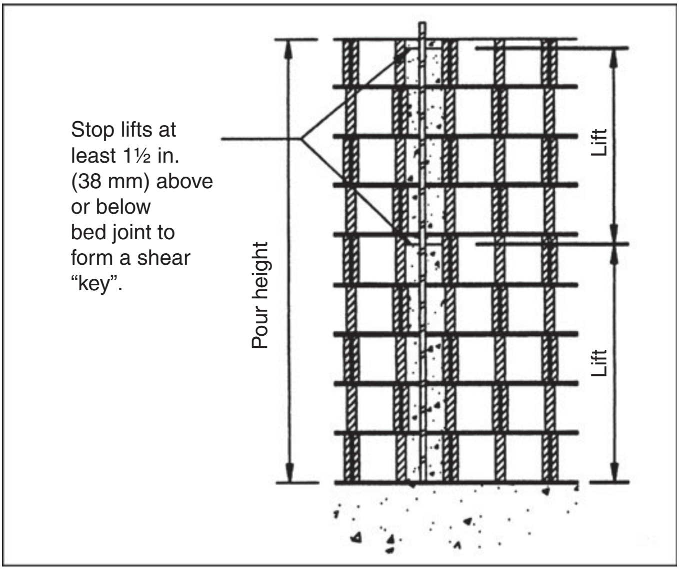


FIGURE 9.12 Grout pour versus grout lift [9.6]. (*Courtesy: NCMA*.)

TABLE 9.2 Grout Pour Heights and Space Requirements [9.11]

Specified grout type	Maximum grout pour height, ft (m)	Maximum width of grout space in. (mm)*,†	Min. grout space dimensions for grouting cells of hollow units†,‡ in. × in. (mm × mm)	Cleanout requirements
Fine	1 (0.3)	¾ (19)	1 ½ × 2 (38 × 51)	No
Fine	5 (1.5)	2 (51)	2 × 3 (51 × 76)	No
Fine	12 (3.7)	2 ½ (64)	2 ½ × 3 (64 × 76)	Yes
Fine	24 (7.3)	3 (76)	3 × 3 (76 × 76)	Yes
Coarse	1 (0.3)	1 ½ (38)	1 ½ × 3 (38 × 76)	No
Coarse	5 (1.5)	2 (51)	2 ½ × 3 (64 × 76)	No
Coarse	12 (3.7)	2 ½ (64)	3 × 3 (76 × 76)	Yes
Coarse	24 (7.3)	3 (76)	3 × 4 (76 × 102)	Yes

*For grouting between masonry wythes.
†Grout space dimension is the clear dimension between any masonry protrusion and shall be increased by the diameters of the horizontal bars within the cross sections the grout space.
‡Area of vertical reinforcement shall not exceed 6 percent of the area of the grout space.

grout, excessive pour height, grout shrinkage, and blocked grout spaces. To minimize these problems and to ensure proper grouting, four sequential steps should be properly executed:

1. Preparation of grout space
2. Grout batching
3. Grout placement and consolidation, and
4. Curing and protection.

A discussion on these aspects of grouting is presented in many references such as Refs. 9.5 and 9.11, which is summarized as follows:

1. Preparation of the grout space: In reinforced masonry construction, spaces which may be grouted include cells of hollow masonry units in beams and walls, the collar joint between multiwythe walls, and the core of columns and pilasters. Thus, the configuration and condition of the grout space can vary, and they influence the preparation required for the grout space. In general, the following considerations are required:

a. When hollow masonry units are used, all cells may or may not be grouted, depending on the design requirement. When certain cells are to be left ungrouted (i.e., empty) or filled with insulation, the cross-webs adjacent to those cells must be fully mortared to prevent flow of grout into the cells that are to be left empty. This will also prevent the leakage of insulation into the cells that are to be grouted.

b. When bond beams are used, a wire mesh is installed underneath to prevent the flow of grout into the masonry below the beam (Fig. 9.13).

c. When grout is to be placed between multiwythe walls, vertical barriers spaced not more than 30 ft apart must be constructed across the grout space for the entire height of the wall to contain grout flow. Their purpose is to prevent excessive flowage, which can cause segregation of grout materials. These vertical barriers can be built from partial masonry units to the full height of the wall (Fig. 9.14).

It is important that grout spaces be clear of all mortar droppings and debris. The reinforcement should be cleaned and properly positioned in place. Any mortar splatter on the surface of the grout space or the reinforcement is highly undesirable and must be removed prior to grouting. This is essential for developing proper bond between the masonry units, reinforcement and grout, and also to facilitate proper placement of reinforcing bars prior to grouting. Care should be taken during construction to prevent excess mortar from extruding and falling into the grout space. Mortar that projects more than ½ in. into the grout space should be removed because large protrusions restrict the flow of grout. The grout will tend to bridge at these locations, which might cause incomplete filling of the grout space.

Specification for Masonry Structures [9.16 (Section 3.2F)] requires that

a. *Cleanouts* (openings at the base of walls) be provided in the courses of all spaces to be grouted when grout pour exceeds 5 ft.

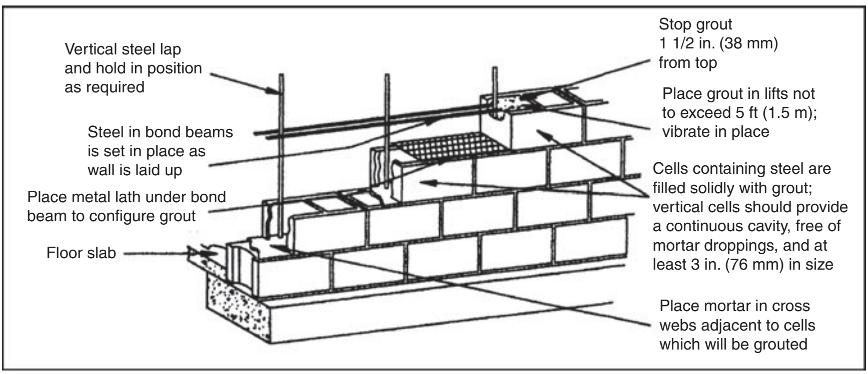


FIGURE 9.13 Typical wall construction with hollow concrete masonry units [9.6]. (*Courtesy: NCMA*.)

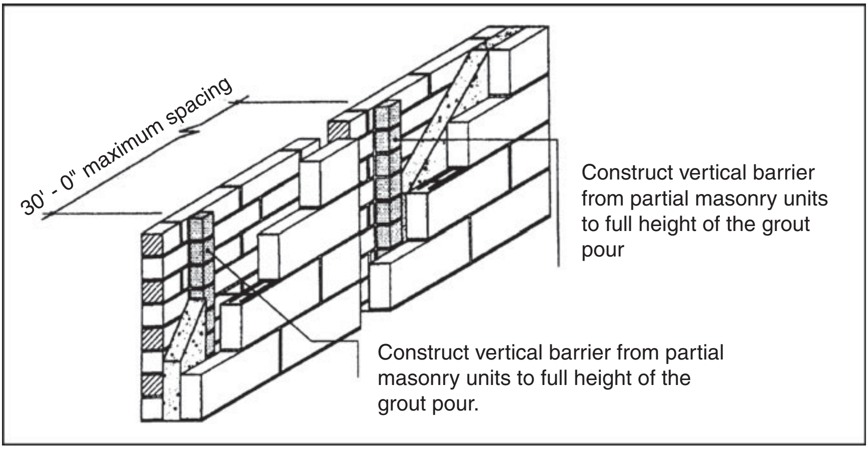


FIGURE 9.14 Vertical grout barriers [9.11]. (*Courtesy: Illinois Masonry Institute.*)

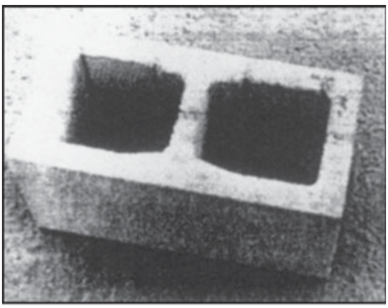
b. In partially grouted masonry, a cleanout must be provided at every vertical bar. In fully (or solidly) grouted masonry, spacing of cleanouts should not exceed 32 in.

c. The openings for cleanouts must be sufficiently large to allow removal of debris from the cells to be grouted. The minimum cleanout opening dimension is required to be 3 in. long and 3 in. high.

d. After cleaning, the cleanouts must be closed with closures to provide bracing to resist grout pressure.

Different methods are used to provide cleanouts for brick and concrete masonry. In solid brick masonry construction, cleanouts may be formed by omitting brick in the bottom course periodically along the base of the element. For hollow brick or hollow concrete masonry, cleanouts can be provided by removing the face shell of the cells to be grouted. In hollow unit concrete masonry construction, a special scored unit (Fig. 9.15*a*) is sometimes used to permit easy removal of part of the face shell to provide for cleanout openings. When special units are not available, saw cutting is commonly used. When both sides of the concrete block wall are to be exposed to view, it may be desirable to remove entire face shell of the unit so that it may be replaced in whole to conceal the opening. Figure 9.15*b* shows typical cleanouts in a reinforced concrete masonry wall.

The cleanouts should be sealed, closed, or covered after the bottom of the grout space is inspected for cleanliness, and reinforcement



(a)

(b)

FIGURE 9.15 (*a*) Hollow concrete unit scored to permit removal of part of face shell for cleanout [9.6], (*b*) cleanout openings in a reinforced concrete masonry wall. (*Courtesy of Mr. Phillip Samblanet, the Masonry Society, Boulder, Colorado*).

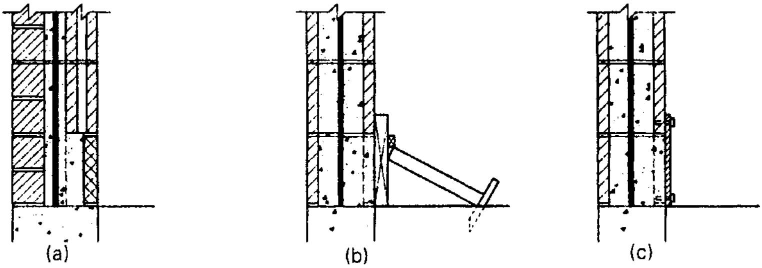


FIGURE 9.16 Methods of sealing cleanout openings: (*a*) reinstall face shell or CMU soap, (*b*) install a 2 × 10 brace against masonry, (*c*) install plywood from board mechanically fastened to masonry [9.11]. (*Courtesy: Illinois Masonry Institute.*)

checked for proper positioning, size, and length. Several methods of closing or sealing cleanouts are shown in Fig. 9.16.

2. Grout batching: Grout is most often supplied in bulk by ready-mix trucks and pumped into place because of the volume and the speed of the placement required. Grout batching on-site is more common for smaller projects. When mixed at site, the grout mix should be batched in multiples of a bag of portland cement as a quality control measure. All materials for grout should be mixed in a mechanical mixer. A discussion on masonry grout is presented in Ref. 9.10.

Grout is specified according to proportions specified in ASTM C476-02 as shown in Table 9.3 [9.16]. Grout may be fine or coarse. The permitted ranges in the required proportions of fine and coarse aggregates are intended to accommodate variations in aggregate type and gradation. The two differ in the size of their constituents. Fine grout is produced with aggregates consisting of natural sand or manufactured sand, with the maximum size aggregate not to exceed 3/16 in. Coarse grout is produced with aggregates consisting of crushed stone or pea gravel, with the maximum size aggregate not to exceed 3/8 in. The selection of grout type depends on the size of the space to be grouted. Fine grout is selected for grout spaces with restricted openings; coarse grout is used where there are no such restrictions.

When the proportion of constituents are not specified and the strength of grout is not specified, then the grout type must be specified and proportions given in Table 9.3 must be used. Grout should conform to ASTM C476-02, and have a minimum grout compressive strength equal to or greater than specified compressive strength of masonry

TABLE 9.3 Grout Proportions by Volume [9.16]

Type of grout	Portland or blended cement	Hydrated lime or lime putty	Aggregate, loose, damp (times the sum of the volumes of the cementitious materials)	
			Fine	Coarse
Fine	1	0-1/10	2 ¼–3	
Coarse	1	0-1/10	2 ¼–3	1–2

Source: MSJC Table SC-7, Ref. 9.16

but not less than 2000 psi [9.16 Section 1.4B]. The compressive strength of grout is to be determined in accordance with ASTM C1019.

Requirements for mixing ingredients of grout are specified in applicable building codes. Water, sand, aggregate, and portland cement should be mixed for a minimum of 3 min, then the hydrated lime (if any) should be added and mixed for no more than 10 min in a mechanical mixer. Grout should have good workability or consistency, which is measured by slump test performed in accordance with ASTM Standard C143/C143M-05a: Test Method for Slump of Hydraulic Cement Concrete (see discussion in Chap. 3). In most cases, a slump of 8 to 11 in. is recommended. The initial high water-cement ratio which is required to produce such a high slump is rapidly reduced by the highly absorptive masonry into which the grout is poured. Small spaces or cells require grout with higher slump than large grout spaces or cells. As the surface area and shell thickness in contact with grout decreases relative to the volume of the grouted space, the slump should be decreased [9.12]. Table 9.4, excerpted from Ref. 9.15, gives estimated volume of grout needed for 8-in. reinforced hollow concrete masonry walls.

3. Grout placement and consolidation: Masonry surfaces tend to be aesthetically very appealing. Therefore, whenever possible, grouting should be done from the unexposed face of the masonry element. Extreme care should be taken to avoid staining on the exposed faces of the masonry element. In case the grout falls on the surface or sticks to it, it should be cleaned off immediately with water and a bristle brush. Waiting until after curing has occurred would make removal difficult.

As alluded to earlier, there are two methods of grouting masonry. Whether brick masonry or concrete masonry construction, the basic grouting practices for the two are the same except some minor variations. Two methods are generally used in practice: *simultaneous* masonry construction and grouting, and grouting *after* completion of masonry construction. The two methods are commonly referred to as "low lift," and "high lift" grouting. Both have certain advantages and disadvantages. The method of grouting is usually selected by the contractor. However, the specifications can require a particular grouting method. In general, the method ultimately selected depends on many factors which include the type of masonry wall, the size and scope of the project, the availability of equipment, and the experience of the contractor.

a. Low-lift grouting: In low lift grouting, the wall is built to scaffold height, or to a bond beam course, to a maximum of 5 ft. The cells to be grouted are cleared of mortar

TABLE 9.4 Estimated Volume of Grout Needed for 8-in. (203 mm) Reinforced Hollow Units Concrete Masonry Walls*, $ft^3/100\ ft^2$ of wall [9.15]

Bond beams	Grouting at (inches on center)					
	48	40	32	24	16	8
None	5½	6½	8½	11	16½	33
8 ft	8	9	10	13	18	33
4 ft	10	11	12½	14½	19	33
2 ft	14½	15½	16½	19	22	33

*Includes 10 percent allowance for waste. To estimate grout needed for 6 in. (152 mm) units, multiply above figures by 0.63. For 12 in. (302 mm) units, multiply by 1.55.

droppings and other debris. Steel reinforcing bars and other items to be embedded are placed in the designated locations, and the cells are grouted. The grout is usually placed with buckets. This grouting method is the simplest and better suited to smaller projects and multiwythe construction, and in situations when the sequencing prevents the use of high-lift grouting. The primary benefit of low-lift grouting is that cleanouts are not required. Since the grout pour cannot exceed 5 ft in one day, all the visual inspection of the grout spaces can be conveniently made from top of the wall. Figure 9.13 shows typical reinforced concrete masonry construction using low-lift grouting.

The International Masonry Institute [9.11] provides guidelines for two types of procedures for low-lift grouting: pours 12 in. or less, and pours greater than 12 in. and up to 5 ft. Both procedures are schematically illustrated in Fig. 9.17.

i. *Pours 12 in. or less:* In this method, grouting is done as the wall is being constructed. The grout is placed in the grout space in lifts not exceeding 6 times the width of the space or a maximum of 8 in. The grout should be terminated at least 1½ in. below the top bed joint to provide a "grout shear key" for successive lifts (Figs. 9.9b and 9.17d). When grout is placed between multiwythe walls, vertical barriers spaced not more than 30 ft apart must be constructed to contain grout flow. Their purpose is to prevent excessive flowage, which can cause segregation of grout materials. These vertical barriers can be built from partial masonry units to the full height of the wall (Fig. 9.14). Spaces containing grout must be puddled or vibrated during placement to ensure complete filling of the grout space. Puddling with a wooden stick is allowed for grout pours of 12 in. or less. Care should be taken to ensure that masonry units are not dislodged or displaced while consolidating grout.

ii. *Pours greater than 12 in. and up to 5 ft:* Walls, single-wythe or multiwythe, are built to a height of up to 5 ft. For single-wythe walls, units are placed with cells sufficiently vertically aligned and clear of debris and mortar obstructions. Units are laid with mortar on cross-webs to contain grout. Reinforcement and other items to be embedded are placed in designated locations, and then grout is poured into the cells (Fig. 9.11).

The grouting of any section of wall must be completed in one day without any interruptions greater than one hour. When grouting operation is stopped or interrupted for more than an hour, it is considered to be the end of the pour. When a pour is stopped prematurely, grout splatter may adhere to the surfaces of the grout spaces and to the reinforcement contained therein. This may be detrimental to the bond development of the next grout pour, and may be critical if this occurs in an area of probable high stress. If the splatter is excessive, it should be treated as debris and removed. When this splatter is cleaned from or falls from its initial lodging place to the top of the last pour, it must be considered as debris and removed. Removal of this debris may require cutting new cleanout in the masonry [9.12].

Except at the topmost level of a wall, the top of grout pour should not normally be terminated at the top of a course of a masonry wall or other grout space, but rather inside a masonry unit or other grout space at least 1½ in. below the bed joint as required by MSJC-08: Specification for Masonry Structures [9.16 Section 3.5] (Figs. 9.9b and 9.17d). When bond beams occur, the grout pour is required to be stopped a minimum of ½ in. below the top of the masonry. The purpose of this requirement is to provide a grout shear key and eliminate a cold joint between the pours. Note that if grouting without cleanouts is employed, this 1½-in. space left for the shear key will quite likely be filled with mortar droppings and other construction debris. All such debris must be cleaned out completely. If the space is not cleaned out, it will create a plane of discontinuity (essentially a plane of weakness) between the last pour and the new pour. This may weaken

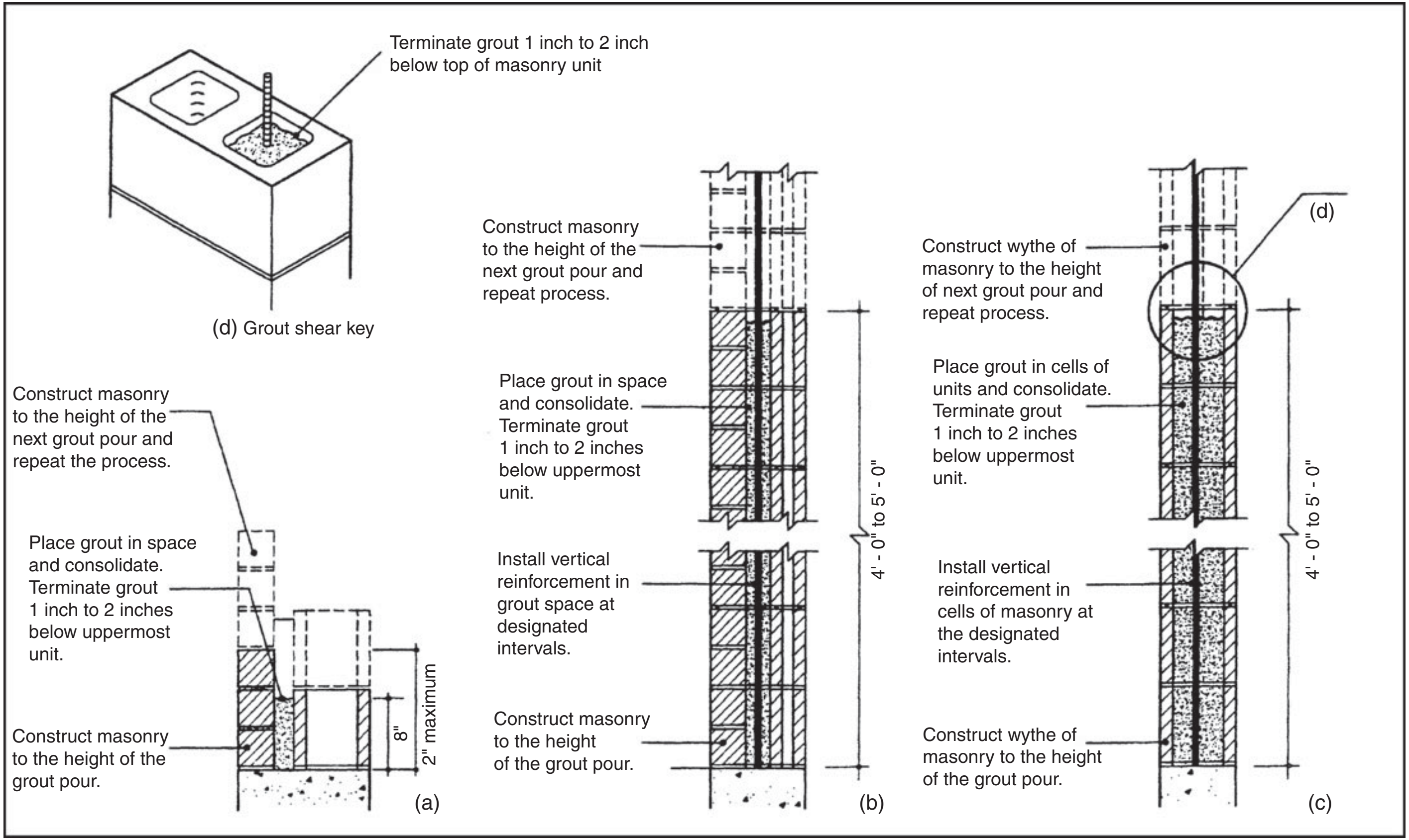


FIGURE 9.17 Methods of low-lift grouting: (*a*) grout pours 12 in. or less for multiwythe wall, (*b*) grout pours up to 5 ft for multiwythe wall, (*c*) grout pours up to 5 ft for single-wythe wall, (*d*) grout shear key [9.11]. (*Courtesy: Illinois Masonry Institute.*)

the wall and create a path for moisture penetration [9.12]. As a minimum, if cleanouts are not provided, special provisions must be made to keep the bottom and sides of the grout spaces, as well as the minimum total clear area as required by Table 9.2, clean and clear prior to grouting.

For multiwythe walls, a minimum of ¾ in. space is required between the two wythes. Vertical barriers must be constructed to contain grout flow. Vertical reinforcement is installed if required, and the space is filled with grout in two or three lifts, evenly distributing the grout throughout the space in each lift. The grout is consolidated shortly after it is placed, and reconsolidated after initial water loss and settlement have occurred.

Freshly placed grout exerts a hydrostatic pressure in the grout space or on the surrounding formwork. This pressure increases with increasing pour height, and grout pours composed of several lifts may develop this pressure for the full pour-height. This hydrostatic pressure may be sufficient to break out the face shells of the hollow masonry units with unbraced ends, or to cause separation of wythes. The grout space should not be filled until the masonry has cured and achieved adequate strength to prevent blowouts.

To resist the grout pressure, and prevent bulging or blowouts, wythes in a multiwythe construction must be bonded together with corrosion-resistant wire ties or joint reinforcement (also called continuous metal ties). However, these ties may not be sufficient to prevent this occurrence [9.16]. The masonry must be allowed to cure for approximately 12 to 18 hours prior to grouting to withstand the hydrostatic grout pressure.

General requirements for ties are specified in MSJC-08 Sections 2.1.5.2.4 (for wythes not bonded by headers) and 2.1.5.3.2 (for walls designed for noncomposite action). Specification for wall ties can be found in MSJC-08 *Specification* [9.16, (Section 3.4C)]. A discussion on metal ties can be found in Ref. 9.1. Ties in alternate courses are required to be staggered. The maximum vertical distance between the ties (for bonded two-wythe walls not bonded by headers) is limited to 24 in. and the maximum horizontal distance to 36 in. When adjustable ties are provided, the vertical or horizontal spacing should not exceed 16 in., unless otherwise required.

Wall ties resist the grout pressure by their tensile capacity. Additionally, they provide the benefit of positive mechanical anchorage between the grout core and the surrounding masonry. Wall ties for grouted multiwythe wall construction are required to be at least 3/16-in. in diameter for every two square feet of area. Ties of different size and spacing that provide equivalent strength between the wythes are permitted. Several commercial tie sizes are shown in Fig. 9.18. Ties used with solid masonry are sometimes bent in a “Z” shape with 2-in. legs bent at 90°. This type of metal tie is not permitted for hollow unit masonry. Unit metal ties may be of the adjustable type as shown in Fig. 9.18b. Adjustable ties offer advantages over the ordinary (nonadjustable) types in that they simplify the erection of multiwythe walls by allowing them to erect the wythes independently instead of simultaneously, as with ordinary ties. They also permit adjustment for differences in level between courses.

As an alternative to metal ties, continuous metal ties, variously called as prefabricated joint reinforcement, mesh, or more commonly, *joint reinforcement*, can be used. It consists of two or more parallel longitudinal wires to which cross wires are welded. Joint reinforcement can be used for the following purposes:

i. To serve as horizontal reinforcement.
ii. To serve as ties to bond masonry wythes.
iii. To serve as longitudinal reinforcement for control of cracking caused by drying shrinkage and temperature changes.

Joint reinforcement can be of ladder-type or truss-type; both can also be of adjustable type. Several types of commercially available joint reinforcement are shown in Fig. 9.19.

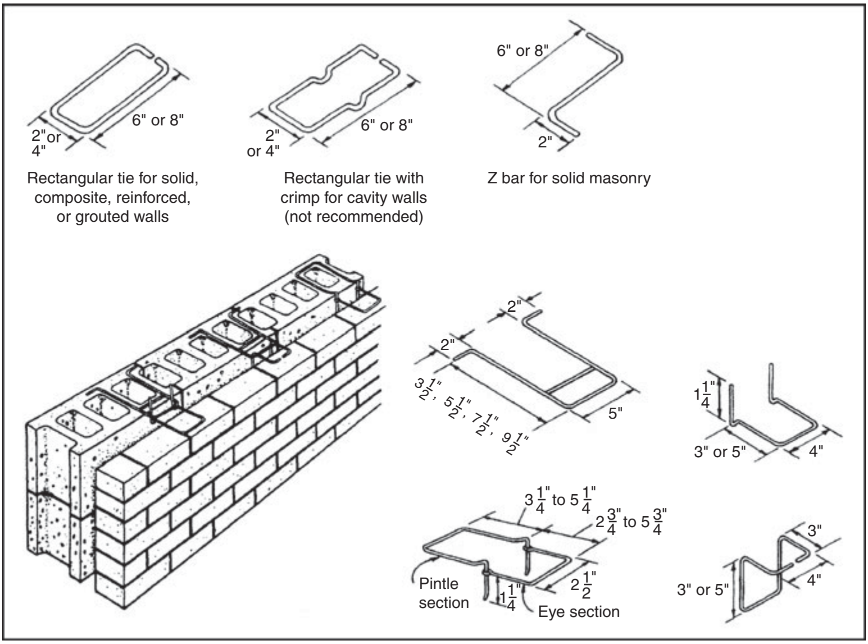


FIGURE 9.18 Metal ties [9.1]. (*Courtesy: Portland Cement Association.*)

In all cases of low-lift grouting, care should be taken to ensure that reinforcing bars project sufficiently above the top course to provide for the minimum lap length as shown in Fig. 9.13. Grout should be moved from the mixer to the point of deposit as far as practical. Pumping or other placing methods which prevent the segregation of the mix should be used. Care should be taken to minimize grout splatter on reinforcement and masonry unit surfaces not being encased in the grout fill.

b. *High-lift grouting:* High-lift grouting involves placement of grout after the masonry is laid up to story height or to the maximum height as specified in Table 9.2. High-lift grouting offers certain advantages over the low-lift grouting, and is especially suitable for large projects. A larger volume of grout can be placed at one time thereby increasing the overall speed of construction and resulting in consistent workmanship. Productivity is increased because the mason does not have to lift and place the unit over the reinforcing bars for single-wythe grouting. Less steel is used for splices, and the location of the steel can be easily checked by the inspector prior to grouting. Figure 9.20 illustrates the schematics of high-lift grouting procedures.

As with low-lift grouting, the International Masonry Institute suggests two types of procedures for high-lift grouting as follows [9.11]. Both procedures allow building the wall to its full "pour" height (limited to 24 ft maximum) as shown in Fig. 9.18. The two differ in that one procedure allows grouting the wall successively in lifts of no more than 5 ft, whereas the other allows the wall to be grouted to its full "pour" height (24 ft maximum) in a single lift.

i. *Pours greater than 5 ft to 24 ft walls*: This procedure allows for the construction of the wall to its total "pour" height (not to exceed 24 ft) and can be used for grout pours

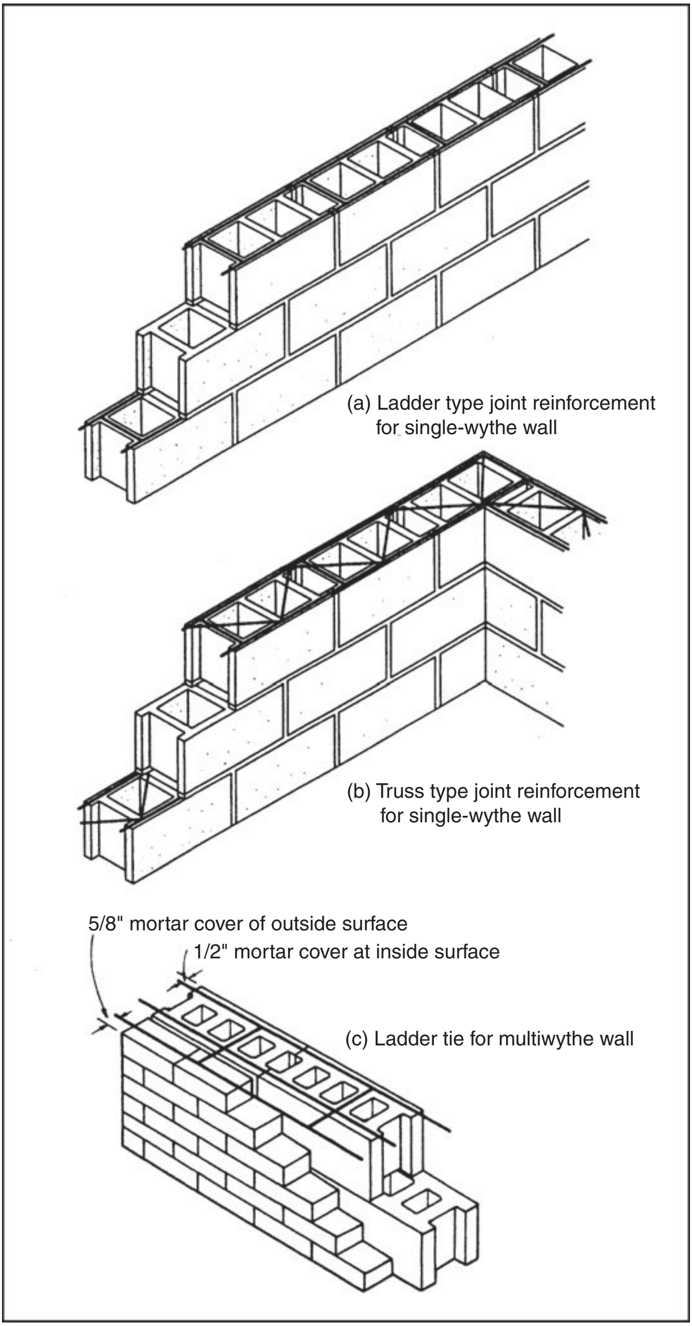


FIGURE 9.19 Various types of joint reinforcement (a-c) [9.1]. (*Courtesy: Portland Cement Association*.)

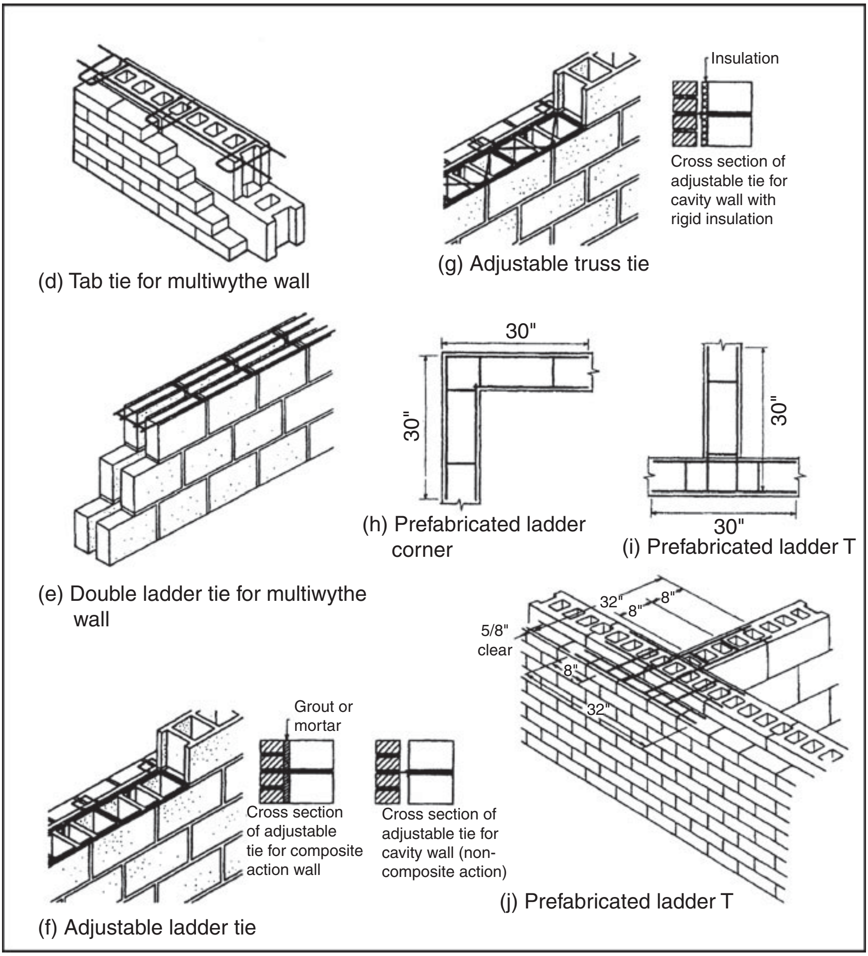


FIGURE 9.19 Various types of joint reinforcement (d-j) [9.1]. (*Courtesy: Portland Cement Association.*)

of 6 to 24 ft for multiwythe (Fig. 9.20*a*) as well as single-wythe walls (Fig. 9.20*b*). The walls are built to their total "pour" heights, with cleanouts provided at the base of the wall. The grout space is cleaned of excessive mortar, mortar droppings, and construction debris, all of which can be removed at the base of the wall through the cleanouts. The cleanouts are sealed after the final inspection and prior to grouting. Next, vertical reinforcing bars are placed at the designated locations, and grout is placed in uniform lifts not to exceed 5 ft. Grout should be fluid and placed before any initial set has occurred. It should be consolidated by mechanical vibrator during placement before loss of plasticity, in a manner to fill the grout space, and reconsolidated after water loss and settlement have occurred.

The next lift of grout is placed after waiting 30 to 60 min and while the grout of the first lift is still plastic. The waiting period allows for additional grout consolidation, and reduces the hydrostatic pressure on the existing masonry due to grout stiffening. Again, the

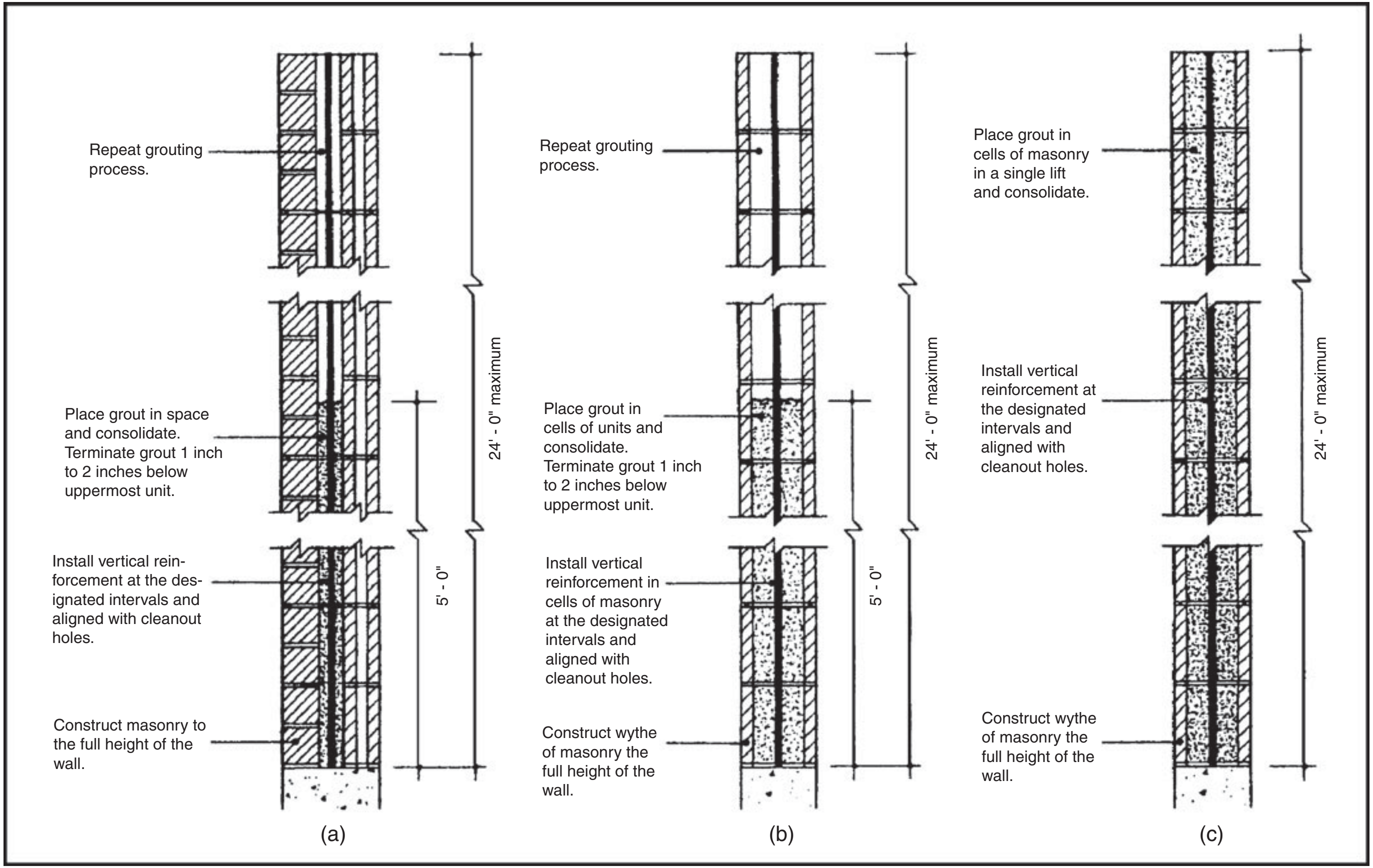


FIGURE 9.20 High-lift grouting procedures (*a*) grout pours 6 to 24 ft for multiwythe wall, (*b*) grout pours 6 to 24 ft for single-wythe wall, (*c*) single lift pour up to 24 ft [9.11]. (*Courtesy: Illinois Masonry Institute.*)

grout is consolidated by extending the vibrator completely through the new lift and 12 to 24 in. into the previous lift. Penetrating the previous list bond the two lifts together. The full height of any section of the wall should be grouted, without any excessive interruptions, in one day.

For multiwythe walls, the wythes are bonded together with ties (as explained in the previous section) to prevent bulging or blowouts. Vertical barriers must be constructed within the grout space to contain the horizontal grout flow. The masonry should be allowed to cure for three days in the warm weather and five days in cold weather to attain sufficient strength prior to grouting. For single-wythe walls, the units are laid with vertical cores sufficiently aligned to facilitate the placement of reinforcing bars and grout. Cross-webs of hollow units are bedded with mortar to contain the flow of grout. The masonry must be allowed to cure for 12 to 18 hours prior to grouting. If the grouting is interrupted for more than 1 hour, a shear key should be formed by terminating the grout lift at least 1½ in. below the top of the uppermost grout lift (Figs. 9.9*b* and 9.17*d*).

ii. *Single-lift pour in cells of single-wythe wall:* This procedure can be adopted when approved by the building officials, and if it can be demonstrated that cells can be properly filled. It allows grouting the full "pour" height of the wall (not to exceed 24 ft) in a single lift (Fig. 9.20*c*). Therefore, it is quicker and more economical. The continuity of the pour eliminates cold joints and maximizes the bond between the reinforcing bars and the grout. The general requirements for preparation of grout space are similar to those of the pervious section. The full height of the wall is grouted in one lift. The grout is consolidated with a vibrator shortly after placement.

Consolidation and reconsolidation of grout by mechanical vibration is very important for good construction. Consolidation causes the grout to flow immediately into every opening and space within a wall. Reconsolidation vibrating also densifies and strengthens the grout because it remixes the grout after initial vibration has forced the free water to migrate to the absorbing surfaces of the masonry units and closes all of the capillaries or tunnels formed by this free water migration.

4. *Curing and protection of reinforced masonry:* Protection requirements for masonry vary with temperature and weather conditions (temperature, humidity, and wind velocity). In general, the masonry work, particularly the top grout pour, should be kept covered and damp to prevent excessive drying. The newly grouted masonry should be fog sprayed three times each day for a period of three days following construction when the ambient temperature exceeds 100°F, or 90ºF with a wind in excess of 8 mph. For walls, columns, and pilasters, at least 12 hours should elapse after construction before application of roof members, except that 72 hours should elapse prior to application of heavy, concentrated loads such as trusses, girders, or beams [9.5].

9.4 MOVEMENTS OF CONSTRUCTION MATERIALS, THEIR CAUSES AND EFFECTS

9.4.1 Types of Movements

Elements and components comprising a structure experience movements that can be attributed to a variety of causes. Primary causes of these movements are the applied loads, material properties, and environmental effects. Applied loads are responsible for deflection of building elements, and drift and settlement of a building. Material properties are responsible for deformation of building elements under sustained loads over time, a phenomenon known as *creep*.

The environmental effects include expansion and contraction due to temperature and moisture content, freezing expansion, shrinkage, and carbonation of concrete and mortars. Essentially, these phenomena cause changes in the volume of materials. Restrained movements normally manifest themselves in the form of structural damage, usually in the form of cracks. To minimize or eliminate this damage, the design should minimize volume changes, prevent movement or accommodate differential movement between materials and assemblies. Schematics of several types of cracking in masonry structures and their causes are shown in Fig. 9.21, which can be minimized or eliminated by proper design. Table 9.5 summarizes various types of movements in building materials. Causes and effects of various types of movements are summarized in Table 9.6. A discussion on movements in masonry structures and consequences of restrained movements can be found in the literature [9.17–9.22, 9.26–9.28].

9.4.2 Causes of Movements

1. *Thermal changes:* All building materials expand and contract due to variations in temperature. The resulting movements are referred to as thermal movements and cause changes in the sizes of building elements. If these movements are restrained, the elements will be subjected to stresses in addition to those due to loads. Thermal movements can occur in both horizontal and in vertical directions.

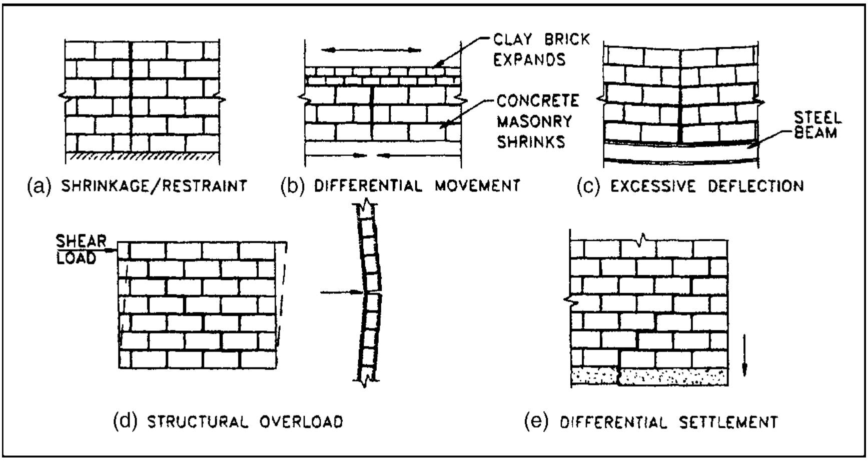


FIGURE 9.21 Causes of cracking and cracking patterns in masonry structures [9.17].

TABLE 9.5 Types of Movements of Building Materials [9.21]

Building material	Thermal	Reversible moisture	Irreversible moisture	Elastic deformation	Creep
Brick masonry	x	–	x	x	x
Concrete masonry	x	x	–	x	x
Concrete	x	x	–	x	x
Steel	x	–	–	x	–
Wood	x	x	–	x	x

TABLE 9.6 Causes of Movements in Buildings and Their Effects [9.21].

Causes	Effects
Temperature changes	
Heating	Expansion
Cooling	Contraction
Change in moisture content	
Drying	Shrinkage
Wetting	Expansion
Overloads on structure	
Dead load, live load	Deflection and
Impact, vibration	Distortion
Load on soil	Settlement

Both clay and concrete masonry are known to experience linear movements that are proportional to temperature changes. Stresses caused by thermal movement can be predicted from the knowledge of its magnitude. The latter can be calculated based on the coefficient of thermal expansion. The change in length of a member, ΔL, subjected to a temperature variation of ΔT can be expressed as

$$\Delta L = \alpha L \Delta T \tag{9.1}$$

where ΔL = thermal movement
α = coefficient of thermal expansion
L = length of member subjected to temperature change
ΔT = change in temperature

Values of coefficients of thermal expansion of various construction materials are given in Table 9.7. Coefficients of thermal expansion for masonry materials may vary from 2.5×10^{-6} to 5.5×10^{-6} in./in./°F. According to the MSJC-08 *Code* [9.4], the thermal expansion coefficients for clay and concrete masonry, respectively, are taken as 4×10^{-6} and 4.5×10^{-6} in./in./°F. It should be noted that materials differ in their response to temperature variations. When materials of different thermal characteristics are used together in construction (e.g., clay and concrete masonry units used together), the design should account for potential differential movement.

For typical design purposes, surface wall temperatures are assumed to vary between 0 and 140°F although the variation can be larger [9.17]. Expansion and contraction of a wall will occur within this range depending on the temperature of the wall at the time of construction, which during winter can be as low as 50°F [9.22]. For example, a 100-ft long concrete masonry wall constructed during a 70°F weather and subjected to a minimum temperature of 0°F will experience a shortening of 0.38 in. based on a thermal coefficient of expansion of 4.5×10^{-6} in./in./°F. The same wall will also experience an elongation of 0.38 in. if the temperature reaches 140°F. The shortening or elongation of the wall due to a temperature change of 70°F be calculated from Eq. (9.1):

$$\Delta L = \alpha L \Delta T = (4.5 \times 10^{-6})(100 \times 12)(70) = 0.38 \text{ in.}$$

Given in Table 9.7 are movements in 100-ft long walls [calculated from Eq. (9.1)] that can be caused due to a change in temperature of 100°F.

TABLE 9.7 Coefficients of Thermal Expansion and Thermal Movement [9.19].

Materials	Average coefficients of lineal thermal expansion millionths (0.000001) in. per in. per degree F	Thermal expansion, inches per 100 ft for 100°F temperature increase
Dense aggregate concrete masonry	5.2	0.62
Lightweight aggregate concrete masonry	4.3	0.52
Clay or shale brick masonry	3.6	0.43
Granite	4.7	0.56
Limestone	4.4	0.53
Marble	7.3	0.88
Gravel aggregate concrete	6.0	0.72
Lightweight structural concrete	4.5	0.54
Aluminum	12.8	1.54
Bronze	10.1	1.21
Stainless steel	9.6	1.15
Structural steel	6.7	0.80
Gypsum plaster	7.6	0.91
Perlite plaster	5.2	0.62
Vermiculite plaster	5.9	0.71

If the thermal movement of any structural element is restrained, it will induce stresses (referred to as thermal stresses) in the member. These stresses will be in addition to those induced by the applied loads, and can be calculated if the thermal movement (ΔL) is known. The change in the length of a member subjected to an axial force can be expressed based on Hooke's law as

$$\Delta L = \frac{PL}{AE} \tag{9.2}$$

where P = axial load
L = member length
A = area of cross-section of member
E = modulus of elasticity of the material

Eq. 9.2 can be used also to calculate the normal stress, $f\,(= P/A)$ caused by the change in member length, by rewriting Eq. 9.2 as follows:

$$f = \frac{(\Delta L)E}{L} \tag{9.3}$$

Alternatively, the normal stress caused by the temperature change can be expressed directly in terms of temperature change by combining Eqs. (9.1) and (9.2):

$$\frac{PL}{AE} = \alpha L \Delta T$$

from which the resulting stress, f, can be calculated as

$$f = \frac{P}{A} = \alpha \Delta T E \tag{9.4}$$

Equation (9.4) is general and can be used to predict stresses caused by restrained movements in any type of material. For example, for a 100-ft long wall constructed from dense aggregate concrete masonry having $f'_m = 1800$ psi and experiencing a temperature change of 100°F, the induced stresses can be calculated as follows:

$$E_m = 900 f'_m = 900(1800) = 1.62 \times 10^6 \text{ psi}$$

$$f = \alpha \Delta T E = (5.2 \times 10^{-6})(100)(1.62 \times 10^6) = 842 \text{ psi}$$

A 842-psi stress increase represents a 46.8% increase above the 1800-psi design compressive strength of concrete masonry, not a desirable situation.

2. *Creep:* Also known as a plastic flow, creep is the deformation exhibited by some materials when continuously stressed, and is manifested by movement in the direction of the applied stress. It is a function of stress and time, and is influenced by physical properties of materials and exposure conditions. *Creep coefficient* is defined as the ratio of creep strain to elastic strain. *Specific creep* is defined as the creep strain per unit of applied stress, and is much greater for clay masonry than for concrete masonry. The MSJC-08 *Code* Section 1.8.6 [9.4] specifies the following values of creep coefficients, k, for clay and concrete masonry:

Clay masonry	$k_c = 0.7 \times 10^{-7}$ per psi
Concrete masonry	$k_c = 2.5 \times 10^{-7}$ per psi

3. *Carbonation:* Carbonation is an irreversible reaction between cementitious materials and atmospheric carbon dioxide, which occurs over a period of several years. Currently there is no standard test method for carbonation shrinkage. A suggested value of carbonation shrinkage coefficient is 2.5×10^{-4} in./in., which results in a shortening of 0.3 in. in a 100-ft long wall [9.17].

4. *Excessive deflection:* Cracking of unreinforced masonry designed according to MSJC-08 Section 2.2 may occur as walls and beams deflect excessively under loads. Cracking may also be induced in walls due to deflection of supporting elements. To guard against excessive deflection, the deflection of elements (beams and lintels) supporting unreinforced masonry due to dead and live load is limited to lesser of 1/600 of span (MSJC-08 Section 1.13.3.1 [9.4]). Also, movement joints can be used to panelize the masonry so that it can articulate with the deflected shape of the supporting member [9.17].

5. *Differential movement:* Differential movement refers to movements resulting from different responses of two materials in contact to loading, moisture, and temperature change. In reinforced masonry construction, three materials are of particular concern: clay bricks, concrete masonry units, and steel.

Differential movement between concrete and brick masonry must be considered when the two are in contact since concrete masonry has an overall tendency to shrink, while clay brick masonry has a tendency to expand. These differential movements may cause distress in masonry, especially in composite construction and in walls that incorporate clay brick and concrete block in the same wythe.

Differential movement can also occur when clay brick is used as an accent in a concrete masonry wall, or vice versa. Cracking may occur as the two materials respond differently to change in temperature unless provisions are made to accommodate the differential movement (shrinkage of concrete masonry and expansion of clay brick). Horizontal reinforcement and frequent control joints would help minimize cracking.

Thermal movements need to be accounted for when using masonry in conjunction with steel. The coefficient of thermal expansion for steel (6.5×10^{-6} in./in./°F) is more than 50 percent greater than that for clay and concrete masonry (4×10^{-6} and 4.5×10^{-6} in./in./°F, respectively). In addition, steel shapes typically have a much higher surface area to volume ratio and tend to respond to temperature changes much more quickly. These differential movements can be accommodated through slotted and flexible connections. A discussion on this topic can be found in the literature [9.23–9.25].

6. *Settlements:* Differential settlement occurs as a result of uneven settlement of the foundation. This may occur due to weak or improperly compacted soil. The resulting distress is manifested usually in the form of a stair-step crack along the mortar joints in the settled portion of the masonry wall. This type of damage can be prevented by proper design and construction of foundation. Unsuitable soil should be removed and replaced with compacted, approved soil, gravel, or concrete. Tree roots, debris, and ice should be removed prior to placement of footings. Adding reinforcement to foundations can also minimize potential of differential settlement.

7. *Moisture movement:* Clay and concrete masonry respond differently to changes in moisture content. Concrete masonry units experience *shrinkage* as result of moisture loss and carbonation. Clay masonry units *expand* with an increase in moisture content. The amount of movement due to moisture changes (i.e., shrinkage of concrete masonry and expansion of clay masonry) can be estimated based on the same principle as used for thermal expansion:

$$\Delta L = kL \tag{9.5}$$

where $k = k_e$, moisture expansion coefficient for clay masonry
$= k_m$, shrinkage coefficient for concrete masonry

a. Concrete masonry: Shrinkage cracks occur when contraction of concrete masonry unit is restrained. If the tensile stress resulting from contraction exceeds the tensile strength of the unit or exceeds the bond strength between the unit and the mortar, shrinkage cracks will occur. Shrinkage cracks are not structural cracks, but they do disfigure the wall in which they occur rendering it unsightly.

Shrinkage of concrete masonry units is affected by many factors. For an individual unit, the amount of drying shrinkage is influenced by the wetness of the unit at the time of placement, the amount and characteristics of cementitious materials, method of curing, aggregate type, consolidation, and wetting and drying cycles. Total shrinkage is determined by ASTM C426 Standard Test Method for Linear Drying Shrinkage of Concrete Masonry Units which measures shrinkage from a saturated condition to a moisture content of 17 percent. Typical total shrinkage values range between 0.0002 in./in. to 0.00045 (i.e., 0.24 to 0.54 in. in 100 ft) [9.17]. The MSJC-08 Section 1.8 [9.4] specifies the value of the *shrinkage coefficient* of concrete masonry, $k_m = 0.5s_l$ where s_l = total linear drying shrinkage of concrete masonry units measured in accordance with ASTM C426, a value not exceeding 0.00065 (or 0.065 percent).

Some restraints can be placed on the moisture content of the unit. ASTM C90-06 for Loadbearing Concrete Masonry Units puts limits on moisture content for Type I units (Table 9.8) depending on linear shrinkage potential and average relative humidity at the place where the units will be installed. The maximum allowable moisture content is related to the drying shrinkage potential of the unit. This is based on the average annual relative humidity expected at the jobsite. The table does not list specific requirements; rather it lists a range of shrinkage values. The objective is to limit residual in-the-wall movement to a tolerable level. By requiring units with higher drying shrinkage properties to contain a lower percentage of moisture, satisfactory in-the-wall performance is achieved [9.25].

It should be noted that the moisture content of concrete masonry units is difficult to maintain. Special packaging requirements might be considered when shipping the

TABLE 9.8 Moisture Content Requirements for Type I Units [ASTM C90-06]

	Moisture content, maximum percent of total absorption (average of 3 units)		
	Humidity* conditions at jobsite or point of use		
Linear shrinkage, percent	Humid	Intermediate	Arid
0.03 or less	45	40	35
0.03–0.045	40	35	30
0.045–0.065 max	35	30	25

**Arid*—Average annual relative humidity less than 50 percent.
Intermediate—Average annual relative humidity between 50 to 75 percent.
Humid—Average annual relative humidity above 75 percent.

units to a jobsite. It is also important to protect the units from rain after they have been delivered at the jobsite.

b. *Clay masonry:* Clay masonry units will expand with an increase in moisture content. However, brick units will not contract when moisture content decreases, that is, this expansion is not reversible by drying at atmospheric temperatures. The amount of moisture in a brick depends on the type of raw materials and the range of firing temperatures [9.18]. A brick unit is smallest in size when it comes from the kiln; it expands with increase in moisture content from that time on. The expansion of a clay brick begins when exposed to humidity in the air. Although it experiences its most expansion during the first few weeks, the expansion continues at a much slower rate for several years. Based on past research, the long-term moisture expansion of brick can be estimated at between 0.0002 and 0.0009 [9.18]. The MSJC-08 Section 1.8 [9.4] specifies the value of coefficient of moisture expansion for clay masonry (k_e) as 0.0003. Reference [9.18] recommends using a value of 0.0003 when designing composite walls, and a value of 0.0005 for veneer walls where an upper bound of movement is estimated. Some moisture is also generated due to rain and from the water used in construction process. Figure 9.22 shows the moisture expansion characteristics of

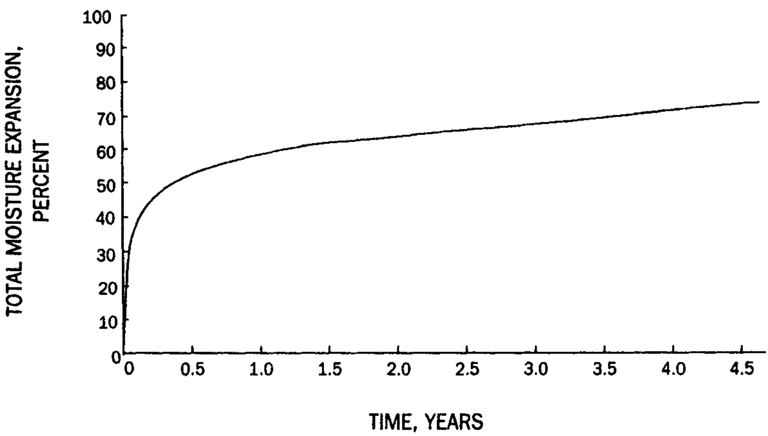


FIGURE 9.22 Projected moisture expansion of fired brick versus time [9.18].

fired bricks. It will be noted that most of the moisture expansion occurs within the first six months.

Freezing expansion occurs as result of increase in volume of moisture as it freezes. The coefficient of freezing expansion is taken as 0.0002 in./in. Freezing expansion does not occur until temperature goes below 14°F. Furthermore, the units must be saturated when frozen to cause expansion. Local conditions must be taken into account to determine if freezing expansion will occur, but is usually considered negligible [9.22].

8. *Shortening of structural frames:* In framed structures, especially concrete frame buildings, vertical shortening due to creep or shrinkage of the structural frame may impose high stresses on masonry. These stresses may develop at window heads, shelf angles, and other points where stresses are concentrated.

9.4.3 Estimating Movement of Exterior Masonry

Total unrestrained movement of masonry walls can be estimated by summing up the movements due to changes in temperature and moisture content. Movement due to temperature change can be calculated from Eq. (9.1), whereas that due to moisture expansion can be calculated from Eq. (9.2).

1. *Brick masonry:* The total unrestrained movement, ΔL, of a wall can be estimated from the following expression:

$$\Delta L = [0.0003 + 4 \times 10^{-6}(\Delta T)]L \tag{9.6}$$

where 0.0003 and 4×10^{-6}, respectively, are the coefficients of moisture and thermal expansion, and ΔT the change in temperature. For example, total movement in a 100-ft long clay brick wall due to a change in temperature of 100°F will be (from Eq. 9.6)

$$\Delta L = [0.0003 + 4 \times 10^{-6}(100)](100 \times 12) = 0.84 \text{ in.}$$

It must be recognized that a 0.84-in. movement is not trivial. For example, in a 100-ft long wall of 2000-psi compressive strength brick masonry, the additional stress, f, caused by an 0.84 in. movement can be calculated from Eq. 9.3:

$$E_m = 700 f'_m = (700)(2000) = 1.4(10^6) \text{ psi}$$

$$f = \frac{(\Delta L)E}{L} = \frac{(0.834)(1.4x10^6)}{(100x12)} = 980 \text{ psi}$$

A stress increase of 980 psi in a 2000-psi compressive stress masonry represents an increase of 49 percent, which is considerable and must be accommodated by suitable construction details (joints).

2. *Concrete masonry:* Concrete masonry will also be subjected to movement due to change in temperature as well as shrinkage. However, thermal expansion of concrete masonry walls is offset partially by shrinkage from carbonation and drying of units. The primary cause of movement in concrete masonry walls is due to initial moisture loss.

9.4.4 Distress in Masonry due to Restricted Movements

It is instructive to study the distress in masonry walls that may be caused as a result of restrained wall movements. Cracking is probably the distress of frequent occurrence in masonry walls. A discussion on failures resulting from restrained movements in masonry

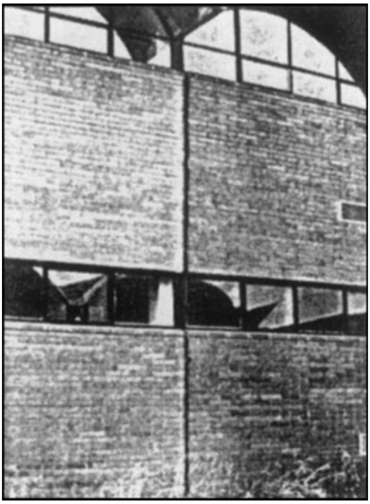


FIGURE 9.23 Damage at the expansion joint between two walls [9.18].

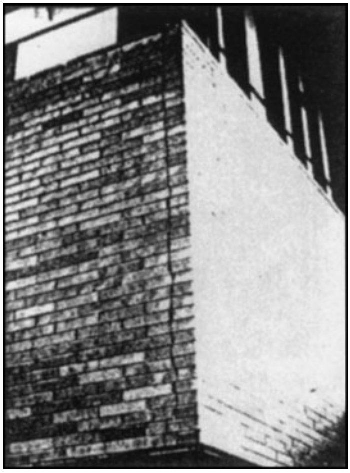


FIGURE 9.24 Vertical crack at the corners of wall intersections [9.18]. (*Courtesy: Portland Cement Association.*)

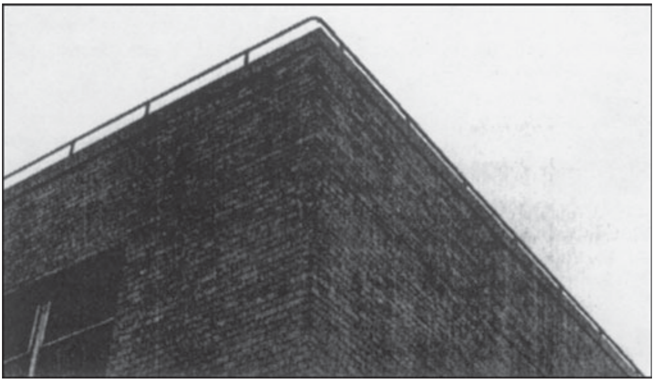
structures, and several case histories, can be found in Refs. 9.19 and 9.20. Examples of distress in brick masonry are reviewed in Ref. 9.18. A summary from these references is presented here. Figures 9.23 through 9.29 identify but a few typical locations where cracks are likely to occur in masonry walls and their causes.

Long walls or walls with large distances between expansion joints may cause distress within the wall. The expansion of the brickwork may force the sealant out of the expansion joint or crack the brickwork between the expansion joint (Fig. 9.23). An insufficient or improper location of expansion joints (discussed hereafter) can lead to cracking at the corners. Perpendicular walls will tend to expand toward the corners causing rotation and cracking near the corners (Fig. 9.24).

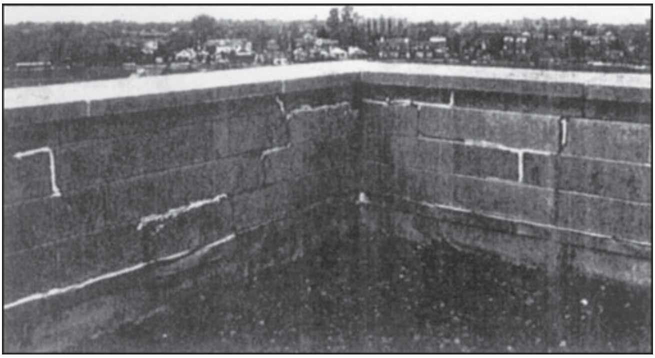
Distress may manifest in the form of bowing of the parapets. Parapets are exposed to extremes of environmental changes (moisture and temperature) on three sides, which may be significantly different from those of the wall below. Expansion can cause the parapets to bow if restrained at both corners (Fig. 9.25), or move away from corners if restrained only at one end. Figure 9.26 shows a crack in reinforced concrete foundation of a brick masonry wall due to differential settlement. Masonry walls built above grade will expand whereas the concrete foundation will shrink. If bonded together, this differential movement causes shear at the wall-foundation interface. The result often is the movement of the brick away from the corner or the cracked corner of the concrete foundation.

Since masonry is inherently weak in tension, walls may develop flexural cracking on account of excessive deflection. Figure 9.27 shows a cracked wall due to inadequate stiffness of the supporting lintel. The crack is typically of flexural type—wider at the bottom, tapering to nothing at some distance above. To safeguard against such a distress, deflections of elements supporting masonry elements are limited by codes as pointed out earlier.

Differential settlement of the foundation may be manifested by diagonal cracking of a wall. If the settlement is uniform along the entire foundation, no damage is likely to occur. However, cracking will result if one part of the foundation settles more than the adjacent part, forming a characteristic stair-step crack pattern along the mortar joints in the settled area (Fig. 9.28).



(a)



(b)

FIGURE 9.25 (*a*) Bowing of a parapet due to expansion. Because of lack of vertical expansion joints in this long parapet wall, the corners pushed out and cracked, with horizontal separation lines and 45° cracking at the exterior corner, (*b*) this parapet wall bowed and cracked in place as result of thermal expansion without relief joints [9.19].

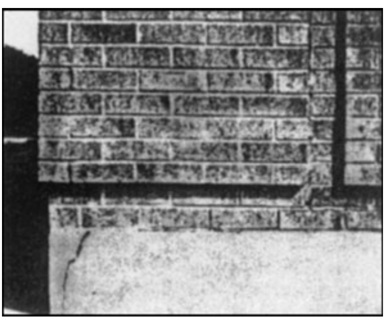


FIGURE 9.26 Cracking at a foundation corner due to differential movement at the wall-foundation interface [9.18].

FIGURE 9.27 Cracking in masonry wall due to excessive deflection of supporting beam or lintel [9.18].

FIGURE 9.28 Stair-step cracking in a masonry wall due to differential settlement [9.19].

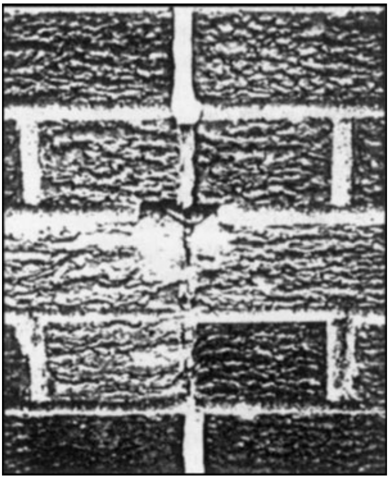


FIGURE 9.29 Spalling due to buckling of joint reinforcement across an expansion joint [9.18].

Cracking or spalling of masonry may result due to expansion of items embedded in or attached to masonry. For example, corrosion of metal reinforcement within the masonry may cause volume increases large enough to cause spalling. Joint reinforcement that is continuous across an expansion joint may buckle and push out the adjacent mortar (Fig. 9.29).

9.5 CONTROL OF CRACKING AND MOVEMENT JOINTS

9.5.1 Function and Types of Movement Joints

As pointed out earlier, restrained movements will be manifested in the form of structural damage or other kind of distress, usually in the form of cracking and spalling. To avoid this damage, the design should minimize volume changes, prevent movement, or accommodate differential movement between materials and assemblies. The last of these can be accomplished by providing horizontal reinforcement and movement joints.

1. *Horizontal reinforcement:* Horizontal reinforcement in walls increases the tensile strength of masonry and helps minimize the formation of relatively wide shrinkage cracks in concrete masonry. It also serves to increase the resistance of walls to unanticipated loads such as impact loads [9.21]. Horizontal reinforcement is provided in one of the two ways: use of bond beams or prefabricated joint reinforcement.

   Bond beams are essentially horizontal structural elements which effectively tie the components of a wall into integral units. As discussed in Chap. 3, bond beams are constructed from special units which facilitate placement of horizontal reinforcement which is embedded in concrete or grout. In situations where vertical steel will not be used in a wall, bond beams are made from special channel-shaped units which are placed end-to-end resulting in a continuous channel. In case vertical reinforcement is required in a wall, as in fully reinforced construction, the bond beam consists of special blocks that resemble standard hollow blocks except that their end and middle webs are recessed or depressed to accommodate horizontal reinforcement (Fig. 9.5*a*).

Bond beams can serve several functions. They can serve as gravity load-carrying elements as well as a means of crack control. Typically, they are placed to (1) serve as lintels over door and window openings, (2) below the sill in walls with openings, (3) at the top of walls and at floor levels to distribute gravity loads, and (4) as horizontal stiffeners incorporated into masonry to transfer flexural stresses to columns and pilaster when unusually high lateral loading is encountered. As a means of crack control, a bond beam is presumed to offer resistance to horizontal movement of an area 24 in. above and below its location in the wall. Thus, bond beams located 4 ft apart serve to influence the entire wall height as a means of crack control. In walls without openings, bond beams are spaced 4 ft apart vertically, and may be of any length up to a maximum 60 ft [9.21, 9.26].

Primarily developed for crack control, the horizontal reinforcement functions much the same as bond beams. Horizontal reinforcement consists of a prefabricated arrangement of a pair of longitudinal bars joined by cross rods designed for embedment into the horizontal mortar joints. When used in concrete masonry walls, it is typically placed at a minimum distance of 8 in. and a maximum distance of 24 in. apart depending on the wall height and spacing of control joints.

2. *Movement joints:* Most building materials and structural assemblages require movement joints to perform satisfactorily and to minimize the potential for distress as discussed in preceding paragraphs. Generally speaking, four types of joints are used in building construction such as concrete building systems: expansion joints, control joints, building expansion joints, and construction joints. These joints cannot be used interchangeably since each type is provided to perform a specific task.

An *expansion joint* is used to separate brick masonry into segments to prevent cracking due to changes in temperature, moisture expansion, elastic deformation due to loads, differential structural movement, and creep. A *control joint* is used in concrete or concrete masonry to create a plane of weakness, which used in conjunction with reinforcement or joint reinforcement, controls the location of cracks due to volume changes resulting from shrinkage and creep. A *building expansion joint* is used to create a plane of isolation separating a building into discrete sections so that stresses developed in one section will not affect the integrity of the entire section. A *construction joint*, also called a *cold joint*, is used primarily in concrete structures where construction is interrupted. It is located where it will least impair the strength of the structure.

In masonry building systems, two types of movement joints are used: expansion joints for brick masonry, and control joints for concrete masonry. The two types of joint serve different purposes. During construction, these joints are detailed based on industry practices and recommendations. A discussion on this topic can be found in the literature [9.1, 9.22, 9.25–9.28]; a brief description of these joint types follows.

a. Expansion joints for brick masonry: An expansion joint is a soft joint, completely void of any rigid material, placed in the exterior wythe of masonry. These joints can be vertical or horizontal. Vertical expansion joints are placed in the outer wythes of long walls, to divide them into segments to accommodate movement caused by changes in temperature and moisture. Horizontal expansion joints are placed directly below shelf angles to compensate for structural movement of the shelf angle resulting from deflection, rotation, and frame shortening, and vertical expansion of masonry.

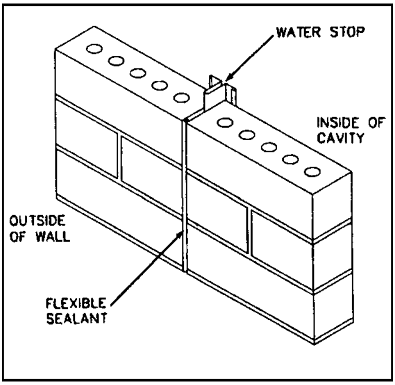


FIGURE 9.30 Expansion joint with copper or plastic water stop filler [9.25].

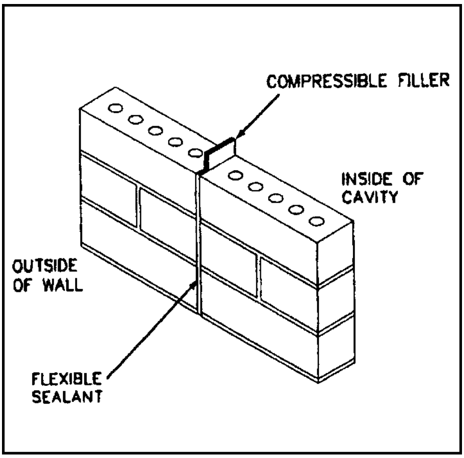


FIGURE 9.31 Expansion joint with compressible premolded filler [9.25].

Although the primary purpose of providing expansion joints is to allow for movement, they also must provide resistance to water penetration and air infiltration. Three types of expansion joints are normally used, which differ in the type of material placed in the joints. Figure 9.30 shows an expansion joint with a copper or plastic water stop filler, which is placed as the wall is constructed. The joint acts as a bellows, compressing and opening as the joint expands and contracts. The plastic filler is more economical. An expansion joint with compressible premolded filler is shown in Fig. 9.31. It is usually made from neoprene and cut into sections. An adhesive on one surface allows the filler to adhere to the brick as the wall is constructed. This compressible premolded filler is ¼ in. wide when installed, but will expand and completely fill the joint over time. The filler also prevents mortar from bridging or dropping into the joint, ensuring a compressible joint. A third type of joint which uses a foam rod is shown in Fig. 9.32. This type of joint is used very widely. The foam rod is inserted after the wall is completed. Before inserting the foam rod, the joint should be cleared of any obstructions resulting from mortar droppings or debris. The horizontal joint reinforcement should be removed if erroneously extended through the joint, failing which the movement will be restrained. If an expansion joint is accidentally omitted during construction, a joint could be cut through the masonry with a saw, and a foam rod inserted to create a functional expansion joint.

A common requirement, regardless of the type of joint used, is that all expansion joints must be properly sealed on the exterior side of the joint with an appropriate elastomeric compound sealant to prevent water penetration and ensure adequate performance of the joint.

b. *Control joints for concrete masonry:* Control joints are used as a means to relieve horizontal tensile stresses by reducing restraint and permitting longitudinal movement to occur. They are essentially vertical separations built into the wall where stress concentrations may occur. They are incorporated to accommodate movements of concrete masonry walls, or those of structural elements adjacent to walls. A typical control joint is shown in Fig. 9.33.

The type of control joint provided depends on the type of wall construction. A control joint built into a single-wythe concrete masonry wall should allow free longitudinal movement as well as have sufficient strength to resist flexural and lateral loads. Figure 9.34 shows a control joint that uses a cross-shaped preformed gasket

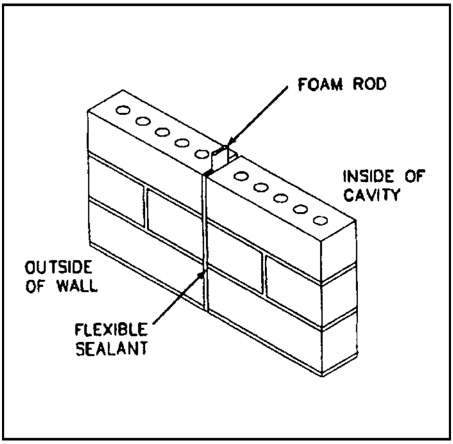


FIGURE 9.32 Expansion joint with foam rod filler [9.25].

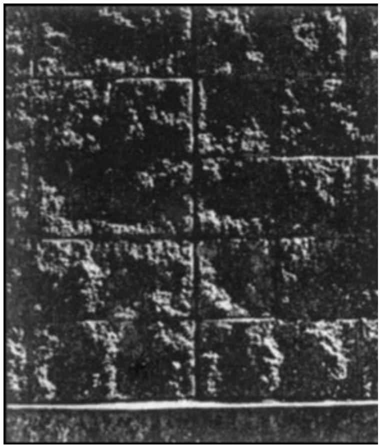


FIGURE 9.33 A typical control joint in a concrete masonry wall [9.1]. (*Courtesy: Portland Cement Association.*)

which slips into a notched concrete unit or a sash unit. The control joint acts as shear key. It should be noted that, unlike expansion joint filler, a preformed gasket is not compressible and should not be used as expansion joint filler.

Alternatively, special interlocking units can be used to create a control joint. These units have concave and convex ends that lock together to form a shear key (Fig. 9.35). After constructing the control joint, it should be filled with elastomeric compound sealant to prevent water penetration.

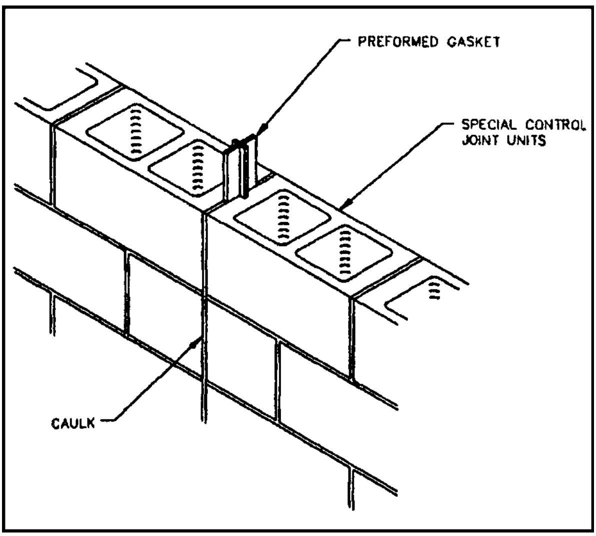


FIGURE 9.34 A hard rubber gasket control joint [9.25].

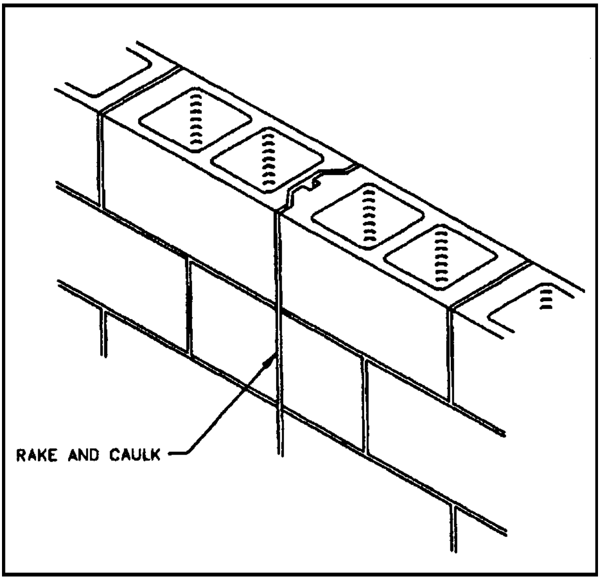


FIGURE 9.35 Control joint using interlocking units [9.25].

A control joint is also required where concrete units are used for the inner wythe of a cavity wall and the concrete masonry is left exposed. In such construction, a simple control joint can be created by racking the mortar joint back approximately ¾ in. and sealing (Fig. 9.36). This creates a vertical plane of weakness in the wall, a location where shrinkage cracks are most likely form.

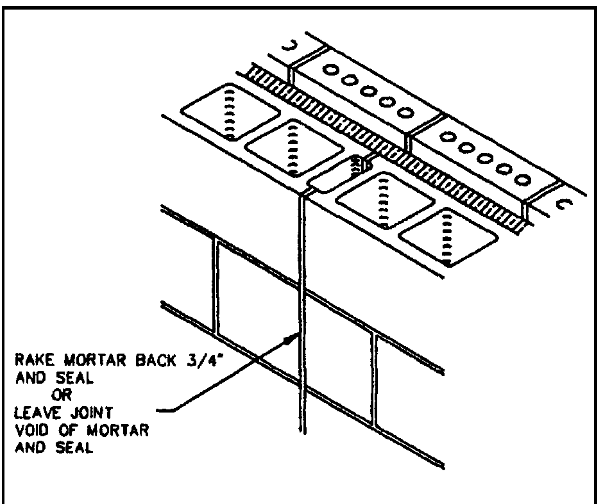


FIGURE 9.36 Control joint formed by a raking joint in the concrete unit wythe of a cavity wall [9.25].

9.5.2 Spacing and Location of Movement Joints

1. *Expansion joint spacing:* There is no specific code requirement for the size and location of expansion joints for brick masonry. The Brick Institute of America [9.22] suggests Eq. (9.7) to estimate unrestrained expansion of brickwork:

$$m_u = (k_e + k_f + k_t \Delta T)L \tag{9.7}$$

where m_u = total unrestrained movement of brickwork, in.
k_e = coefficient of moisture expansion, in./in.
k_f = coefficient of freezing expansion, in./in.
k_t = coefficient of thermal expansion, in./in.
ΔT = change in temperature, °F
L = length of wall, in.

Equation (9.7) provides an estimate of the amount of wall movement that may occur in a clay brick wall system. In addition to the amount of movement, there are a number of other factors that may affect the size and spacing of expansion joints. These include wall restraint, wall orientation, elastic deformation due to loads, shrinkage and creep of mortar, and construction tolerances. The Brick Institute of America suggests Eq. (9.8) to relate spacing between expansion joints to total unrestrained movement of brickwork and the expansion joint width [9.22]:

$$S_e = \frac{w_j e_j}{(k_e + k_f + k_t \Delta T)100} \tag{9.8}$$

where S_e = spacing between the expansion joints, in.
w_j = width of expansion joint, in.
e_j = extensibility of expansion joint material, %

The expansion joint is typically sized to resemble a mortar joint, usually 3/8 to ½ in. wide. The maximum size of the expansion joint depends on the elastic properties of sealant. Extensibilities of highly elastic expansion joints typically vary between 25 and 50 percent. Compressibility of backing materials can range up to 75 percent [9.22].

The temperature change, ΔT, to be used in Eqs. (9.7) and (9.8) depends on the mean wall temperatures. Theoretically, this can be calculated considering the maximum and minimum mean wall temperatures and the mean wall temperature at the time of construction. However, it is difficult to accurately predict these temperature values. Therefore, it is conservative to estimate the temperature variation based on the difference between the maximum and minimum mean wall temperature.

The maximum mean wall temperature may vary from the maximum ambient air temperature due to several factors such as wall orientation, location of insulation, color and density of wall, and can reach as high as 140°F. Minimum mean wall temperature may be close to the winter construction temperature of, say 50°F. Thus, the temperature difference, ΔT, can be as much as 100°F. Assuming that the color of the brick is light red, the desired joint width as 3/8 in., extensibility of the expansion joint material (e_j) as 50 percent, and negligible freezing expansion (i.e., $k_f = 0$) and appropriate values of k_e and k_f, the spacing of the expansion joint can be determined from Eq. (9.4):

$$S_e = \frac{(0.375)(50)}{[0.0003 + 4.0x10^{-6}](100)} = 268 \text{ in. or } 22 \text{ ft } 4 \text{ in.}$$

Therefore, the maximum spacing for vertical expansion joints in a straight clay brick wall should be 22 ft 4 in. For practical purposes, this spacing should be matched with other distances such as spacing of columns for aesthetic reasons. The maximum recommended spacing for vertical expansion joints is 30 ft without openings.

There are several factors that may be considered to reduce the number of vertical expansion joints. These include age and color of the brick, building orientation, and screening. If the brick is more than six months old, then most of the moisture expansion may have already taken place (see Fig. 9.22). Light-colored bricks reflect solar heat better than dark-colored bricks. Walls with southern exposure will experience the most thermal expansion and walls with a northern exposure will experience the least. Temperature variations can be reduced by providing shade to the masonry walls with trees and other foliage.

2. *Location of expansion joints:* Once the spacing of vertical expansion joints has been determined, the next step is to find ideal locations for them. Several factors should be considered for their locations. These include building size, configuration, structure, wall exposures, wall heights, offsets, openings, wall intersections, and the number of expansion joints required. Figure 9.37 shows suggested locations for placement of vertical expansion joints. A discussion on locations for expansion joints can be found in Ref. 9.22.

3. *Spacing and placement of control joints:* No specific requirements can be stated for spacing and placement of control joints for concrete masonry walls. It has been demonstrated in practice that control joints should not be more than 20 ft apart in exterior walls with frequent openings. This may be exceeded in walls with openings, but the joint spacing should never be more than 25 ft to be most effective [9.1]. The National

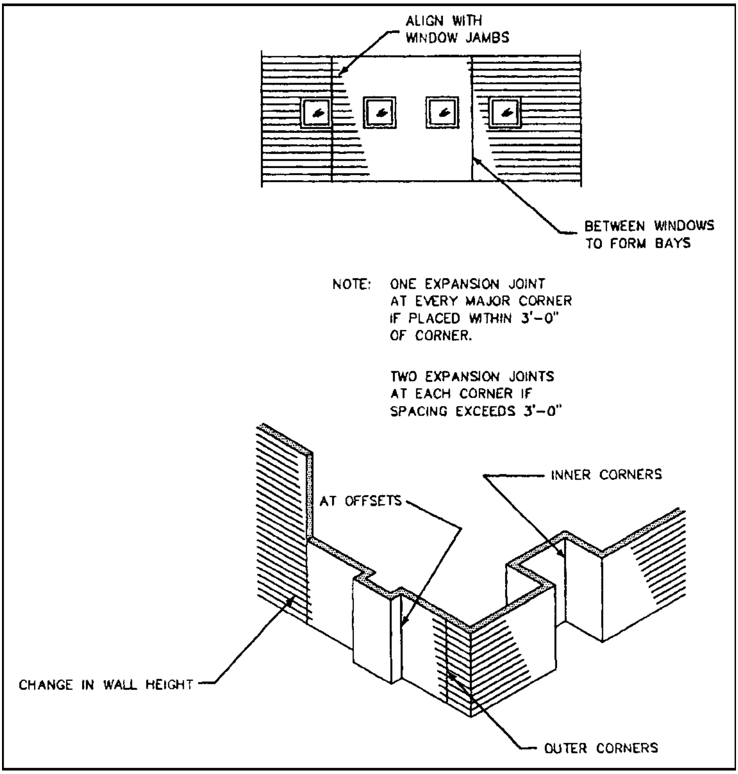


FIGURE 9.37 Suggested placement of vertical expansion joints [9.25].

Concrete Masonry Association suggests locating control joints at the following locations in a concrete masonry wall [9.1, 9.26]:

a. At abrupt changes in wall height.
b. At all changes in wall thickness, such as those at pipe or duct chases and those adjacent to columns or pilasters.
c. Above joints in foundations and floors.
d. Below joints in roofs and floors that bear on the wall.
e. At a distance of not over one-half the allowable joint spacing from bonded intersections and corners.
f. At one or both sides of all door and window openings, unless other crack control measures (such as joint reinforcement or bond beams) are taken.

Control joints should extend through plaster applied directly to masonry units. Plaster applied to lath which is furred from masonry may not require vertical separation at control joints. Figures 9.38 through 9.41 show typical locations for control joints.

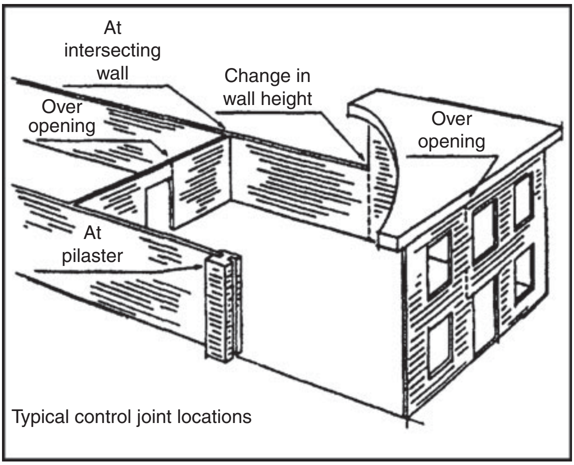


FIGURE 9.38 Typical control joint locations [9.22].

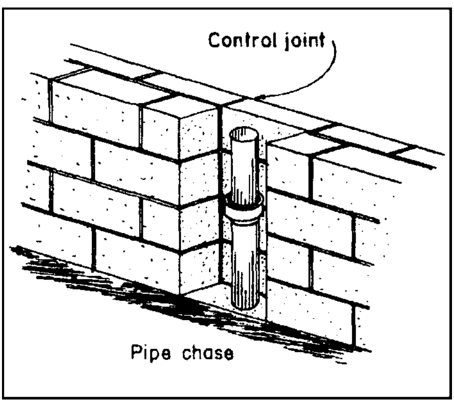


FIGURE 9.39 A control joint at abrupt change in wall thickness [9.1]. (*Courtesy: Portland Cement Association.*)

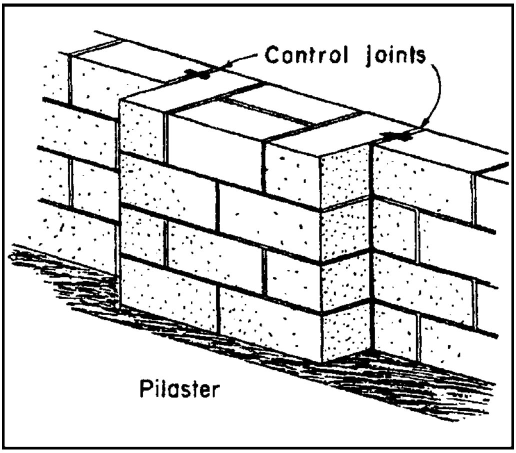


FIGURE 9.40 Control joint at a pilaster edge [9.1]. (*Courtesy: Portland Cement Association.*)

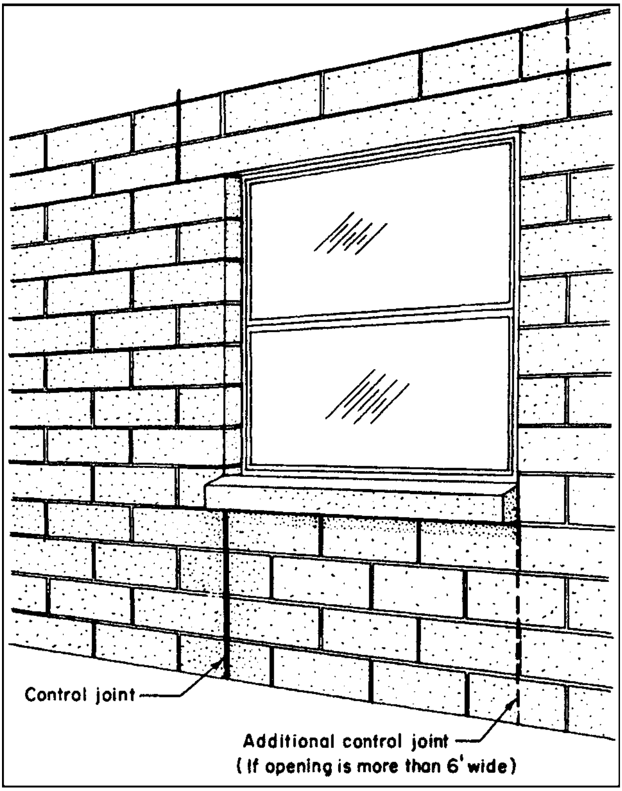


FIGURE 9.41 Control joints located at window openings to avoid random cracking [9.1]. (*Courtesy: Portland Cement Association.*)

TABLE 9.9 Maximum Horizontal Spacing of Vertical Control Joints in Concrete Masonry Walls, feet [9.25]

Average annual relative humidity	Wall location	vertical spacing of bed joint reinforcement, in.	Types of concrete masonry: I Moisture controlled	II Nonmoisture controlled
Greater than 75%	Exterior	None	12	6
		16	18	10
		8	24	14
	Interior	None	16.5	9
		16	24	14
		8	31.6	19
Between 50% and 75%	Exterior	None	18	12
		16	24	16
		8	30	20
	Interior	None	22.5	15
		16	30	20
		8	37.6	25
Less than 50%	Exterior	None	24	18
		16	30	22
		8	36	26
	Interior	None	28.5	21
		16	36	26
		8	43.6	31

The spacing of vertical control joints is influenced by several factors including wall exposure to weather, the average annual relative humidity, the amount and spacing of horizontal reinforcement, and the type of concrete masonry units (moisture controlled or not). Table 9.9 shows spacing for control joints based on these criteria [9.25].

9.6 QUALITY ASSURANCE

The matter of quality control was discussed in Section 3.8 (Chap. 3). Provisions for quality assurance are covered in 2009 IBC Section 2105. It requires establishment of a quality assurance program to ensure that constructed masonry is in compliance with the construction documents. MSJC-08 [9.4] defines quality assurance as administrative and procedural requirements to ensure that constructed masonry is in compliance with the contract documents (see MSJC-08 Section 1.18 for details).

All masonry constructed in compliance with the provisions of 2009 IBC and/or MSJC-08 *Code* must be inspected to ensure quality of construction conforming to contract documents. This is in variance with the 97-UBC requirements which permitted allowable stress design of masonry to be constructed either without or with special inspection. The 97-UBC permitted masonry construction under two different conditions when using allowable stress design: masonry construction with inspection, or without inspection. In the latter case, the allowable stresses were reduced by one-half. However, the strength design provisions required mandatory inspection of the masonry construction.

To ensure uniform application of inspection criteria and measures for quality assurance, the MCJC-08 *Code* [9.4] and *Specifications* [9.16] specify the minimum requirements as shown in Ref. [9.16], Tables 3, 4, and 5 (Level A, Level B, and Level C Quality Assurance). The architect/engineer may increase the amount of testing and inspection required.

9.7 FLASHING FOR MASONRY CONSTRUCTION

9.7.1 General Considerations

Water, in any of its three forms—liquid, solid, or vapor—may cause damage to masonry structures. In the liquid state, water penetrating to the interior of a building may cause damage to its contents. In the solid form (i.e., ice), it can cause cracking and spalling of masonry. In the vapor form, it can lead to condensation inside the cores and on the surfaces of masonry if the dew point temperature is reached. The purpose of flashing is to intercept the flow of water through masonry, direct it to the exterior of the structure, and prevent upward migration of water from below the grade level. Most buildings require flashing.

Water penetration in buildings is a complex phenomenon, which is influenced by many factors. These include wind pressure and direction, gravity, and absorption. Wind-driven rain may enter into masonry through cracks at the interface between mortar and units, and migrate downward through the wall due to the force of gravity. Alternatively, it may be transferred horizontally through the wall either by pressure or by bridging action. Wind-driven rain may also be absorbed by masonry units and carried from the exterior surface to the interior surface by capillary action. Ground water may be drawn upward by the wicking action of units placed on porous and wet foundations or by contact with moist soil.

Considerable information on flashing for the purpose of protection from moisture is provided in industry publications [9.25, 9.29, 9.30]. The National Concrete Masonry Association suggests guidelines that should be followed during the design stage. The areas of structure conducive to water flow should be identified and examined individually during the design stage. Such areas include, but are not limited to, wall bases, opening sills, opening heads, wall-floor junctures, wall-roof junctures, projections and recesses, parapets, landscaping, etc.

The design of masonry in order to eliminate the potential for water penetration needs considerations of the many types of construction, such as single wythe, cavity, curtain, infill, veneer, etc. The most important aspect of flashing design is to provide proper detailing to ensure that it will serve the intended purpose over time.

9.7.2 Flashing Materials

The selection of proper flashing material is of utmost importance since flashing is normally designed and installed for the life of the structure. Repair or replacement of flashing is very labor-intensive and expensive. Service conditions, projected life of the structure, and the past performance of the flashing material should be given due consideration in the selection of the flashing material.

Many varieties of flashing are commercially available. These may be composed of a wide variety of materials such as stainless steel, copper, a combination of copper and lead, polyvinyl chloride (PVC), and other plastics. The criteria for the selection of the flashing material should include resistance to puncture during installation, resistance to degradation by ultraviolet light, performance at elevated temperatures, and compatibility with joint sealants. A brief discussion of these devices can be found in Ref. 9.1.

9.7.3 Construction Practices

For a satisfactory performance, it is important that flashings be installed properly or they may aggravate rather than eliminate the water penetration problems. Reference [9.29] suggests conducting complete evaluation of each detail to ensure that:

1. Flashing should not be penetrated by shelf angle bolt-nut anchorage.
2. Flashing should not be penetrated with masonry anchors.
3. Flashing should be continuous around building or terminated with an end-dam (an upward terminus of the flashing within a head joint.)
4. Flashing requiring lapping of sections should be joined using the proper adhesive to prevent water flow around the lapped joint.
5. Flashing should lead to weep holes, and the collar joint or cavity should be free of mortar that might bridge the collar joint.
6. Flashing of the through-wall type should be bedded and covered with a thin layer of mortar before laying the next course or cap.
7. Flashing in walls of the through-wall type should extend into the interior sufficiently to allow a 1-in. lip between the interior surface and the furring or adhered board insulation.
8. Flashing terminating as a drip ledge should extend ½ in. beyond the wall surface and be bent downward at a 45° angle.
9. Isolate dissimilar metals in contact where galvanic corrosion may occur in the presence of an electrolyte.
10. Terminate flashing within the interior masonry wythe or in reglets installed and provided by other trades.
11. Be attentive to tolerances of all trades so that masonry may work and flashing will not require the use of unusual construction methods.

9.7.4 Typical Details

Detailing flashing for masonry structures depends on the type of masonry construction and location where provided. Figures 9.42 through 9.44 show a few typical flashing details as suggested in Ref. 9.30. Figure 9.42 shows the flashing detail for a cavity wall. The flashing should extend from the midpoint to the inner face shell across the unit surface, project

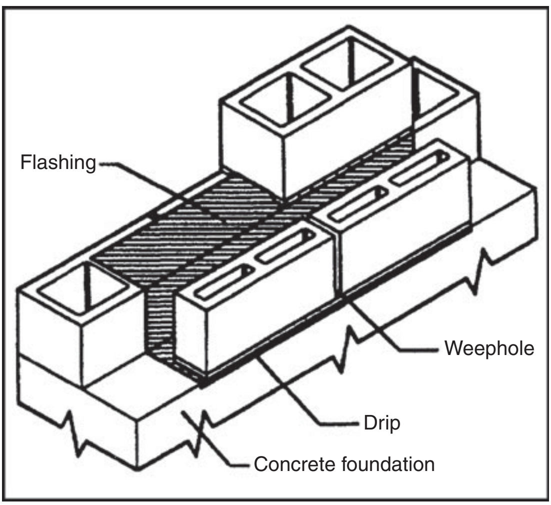


FIGURE 9.42 Flashing for cavity wall construction [9.30].

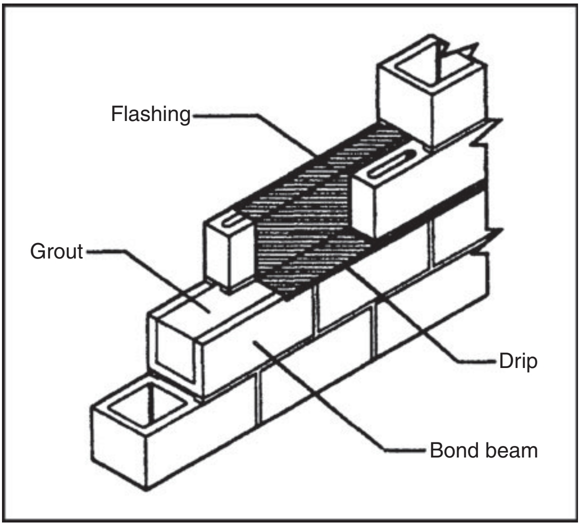


FIGURE 9.43 Flashing at bond beam locations [9.30].

downward to the foundation surface, outward to the exterior face of the wall, and terminate with a slopped joint. Weep holes should be spaced not more than 32 in. apart.

Figure 9.43 illustrates flashing at bond beams. Like cavity walls, they are positioned to direct water to the exterior. Care should be taken to ensure that grout in the bond beam is level with the surface of the units. This detail may be modified for single-wythe construction. Filling in the cores of the first course below the flashing (shown as bond beam in Fig. 9.43)

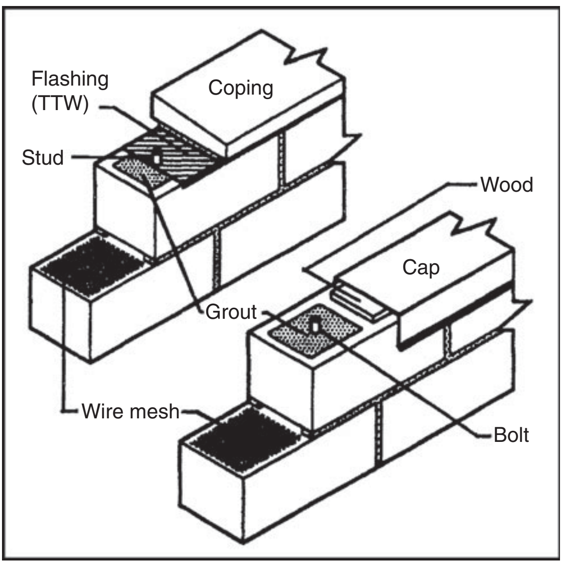


FIGURE 9.44 Flashing at copings and caps [9.30].

with gravel or other similar material will support the flashing from beneath and prevent sagging of the flashing and formation of reservoirs on the flashing at the core locations.

Figure 9.44 shows details for flashing for copings and caps. During placement of final courses of masonry in parapets, and commencing with the second course below the coping/cap location, wire mesh should be placed over cores so that grout can be placed for the positioning of anchor bolts.

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CHAPTER 10
ANCHORAGE TO MASONRY

10.1 INTRODUCTION

Reinforced masonry consists of precast units. Anchor bolts are used as primary means of anchoring (attaching) of various structural (loadbearing) and nonstructural (nonloadbearing) elements that form parts of reinforced masonry structures. Structural elements that need to be anchored to masonry include roof and floor diaphragms, ledger beams, girders, and any other element that might be required to transfer their loads to masonry. For example, flexible roof and floor diaphragms are typically supported on ledger beams which are anchored to the masonry walls with anchor bolts. This type of connection transfers gravity load as well as lateral load from diaphragms to ledger beams, and then to the walls. Anchor bolts are used to support girders supporting roof or floor diaphragms, which transfer their loads to supporting walls through anchorage.

Provisions for design for anchorage using anchor bolts embedded in grout are given in MSJC-08 *Code* [10.1] Sec. 2.1.4 for allowable stress design and in Sec. 3.1.6 for strength design; the latter is discussed in this chapter. The design philosophy that forms the basis of MSJC *Code* provisions is discussed in the MSJC-08 *Commentary* [10.2], which is referred to in the discussion presented herein. Regardless of the method used for designing anchorage to masonry, anchor bolts are required to be grouted with specified minimum grouted space around them.

10.2 TYPES OF ANCHOR BOLTS

Figure 10.1 shows the types of anchor bolts covered in the MSJC-08 *Code.* These can be classified as follows:

1. Headed anchor bolts with hexagonal or square head
2. Bent-bar anchor bolts (J- or L-bolts, the alphabets describe the bolt shapes)
3. Plate anchor bolts with square or round plates at the unthreaded ends

The term “bent-bar” anchor bolts has been used in this book to denote both the J- and L-anchor bolts.

According to *Specification for Masonry Structures* [10.3], anchor bolts are required to conform to the flowing ASTM Specifications:

1. *Headed anchor bolts:* ASTM A307 Grade A. (ASTM does not defines a yield strength for this product; $f_y = 36$ ksi used in this chapter unless otherwise specified.)
2. *Plate and bent-bar anchors:* ASTM A36/A36 M ($f_y = 36$ ksi)

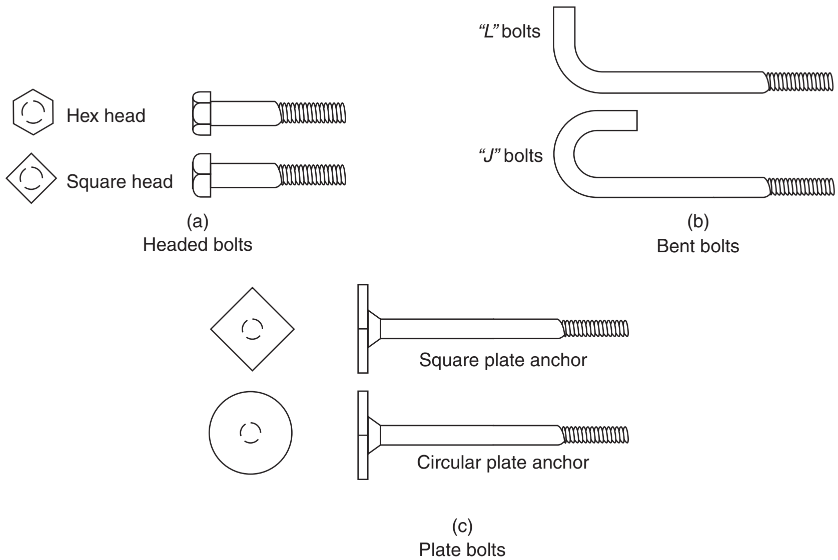


FIGURE 10.1 Types of anchor bolts.

MSJC-08 *Code* addresses only the headed and bent-bar anchor bolts for calculating nominal strengths of anchor bolts. Other types, such as plate anchor bolts, toggle bolts, expansion anchors, etc., are not covered in MSJC-08. However, these other types of anchor bolts can be used provided their strengths could be substantiated by load tests performed in accordance with ASTM E488 [10.4]. A minimum of five tests must be performed in order to satisfy this requirement (this is an exception; ASTM E488 requires only three tests); the allowable loads may not exceed 20 percent of the average tested strength.

10.3 PLACEMENT AND EMBEDMENT OF ANCHOR BOLTS IN MASONRY GROUT

The length of anchor bolt embedment in the grout and the thickness of grout surrounding it have a significant influence on strength of anchor bolt. The grout serves as the medium through which the loads are transferred from the attached component to the masonry. Therefore, the anchor bolt should have enough grout surrounding it. The requirements pertaining to the placement and embedment of anchor bolts in masonry grout are specified in MSJC-08 *Code* Sec. 1.16, which are summarized as follows:

1. Regardless of the type of anchor bolt used for anchorage to masonry, all embedded anchor bolts must be grouted in place with at least ½ in. of grout between the anchor bolt and the masonry, except that ¼ in.-diameter anchor bolts are permitted to be placed in bed joints that are at least ½ in. in thickness (the latter is a less common practice).

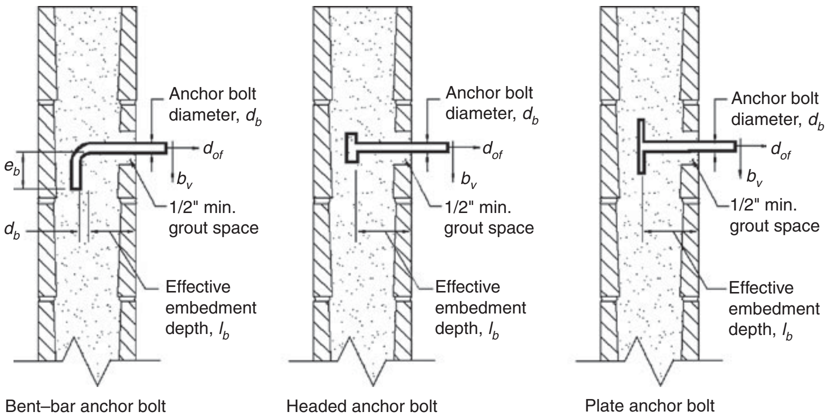


FIGURE 10.2 Effective embedment length, l_b, for various types of anchor bolts. As a minimum, the effective embedment length must be the greater of 4 bolt diameters or 2 in. (*Courtesy: CMACN.*)

2. For all types of anchor bolts, the strength of anchorage depends on their effective embedment lengths, l_b (Fig. 10.2). The effective embedment lengths for headed and bent-bar anchor bolts are measured differently as follows:
   a. *Headed anchor bolt:* l_b = effective length of embedment, measured perpendicular from the surface of the masonry to the compression bearing surface of the anchor bolt head.
   b. *Bent-bar anchor bolts*: l_b = effective length of embedment measured perpendicular from the surface of the masonry to the compression bearing surface of the bent end minus one bolt diameter.
3. Regardless of the type of anchor bolt and the type of loading (axial tension, shear, or combined axial tension and shear, discussed later), the minimum effective embedment length must be the greater of 4 diameters or 2 in. This minimum embedment length requirement is considered a practical minimum, based on typical construction methods used for embedding anchor bolts in grout. The validity of allowable shear and tension equations for small embedment depths of less than 4 diameters has not been verified by tests [10.2].

10.4 NOMINAL STRENGTH OF ANCHOR BOLTS

10.4.1 Factors Affecting the Nominal Strength of Anchor Bolts

The strength of anchorage when using anchor bolts depends on many factors:

1. Type of anchor bolt
2. Modes of failure
3. Type of applied load (tension, shear, or combined tension and shear)
4. Compressive strength of masonry
5. Diameter of anchor bolt

6. Effective embedment length of the anchor bolt
7. Edge distance of the anchor bolt
8. Projected extension of bent-bar anchor bolts
9. Yield strength of anchor bolt steel
10. Condition of the anchor bolt

Anchor bolts can be loaded in

1. Tension
2. Shear
3. Combined tension and shear

Determination of nominal strengths of anchor bolts embedded in masonry grout is based on the mode of failure. The results of test on anchor bolts in tension have shown that anchors failed by pullout of a conically shaped section of masonry, or by failure of the anchor itself. The cone originates at the bearing point of the embedment and radiates at 45° in the direction of the pull. Bent-bar anchor bolts (J-bolts) often failed by completely slipping out of the specimen as the bent portion of the bolt straightened out.

Each mode of failure is designated as a limit state. Thus, design for anchorage in masonry grout is based on the philosophy of *limit states design*.

Typical limit states corresponding to these loading conditions are considered as follows:

1. Masonry breakout
2. Masonry crushing
3. Anchor bolt pryout (for bent-bar anchor bolt)
4. Anchor bolt pullout
5. Tension yielding of the anchor bolt steel
6. Shear yielding of the anchor bolt steel

The force required to reach the above limit states is different in each case; the limit states are also different for headed anchor bolts (which are straight) and the bent-bar anchor bolts. The nominal strength of an embedded anchor bolt is taken as the smallest of the forces determined by considering applicable limit states. In the case of combined limit states, an interaction equation must be satisfied.

Limit states and expressions governing the nominal strengths of anchor bolts corresponding to them are specified in MSJC-08 *Code* Sec. 3.1.6. Different limit states are considered for headed and bent-bar anchor bolts as follows:

1. Headed anchor bolts:
   a. Limit state of masonry breakout
   b. Tensile yielding of anchor bolt steel
2. Bent-bar anchor bolts (J- or L-bolts):
   a. Limit state of masonry breakout
   b. Limit state of masonry crushing
   c. Tensile yielding of anchor bolt steel
3. Headed and bent-bar bolts (J- or L-bolts) in shear:
   a. Limit state of masonry breakout
   b. Limit state of masonry crushing

c. Limit state of anchor bolt pryout
d. Limit state of shear yielding of steel

The limit state of bent-bar anchor bolt pryout occurs as the bent portion of the bolt straightens and pulls out due to slippage under the action of the applied tensile force.

Nominal strengths of anchor bolts corresponding to these loading conditions and limit states are discussed in the following sections. In all cases of determination of design strength of anchor bolts, a strength reduction factor ϕ (a dimensionless number less than 1.0) is applied to the *nominal* strengths to obtain design strengths. Similarity exists between the behaviors of anchors embedded in concrete and those embedded in masonry grout, which is evidenced by the research data for anchor bolts embedded in grout. Therefore, strength reduction factors associated with various limit states corresponding to the failure of anchorage in grout are derived from the expressions based on research on the performance of anchor bolts embedded in concrete [10.2]. The values of ϕ corresponding to various limit states are listed in Table 10.1.

TABLE 10.1 Strength Reduction Factors, ϕ, for Anchor Bolts (MSJC-08 Sec. 3.1.4.4)

Limit states	Strength reduction factor, ϕ
Masonry breakout,	0.5
Masonry crushing	0.5
Anchor bolt pryout	0.5
Anchor bolt pullout	0.65
Anchor bolt steel: tension yielding	0.9
Anchor bolt steel: shear yielding	0.9

10.5 NOMINAL AXIAL STRENGTH OF ANCHOR BOLTS LOADED IN TENSION AND IN COMBINED TENSION AND SHEAR

10.5.1 Headed Anchor Bolts Loaded in Tension

Two limit states (masonry breakout and tensile yielding of anchor bolt steel) are considered for determination of the nominal axial tensile strengths of headed anchor bolts; the smaller value of the force required to reach any of these two limit states is taken as the nominal axial strength of the anchor bolt.

10.5.1.1 Limit State of Masonry Cone Breakout The limit state of masonry breakout assumes that failure is manifested by breakout of right circular cone as a result of tensile force in the anchor bolt. The exposed area of the masonry breakout cone, A_{pt}, is calculated simply as the projected (or exposed) area of a right circular cone having a radius equal to the effective embedment length, l_b:

$$A_{pt} = \pi l_b^2 \tag{10.1}$$

where l_b = effective embedment length of the headed or bent-bar anchor bolt in tension, (in.), defined earlier (Fig. 10.2).

Equation (10.1) is based on the implicit assumption that the anchor bolt is located at least a distance "l_b" from the edge of the masonry so that a full masonry breakout cone would develop as a result of failure. When the edge distance, l_{be}, is smaller than the effective embedment length, l_b, the full breakout cone of radius l_b cannot develop; therefore, a smaller radius, l_{be}, should be used to calculate the projected are of the breakout cone. In such cases, the actual bolt edge distance, l_{be}, should be used in lieu of l_b in Eq. (10.1) (because the radius of the right circular cone cannot exceed l_{be}):

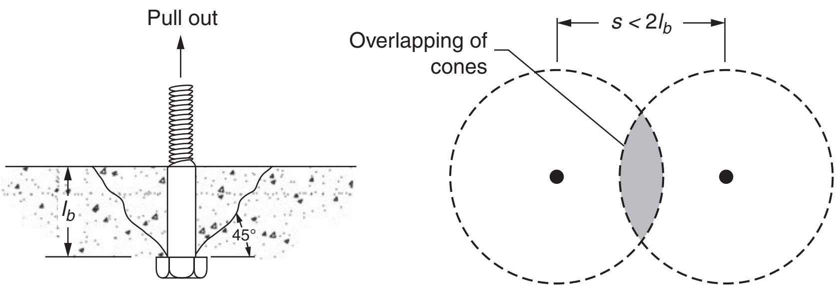
$$A_{pt} = \pi l_{be}^2 \tag{10.2}$$

In Eq. (10.1), A_{pt} is the projected area on masonry surface of a right circular cone (Fig. 10.3) that is assumed to pull out independently, that is, without overlapping the breakout masonry cones of the adjacent bolts. If the projected areas of the two masonry breakout cones overlap as result of a closer spacing s between the adjacent anchor bolts such that $s < 2\ l_b$ as shown in Fig. 10.3, then the projected area, A_{pt}, of each pullout cone is to be reduced by one-half of the overlapping area. The effective (or adjusted) projected area, A'_{pt}, is given by Eq. (10.3):

$$A'_{pt} = \pi l_b^2 - \frac{1}{2} l_b^2 (\theta - \sin\theta) \tag{10.3}$$

where angle θ (radians) is given by Eq. (10.4):

$$\theta = 2\cos^{-1}\left(\frac{s}{2l_b}\right) \tag{10.4}$$



(a)
Masonry cone type failure

(b)
Overlapping pull out cone areas

FIGURE 10.3 Masonry breakout cones and overlapping cone areas.

The second term on the right-hand side in Eq. (10.3) represents one-half the overlapping projected area of the masonry breakout cones, and depends on the ratio of the bolt spacing (s) to twice the effective embedment length ($2l_b$). For example, for an anchor bolt spacing of twice the effective embedment length ($s = 2l_b$), Eq. (10.4) yields $\theta = 0$ radian, so that reduction due to overlapping breakout cones is equal to zero. Similarly, for an anchor bolt spacing (s) equal to the effective embedment length (i.e., $s = l_b$), Eq. (10.4) yields $\theta = 2\pi/3$, which would result in the reduction due to the overlapping projected area equal to approximately 20 percent [equal to the second term in Eq. (10.3)], and so on.

The nominal strength of a headed anchor bolt based on the limit state of masonry breakout, B_{anb}, depends on the projected area of the masonry breakout cone and the compressive strength of masonry, and is determined from Eq. (10.5):

$$B_{anb} = 4A_{pt}\sqrt{f'_m} \tag{10.5}$$

where f'_m = compressive strength of masonry.

The strength reduction factor for this limit state is: $\phi = 0.5$.

10.5.1.2 Limit State of Tensile Yielding of Bolt Steel The nominal axial tensile strength of a headed anchor bolt B_{ans}, based on the limit state of tensile yielding of anchor bolt steel, depends on the yield strength of the bolt steel and the cross-sectional area of bolt, and is determined from Eq. (10.6). It is assumed that bolt threads lie in the plane of shear.

$$B_{ans} = A_b f_y \tag{10.6}$$

where A_b = cross-sectional area of the anchor bolt
f_y = yield strength of steel

The strength reduction factor for this limit state is: $\phi = 0.65$.

Example 10.1 Design axial tensile strength of a headed anchor bolt.

Calculate the design axial tensile strength of a $^5/_8$-in.-diameter A307 headed anchor bolt ($f_y = 36$ ksi) having an effective embedment length of 6 in. The bolt is at least 12 in. from adjacent bolts or the edge of masonry. The compressive strength of masonry = 1500 lb/in.^{2}.

Solution

1. Nominal axial tensile strength of the anchor bolt based on the limit state of masonry breakout.

   Check if the projected area of the masonry breakout cones need to be reduced. In this example, l_b = 6 in. and spacing, s = 12 in. Since $s \geq 2l_b$, the masonry breakout cones would not overlap. Therefore, the projected area of the masonry breakout cone need not be reduced. Calculate A_{pt} from Eq. (10.1):

   $$A_{pt} = \pi l_b^2 = \pi(6)^2 = 113.1\ \text{in}^2$$

   Based on the limit state of masonry breakout cone, the allowable load on the anchor bolt is determined from Eq. (10.5):

   $$B_{anb} = 4A_{pt}\sqrt{f'_m} = (4)(113.1)(\sqrt{1500}) = 17{,}521\ \text{lb}$$

   The strength reduction factor for this limit state is 0.5. Thus, the design axial strength is

   $$\phi B_{anb} = 0.5(17{,}521) = 8761\ \text{lb}$$

2. Nominal axial tensile strength of the anchor bolt corresponding to the tension yielding limit state of the anchor bolt steel.

   Based on the limit state of tensile yielding of the anchor bolt steel, the nominal axial strength of the anchor bolt is determined from Eq. (10.6):

   $$B_{ans} = A_b f_y$$

For a ⅝-in.-diameter anchor bolt, the cross-sectional area is

$$A_b = \frac{\pi}{4}(d)^2 = \frac{\pi}{4}(0.625)^2 = 0.31\ \text{in}^2$$

$$B_{ans} = A_b f_y = (0.31)(36{,}000) = 11{,}160\ \text{lb}$$

The strength reduction factor for this limit state is 0.9. Thus, the design strength is

$$\phi B_{ans} = 0.9(11{,}160) = 10{,}044\ \text{lb}$$

The smaller of the two design strength values calculated governs. The design strength of the anchor bolt is governed by the limit state of the masonry breakout and is equal to 8761 lb.

Answer: $\phi B_{an} = 8761$ lb.

Example 10.2 Design axial tensile strength of a headed anchor bolt.

What would be the design axial tensile strength of the ⅝-in.-diameter bolt of Example 10.1 if the masonry compressive strength were 2500 lb/in.^{2}?

Solution

1. Nominal axial tensile strength of the anchor bolt based on the limit state of masonry breakout.

   Because the anchor bolt positions remain the same as in Example 10.1, A_{pt} remains unchanged. Thus, based on the limit state of masonry breakout cone, the nominal axial strength of the anchor bolt allowable load on the anchor bolt is determined from Eq. (10.5):

$$B_{anb} = 4A_{pt}\sqrt{f'_m} = (4)(113.1)(\sqrt{2500}) = 22{,}620\ \text{lb}$$

   The strength reduction factor for this limit state is 0.5. Thus, the design strength is

$$\phi B_{anb} = 0.5(22{,}620) = 11{,}310\ \text{lb}$$

2. Nominal axial tensile strength of the anchor bolt corresponding to the tensile yielding limit state of the anchor bolt steel.

   Based on the limit state of tensile yielding of the anchor bolt steel, the nominal axial strength of the anchor bolt is determined from Eq. (10.6) as in Example 10.1.

$$B_{ans} = A_b f_y$$

   For a ⅝-in.-diameter anchor bolt, the cross-sectional area is

$$A_b = \frac{\pi}{4}(d)^2 = \frac{\pi}{4}(0.625)^2 = 0.31\ \text{in}^2$$

$$B_{ans} = A_b f_y = (0.31)(36{,}000) = 11{,}160\ \text{lb}$$

   The strength reduction factor for this limit state is 0.9. Thus, the design strength is

$$\phi B_{ans} = 0.9(11{,}160) = 10{,}044\ \text{lb}$$

   The smaller of the two design strength values calculated above governs. The design strength of the anchor bolt is governed by the limit state of tensile yielding of anchor bolt steel: $\phi B_{an} = 10{,}044$ lb.

Answer: $\phi B_{an} = 10{,}044$ lb
Commentary: This represents an increase of 14.2 percent over the nominal strength value of the anchor bolt determined in Example 10.2, and represents increase in the design strength of the anchor bolt due to higher compressive strength of masonry.

Example 10.3 Design axial tensile strength of a headed anchor bolt.

Calculate the design axial tensile strength of a ⅝-in.-diameter A307 headed anchor bolt ($f_y = 36$ ksi) having an effective embedment length of 6 in. The bolt is at least 9 in. from adjacent bolts or the edge of masonry. The compressive strength of masonry = 1500 lb/in.^{2}.

Solution

1. Nominal axial tensile strength of the anchor bolt based on the limit state of masonry breakout.

   Check if the projected area of the masonry breakout cone needs to be reduced because of overlapping. In this example, $l_b = 6$ in. and spacing, $s = 9$ in. Since $s \le 2l_b$, the breakout masonry cones would overlap; therefore, the projected area of the masonry breakout cone needs be reduced (adjusted). Calculate the effective masonry breakout area A'_{pt} from Eq. (10.3):

$$A'_{pt} = \pi l_b^2 - \frac{1}{2} l_b^2 (\theta - \sin\theta)$$

   Calculate angle θ from Eq. (10.4):

$$\theta = 2Cos^{-1}\left(\frac{s}{2l_b}\right) = 2Cos^{-1}\left(\frac{9}{2(6)}\right) = 82.82^{\circ} = 1.4455 \text{ rad.}$$

$$A'_{pt} = \pi l_b^2 - \frac{1}{2} l_b^2 (\theta - \sin\theta) = \pi(6)^2 - \frac{1}{2}(6)^2(1.4455 - \sin 82.82^{\circ}) = 104.93 \text{ in}^2$$

   The nominal axial strength of the anchor bolt is determined form Eq. (10.5).

$$B_{anb} = 4A'_{pt}\sqrt{f'_m} = (4)(104.93)(\sqrt{1500}) = 16{,}256 \text{ lb}$$

   The strength reduction factor for this limit state is 0.5. Thus,

$$\phi B_{anb} = 0.5(16{,}256) = 8128 \text{ lb}$$

2. Nominal axial tensile strength of the anchor bolt corresponding to the tensile yielding limit state of the anchor bolt steel.

   For a 5/8-in.-diameter anchor bolt, the cross-sectional area is

$$A_b = \frac{\pi}{4}(d)^2 = \frac{\pi}{4}(0.625)^2 = 0.31 \text{ in}^2$$

   Based on the tensile yielding strength of the anchor bolt steel, the nominal axial tensile strength is determined from Eq. (10.6):

$$B_{ans} = A_b f_y = (0.31)(36{,}000) = 11{,}160 \text{ lb}$$

   The strength reduction factor for this limit state is 0.9. Thus,

$$\phi B_{ans} = 0.9(11{,}160) = 10{,}044 \text{ lb} > \phi B_{anb} = 8128 \text{ lb.}$$

Therefore, the design axial tensile strength of the anchor bolt is governed by the masonry breakout limit state and is equal to 8128 lb.

Answer: $\phi B_{an} = 8128$ lb.
Commentary: The allowable tensile load of 8128 lb represents a 7.2 percent reduction as compared to 8761 lb in Example 10.1 in which case the areas of right circular masonry breakout cones did not overlap.

Example 10.4 Design axial tensile strength of a headed anchor bolt.

Determine the design axial strength of a 5/8-in.-diameter A307 headed anchor bolt ($f_y = 36$ ksi) embedded in a nominal 8-in. concrete masonry wall as shown in Fig. E10.4. The bolts have an effective embedment length of 6 in. and are spaced at 12 in. on center. The compressive strength of the masonry is 1500 lb/in.^{2}.

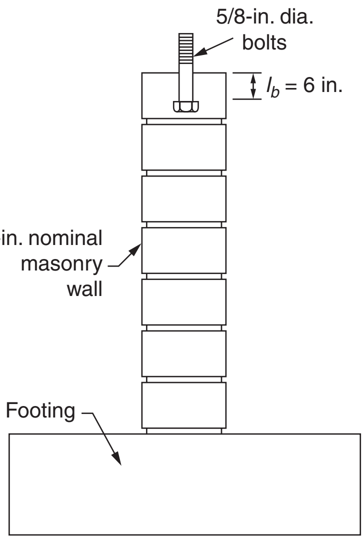


FIGURE E10.4

Solution

1. Nominal axial tensile strength of the anchor bolt corresponding to masonry breakout limit state.

$$\text{Wall thickness, } t = 7.625 \text{ in. (8-in. nominal)}$$

Calculate A_{pt}. Effective embedment length of anchor bolt,

$$l_b = 6 \text{ in.}$$

Edge distance,

$$l_{be} = ½\, t = ½\,(7.625) = 3.81 \text{ in.} < l_b = 6 \text{ in.}$$

The radius of the projected area of the masonry breakout cannot exceed $l_{be} = 3.81$ in. Therefore, $l_{be} = 3.81$ in. governs. From Eq. (10.2),

$$A_{pt} = \pi(l_{be})^2 = \pi(3.81)^2 = 45.6 \text{ in}^2$$

Based on the limit state of masonry breakout, the nominal axial strength in of the anchor bolt is given by Eq. (10.5).

$$B_{anb} = 4A'_{pt}\sqrt{f'_m} = (4)(45.6)(\sqrt{1500}) = 7064 \text{ lb}$$

The strength reduction factor for this limit state is 0.5. Thus,

$$\phi B_{anb} = 0.5(7064) = 3532 \text{ lb}$$

2. Nominal axial tensile strength of the anchor bolt corresponding to the tensile yielding limit state of the anchor bolt steel.

   For a $^5/_8$-in.-diameter anchor bolt, the cross-sectional area is

$$A_b = \frac{\pi}{4}(d)^2 = \frac{\pi}{4}(0.625)^2 = 0.31 \text{ in}^2$$

Based on the tensile yielding limit state of the anchor bolt, the nominal strength of the anchor bolt is determined from Eq. (10.6):

$$B_{ans} = A_b f_y = (0.31)(36{,}000) = 11{,}160 \text{ lb}$$

The strength reduction factor for this limit state is 0.9. Thus,

$$\phi B_{ans} = 0.9(11{,}160) = 10{,}044 \text{ lb} > \phi B_{anb} = 3532 \text{ lb}$$

Therefore, the design axial strength of the anchor bolt is governed by the limit state of masonry breakout and is equal to 3532 lb.

Answer: $\phi B_{an} = 3532$ lb.

10.5.2 Bent-Bar Anchor Bolts Loaded in Tension

The nominal axial tensile strength of a bent-bar anchor bolt embedded in grout, B_{an}, is determined by considering the three limit states described earlier. The nominal axial strength of the bent-bar anchor bolt is taken as the *smallest* of the nominal axial strengths corresponding to those limit states

10.5.2.1 Nominal Axial Strength Based on Masonry Breakout Limit State The nominal axial strength based on the anchor bolt pullout limit state, B_{anb}, is determined similar to that for the headed anchor bolts:

$$B_{anb} = 4A_{pt}\sqrt{f'_m} \qquad \text{(10.5 repeated)}$$

The strength reduction factor corresponding to the limit state of masonry breakout is 0.5.

10.5.2.2 Nominal Axial Strength Based on Anchor Bolt Pullout Limit State The nominal axial strength of a bent-bar anchor bolt based on the anchor bolt pullout limit state, B_{anp}, is determined from Eq. (10.7):

$$B_{anp} = 1.5 f'_m e_b d_b + \left[300\pi\left(l_b + e_b + d_b\right)d_b\right] \qquad (10.7)$$

where e_b = projected leg extension of bent bar anchor bolt measured from the inside edge of anchor bolt at bend to farthest point of anchor bolt in the plane of the hook, in. (Fig. 10.2)

d_b = nominal diameter of the bent bar anchor bolt, in.

l_b = effective embedment length of the bent bar bolt, in., measured perpendicular from the surface of the masonry to the bearing surface of the bent end minus one bolt diameter (Fig. 10.2)

The minimum permissible effective embedment length for bent-bar anchor bolts is the greater of four bolt diameters or 2 in.

The strength reduction factor ϕ corresponding to the anchor pullout limit state is 0.65.

Commentary: MSJC-05 *Code* stipulated that the second term in Eq. (10.7) could be included only if the specified quality assurance program included verification that shanks of J- and L-bolts would be free of undesirable materials such as oil, debris, and grease when installed, which would otherwise interfere with the bond between the bolt and the grout. This stipulation has been deleted in MSJC-08 *Code*.

10.5.2.3 Nominal Axial Strength Based on Limit State of Anchor Bolt Steel Yielding The nominal axial strength based on limit state of anchor bolt steel yielding, B_{ans}, is determined similar to that of headed anchor bolts embedded in grout:

$$B_{ans} = A_b f_y \qquad \text{(10.6 repeated)}$$

The strength reduction factor ϕ corresponding to the anchor pullout is 0.9.

Example 10.5 Design axial tensile strength of a bent-bar anchor bolt.

Determine the design axial tensile strengths of ¾-in.-diameter A307 bent-bar anchor bolts (f_y = 36 ksi) embedded 4 in. apart on the top of a nominal 12-in. wide concrete masonry wall as shown in Fig. E10.5. The distance between the masonry surface and compression bearing surface of the bent end is 5.25 in. and the projected extension of the bent bar is 4 in. The compressive strength of the masonry is 1500 lb/in.^{2}.

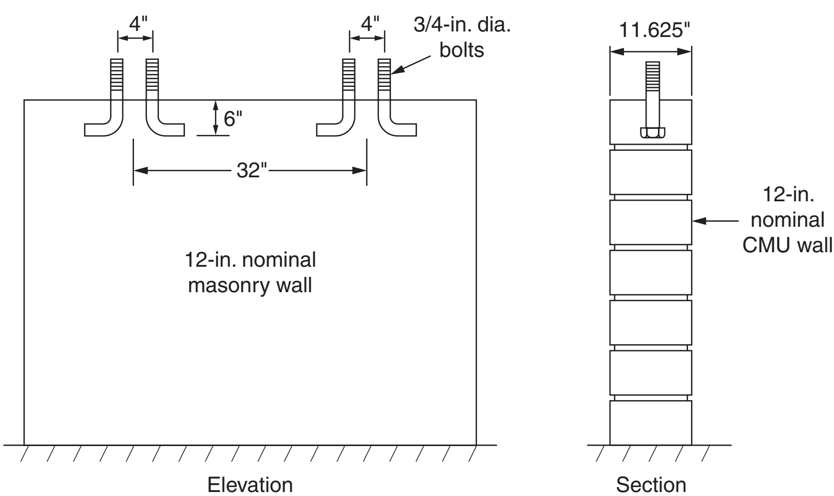


FIGURE E10.5

Solution

1. Nominal axial strength of the anchor bolt based on the limit state of masonry breakout.

   Determine the effective embedment length l_b. For a bent-bar anchor bolt, the effective embedment length is measured as the distance between the masonry surface and the compression bearing surface of the bent bar minus one bolt diameter.

$$\text{Bolt diameter} = 0.75 \text{ in.}$$

Effective embedment length,

$$l_b = 5.25 - (0.75) = 4.5 \text{ in.}$$

Thickness of wall,

$$t = 11.625 \text{ in. (12 in. nominal)}$$

Edge distance,

$$l_{be} = ½\,(11.625) = 5.812 \text{ in.} > 4.5 \text{ in.}$$

Therefore,

$$l_b = 4.5 \text{ in. governs.}$$

First, check if the projected areas of masonry breakout cones overlap. Because the bolts are spaced 4 in. apart, which is less than $2l_b = 2(4.5) = 9$ in., the projected areas of the masonry breakout cones would overlap; therefore, the projected area of the masonry breakout cone needs to be reduced. Calculate the effective projected area of the masonry breakout cone, A'_{pt}, from (Eq. 10.3):

$$A'_{pt} = \pi l_b^2 - \frac{1}{2} l_b^2 (\theta - \sin\theta)$$

Calculate θ from Eq. (10.4):

$$\theta = 2\,Cos^{-1}\left(\frac{s}{2l_b}\right) = 2\,Cos^{-1}\left(\frac{4}{2(4.5)}\right) = 127.22^\circ = 2.22 \text{ rad.}$$

$$A'_{pt} = \pi l_b^2 - \frac{1}{2} l_b^2 (\theta - \sin\theta) = \pi(4.5)^2 - \frac{1}{2}(4.5)^2(2.22 - \sin 127.22^\circ) = 49.20 \text{ in}^2$$

$$B_{anb} = 4A'_{pt}\sqrt{f'_m} = (4)(49.20)(\sqrt{1500}) = 7622 \text{ lb}$$

The strength reduction factor for this limit state is 0.5. Thus,

$$\phi B_{anb} = 0.5(7622) = 3811 \text{ lb}$$

2. Nominal axial tensile strength of the anchor bolt based on the limit state of anchor bolt pullout.

   The nominal axial strength of the anchor bolt corresponding to anchor bolt pullout limit state is determined from Eq. (10.7):

$$B_{anp} = 1.5 f'_m e_b d_b + [300\pi(l_b + e_b + d_b)d_b]$$

For this example,

$$f'_m = 1500 \text{ lb/in}^2 \quad e_b = 4 \text{ in.} \quad l_b = 4.5 \text{ in. (calculated above)} \quad d_b = 0.75 \text{ in.}$$

$$\begin{aligned} B_{anp} &= 1.5 f'_m e_b d_b + [300\pi(l_b + e_b + d_b)d_b] \\ &= 1.5(1500)(4)(0.75) + [300\pi(4.5 + 4 + 0.75)(0.75)] \\ &= 6750 + 6538 = 13,288 \text{ lb} \end{aligned}$$

The strength reduction factor for this limit state is 0.5. Thus,

$$\phi B_{anp} = 0.5(13,288) = 6644 \text{ lb.}$$

3. Nominal axial tensile strength of the anchor bolt based on the limit state of tension yielding of bolt steel.

   For a ¾-in.-diameter anchor bolt, the cross-sectional area is

$$A_b = \frac{\pi}{4}(d)^2 = \frac{\pi}{4}(0.75)^2 = 0.44 \text{ in}^2$$

From Eq. (10.6),

$$B_{ans} = A_b f_y = (0.44)(36.000) = 15,840 \text{ lb}$$

The strength reduction factor for this limit state is 0.9. Thus,

$$\phi B_{ans} = 0.9(15,840) = 14,256 \text{ lb}$$

The smallest of the above three values is selected as the design axial strength of the bent-bar anchor bolt. Therefore, the design axial strength of the anchor bolt is governed by the limit state of masonry breakout, and is equal to 3811 lb.

Answer: $\phi B_{an} = 3811$ lb.

10.6 NOMINAL SHEAR STRENGTH OF HEADED AND BENT-BAR ANCHOR BOLTS IN SHEAR

The nominal shear strength of a headed or a bent-bar anchor bolt, B_{vn}, is taken as the *smallest* of shear strengths corresponding to the following four limit states:

1. Limit state of masonry breakout
2. Limit state of masonry crushing
3. Limit State of anchor bolt pryout
4. Limit state of shear yielding of steel

10.6.1 Nominal Shear Strength Based on Limit State of Masonry Breakout

The nominal strength of an anchor bolt governed by the limit state of masonry breakout as determined from Eq. (10.8):

$$B_{vnb} = 4 A_{pv} \sqrt{f'_m} \tag{10.8}$$

where A_{pv} is one-half of projected area of a right circular cone used for calculating shear breakout capacity of an anchor bolt, and is given by Eq. (10.9):

$$A_{pv} = \frac{1}{2}\pi l_{be}^2 \tag{10.9}$$

where l_{be} = anchor bolt edge distance, measured in the direction of the load, from edge of masonry to center of cross section of anchor bolt. A_{pv} equals one-half of area of stress cone (i.e., area of a semicircle with a radius equal to l_{be}) directed toward the free edge. This is because the failure would occur as soon as one-half of the masonry breaks out from the nearest edge of the masonry. Quite often, the edge distance, l_{be}, is too large in the direction of the applied force (shear); in such cases, the limit state of masonry breakout might not govern.

The strength reduction factor corresponding to the limit state of masonry breakout is $\phi = 0.5$.

10.6.2 Nominal Shear Strength Based on Limit State of Masonry Crushing

The nominal shear strength corresponding to the limit state of the masonry crushing, B_{vnc}, is determined from Eq. (10.10):

$$B_{vnc} = 1050(f'_m A_b)^{0.25} \tag{10.10}$$

The strength reduction factor corresponding to the limit state of masonry crushing is $\phi = 0.5$.

10.6.3 Nominal Shear Strength Based on Limit State of Anchor Bolt Pryout

The nominal shear strength of an anchor bolt corresponding to the limit state of anchor pullout, B_{vpry}, is determined from Eq. (10.11):

$$B_{vpry} = 2.0 B_{anb} \tag{10.11}$$

Equation (10.11) can be expressed in terms of the area of right circular breakout cone and the compressive strength of masonry by substituting for B_{anb} from Eq. (10.5):

$$B_{anb} = 4 A_{pt} \sqrt{f'_m} \tag{10.5 repeated}$$

$$B_{vpry} = 8 A_{pt} \sqrt{f'_m} \tag{10.12}$$

The strength reduction factor corresponding to the limit state of anchor bolt pryout is $\phi = 0.5$.

10.6.4 Nominal Shear Strength Based on Limit State of Shear Yielding of Anchor Bolt Steel

The nominal shear strength of an anchor bolt governed by the limit state of shear yielding of anchor bolt steel is determined from Eq. (10.13), in which the shear yield strength of steel is taken as $0.6f_y$.

$$B_{vns} = 0.6 A_b f_y \tag{10.13}$$

The strength reduction factor corresponding to the shear yielding state is: $\phi = 0.9$

10.7 HEADED AND BENT-BAR ANCHOR BOLTS IN COMBINED AXIAL TENSION AND SHEAR

In certain situations, anchor bolts are loaded in combined axial tension and shear. Typical examples are roof and floor diaphragms, which transfer gravity and lateral loads to supporting walls. Based on tests, the allowable loads on such anchor bolts can be determined from the interaction relationship expressed as Eq. (10.14) (MSJC-08 Section 3.1.6.3.3):

$$\frac{b_{af}}{\phi B_{an}} + \frac{b_{vf}}{\phi B_{vn}} \leq 1.0 \tag{10.14}$$

where b_{af} = factored axial force in the anchor bolt
b_{vf} = factored shear force in the anchor bolt

In Eq. (10.14), quantities ϕB_{an} and ϕB_{vn} are the governing values of the design strengths in axial tension, and shear, respectively, as defined earlier (consequently, the values of the strength reduction factor-ϕ associated with B_{an} and B_{vn} could be different). Example 10.6 illustrates calculations for bent-bar anchor bolts in combined axial tension and shear, for which Eq. (10.14) needs to be satisfied.

10.8 STRUCTURAL WALLS AND THEIR ANCHORAGE REQUIREMENTS

10.8.1 Force Transfer Mechanism to Walls Through Anchorage of Diaphragms

Bent-bar anchor bolts are used for attachment of the diaphragms to masonry walls. Requirements for anchorage of structural walls are specified in ASCE 7-05 Sec. 12.11. Typically, diaphragms transfer both the gravity loads as well as their in-plane (lateral) loads to the supporting walls through ledger beams which are bolted to walls. As a result, anchor bolts are subjected to both axial tension and shear. Axial tension in anchor bolts results from lateral (wind or seismic) loads that the diaphragm transfers to supporting walls. Shear in the anchor bolt results from the gravity loads that the diaphragm transfers to the supporting walls.

10.8.2 Gravity Loads to be Transferred Through Anchorage

Typically, the gravity loads transferred from diaphragms are as follows:

1. *Roof diaphragm:* Roof dead load (D), roof live load (L_r), snow load (S), and rain load (R)
2. *Floor diaphragm:* Floor dead load (D) and floor live load (L)

ASCE 7-05 Sec. 2.3.2 specifies the following load combinations for these loads with wind load W and the earthquake load E:

$$\text{Combination 4: } U = 1.2D + 1.6W + 0.5(L_r \text{ or } S \text{ or } R) \tag{10.15}$$

$$\text{Combination 5: } U = 1.2D + 1.0E + L + 0.2S \tag{10.16}$$

The earthquake force, E, in Eq. (10.16) is stipulated in ASCE 7-05 Sec. 12.4 as follows:

$$E = E_h + E_v \tag{10.17}$$

where E = seismic load effect
E_h = effect of horizontal seismic force
E_v = effect of vertical seismic force

Forces E_h and E_v are defined as follows:

$$E_h = \rho Q_E \tag{10.18}$$

$$E_v = 0.2S_{\mathrm{DS}}D \tag{10.19}$$

where Q_E = effect of horizontal forces from V or F_p
ρ = redundancy factor (ASCE 7-05 Sec. 12.3.4)
S_{DS} = design spectral response acceleration parameter at short periods (0.2 s)
D = dead load

Substitution of Eq. (10.17) through Eq. (10.19) in Eq. (10.16) yields Eq. (10.20):

$$U = (1.2 + 0.2S_{DS})D + \rho Q_E + L + 0.2S \tag{10.20}$$

Equation (10.20) is the same equation as ASCE 7-05 Sec. 12.4.2.3, Eq. 5 for seismic load combination for strength design. It is noted that Eq. (10.20) does not include the roof live load because of the improbability of its presence during a design earthquake event. Equation (10.15) or Eq. (10.20) should be used to calculate shear force in anchor bolts caused by gravity loads in combination with lateral loads.

In the absence of a roof diaphragm subjected to seismic loads, the shear force in the anchor bolts can be determined by substituting floor live load $L = 0$ in Eq. (10.20), resulting in Eq. (10.21):

$$U = (1.2 + 0.2S_{DS})D + \rho Q_E + 0.2S \tag{10.21}$$

In the absence of snow load, Eq. (10.21) becomes

$$U = (1.2 + 0.2S_{DS})D + \rho Q_E \tag{10.22}$$

10.8.3 Lateral Seismic Loads to be Transferred Through Anchorage

The anchorage of concrete or masonry structural walls to supporting construction (i.e., diaphragms that are not flexible) is required to be designed so as to provide a direct connection capable of resisting the *greater* of the following:

1. A force equal to $0.4S_{DS}IW_p$
2. A force of $400S_{DS}I$
3. 280 lb per linear foot of the wall

In the case of flexible diaphragms, the anchorage requirements are much more stringent because of amplification resulting from diaphragm flexibility. ASCE 7-05 Sec. 12.11.2.1 requires that anchorage of concrete or masonry walls to flexible diaphragms in structures assigned to seismic design Category C, D, E, or F have the strength to develop out-of-plane force, F_p, given by Eq. (10.23):

$$F_p = 0.8S_{DS}IW_p \tag{10.23}$$

where F_p = out-of-plane force to be resisted by the anchorage
S_{DS} = the design spectral acceleration parameter at short periods (0.2 second)
I = occupancy importance factor
W_p = the weight of the wall tributary to the anchor

Structural walls are required to be designed to resist bending between the anchors where the anchor spacing exceeds 4 ft.

Examples 10.6 and 10.7 illustrate typical calculations for anchor bolts subjected simultaneously to tension and shear.

Example 10.6 Design strength of bent-bar anchor bolts under combined tension and shear.

A flexible roof diaphragm is attached to a reinforced concrete masonry wall with ⅝-in.-diameter A307 L-bolts (bent-bar bolts, $f_y = 36$ ksi) as shown in Fig. E10.6. The anchor bolts have an effective length of 6 in. and are spaced 8 in. on each side of the roof rafters which are spaced at 4 ft on center. The roof diaphragm transfers dead load and live loads of 64 lb per linear foot and 80 lb per linear foot of the wall, respectively. The following information is provided: $f'_m = 1500$ lb/in.^{2}, dead weight of wall = 78 lb/ft^2, and $S_{DS} = 1.25g$, $I = 1.0$, Seismic Design Category C. Check if the anchor bolts are adequate to support the loads from the diaphragm.

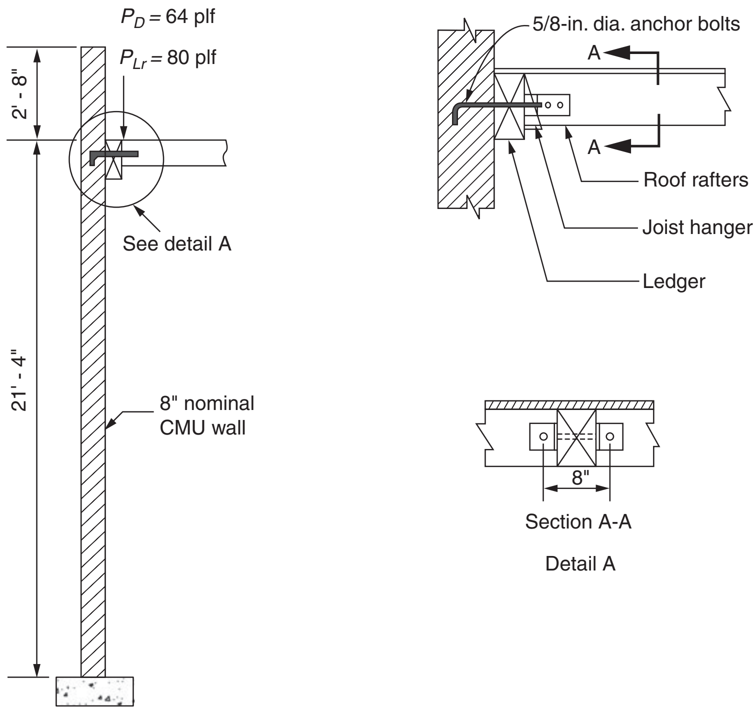


FIGURE E10.6 Bent-bar anchor bolts in tension and shear. (*Courtesy: CMACN.*)

Solution

Calculate the out-of-plane force, F_p, as a result of the seismic force transferred from the diaphragm to the anchor bolts, which causes tensile force in the anchor bolts.

The out-of-plane anchorage force, F_p, for a wall connected to a *flexible* diaphragm is determined from Eq. (10.23) [ASCE 7-05 Sec. (Eq. 12.11-1)]:

$$F_p = 0.8\, S_{DS} I W_p = 0.8(1.25)(1.0)W_p = 1.0W_p$$

The tributary wall weight is

$$W_p = 78\left(\frac{21.33}{2} + 2.67\right) = 1040 \text{ lb/ft}$$

Therefore, the factored axial tensile force in the anchor bolt is calculated as

$$F_p = 1.0W_p = 1.0(1040) = 1040 \text{ lb/ft}$$

Using the seismic load combination Eq. (10.20), the design shear force is determined from Eq. (10.22):

$$(1.2 + 0.2S_{\mathrm{DS}})D = (1.2 + 0.2 \times 1.25)\ (64) = 93 \text{ lb/ft}$$

Thus, the anchor bolts are required to resist a tensile force of 1040 lb per linear foot of the wall and a shear force of 93 lb per linear foot of the wall. Because the framing members (roof rafters) are spaced at 4 ft intervals, and each rafter transfers the force to the wall through two anchor bolts, the factored forces per bolt are

Tensile force,

$$b_{af} = ½(1040)(4) = 2080 \text{ lb}$$

Shear force,

$$b_{vf} = ½(93)(4) = 186 \text{ lb}$$

Commentary: The bent-bar anchor is subjected to axial tension and shear. In order to check its adequacy to resist these loads, we need to calculate its nominal strength in (1) axial tension, (2) shear, and (3) check if the interaction equation is satisfied.

1. Strength in axial tension

   a. Nominal strength based on limit state of masonry cone breakout, B_{anb} [Eq. (10.5)]:

$$B_{anb} = 4A_{pt}\sqrt{f'_m}$$

Check if the areas of masonry breakout cones overlap. Since bolts are spaced 8 in. apart, which is less than $2l_b = 2(6.0) = 12$ in., the breakout cones would overlap, and the effective area of the masonry breakout cone needs to be reduced. Calculate the effective masonry breakout area A'_{pt} from Eq. (10.3):

$$A'_{pt} = \pi l_b^2 - \frac{1}{2} l_b^2\,(\theta - \sin\theta)$$

Effective embedment length,

$$l_b = 6 \text{ in.}$$

Bolt spacing,

$$S = 8 \text{ in.}$$

Calculate θ from Eq. (10.4):

$$\theta = 2\,Cos^{-1}\left(\frac{s}{2l_b}\right) = 2\,Cos^{-1}\left(\frac{8}{2(6)}\right) = 96.38^{\circ} = 1.682 \text{ rad.}$$

$$A'_{pt} = \pi l_b^2 - \frac{1}{2} l_b^2\,(\theta - \sin\theta) = \pi(6)^2 - \frac{1}{2}(6)^2(1.682 - \sin 96.38^{\circ}) = 100.7 \text{ in}^2$$

$$B_{anb} = 4A'_{pt}\sqrt{f'_m} = 4(100.7)(\sqrt{1500}) = 15{,}600 \text{ lb}$$

The strength reduction factor for this limit state is 0.5. Thus,

$$\phi B_{anb} = 0.5(15{,}600) = 7800 \text{ lb}$$

b. Axial strength based on the limit state of anchor pull out, B_{anp}
Nominal axial strength based on the limit state of anchor pullout is given by Eq. (10.7):

$$B_{anp} = 1.5 f'_m e_b d_b + [300\pi(l_b + e_b + d_b)d_b]$$

In this example,

$$f'_m = 1500 \text{ lb/in}^2 \qquad e_b = 4 \text{ in.} \qquad l_b = 6 \text{ in.} \qquad d_b = 0.625 \text{ in.}$$

$$\begin{aligned} B_{anp} &= 1.5 f'_m e_b d_b + [300\pi(l_b + e_b + d_b)d_b] \\ &= 1.5(1500)(4)(0.625) + 300\pi(6 + 4 + 0.625)(0.625) \\ &= 5625 + 6259 = 11{,}884 \text{ lb} \end{aligned}$$

The strength reduction factor for this limit state is 0.65. Thus,

$$\phi B_{anp} = 0.65(11{,}884) = 7725 \text{ lb}$$

c. Axial strength based on limit state of tension yielding of bolt steel
Based on the tensile yielding strength of the anchor bolt, the allowable load is determined from Eq. (10.6). For a $^5/_8$-in.-diameter anchor bolt, the cross-sectional area is

$$A_b = \frac{\pi}{4}(d)^2 = \frac{\pi}{4}(0.625)^2 = 0.31 \text{ in}^2$$

$$B_{ans} = A_b f_y = (0.31)(36{,}000) = 11{,}160 \text{ lb}$$

The strength reduction factor for this limit state is $\phi = 0.9$. Therefore, the design strength is

$$\phi B_{vns} = 0.9(11{,}160) = 10{,}044 \text{ lb}$$

The smallest of the above three values is taken as the design axial strength of the bolt. Thus,

$$\phi B_{an} = 7725 \text{ lb}$$

2. Calculate the nominal shear strength of bolts, B_{vn}, based on the four limit states described earlier [Eqs. (10.8) through (10.13)].
   a. Shear strength based on limit state of masonry breakout, B_{vnb} [Eq. (10.8)]:

$$B_{vnb} = 4A_{pv}\sqrt{f'_m}$$

$$A_{pv} = \frac{1}{2}\pi l_{be}^2$$

Because the distance to the edge of the masonry, l_{be}, measured in the direction (downward) of the applied shear, is very large, Eq. (10.8) does not apply.
b. Shear strength based on limit state of masonry crushing, B_{vnc} [Eq. 10.10)]:

$$f'_m = 1500\, lb/ft^2,\ A_b = 0.31 \text{ in.}^2$$

$$B_{vnc} = 1050(f'_m A_b)^{0.25} = (1050)[(1500)(0.31)]^{0.25} = 4876 \text{ lb}$$

The strength reduction factor for this limit state is $\phi = 0.5$. Therefore, the design shear strength is

$$\phi B_{vnc} = 0.5(4876) = 2438 \text{ lb}$$

c. Shear strength based on limit state of masonry crushing, B_{vpry}

$$B_{vpry} = 8A'_{pt}\sqrt{f'_m}$$

$$A'_{pt} = 100.7 \text{ in}^2 \qquad \text{(calculated earlier)}$$

$$B_{vpry} = 8A'_{pt}\sqrt{f'_m} = 8(100.7)\sqrt{1500} = 31{,}201 \text{ lb}$$

The strength reduction factor for this limit state is $\phi = 0.5$. Therefore, the design shear strength is

$$\phi B_{vpry} = 0.5(31{,}200) = 15{,}600 \text{ lb}$$

d. Shear strength of anchor bolt based on limit state of shear yielding of bolt steel [Eq. (10.13)]:

$$B_{vn} = 0.6A_b f_y = 0.6(0.31)(36{,}000) = 6696 \text{ lb}$$

The strength reduction factor for this limit state is $\phi = 0.9$. Therefore, the design shear strength is

$$\phi B_{vn} = 0.9(6696) = 6026 \text{ lb}$$

The smallest of the above four design shear strengths governs. Therefore, the design shear strength of the anchor bolt is governed by the limit state of masonry crushing.

$$\phi B_{vn} = 0.5(4876) = 2438 \text{ lb}$$

3. Check the interaction equation, Eq. (10.14):

$$\frac{b_{af}}{\phi B_{an}} + \frac{b_{vf}}{\phi B_{vn}} \le 1.0$$

$$\frac{b_{af}}{\phi B_{an}} + \frac{b_{vf}}{\phi B_{vn}} = \frac{2080}{(7725)} + \frac{186}{(2438)} = 0.35 \le 1.0, \text{ OK.}$$

Answer: The ⅝-in. L-anchor bolts are adequate for anchorage of roof diaphragm to the masonry wall.

Example 10.7 Design strength of bent-bar anchor bolts under combined tension and shear.

A steel girder is attached to an 8-in. (nominal) wide masonry wall through four ⅝-in.-diameter A307 L-bolts (bent-bar anchor bolts), which transfers dead and live loads of 2000 lb each, and a moment of 300 lb-ft due to wind loads to the wall as shown in Fig. E10.7A. The gravity load has an eccentricity of 4 in. The bolts are spaced 8 in. on center vertically and 6 in. on center horizontally. Each bolt has an effective embedment length of 6 in. and a projected extension of 4 in. Check if the anchorage is adequate to transfer loads from the girder to the supporting wall. The compressive strength of masonry, $f'_m = 1500$ lb/in.^{2}.

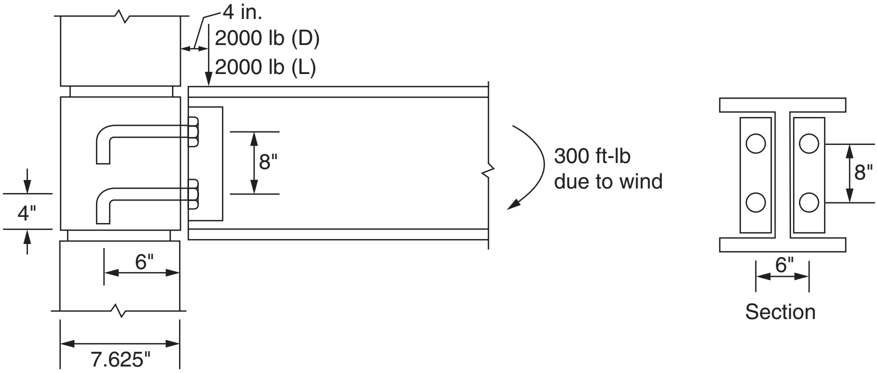


FIGURE E10.7A

Solution

Calculate bolt forces. The anchor bolts are subjected to axial tension caused by the applied moment, and shear due to applied gravity loads. Because of the presence of wind loads, the appropriate load combination is given by Eq. (10.15) [Eq. (4) of ASCE 7-05 Sec. 2.3.2]:

$$U = 1.2D + 1.6W + L + 0.5(L_r \text{ or } S \text{ or } R)$$

$$\text{Eccentricity of gravity loads} = 4 \text{ in.}$$

Anchor bolts are subjected to moment due to eccentricity of dead and live load, and due to wind. Therefore, the factored moment is determined from the above load combination equation:

$$M_u = 1.2(2000)(4) + 1.6\,(300)(12) + 2000\,(4) = 23{,}360 \text{ lb-in.}$$

Since bolts are spaced 8-in. apart vertically and two bolts develop axial tension, the force in each bolt is (Fig. E10.7B)

$$b_{af} = \frac{23{,}360}{2(8)} = 1460 \text{ lb}$$

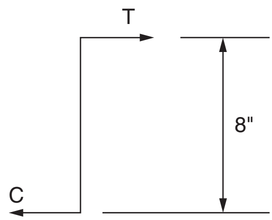


FIGURE E10.7B

The factored gravity load $= 1.2D + L = 1.2(2000) + 2000$
$= 4400$ lb

Shear per bolt,

$$b_{vf} = \frac{4400}{4} = 1100 \text{ lb}$$

Commentary: The bent-bar anchor is subjected to axial tension and shear. In order to check its adequacy to resist these loads, we need to calculate its nominal strength in (1) axial tension, (2) shear, and (3) check if the interaction equation is satisfied.

1. Axial strength of anchor bolt in tension:
   a. Axial strength based on limit state of masonry cone breakout [Eq. (10.5)]:
   Effective embedment length,

$$l_b = 6 \text{ in.}$$

Check if the areas of masonry breakout cones overlap. Since bolts are spaced 8 in. apart vertically and 6 in. apart horizontally, 6 in. (smaller distance) governs, which is less than $2l_b = 2(6.0) = 12$ in., the effective area of the masonry breakout cone needs to be reduced. Calculate the effective masonry breakout area A'_{pt} from Eq. (10.3):

$$A'_{pt} = \pi l_b^2 - \frac{1}{2} l_b^2 (\theta - \sin\theta) \tag{10.3}$$

Calculate θ from Eq. (10.4):

$$\theta = 2\,Cos^{-1}\left(\frac{s}{2l_b}\right) = 2\,Cos^{-1}\left(\frac{6}{2(6)}\right) = 96.38^o = 1.682 \text{ rad.}$$

$$A'_{pt} = \pi l_b^2 - \frac{1}{2} l_b^2 (\theta - \sin\theta) = \pi(6)^2 - \frac{1}{2}(6)^2(1.682 - \sin 96.38^\circ) = 100.7 \text{ in}^2$$

$$B_{anb} = 4A'_{pt}\sqrt{f'_m} = (4)(100.7)(\sqrt{1500}) = 15{,}600 \text{ lb}$$

The strength reduction factor for this limit state is 0.5. Therefore, the design axial strength of the anchor bolt is

$$\phi B_{anb} = 0.5(15{,}600) = 7800 \text{ lb}$$

b. Axial strength based on limit state of anchor bolt pullout
The bolt tension capacity due to anchor pullout is given by Eq. (10.7):

$$\begin{aligned} B_{anp} &= 1.5 f'_m e_b d_b + 300\pi(l_b + e_b + d_b)d_b \\ &= 1.5(1500)(4)(0.625) + 300\pi(6 + 4 + 0.625)(0.625) \\ &= 11{,}884 \text{ lb} \end{aligned}$$

The strength reduction factor for this limit state is 0.65. Therefore, the design strength is

$$\phi B_{an} = 0.65(11{,}884) = 7725 \text{ lb} < \phi B_{anb} = 7800 \text{ lb}$$

c. Axial strength based on limit state of steel yielding
Based on the tensile yield strength of the anchor bolt, the allowable load is determined from Eq. (10.6). For a $^5/_8$-in.-diameter anchor bolt, the cross-sectional area is

$$A_b = \frac{\pi}{4}(d)^2 = \frac{\pi}{4}(0.625)^2 = 0.31 \text{ in}^2$$

$$B_{ans} = A_b f_y = (0.31)(36.000) = 11{,}600 \text{ lb}$$

The strength reduction factor for this limit state is 0.9. Therefore, the design strength is

$$\phi B_{ans} = 0.9(11{,}600) = 10{,}004 \text{ lb} > \phi B_{anb} = 7725 \text{ lb}$$

Therefore, $\phi B_{an} = 7725$ lb (smallest value governs).

2. Calculate the strength of the bolt in shear.
   a. Shear strength based on limit state of masonry breakout [Eq. (10.8)]

Because the edge distance, l_{be}, in the direction of the applied loading is too large, probability of masonry breakout is not required to be considered.

b. Shear strength based on limit state of masonry crushing [Eq. (10.10)]

$$B_{vnc} = 1050(f'_m A_b)^{0.25} 1050[(1500)(0.31)]^{0.25} = 4876 \text{ lb}$$

The strength reduction factor for this limit state is 0.5. Therefore, the design strength is

$$\phi B_{vnc} = 0.5(4876) = 2438 \text{ lb}$$

c. Shear strength based on limit state of anchor bolt pryout [Eq. (10.11)]

$$B_{vpry} = 2.0B_{anb}$$

$$B_{anb} = 15{,}600 \text{ lb (calculated earlier)}$$

$$B_{vpry} = 2.0B_{anb} = 2(15{,}600) = 31{,}200 \text{ lb}$$

The strength reduction factor for this limit state is 0.5. Therefore, the design strength is

$$\phi B_{vpry} = 0.5(31{,}000) = 15{,}600 \text{ lb}$$

d. Shear strength based on limit state of shear yielding of anchor bolt [Eq. (10.13)]

$$B_{vns} = 0.6A_b f_y = (0.6)(0.31)(36{,}000) = 6696 \text{ lb}$$

The strength reduction factor for this limit state is 0.9. Therefore, the design strength is

$$\phi B_{vns} = 0.9(6696) = 6026 \text{ lb}$$

The smallest of the above three shear strength values governs. Therefore, $\phi B_{vn} = 2438$.

3. Check the interaction equation.

$$\frac{b_{af}}{\phi B_{an}} + \frac{b_{vf}}{\phi B_{vn}} \leq 1.0$$

$$\frac{b_{af}}{\phi B_{an}} + \frac{b_{vf}}{\phi B_{vn}} = \frac{1460}{7725} + \frac{1100}{2438} = 0.64 \leq 1.0, \text{ OK.}$$

Answer: The ⅝-in.-diameter bolts are adequate to transfer loads from the girder to the supporting wall.

Problems

10.1 Calculate the allowable tensile load on a ¾-in.-diameter A307 headed anchor bolt (f_y = 36 ksi) having an effective embedment length of 6 in. The bolt is at least 12 in. from adjacent bolts or the edge of masonry. The compressive strength of masonry = 1500 lb/in.^{2}.

10.2 What would be the allowable load on the ¾-in.-diameter bolt of Example 10.1 if the masonry compressive strength were 2000 lb/in.^{2}?

10.3 Calculate the allowable tensile load on a ¾-in.-diameter A307 headed anchor bolt (f_y = 36 ksi) having an effective embedment length of 6 in. The bolt is at least

9 in. from adjacent bolts or the edge of masonry. The compressive strength of masonry = 1800 lb/in.[2].

10.4 Determine the design tensile load for a ¾-in.-diameter headed A307 bolt embedded in a nominal 8-in. concrete masonry wall as shown in Fig. P10.4. The bolts are spaced at 12 in. on center. The compressive strength of the masonry is 2000 lb/in.[2].

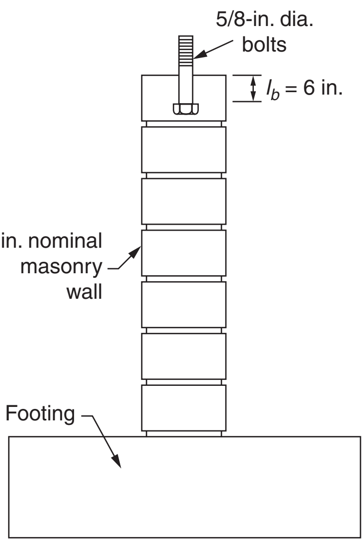


FIGURE P10.4

10.5 Determine the allowable tensile capacity of a 5/8-in.-diameter A307 L-bolt embedded in a nominal 12-in. concrete masonry wall as shown in Fig. P10.5. The bolts are spaced at 12 in. on center. The compressive strength of the masonry is 2000 lb/in.[2].

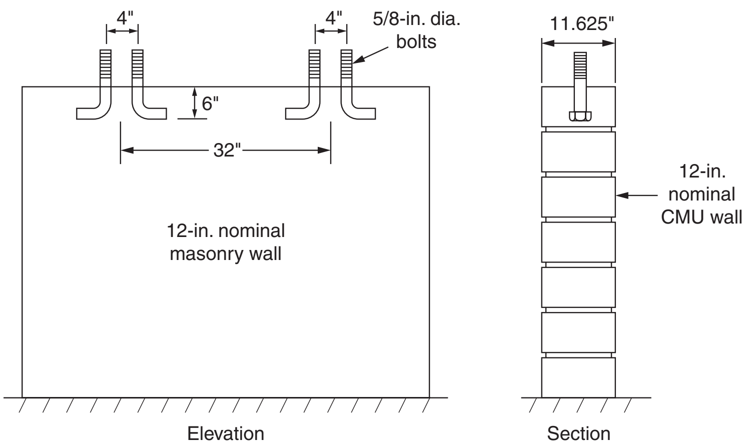


FIGURE P10.5

10.6 A flexible diaphragm is attached to a reinforced concrete masonry wall with $\frac{5}{8}$-in.-diameter A307 L-bolts (bent-bar bolts) as shown in Fig. P10.6. The anchor bolts have an effective length of 6 in. and are spaced 8 in. on each side of the roof rafters which are spaced at 4 ft on center. The projected extension of the bent bar is 4 in. The diaphragm transfers dead load and live loads of 80 lb per linear foot and 80 lb per linear foot of the wall, respectively. The following information is provided: $f'_m = 2000$ lb/in.^{2}, dead weight of wall = 78 lb/ft^2, and $S_{DS} = 1.2g$, $I = 1.0$, Seismic Design Category C. Check if the anchor bolts are adequate to support the loads from the diaphragm.

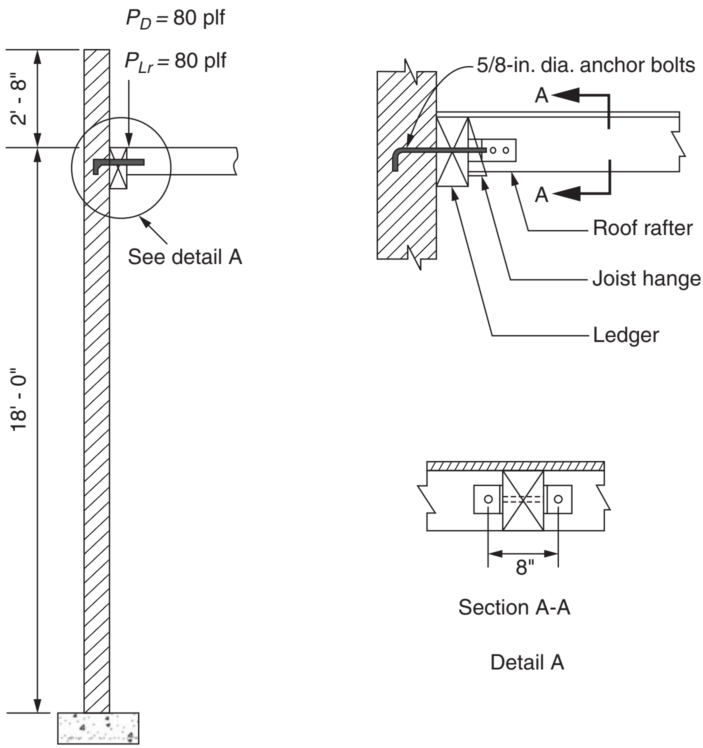


FIGURE P10.6 Bent-bar anchor bolts in tension and shear.

10.7 A steel girder is attached to an 8-in. (nominal) wide masonry wall through four ¾-in.-diameter A307 L-bolts (bent-bar anchor bolts), which transfers gravity and wind loads as shown in Fig. P10.7. The gravity load has an eccentricity of 4 in. The bolts are spaced 8 in. on center vertically and 6 in. on center horizontally. Each bolt has an effective embedment length of 6 in. and a projected extension of 4 in. Check if the anchorage is adequate to transfer loads from the girder to the supporting wall. The compressive strength of masonry, $f'_m = 1800$ lb/in.^{2}.

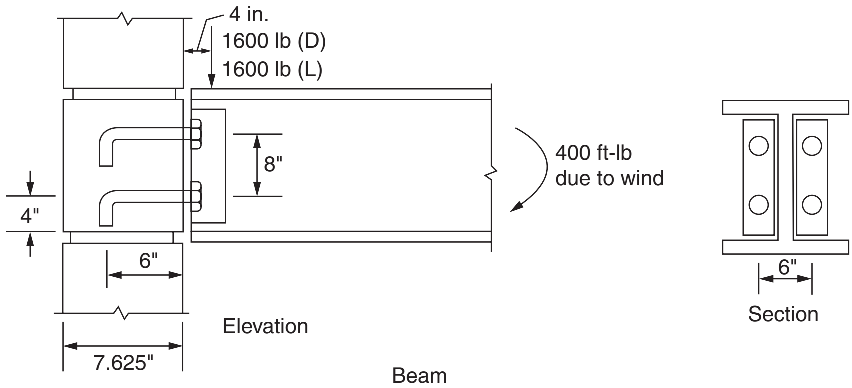


FIGURE P10.7

REFERENCES

10.1 MSJC (2008). *Building Code Requirements for Masonry Structures* (TMS 402-08/ACI 530-08/ASCE 5-08), Reported by The Masonry Standards Joint Committee, American Concrete Institute, the Masonry Society, Boulder, CO.

10.2 MSJC (2008). *Commentary on Building Code Requirements for Masonry Structures* (TMS 402-08/ACI 530-08/ASCE 5-08), Reported by The Masonry Standards Joint Committee, American Concrete Institute, the Masonry Society, Boulder, CO.

10.3 MSJC (2008). *Specification for Masonry Structures* (TMS 602-08/ACI 530.1-08/ASCE 6-08), Reported by The Masonry Standards Joint Committee, American Concrete Institute, the Masonry Society, Boulder, CO.

10.4 ASTM (2003). *Test Methods for Strength of Anchors in Concrete and Masonry*, Standard E488-96, Vol. 04.11, ASTM International, West Conshohocken, PA.

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APPENDIX

DESIGN AIDS: TABLES

TABLE A.1 ASTM C270—Mortar Proportion Specification Requirements (MSJC-08 Table SC-1) (Reprinted with permission.)

		Proportions by volume (cementitious materials)									
		Portland cement or blended cement	Masonry cement			Mortar cement			Hydrated lime or lime putty	Aggregate measured in damp, loose condition
Mortar	Type		M	S	N	M	S	N		
Cement-lime	M	1	-	-	-	-	-	-	¼	Not less than 2¼ and not more than 3 times the volume of cementitious materials
	A	1	-	-	-	-	-	-	Over ¼–½	
	N	1	-	-	-	-	-	-	Over ½–¼	
	O	1	-	-	-	-	-	-	Over 1¼–2½	
Mortar cement	M	1	-	-	-	-	-	1	-	
	M	-	-	-	-	1	-	-	-	
	S	1½	-	-	-	-	-	1	-	
	S	-	-	-	-	-	1	-	-	
	N	-	-	-	-	-	-	1	-	
	O	-	-	-	-	-	-	1	-	
Masonry cement	M	1	-	-	1	-	-	-		
	M	-	1	-	-	-	-	-		
	S	1½	-	-	1	-	-	-		
	S	-	-	1	-	-	-	-		
	N	-	-	-	1	-	-	-		
	O	-	-	-	1	-	-	-		

TABLE A.2 ASTM C270—Property Verification Requirements for Laboratory Prepared Mortar (MSJC-08 Table SC-2)

Mortar	Type	Average compressive[†] strength at 28 days minimum (psi)	Water retention minimum (%)	Air content maximum (%)
Cement-lime	M	2,500	75	12
	S	1,800	75	12
	N	750	75	14[‡]
	O	350	75	14[‡]
Mortar cement	M	2,500	75	12
	S	1,800	75	12
	N	750	75	14[‡]
	O	350	75	14[‡]
Masonry cement	M	2,500	75	18
	S	1,800	75	18
	N	750	75	20[§]
	O	350	75	20[§]

[*]When structural reinforcement is incorporated in cement-lime or mortar cement mortars, the maximum air content shall not exceed 12 percent.

[†]When structural reinforcement is incorporated in masonry cement mortar, the maximum air content shall be 18 percent.

TABLE A.3 Grout Proportion by Volume for Masonry Construction (Adapted from MSJC-08 Table SC-7)

Type	Parts by volume of portland cement or blended cement	Parts by volume of hydrated lime or lime putty	Aggregate, measured in damp, loose condition	
			Fine	Coarse
Fine grout	1	0–1/10	2¼–3 times the sum of the volumes of the cementitious materials	-
Coarse grout	1	0–1/10	2¼–3 times the sum of the volumes of the cementitious materials	1–2 times the sum of the volumes of the cementitious materials

TABLE A.4 Compressive Strength of Clay Masonry Based on the Compressive Strength of Clay Masonry Units and Type of Mortar used in Construction (MSJC-08 Table 1) (Reprinted with permission.)

Net area compressive strength of clay masonry units (psi)		Net area compressive strength of masonry (psi)
Type M or S mortar	Type N mortar	
1,700	2,100	1,000
3,350	4,150	1,500
4,950	6,200	2,000
6,600	8,250	2,500
8,250	10,300	3,000
9,900	-	3,500
11,500	-	4,000

For SI: 1 lb per square in. = 0.00689 MPa.

TABLE A.5 Compressive Strength of Concrete Masonry Based on the Compressive Strength of Concrete Masonry Units and Type of Mortar Used in Construction (MSJC-08 Table 2) (Reprinted with permission.)

Net compressive strength of concrete masonry units (psi)		Net area compressive strength of masonry (psi)
Type M or S mortar	Type N mortar	
	1,900	1,350
1,900	2,150	1,500
2,800	3,050	2,000
3,750	4,050	2,500
4,800	5,250	3,000

For SI: 1 in. = 25.4 mm, 1 lb per square in. = 0.00689 MPa.

TABLE A.6 Minimum Thickness of Face Shell and Webs*

Nominal width (W) of units, in. (mm)	Face shell thickness (FST), min, in. (mm)	Wall thickness	
		Webs, min, in. (mm)	Equivalent web thickness, min, in./linear ft* (mm/linear m)
3 (76.2) and 4 (102)	¾ (19)	¾ (19)	1 5/8 (136)
6 (152)	1 (25)	1 (25)	2¼ (188)
8 (203)	1¼ (32)	1 (25)	2¼ (188)
10 (254)	1 3/8 (35)	1 1/8 (29)	2½ (209)
12 (305)	1½ (38)	1 1/8 (29)	2½ (209)

*Sum of the measured thickness of all webs in the unit, multiplied by 12 and divided by the length of the unit.

TABLE A.7 Modulus of Rupture (f_r), psi (MSJC-08 Table 3.1.8.2.1) (Reprinted with permission.)

Direction of flexural tensile stress and masonry type	Mortar types: Portland cement/lime or mortar cement, M or S	Portland cement/lime or mortar cement, N	Masonry cement or air entrained portland cement/lime, M or S	Masonry cement or air entrained portland cement/lime, N
Normal to bed joints in running bond or stack bond				
Solid units	100	75	60	38
Hollow units				
Ungrouted	63	48	38	23
Fully grouted	163	158	153	145
Parallel to bed joints in running bond				
Solid units	200	150	120	75
Hollow units				
Ungrouted and partially grouted	125	95	75	48
Fully grouted	200	15	120	145
Parallel to bed joints in stack bond	0	0	0	0

Note: For partially grouted masonry, the modulus of rupture values shall be determined on the basis of linear interpolation between fully grouted hollow units and ungrouted hollow units based on amount (percentage) of grouting.

TABLE A.8 Size, Dimensions, and Weights of Standard Reinforcing Bars

Bar size	Nominal dimensions: Diameter, in.	Nominal dimensions: Cross-sectional area, in.^{2}	Weight, lb/ft
No. 3	0.375	0.11	0.376
No. 4	0.5	0.20	0.668
No. 5	0.625	0.31	1.043
No. 6	0.75	0.44	1.502
No. 7	0.875	0.61	2.044
No. 8	1.0	0.79	2.67
No. 9	1.128	1.00*	3.40
No. 10	1.27	1.27*	4.30
No. 11	1.41	1.56*	5.31
No. 14	1.693	2.25*	7.65
No. 18	2.257	4.0*	13.60

Source: CRSI: Reinforcing Bars: Anchorages and Splices, 5th ed. 2008.

*These cross-sectional areas are based on the equivalent areas of squares as follows:

Bar size	Squares for cross-sectional areas
No. 9	1 × 1 in.
No. 10	1⅛ × 1⅛ in.
No. 11	1¼ × 1¼ in.
No. 14	1½ × 1½ in.
No. 18	2.0 × 2.0 in

TABLE A.9 Areas of Groups of Standard Deformed Reinforcing Bars

	Number of bars									
Bar no.	1	2	3	4	5	6	7	8	9	10
3	0.11	0.20								
4	0.20	0.39	0.58	0.78	0.98	1.18	1.37	1.57	1.77	1.96
5	0.31	0.61	0.91	1.23	1.53	1.84	2.15	2.45	2.76	3.07
6	0.44	0.88	1.32	1.77	2.21	2.65	3.09	3.53	3.98	4.42
7	0.61	1.20	1.80	2.41	3.01	3.61	4.21	4.81	5.41	6.01
8	0.79	1.57	2.35	3.14	3.93	4.71	5.50	6.28	7.07	7.85
9	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00
10	1.27	2.53	3.79	5.06	6.33	7.59	8.86	10.12	11.39	12.66
11	1.56	3.12	4.68	6.25	7.81	9.37	10.94	12.50	14.06	15.62
14	2.25	4.50	6.75	9.00	11.25	13.50	15.75	18.00	20.25	22.50
18	4.0	8.00	12.00	16.00	20.00	24.00	28.00	32.00	36.00	40.00

	Number of bars									
Bar no.	11	12	13	14	15	16	17	18	19	20
4	2.16	2.36	2.55	2.75	2.95	3.14	3.34	3.53	3.73	3.93
5	3.37	3.68	3.99	4.30	4.60	4.91	5.22	5.52	5.83	6.14
6	4.86	5.30	5.74	6.19	6.63	7.07	7.51	7.95	8.39	8.84
7	6.61	7.22	7.82	8.42	9.02	9.62	10.22	10.82	11.43	12.03
8	8.64	9.43	10.21	11.00	11.78	12.57	13.35	14.14	14.92	15.71
9	11.00	12.00	13.00	14.00	15.00	16.00	17.00	18.00	19.00	20.00
10	13.92	15.19	16.45	17.72	18.98	20.25	21.52	22.78	24.05	25.31
11	17.19	18.75	20.31	21.87	23.44	25.00	26.56	28.13	29.69	31.25
14	24.75	27.00	29.25	31.50	33.75	36.00	38.25	40.50	42.75	45.00
18	44.00	48.00	52.00	56.00	60.00	64.00	68.00	72.00	76.00	80.00

TABLE A.10 Properties of Steel Reinforcing Wire

United States Steel Wire Gage; ASTM A82							
Steel wire gage	Diameter, in.	Perimeter, in.	Area, square in.	Ultimate strength, lb	Weight, plf	Allowable tension load @ 30,000 psi lb	Allowable tension load @ 40,000 psi (1/3 increase) lb
22	0.0286	0.090	0.0006	51	0.00218	19	26
21	0.0317	0.100	0.0008	63	0.00268	24	32
20	0.0348	0.109	0.0010	76	0.00323	29	38
19	0.0410	0.129	0.0013	106	0.00484	40	53
18	0.0475	0.149	0.0017	142	0.00602	53	71
17	0.0540	0.170	0.0023	183	0.00778	69	92
16	0.0625	0.196	0.0031	245	0.01042	92	123
15	0.0720	0.226	0.0041	326	0.01383	122	163
14	0.0800	0.251	0.0050	402	0.01707	150	200
13	0.0915	0.287	0.0066	529	0.02233	198	264
12	0.1055	0.330	0.0087	700	0.02969	261	348
11	0.1205	0.377	0.0114	912	0.03873	342	456
10	0.1350	0.424	0.0143	1146	0.04861	429	572
*9	0.1483	0.465	0.0173	1381	0.05866	519	692
*8	0.1620	0.509	0.0206	1649	0.07000	618	824
7	0.1770	0.556	0.0246	1968	0.08356	738	984
*3/16″	0.1875	0.587	0.0276	2209	0.09377	828	1104
6	0.1920	0.603	0.0289	2316	0.09832	867	1156
5	0.2070	0.650	0.0336	2692	0.1143	1008	1344
4	0.2253	0.707	0.0399	3189	0.1354	1197	1596
3	0.2437	0.763	0.0466	3732	0.1584	2214	1864
*1/4″	0.2500	0.785	0.0491	3927	0.1667	2301	1964
2	0.2625	0.823	0.0541	4330	0.1838	2580	2163
1	0.2830	0.889	0.0629	5033	0.2136	3096	2515
1-0	0.3065	0.961	0.0738	5903	0.2506	2214	2952
*5/16″	0.3125	0.980	0.0767	6135	0.2605	2301	3068
2-0	0.3310	1.040	0.0860	6880	0.2922	2580	3440
3-0	0.3625	1.137	0.1032	8257	0.3505	3096	4128

*Used for joint reinforcing, there are two wires or more per joint depending on type of reinforcing.

Note: f_u = 80,000 psi min.; f_v = 70,000 psi min.; $f_{a,\text{ allowable}}$ = 30,000 psi allowable for wind and seismic loads: f_a = 40,000 psi.

TABLE A.11 Values of $0.375\rho_b$ and $0.50\rho_b$ for Various Values of f'_m and f_y for Concrete Masonry

	$f_y = 40{,}000$ psi			$f_y = 60{,}000$ psi			
f'_m, psi	$0.375\rho_b$	$0.5\rho_b$	ρ_{max}	0.375	$0.5\rho_b$	ρ_{max}	f'_m, psi
1350	0.0052	0.0070	0.0118	0.0030	0.0039	0.0064	1350
1500	0.0058	0.0077	0.0131	0.0033	0.0044	0.0071	1500
1600	0.0062	0.0082	0.0140	0.0035	0.0047	0.0076	1600
1800	0.0070	0.0093	0.0158	0.0039	0.0053	0.0086	1800
2000	0.0077	0.0103	0.0175	0.0044	0.0058	0.0095	2000
2500	0.0097	0.0129	0.0219	0.0055	0.0073	0.0119	2500
3000	0.0116	0.0155	0.0263	0.0066	0.0088	0.0143	3000
3500	0.0135	0.0180	0.0306	0.0077	0.0102	0.0167	3500
4000	0.0155	0.0206	0.0350	0.0088	0.0117	0.0190	4000
4500	0.0174	0.0232	0.0394	0.0098	0.0131	0.0214	4500
5000	0.0193	0.0258	0.0438	0.0109	0.0146	0.0238	5000
5500	0.0213	0.0284	0.0482	0.0120	0.0161	0.0262	5500
6000	0.0232	0.0309	0.0525	0.0131	0.0175	0.0286	6000

TABLE A.12 Values of $0.375\rho_b$ and $0.50\rho_b$ for Various Values of f'_m and f_y for Clay Masonry

	$f_y = 40{,}000$ psi			$f_y = 60{,}000$ psi			
f'_m, psi	$0.375\rho_b$	$0.5\rho_b$	ρ_{max}	0.375	$0.5\rho_b$	ρ_{max}	f'_m, psi
1350	0.0058	0.0077	0.0136	0.0034	0.0045	0.0076	1350
1500	0.0065	0.0086	0.0151	0.0038	0.0050	0.0085	1500
1600	0.0069	0.0092	0.0161	0.0040	0.0054	0.0090	1600
1800	0.0077	0.0103	0.0181	0.0045	0.0060	0.0102	1800
2000	0.0086	0.0115	0.0201	0.0050	0.0067	0.0113	2000
2500	0.0108	0.0143	0.0251	0.0063	0.0084	0.0141	2500
3000	0.0129	0.0172	0.0302	0.0075	0.0101	0.0170	3000
3500	0.0151	0.0201	0.0352	0.0088	0.0117	0.0198	3500
4000	0.0172	0.0230	0.0402	0.0101	0.0134	0.0226	4000
4500	0.0194	0.0258	0.0453	0.0113	0.0151	0.0254	4500
5000	0.0215	0.0287	0.0503	0.0126	0.0168	0.0283	5000
5500	0.0237	0.0316	0.0553	0.0138	0.0184	0.0311	5500
6000	0.0258	0.0344	0.0603	0.0151	0.0201	0.0339	6000

TABLE A.13 Values of ϕk_n for Various Reinforcement Ratios

$$\phi k_n = \phi[f'_m \omega(1 - 0.625\omega)],\ \omega = \rho f_y / f'_m,\ f_y = 60{,}000 \text{ psi}$$

$\rho = A_s/bd$	f'_m (psi) 1500	1800	2000	2500	3000	3500	4000	4500	5000	5500	6000	$\rho = A_s/bd$
0.001	53	53	53	53	53	53	53	54	54	54	54	0.001
0.0011*	58	58	58	58	59	59	59	59	59	59	59	0.0011*
0.002	103	104	104	105	105	106	106	106	106	107	107	0.002
0.003	150	152	153	155	156	157	157	158	158	159	159	0.003
0.004	194	198	200	203	205	207	208	209	210	210	211	0.004
0.005	236	242	245	250	253	256	257	259	260	261	262	0.005
0.006	275	284	288	295	300	303	306	308	309	311	312	0.006
0.007	312	323	328	338	345	350	353	356	358	360	361	0.007
0.008	346	360	367	380	389	395	400	403	406	408	410	0.008
0.009	377	395	404	420	431	439	445	450	453	456	459	0.009
0.010	405	428	439	459	473	482	489	495	500	503	506	0.010
0.011	431	458	471	496	512	524	533	540	545	549	553	0.011
0.012	454	486	502	531	551	565	575	583	590	595	599	0.012
0.013	474	512	531	565	588	604	616	626	634	640	645	0.013
0.014	491	536	558	597	624	643	657	668	677	684	690	0.014
0.015	506	557	582	628	658	680	696	709	719	727	734	0.015
0.016	518	576	605	657	691	716	734	749	760	770	778	0.016
0.017	528	593	625	684	723	751	772	788	801	812	820	0.017
0.018	535	608	644	710	753	785	808	826	841	853	863	0.018
0.019	539	620	660	734	782	817	843	864	880	893	904	0.019
0.020	540	630	675	756	810	849	878	900	918	933	945	0.020
0.021	539	638	687	777	836	879	911	936	955	972	985	0.021
0.022	535	644	698	796	861	908	943	970	992	1010	1025	0.022
0.023	528	647	706	814	885	936	974	1004	1028	1047	1063	0.023
0.024	518	648	713	829	907	963	1004	1037	1063	1084	1102	0.024
0.025	506	647	717	844	928	988	1034	1069	1097	1120	1139	0.025
0.026	491	644	720	856	948	1013	1062	1100	1130	1155	1176	0.026
0.027	474	638	720	868	966	1036	1089	1130	1163	1190	1212	0.027
0.028	454	630	718	877	983	1058	1115	1159	1194	1223	1247	0.028
0.029	431	620	714	885	998	1079	1140	1188	1225	1256	1282	0.029
0.030	405	608	709	891	1013	1099	1164	1215	1256	1289	1316	0.030

*$\rho_{\min}$ corresponding to $M_n = 1.3M_{cr}$.

TABLE A.14 Values of Modular Ratio ($n = E_s/E_m$) for Clay and Concrete Masonry ($E_s = 29 \times 10^6$ psi)

	Clay masonry		Concrete masonry		
f'_m, psi	$E_m = 700\ f'_m$ (10^6 psi)	Modular ratio, $n = E_s/E_m$	$E_m = 900\ f'_m$ (10^6 psi)	Modular ratio, $n = E_s/E_m$	f'_m, psi
1350	0.945	30.69	1.215	23.87	1350
1500	1.060	27.36	1.350	21.48	1500
1800	1.260	23.02	1.620	17.91	1800
2000	1.400	20.71	1.800	16.11	2000
2500	1.750	16.57	2.250	12.89	2500
3000	2.100	13.81	2.700	10.74	3000
3500	2.450	12.08	3.150	9.21	3500
4000	2.800	10.36	3.600	8.06	4000
4500	3.15	9.21	4.050	7.17	4500
5000	3.50	8.29	4.500	6.44	5000
5500	3.85	7.53	4.950	5.86	5500
6000	4.20	6.90	5.400	5.37	6000

TABLE A.15 Values of Coefficients for Elastic Analysis of Reinforced Concrete Masonry Sections

$$\rho = \frac{A_s}{bd}, n = \frac{E_s}{E_m}, k = \sqrt{(n\rho)^2 + 2n\rho} - n\rho,\ j = 1 - \frac{k}{3},\ f_m = \frac{M}{bd^2}\frac{2}{kj}$$

$n\rho$	k	j	$2/kj$	$n\rho$	k	j	$2/kj$	$n\rho$	k	j	$2/kj$
0.001	0.0437	0.9854	46.41	0.046	0.2608	0.9131	8.40	0.091	0.3452	0.8849	6.55
0.002	0.0613	0.9796	33.32	0.047	0.2632	0.9123	8.33	0.092	0.3467	0.8844	6.52
0.003	0.0745	0.9752	27.52	0.048	0.2655	0.9115	8.26	0.093	0.3482	0.8839	6.50
0.004	0.0855	0.9715	24.07	0.049	0.2679	0.9107	8.20	0.094	0.3497	0.8834	6.47
0.005	0.0951	0.9683	21.71	0.050	0.2702	0.9099	8.14	0.095	0.3511	0.8830	6.45
0.006	0.1037	0.9654	19.98	0.051	0.2724	0.9092	8.07	0.096	0.3526	0.8825	6.43
0.007	0.1115	0.9628	18.63	0.052	0.2747	0.9084	8.02	0.097	0.3540	0.8820	6.41
0.008	0.1187	0.9604	17.54	0.053	0.2769	0.9077	7.96	0.098	0.3554	0.8815	6.38
0.009	0.1255	0.9582	16.64	0.054	0.2790	0.9070	7.90	0.099	0.3569	0.8810	6.36
0.010	0.1318	0.9561	15.87	0.055	0.2812	0.9063	7.85	0.100	0.3583	0.8806	6.34
0.011	0.1377	0.9541	15.22	0.056	0.2833	0.9056	7.80	0.101	0.3597	0.8801	6.32
0.012	0.1434	0.9522	14.65	0.057	0.2854	0.9049	7.74	0.102	0.3610	0.8797	6.30
0.013	0.1488	0.9504	14.15	0.058	0.2875	0.9042	7.69	0.103	0.3624	0.8792	6.28
0.014	0.1539	0.9487	13.70	0.059	0.2895	0.9035	7.65	0.104	0.3638	0.8787	6.26
0.015	0.1589	0.9470	13.29	0.060	0.2916	0.9028	7.60	0.105	0.3651	0.8783	6.24
0.016	0.1636	0.9455	12.93	0.061	0.2936	0.9021	7.55	0.106	0.3665	0.8778	6.22
0.017	0.1682	0.9439	12.60	0.062	0.2956	0.9015	7.51	0.107	0.3678	0.8774	6.20
0.018	0.1726	0.9425	12.30	0.063	0.2975	0.9008	7.46	0.108	0.3691	0.8770	6.18
0.019	0.1769	0.9410	12.02	0.064	0.2995	0.9002	7.42	0.109	0.3705	0.8765	6.16
0.020	0.1810	0.9397	11.76	0.065	0.3014	0.8995	7.38	0.110	0.3718	0.8761	6.14
0.021	0.1850	0.9383	11.52	0.066	0.3033	0.8989	7.34	0.111	0.3731	0.8756	6.12
0.022	0.1889	0.9370	11.30	0.067	0.3051	0.8983	7.30	0.112	0.3744	0.8752	6.10
0.023	0.1927	0.9358	11.09	0.068	0.3070	0.8977	7.26	0.113	0.3756	0.8748	6.09
0.024	0.1964	0.9345	10.90	0.069	0.3088	0.8971	7.22	0.114	0.3769	0.8744	6.07
0.025	0.2000	0.9333	10.71	0.070	0.3107	0.8964	7.18	0.115	0.3782	0.8739	6.05
0.026	0.2035	0.9322	10.54	0.071	0.3125	0.8958	7.15	0.116	0.3794	0.8735	6.03
0.027	0.2069	0.9310	10.38	0.072	0.3142	0.8953	7.11	0.117	0.3807	0.8731	6.02
0.028	0.2103	0.9299	10.23	0.073	0.316	0.8947	7.07	0.118	0.3819	0.8727	6.00
0.029	0.2136	0.9288	10.08	0.074	0.3178	0.8941	7.04	0.119	0.3832	0.8723	5.98
0.030	0.2168	0.9277	9.94	0.075	0.3195	0.8935	7.01	0.120	0.3844	0.8719	5.97
0.031	0.2199	0.9267	9.81	0.076	0.3212	0.8929	6.97	0.121	0.3856	0.8715	5.95
0.032	0.2230	0.9257	9.69	0.077	0.3229	0.8924	6.94	0.122	0.3868	0.8711	5.94
0.033	0.2260	0.9247	9.57	0.078	0.3246	0.8918	6.91	0.123	0.3880	0.8707	5.92
0.034	0.2290	0.9237	9.46	0.079	0.3263	0.8912	6.88	0.124	0.3892	0.8703	5.90
0.035	0.2319	0.9227	9.35	0.080	0.3279	0.8907	6.85	0.125	0.3904	0.8699	5.89
0.036	0.2347	0.9218	9.24	0.081	0.3296	0.8901	6.82	0.126	0.3916	0.8695	5.87
0.037	0.2375	0.9208	9.14	0.082	0.3312	0.8896	6.79	0.127	0.3927	0.8691	5.86
0.038	0.2403	0.9199	9.05	0.083	0.3328	0.8891	6.76	0.128	0.3939	0.8687	5.84
0.039	0.2430	0.9190	8.96	0.084	0.3344	0.8885	6.73	0.129	0.3951	0.8683	5.83
0.040	0.2457	0.9181	8.87	0.085	0.3360	0.8880	6.70	0.130	0.3962	0.8679	5.82
0.041	0.2483	0.9172	8.78	0.086	0.3376	0.8875	6.68	0.131	0.3974	0.8675	5.80
0.042	0.2509	0.9164	8.70	0.087	0.3391	0.8870	6.65	0.132	0.3985	0.8672	5.79
0.043	0.2534	0.9155	8.62	0.088	0.3407	0.8864	6.62	0.133	0.3996	0.8668	5.77

(*Continued*)

TABLE A.15 Values of Coefficients for Elastic Analysis of Reinforced Concrete Masonry Sections (*Continued*)

$$\rho = \frac{A_s}{bd},\ n = \frac{E_s}{E_m},\ k = \sqrt{(n\rho)^2 + 2n\rho} - n\rho,\ j = 1 - \frac{k}{3},\ f_m = \frac{M}{bd^2}\frac{2}{kj}$$

$n\rho$	k	j	$2/kj$	$n\rho$	k	j	$2/kj$	$n\rho$	k	j	$2/kj$
0.044	0.2559	0.9147	8.54	0.089	0.3422	0.8859	6.60	0.134	0.4007	0.8664	5.76
0.045	0.2584	0.9139	8.47	0.090	0.3437	0.8854	6.57	0.135	0.4019	0.8660	5.75
0.136	0.4030	0.8657	5.73	0.181	0.4473	0.8509	5.25	0.36	0.5617	0.8128	4.38
0.137	0.4041	0.8653	5.72	0.182	0.4482	0.8506	5.25	0.37	0.5664	0.8112	4.35
0.138	0.4052	0.8649	5.71	0.183	0.4491	0.8503	5.24	0.38	0.5710	0.8097	4.33
0.139	0.4063	0.8646	5.69	0.184	0.4499	0.8500	5.23	0.39	0.5755	0.8082	4.30
0.140	0.4074	0.8642	5.68	0.185	0.4508	0.8497	5.22	0.40	0.5798	0.8067	4.28
0.141	0.4084	0.8639	5.67	0.186	0.4516	0.8495	5.21	0.41	0.5840	0.8053	4.25
0.142	0.4095	0.8635	5.66	0.187	0.4525	0.8492	5.20	0.42	0.5882	0.8039	4.23
0.143	0.4106	0.8631	5.64	0.188	0.4534	0.8489	5.20	0.43	0.5922	0.8026	4.21
0.144	0.4116	0.8628	5.63	0.189	0.4542	0.8486	5.19	0.44	0.5961	0.8013	4.19
0.145	0.4127	0.8624	5.62	0.190	0.4551	0.8483	5.18	0.45	0.6000	0.8000	4.17
0.146	0.4137	0.8621	5.61	0.191	0.4559	0.8480	5.17	0.46	0.6038	0.7987	4.15
0.147	0.4148	0.8617	5.60	0.192	0.4567	0.8478	5.17	0.47	0.6075	0.7975	4.13
0.148	0.4158	0.8614	5.58	0.193	0.4576	0.8475	5.16	0.48	0.6111	0.7963	4.11
0.149	0.4169	0.8610	5.57	0.194	0.4584	0.8472	5.15	0.49	0.6146	0.7951	4.09
0.150	0.4179	0.8607	5.56	0.195	0.4592	0.8469	5.14	0.50	0.6180	0.7940	4.08
0.151	0.4189	0.8604	5.55	0.196	0.4601	0.8466	5.13	0.51	0.6214	0.7929	4.06
0.152	0.4199	0.8600	5.54	0.197	0.4609	0.8464	5.13	0.52	0.6247	0.7918	4.04
0.153	0.4209	0.8597	5.53	0.198	0.4617	0.8461	5.12	0.53	0.6280	0.7907	4.03
0.154	0.4219	0.8594	5.52	0.199	0.4625	0.8458	5.11	0.54	0.6312	0.7896	4.01
0.155	0.4229	0.8590	5.50	0.200	0.4633	0.8456	5.11	0.55	0.6343	0.7886	4.00
0.156	0.4239	0.8587	5.49	0.205	0.4673	0.8442	5.07	0.56	0.6373	0.7876	3.98
0.157	0.4249	0.8584	5.48	0.210	0.4712	0.8429	5.03	0.57	0.6403	0.7866	3.97
0.158	0.4259	0.8580	5.47	0.215	0.4751	0.8416	5.00	0.58	0.6433	0.7856	3.96
0.159	0.4269	0.8577	5.46	0.220	0.4789	0.8404	4.97	0.59	0.6462	0.7846	3.94
0.160	0.4279	0.8574	5.45	0.225	0.4825	0.8392	4.94	0.60	0.6490	0.7837	3.93
0.161	0.4288	0.8571	5.44	0.230	0.4862	0.8379	4.91	0.61	0.6518	0.7827	3.92
0.162	0.4298	0.8567	5.43	0.235	0.4897	0.8368	4.88	0.62	0.6545	0.7818	3.91
0.163	0.4308	0.8564	5.42	0.240	0.4932	0.8356	4.85	0.63	0.6572	0.7809	3.90
0.164	0.4317	0.8561	5.41	0.245	0.4966	0.8345	4.83	0.64	0.6598	0.7801	3.89
0.165	0.4327	0.8558	5.40	0.250	0.5000	0.8333	4.80	0.65	0.6624	0.7792	3.87
0.166	0.4336	0.8555	5.39	0.255	0.5033	0.8322	4.77	0.66	0.6650	0.7783	3.86
0.167	0.4346	0.8551	5.38	0.260	0.5066	0.8311	4.75	0.67	0.6675	0.7775	3.85
0.168	0.4355	0.8548	5.37	0.265	0.5097	0.8301	4.73	0.68	0.6700	0.7767	3.84
0.169	0.4364	0.8545	5.36	0.270	0.5129	0.8290	4.70	0.69	0.6724	0.7759	3.83
0.170	0.4374	0.8542	5.35	0.275	0.5160	0.8280	4.68	0.70	0.6748	0.7751	3.82
0.171	0.4383	0.8539	5.34	0.280	0.5190	0.8270	4.66	0.71	0.6771	0.7743	3.81
0.172	0.4392	0.8536	5.33	0.285	0.5220	0.8260	4.64	0.72	0.6794	0.7735	3.81
0.173	0.4401	0.8533	5.33	0.290	0.5249	0.8250	4.62	0.73	0.6817	0.7728	3.80
0.174	0.4410	0.8530	5.32	0.295	0.5278	0.8241	4.60	0.74	0.6839	0.7720	3.79
0.175	0.4419	0.8527	5.31	0.300	0.5307	0.8231	4.58	0.75	0.6861	0.7713	3.78
0.176	0.4429	0.8524	5.30	0.310	0.5362	0.8213	4.54	0.76	0.6883	0.7706	3.77
0.177	0.4437	0.8521	5.29	0.320	0.5416	0.8195	4.51	0.77	0.6904	0.7699	3.76
0.178	0.4446	0.8518	5.28	0.330	0.5469	0.8177	4.47	0.78	0.6925	0.7692	3.75
0.179	0.4455	0.8515	5.27	0.340	0.5520	0.8160	4.44	0.79	0.6946	0.7685	3.75
0.180	0.4464	0.8512	5.26	0.350	0.5569	0.8144	4.41	0.80	0.6967	0.7678	3.74

TABLE A.16 Values of Column Stability Factor C_P for Various h/t Ratios

h/t	10	11	12	13	14	15	16	17	18	19
C_P	0.939	0.926	0.912	0.897	0.880	0.862	0.843	0.823	0.802	0.779
h/t	20	21	22	23	24	25	26	27	28	28.577
C_P	0.755	0.730	0.704	0.676	0.647	0.617	0.586	0.554	0.520	0.500
h/t	28.577	29	30	31	32	33	34	35	36	37
C_P	0.500	0.486	0.454	0.425	0.399	0.375	0.353	0.333	0.315	0.298

Notes:

1. $h/r \le 99$ corresponds to $h/t \le 28.577$. For these values, $C_P = 1-\left(\frac{h}{140r}\right)^2 = 1-0.000612244898\left(\frac{h}{t}\right)^2$

2. $h/r > 99$ corresponds to $h/t > 28.577$. For these values, $C_P = \frac{408.33}{\left(h/t\right)^2}$

TABLE A.17 Spacing Requirements to Accommodate Longitudinal Column Reinforcing Bars

	Maximum width (in.) for number of bars at one face				
Bar no.	2	3	4	5	6
4	4.0	6.0	8.0	10.0	12.0
5	4.25	6.375	8.5	10.625	12.75
6	4.75	6.75	9.0	11.25	13.5
7	4.75	7.125	9.5	11.875	14.25
8	5.0	7.5	10.0	12.5	15.0
9	5.45	8.25	11.10	13.90	16.75
10	5.96	9.125	12.30	15.50	18.65
11	6.45	9.95	13.50	17.00	20.55

Note: The values in the table are based on the assumption that the maximum size of coarse aggregate will not exceed 1 in. and lateral reinforcement consists of No. 3 bars.

TABLE A.18 Average Weight of Concrete Masonry Units, Pounds per Unit

Unit 16″ long		Lightweight units: 103 pcf					Medium weight units: 115 pcf					Normal weight units: 135 pcf				
Thickness of units		4″	6″	8″	10″	12″	4″	6″	8″	10″	12″	4″	6″	8″	10″	12″
Individual block	4″ high units	8	11	13	15	20	9	13	15	17	22	10	16	18	20	26
	8″ high units	16	23	27	32	42	18	28	32	36	47	21	33	37	42	55

ASTM C90 classifies masonry units as follows: Lightweight less than 105 pcf, medium weight 105 pcf to less than 125 pcf, and normal weight 125 pcf or more.

TABLE A.19 Average Weight of Completed Wall*, lb/ft^2, and Equivalent Solid Thickness (Weight of Grout 140 pcf)

		Hollow concrete block												Hollow clay block, 120 pcf			Equivalent solid thickness†, in.			
		Light weight, 103 pcf				Medium weight, 115 pcf				Normal weight, 135 pcf										
Wall thickness		6″	8″	10″	12″	6″	8″	10″	12″	6″	8″	10″	12″	4″	6″	8″	6″	8″	10″	12″
Solid grouted wall		52	75	93	118	58	78	98	124	63	84	104	133	38	56	77	5.6	7.6	9.6	11.06
Vertical Cores Grouted At	16″ o.c.	41	60	69	88	47	63	80	94	52	66	86	103	33	45	59	4.5	5.8	7.2	8.5
	24″ o.c.	37	55	61	79	43	52	72	85	48	61	78	94	31	42	54	4.1	5.2	6.3	7.5
	32″ o.c.	36	52	57	74	42	55	68	80	47	58	74	89	30	40	51	4.0	4.9	5.9	7.0
	40″ o.c.	35	50	55	71	41	53	66	77	46	56	72	86	29	39	49	3.8	4.7	5.7	6.7
	48″ o.c.	34	49	53	69	40	45	64	75	45	55	70	83	28	38	48	3.7	4.6	5.5	6.5
No grout in wall		26	33	36	47	32	36	41	53	37	42	47	62	25	30	35	3.4	4.0	4.7	5.5

*The above table gives the average weights of completed walls of various thickness in pounds per square foot of wall face area. An average amount has been added into these values to include the weight of bond beams and reinforcing steel.

†Equivalent solid thickness means the calculated thickness of the wall if there were no hollow cores, and is obtained by dividing the volume of solid material in the wall by the face area of the wall.

This Equivalent Solid Thickness (EST) is for the determination of area for structural design only, e.g., $f_s = P/(\text{EST})b$. It is *not* to be used to obtain fire ratings. Fire rating thickness is based either on equivalent solid thickness of ungrouted units only or solid grouted walls.

TABLE A.20 Average Weight of Completed Wall*, lb/ft^2 and Equivalent Solid Thickness (Weight of Grout 105 pcf)

	Hollow concrete block												Hollow clay block, 120 pcf		
	Lightweight, 103 pcf				Medium weight, 115 pcf				Normal weight, 135 pcf						
Wall thickness	6″	8″	10″	12″	6″	8″	10″	12″	6″	8″	10″	12″	4″	6″	8″
Solid grouted wall	45	65	79	100	51	68	84	106	56	74	90	115	35	49	66
16″ o.c.	37	51	61	78	43	54	66	84	48	60	72	93	31	39	49
24″ o.c.	35	47	55	71	41	50	60	77	46	56	66	86	30	39	49
32″ o.c.	33	45	52	67	39	48	57	73	44	54	63	82	29	37	47
40″ o.c.	32	43	50	65	38	46	55	71	43	52	61	78	27	35	44
48″ o.c.	31	42	49	63	37	45	54	69	42	51	60	78	27	35	44
No grout in wall	26	33	36	47	32	36	41	53	37	42	47	62	25	30	35

*The above table gives the average weights of completed walls of various thickness in lb per square foot of wall face area. An average amount has been added into these values to include the weight of bond beams and reinforcing steel.

TABLE A.21 Average Weight of Reinforced and Grouted Brick Walls (psf)

Wall thickness	Weight, psf	Wall thickness	Weight, psf	Wall thickness	Weight, psf	Wall thickness	Weight, psf
8″	80	9″	90	10″	100	12″	120
8½″	85	9½″	95	11″	110	13″	130

TABLE A.22 Ratio of Area of Steel to Gross Cross-Sectional Area of Masonry

Nom. wall thickness	Bar size	Spacing of steel reinforcing										
		8″	12″	16″	20″	24″	28″	32″	36″	40″	44″	48″
6″	#3	0.00230	0.00153	0.00115	0.00092	0.00076	0.00065					
	#4	0.00417	0.00278	0.00208	0.00167	0.00139	0.00119	0.00104	0.00093	0.00083	0.00076	0.00069
	#5	0.00646	0.00431	300324	0.00258	0.00216	0.00185	0.00162	0.00144	0.00129	0.00117	0.00108
	#3	0.00172	0.00115	0.00086	0.00069	0.00057						
	#4	0.00312	0.00208	0.00156	0.00125	0.00104	0.00089	0.00078	0.00069	0.00063		
8″	#5	0.00485	0.00323	0.00242	0.00194	0.00161	0.00138	0.00121	0.00108	0.00097	0.00088	0.00081
	#6	0.00688	0.0046	0.00344	0.00276	0.00229	0.00197	0.00172	0.00153	0.00138	0.00126	0.00115
	#7	0.00938	0.00626	0.00469	0.00376	0.00313	0.00268	0.00234	0.00209	0.00188	0.00171	0.00156
	#4	0.0025	0.00167	0.00125	0.00100	0.00083	0.00071	0.00063				
	#5	0.00388	0.00258	0.00194	0.00155	0.00129	0.00111	0.00097	0.00086	0.00078	0.00070	0.00065
10″	#6	0.00550	0.00368	0.00275	0.00221	0.00183	0.00158	0.00138	0.00123	0.00110	0.00100	0.00092
	#7	0.00750	0.00501	0.00375	0.00301	0.0025	0.00215	0.00188	0.00167	0.00150	0.00137	0.00125
	#8	0.00988	0.00655	0.00494	0.00393	0.00329	0.00281	0.00247	0.00218	0.00198	0.00179	0.00165
	#9	0.01250	0.00833	0.00625	0.00500	0.00417	0.00357	0.00313	0.00278	0.00250	0.00227	0.00208
	#4	0.00208	0.00139	0.00104	0.00083	0.00069	0.00060					
	#5	0.00323	0.00215	0.00161	0.00129	0.00109	0.00092	0.00081	0.00072	0.00065		
12″	#6	0.00459	0.00307	0.00232	0.00184	0.00153	0.00132	0.00116	0.00102	0.00092	0.00084	0.00077
	#7	0.00625	0.00417	0.00313	0.0025	0.00208	0.00179	0.00156	0.00139	0.00125	0.00114	0.00104
	#8	0.00823	0.00546	0.00411	0.00328	0.00274	0.00234	0.00206	0.00182	0.00165	0.00149	0.00137
	#9	0.01042	0.00694	0.00521	0.00417	0.00347	0.00298	0.0026	0.00231	0.00208	0.00189	0.00174

TABLE A.23 Area and Perimeters of Group of Reinforcing Bars

		Reinforcing bar spacing, in.										
Size		8	12	16	20	24	28	32	36	40	44	48
#2	A_s	0.07	0.05	0.04	0.03							
	Σ_o	1.2	0.8	0.6	0.5							
#3	A_s	0.17	0.11	0.08	0.07	0.06	0.05	0.04				
	Σ_o	1.8	1.2	0.9	0.7	0.6	0.5	0.4				
#4	A_s	0.29	0.2	0.15	0.12	0.1	0.08	0.08	0.07	0.06	0.05	0.05
	Σ_o	2.4	1.6	1.2	0.9	0.8	0.7	0.6	0.5	0.5	0.4	0.4
#5	A_s	0.46	0.31	0.23	0.19	0.15	0.13	0.12	0.10	0.09	0.09	0.08
	Σ_o	2.9	2.0	1.5	1.2	1.0	0.8	0.7	0.7	0.6	0.5	0.5
#6	A_s	0.66	0.44	0.33	0.27	0.22	0.19	0.17	0.15	0.13	0.12	0.11
	Σ_o	3.5	2.4	1.8	1.4	1.2	1	0.9	0.8	0.7	0.6	0.6
#7	A_s	0.90	0.60	0.45	0.36	0.30	0.26	0.24	0.20	0.18	0.16	0.15
	Σ_o	4.1	2.8	2.1	1.70	1.40	1.20	1.00	0.90	0.80	0.80	0.70
#8	A_s	1.18	0.79	0.59	0.47	0.39	0.34	0.31	0.26	0.24	0.21	0.20
	Σ_o	4.7	3.1	2.4	1.90	1.60	1.40	1.20	1.00	0.90	0.90	0.80
#9	A_s	1.5	1.00	0.75	0.60	0.50	0.43	0.40	0.33	0.30	0.27	0.25
	Σ_o	5.3	3.5	2.7	2.10	1.80	1.50	1.30	1.20	1.00	1.00	0.90
#10	A_s	1.91	1.27	0.95	0.76	0.64	0.54	0.51	0.42	0.38	0.35	0.32
	Σ_o	6.00	4.00	3.00	2.40	2.00	1.70	1.50	1.30	1.20	1.10	1.00
#11	A_s	2.34	1.56	1.17	0.94	0.78	0.67	0.62	0.52	0.47	0.43	0.39
	Σ_o	6.60	4.40	3.30	2.70	2.20	1.90	1.70	1.50	1.30	1.20	1.10

A_s = area of steel per foot length for the given bar size.
Σo = perimeter of reinforcing bars per foot of wall for the given bars size.

TABLE A.24 Section Properties for Horizontal Cross Sections

4-in. single-wythe walls*										
			A_n		I_x		S_x		r	
Units	Grouted cores	Mortar bedding	in.^{2}/ft	(10^3mm^2/m)	in.^{4}/ft	(10^6 mm^4/m)	in.^{3}/ft	(10^6 mm^3/m)	in.	(mm)
Hollow	None	Face shell	18.0	(38.1)	39.4	(53.8)	21.0	(1.13)	1.35	(34.3)
Hollow	None	Full	21.6	(45.7)	39.4	(53.8)	21.7	(1.17)	1.35	(34.3)
Solid	None	Full	43.5	(92.1)	47.4	(64.7)	26.3	(1.41)	0.04	(26.5)

6-in. single-wythe walls										
			A_n		I_x		S_x		r	
Units	Grouted cores	Mortar bedding	in.^{2}/ft	(10^3 mm^2/m)	in.^{4}/ft	(10^6 mm^4/m)	in.^{3}/ft	(10^6 mm^3/m)	in.	(mm)
Hollow	None	Face shell	24.0	(50.8)	3.3	(190)	46.3	(2.49)	2.08	(52.9)
Hollow	None	Full	32.2	(68.1)	3.4	(190)	493.5	(2.66)	2.08	(52.9)
Solid	None	Full	67.5	(143)	176.9	(242)	63.3	(3.40)	1.62	(41.1)
Hollow	8″ o.c.	Full	67.5	(143)	176.9	(242)	63.3	(3.40)	1.62	(41.1)
Hollow	16″ o.c.	Face shell	46.6	(98.6)	158.1	(216)	55.1	(2.96)	1.79	(45.5)
Hollow	24″ o.c.	Face shell	39.1	(82.7)	151.8	(207)	52.2	(2.81)	1.87	(47.4)
Hollow	32″ o.c.	Face shell	35.3	(74.7)	148.7	(203)	50.7	(2.73)	1.91	(48.5)
Hollow	40″ o.c.	Face shell	33.0	(69.9)	146.8	(200)	49.9	(2.68)	1.94	(49.3)
Hollow	48″ o.c.	Face shell	31.5	(66.7)	145.5	(199)	49.3	(2.65)	1.96	(49.8)
Hollow	56″ o.c.	Face shell	30.5	(64.5)	144.6	(198)	48.9	(2.63)	1.98	(50.2)
Hollow	64″ o.c.	Face shell	29.6	(62.8)	144.0	(197)	48.5	(2.61)	1.99	(50.5)
Hollow	72″ o.c.	Face shell	29.0	(61.4)	143.5	(196)	48.3	(2.60)	2.00	(50.7)

(*Continued*)

TABLE A.24 Section Properties for Horizontal Cross Sections (*Continued*)

8-in. single-wythe walls										
			A_n		I_x		S_x		r	
Units	Grouted cores	Mortar bedding	in.^{2}/ft	(10^3 mm^2/m)	in.^{4}/ft	(10^6 mm^4/m)	in.^{3}/ft	(10^6 mm^3/m)	in.	(mm)
Hollow	None	Face shell	30.0	(63.5)	334.0	(456)	81.0	(4.35)	2.84	(72.0)
Hollow	None	Full	41.5	(87.9)	334.0	(456)	87.6	(4.71)	2.84	(72.0)
Solid	None	Full	91.5	(194)	440.2	(601)	116.3	(6.25)	2.19	(55.7)
Hollow	8″ o.c.	Full	91.5	(194)	440.2	(601)	116.3	(6.25)	2.19	(55.7)
Hollow	16″ o.c.	Face shell	62.0	(131)	387.1	(529)	99.3	(5.34)	2.43	(61.6)
Hollow	24″ o.c.	Face shell	51.3	(109)	369.4	(504)	93.2	(5.01)	2.53	(64.3)
Hollow	32″ o.c.	Face shell	46.0	(97.3)	360.5	(492)	90.1	(4.85)	2.59	(65.8)
Hollow	40″ o.c.	Face shell	42.8	(90.6)	355.2	(485)	88.3	(4.75)	2.63	(66.9)
Hollow	48″ o.c.	Face shell	40.7	(86.0)	351.7	(480)	87.1	(4.68)	2.66	(67.6)
Hollow	56″ o.c.	Face shell	39.1	(82.8)	349.1	(477)	86.2	(4.64)	2.68	(68.2)
Hollow	64″ o.c.	Face shell	38.0	(80.4)	347.2	(474)	85.6	(4.60)	2.70	(68.6)
Hollow	72″ o.c.	Face shell	37.1	(78.5)	345.8	(472)	85.0	(4.57)	2.71	(69.0)

10-in. single-wythe walls										
			A_n		I_x		S_x		r	
Units	Grouted cores	Mortar bedding	in.^{2}/ft	(10^3 mm^2/m)	in.^{4}/ft	(10^6 mm^4/m)	in.^{3}/ft	(10^6 mm^3/m)	in.	(mm)
Hollow	None	Full	50.4	(107)	635.3	(868)	132.0	(7.10)	3.55	(90.2)
Solid	None	Full	115.5	(244)	884.1	(1210)	185.3	(9.96)	2.77	(70.3)
Hollow	8″ o.c.	Full	115.5	(244)	884.1	(1210)	185.3	(9.96)	2.77	(70.3)
Hollow	16″ o.c.	Face shell	76.2	(161)	759.7	(1040)	153.1	(8.23)	3.04	(77.3)
Hollow	24″ o.c.	Face shell	61.8	(131)	718.2	(981)	141.3	(7.60)	3.17	(80.5)
Hollow	32″ o.c.	Face shell	54.6	(116)	697.5	(952)	135.4	(7.28)	3.25	(82.5)
Hollow	40″ o.c.	Face shell	50.3	(106)	685.0	(935)	131.9	(7.09)	3.30	(83.7)
Hollow	48″ o.c.	Face shell	47.4	(100)	676.7	(924)	129.5	(6.96)	3.33	(84.7)

Hollow	56″ o.c.	Face shell	45.3	(96.0)	670.8	(916)	127.9	(6.87)	3.36	(85.3)
Hollow	64″ o.c.	Face shell	43.8	(92.7)	666.4	(910)	126.6	(6.81)	3.38	(85.9)
Hollow	72″ o.c.	Face shell	42.6	(90.1)	662.9	(905)	125.6	(6.75)	3.34	(86.3)
12-in. single-wythe walls										
			A_n		I_x		S_x		r	
Units	Grouted cores	Mortar bedding	in.^{2}/ft	(10^3 mm^2/m)	in^4/ft	(10^6 mm^4/m)	in.^{3}/ft	(10^6 mm^3/m)	in.	(mm)
Hollow	None	Face shell	36.0	(76.2)	1064.7	(1450)	159.9	(8.60)	4.29	(109)
Hollow	None	Full	57.8	(122)	1064.7	(1450)	183.2	(9.85)	4.29	(109)
Solid	None	Full	139.5	(295)	1556.0	(2120)	270.3	(14.5)	3.34	(84.8)
Hollow	8″ o.c.	Full	139.5	(295)	1556.0	(2120)	270.3	(14.5)	3.34	(84.8)
Hollow	16″ o.c.	Face shell	90.2	(191)	1310.4	(1790)	217.7	(11.7)	3.67	(93.1)
Hollow	24″ o.c.	Face shell	72.1	(153)	1228.5	(1680)	198.4	(10.7)	3.82	(96.7)
Hollow	32″ o.c.	Face shell	63.1	(134)	1187.5	(1620)	188.8	(10.1)	3.91	(99.3)
Hollow	40″ o.c.	Face shell	57.7	(122)	1163.0	(1590)	183.0	(9.84)	3.97	(101)
Hollow	48″ o.c.	Face shell	54.1	(114)	1146.6	(1570)	179.2	(9.63)	4.02	(102)
Hollow	56″ o.c.	Face shell	51.5	(109)	1134.9	(1550)	176.4	(9.48)	4.05	(103)
Hollow	64″ o.c.	Face shell	49.5	(105)	1126.1	(1540)	174.3	(9.37)	4.08	(104)
Hollow	72″ o.c.	Face shell	48.0	(102)	1119.3	(1530)	172.7	(9.29)	10.10	(104)

*Values in these tables are based on minimum face shell and web thicknesses defined in ASTM C90. As shown in the table, manufactured units generally exceed these dimensions, making 4-in. concrete masonry units difficult or impossible to grout.

TABLE A.25 Section Properties for Vertical Cross Sections

4-in. single-wythe walls*

Units	Grouted cores	Mortar bedding	A_n in.^{2}/ft	A_n (10^3 mm^2/m)	I_y in.^{4}/ft	I_y (10^6 mm^4/m)	S_y in.^{3}/ft	S_y (10^6 mm^3/m)
Hollow	None	Face shell	8.0	(38.1)	38.0	(51.9)	21.0	(1.13)
Solid	None	Full	43.5	(92.1)	47.6	(65.0)	26.03	(1.41)

6-in. single-wythe walls

Units	Grouted cores	Mortar bedding	A_n in.^{2}/ft	A_n (10^3 mm^2/m)	I_y in.^{4}/ft	I_y (10^6 mm^4/m)	S_y in.^{3}/ft	S_y (10^6 mm^3/m)
Hollow	None	Face shell	24.0	(50.8)	130.3	(178)	46.3	(2.49)
Solid	None	Full	67.5	(143)	178.0	(243)	63.3	(3.40)
Hollow	8″ o.c.	Full	67.5	(143)	178.0	(243)	63.3	(3.40)
Hollow	16″ o.c.	Face shell	44.7	(94.7)	154.2	(211)	54.8	(2.95)
Hollow	24″ o.c.	Face shell	37.8	(80.1)	146.2	(200)	52.0	(2.80)
Hollow	32″ o.c.	Face shell	34.4	(72.7)	142.3	(194)	50.6	(2.72)
Hollow	40″ o.c.	Face shell	32.3	(68.4)	139.9	(191)	49.7	(2.67)
Hollo	48″ o.c.	Face shell	30.9	(65.4)	138.3	(189)	49.2	(2.64)
Hollow	56″ o.c.	Face shell	29.9	(63.3)	137.1	(187)	48.8	(2.62)
Hollow	64″ o.c.	Face shell	29.2	(61.8)	136.3	(186)	48.5	(2.61)
Hollow	72″ o.c.	Face shell	28.6	(60.6)	135.6	(185)	48.2	(2.59)

8-in. single-wythe walls								
			A_n		I_y		S_y	
Units	Grouted cores	Mortar bedding	in.^{2}/ft	(10^3mm^2/m)	in.^{4}/ft	(10^6 mm^4/m)	in.^{3}/ft	(10^6 mm^3/m)
Hollow	None	Face shell	30.0	(63.5)	308.7	(422)	81.0	(4.35)
Solid	None	Full	91.5	(194)	443.3	(605)	116.3	(6.25)
Hollow	8″ o.c.	Full	91.5	(194)	443.3	(605)	116.3	(6.25)
Hollow	16″ o.c.	Face shell	59.3	(126)	376.0	(513)	98.6	(5.30)
Hollow	24″ o.c.	Face shell	49.5	(105)	353.6	(483)	92.7	(4.99)
Hollow	32″ o.c.	Face shell	44.7	(94.5)	342.4	(468)	89.8	(4.83)
Hollow	40″ o.c.	Face shell	41.7	(88.3)	335.6	(458)	88.0	(4.73)
Hollow	48″ o.c.	Face shell	39.8	(84.2)	331.1	(452)	86.9	(4.67)
Hollow	56″ o.c.	Face shell	38.4	(81.2)	327.9	(448)	86.0	(4.62)
Hollow	64″ o.c.	Face shell	37.3	(79.0)	325.5	(445)	85.4	(4.59)
Hollow	72″ o.c.	Face shell	36.5	(77.3)	323.7	(442)	84.9	(4.56)

10-in. single-wythe walls								
			A_n		I_y		S_y	
Units	Grouted cores	Mortar bedding	in.^{2}/ft	(10^3 mm^2/m)	in.^{4}/ft	(10^6 mm^4/m)	in.^{3}/ft	(10^6 mm^3/m)
Hollow	None	Face shell	33.0	(69.9)	566.7	(774)	117.8	(6.33)
Solid	None	Full	115.5	(244)	891.7	(1220)	185.3	(9.96)
Hollow	8″ o.c.	Full	115.5	(244)	891.7	(1220)	185.3	(9.96)
Hollow	16″ o.c.	Face shell	72.3	(153)	729.2	(996)	151.5	(8.15)
Hollow	24″ o.c.	Face shell	59.2	(125)	675.0	(922)	140.3	(7.54)
Hollow	32″ o.c.	Face shell	52.7	(111)	648.0	(885)	134.6	(7.24)
Hollow	40″ o.c.	Face shell	48.7	(103)	631.7	(863)	131.3	(7.06)
Hollow	48″ o.c.	Face shell	46.1	(97.6)	620.9	(848)	129.0	(6.94)
Hollow	56″ o.c.	Face shell	44.2	(93.6)	613.1	(837)	127.4	(6.85)
Hollow	64″ o.c.	Face shell	42.8	(90.7)	607.3	(829)	126.2	(6.78)
Hollow	72″ o.c.	Face shell	41.7	(88.3)	602.8	(823)	125.3	(6.73)

(*Continued*)

TABLE A.25 Section Properties for Vertical Cross Sections (*Continued*)

12-in. single-wythe walls								
			A_n		I_y		S_y	
Units	Grouted cores	Mortar bedding	in.^{2}/ft	(10^3 mm^2/m)	in.^{4}/ft	(10^6 mm^4/m)	in.^{3}/ft	(10^6 mm^3/m)
Hollow	None	Face shell	36.0	(76.2)	929.4	(1270)	159.9	(8.60)
Solid	None	Full	139.5	(295)	1571.0	(2150)	270.3	(14.5)
Hollow	8″ o.c.	Full	139.5	(295)	1571.0	(2150)	270.3	(14.5)
Hollow	16″ o.c.	Face shell	85.3	(181)	1250.2	(1710)	215.1	(11.6)
Hollow	24″ o.c.	Face shell	68.9	(146)	1143.3	(1560)	196.7	(10.6)
Hollow	32″ o.c.	Face shell	60.7	(128)	1089.8	(1490)	187.5	(10.1)
Hollow	40″ o.c.	Face shell	55.7	(118)	1057.7	(1440)	182.0	(9.78)
Hollow	48″ o.c.	Face shell	52.4	(111)	1036.3	(1420)	178.3	(9.59)
Hollow	56″ o.c.	Face shell	50.1	(106)	1021.1	(1390)	175.7	(9.44)
Hollow	64″ o.c.	Face shell	48.3	(102)	1009.6	(1380)	173.7	(9.34)
Hollow	72″ o.c.	Face shell	47.0	(99.4)	1000.7	(1370)	172.2	(9.26)

TABLE A.26 Rigidity Coefficients for Cantilever Walls

$$R_C = \frac{1}{0.4\left(\frac{h}{d}\right)^3 + 0.3\left(\frac{h}{d}\right)}$$

h/d	R_c	h/d	R_c	h/d	R_c	h/d	R_c	h/d	R_c	h/d	R_c
9.90	0.003	5.20	0.017	1.85	0.324	1.38	0.682	0.91	1.741	0.45	5.833
9.80	0.003	5.10	0.018	1.84	0.329	1.37	0.695	0.90	1.781	0.44	6.021
9.70	0.003	5.00	0.019	1.83	0.333	1.36	0.707	0.89	1.822	0.43	6.219
9.60	0.003	4.90	0.021	1.82	0.338	1.35	0.720	0.88	1.864	0.42	6.425
9.50	0.003	4.80	0.022	1.81	0.343	1.34	0.733	0.87	1.907	0.41	6.641
9.40	0.003	4.70	0.023	1.80	0.348	1.33	0.746	0.86	1.952	0.40	6.868
9.30	0.003	4.60	0.025	1.79	0.353	1.32	0.760	0.85	1.997	0.39	7.106
9.20	0.003	4.50	0.026	1.78	0.358	1.31	0.774	0.84	2.045	0.38	7.356
9.10	0.003	4.40	0.028	1.77	0.364	1.30	0.788	0.83	2.093	0.37	7.618
9.00	0.003	4.30	0.030	1.76	0.369	1.29	0.803	0.82	2.143	0.36	7.895
8.90	0.004	4.20	0.032	1.75	0.375	1.28	0.818	0.81	2.195	0.35	8.187
8.80	0.004	4.10	0.035	1.74	0.380	1.27	0.833	0.80	2.248	0.34	8.495
8.70	0.004	4.00	0.037	1.73	0.386	1.26	0.849	0.79	2.303	0.33	8.820
8.60	0.004	3.90	0.040	1.72	0.392	1.25	0.865	0.78	2.359	0.32	9.165
8.50	0.004	3.80	0.043	1.71	0.398	1.24	0.881	0.77	2.418	0.31	9.531
8.40	0.004	3.70	0.047	1.70	0.404	1.23	0.898	0.76	2.478	0.30	9.921
8.30	0.004	3.60	0.051	1.69	0.410	1.22	0.915	0.75	2.540	0.29	10.335
8.20	0.004	3.50	0.055	1.68	0.417	1.21	0.933	0.74	2.604	0.28	10.778
8.10	0.005	3.40	0.060	1.67	0.423	1.20	0.951	0.73	2.669	0.27	11.252
8.00	0.005	3.30	0.065	1.66	0.430	1.19	0.970	0.72	2.737	0.26	11.760
7.90	0.005	3.20	0.071	1.65	0.436	1.18	0.989	0.71	2.808	0.25	12.308
7.80	0.005	3.10	0.078	1.64	0.443	1.17	1.008	0.70	2.880	0.24	12.898
7.70	0.005	3.00	0.085	1.63	0.450	1.16	1.028	0.69	2.955	0.23	13.538

(Continued)

TABLE A.26 Rigidity Coefficients for Cantilever Walls (*Continued*)

$$R_C = \frac{1}{0.4\left(h/d\right)^3 + 0.3\left(h/d\right)}$$

h/d	R_c	*h/d*	R_c	*h/d*	R_c	*h/d*	R_c	*h/d*	R_c	*h/d*	R_c
7.60	0.006	2.90	0.094	1.62	0.457	1.15	1.049	0.68	3.032	0.22	14.233
7.50	0.006	2.80	0.104	1.61	0.465	1.14	1.070	0.67	3.112	0.21	14.992
7.40	0.006	2.70	0.115	1.60	0.472	1.13	1.092	0.66	3.195	0.20	15.823
7.30	0.006	2.60	0.128	1.59	0.480	1.12	1.114	0.65	3.280	0.195	16.269
7.20	0.007	2.50	0.143	1.58	0.487	1.11	1.136	0.64	3.369	0.190	16.738
7.10	0.007	2.40	0.160	1.57	0.495	1.10	1.160	0.63	3.460	0.185	17.232
7.00	0.007	2.30	0.180	1.56	0.503	1.09	1.183	0.62	3.555	0.180	17.752
6.90	0.007	2.20	0.203	1.55	0.512	1.08	1.208	0.61	3.652	0.175	18.300
6.80	0.008	2.10	0.231	1.54	0.520	1.07	1.233	0.60	3.754	0.170	18.880
6.70	0.008	2.00	0.263	1.53	0.529	1.06	1.259	0.59	3.859	0.165	19.494
6.60	0.009	1.99	0.267	1.52	0.537	1.05	1.285	0.58	3.968	0.160	20.146
6.50	0.009	1.98	0.270	1.51	0.546	1.04	1.312	0.57	4.080	0.155	20.838
6.40	0.009	1.97	0.274	1.50	0.556	1.03	1.340	0.56	4.197	0.150	21.575
6.30	0.010	1.96	0.278	1.49	0.565	1.02	1.369	0.55	4.319	0.145	22.362
6.20	0.010	1.95	0.282	1.48	0.574	1.01	1.398	0.54	4.445	0.140	23.203
6.10	0.011	1.94	0.286	1.47	0.584	1.00	1.429	0.53	4.576	0.135	24.106
6.00	0.011	1.93	0.289	1.46	0.594	0.99	1.460	0.52	4.712	0.130	25.076
5.90	0.012	1.92	0.293	1.45	0.604	0.98	1.491	0.51	4.853	0.125	26.122
5.80	0.013	1.91	0.298	1.44	0.615	0.97	1.524	0.50	5.000	0.120	27.254
5.70	0.013	1.90	0.302	1.43	0.626	0.96	1.558	0.49	5.153	0.115	28.483
5.60	0.014	1.89	0.306	1.42	0.636	0.95	1.592	0.48	5.312	0.110	29.822
5.50	0.015	1.88	0.310	1.41	0.648	0.94	1.628	0.47	5.479	0.105	31.286
5.40	0.015	1.87	0.315	1.40	0.659	0.93	1.665	0.46	5.652	0.100	32.895
5.30	0.016	1.86	0.319	1.39	0.671	0.92	1.702				

TABLE A.27 Rigidity Coefficients for Fixed Walls

$$R_f = \frac{1}{0.1\left(\frac{h}{d}\right)^3 + 0.3\left(\frac{h}{d}\right)}$$

h/d	R_f	h/d	R_f	h/d	R_f	h/d	R_f	h/d	R_f	h/d	R_f
9.90	0.010	5.20	0.064	1.85	0.842	1.38	1.478	0.91	2.871	0.45	6.939
9.80	0.010	5.10	0.068	1.84	0.851	1.37	1.497	0.90	2.916	0.44	7.117
9.70	0.011	5.00	0.071	1.83	0.861	1.36	1.516	0.89	2.963	0.43	7.302
9.60	0.011	4.90	0.076	1.82	0.870	1.35	1.536	0.88	3.011	0.42	7.496
9.50	0.011	4.80	0.080	1.81	0.880	1.34	1.556	0.87	3.060	0.41	7.699
9.40	0.012	4.70	0.085	1.80	0.890	1.33	1.577	0.86	3.109	0.40	7.911
9.30	0.012	4.60	0.090	1.79	0.900	1.32	1.597	0.85	3.160	0.39	8.135
9.20	0.012	4.50	0.096	1.78	0.911	1.31	1.619	0.84	3.213	0.38	8.369
9.10	0.013	4.40	0.102	1.77	0.921	1.30	1.640	0.83	3.266	0.37	8.616
9.00	0.013	4.30	0.108	1.76	0.932	1.29	1.662	0.82	3.321	0.36	8.876
8.90	0.014	4.20	0.115	1.75	0.943	1.28	1.684	0.81	3.377	0.35	9.150
8.80	0.014	4.10	0.123	1.74	0.953	1.27	1.707	0.80	3.434	0.34	9.440
8.70	0.015	4.00	0.132	1.73	0.965	1.26	1.730	0.79	3.493	0.33	9.747
8.60	0.015	3.90	0.141	1.72	0.976	1.25	1.753	0.78	3.553	0.32	10.073
8.50	0.016	3.80	0.151	1.71	0.987	1.24	1.777	0.77	3.615	0.31	10.419
8.40	0.016	3.70	0.162	1.70	0.999	1.23	1.802	0.76	3.678	0.30	10.787
8.30	0.017	3.60	0.174	1.69	1.010	1.22	1.826	0.75	3.743	0.29	11.181
8.20	0.017	3.50	0.187	1.68	1.022	1.21	1.851	0.74	3.809	0.28	11.602
8.10	0.018	3.40	0.202	1.67	1.034	1.20	1.877	0.73	3.877	0.27	12.053
8.00	0.019	3.30	0.218	1.66	1.047	1.19	1.903	0.72	3.948	0.26	12.538
7.90	0.019	3.20	0.236	1.65	1.059	1.18	1.929	0.71	4.019	0.25	13.061
7.80	0.020	3.10	0.256	1.64	1.072	1.17	1.956	0.70	4.093	0.24	13.627
7.70	0.021	3.00	0.278	1.63	1.085	1.16	1.984	0.69	4.169	0.23	14.242

(*Continued*)

TABLE A.27 Rigidity Coefficients for Fixed Walls (*Continued*)

$$R_f = \frac{1}{0.1\left(h/d\right)^3 + 0.3\left(h/d\right)}$$

h/d	R_f	h/d	R_f	h/d	R_f	h/d	R_f	h/d	R_f	h/d	R_f
7.60	0.022	2.90	0.302	1.62	1.098	1.15	2.012	0.68	4.247	0.22	14.911
7.50	0.023	2.80	0.329	1.61	1.111	1.14	2.040	0.67	4.328	0.21	15.643
7.40	0.023	2.70	0.360	1.60	1.124	1.13	2.069	0.66	4.410	0.20	16.447
7.30	0.024	2.60	0.394	1.59	1.138	1.12	2.099	0.65	4.495	0.195	16.880
7.20	0.025	2.50	0.432	1.58	1.152	1.11	2.129	0.64	4.583	0.190	17.335
7.10	0.026	2.40	0.476	1.57	1.166	1.10	2.159	0.63	4.673	0.185	17.815
7.00	0.027	2.30	0.524	1.56	1.180	1.09	2.191	0.62	4.766	0.180	18.321
6.90	0.029	2.20	0.580	1.55	1.194	1.08	2.222	0.61	4.861	0.175	18.855
6.80	0.030	2.10	0.643	1.54	1.209	1.07	2.255	0.60	4.960	0.170	19.421
6.70	0.031	2.00	0.714	1.53	1.224	1.06	2.288	0.59	5.062	0.165	20.020
6.60	0.033	1.99	0.722	1.52	1.239	1.05	2.321	0.58	5.168	0.160	20.657
6.50	0.034	1.98	0.730	1.51	1.254	1.04	2.356	0.57	5.277	0.155	21.335
6.40	0.036	1.97	0.738	1.50	1.270	1.03	2.391	0.56	5.389	0.150	22.057
6.30	0.037	1.96	0.746	1.49	1.286	1.02	2.426	0.55	5.505	0.145	22.829
6.20	0.039	1.95	0.754	1.48	1.302	1.01	2.463	0.54	5.626	0.140	23.655
6.10	0.041	1.94	0.762	1.47	1.318	1.00	2.500	0.53	5.751	0.135	24.542
6.00	0.043	1.93	0.770	1.46	1.335	0.99	2.538	0.52	5.880	0.130	25.497
5.90	0.045	1.92	0.779	1.45	1.352	0.98	2.577	0.51	6.014	0.125	26.528
5.80	0.047	1.91	0.788	1.44	1.369	0.97	2.616	0.50	6.154	0.120	27.645
5.70	0.049	1.90	0.796	1.43	1.386	0.96	2.656	0.49	6.299	0.115	28.858
5.60	0.052	1.89	0.805	1.42	1.404	0.95	2.697	0.48	6.449	0.110	30.181
5.50	0.055	1.88	0.814	1.41	1.422	0.94	2.739	0.47	6.606	0.105	31.630
5.40	0.058	1.87	0.823	1.40	1.440	0.93	2.782	0.46	6.769	0.100	33.223
5.30	0.061	1.86	0.832	1.39	1.459	0.92	2.826				

TABLE A.28 Conversion Factors: U.S. Customary Units to SI Units

	Multiply		By		To obtain
Length	inches	×	25.4	=	millimeters
	feet	×	0.3048	=	meters
	yards	×	0.9144	=	meters
	miles (statute)	×	1.609	=	kilometers
Area	square inches	×	645.2	=	square millimeters
	square feet	×	0.0929	=	square meters
	square yards	×	0.8361	=	square meters
Volume	cubic inches	×	16 387	=	cubic millimeters
	cubic feet	×	0.028 32	=	cubic meters
	cubic yards	×	0.7646	=	cubic meters
	gallons (U.S. liquid)	×	0.003 785	=	cubic meters
Force	pounds	×	4.448	=	newtons
	kips	×	4448	=	newtons
Force per unit length	pounds per foot	×	14.594	=	newtons per meter
	kips per foot	×	14 594	=	newtons per meter
Load per unit volume	pounds per cubic foot	×	0.157 14	=	kilonewtons per cubic meter
Bending moment or torque	inch-pounds	×	0.1130	=	newton meters
	foot-pounds	×	1.356	=	newton meters
	inch-kips	×	113.0	=	newton meters
	foot-kips	×	1356	=	newton meters
	inch-kips	×	0.1130	=	kilonewton meters
	foot-kips	×	1.356	=	kilonewton meters
Stress, pressure, loading (force) per unit area)	pounds per sq inch	×	6895	=	pascals
	pounds per sq inch	×	6.895	=	kilopascals
	pounds per sq inch	×	0.006 895	=	megapascals
	kips per sq inch	×	6.895	=	megapascals
	pounds per sq foot	×	47.88	=	pascals
	pounds per sq foot	×	0.047 88	=	kilopascals
	kips per sq foot	×	47.88	=	kilopascals
	kips per sq foot	×	0.047 88	=	megapascals
Mass	pounds	×	0.454	=	kilograms
Mass per unit volume (density)	pounds per cubic foot	×	16.02	=	kilograms per cubic meter
	pounds per cubic yard	×	0.5933	=	kilograms per cubic meter
Moment of inertia	inches	×	416.231	=	millimeters
Mass per unit length	pounds per foot length	×	1.488	=	kilograms per meter
Mass per unit area	pounds per square foot	×	4.882	=	kilograms per square meter

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GLOSSARY

The following terms and definitions have been adapted from several references as cited at the end of this section and are used extensively throughout this book. All terms and definitions are listed alphabetically.

A

AAC Units Autoclaved aerated concrete units, fabricated from autoclaved aerated concrete, which possesses high air content—80 percent of its volume.

Absorption The weight of water a brick absorbs when immersed in either cold or boiling water for a stated length of time, expressed as a percentage of the weight of the dry unit. See ASTM C67.

Active Fault A fault determined to be active by the authority having jurisdiction from properly substantiated data (e.g., most recent mapping of active faults by the United States Geological Survey).

Adhered Attached by adhesion rather than mechanical anchorage, as in adhered veneer.

Admixtures Materials added to mortar to impart special or intended properties to it.

Adobe Soil of diatomaceous content mixed with sufficient water so that plasticity can be developed for molding into masonry units.

Aggregate Granular mineral material such as natural sand, manufactured sand, gravel, crushed stone, and air-cooled blast furnace slag.

Allowable Stress Design A method of proportioning structural members such that elastically computed stresses produced in the members by nominal loads do not exceed specified allowable stresses (also called "working stress design").

Alumina The oxide of aluminum; an important constituent of the clays, used in bricks, tiles, and refractories.

Anchor A piece or assemblage, usually metal, used to attach building parts (e.g., plates, joists, trusses, etc.) to masonry or masonry materials.

Arch A vertically curved structural member spanning openings or recesses; may also be built flat by using special masonry shapes or placed units.

Arching Action The distribution of loads in masonry over an opening. The load is usually assumed to occur in a triangular pattern above the opening, varying from a maximum at the center and zero at the supports.

Areas

Bedded Area The area of the surface of a masonry unit which is in contact with the mortar in the plane of joint.

Effective Area of Reinforcement (A_s) The cross-sectional area of reinforcement multiplied by the cosine of the angle between the reinforcement and the direction for which the effective area is to be determined.

Gross Area The total cross-sectional area of a specified section.

Net Area The gross cross-sectional area minus the area of ungrouted cores, notches, cells, and unbedded areas. Net area is the actual surface area of a cross section of masonry.

Transformed Area The equivalent area of one material to a second, based on the ratio of modulus of elasticity of the first material to that of the second.

Ashlar Masonry Masonry composed of rectangular units of burned clay or shale, or stone, generally larger in size than the brick and properly bonded, having sawed, dressed, or squared beds, and joints laid in mortar. Often the unit size varies to provide a random pattern, *random ashlar*.

Attachments Means by which components and their supports are secured or connected to the seismic force-resisting system of the structure. Such attachments include anchor bolts, welded connections, and mechanical fasteners.

B

Backfilling 1. Rough masonry built behind a facing or between two faces. 2. Filling over the extrados of an arch. 3. Brickwork in progress in spaces between structural timbers, sometimes called *brick nagging*.

Backup The part of a masonry wall behind the exterior facing.

Base Shear Total design lateral force or shear at the base.

Basic Wind Speed Three-second gust speed at 33 ft (10 m) above the ground in Exposure C (see ASCE 7-05 Sec. 6.5.6.3) as determined in accordance with ASCE 7-05 Sec. 6.5.4.

Bat A piece of brick.

Batter Recessing or sloping masonry back in successive courses, the opposite of corbel.

Bearing Plate A plate placed under a beam, girder, or truss to distribute the reaction load to the supporting medium.

Bearing Wall See wall, bearing wall.

Bed Joint The horizontal layer of the mortar on which masonry is laid.

Belt Course A narrow horizontal course of masonry, sometimes slightly projected such as windowsills, which are made continuous. Sometimes called *string course* or *sill course*.

Blocking A method of bonding two adjoining or intersecting walls not built at the same time, by means of offsets whose vertical dimensions are not less than 8 in.

Bond 1. Tying various parts of a masonry wall by lapping one unit over the other or by connecting with metal parts. 2. Patterns formed by exposed faces of masonry units. 3. Adhesion of mortar and grout to the units or reinforcements.

Adhesion Bond The adhesion between masonry units and mortar or grout.

American Bond A form of bond in which every sixth course is a header course and the intervening courses are the stretcher courses.

Basketweave Bond A form of bond in which module groups of brick laid at right angles to those adjacent.

Dutch Bond A bond having courses made up alternately of headers and stretchers (same as English cross bond).

Flemish Bond A bond consisting of headers and stretchers alternating in every course, so laid as always to break joints, each header being placed in the middle of the stretchers in courses above and below.

Gothic Bond Another name for Flemish bond reflecting its use in northern Europe throughout the Middle Ages.

Grout Bond The adhesion to, and/or the interlocking of grout with the masonry units and the reinforcement.

Header Bond A bond consisting entirely of headers. A *header course* is a course of bricks all of which are headers.

Herring Bone Bond In masonry, the arrangement of bricks in a course in a zigzag fashion, with the end of one brick laid at right angles against the side of a second brick.

Mechanical Bond Tying masonry units together with metal ties or reinforcing steel or keys.

Mortar Bond The adhesion of mortar to masonry units.

Reinforcing Bond The adhesion between steel reinforcement and mortar or grout.

Running Bond Lapping of units in successive courses so that the vertical head joints lap. Placing vertical mortar joints centered over the unit below is called center bond, or half bond, while lapping 1/3 or ¼ is called *third bond* or *quarter bond*.

Stacked Bond A bonding pattern where no unit overlaps either the one above or below, all head joints form a continuous vertical line. Also called plumb joint bond, straight stack, jack bond, jack on jack, and checkerboard bond.

Stretcher Bond A bond pattern consisting entirely of stretchers.

Yorkshire Bond A variation of Flemish bond with two stretches between every header; also called *flying bond* or *monk bond.*

Bond Beam 1. A horizontal reinforced masonry beam, serving as an integral part of the wall. 2. Course or courses of a masonry wall grouted and usually reinforced in the horizontal direction. Serves a horizontal tie of wall, bearing course for structural members, or as a flexural member itself.

Bond Course The course consisting of units which overlap more than one wythe of masonry.

Bonder A bonding unit. See header.

Boundary Members Portions along the wall and diaphragm edges strengthened by longitudinal and transverse reinforcement. Boundary members include chords and drag struts at diaphragm and shear wall perimeters, interior openings, discontinuities, and reentrant corners.

Breaking Joints Any arrangements of masonry units which prevents continuous vertical joints from occurring in adjacent courses.

Brick A solid unit of clay or shale, formed into a rectangular prism while plastic and burned or fired in a kiln. Similarly shaped units made of portland cement mixes are generally called concrete brick.

Acid-Resistant Brick Brick suitable for use in contact with chemicals, usually in conjunction with acid-resistant mortars.

Adobe Brick Large roughly molded, sun-dried clay brick of varying size.

Angle Brick Any brick shaped to an oblique angle to fit a salient corner.

Arch Brick A wedge-shaped brick for special use in an arch.

Building Brick Brick for building purposes not specially treated for color or texture. Formerly called *common brick*. (See ASTM Specification C62). Also, a poor quality brick, often with surface blemishes which is unsuitable for facing, but is still adequate for the interior of a wall. (Converse facing brick.)

Bull Nose Brick A brick having an especially molded semicircular edge.

Clinker Brick A very hard-burned brick whose shape is distorted or bloated due to nearly complete vitrification. Adds texture and color to a wall.

Common Brick See Building Brick.

Dry-Press Brick Bricks formed in molds under high pressure from relatively dry clay (5 to 7 percent moisture content).

Economy Brick Brick whose dimensions are $4 \times 4 \times 8$ in.

Engineered Brick Brick whose nominal dimensions are $4 \times 3.2 \times 8$ in. Also dense, uniform, hard, machine-made brick which lacks porosity and used for engineering construction.

Facing Brick Brick made especially for facing purposes, often treated to produce surface texture. They are made of selected clays, or treated to produce desired color. See ASTM Specification C216.

Fire Brick Brick of refractory ceramic material, which will resist high temperatures.

Floor Brick Smooth dense brick, highly resistant to abrasion, used as finished floor surfaces. See ASTM Specification C410.

Gauged Brick 1. Brick which has been ground or otherwise produced to accurate dimensions. 2. A tapered arch brick.

Glazed Brick Brick with one end and one face glazed, made by coating the surfaces with a salt before firing. Became very popular in the late 1900s because of its easy-cleaning and light reflecting qualities.

Green Brick Brick before it is fired.

Hollow Brick A masonry unit of clay or shale whose net cross-sectional area in any plane parallel to the bearing surface is not less than 60 percent of the gross cross-sectional area measured in the same plane. See ASTM Specification C652.

Jumbo Brick A brick larger in size than the "standard." Some producers use this term to describe over-size brick of specific dimensions manufactured by them.

Norman Brick A brick whose nominal dimensions are 4 × 2 2/3 × 12 in.

Paving Brick Vitrified brick especially suitable for use in pavements where resistance to abrasion is important. See ASTM Specification C7.

Perforated Brick A brick with vertical holes passing through the thickness.

Roman Brick Brick whose nominal dimensions are 4 × 2 × 12 in.

Salmon Brick Generic term for under-burned bricks, which are more porous slightly larger, and lighter colored than hard-burned brick, usually pinkish-orange color.

SCR Brick Brick whose nominal dimensions are 6 × 2 2/3 × 12 in.

Sewer Brick Low absorption, abrasion-resistant brick intended for use in drainage structures.

Soft-Mud Brick Brick prepared molding relatively wet clay (20–30 percent moisture), often a hand process. When insides of molds are sanded to prevent or to avoid the sticking of clay, the product is called *sand-struck brick.* When molds are wetted to prevent sticking, the result is the product is *water-struck* brick.

Stiff-Mud Brick Brick produced by extruding stiff but plastic clay (12–15 percent moisture) through a die.

Brick and Brick A method of laying brick so that units touch each other with only enough mortar to fill surface irregularities.

Brick Grade Designation for the durability of the unit expressed as SW for severe weathering, MW for moderate weathering, or NW for negligible weathering. See ASTM Specifications C216, C62, and C652.

Brick Type Designation for facing brick which controls tolerance, clippage and distortion. Expressed as FBS, FBX and FBA for solid bricks, and HBS, HBX, HBA and HBB for hollow bricks. See ASTM Specifications C216 and C652.

Building Any structure whose intended use includes shelter of human occupants.

Building, Enclosed A building that does not comply with the requirements for open or partially enclosed buildings.

Building Envelope Cladding, roofing, exterior walls, glazing, door assemblies, window assemblies, skylight assemblies, and other components enclosing the building.

Building, Simple Diaphragm A building in which both windward and leeward wind loads are transmitted through floor and roof diaphragms to the same vertical MWFRS (e.g., no structural separations).

Buttering Placing mortar on a masonry unit with a trowel.

C

Cantilevered Column Systems A seismic force resisting system in which lateral forces are resisted entirely by columns acting as cantilevers from the base.

Capacity Insulation The ability of masonry to store heat as a result of its mass, density, and specific heat.

Carbonation A process of chemical weathering whereby minerals that contain sodium oxide, calcium oxide, potassium oxide, or other basic oxides are changed to carbonates by the action of carbonic acid derived form atmospheric carbon dioxide and water.

Cavity An unfilled space.

C/B Ratio The ratio of the weight of water absorbed by a masonry unit during immersion in cold water to the weight absorbed during immersion in boiling water. An indication of the probable resistance of brick to freezing and thawing. Also called *saturation coefficient*.

Cell A void space having a gross cross-sectional area greater than 1½ square inches.

Cement A binding material used for concrete, mortar and grout. The cements provided for use in masonry construction are Portland cement, masonry cement and mortar cement.

Centering Temporary formwork for the support of masonry arches or lintels during construction. Also called *center(s)*.

Characteristic Earthquake An earthquake assessed for an active fault having a magnitude equal to the best estimate of the maximum magnitude capable of occurring on the fault, but not less than largest magnitude that has occurred historically on the fault.

Chase A continuous recess built into a wall to receive pipes, ducts, etc.

Clay A natural mineral aggregate consisting essentially of hydrous aluminum silicate; it is plastic when sufficiently wetted, rigid when dried and vitrified when fired to a sufficiently high temperature.

> **Fire Clay** Clay that is formed at greater depths than surface clay or shale. It contains only 2 to 10 percent oxides.
>
> **Shale** A metamorphic form of clay, hardened and layered under natural geologic conditions.
>
> **Surface Clay** Clay that occurs quite close to the surface of the earth. Being easily accessible, they are easily mined and least expensive. It contains high oxide content, ranging from 10 to 25 percent.

Cleanout An opening to the bottom of a grout space of sufficient size and spacing to allow the removal of debris.

Clip A portion of brick cut to length.

Closer The last masonry unit laid in a course. It may be whole or a portion of a unit.

Closure Supplementary or short length units used at corners used at corners or jambs to maintain bond patterns.

Coefficient of Creep The ratio of deformation due to sustained load to elastic deformation. See creep.

Collar Joint The vertical, longitudinal, mortar, or grouted joint between wythes.

Collector A diaphragm or shear wall element parallel to the applied force that collects and transfers shear forces to vertical-force-resisting elements or distributes within a diaphragm or a shear wall.

Column An isolated compressive load carrying vertical member whose horizontal dimensions measured at right angles to the thickness does not exceed three times its thickness and whose height is at least three times its thickness.

Column, Reinforced A vertical structural member in which both the steel and masonry resist the imposed compressive load.

Column, Unreinforced A vertical structural member which carries compressive load without any steel reinforcement.

Component A part of element of an architectural, electrical, mechanical, or structural system.

Component, Equipment A mechanical or electrical component or element that is part of mechanical and/or electrical system within or without a building system,

Component, Flexible Component, including its attachments, having a fundamental period is greater than 0.06 second.

Component, Rigid Component, including its attachments, having a fundamental period less than or equal to 0.06 second.

Component Support Those structural members or assemblies or members, including braces, frames, struts, and attachments that transmit all loads and forces between systems, components, or elements and the structure.

Components and Cladding Elements of the building envelope that do not qualify as part of the MWFRS.

Composite Action Transfer of stress between components of a member designed so that the combined components act together in resisting the imposed load.

Composite Masonry Multi-component masonry members acting with composite action.

Concrete Masonry Unit In masonry, precast, hollow block, or solid brick of concrete used in the construction of buildings.

Bond Beam Block A hollow unit with web portions depressed 1¼ inches or more to form a continuous channel, or channels, for reinforcing steel and grout. U-blocks are sometimes used to form Bond Beams, especially as over openings.

Bullnose Block A concrete masonry unit that has one or more rounded exterior corners.

Channel Block A hollow unit with web portions depressed less than 1¼ inches to form a continuous channel for reinforcing steel and grout.

Concrete Block A hollow concrete masonry unit made from portland cement and suitable aggregates such as sand, gravely crushed stone, bituminous or anthracite cinders, burned clay or shale, pumice, volcanic scoria, air-cooled or expanded blast furnace slag, with or without the inclusion of other materials.

Lintel Block A masonry unit consisting of one core with one side open, usually placed with the open side up to form a continuous beam over openings.

Pilaster Block Concrete masonry units designed for use in construction of plain or reinforced concrete masonry pilasters and columns.

Slump Block Concrete masonry units produced so they “slump” or sag in irregular fashion before they harden.

Solid Block A concrete masonry unit whose net cross-sectional area in every plane parallel to the bearing surface is 75 percent or more of its gross cross-sectional area measure in the same plane.

U-Block See Lintel block.

Coping The material or masonry units forming a cap or finish on top of a wall, pier, pilaster, chimney, etc., sometimes projected out from the wall to provide decorative as well as protective features. It protects masonry below from penetration of water from above.

Corbel A shelf or ledge formed by projecting successive courses of masonry out from the face of the wall.

Coupling Beam A beam that is used to connect adjacent concrete wall elements to make them act together as a unit to resist lateral forces.

Course One of the continuous horizontal layer of units, bonded with mortar in masonry.

Creep Deformation due to sustained load. Deformation creep is a property of concrete and other materials by which they deform under sustained loads at stresses well below those in the elastic range.

Culls Masonry units that do not meet the standards or specifications, and have been rejected.

D

Damp Course A course or layer of impervious material, which prevents capillary entrance of moisture from the ground to a lower course.

Dampproofing Prevention of moisture penetration by capillary action.

Deformability The ratio of the ultimate deformation to the limit deformation.

High-Deformability Element An element whose deformability is not less than 3.5 where subjected to four fully reversed cycles at the limit deformation.

Limited-Deformability Elements An element that is neither a low-deformability or high-deformability element.

Low-Deformability Element An element whose deformability is 1.5 or less.

Deformation

Limit Deformation Two times the initial deformation that occurs as a load equal to 40 percent of the maximum strength.

Ultimate Deformation The deformation at which failure occurs and that shall be deemed to occur if the sustainable load reduces to 80 percent or less of the maximum strength.

Design Earthquake The earthquake effects that are 2/3 of maximum considered earthquake (MCE).

Design Earthquake Ground Motion The earthquake ground motions that are two-thirds of the corresponding MCE ground motions.

Design Strength The product of the nominal strength and the resistance factor.

Designated Seismic Systems The seismic force-resting system and those architectural, electrical, and mechanical systems or their components that require design in accordance with Chapter 13 and for which the component importance factor, I_p is greater than 1.0.

Diaphragm Roof, floor, or other membrane or bracing system acting to transfer the lateral forces to the vertical resisting elements.

Diaphragm, Flexible A diaphragm is flexible for the purpose of distribution of story shear and torsional moment when the lateral deformation of the diaphragm is more than two times the average story drift of the associated story, determined by comparing the calculated maximum in-plane deflection of the diaphragm itself under lateral force with the story drift of the adjoining vertical lateral force resisting elements under equivalent tributary force.

Diaphragm, Rigid A diaphragm that is not flexible.

Diaphragm Boundary A location where shear is transferred into or out of the diaphragm element. Transfer is either to a boundary element or to another force-resisting element.

Diaphragm Chord A diaphragm boundary element perpendicular to the applied load that is assumed to take axial stresses due to the diaphragm movement.

Dimensions

Actual Dimensions The measured dimensions of a designated item, for example, a designated masonry unit or wall, as used in the structure. The actual dimension shall not vary from the specified dimension by more than the amount allowed in the appropriate standard of quality in MSJC Standard TMS 602/ACI 530.1/ASCE 6.

Modular Dimensional Standards Dimensional standards approved by the American Standards Association for all building material and equipment, based upon a common unit or measure of four inches in case of brick masonry and eight inches in case of concrete block masonry, known as the module. This module is used as a basis for the grid, which is essential for dimensional coordination of two or more different materials.

Nominal Dimension A dimension greater than a specified masonry unit dimension by the thickness of a mortar joint. Nominal dimension is measured from center-to-center of adjacent joints. A hollow concrete block $8 \times 8 \times 16$ in. (nominal dimensions) has actual dimensions as 7 5/8 × 7 5/8 × 15 5/8 in. to allow for 3/8 in.-wide joints.

Specified Dimensions The dimensions specified for the manufacture or construction of masonry, masonry units, joints or any other component of a structure. Unless otherwise stated, all calculations shall be made using or based on specified dimensions.

For example, width b of an 8 in. (nominal) wide beam will be taken as 7 5/8 in. (actual width) for calculations. Similarly, the area of a 24×24 in. (nominal) column will be taken as 23 5/8 × 23 5/8 in. (= 558 in.^{2}).

Drag Strut (Collector, Tie, Diaphragm Strut) A diaphragm or shear wall boundary element parallel to the applied load that collects and transfers diaphragm shear forces to the vertical force-resisting elements, or distributes forces to the diaphragm or shear wall.

Drying Shrinkage Also called *shrinkage*. Volume change during hardening and curing of concrete. The primary cause of shrinkage is loss of adsorbed and capillary water.

E

Eccentricity The normal distance between the centroidal axis of a member and the parallel resultant load.

Effective Height The height of a member to be assumed for computing slenderness ratio of vertical element, e.g., a column or a wall.

Effective Thickness The thickness of a member to be used for computing slenderness ratio.

Efflorescence A powder or stain found on the surface of the masonry, resulting from deposition of water-soluble salts.

Emperical Design Design based on the physical limitations learned from experience or observations gained through experience, without structural analysis.

Enclosure An interior space surrounded by walls.

Engineered Brick Masonry (EBM) Masonry in which design is based on a rational structural analysis.

Equivqlent Solid Thickness Thickness of a partially grouted wall used to calculate axial stress in the wall due to gravity loads.

Exfoliation Peeling or scaling of stone or clay brick surfaces caused by chemical or physical weathering.

Extrados The exterior curve of an arch or a vault.

F

Face (1) The exposed surface of a wall or masonry unit. (2) The surface of a unit designed to be exposed in the finished masonry.

Face Shell The sidewall of a hollow concrete masonry unit.

Face Shell Bedding Application of mortar to only the horizontal surface of the face shells of hollow masonry units and in the head joints to a depth equal to the thickness of the face shell.

Facing Any material, forming a part of a wall, used as a finished surface.

Factored Load The product of a service load and the load factor.

Field The expanse of wall between openings, corners, etc., principally composed of stretchers.

Filter Block A hollow, vitrified clay masonry unit, sometimes salt-glazed, designed for trickling filter floors in sewage disposal plants. See ASTM Specification C159.

Fireproofing Any material or combination protecting structural members to increase their fire resistance.

Flagging 1. A collective term for flagstones. 2. A surface paved with flagstones. 3. The process of setting flagstones.

Flagstones A type of stones that easily splits into flags or slabs; also a term applied to irregular pieces of such stone split into slabs from 1 to 3 in. thick, and used for walks, patios, etc.

Flashing 1. A thin impervious material placed in mortar joints and through air spaces in masonry to prevent water penetration and/or provide water drainage. 2. Manufacturing method to produce specific color tones.

Flat Arch A lintel made from tapering bricks, held in place by the shape and friction. Also called a *straight arch*, a *jack arch* or *French arch*.

Frame

Concentrically Braced Frame (CBF) A braced frame in which the members are subjected primarily to axial forces. CBFs are categorized as ordinary concentrically brace frames (OCBF) or special concentrically braced frames (SCBF).

Eccentrically Braced Frame (EBF) A diagonally braced frame in which at least one end of each brace frames into a beam a short distance from a beam-column or from another diagonal brace.

Frog A depression in the bed surface of brick. Sometimes called a *panel*.

Furring A method of finishing the interior face of a masonry wall to provide space for insulation, prevent moisture transmittance, or to provide a level surface for finishing.

G

Glass Unit Masonry Nonload-bearing masonry composed of glass units bonded by mortar.

Glazed Structural Unit, GSU A solid or hollow unit with a surface of applied smooth glossy nature, e.g., a tile with a fired glaze finish.

Grade (of Bricks)

Grade MW (Moderate Weathering) Grade of brick for moderate resistance to freezing.

Grade NW (Negligible Weathering) Brick intended for use as back-up or interior masonry.

Grade SW (Severe Weathering) ASTM grade of brick intended for use where high resistance to freeze-thaw is desired.

Grout A high-slump (i.e., fluid) mixture of portland cement, aggregate, and water which is poured into hollow cells, or joints of the masonry walls, to encase steel and bond units together. Derived from Swedish word "groot" meaning "porridge," which indicates some indication of its consistency.

Grout Lift An increment or grout height within the total pour; a pour may consist of one or more grout lifts.

Grout Pour The total height of masonry wall to be grouted before the erection of additional masonry. A grout pour will consist of one or more grout lifts.

Grouted Masonry

Grouted Hollow Unit Masonry The form of grouted masonry construction in which certain designated cells of hollow units are continuously filled with grout.

Grouted Multiwythe Masonry The form of grouted masonry construction in which the space between the wythes is solidly or periodically filled with grout.

Grouting, High-Lift A technique of grouting; grouting is placed in pours over 5 ft, up to story height and cleanout holes are required at the bottom of each grout space containing reinforcement.

Grouting, Low-Lift A technique of grouting; the wall is constructed to pour heights up to 5 ft and no cleanouts are needed.

H

Header 1. A masonry unit which overlaps two or more adjacent wythes of masonry to tie them together; often called *bonder*. 2. Refers to a brick laid on its wide face with the end showing in the plane of the wall with larger dimension or the width parallel to the wall face (and thickness perpendicular to the wall).

Blind Header A concealed brick header in the interior of a wall, not showing on the faces.

Clipped Header A bat placed to look like a header for purposes of establishing a pattern.

Flare Header A header of darker color than the field of the wall.

Hollow Masonry Unit A masonry unit whose net cross-sectional area in any plane parallel to the bearing surface is less than 75 percent of its cross-sectional area measured in the same plane.

I

Importance Factor A factor assigned to each structure according to its occupancy category as specified in ASCE 7-05 Standard.

Initial Rate of Absorption (IRA) The weight of water absorbed expressed in grams per 30 sq. in. of contact surface when a brick is partially immersed for one minute.

J

Joints In building construction, the space or opening between two or more adjoining surfaces.

Bed Joint Horizontal layer of mortar on which a masonry unit is laid.

Collar Joint Vertical longitudinal joint between wythes of masonry or between masonry wythe and backing.

Control Joint[1] (1) In concrete masonry, a continuous joint or plane to accommodate unit shrinkage; may contain mortar or grout. Typically provided in concrete masonry walls for the full height, to allow for contraction due to permanent drying shrinkage.

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[1]It should be noted that terms *control joint* and *expansion joint* are not interchangeable. The two types of joints are completely different in function and configuration.

(2) In building construction, a formed sawed, tooled, or assembled joint acting to regulate the location and degree of cracking and separation resulting from the dimensional change of different elements of a structure. (3) In concrete, concrete masonry, stucco, or coating systems; a formed, sawed, or assembled joint acting to regulate the location of cracking, separation, and distress resulting from dimension or positional change.

Expansion Joint (1) A continuous joint or plane to accommodate expansion, contraction, and differential movement; does not contain mortar, grout, reinforcement, or other hard materials. (2) In building construction, a structural separation between building elements that allows independent movement without damage to the assembly. (3) Discontinuity between two constructed elements or components, allowing for differential movement (such as expansion) between them without damage. Typically provided in brick walls to allow expansion due to reabsorption of moisture. Expansion joints do not transfer stress; consequently, they must be located at points where both the shear and moment are expected to remain zero.

Head Joint Vertical transverse mortar joint placed between masonry units within the wythe at the time the masonry units are laid.

Mortar Joint In mortared masonry construction, the joints between units that are filled with mortar.

Movement Joint In building construction, a joint designed to accommodate movement of adjacent elements.

Raked Joint A mortar joint where ¼ to ½ in. of mortar is removed from the outside surface of the joint.

Saddle Joint A vertical joint along which the stone is lapped on either side to rise above the level of the wash on a coping or sill, thus diverting water from the joint.

Shoved Joint Vertical joint filled by shoving a unit against the next unit when it is being laid in a bed of mortar.

Shushed Joint Head or collar joint constructed by "throwing" mortar in with the edge of a trowel.

Struck Joint A joint from which excess mortar has been removed by a stroke of the trowel, leaving an approximately flush joint.

Tooled Joint A mortar joint between two masonry units manually shaped or compressed with a jointing tool such as a concave or V-notched jointer.

Joint Reinforcement Metal bars or wires, usually prefabricated, to be placed in mortar bed joints.

Jointing The finishing of joints between courses of masonry units before the mortar has hardened.

K

Keystone The wedge-shaped piece at the top of an arch, which is regarded as the most important member because it binds, or locks, all the other members together.

Kiln A furnace or oven or heated enclosure used for burning brick or other dry material.

Kiln Run Brick from one kiln which has not been sorted or graded for size or color variation.

King Closer A brick cut diagonally to have one 2-in. end and one full width end.

L

Ladder Bar A prefabricated reinforcement designed for embedment in the horizontal mortar joints of masonry. Parallel deformed side rods connected in a single plane, by cross wires, thus forming a ladder design.

Lateral Support Means whereby walls are braced either vertically or horizontally by columns, pilasters, cross-walls, beams, floors, roofs, etc.

Lift See grout lift.

Limit State A condition beyond which a structure or member becomes unsuitable or unfit for service and is judged no longer useful for its intended function (serviceability state) or to be unsafe (strength limit state).

Lintel A beam placed over an opening in a wall to carry the superimposed loads and the masonry above the opening.

Load Factors Factors used as multipliers to service loads to obtain factored loads for use in limit states design (or load and resistance design, LRFD).

Load and Resistance Factor Design (LRFD) A method of proportioning structural members and their connections using load and resistance factor factors such that no applicable limit state is exceeded when the structure is designed to appropriate load combinations.

Loads, Factored Loads obtained by multiplying service loads with code-specified load factors, for use in limit states design (or load and resistance factor design).

Loads, Service Loads that are specified in design codes for use in allowable stress design (ASD).

Loads, Ultimate Loads used for structural design performed according to limit states design (or load and resistance design) principles. Often used synonymously with *factored loads.*

M

Main Wind-Force Resisting System (MWFRS) An assemblage of structural elements assigned to provide support and stability for the overall structure. The system generally receives wind loading from more than one surface.

Masonry A built-up construction or combination of building units or materials of clay, shale, concrete glass, gypsum, stone, or other units bonded together with or without mortar or grout or other accepted method of joining (2009 IBC).

Masonry Cement A mill-mixed cementitious material to which sand and water must be added for use as binder in concrete and masonry construction. See also mortar cement.

Masonry Unit Natural or manufactured building units of burned clay, concrete, stone, glass, gypsum, or other similar building units, conforming to the requirements specified in applicable building codes, or combination thereof, made to be bonded together by a cementitious agent.

Modular Masonry Unit One whose nominal dimensions are based on the 4-in. module.

Hollow-Masonry Unit A masonry whose net cross-sectional area in every plane parallel to the bearing surface is less than 75 percent or more of the gross cross-sectional area in the same plane.

Solid-Masonry Unit A masonry unit whose net cross-sectional area in every plane parallel to the bearing surface is 75 percent or more of the gross cross-sectional area in the same plane.

Maximum Considered Earthquake The most severe earthquake effects considered in ASCE 7-05 Standard for design purposes.

Mold The box or container into which clay or brick earth is pushed to produce a regular shape.

Mortar A plastic mixture of cementitious materials, fine aggregate, and water. See ASTM Specifications C270, C476, or BIA M1-72.

Cement Mortar Mortar made from Portland cement, sand, and water, instead of or in addition to lime. (The term is used to distinguish from lime mortar).

Fat Mortar Mortar containing a high percentage of cementitious components. It is sticky mortar that adheres to a trowel.

High Bond Mortar Mortar which develops higher bond strengths with masonry units than normally developed with conventional mortar.

Lean Mortar Mortar which is deficient in cementitious components. It is usually harsh and difficult to spread.

Lime Mortar Mortar made from slaked lime. Lime itself is naturally occurring as limestone and chalk. It is burnt in a kiln to produce quicklime which is then mixed with water to produce slaked lime.

Mortar Cement Cement manufactured in compliance with ASTM C270. See also masonry cement.

N

Net Cross-Sectional Area The gross cross-sectional area of a unit minus the area of cores or cellular spaces.

Nominal Dimension See dimensions, nominal.

Nominal Loads The magnitudes of the loads specified in ASCE 7-05 Standard for dead, live, soil, wind, snow, rain, flood, and earthquake.

Nominal Strength The capacity of a structure or member to resist the effects of loads, as determined by computations using specified material strengths and dimensions and formulas derived from accepted principles of structural mechanics or by field tests or laboratory tests of scaled models, allowing for modeling effects and differences between laboratory and field conditions.

Noncombustible 1. Not combustible. 2. Any material that will neither ignite nor actively support combustion in air at a temperature of 1200°F when exposed to fire.

O

Occupancy The purpose for which a building or other structure, or part thereof, is used or intended to be used.

Other Structures Structures, other than buildings, for which loads are specified in this standard.

Overhand Work Laying brick from inside a wall rather than exterior side of the wall by men standing on a floor or on a scaffold.

P

P-Delta Effect The second order effect on shears and moments of frame members induced by axial loads on a laterally displaced building frame.

Pargeting The process of applying a coat of cement mortar to masonry. Often spelled and/or pronounced parging.

Partition A partition wall, one story or less in height.

Paver A paving stone, brick, or concrete masonry unit.

Permeability Property of allowing passage of fluids.

Pick and Dip A method of laying brick whereby the bricklayer picks up a brick with one hand, and with the other hand mortar on a trowel to lay the brick. Sometimes called the *Eastern* or *New England* method.

Pier An isolated column of masonry.

Pilaster A wall portion projecting from either or both wall faces and serving as a vertical column and/or beam.

Pilaster Block Concrete masonry units designed for use in construction of plain or reinforced concrete masonry pilasters and columns.

Pointing Troweling mortar into joints after masonry units are laid.

> **Repointing** Filling in cut-out or defective mortar joints in masonry with fresh mortar.
>
> **Tuckpointing** Decorative method of pointing masonry with a surface mortar that is different from the bedding mortar.

Pointing Trowel A small hand instrument used by stone masons or bricklayers for pointing up joints.

Prefabricated Masonry Masonry fabricated at a location other than its final location in the structure. Also known as preassembled, penalized, or sectionalized masonry.

Prism An assemblage of masonry units and mortar with or without grout used as a test specimen for determining properties of the masonry.

Pour See grout pour.

Q

Quality Assurance The administrative and procedural requirements established by the contract documents to assure that constructed masonry is in compliance with the contract documents.

Quoins Projecting bricks at the corner of a building shaped to look like projecting stone features.

R

Racking A method entailing stepping back successive courses of masonry.

Random Rubble Masonry wall built of unsquared or rudely squared stones, irregular in size and shape.

Random Rubble Dry Masonry A form of construction in which stones of various sizes are randomly stacked on top of each other to build a wall without the use of any mortar.

Reinforced Masonry A form of masonry construction in which reinforcement acting with the masonry is used to resist forces and is designed according to the principles of structural mechanics. Reinforced Grouted Masonry and Reinforced Hollow Unit Masonry are subheads that are sometimes used in building codes.

> **Reinforced Filled Masonry** A type of construction made with a single wythe of hollow masonry units, reinforced vertically and horizontally with deformed steel reinforcement, and all cores and voids are filled solidly with grout after the wall is laid.
>
> **Reinforced Grouted Masonry** A type of masonry construction made with two wythes of masonry units in which the collar joint between the two wythes is reinforced and filled solidly with concrete grout. The grout may be placed as the work progresses or after the masonry units are placed.
>
> **Reinforced Hollow Masonry** A type of construction made with a single wythe of hollow masonry units, reinforced vertically and horizontally with steel bars at certain intervals. Cores and voids containing steel bars are grouted as the work progresses.

Relieving Arch An arch constructed in the face of a wall rather than over an opening, typically used to transfer loads to piers within a wall.

Resistance Factor A factor that accounts for deviations of the actual strength from the nominal strength and the manner and consequences of failure. Also called strength reduction factor.

Rowlock Bricks laid on edge; a *header* turned 90º in the plane of the wall.

Rubble Rough broken stones or bricks used to fill courses of walls or for other filling; also, rough broken stone direct from the quarry.

S

Sailor A brick laid on edge with its wide face vertical

SCR Masonry Process A construction aid providing greater efficiency, better workmanship, and increased production in masonry construction. It utilizes story poles, marked lines, and adjustable scaffoldings.

Shale Clay which has been subjected to high pressures until it has hardened.

Shell The outer portion of a hollow masonry unit as placed in masonry.

Shrinkage See drying shrinkage.

Slenderness Ratio Ratio of the effective height of a member to its effective thickness.

Soffit The underside of a beam, lintel, or an arch.

Soldier A brick laid on end with its narrow face vertical.

Solid Masonry Unit A unit whose net cross sectional area in every plane parallel to the bearing surface is 75 percent or more of its gross cross-sectional area measured in the same plane.

Spandrel That part of a panel wall above the top of a window in one story and below the sill of the window in the story above.

Specific Stiffness Ratio of modulus of elasticity of a material to its unit weight.

Specific Strength Ratio of tensile strength of a material to its unit weight.

Straight Arch An arch made of cut bricks which has no perceptible rise in the center. Also called *jack arch* or *flat arch*, or *French arch*.

Strength Design A method of proportioning structural members such that the calculated member forces based on factored loads do not exceed the member design strength (also called load and resistance factor design, LRFD). The term "strength design" is used in the design of masonry and concrete elements.

Strength, Nominal The capacity of a structure or member to resist the effects of loads as determined from calculations using specified material strengths and dimensions, and formulas derived from accepted principles of structural mechanics or from field or laboratory tests of scaled models, allowing for modeling effects and differences between laboratory and field conditions.

Strength, Required Strength of a member or connection required to resist factored loads or related internal forces and moments in such combinations as stipulated by pertinent code provisions.

Stretcher A masonry unit laid with its largest dimension horizontal and its narrow face parallel to the wall face. A *rowlock stretcher* is a brick laid on its narrow face, with wide face parallel to the wall.

T

Temper To moisten and mix clay, plaster, or mortar to a proper consistency.

Terra Cotta A clay product used for ornamental work on the exterior of buildings; also used extensively in making vases and for decorations on statuettes. It is made of hard-baked clay in variable colors with a fine glazed surface.

Tie Any unit of material, usually a mechanical metal fastener, which connects masonry to masonry or to other materials.

Wall Tie A bonder or metal piece which connects wythes of masonry to each other or to other materials.

Wall Tie, Cavity A rigid, corrosion resistant metal tie which bonds two wythes of a cavity wall. It is usually steel, 3/16 in. in diameter, and formed in a "Z" shape or a rectangle.

Wall Tie, Veneer A strip or piece of metal used to tie a facing veneer to the backing.

Toothing Constructing the temporary end of a wall with the end stretchers of every alternate course projecting. Projecting units are called toothers.

Tuckpointing See pointing.

U

Ultimate Loads See loads, ultimate.

Unreinforced Masonry

Green Masonry In masonry, a molded clay unit before it has been burned in preparation for building purposes, an uncured concrete masonry unit.

Stone Masonry A form of construction with natural or cast stones in which the units are laid and set in mortar with all joints filled.

V

Veneer A single wythe of masonry that provides the exterior finish of a wall system and transfers out-of-plane load directly to a backing, but not considered as contributing to the structural strength the wall system.

Vitrification The condition resulting when kiln temperatures are sufficient to fuse grains and close pores of a clay product, making the mass impervious.

Voussoir Arch An arch built with wedge-shaped stones. Usually the stones are deeper than their width on the surface.

W

Wall A vertical member of a structure whose horizontal dimension measured at right angles to thickness exceeds three times its thickness.

Area Wall 1. The masonry surrounding or party surrounding an area. 2. The retaining wall around basement windows below grade.

Bonded Wall A masonry wall in which two or more wythes are bonded to act as a structural unit.

Cavity Wall A wall built of masonry units so arranged as to provide a continuous air space within the wall (with or without insulating material), and in which the inner and outer wythes of the wall are tied together with metal ties.

Composite Wall A multiple-wythe wall in which at least one of the wythes is dissimilar to the other wythe or wythes with respect to type or grade of masonry unit or mortar.

Curtain Wall An exterior nonload-bearing wall not wholly supported at each story. Such walls may be anchored to columns, spandrel beams, floors or bearing walls, but not necessarily built between structural elements.

Dwarf Wall A wall or partition which does not extend to the ceiling.

Enclosure Wall An exterior nonbearing wall in skeleton frame construction. It is anchored to columns, piers or floors, but not necessarily built between columns or piers nor wholly supported at each story.

Exterior Wall Any outside wall or vertical enclosure of a building other than a party wall.

Faced Wall A composite wall in which the masonry facing and backing are so bonded as to exert a common reaction under load.

Fire Division Wall Any wall which subdivides a building so as to resist the spread of fire. It is not necessarily continuous through all stories to and above the roof.

Fire Wall Any wall which subdivides a building to resist the spread of fire and which extends continuously from the foundation through the roof.

Foundation Wall The portion of a load-bearing wall below the level of the adjacent grade, or below first floor beams or joists.

Hollow Wall A wall built of masonry units arranged to provide an air space within the wall. The separated facing and backing are bonded together with masonry units.

Insulated Cavity Wall See *SCR Insulated Cavity Wall*.

Load-bearing Wall See wall, load-bearing.

Nonload-bearing Wall See wall, nonload-bearing.

Panel Wall An exterior, nonload-bearing wall wholly supported at each story.

Parapet Wall That part of any wall entirely above the roof line.

Party Wall A wall used for joint service by adjoining buildings.

Perforated Wall One which contains a considerable number of relatively small openings. Often called *pierced wall or screen wall*.

SCR Insulated Cavity Wall Any cavity wall containing insulation which meets rigid criteria established by the Structural Clay Institute (BIA).

Shear Wall A wall which resists horizontal forces applied in the plane of the wall. See Chapt. 7 for the many different types of shear wall designs (unreinforced, reinforced, prestressed and AAC shear walls).

Single-Wythe Wall A wall containing only one masonry unit in wall thickness.

Solid Masonry Wall A wall built of solid masonry units, laid contiguously, with joints between units completely filled with mortar or grout.

Spandrel Wall The part of a curtain wall above the top of a window in one story and below the sill of the window in the story above.

Veneered Wall A wall having a facing of masonry units or other weather-resisting non-combustible materials securely attached to the backing, but not so bonded as to intentionally exert common action under load (i.e., it is not a *composite wall*).

Wall, load-bearing A wall that supports gravity load from floor or roof in addition to its own weight. Often, the definition of a bearing wall is code-specific for design purposes.

Wall, nonload-bearing Any wall that is not a load-bearing wall.

Wall Tie See tie.

Waterproofing Prevention of moisture flow through masonry due to water pressure.

Web An interior solid portion of a hollow-masonry unit as placed in masonry.

Weathered In masonry, stonework which has been cut with sloped surfaces so it will shed water from rain or snow.

Weep Holes Openings placed in mortar joints of facing material at the level of flashing, to allow the moisture to escape, or openings in retaining walls to allow water to escape.

Weeping Joints A masonry joint treatment, in which mortar extruding from the joint in laying is not cut off, but is allowed to harden. Gives informal rustic appearance but difficult to waterproof.

Wythe 1. Each continuous vertical section of masonry one unit in thickness. 2. The thickness of masonry separating flues in a chimney. Also called *withe* or *tier*.

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INDEX

AAC. *See* Autoclaved aerated concrete (AAC) blocks
Accidental eccentricity, 7.65–66
Accidental horizontal torsional moment, 7.65–67
Actual dimension, 2.13
Adhered veneers, 2.19, 6.14
Admixtures, 3.3–4
Adsorbed water, 3.25
Aggregates, 3.3
Allowable stress design-level (ASD-level) force, 6.34
Allowable stress design (ASD) method, 1.9–10, 4.1
  bar sizes and, 3.15
  load combinations for, 1.12–13
  MSJC-08 on, 6.38
All purpose unit (Kerf unit), 2.20, 2.29
Alternate courses, 5.3
Alternate weephole detail, 8.46
American Concrete Institute (ACI), 1.8–9
American Society for Testing and Materials (ASTM), 2.2
  brick classification from, 2.8–9
  on cores, 2.7–8
  on mortar, 3.2–3
American Society of Civil Engineers (ASCE), 1.8–9
Amplification factor, 6.33
Anchorage. *See also* Bent-bar anchor bolts (J- or L-bolts)
  anchor bolts
    grout placement/embedment of, 10.2–3
    in grout *v.* concrete, 10.5
    loading of, 10.4
    nominal axial strength of, in tension, 10.5–16
    nominal strength of, 10.3–5
    strength reduction factors and, 10.5
    types of, 10.1–2
Anchorage. (*Cont.*):
  introduction to, 10.1
  limit states design and, 10.4–5
  problems on, 10.24–27
  structural walls and anchorage requirements
    force transfer mechanism to walls through anchorage of diaphragms, 10.16
    gravity loads to be transferred through anchorage, 10.16–17
    lateral seismic loads to be transferred through anchorage, 10.17–24
    problems on, 10.24–27
Anchored veneers, 2.19, 6.14
Angular faces, 2.31
Arch, 1.3
Architectural components
  SDC exceptions in, 6.32
  seismic force and, 6.34
Architectural units, 2.29, 2.31
ASCE. *See* American Society of Civil Engineers
ASD. *See* Allowable stress design (ASD) method
ASD-level. *See* Allowable stress design-level (ASD-level) force
Aspect ratio, 3.13–15, 7.3–5, 7.18
ASTM. *See* American Society for Testing and Materials
Autoclaved aerated concrete (AAC) blocks, 7.82
  availability of, 2.55
  joint reinforcement and, 2.54
Autoclaved cellular concrete. *See* Autoclaved aerated concrete
Axial loads, 7.95–108. *See also* Columns; Combined axial load and bending
  under column combined axial load and bending, 5.28–57
    analysis of, 5.28–29

Axial loads, under column combined axial load and bending (*Cont.*):
carry capacity of, 5.28–29
example 5.8, 5.29–33
example 5.9, 5.33–36
example 5.10, 5.37–39
at maximum positive moment, 8.41
out-of-plane loads and, 6.69
Axial strength, 5.7–10. *See also* Nominal axial strength
Axial stress
out-of-plane loads and, 6.54
retaining walls and, 8.29
Axial tensile strength design, 10.7–14

Backfill
material selection for, 8.42–43
maximum depth of, 8.42
operation for, 8.43–45
surcharge effects of, 8.21–22
swelling pressures from, 8.43
Backfilling operation
bracing in, 8.43–44
surcharge loads in, 8.44
timing of, 8.43
water and, 8.44–45
Backup walls, 6.13–14
Balanced condition, 4.31, 5.41, 5.43
Balanced point, in interaction diagrams, 5.58
Balanced steel ratio, p_b, 4.31–34
Bars, 6.45. *See also* Anchorage; Bent-bar anchor bolts
bar positioners, 6.47, 6.49–50, 9.5–6
bar sizes
ASD and, 3.15
for columns longitudinal reinforcement, 5.12–13
in design procedure (columns), 5.22
for vertical reinforcement, 6.41, 6.48, 9.2
bar spacing devices, 9.5
bend of, 3.17–18
bundling of, 4.42
deformed bars, 3.15–17, 3.18–19
development lengths for, 4.117–119
effective compressive width per bar, 6.52
epoxy-coated steel bars, 3.19
grades of, 3.16–18
in hollow concrete block, 2.27–29
lateral ties and, 5.13–16
plain bars, 3.15, 3.18
Basement walls, 6.48. *See also* Subterranean (basement) walls
Basket weave, 2.31
diagonal basket weave, 6.22–23
Beams, 4.12–14, 4.53–55, 4.69–74, 6.65–66, 7.6, 7.18. *See also* Bond beams; Reinforced masonry beam design; T-beams
deep wall beams, 4.101–108
ledger beam, 6.40, 6.42
underreinforced beams, 4.31
wall beams, 4.101–108, 6.15
Beams flexural design procedure, general procedure, 4.41–51
example 4.14, 4.44–46
example 4.15, 4.47–49
example 4.16, 4.49–51
Bearing pressure, 8.32
Bearing walls, 6.1
nonbearing *v.*, 6.2–3
Bearing wall system, 7.1, 7.45
Bedding areas, 2.3
Bed joints, 2.3
ties in, 5.3
Bend, of bars, 3.17–18
Bent-bar anchor bolts (J- or L-bolts), 10.1–2
in combined axial tension and shear, 10.15–16
design axial tensile strength of, 10.12–14
failure of, 10.4
in grout, 10.3
headed anchor bolts/combined axial tension and shear, 10.15–16
limit states design and, 10.4–5
nominal axial strength
anchor bolt pullout limit state and, 10.11–12
limit state of anchor bolt steel yielding and, 10.12–14
loaded in tension, 10.11–14
masonry breakout limit state and, 10.11
nominal shear strength
limit state of anchor bolt pryout and, 10.15
limit state of anchor bolt steel shear yielding and, 10.15
limit state of masonry breakout and, 10.14–15
limit state of masonry crushing and, 10.15
in shear, 10.14–15
Beveled-end units, 2.29
Bevelled units, 2.29
Bevel siding, 2.31
Blended hydraulic cement, 3.3
BOCA National Building Code, 1.8
Bolted connections, 6.42
Bolts. *See* Anchorage

Bond beams, 4.30, 4.109–110, 4.112–115, 7.10, 9.12
  flashing at, 9.45
  formation of, 9.5
  horizontal bond beam reinforcement, 2.44, 9.5, 9.33–34
  in partially grouted walls, 9.5
  in pilasters, 5.2
Bond beam stretcher, 2.7
Bond beam units, 2.28–29
Bond course, 2.36
Bonded wall, 6.9
Bond patterns, 2.36–40, 6.17–23, 6.44–45. *See also* Running bond; Stack bond
  definition of, 6.16
  in screen walls, 6.4
  in single-wythe walls, 6.4
  in two-wythe brick walls, 6.4
  wythe and, 6.21
Bonds. *See also* Stack bond
  diagonal bond, 6.22–23
  half running bond, 6.18–20
  masonry wall bond patterns, 6.16–23
  reinforcing bond, 3.1
Box system, 7.1
Braced frames, 7.1
Brick(s), 2.36–39, 4.67–68, 6.4. *See also* Masonry units and their applications
  advantages of, 2.1–2
  ceramic glazed, 2.9, 2.19
  chemical resistant brick, 2.9
  classification of, 2.8–9
  clay building brick, 2.3, 2.4–23
  CMUs *v.*, for columns, 5.1, 5.3
  common shapes of, 2.5
  expansion joint for, 9.34–35
  exterior, estimating movement of, 9.30
  facing, 2.5, 2.9, 2.15
  fired brick, 1.2, 9.29–30
  history of, 1.2–1.4
  hollow brick, 2.3, 2.6–12, 2.15–16
  industrial floor brick, 2.9
  modular bricks, 2.10–11, 6.51
  mortorless brick paving, 2.17–18
  nonmodular bricks, 2.10, 2.12, 6.51
  Norman brick, 2.5, 2.10, 2.11
  paving, 2.9, 2.17–18
  positions of, in wall construction, 2.6
  reclaimed, 2.21–23
  reinforced brick construction, 1.4, 2.44, 9.5
  Roman brick, 2.10, 2.11
  salmon brick, 2.22–23
Brick(s) (*Cont.*):
  salvaged (second-hand), 2.21–23
  sewer and manhole brick, 2.9
  solid bricks, 6.5
  thin brick veneer, 2.18–19
  time *v.* moisture expansion of, 9.29–30
  weathering and, 2.13–14
Brick floaters, 5.3
Brunel, Marc Isambard, 1.4
Buckling load, 5.4
Buckling strength, 5.4–6
Building brick (solid), 2.9
Building codes
  Code of Hammurabi, 1.6–7
  definition of, 1.6
  history of, 1.6–1.9
  MCJC-08 Code and Specification, 1.8–9
  model codes, 1.7–8
  MSJC code, 5.1, 5.4, 5.10–16
  reinforcement size and, 9.2
  topics of, 1.9
Building configurations, 7.57
  horizontal structural irregularities and, 7.58–62
Building expansion joint, 9.34
Building frame systems, 7.1, 7.45
Bull-nosed units, 2.29

CABO. *See* Council of American Building Officials
Cantilevered shear walls, 7.11–15
Cantilever elements, 6.34
Cantilever type walls, 6.14–15, 7.112
  vertical, subterranean walls as, 8.39
Cantilever walls, 7.6, 7.11–13, 7.18, 7.89, 8.39
  as retaining walls, 8.3–4
Caps, flashing at, 9.45–46
Carbonation shrinkage
  definition of, 3.25–26
  materials and, 9.27
Cavity, 6.5, 6.11
Cavity walls, 2.43–44, 9.37
  advantages of, 6.11
  composite walls and, 6.11
  description of, 6.5
  details of, 2.45
  flashing for, 9.44–45
Cells, 2.3, 2.26, 2.28
  CMU *v.*, 2.30, 2.33
  configurations of, 2.7
  cores *v.*, 2.6
  description of, 2.15–16
  grout and, 3.1, 3.6

Cells (*Cont.*):
  reinforcing bars and, 6.49–50
  web *v.*, 2.33
Cement lime types, 3.2
Center of mass, 7.62–64
Center of rigidity, 7.62–64
Ceramic glazed brick, 2.9, 2.19
Ceramic glazed tiles, 2.19–21
Channel-type restraint construction, 2.50
Chemical resistant brick, 2.9
Chimneys, 2.32
Cladding, 6.26
Clay building brick, 2.3, 2.4–23
  visual inspection, 2.15
Clay masonry, 1.4, 2.16–17, 2.36–39, 4.16–17
  CMU *v.*, columns size in, 5.11
  compressive strength of, concrete *v.*, 5.11
  moisture movement and, 9.29–30
Clay units, 2.2
Cleanouts
  grouting requirements for, 9.10–13
  grout types and, 9.11
  requirements
    for brick *v.* concrete masonry, 9.13
    for closing/sealing, 9.13–14
    grouting and, 9.10–13, 9.16, 9.18
Clear height, 5.5
CMUs. *See* Concrete masonry units
Coarse grout, 3.7–9, 3.11
Code of Hammurabi, 1.7
Codes, building, 1.6–9, 9.2
Coefficient of creep, 3.27–28, 9.27
Coefficient of thermal expansion, 3.23
  materials and, 9.27–28
  thermal movement and, 9.25–27
Cold joint, 9.34
Collar joint, 3.6, 6.5, 6.9–11
  horizontal reinforcement at, 9.5
Columns, 2.32. *See also* Axial loads; Reinforced masonry column
  axial load-bending moment interaction diagrams, 5.57–58
  axially loaded column behavior
    buckling strength, 5.4
    effective column height: end restraints effects, 5.5–6
  brick floaters in, 5.3
  buckling strength
    buckling load in, 5.4
    critical load in, 5.4
    elastic buckling failure in, 5.4
    end restraints and, 5.5–6
Columns, buckling strength (*Cont.*):
    Euler's load in, 5.4
    Hooke's law in, 5.4
    materials *v.*, 5.4
  clear height of, 5.5
  CMUs for, 5.3
  CMUs *v.* bricks for, 5.1, 5.3
  cross-sectional areas of masonry columns, 5.11
  definition of, 5.1
  design of, 5.1, 5.4
  flush wall columns, 5.1
  footing in, 5.2
  interaction diagram for walls under combined loading (axial load and bending), 5.58–60
  introduction on, 5.1–4
  in-wall columns, 5.1
  masonry columns shear strength, 5.60–64
  masonry piers, 5.64–68
  pilasters *v.*, 5.1–3
  problems on, 5.65–68
  reinforcement
    lateral ties reinforcement, 5.13–16
    longitudinal reinforcement, 5.12–13
    shear walls *v.*, 5.59–60
    vertical reinforcement for, 5.2
  running bond for, 5.1
  strength design for, 5.1, 5.4
  wall columns, 5.1
Column units, 2.29
Combined axial load and bending
  carrying capacity and, 5.28–29
  interaction diagram for, 5.39–58
    balanced condition in, 5.41, 5.43
    balanced point in, 5.58
    calculations on, 5.40–57
    compression reinforcement strain case in, 5.47–48
    compressive face in, 5.44–45
    compressive reinforcement tensile strain case in, 5.48–49
    example 5.11, 5.49–57
    principle of superposition in, 5.44
    tension face zero strain case in, 5.46–47
    tension reinforcement zero strain case in, 5.47
    values table for, 5.46
Commentary on Building Code Requirements, 5.4
Commentary on Specification, 5.4
Common bond, 2.38, 6.4

Common bond pattern, 2.38, 2.40, 6.4
Components and cladding, 6.26
Composite action, 6.9
Composite masonry walls, 6.10
Composite walls, 6.1
  advantages of, 6.9
  cavity walls and, 6.11
  collar joints of, 6.9–11
  description of, 6.9
  headers in, 6.11
  metal ties in, 6.11
Compressive strength, 3.12–15
Compression block, 6.51
Compression reinforcement strain case, 5.47–48
Compressive reinforcement tensile strain case, 5.48–49
Compressive strength
  of CMU, unit strength *v.*, 2.16–17
  CMU unit strength *v.* concrete masonry, 2.25
  of concrete *v.* clay masonry, 5.11
  grout *v.*, 3.9–11
  of masonry *v.* masonry units, 3.12–15
  prism test method for, 3.13–15
Concave joint, 2.42
Concrete, 2.27–29, 2.35, 2.54, 2.55
  composition of, 3.25
  compressive strength of, clay masonry *v.*, 5.11
  grout *v.*, 3.11–12, 10.5
Concrete block walls, 6.41
Concrete building brick, 2.26
  blocks and, 2.3
Concrete masonry, 2.25, 4.16, 5.11, 6.82, 9.13
  control joint for, 9.35–37
  exterior, estimating movement of, 9.30
  moisture movement and, 9.28–29
Concrete masonry units (CMUs), 1.4, 2.1, 6.71, 9.12–13. *See also* Masonry units and their applications
  bricks *v.*, for columns, 5.1, 5.3
  cells *v.*, 2.30, 2.33
  clay *v.*, columns size in, 5.11
  for columns, 5.3
  column shear strength and, 5.61–64
  compressive strength of, 2.16–17, 2.25
  for fence wall, 6.80
  for pilasters, 5.3
  sound absorbing property of, 2.35
  for sound control, 2.29–30
Concrete walls, 9.42
Cone breakout
  limit state of masonry for, 10.5–7
  overlapping cone areas and, 10.6
Consolidation
  of grout, 9.15–23
  reconsolidation and, 9.23
Construction aspects. *See also* Grouting; Materials movements
  cracking and movement joints control
    function/types of movement joints, 9.33–37
    horizontal reinforcement and, 9.33–34
    movement joints and, 9.34–37
    movement joints spacing/location, 9.38–42
  flashing for masonry construction
    construction practices, 9.43–44
    flashing materials, 9.43
    general considerations, 9.43
    typical details, 9.44–46
  introduction to, 9.1
  materials movements
    causes of, 9.26–30
        moisture changes as, 9.24–25
        structural overloads as, 9.24–25
        thermal changes as, 9.24–25
    coefficients of thermal expansion and, 9.25–27
    cracking from, 9.24
    distress from restrictions of, 9.30–33
    estimating, of exterior masonry, 9.30
    types of, 9.23–24
  quality assurance, 9.42–43
  steel reinforcement placement, 9.2–7
    flexure in, 9.2
    installation of, 9.4–6
    limits for, 9.2, 9.4
Construction joint, 9.34
Contec's AAC blocks, 2.55
Continuous air space, 6.11
Contraction, 9.25
Control joint, 9.34
  for concrete masonry, 9.35–37
  cracking and, 9.41
  for inner wythe of cavity wall, 9.37
  interlocking units for, 9.36–37
  pilasters and, 9.41
  random cracking and, 9.41
  spacing/placement of, 9.39–42
  wall thickness and, 9.40
Control joint units, 2.29
Conventional SRWs, 8.6
Copings, flashing at, 9.45–46
Cores, 2.3, 2.7–8, 2.12, 2.16
  cells *v.*, 2.6
  definition of, 2.15
Coring pattern, 2.4, 2.7–8

Corner pilaster, 5.3
Council of American Building Officials (CABO), 1.8
Coupled wall, 7.6–7
Course, 2.46
  alternate courses, 5.3
  bond course, 2.36
Coursed ashlar bond pattern, 2.40
Coursed ashlar masonry, 1.2, 6.22–23
Course grout, fine grout *v.*, 3.7–8
Cover, 3.21–22
Cracking, 9.26–40, 9.42
  control joint and, 9.41
  from materials movements, 9.24
Cracking moment, 7.96. *See also specific examples*
  nominal cracking moment, 4.26–31
  stress design and, 6.55
Creep, 9.23
  coefficient of creep, 3.27–28, 9.27
  materials and, 9.24
  specific creep, 9.27
Critical load, in columns, 5.4
Curbs, 8.9
Curtain wall, 6.3, 6.12
Curved faces, 2.31
Customized masonry, 2.23
  columns, chimney units and caps, 2.32

Dead load, 6.24–25
Deep frogs, 2.3, 2.15–16
Deep wall beams, 4.101–108
Deflection, 4.91, 4.126–131, 7.11, 7.17, 9.25
  excessive deflection, 9.24, 9.27, 9.31–32
  horizontal midheight deflection, 6.54
  out-of-plane deflection, 6.38–40
Deformation
  instantaneous deformation, 3.27
  from shrinkage, 3.25–26
Deformed reinforcing wire, 3.19–21
Demand, 1.10
Designed walls, 6.14
Design loads
  lintels and, 4.83–87
  for subterranean walls, 8.36
Design methods
  allowable stress design and, 1.9–10
  factored loads and, 1.10
  load factors and, 1.10
  strength design and, 1.10–1.11
Design procedure (columns), 5.25–28
  of longitudinal steel for given column sizes, 5.21–24
    bar sizes in, 5.22
    lateral ties in, 5.23–24
    nominal column size in, 5.22
Design procedure (retaining walls)
  bearing pressure in, 8.32
  cross section of wall in, 8.29–30
  initial steps in, 8.29
  lapped splices in, 8.34–35
  moisture and, 8.34
  shear strength in, 8.33
  sliding in, 8.32
  stability in, 8.31
  strength design in, 8.29–30
  structural design in, 8.32–33
Design strength. *See* Strength design
Design wind pressure, 6.35–36
Detailed plain (unreinforced) AAC masonry shear wall, 7.82
Detailed plain (unreinforced) masonry shear wall, 7.46, 7.82, 7.84
Detailed plain masonry shear walls, 7.83–84
Development length, 4.117–119
Diagonal basket weave, 6.22–23
Diagonal bond, 6.22–23
Diaphragm discontinuity irregularity, 7.61
Diaphragm forces, 7.2–3
Diaphragms, 2.17–18, 4.111–115, 6.44, 7.2–6, 7.52–57, 7.63–66, 7.69–81, 10.16
  flexible *v.* rigid, 7.5, 7.50–51
Differential movement, 9.24
  materials and, 9.27–28
Differential settlement, 9.24
  distress and, 9.31–33
  stair-step cracking from, 9.33
Direct shears, 7.64
Distortion, 9.25
Distress
  differential settlement and, 9.31–33
  excessive deflection and, 9.31–32
  in long walls, 9.31
  in parapets bowing, 9.31–32
  from restricted movements, 9.30–33
  spalling, 9.33
Distribution factor, 7.19
D-number, 3.21
Double bullnose pier block, 5.3
Double ladder tie, 9.21
Double opening strip, 7.7
Double-projection pilaster, 6.70
  standard concrete masonry units for, 6.71

Double punched opening, 7.7
Double stretcher garden wall bond, 2.39
Drainage provisions
  for retaining and subterranean walls, 8.45
  weep holes as, 8.45–46
Drying shrinkage, 3.25
Dry masonry, 1.2
Dual systems, 7.1, 7.45, 7.47
Ductile response, 7.112
Durability grades, 2.13–14
Dutch bond, 2.38–39
Dynamic analysis, 7.40
Dynamic force irregularities, 7.58

Earth pressure basic concepts, 8.9–11
Earthquakes, 1.5
  Northridge, California (January 17, 1994), 7.113–114
  unreinforced masonry and, 1.3
Earthquake loads. *See also* Out-of-plane loads
  effects and combinations of, 1.13
  on nonstructural walls/parapets, 6.31–38
  occupancy categories and, 6.30–31, 6.35–37
Eccentricity, 5.7, 6.40, 6.43, 7.62, 7.65–67
Edge restraint conditions
  subterranean walls and, 8.36–37
  vertical *v.* horizontal, 6.75
Effective column height: end restraints effects, 5.5–6
Effective compressive width per bar, 6.52
Effective depth
  of reinforcement, 6.49–51
  T-beam analysis, 6.51–54
Effective length factor, 5.5–6
Effective lengths, 6.16
Effective moment of inertia concept, 4.126–131
Effective span, of lintels, 4.77–80
Effective width, 6.52
Elastic buckling failure, 5.4
Elasticity, 3.16
  modulus of elasticity, 3.22–23, 3.25
Electrical equipment, 6.32
Empirical design, 6.14
Empirically designed shear wall, 7.83
End-dam, 9.44
End restraints, column buckling strength and, 5.5–6
End restraints effects, 5.5–6
End zones, 6.28–29
Energy efficiency, 2.29, 2.30
Engineered walls, 6.14
English bond, 2.36–38
English cross bond, 2.38, 2.39
Epoxy-coated steel bars, 3.19
Equivalent lateral force procedure, 7.40
Euler's load, 5.4
Excessive deflection, 9.24
  distress and, 9.31–32
  materials and, 9.27
Expansion, 9.25
Expansion joint, 3.26
  for brick masonry, 9.34–35
  building expansion joint, 9.34
  fillers for, 9.35
  foam rod for, 9.35–36
  location of, 9.39–41
  maximum horizontal spacing of, in concrete walls, 9.42
  moisture and, 9.42
  spacing of, 9.38–39, 9.42
Exterior veneers, 6.14
Extreme torsional irregularity, 7.61–62, 7.78–81

Faces, 2.4
  in architectural units, 2.31
  compressive face, 5.44–45
  tension face zero strain case, 5.46–47
Face shells, 2.7, 2.33, 6.53
Facing brick (solid), 2.5, 2.15
  ASTM classification of, 2.9
Factored loads, 1.10, 4.1
Fence walls, 6.14–15
  CMUs for, 6.80
  design of, 6.78–81
  serpentine, 6.80–81
  stability of, 6.78–79
Fiber-reinforced polymer (FRP) reinforcement, 3.19
Fillers, 9.35
Filler wall, 6.3, 6.12
Fine grout, 3.9, 3.11
  course grout *v.*, 3.7–8
Fixed-ended shear wall, 7.13–18
Flange, 6.51–53, 7.8, 7.110
  pilaster as, 6.77
Flange walls, 6.49, 7.8
Flashing
  at bond beams, 9.45
  for cavity wall construction, 9.44–45
  construction aspects for, 9.43–46
  at copings/caps, 9.45–46
  end-dam for, 9.44
  general considerations on, 9.43
  materials for, 9.43
  typical details on, 9.44–46
  weep holes and, 9.44

Flemish bond, 2.36–37, 2.38, 6.4
Flexible brick pavements, 2.17–18
Flexural deflections, 7.17
Flexural design, 4.41–53
Flexural failure, 7.110–111
Flexural resistance, 6.50
Flexural stresses, 6.40, 6.48
Flexure
  in steel reinforcement placement, 9.2
  vertical reinforcement and, 6.47–49
Flexural analysis, 4.7–12
Flexural loads, 5.28, 7.105–108
Flexural strength, 4.115–117
  element depth *v.*, 9.2
Flush joint, 2.42
Flush wall columns, 5.1
Fluted faces, 2.31
Foam rod, for expansion joint, 9.35–36
Footing(s)
  bearing pressure under, 8.28
  in columns, 5.2
  placement of, 8.43
  for retaining and subterranean walls, 8.42
Forces, 8.31, 8.39
  force transfer mechanism, 10.16
  inertial forces, 7.3–4
  in-plane forces, 6.2, 6.25, 7.2
  LFRS, 5.11, 6.2, 6.25, 7.1
  MWFRS, 6.25, 7.1
  out-of-plane forces, 6.2–3
  seismic force, 6.34–38, 7.50
  seismic lateral forces, 7.39–50
  SFRS, 6.25, 7.1
  wall forces, 6.35–37
Frogged ends, 2.3
Frogs, deep frogs, 2.3, 2.15–16
FRP. *See* Fiber-reinforced polymer (FRP) reinforcement
Fully grouted masonry construction, 8.42
Fully grouted wall, 6.5

Glass block, 2.46–47
Grade MW, 2.15
Grade N, 2.25, 2.26
Grade S, 2.25, 2.26
Grades, of reinforcing bars, 3.16–18
Grade SW, 2.15
Gravity loads, 10.16–17
  lateral loads and, 6.2–3, 6.23–24, 6.44–45, 7.95
  transverse loads and, 6.44–69
Gravity walls, 8.3
Great Wall of China, 1.3
Gross transformed moment of inertia, 4.124
Grout, 2.7, 2.26, 6.21, 6.45, 9.5, 9.7–8
  anchor bolts and, 10.2–3, 10.5
  cells and, 3.1, 3.6
  compressive strength and, 3.9–11
  concrete *v.*, 3.11–12, 10.5
  consistency of, 3.6–7
  construction aspects and, 9.9–23
  fully grouted masonry construction, 8.42
  functions of, 3.6
  hollow masonry units and, 6.5
  methods of specifying, 3.7–9
  modulus of elasticity, 3.23
  mortar *v.*, 3.11–12
  properties specification for, 3.9
  proportions specification for, 3.9
  slump test for, 3.7, 3.8
  strength of, 3.7, 3.9–11
  temperature and, 9.23
  ties in, 5.3
  types of, 3.7–9, 9.14
  in wall reinforcement, 6.47
Grouted pocket, 5.3
Grouted wall, 6.5
Grouting
  cleanout requirements for, 9.10–13, 9.16, 9.18
  general considerations on, 9.7–8
  lifts and, 9.9–10
  low-lift *v.* high-lift, 9.8, 9.15–16, 9.19
  methods of
    curing/protection in, 9.23
    grout matching in, 9.14–15
    grout placement/consolidation in, 9.15–23
    grout pour *v.* grout lift in, 9.11
    height/space requirements and, 9.11
    importance of, 9.10–11
    preparation of grout space in, 9.12–14
Grout lift, 3.6, 9.9–11. *See also* High-lift grouting; Low-lift grouting
  timing of, 9.16, 9.21, 9.23
Grout pour, 3.6
  grout lift *v.*, 9.11
  heights for, 3.9, 9.10–11, 9.16–17, 9.19, 9.21
  shear key in, 9.16–17
  timing in, 9.16, 9.21.9.23
Grout reinforcement, 6.47
Grout slump, 3.7

Grout space preparation
for bond beams, 9.12
estimated grout volume in, 9.15
grout batching in, 9.13
grout types in, 3.7–9, 9.14
hollow CMUs and, 9.12–13
for multiwythe walls, 9.12
placement and consolidation in, 9.15
pour timing in, 9.16
requirements for, 9.12–13
slump in, 9.15

H40V, 2.16
H60V, 2.16
Half running bond, 6.18
corner details in, 6.19
corner layout details in, 6.20
Headed anchor bolts, 10.1–2, 10.16
axial tensile strength design for, 10.7–11
in grout, 10.3
limit state of masonry cone breakout for, 10.5–7
limit state of tensile yielding of bolt steel
example 10.1, 10.7–8
example 10.2, 10.8–9
example 10.3, 10.9–10
example 10.4, 10.10–11
limit states design and, 10.4–5
nominal axial strength of, in tension, 10.5–11
nominal shear strength of, in shear, 10.14–15
Header, 2.5–6, 6.11
Head joints, 6.46
Hebel wall system, 2.55
Hidden (in-wall) pilaster, 6.70
High-lift grouting
low-lift *v.*, 9.8, 9.15–16, 9.19
for multiple-wythe walls, 9.10, 9.22–23
for single-wythe walls, 9.23
of two-wythe wall, 9.10
types of, 9.19, 9.21
Highway noise barrier walls, 6.14–15
Highway sound barrier walls
concrete masonry, advantages of, 6.82
design considerations of, 6.82–83
design tables on, 6.83
"insertion loss" of, 6.82
pier and panel walls as, 6.82–83
problems on, 6.84–85
purpose of, 6.81–82
transmission loss of, 6.82–83

Hollow brick, 2.3, 2.10–12, 2.15
ASTM classification of, 2.9
configurations of, 2.6–8
types of, 2.16
Hollow concrete block, 2.27–29
Hollow masonry units, 2.4, 2.25, 3.14
alignment of, for vertical reinforcement, 9.3–9.4
grout and, 6.5
solid bricks *v.*, 6.5
Hollow unit masonry construction, 8.42
Hooke's law, 5.4
Horizontal bond beam reinforcement, 2.44, 9.5, 9.33–34
Horizontal diaphragms, 7.50–57
Horizontal eccentricity, 7.65
Horizontal edge restraint conditions, 6.75
Horizontally spanning walls, 6.15–16, 6.24
Horizontal midheight deflection, 6.54
Horizontal reinforcement, 2.41, 5.2, 6.5–9, 6.19, 6.21, 9.5
bond beams and, 2.44, 9.5, 9.33–34
cracking and movement joints control and, 9.33–34
Horizontal stack, 6.22–23
Horizontal structural irregularities, 7.58–62
Horizontal torsional eccentricity, 7.62
Hydrated lime, 3.3

Importance factor, 6.27, 6.32
Industrial floor brick, 2.9
Inertial forces, 7.3–4
Initial rate of absorption (IRA), 3.7
In-plane forces, 6.2, 6.25, 7.2
In-plane loads, 2.50, 4.111–112, 4.115–117, 6.1
In-plane shear and moment, 6.3
Instantaneous deformation, 3.27
Interaction diagrams. *See* Combined axial load and bending
Interior veneers, 6.14
Interior zone, 6.28–29
Intermediate reinforced masonry shear wall, 7.46–47, 7.82, 7.84–85
Intermediate reinforced prestressed masonry shear wall, 7.82
International Building Code, 1.6, 1.8, 7.39
In-wall columns, 5.1
IRA. *See* Initial rate of absorption

J-bolts. *See* Anchorage
Joint finish, 2.42

Joint reinforcement, 3.21, 6.10–11
AAC blocks and, 2.54
ladder type joint reinforcement, 3.19–20, 9.20
metal ties *v.*, 9.18–19
for multiple-wythe wall, 9.20–21
for single-wythe wall, 9.20
truss type joint reinforcement, 3.19–20, 9.20
Joints, 2.54. *See also* Construction aspects; *specific joints*
Joist supports, 6.42
J- or L-bolts. *See* Anchorage

Kanamori, Skigeyuki, 1.4
Kerf unit, 2.20, 2.29
Key/wood jamb block, 5.3

Ladder tie, 2.37, 9.20
double ladder tie, 9.21
Ladder type joint reinforcement, 3.19–20, 9.20
L-anchor bolts, 10.1–2
Lapped splices
in design procedure (retaining walls), 8.34–35
length of, 9.6
Lateral force-resisting systems (LFRS), 5.11, 6.2, 6.25, 7.1
Lateral loading conditions, 8.36–38
Lateral loads, 6.2–3, 6.23–25, 6.44–45, 7.95
walls/pilasters and, 6.75–76
Lateral pressures (retaining walls)
backfill surcharge effects, 8.21–22
earth pressure basic concepts, 8.9–11
earth pressure theories, 8.12–20
seismic earth pressure, 8.22–25
selection of applicable earth pressure theory, 8.20–21
Lateral seismic loads, 10.17–24
Lateral soil pressure, 8.38
Lateral support, 6.44
Lateral ties (columns), 5.1–2
in columns design procedure, 5.23–24
configurations of, 5.14
elevation placement of, 5.13, 5.15–16
longitudinal reinforcement bars and, 5.13–16
sizes of, 5.13, 5.15–16
L-bolts. *See also* Anchorage
limit states design and, 10.4–5
LFRD-level. *See* Strength level (LFRD-level) force
LFRS. *See* Lateral force-resisting systems
Lifting, 2.52
Lifts. *See* High-lift grouting; Low-lift grouting
Lightly reinforced wall, 6.9
Light transmission, 2.47
Lightweight, 2.24
Limit states design
anchor bolts and, 10.4–5
loading conditions for, 10.4
Limit states design standard, 1.5
Lintels
design considerations
arching action in masonry walls, 4.81–83
deflection considerations, 4.91
design loads in, 4.83–87
design moments, 4.89–91
effective span, 4.77–80
example 4.26, 4.91–96
example 4.27, 4.96–101
lintel depth considerations, 4.87–89
general description, 4.74–77
Lintel units, 2.28–29
Live load, 6.43
Load-bearing walls, 6.4–5, 6.23
nonload-bearing *v.*, 6.2–3
Load bearing wall tile, 2.9
Load combinations. *See* Masonry
Load dispersion, 6.46
Load factors, 1.10, 4.3–5
Loading, 6.21
Loading conditions, 8.36–38, 10.4
Longitudinal reinforcement (columns)
bar sizes in, 5.12–13
determination of, 5.22–24
lateral ties and, 5.13–16
Low-lift grouting
high-lift *v.*, 9.8, 9.15–16, 9.19
for multiple-wythe walls, 9.17
types of, 9.16–17

Main wind force-resisting systems (MWFRS), 6.25, 7.1
Masonry
building codes and standards, 1.6–9, 9.2
coursed ashlar masonry, 1.2
design methods in, 1.9–11
dry masonry, 1.2
history of, 1.2–3
load combinations in
for allowable strength design, 1.12–13
earthquake load effects and combinations, 1.13
general, 1.11
seismic load combinations, 1.13–14
for strength design, 1.11–12
plain masonry, 1.1, 1.4, 1.5

Masonry (*Cont.*):
random rubble dry masonry, 1.2
reinforced masonry, 1.1, 1.3–5
solid masonry, 1.2
unreinforced masonry, 1.1, 1.4–6
Masonry cement, 3.3
Masonry piers, 5.67–68
definition of, 5.64
reinforcement requirements of, 5.65
section/elevation of, 5.64
Masonry prisms, 3.13, 4.123
Masonry Standards Joint Committee (MSJC), 1.8–9
Masonry Systems, Inc., 2.52
Masonry units, 3.12
Masonry units and their applications. *See also* Brick(s)
AAC, 2.54–55
application of, 2.1–2
bonds and patterns in masonry work, 2.41
in clay masonry (brick) walls, 2.36–39
common bond pattern, 2.38, 2.40
course and, 2.36
double stretcher garden wall bond, 2.39
Dutch bond, 2.38–39
English bond, 2.36–38
English cross bond, 2.38–39
Flemish bond, 2.36–38, 6.4
garden wall bond, 2.39
horizontal bond beam reinforcement, 2.44
mortar bond, 2.35, 3.2
one-third running bond, 2.38–39, 6.19
pattern bond, 2.35–37
pattern bonds, 2.35–36
patterns of, in concrete masonry (block) walls, 2.39–40
structural bond, 2.35–36
types of, 2.35
clay building brick
ASTM classification of brick units, 2.8–9
brick dimensions, 2.12–13
brick sizes and nomenclature, 2.9–12
durability grades, 2.13–15
general description, 2.4–6
hollow brick configurations, 2.6–8
CMUs
ASTM standards for, 2.25
concrete blocks sound absorbing properties, 2.35
concrete building brick, 2.26
general description, 2.23–25
nonload-bearing CMUs, 2.26
Masonry units and their applications, CMUs (*Cont.*):
second-hand, 2.35
shapes and applications of, 2.26–30
sizes of, 2.30–33
solid and hollow load-bearing, 2.25
surface texture of, 2.33–35
unit strength *v.* concrete masonry specified compressive strength, 2.25
functional aspects of
architectural terra-cotta, 2.21
ceramic glazed brick and facing tiles, 2.19–21
customized masonry, 2.23
facing brick, 2.15
hollow brick, 2.15–16
paving bricks, 2.17–18
second-hand CMUs, 2.21–23
thin brick veneer, 2.18–19
unit strength *v.* clay masonry specified compressive strength, 2.16–17
visual inspection, 2.17
general description of, 2.2–4
glass unit masonry, 2.51, 6.3
channel-type restraint construction and, 2.50
description of, 2.46–47
in-plane loads and, 2.50
light transmission and, 2.47
mortar for, 2.48
panel anchor construction and, 2.49
panel sizes for, 2.48
planning for, 2.47
weight of, 2.50
wind load resistance and, 2.48–49
introduction to, 2.1
mortar joints, 2.42–43
mortarless block systems, 2.51
prefabricated masonry, 2.53
advantages of, 2.54
design of, 2.52
disadvantages of, 2.54
evolution of, 2.51–52
production of, 2.52
structural requirements for masonry in stack bond, 2.41
wall construction types
modular construction of walls, 2.46
single and cavity walls, 2.43–45
Masonry veneers, 2.19
Masonry wall bond patterns, 6.16–23

Masonry wall types
  cavity walls, 6.5, 6.11
  classifications of, 6.2–16
  composite *v.* noncomposite, 6.9–11
  curtain wall, 6.3, 6.12
  engineered walls, 6.14
  filler wall/panel wall, 6.3, 6.12
  hollow masonry units *v.* solid bricks, 6.5
  load-bearing *v.* nonload-bearing, 6.2–3
  load-carrying function of, 6.2–3
  reinforcement of, 6.5–9
  screen walls, 6.4, 6.12–13
  support conditions and behavior of, 6.14–17
  veneer walls, 6.13–14
  wythes *v.*, 6.3–5
Materials movements
  causes of, 9.26–30
    moisture changes as, 9.24–25
    structural overloads as, 9.24–25
    thermal changes as, 9.24–25
  coefficients of thermal expansion and, 9.25–27
  cracking from, 9.24
  distress from restrictions of, 9.30–33
  estimating, of exterior masonry, 9.30
  types of, 9.23–24
Materials of masonry construction. *See also* Construction aspects
  carbonation shrinkage and, 9.27
  coefficient of thermal expansion, 9.27–28
    thermal movement and, 9.26
  column failure and, 5.4
  compressive strength of, masonry *v.* masonry units, 3.12–15
  creep of, 3.27–28
  differential movement and, 9.27–28
  excessive deflection and, 9.27
  for flashing, 9.43
  grout, 3.6–12
  influence of moisture (shrinkage), 3.25–26
  introduction on, 3.1
  modulus of elasticity of, 3.22–23
  mortar, 3.1–3.6
  mortar *v.* grout *v.* concrete, 3.11–12
  shortening of structural frames and, 9.30
  steel reinforcement, 3.15–22
  thermal effects on masonry, 3.23–25
Mechanical cutters, 2.29–30
Mechanical equipment, 6.32
Mechanically stabilized earth (MSE) system. *See* Reinforced soil SRWs
Mechanical reinforcement ratio, 4.22
Mechanical splices, 9.6
Medium weight, 2.24
Metal accessories, 3.21
Metal ties, 2.37, 6.4
  in cavity wall, 2.45
  in composite walls, 6.11
  joint reinforcement *v.*, 9.18–19
Minimum yield stress, 3.16
Model codes, 1.7–8
Moderate weathering (MW), 2.13–14
Modular bricks, 2.10–11, 6.51
Modular construction, 2.13
  of walls, 2.46
Modular ratio, 3.23
Modulus of elasticity
  of grout, 3.23
  of masonry, 3.22–23, 3.25
  of masonry materials, 3.22–23
  of steel reinforcement, 3.22
Modulus of rupture, 4.26–27
Moisture
  content requirements of, for type I units, 9.29
  design procedure (retaining walls) and, 8.34
  expansion joint spacing and, 9.42
  influence of, on materials, 3.25–26
  as materials movements cause, 9.24–25
Moisture movement, 3.25–26, 9.24–25
  clay masonry and, 9.29–30
  concrete masonry and, 9.28–29
  fired brick *v.* time and, 9.29–30
  shrinkage *v.* expansion in, 9.28
Moment frames, 7.1
Moments, 4.124, 5.57–58, 6.3, 6.38–40, 6.43–44, 7.65–67, 7.69–81, 7.108–109, 8.41
  cracking moment, 4.27–31, 6.55, 7.96
  design moments, 4.89–91
  wall panels and, 6.75–76
Mortar
  ASTM on, 3.2–3
  functions of, 3.1–2
  for glass unit masonry, 2.48
  grout *v.*, 3.11–12
  history of, 3.1
  materials in, 3.2–4
  methods of specifying, 3.4–5
  MSJC code and, 5.11
  selection of right type of, 3.5–6
  types of, 2.48, 3.2–3, 3.5–6, 5.11, 6.2–3, 6.21
Mortar bond, 2.35, 3.2
Mortar cement, 3.3

Mortar joints, 2.42
  ties in, 5.3
  tooling of, 2.43
Mortarless block systems, 2.51
Mortorless brick paving, 2.17–18
Movement joints, 9.33
  building expansion joint, 9.34
  construction joint/cold joint, 9.34
  control joint, 9.34
    spacing/placement of, 9.39–42
  cracking and movement joints control and, 9.34–42
  expansion joint, 9.34
    location of, 9.39–41
    spacing of, 9.38–39, 9.42
MSJC-08 Code and Specification, 1.8–9
MSJC. *See* Masonry Standards Joint Committee
MSJC-08, 6.2, 6.3, 6.19, 6.21, 6.40, 6.45, 6.52, 6.77–78
  on ASD, 6.38
  out-of-plane loads design and, 6.54–69
  on strength design, 6.54–69
MSJC-08 code and specifications, 1.8–9
MSJC code
  definition in, 5.1
  on dimensional limits, 5.11
  lateral ties, 5.13–16
  longitudinal reinforcement, 5.12–13
  mortar and, 5.11
  reinforced masonry columns and, 5.1, 5.4, 5.10–16
Multiple-wythe walls, 6.3, 6.5, 6.10–11
  bond patterns in, 6.4
  high-lift grouting for, 9.10, 9.22–23
  joint reinforcement for, 9.20–21
  low-lift grouting for, 9.17
  reinforcement in, 9.2–3
MW. *See* Moderate weathering
MWFRS. *See* Main wind force-resisting systems

National Concrete Masonry Association, 9.2
Net cross-sectional area, 2.4
NL. *See* No limit in height
No limit in height (NL), 7.88
Nominal axial strength
  of anchor bolts in tension, 10.5–16
    headed anchor bolts and, 10.5–11
  of bent-bar anchor bolts, 10.11–14
Nominal axial strength design, 10.11–14
Nominal cracking moment, 4.27–31
Nominal diameters, 3.15
Nominal dimension, 2.10, 2.11, 2.13
Nominal shear strength, 4.58–59, 6.54, 10.4, 10.14–15
Nominal strength, 4.21–26, 8.40
  of anchor bolts, 10.3–5
  shear *v.* tension in, 10.4
  tension *v.*, 10.4
Nominal yield strength, 5.11
Nonbearing walls, 6.2–3
Noncomposite walls, 6.9–11
Noncontact splice, 9.6
Noncoupled walls, 7.6–7
Nonload-bearing CMUs, 2.26
Nonload-bearing walls, 6.4, 6.12–13, 6.23
  axial and transverse loads and, 6.77–78
  fence walls design, 6.78–81
  highway sound barrier walls, 6.81–85
  load-bearing *v.*, 6.2–3
Nonload-bearing wall tile, 2.9
Nonmodular bricks, 2.10, 2.12, 6.51
Nonparallel systems irregularity, 7.61
Nonstructural cladding, 6.13–14
Nonstructural walls
  amplification factor in, 6.33
  earthquake loads on, 6.31–38
  mechanical/electrical equipment in, 6.32
  seismic force and, 6.32–33
Normal weight, 2.24
No weathering (NW), 2.13–14
NW. *See* No weathering

Occupancy categories
  earthquake loads and, 6.30–31, 6.35–37
  SDCs and, 6.31
  wind loads and, 6.27–28, 6.35–37
One-third running bond, 2.38–39, 6.19
Open-ended A or H units, 2.26, 2.28, 6.47
  convenience of, 9.4
Ordinary plain (unreinforced) AAC masonry shear wall, 7.82
Ordinary plain (unreinforced) masonry shear wall, 7.46, 7.82
Ordinary plain masonry walls, 7.83
Ordinary reinforced AAC masonry shear wall, 7.82
Ordinary reinforced masonry shear wall, 7.46–47, 7.82, 7.84
Out-of-offsets irregularity, 7.61
Out-of-plane deflection, 6.38–40
Out-of-plane forces, 6.2–3
Out-of-plane lateral loads, 6.25

Out-of-plane loads, 6.1, 6.53. *See also* Walls under gravity and transverse loads
  analysis of, 6.38–43
    assumptions in, 6.38–40
    bolted connections in, 6.42
    eccentricity in, 6.40, 6.43
    ledger beam in, 6.40, 6.42
    moment in, 6.39–40
    procedures in, 6.40–44
  axial loads and, 6.69
  axial stress and, 6.54
  earthquake loads, 6.3, 6.29–38
    amplification factor and, 6.33
    on nonstructural walls/parapets, 6.31–38
    occupancy categories and, 6.30–31, 6.35–37
    out-of-plane loads, 6.29–38
    seismic force and, 6.32–33
    structural walls/out-of-plane loads and, 6.29–31
  introduction to, 6.25
  MSJC-08 design provisions for
    example 6.2, 6.55–59
    example 6.3, 6.60–65
    example 6.4, 6.65–66
    example 6.5, 6.67–69
  MWFRS/LFRS and, 6.25
  strength design and, 6.54–69
  structural walls and, 6.29–31
  wind loads, 6.26–29
Overreinforced beams, 4.53–55
Overturning moment, 7.108–109

Panel anchor construction, 2.49
Panel sizes for glass unit masonry, 2.48
Panel wall, 6.12, 6.82–83. *See also* Wall panels
Parapets, 6.16, 6.29
  amplification factor in, 6.33
  distress of, from bowing, 9.31–32
  earthquake loads on, 6.31–38
Partially grouted wall, 6.5, 6.7, 6.52
  bond beams in, 9.5
  as T-beams, 6.51
Partially reinforced wall, 6.9
Pattern bond, 2.35–37
Pavers, 2.17–18
Paving brick, 2.9
Perforated walls, 7.6, 7.17–39, 7.112
Pier and panel walls, 6.82–83
Pier failure, 7.112–113
Piers, 5.3, 5.64–68, 7.6, 7.18–19, 7.21–22, 7.32, 7.35–36. *See also* Masonry piers
Pilaster(s), 2.28–29
  in bond beams, 5.2
  bonded *v.* unbonded, 6.72–73
  clear height of, 5.5
  CMUs for, 5.3
  columns *v.*, 5.1–3
  control joint and, 9.41
  definition of, 6.69
  design considerations for, 6.70–77
  double-projection pilaster, 6.70–71
  as flange, 6.77
  functions of, 6.69–70
  hidden (in-wall) pilaster, 6.70
  horizontal reinforcement in, 5.2
  introduction to, 6.69–70
  lateral loads and, 6.75–76
  shapes and sizes of, 2.33
  shear and, 6.75–77
  single-projection pilaster, 6.70–71
  T-beam and, 6.77
  vertical load and, 6.72
  wall panels and, 6.73–77
Placement, 9.6–7. *See also* Steel reinforcement
  of control joint, 9.39–42
  of footing, 8.43
  grout and, 9.15–23, 10.2–3
Plain masonry, 1.1, 1.4, 1.5
Planning
  correctness in, 2.47
  mistakes in, 2.46
Plastic flow. *See* Creep
Plate anchor bolts, 10.1–2
  in grout, 10.3
Ponding failure, 4.120
Portland cement, 3.3
Precast concrete slab, 6.41
Precast floor planks, 6.41
Prefabricated ladder T, 9.21
Prefabricated masonry, 2.51–54
Prestressed floor planks, 6.41
Prestressed masonry shear walls, 7.46, 7.82–83
Prism strength, 3.13–15
Prism test method
  calculation in, 3.14–15
  for compressive strength, 3.13–15
  masonry prism examples for, 3.14
Properties specification, 3.4–5, 3.9
Proportioned walls, 6.14
Proportions specification, 3.4–5, 3.9

Quicklime, 3.3

Radial units, 2.31
Raked and tooled joint, 2.42
Random ashlar, 1.2
Random ashlar bond pattern, 2.40
Random cracking, 9.41
Random rubble dry masonry, 1.2
Recessed faces, 2.31
Reclaimed brick, 2.21–23
Rectangular beam analysis, 6.65–66
Rectangular sections in flexure analysis, 4.7–26
  conditions for yielding of tension reinforcement based on strain compatibility, 4.12–21
    conditions for yielding of tension reinforcement in beams at balanced conditions, 4.12–14
    general conditions (c/d ratios) for post-yielding of tension reinforcement based on strain compatibility, 4.14–21
  flexural analysis in strength design principles, 4.7–12
  nominal strength and reinforcement ratio, 4.21–26
Rectangular tie, 2.37
Reentrant corner irregularity, 7.61
Reinforced brick construction, 1.4, 2.44
  bar spacing devices, 9.5
Reinforced masonry, 1.1
  evolution of, 1.3–5
  history of, 1.4–5
  unreinforced *v.*, 1.5–6
Reinforced masonry beam design, beams flexural design procedure. *See also* Beams flexural design procedure; Lintels; Rectangular sections in flexure analysis
  analysis of doubly-reinforced beams, 4.69–74
    example 4.25, 4.73–74
    introduction to, 4.69–70
  bond beams
    definition and functions of, 4.109
    typical reinforcement in, 4.109–110
  design for shear in reinforced masonry beams
    example 4.22, 4.63–65
    example 4.23, 4.65–67
    example 4.24, 4.67–69
  development lengths for reinforcing bars, 4.117–118
    example 4.31, 4.118–119
  diaphragm action
    bond beam and diaphragm action analysis, 4.112–115
    in-plane loads in, 4.111–112
Reinforced masonry beam design, beams flexural design procedure (*Cont.*):
  flexural strength of a wall under in-plane loads, 4.115–117
  historical development of, 4.2
  introduction to, 4.1–2
  lateral support of masonry beams, 4.69
  masonry beam determination of shear reinforcement, 4.61–62
  masonry beams design, 4.31–41
    determination of balanced steel ratio p_b, 4.31–34
    ductility in reinforced concrete beams: balanced, underreinforced, and overreinforced beams, 4.31
    minimum and maximum tensile reinforcement in, 4.34–41
  masonry wall beams (deep wall beams), 4.101–108
    deep beams concept, 4.101–102
    deep beams examples, 4.102–108
    wall beams defined, 4.101
  modulus of rupture and nominal cracking moment of masonry beam, 4.26–31
    modulus of rupture, 4.26–27
    nominal cracking moment of masonry beam examples, 4.27–31
  MSJC *Code* provisions
    for estimating masonry beam shear strength, 4.56–58
    for transverse (shear) reinforcement, 4.63
  overreinforced beams, 4.53–56
  problems on, 4.131–139
  reinforced masonry beams
    deflections, effective moment of inertia concept in, 4.126–131
    service load analysis for concept of transformed section, 4.120–122
    service load analysis for modular ratio, *n*, 4.124–126
    service load analysis for neutral axis location in, 4.122–124
  reinforced masonry nominal shear strength, 4.58
    example 4.20, 4.58–59
  serviceability criteria for beams, 4.119–120
  shear strength with shear reinforcement, 4.59–60
    example 4.21, 4.60–61
  strength design philosophy
    assumptions in, 4.5–7
    basic concepts, 4.2–3
    strength reduction factors and load factors, 4.3–5

Reinforced masonry column
  analysis of, 5.16–21
    example 5.2, 5.17–18
    example 5.3, 5.18–19
    example 5.4, 5.19–21
  axial strength of
    column height category and, 5.9–10
    eccentricity and, 5.7
    practical calculations on, 5.7–10
    slenderness and, 5.7–9
    in theory, 5.7
  balanced point in, 5.58
  design procedure
    for longitudinal steel for given column sizes, 5.21–24
    for masonry column size for given axial loads, 5.24–28
  interaction diagrams for, 5.57–58
  MSJC code and, 5.1, 5.4, 5.10–16
Reinforced masonry walls, 6.5–9
Reinforced soil SRWs, 8.6–9
Reinforcement. *See also* Horizontal reinforcement; Joint reinforcement
  for columns, 5.12–16
  doubly reinforced beams, 4.69–74
  dowels, 9.6
  FRP reinforcement, 3.19
  horizontal bond beam reinforcement, 2.44
  of masonry wall types, 6.5–9
  singly reinforced beams, 4.69
  splices for, 9.6
  steel reinforcement, 3.12, 3.15–22, 5.11, 9.2, 9.4–7
  tensile reinforcement, 4.34–41, 7.96–97, 8.40
  timing of, 9.4–5
  of walls, 6.45–47
Reinforcement bars, 2.27–29, 3.15–19, 6.45, 6.47–50. *See also* Anchorage
Reinforcement depth, 6.52–53
  brick sizes and, 6.51
  dimensions for calculation of, 6.50
  measurement of, 6.49
  position in cells and, 6.50
Reinforcement protection, 3.21–22
Reinforcement ratio, 4.21–26
Reinforcing bars, 3.16–18, 4.117–119, 6.49–50
Relative humidity, 3.25
Resistance factor, 4.1
Retaining and subterranean walls, 8.5–6
  backfilling operation for, 8.43–45
  backfill material selection for, 8.42–43
Retaining and subterranean walls (*Cont.*)
  bridge abutment, 8.2
  construction considerations for, 8.42–48
  drainage provisions for, 8.45
  footings for, 8.42
  introduction to, 8.1–2
  problems on, 8.46–48
  retaining embankment example, 8.2
Retaining wall external stability
  bearing pressure under the footing, 8.28
  considerations of, 8.25–26
  against overturning, 8.26
  against sliding, 8.26–28
Retaining walls, 6.14–15, 6.48. *See also* Design procedure (retaining walls)
  axial stresses and, 8.29
  basement/subterranean, 8.5–6
  bridge abutments, 8.5
  cantilever walls, 8.3–4
  counterforted and buttressed, 8.4–5
  curbs, 8.9
  forces acting on, 8.31
  gravity walls, 8.3
  lateral pressures on, 8.9–25
  masonry, design procedure for, 8.29–35
  principle types of, 8.2–9
  segmental retaining walls, 8.6–9
  SRW, 8.6–9
Ribbed faces, 2.31
Rigid brick pavements, 2.17–18
Rigid frames, 7.1
Rigidity
  center of rigidity, 7.62–64
  of diaphragms, 7.5, 7.50–51
  relative rigidity *v.*, of shear walls, 7.10–17, 7.35–36
  of shear wall with openings, 7.17–39
Rowlock, 2.5–6
Rowlock stretcher, 2.6
Running bond, 2.38–39, 6.4, 6.18–23, 7.9
  for columns, 5.1
  in concrete masonry (block) walls, 2.40
  description of, 2.37
  gravity loads distribution and, 6.44–45
  half running bond, 6.18–20
  head joints and, 6.46
  load dispersion and, 6.46
  one-third running bond, 2.38–39, 6.19
  T-beams and, 6.52
  wall reinforcement and, 6.46, 6.52
Sailor, 2.6
Sash blocks, 2.29

Scored faces, 2.31
SCR. *See* Structural Clay Products Research Foundation
Screen walls
  bond patterns in, 6.4
  design of, 6.12–13
  purpose of, 6.12
SDCs. *See* Seismic Design Categories
SEAOC. *See* Structural Engineers Association of California
Second-hand bricks, 2.21–23
Second-hand CMUs, 2.35
Segmental retaining walls (SRW)
  conventional SRWs, 8.6
  reinforced soil SRWs, 8.6–9
Seismic Design Categories (SDCs), 2.41, 3.3, 5.11, 7.67, 7.69
  determination of, 6.36, 7.88–89
  exceptions and, 6.31–32
  importance factor in, 6.32
  mechanical/electrical equipment and, 6.32
  mortar/cement and, 6.2–3
  occupancy categories and, 6.31
  shear walls and, 7.87–89
Seismic Design Category C, 3.3
Seismic Design Category D, 2.41, 3.3, 5.11, 7.67, 7.69
Seismic Design Category E, 2.41, 3.3, 5.11, 7.67, 7.69
Seismic Design Category F, 2.41, 3.3, 5.11, 7.67, 7.69
Seismic earth pressure, 8.22–25
Seismic force, 6.38, 7.50
  architectural components/elements and, 6.34
  cantilever elements and, 6.34
  earthquake loads and, 6.32–33
  wall forces *v.*, 6.35–37
  wind loads and, 6.35–37
Seismic force-resisting systems (SFRS), 6.25, 7.1
Seismic lateral forces, 7.39–50
Seismic load combinations, 1.13–14
Seismic loads, 6.24–25, 6.29–38, 7.40–41, 10.17–24
Seismic Performance Category (SPC) E and F, 6.21
Semirigid brick pavements, 2.17–18
Serpentine (face), 2.31
Serpentine walls, 6.80–81
Service load method. *See* Allowable stress design (ASD) method
Settlement, 9.25, 9.28
  differential settlement, 9.24, 9.31–33
Severe weathering (SW), 2.13–14
Sewer and manhole brick, 2.9
SFRS. *See* Seismic force-resisting systems
Shear
  limit states design and, 10.4–5
  nominal strength *v.*, 10.4
  pilasters and, 6.75–77
  wall panels and, 6.75–77
Shear key, 9.17
  in grout pour, 9.16
Shear lag, in T-beams, 6.52
Shear panels, 7.1
Shear strength
  in design procedure (retaining walls), 8.33
  determination of, 7.90–92
  for masonry beams, 4.56–58
  of masonry columns, 5.60–64
    of CMU column, 5.61–64
  nominal shear strength, 4.58–59, 6.54, 10.4, 10.14–15
  with shear reinforcement, 4.59–61
Shear stress, 6.11
Shear walls, 5.59–60
  building configuration on lateral force distribution to
    accidental eccentricity and accidental torsion in diaphragms, 7.65–66
    accidental torsional moments amplification, 7.66
    effects of horizontal torsional eccentricity, 7.62
    general considerations on, 7.57–59
    horizontal and vertical structural irregularities ramifications, 7.67–69
    horizontal structural irregularities, 7.58–62
    inherent torsion in diaphragms: direct and torsional shears, 7.63–65
    shear walls and diaphragms under direct shear and torsional moments analysis, 7.69–81
    vertical structural irregularity, 7.62–63
  cantilevered shear walls, 7.11–15
  column reinforcement *v.*, 5.59–60
  configurations of, 7.68
  definition of, 5.58
  design considerations for
    design for flexure, 7.89
    design shear strength, 7.90–92

Shear walls, design considerations for (*Cont.*):
minimum thickness requirements for, 7.89–90
other reinforcement requirements for, 7.92–95
reinforced masonry shear walls types, 7.81–86
seismic design categories, 7.87–89
failure modes of, 7.115–120
flexural failure, 7.110–111
in multistory buildings, 7.112–114
openings and, 7.112–113
rocking, 7.110, 7.112
shear failure, 7.110, 7.112
TCCMAR testing on, 7.112, 7.114
torsional failure, 7.113
fundamental concepts on, 7.2–6
horizontal diaphragms and
chords and collector elements design forces, 7.52–54
diaphragm design forces, 7.51–52
diaphragm force for one-story building, 7.54–57
flexible *v.* rigid diaphragms, 7.50–51
interaction diagram for, 5.59–60
introduction to, 7.1–2
multistory, 7.2
design of, 7.108–110
perforated walls, 7.6, 7.17–39, 7.112
problems on, 7.115–120
rigidity of shear wall with openings, 7.17–20
example 7.4, 7.24–27
example 7.5, 7.27–29
example 7.6, 7.29–34
example 7.7, 7.34–36
example 7.8, 7.36–39
method A, 7.21
method B, 7.21–22
method C, 7.22
springs in parallel concept, 7.22–23
springs in series concept, 7.23–24
rigidity *v.* relative rigidity of
cantilevered shear wall and, 7.11–13
fixed-ended shear wall and, 7.13–17
flexural *v.* shear deflections and, 7.17
general concept of, 7.10–11
piers and, 7.35–36
seismic lateral forces on
base shear distribution over building height—F_x-story forces, 7.47–50
equivalent lateral force procedure, 7.41–47
Shear walls, seismic lateral forces on (*Cont.*):
horizontal distribution of seismic forces, 7.50
references for, 7.39
response modification factor R for shear wall buildings, 7.45–47
seismic analysis methods/design forces, 7.39–40
seismic loads effects/load combinations, 7.40–41
seismic response coefficient, 7.41–45
shear wall building systems, 7.46
types of, 7.6–10, 7.82–83
Shear walls analysis, 5.59–60, 6.21
diaphragms under direct shear and torsional moments and, 7.69
example 7.12, 7.70–73
example 7.13, 7.74–78
example 7.14, 7.78–81
for flexure and axial loads
flexural capacity of axially loaded shear walls, 7.105–108
shear walls under flexure, 7.95–104
Shiner, 2.6
Shortening of structural frames, 9.30
Shrinkage, 9.24, 9.25
carbonation shrinkage, 3.25–26, 9.27
deformation from, 3.25–26
drying shrinkage, 3.25
expansion *v.*, 9.28
Simplified lateral force procedure, 7.40
Single-lift pours, 9.22–23
Single opening strip, 7.7
Single-projection pilaster, 6.70
standard concrete pilaster units for, 6.71
Single punched opening, 7.7
Single walls, 2.43–44
Single-wythe walls, 2.43–44, 6.3, 6.5
bond patterns in, 6.4
high-lift grouting for, 9.23
joint reinforcement for, 9.20
low-lift grouting for, 9.17
reinforcement in, 9.2
single-lift pours for, 9.23
Skigeyuki Kanamori, 1.4
Slenderness, 4.2
of reinforced masonry columns, 5.7–9
Slump, in grout space preparation, 9.15
Slump blocks, 2.35
description of, 2.33–34
Slump test, for grout, 3.7, 3.8
Soaps, 2.5

Soldier, 2.6
Solid bricks, 2.5, 2.9, 2.15, 6.5
Solid cantilever wall, 7.6
Solid CMUs, 2.25
Solid masonry, 1.2
Solid masonry unit, 2.4, 2.25
Solid unit masonry construction, 8.42
Solid wall, 6.5, 7.6–7
Sound control, 2.35, 6.81–85
  CMUs for, 2.29–30
Spalling, 9.33
Spandrel, 7.6
  failure of, 7.112–113
Spandrel beams, 7.18
SPC. *See* Seismic Performance Category (SPC) E and F
Special reinforced masonry shear wall, 7.46–47, 7.83, 7.85–86
Special reinforced prestressed masonry shear wall, 7.83
Specifications, 1.6, 1.8–9, 3.4–5, 3.9, 5.4
Specific creep, 9.27
Specific modulus, 3.19
Specific strength, 3.19
Specified compressive strength, 2.16–17, 2.25
Specified dimension, 2.13
Splices
  lapped splices, 8.34–35, 9.6
  mechanical splices, 9.6
  noncontact splice, 9.6
  for reinforcement, 9.6
  welded splices, 9.6
Split blocks, 2.35
  description of, 2.34
Squat walls, 7.3–4
Squeezed joint, 2.42
SRW. *See* Segmental retaining walls
Stability. *See also* Retaining wall external stability
  in design procedure (retaining walls), 8.31
  of fence walls, 6.78–79
Stack bond, 2.37–38, 2.40, 2.48, 6.18–19, 6.21, 6.23, 6.44
  head joints and, 6.46
  load dispersion and, 6.46
  strength limitations for, 2.41
  structural requirements and, 2.41
  T-beams and, 6.53
The Standard Building Code, 1.8
Standards, 1.6–9, 9.2
Steel reinforcement, 3.12, 3.15–21
  modulus of elasticity, 3.22
  nominal yield strength of, 5.11
  placement of
    flexure in, 9.2
    installation of, 9.4–6
    limits for, 9.2, 9.4
    requirements of, 9.6–7
    summary about, 9.2
Steel wire, 3.18
Story drift, 7.62
Strain gradient, 7.97
Strength, 3.7, 3.12–15, 4.2–3. *See also* Shear strength
  buckling strength, 5.4–6
  glass units and, 2.47–48
  of grout, 3.9–11
  horizontal stack and, 6.22–23
  specified compressive strength, 2.16–17, 2.25
  stack bond limitations, 2.41
Strength design, 6.38–40. *See also* Reinforced masonry beam design
  axial tensile strength design, 10.7–11
  for columns, 5.1, 5.4
  concept of, 1.10
  cracking moment, 6.55
  design methods and, 1.10–1.11
  in design procedure (retaining walls), 8.29–30
  flexural analysis in, 4.1, 4.7–12
  horizontal midheight deflection in, 6.54
  load combinations for, 1.11–12
  MSJC-08 on, 6.54–69
  nominal axial strength design, 10.11–14
  nominal moment strength in, 6.54–55
  nominal shear strength in, 6.54
  out-of-plane loads and, 6.54–69
  strength design philosophy, 4.2–7
  vertical reinforcement in, 6.55
Strength level (LFRD-level) force, 6.34
Strength reduction factors
  anchorage and, 10.5
  load factors and, 4.3–5
Stresses, 3.18, 6.10, 6.12. *See also specific types of stress*
Stretcher, 2.3, 2.4, 2.5–6, 2.12
Structural bond, 2.35–36
Structural Clay Products Research Foundation (SCR), 1.4
Structural Engineers Association of California (SEAOC), 4.2
Structural overload, 9.24

Structural walls
  anchorage requirements and, 10.16–27
  nonstructural walls, 6.31–38
  out-of-plane loads and, 6.29–31
Stussi, F., 4.1
Subterranean (basement) walls, 8.5–6. *See also* Retaining and subterranean walls
  cross section of, 8.38
  design considerations for, 8.36–41
  design loads for, 8.36
  design of, 8.38–41
  empirical design of, 8.41
  example of, 8.38–41
  forces on, 8.39
  lateral loading conditions for, 8.36–38
  lateral soil pressure on, 8.38
  plain *v.* reinforced, 8.35–36
  purpose/advantages of, 8.35
  support conditions for, 8.36–37
  as vertical cantilever type walls, 8.39
  water penetration prevention in, 8.45–48
Surcharge effects, 8.21–22
Surcharge loads, 8.44
Surface-coated wall, 6.4
Surface texture, 2.33–35
Suspended diaphragm pavements, 2.17–18
SW. *See* Severe weathering
Swelling pressures, from backfill, 8.43

Taper block, 2.31
T-beams, 6.54
  analysis of, effective depth of, 6.51–54
  effective width in, 6.52
  example of, 6.67–69
  face-shell thickness and, 6.53
  flange *v.* web in, 6.51, 6.53
  partially grouted wall as, 6.51
  pilasters and, 6.77
  running bond *v.* other bonds, 6.52
  shear lag in, 6.52
  stack bond and, 6.53
  width in, 6.52–53
TCCMAR, 7.112, 7.114
Temperature, 9.1
  grout and, 9.23
Tensile reinforcement, 7.96–97, 8.40
  in beams, 4.34–41
Tension
  anchor bolts in, 10.5–16
  axial tension, 10.15–16
  nominal axial strength and, 10.5–16
Tension (*Cont.*):
  nominal strength *v.*, 10.4
  shear *v.*, 10.4
  tension face zero strain case, 5.46–47
  tension reinforcement
    in beams, 4.12–14
    strain compatibility and, 4.12–21
  tension reinforcement zero strain case, 5.47
Tension stiffening, 4.127
Terra-cotta, 2.21
The Masonry Society (TMS), 1.8–9, 4.2
Thermal changes, 6.12, 9.24
  coefficient of thermal expansion, 3.23, 9.25–28
Thermal effects, 3.23–25
Thermal movement, 9.25–27
Thin brick veneer, 2.18–19
  ASTM classification of, 2.9
Through-the-wall (TTW) units, 2.12
Ties, 3.21. *See also* Lateral ties; Metal ties
  in bed joints, 5.3
  in grout, 5.3
  in grouted pocket, 5.3
  in mortar joints, 5.3
  wall ties, 9.18–19
  wire ties, 9.18
Time, fired brick moisture expansion *v.*, 9.29–30
Timing
  of backfilling operation, 8.43
  of grout lift, 9.16, 9.21, 9.23
  in grout pour, 9.16, 9.21.9.23
  in grout space preparation, 9.16
  of reinforcement, 9.4–5
TL. *See* Transmission loss
TMS. *See* The Masonry Society
Tooling, 2.42–43
Torsional eccentricity, 7.62
Torsional failure, 7.113
Torsional irregularity, 7.61–62, 7.68, 7.78–81
Torsional moments, 7.69–81
  accidental horizontal torsional moment, 7.65–67
Torsional shears, 7.65, 7.69
Transformed section, 4.120–122
Transmission loss (TL)
  of highway sound barriers, 6.82–83
  materials and, 6.83
Truss tie, 2.37, 9.21
Truss type, 6.13
Truss type joint reinforcement, 3.19–20, 9.20
TTW. *See* Through-the-wall (TTW) units
Tuck pointer, 2.42
Two-cell hollow concrete masonry units, 2.26, 2.28

Type FBA, 2.15
Type FBS, 2.15
Type FBX, 2.15
Type HBA, 2.16
Type HBB, 2.16
Type HBS, 2.16
Type HBX, 2.16
Type K mortar, 3.2
Type M mortar, 3.2–3, 3.5–6, 6.21
Type N mortar, 2.48, 3.2–3, 3.5, 5.11, 6.2–3
Type O mortar, 3.2, 3.5
Types, thin brick veneer, 2.18–19
Type S mortar, 2.48, 6.2–3, 6.21
  designation of, 3.2
  requirement of, 3.3
  selection of, 3.5–6

Underreinforced beams, 4.31
Uniform Building Code, 1.7–8
Units for key/wood jamb block, 5.3
Unit shear, 7.5
Units sizes, for vertical reinforcement, 6.41, 6.48, 9.2
Unit strength method, 3.12–13
Unreinforced masonry, 1.1, 1.4
  earthquakes and, 1.3
  grouting in, 9.8
  reinforced masonry *v.*, 1.5–6
Unreinforced masonry walls, 6.5, 7.46, 7.82–84
Used brick, 2.21–23

Veneer walls, 2.9, 2.18–19, 6.13–14
Vertical cantilever, subterranean walls as, 8.39
Vertical edge restraint conditions, horizontal *v.*, 6.75
Vertical load, pilasters and, 6.72
Vertically spanning walls, 6.15–17, 6.22, 6.24
Vertical reinforcement, 2.44, 6.5–9, 6.21
  bar positioners for, 9.5–6
  bar sizes/units sizes for, 6.41, 6.48, 9.2
  block shapes for, 6.47
  for columns, 5.2
  flexure and, 6.47–49
  hollow masonry units and, 9.3–9.4
  open-ended units in, 6.47, 6.49
  in retaining walls, 6.48
  in strength design, 6.55
Vertical stack, 6.22–23
Vertical structural irregularity, 7.62–63
Visual inspection, 2.15, 2.17
V joint, 2.42
Volume change joints, 3.24, 3.26

Wall beams, 6.15
  deep wall beams, 4.101–108
Wall columns, 5.1
Wall construction. *See also* Walls under gravity and transverse loads
  brick positions in, 2.6
  cavity wall construction, 9.44–45
  modular construction, 2.46
  single and cavity walls, 2.43–45
Wall forces, wind/seismic forces *v.*, 6.35–37
Wall intersections, 7.8–10
Wall panels, 6.15
  design of, 6.74–75
  lateral loads and, 6.75–76
  moments and, 6.75–76
  pilasters and, 6.73–77
  shear and, 6.75–77
  vertical *v.* horizontal edge restraint conditions and, 6.75
  wind load and, 6.74–75
Wall reinforcement
  under gravity and transverse loads, 6.45–47
  grout in, 6.47
  running bond and, 6.46
  smaller bar spacings in, 6.45
Walls
  clear height of, 5.5
  modular construction of, 2.46
Walls under gravity and transverse loads
  analysis of
    dead load of masonry walls, 6.24–25
    general considerations, 6.23–25
  bond patterns, 6.16–23
  introduction to, 6.1
  masonry wall types, 6.1–16
  nonload-bearing walls, 6.77–85
  out-of-plane loads, 6.25–38, 6.54–68
    analysis of, for, 6.38–44
    axial loads and, 6.69
  problems on, 6.84–85
  wall design for gravity and transverse loads
    effective depth: T-beam analysis in, 6.51–54
    gravity loads distribution in, 6.44–45
    lateral support in, 6.44
    MSJC-08 provisions, out-of-plane loads design, 6.54–69
    reinforcement effective depth, 6.49–51
    vertical reinforcement to resist flexure, 6.47–49
    wall reinforcement in, 6.45–47

Wall ties
grout and, 9.18
joint reinforcement *v.* metal ties as, 9.18–19
Wall tile, 2.9
Wall with openings, 7.6. *See also* Shear walls
Water. *See also* Flashing; Moisture
mortar and, 3.3, 3.11–12
Water penetration prevention
alternate weephole detail, 8.46
impermeable backfill, 8.46
longitudinal drain pipe, 8.46
permeable backfill, 8.46
problems on, 8.46–48
in subterranean walls, 8.45–48
Weather, 3.7, 9.1. *See also* Thermal changes
Weathering, brick and, 2.13–14
Weathering index map, 2.14
Weather joint, 2.42
Web of hollow masonry units, 2.7, 2.12, 2.26, 2.29, 2.32, 2.33, 6.52
cells *v.*, 2.33
flange *v.*, in T-beams, 6.51, 6.53
in partially grouted construction, 6.49
Weep holes, 9.45
alternate weephole detail, 8.46
as drainage provisions, 8.45–46
flashing and, 9.44
Weight, of glass unit masonry panels, 2.50
Weight class of concrete masonry unit, 2.24
Welded splices, 9.6
Welded wire fabric (WWF), 3.19–21
Weld plate connections, 6.43
Whitney, Charles, 4.7
Wind forces
MWFRS, 6.25, 7.1
wall forces *v.*, 6.35–37
Wind load resistance, 2.48–49
Wind loads, 6.24–25
building height/exposure and, 6.26–27
components and cladding in, 6.26
end zones and, 6.28–29
importance factor in, 6.27
net design wind pressure and, 6.28
occupancy categories and, 6.27–28, 6.35–37
out-of-plane loads and, 6.26–29
seismic force and, 6.35–37
wall panels and, 6.74–75
Wire ties, grout and, 9.18
W-number, 3.21
Wood jamb block, 5.3
Workability, 3.4
Working stress design. *See* Allowable stress design (ASD) method
WSD. *See* Allowable stress design (ASD) method
W-shape, 4.112
WWF. *See* Welded wire fabric
Wythes, 2.5–6, 2.36, 6.10–11, 6.51. *See also* Multiple-wythe walls; Single-wythe walls
bond patterns and, 6.21
masonry wall types *v.*, 6.3–5

Yield stress, 3.16
Ytong's AAC blocks, 2.55

Zinc-coated steel bars, 3.18–19
Z tie, 2.37